THE PENTAGON

Volume XVI FALL, 1956 Number 1

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Kappa Mu Epsilon, national honorary mathematics society, was founded in 1931. The object of the fraternity is fourfold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievements in the study of mathematics at the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

A Symmetrical Notation*

DeWayne Nymann

Student, Iowa State Teachers College

Throughout history, man has constantly been searching for new ideas and machines which would help in his way of life. This search has also been taking place in mathematics. There have been many changes in the most fundamental part of mathematics—our number notation. The Egyptians had a comparatively poor system, their hieroglyphics. Later the Babylonians introduced a sexagesimal system that had positional value. Finally our present day notation was evolved, which is a very good system and will probably be with us for many years. However, there have been other systems devised in the past few years, one of which is the symmetrical notation. It is my purpose to introduce this notation to you in this paper.

First of all let us express any number N in our present notation:

$$N = \sum_{n=-\infty}^{T} a_n r^n, \qquad 0 \leqq a_n \leqq r-1$$

This means that the number N can be written as the summation of a_n times the base of the system r raised to the power n, where n can range from negative infinity to a definite number T. The symbol a_n can represent any number greater than or equal to zero and less than or equal to the base r, minus one. To illustrate this I will give an example in our present system where r equals ten:

$$478.6359 = 4 \times 10^2 + 7 \times 10^1 + 8 \times 10^0 + 6 \times 10^{-1} + 3 \times 10^{-2} + 5 \times 10^{-3} + 9 \times 10^{-4}$$

In this example r was equal to 10. However, r could be any integer you might wish it to be.

In the symmetrical notation this same number N could be written:

$$N = \sum_{n=-\infty}^{T} a_n r^n, \qquad -(r-1)/2 \leqq a_n \leqq (r-1)/2$$

* A paper presented at the regional meeting of K.M.E. at William Jewell College, Spring, 1955.

The only modification is that a_n is greater than or equal to the negative of the quantity r minus one, divided by two; and it is less than or equal to the quantity r minus one, divided by two. This will make the symbols of this system symmetrical around zero. For convenience let us choose r to be an odd integer such as seven. The symbols for the base seven, in symmetrical notation, would be $-3, -2, -1, 0, 1, 2, 3$. From now on I will write the negative numbers with a bar above them such as $\bar{2}$. An example of this notation in base 7 would be as follows:

$$2\bar{3}1.2\bar{1}3 = 2 \times 7^2 + \bar{3} \times 7^1 + 1 \times 7^0 + 2 \times 7^{-1} + \bar{1} \times 7^{-2} + 3 \times 7^{-3}$$

If an even base had been selected we could not have had a symmetrical notation because we would need to add an extra symbol at one of the ends. This would be necessary since a number system to the base r must have r symbols in it. For the number base six, a 3 or a $\bar{3}$ would have to be added in order to have six symbols. Therefore, for the purpose of this paper, and in the interest of simplicity I will select seven—an odd number—for the base.

Using seven as the base we write the numbers of this system in the positive direction as follows:

$$1, 2, 3, 1\bar{3}, 1\bar{2}, 1\bar{1}, 10, 11, 12, 13, 2\bar{3}, 2\bar{2}, 2\bar{1}, 20, 21, 22, 23, 3\bar{3}, 3\bar{2}, 3\bar{1}, 30, 31, 32, 33, 1\overline{33}, 1\overline{32}, 1\overline{31}, 1\bar{3}0; 1\bar{3}1, 1\bar{3}2, 1\bar{3}3, 1\overline{23}, \cdots$$

The following go in the negative direction and have the characteristic that the first digit is always barred:

$$\bar{1}, \bar{2}, \bar{3}, \bar{1}3, \bar{1}2, \bar{1}1, \bar{1}0, \overline{11}, \overline{12}, \overline{13}, \bar{2}3, \bar{2}2, \bar{2}1, \bar{2}0, \overline{21}, \overline{22}, \overline{23}, \bar{3}3, \bar{3}2, \bar{3}1, \bar{3}0, \overline{31}, \overline{32}, \overline{33}, \bar{1}33, \bar{1}32, \cdots.$$

This is a beginning of the whole numbers. You may wish to write a few more to become used to this system. Of course it should be remembered that between any two numbers there is an infinity of numbers—such as between one and two we would find some intermediate values ($1.12\bar{3}$, $2.\bar{3}$, $2.\bar{1}21$, etc.)

Now we shall explore the use of this system in the arithmetical processes. We shall first construct an addition table using this system.

	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3
$\bar{3}$	$\bar{1}1$	$\bar{1}2$	$\bar{1}3$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0
$\bar{2}$	$\bar{1}2$	$\bar{1}3$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1
$\bar{1}$	$\bar{1}3$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2
0	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3
1	$\bar{2}$	$\bar{1}$	0	1	2	3	$1\bar{3}$
2	$\bar{1}$	0	1	2	3	$1\bar{3}$	$1\bar{2}$
3	0	1	2	3	$1\bar{3}$	$1\bar{2}$	$1\bar{1}$

You will notice that this table is symmetrical with respect to the diagonal from upper left to lower right. This would facilitate memorization of the table. We can demonstrate the use of this table with the following problem:

$$\begin{array}{r} 2\bar{3}\bar{1}2.1 \\ \bar{2}131.23\bar{2} \\ 3\bar{2}3.\bar{1}\bar{1}1 \\ \hline 11\bar{1}.22\bar{1} \end{array}$$

Since the first number of the above problem is not as precise as the other numbers, we must round off the total to $11\bar{1}.2$. The rounding-off process is easy in the symmetrical notation because all that is necessary is to drop the places which you do not need. The reason this can be done is that the value of the first place which is to be dropped is always less than half of the number base.

If you wish to subtract one number from another, you merely remove all the bars from the barred symbols and place bars over all the unbarred symbols in the subtrahend. This is making a negative number out of a positive number or making a positive out of a negative as the case may be. Once this has been done, you proceed by adding the minuend and the altered subtrahend. That is all there is to subtraction.

The multiplication table is as follows:

	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3
$\bar{3}$	12	$1\bar{1}$	3	0	$\bar{3}$	$\bar{1}1$	$\bar{1}\bar{2}$
$\bar{2}$	$1\bar{1}$	$1\bar{3}$	2	0	$\bar{2}$	$\bar{1}3$	$\bar{1}1$
$\bar{1}$	3	2	1	0	$\bar{1}$	$\bar{2}$	$\bar{3}$
0	0	0	0	0	0	0	0
1	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3
2	$\bar{1}1$	$\bar{1}3$	$\bar{2}$	0	2	$1\bar{3}$	$1\bar{1}$
3	$\bar{1}\bar{2}$	$\bar{1}1$	$\bar{3}$	0	3	$1\bar{1}$	12

This table is symmetrical with respect to both diagonals. As an example of the use of the multiplication table, let us multiply $2\bar{1}$ by $\bar{1}3$.

$$
\begin{array}{r}
2\bar{1} \\
\bar{1}3 \\
\hline
1\bar{1}\bar{3} \\
\bar{2}1 \\
\hline
\bar{1}0\bar{3}
\end{array}
$$

As a check we can change these numbers to base ten. The $2\bar{1}$ is thirteen, and the $\bar{1}3$ is minus four. Multiplying these we get minus fifty-two. Thus it can be seen readily that $\bar{1}0\bar{3}$ is the same as minus fifty-two.

In our base ten notation, if we wish to divide one number A, into another number B we want to find a number C such that $A \times C$ will be as nearly equal to B as possible but never larger than B. In the symmetric notation we want $A \times C$ to be as close to being equal to B as possible. This means that the remainder may be negative at times. Under base ten notation, 8 divided by 3 is 2 with a remainder of 2. Using the theory of symmetric division, 8 divided by 3 would be 3 with a remainder of -1. As an example of division in the symmetric notation using base seven, let us divide 120 by $1\bar{3}$.

$$\begin{array}{r} 22 \\ 1\bar{3}\,/\overline{120} \\ 11 \\ \hline 10 \\ 11 \\ \hline \bar{1} \end{array}$$

In the second step of the above example, when we are dividing 10 by $1\bar{3}$, we decide that the quotient is 2 rather than 1 because the remainder for 2 is $\bar{1}$, and for 1 the remainder is 3.

I have constructed the following table of logarithms using the symmetrical notation and the base seven.

The logarithms in the table are to the base seven rather than to the base ten as in the common logarithms. They are also in the number base seven. When using these logarithms for computations we get the mantissa from the table and attach a characteristic in the same way as we do with common logarithms. The following are some examples of the use of this table:

$$132 \times 23 = 1\bar{3}\bar{3}0\bar{1} \qquad \begin{array}{l} \text{Log } 132 = 2.13\overline{211} \\ \text{Log } 23 = 1.3123\bar{1} \\ \hline \end{array}$$

$$\text{Antilog } 1\bar{3}.\overline{23}02\bar{2} = 1\bar{3}300, \text{ approx.}$$

	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3
$1\bar{3}$	$.\bar{2}\bar{3}03\bar{2}$	$.\bar{2}\bar{2}020$	$.\bar{2}\bar{1}00\bar{2}$	$.\bar{2}0\bar{1}23$	$.\bar{2}1\bar{1}\bar{3}\bar{1}$	$.\bar{2}2\bar{2}\bar{3}\bar{3}$	$.\bar{2}2321$
$1\bar{2}$	$.\bar{2}32\bar{1}2$	$.\bar{1}\bar{3}022$	$.\bar{1}2\bar{1}\bar{3}0$	$.\bar{1}\bar{1}\bar{3}\bar{2}\bar{1}$	$.\bar{1}\bar{1}22\bar{3}$	$.\bar{1}0\bar{1}33$	$.\bar{1}\bar{1}\bar{3}12$
$1\bar{1}$	$.\bar{1}12\bar{2}3$	$.\bar{1}2\bar{1}2\bar{3}$	$.\bar{1}\bar{3}\bar{3}\bar{3}0$	$.\bar{1}31\bar{1}\bar{1}$	$.0\bar{3}\bar{2}0\bar{1}$	$.0\bar{3}201$	$.0\bar{2}\bar{1}00$
10	$.0\bar{2}3\bar{1}0$	$.0\bar{1}02\bar{3}$	$.0\bar{1}33\bar{3}$	$.00000$	$.01\bar{3}\bar{3}\bar{1}$	$.01003$	$.01332$
11	$.020\bar{1}\bar{1}$	$.0231\bar{1}$	$.03\bar{1}3\bar{3}$	$.033\bar{3}\bar{2}$	$.1\bar{3}\bar{1}2\bar{3}$	$.1\bar{3}2\bar{2}0$	$.1\bar{2}\bar{2}\bar{2}1$
12	$.1\bar{2}1\bar{2}\bar{1}$	$.1\bar{1}\bar{3}\bar{3}2$	$.1\bar{1}\bar{1}32$	$.1\bar{1}221$	$.10\bar{2}1\bar{3}$	$.101\bar{1}\bar{2}$	$.10333$
13	$.11\bar{1}\bar{1}0$	$.112\bar{2}2$	$.12\bar{3}22$	$.120\bar{1}\bar{1}$	$.1223\bar{3}$	$.13\bar{2}\bar{1}\bar{1}$	$.1302\bar{1}$
$2\bar{3}$	$.133\bar{2}\bar{2}$	$.2\bar{3}\bar{2}02$	$.2\bar{3}01\bar{3}$	$.2\bar{3}3\bar{2}\bar{2}$	$.2\bar{2}\bar{2}0\bar{3}$	$.2\bar{2}012$	$.2\bar{2}23\bar{1}$
$2\bar{2}$	$.2\bar{1}\bar{2}\bar{3}1$	$.2\bar{1}0\bar{2}2$	$.2\bar{1}2\bar{1}2$	$.20\bar{3}00$	$.20\bar{1}1\bar{2}$	$.20111$	$.20312$
$2\bar{1}$	$.21\bar{2}13$	$.21013$	$.21211$	$.22\bar{3}1\bar{1}$	$.22\bar{1}02$	$.2210\bar{2}$	$.223\bar{1}\bar{1}$
20	$.23\bar{2}\bar{2}\bar{1}$	$.230\bar{3}\bar{1}$	$.2313\bar{3}$	$.23312$	$.\bar{3}\bar{3}\bar{2}0\bar{3}$	$.\bar{3}\bar{3}0\bar{2}1$	$.\bar{3}\bar{3}133$
21	$.\bar{3}\bar{3}32\bar{3}$	$.3\bar{2}\bar{2}0\bar{2}$	$.3\bar{2}0\bar{2}\bar{3}$	$.3\bar{2}123$	$.3\bar{2}300$	$.3\bar{1}\bar{2}\bar{2}\bar{2}$	$.3\bar{1}\bar{1}21$
22	$.3\bar{1}1\bar{1}3$	$.3\bar{1}3\bar{3}\bar{2}$	$.30\bar{3}10$	$.30\bar{1}\bar{2}0$	$.30020$	$.302\bar{1}\bar{1}$	$.3033\bar{3}$
23	$.31\bar{2}\bar{1}2$	$.31\bar{1}3\bar{1}$	$.311\bar{1}3$	$.3123\bar{1}$	$.32\bar{3}\bar{1}1$	$.32\bar{2}23$	$.320\bar{1}\bar{3}$
$3\bar{3}$	$.3212\bar{3}$	$.323\bar{2}\bar{3}$	$.33\bar{3}1\bar{3}$	$.33\bar{2}32$	$.330\bar{1}1$	$.3312\bar{2}$	$.333\bar{3}3$
$3\bar{2}$	$1.\bar{3}\bar{3}\bar{3}0\bar{1}$	$1.\bar{3}\bar{3}\bar{2}23$	$1.\bar{3}\bar{3}0\bar{2}\bar{2}$	$1.\bar{3}\bar{3}100$	$1.\bar{3}\bar{3}222$	$1.\bar{3}\bar{2}333$	$1.\bar{3}\bar{2}\bar{2}0\bar{3}$
$3\bar{1}$	$1.\bar{3}\bar{2}\bar{1}2\bar{3}$	$1.\bar{3}\bar{2}1\bar{3}\bar{3}$	$1.\bar{3}\bar{2}2\bar{1}\bar{3}$	$1.\bar{3}\bar{2}302$	$1.\bar{3}\bar{1}\bar{3}21$	$1.\bar{3}\bar{1}\bar{1}\bar{3}\bar{1}$	$1.\bar{3}\bar{1}0\bar{1}\bar{3}$
30	$1.\bar{3}\bar{1}\bar{1}01$	$1.\bar{3}\bar{1}22\bar{2}$	$1.\bar{3}\bar{1}331$	$1.\bar{3}0\bar{2}\bar{2}\bar{3}$	$1.\bar{3}0\bar{1}\bar{1}\bar{1}$	$1.\bar{3}0001$	$1.\bar{3}0113$
31	$1.\bar{3}023\bar{2}$	$1.\bar{3}1\bar{3}\bar{3}\bar{1}$	$1.\bar{3}1\bar{2}\bar{2}\bar{1}$	$1.\bar{3}1\bar{1}\bar{1}0$	$1.\bar{3}1000$	$1.\bar{3}111\bar{1}$	$1.\bar{3}122\bar{2}$
32	$1.\bar{3}133\bar{3}$	$1.\bar{3}2\bar{3}33$	$1.\bar{3}2\bar{1}\bar{3}1$	$1.\bar{3}20\bar{2}\bar{1}$	$1.\bar{3}21\bar{2}3$	$1.\bar{3}22\bar{1}0$	$1.\bar{3}230\bar{3}$
33	$1.\bar{3}3\bar{3}01$	$1.\bar{3}321\bar{3}$	$1.\bar{3}3\bar{1}10$	$1.\bar{3}3012$	$1.\bar{3}312\bar{3}$	$1.\bar{3}322\bar{1}$	$1.\bar{3}3321$

From the table we see that the antilog of $1\bar{3}.\bar{2}\bar{3}02\bar{2}$ is very close to $1\bar{3}\bar{3}$ so our answer without interpolation is $1\bar{3}\bar{3}00$.

$$(1\bar{1})^3 = 1\bar{3}3\bar{1} \qquad \begin{array}{rcl} \text{Log } 1\bar{1} & = & 1.\bar{1}31\bar{1}\bar{1} \\ 3 \text{ Log } 1\bar{1} & = & 3.\bar{2}23\bar{3}\bar{3} \\ \hline \text{Antilog } 3.\bar{2}23\bar{3}\bar{3} & = & 1\bar{3}30. \end{array}$$

The antilog of $3.\bar{2}23\bar{3}\bar{3}$ could be interpolated in order to obtain a more precise value.

I would like to point out some of the advantages of the symmetrical notation. First, there are fewer tables to learn and they are smaller than our present tables. They also are symmetric which makes them easy to learn.

In accounting, separate columns would not have to be kept for income and expenditures. The expenditures could be entered as negative numbers and then the whole column could be added at once and the result would be the balance or the "cash on hand."

These are some of the advantages of a system such as this. However, there are many barriers in switching to this system. The main one is that all of us would need to learn arithmetic over again. In my opinion this system will never be widely accepted because we have a very good system already, and in general, people would not want to change systems. This has been the case throughout history. The leaders of the Middle Ages outlawed the Hindu-Arabic system because they thought the Roman numerals were good enough and because they thought the Hindu-Arabic system was the work of the heathens.

In closing I want to say that I am not presenting this paper with the idea that the symmetrical notation should replace our present system, but I am doing it for its practical and mathematical interest.

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An Introduction to Hyperbolic Geometry

EDWARD Z. ANDALAFTE
Student, Southwest Missouri State College

The purpose of this paper is to investigate briefly a geometry which results from a change in one of the fundamental postulates of Euclidean geometry. More specifically we shall deny Euclid's fifth postulate, regarding parallels, and replace it with a new assumpiton leading to a study of what is known as *hyperbolic geometry.*

Euclid's fifth postulate states:

If a straight line falling on two straight lines make the interior angles on the same side less than two right angles the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles.[1]

A more familiar statement, which can be shown to be equivalent to the fifth postulate, is Playfair's Axiom, which may be stated:

Through a given point not on a given line there can be drawn only one line parallel to the given line.

At this point it is appropriate to state that much of Euclidean geometry will remain valid under our new parallel postulate. In fact all of Euclid's first twenty-six propositions will be true since parallels are neither mentioned in them nor in their proofs. Two other statements from Euclidean geometry which will be true in our new geometry are here stated.

Pasch's Axiom: Let A, B, C be three points not lying in the same straight line and let α be a straight line lying in the plane ABC and not passing through any of points A, B, C. Then, if line α passes through a point of the segment AB, it will also pass through either a point of the segment BC or a point of the segment AC.[2]

Postulate of Dedekind: If all points of a straight line fall into two classes such that every point of the first class lies to the left of every point of the second class, then there exists one and only one point which produces this division of all points into two classes, this severing of the straight line into two portions.[3]

We shall begin our study of hyperbolic geometry with an investigation of the fundamental postulate.

This postulate may be stated:

Through a given point not on a given line more than one line can be drawn not intersecting the given line.

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[1] Harold E. Wolfe, Non-Euclidean Geometry, (New York: Dryden Press, 1945) p. 4.
[2] Ibid., p. 9.
[3] Ibid., p. 10.

If more than one line through P is nonintersecting with l, an infinite number are nonintersecting. If AB and CD are nonintersecting, EF can be drawn within the vertical angles APC and DPB and will not intersect l because if it did PB must intersect l by repeated application of Pasch's Axiom. This contradicts the original conditions. Thus EF does not intersect l.

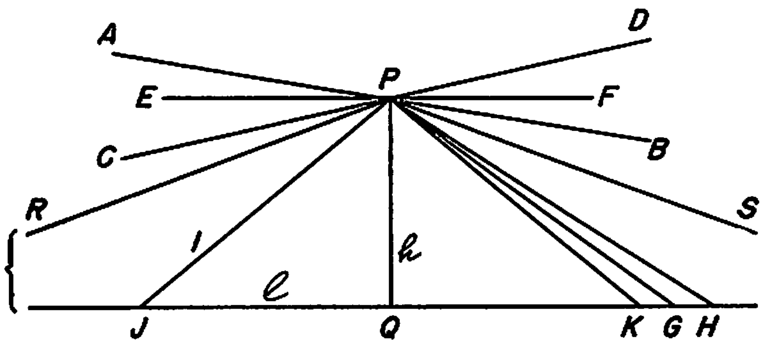

FIGURE I

By rotating PQ about P two sets of lines are obtained, those which intersect l and those which do not intersect l. By Dedekind's Postulate there exists a line which divides the lines into two sets. This cannot be the last intersecting line since, for every line PG assumed to be the last intersecting line, a point H can be located on l so that G is between H and Q and PH intersects l. The division is the first nonintersecting line to l, since there is no last intersecting line. Thus PR and PS do not cut l, but every line in the angle RPS does cut l.

To prove that angles RPQ and SPQ (Fig. 1) are equal and each is less than a right angle, assume that one of the angles, for example RPQ, is the larger. Construct angle IPQ equal to angle QPS. IP intersects l at J since it is within angle RPS. Construct QK equal to QJ and draw PK. Then triangles QPJ and QPK are congruent and angles QPK and QPJ are equal. Then, since angle QPJ equals angle QPS, angle QPK equals angle QPS. But PS does not intersect l. Therefore angle RPQ equals angle SPQ since a contradiction is reached otherwise. If RPQ and QPS were each right angles, then RPS would be a straight line and would be the only line through P which is nonintersecting with l, and a contradiction would be reached.

Thus we are able to compose *Theorem* 1: *If l is any line and* P *is any point not on l there are always two lines through* P *which do not intersect l which make equal acute angles with the perpendicular from* P *to l and which are such that every line through* P *within the angle containing the perpendicular to l intersects l and every other line does not.*

We proceed to define certain terms which will appear throughout the development of this geometry.

Definitions: The lines of the character of *PR* and *PS* on page 11 are defined as *parallels* to *l*. All other lines not within angle *RPS* are *nonintersecting* with reference to *l*. The *angle of parallelism* for distance *h* is angle *QPS* and is denoted by $\Pi(h)$. An *ideal point* is the point which is common to two parallel lines. It has all characteristics of an ordinary point, *e.g.*, two ideal points determine a line; an ideal point and an ordinary point determine a line. It is also referred to as a *point at infinity* or an *infinitely distant point*. Hereafter an ideal point will be represented by the letter Γ.

Theorem 2: *The exterior angles of AB*Γ *at A and B, made by producing AB, are greater than their respective opposite interior angles.*

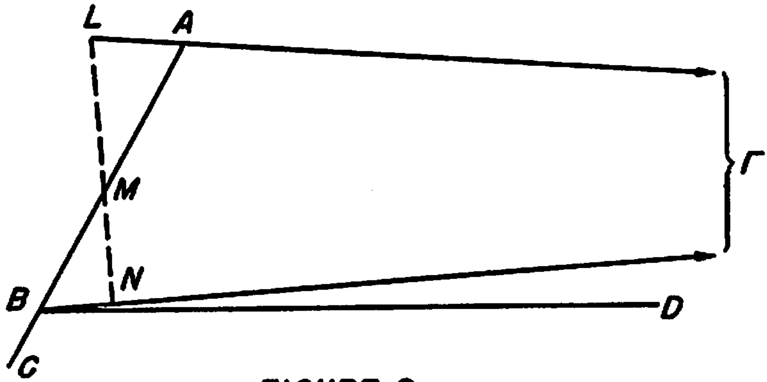

FIGURE 2

Draw angle *CBD* equal to angle *BA*Γ. *BD* cannot cut *A*Γ. Through the midpoint of *AB* draw *MN* perpendicular to *B*Γ. Draw *AL* equal to *BN*. If *BD* coincides with *B*Γ then triangles *BNM* and *ALM* are congruent. *LMN* is a straight line since the vertical angles at *M* are equal. Then $\Pi(LN)$ equals angle *ALM* and equals 90°. This is absurd. Therefore *BD* does not coincide with *B*Γ since the contrary results in an absurdity. Thus angle *CB*Γ is greater than angle *CBD* since BD is within $CB\Gamma$. Therefore angle $CB\Gamma$ is greater than angle $BA\Gamma$ since angle CBD equals angle $BA\Gamma$.

Theorem 3: *If AB and $A'B'$ are equal, and angle $BA\Gamma$ is equal to angle $B'A'\Gamma'$ then angle $AB\Gamma$ is equal to angle $A'B'\Gamma'$ and the figures are congruent.*

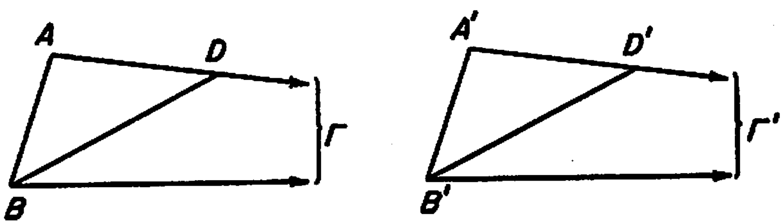

FIGURE 3

Assume angle $AB\Gamma$ is greater than angle $A'B'\Gamma'$. Draw angle ABC equal to angle $A'B'\Gamma'$. BC cuts $A\Gamma$ at D since it is within the angle of parallelism. Draw $A'D'$ equal to AD. Triangles BAD and $B'A'D'$ are congruent and angles ABD and $A'B'D'$ are equal. But angle $A'B'D'$ would thus equal angle $A'B'\Gamma'$. This is absurd. Therefore angle $AB\Gamma$ equals angle $A'B'\Gamma'$ since the contrary results in an absurdity.

As a result of this theorem it can be seen that the angles of parallelism for equal distances are equal, *i.e.*,

if $\qquad a = b$

then $\qquad \Pi(a) = \Pi(b)$.

Definition: The *Saccheri Quadrilateral* is the figure formed by drawing equal perpendiculars at the ends of a line segment and on the same side of it and connecting the extremities of these perpendiculars. The *base* is the side between the two right angles. The *summit* is the side opposite the base. The *summit angles* are the angles adjacent to the summit.

Theorem 4: *The line joining the midpoints of the base and summit of a Saccheri Quadrilateral is perpendicular to both of them; the summit angles are equal and acute.*

Let M and N be the midpoints of AB and CD respectively. Angles MAD and MBC are equal since both are right angles. Then triangles AMD and MBC are congruent and DM equals MC. Thus triangles DMN and MCN are congruent and DNM and CNM are

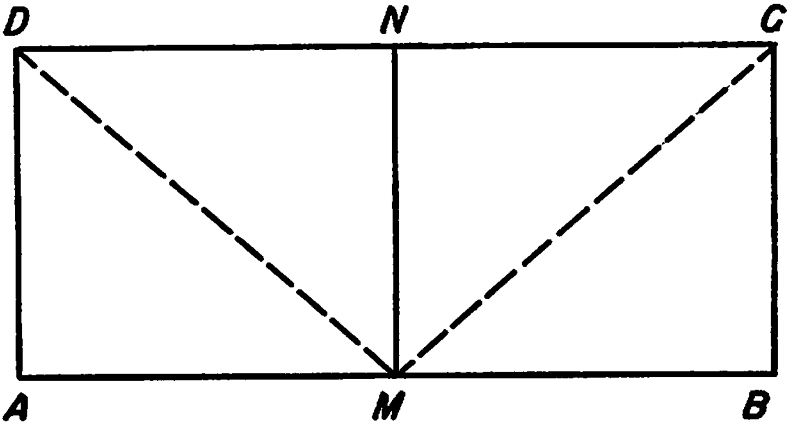

FIGURE 4

both right angles. It follows that angles ADM and BCM, NDM and NCM, DMN and CMN, and AMD and BMC are equal. Therefore angles AMN and BMN are both right angles, and angles ADN and BCN are equal. In order to continue the proof, we must investigate the following

Corollary: The base and summit of a Saccheri Quadrilateral are nonintersecting lines.

Since the base and summit are both perpendicular to the line connecting their midpoints, they are nonintersecting by the definition, p. 12.

Returning to our proof, draw $C\Gamma$ and $D\Gamma$ parallel to AB in the same sense. $C\Gamma$ and $D\Gamma$ are within angles BCE and ADC respectively since DE is nonintersecting with respect to AB. Angles $AD\Gamma$ and $BC\Gamma$ are equal since they are the angles of parallelism for equal distances. Angle $EC\Gamma$ is greater than angle $CD\Gamma$ by Theorem 2, p. 12, thus making angle BCE greater than ADC. Thus angle BCE is greater than angle BCD and angle BCD is acute.

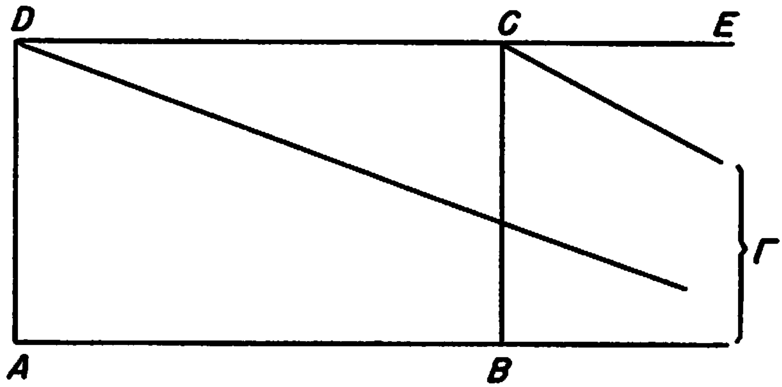

FIGURE 5

Definition: The *Lambert Quadrilateral* is a quadrilateral with three of its angles equal to 90°. It is also called a *trirectangular quadrilateral.*

Let the angles at A, B, and D be right angles. Extend BA to E making EA equal to AB, and draw EF equal to BC perpendicular to EA. Triangles FEA and CBA are congruent and FA equals AC. Angles EAD and DAB are right angles and angles EAF and BAC are

Theorem 5: *In a trirectangular quadrilateral the fourth angle is acute.*

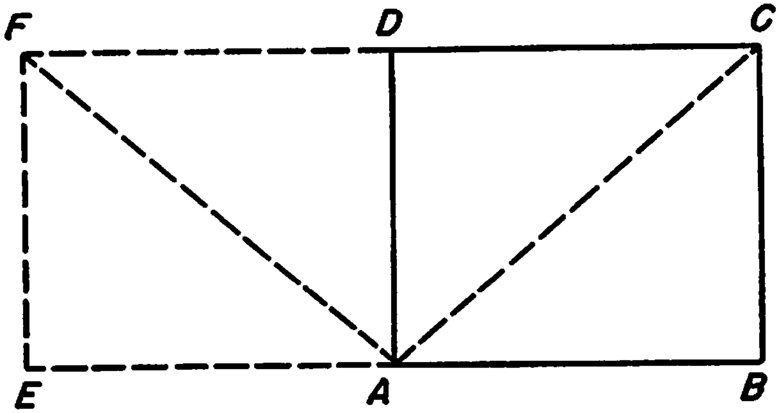

FIGURE 6

equal. Thus angles *FAD* and *CAD* are equal and triangles *FAD* and *CAD* are congruent. Therefore angles *FDA* and *CDA* are right angles and *FDC* is a straight line. It follows that *EBCF* is a Saccheri Quadrilateral and angle *DCB* is an acute angle by Theorem 4, p. 13.

Thus one of the results of the denial of Euclid's fifth postulate has been shown. Needless to say, the study of non-Euclidean geometry may be continued far beyond this point, leading to many other startling theorems. For those who may wish to continue the study of this subject, a brief bibliography has been appended containing only those books to which the author has referred in preparation of this paper. It is hoped that this paper may create in others an interest in a fascinating line of mathematical thought.

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It is well in order to aid the understanding and memory to choose intermediate truths (Which are called lemmas, since they appear to be a digression) which will shorten the major proof and yet appear memorable and worthy in themselves of being demonstrated—and there is real art in this.

—G. W. Leibnitz

The Influence of Mathematics on the Philosophy of Aristotle[1]

R. H. MOORMAN
Tennessee Polytechnic Institute

1. **Introduction.** For a number of years the writer has been trying to study the influence of mathematics on philosophy by studying the most outstanding men who have been famous both as mathematicians and as philosophers. The history of mathematics and the history of philosophy might be thought of as surfaces which intersect in the lives of many men. The writer has published papers on Descartes, Spinoza, Leibniz, Pythagoras, and Plato. The present paper deals with Aristotle, who is known primarily as a philosopher and scientist, but who was an eminent mathematician. Perhaps this is a timely study since the present interest and activity in the philosophy of mathematics can find their parallels only in the days of Plato and Aristotle.[2] Evidence that Aristotle is known by many people even today is given perhaps by the story of the Cockney English woman who taught her children about the famous man "Harry Stotle."

2. **Life** (384 B.C.—322 B.C.). Aristotle was born at Stagira, a Greek colonial town on the Aegean Sea, the son of a doctor. Aristotle studied in Plato's Academy in Athens from the time he was seventeen years of age until he was thirty-seven, leaving because of the death of Plato. In 342 B.C., he went to Macedonia to become the teacher of Alexander, later to be called "the Great."

In 334 B.C., Aristotle returned to Athens and became head of the Peripatetic School of the Lyceum. He taught there for twelve years until shortly before his death.

Aristotle was a prolific writer. His books on logic were called by his disciples the *Organon*, or "Instrument" by means of which knowledge was to be obtained. His works on science included the *Physics* which dealt with astronomy, meteorology, plants, and animals. The writings which Aristotle entitled *First Philosophy* were called *Metaphysics* by his disciples. He dedicated his work on *Ethics* to his son Nicomachus, and thus it was called the *Nicomachean*

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[1] Read before the Mathematics Section of the Tennessee Academy of Science, November 26, 1954.

[2] H. G. Apostle, **Aristotle's Philosophy of Mathematics**, (Chicago: The University of Chicago Press, 1952), p. vii.

Ethics. Other works of importance included the *Rhetoric*, the *Poetics*, and the *Politics*.

3. **Mathematics.** Histories of mathematics usually state that Aristotle was important for his attitude toward mathematics rather than for his contributions to the subject.[9] Aristotle had such wide interests in knowledge of all kinds that it is likely that he never had much time for mathematical writing. Of his extant works none treats of mathematics systematically. A book on mathematics is listed in the Catalogue of Diogenes Laertius but it was probably an incorrect listing. Numerous passages on mathematics run throughout the works that have come down to us.

Just as Plato is said to have placed over the door of his Academy, "Let no one ignorant of geometry enter here," so Aristotle assumed a mastery of elementary mathematics on the part of the students in his Lyceum. He framed his system of logic on models from mathematics.

As tutor to Alexander the Great, Aristotle collected great quantities of scientific information relating to the countries which Alexander conquered. His views on science were adopted by the Church in the Middle Ages, and it became irreligious to disagree with any of Aristotle's conclusions.

Aristotle mastered the mathematics of his day and held its discipline in high regard. He wrote about a method in mathematics called "verging" which was used in attempts to trisect an angle, one of the three famous problems of antiquity. In Aristotle we find many of the concepts of irrationals, the continuum, and the infinite. His students at the Lyceum made many contributions to mathematics. He seemed to have a genius at inspiring his students to pursue his unfinished hints.

Aristotle classified mathematics as one of the "theoretical" sciences as opposed to the productive and practical arts. For him, the theoretical sciences were mathematics, physics, and theology (first philosophy or metaphysics).

His discussion of the errors in measurement would do justice to any twentieth century teacher trying to explain the approximate numbers obtained by measurement.

4. **Philosophy.** Aristotle is generally regarded as the greatest thinker in Greek life. No other philosopher proposed and defended a philosophy as well as he did. He was a universal genius in philosophy.

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[9] Vera Sanford, *A Short History of Mathematics* (New York, Houghton Mifflin Company, 1930), p. 9.

Aristotle was the first philosophical writer to make a strict separation of the branches of philosophy. He divided philosophy into logic, metaphysics, physics, ethics, politics, and the philosophy of art.

Aristotle shares with Plato the distinction of being the most famous of ancient philosophers. He opposed Platos *ideas,* and to replace *ideas,* he introduced *forms*. He held that the form of a building in the mind of the architect was in some way the cause of the building, having a tendency to produce a concrete reality like itself. He introduced four kinds of causes: the formal, the material, the final, and the efficient. In a building the formal cause would be the plan in the architect's mind, the material cause would be the construction material, the final cause would be the purpose of the building, and the efficient cause would be the actual builder.

As Plato had done, Aristotle sought for a spiritual explanation of the universe, but he held that such an explanation could be best attained by investigation and comparison of actual phenomena. Thus he accumulated a large number of facts and observations which were to a great extent the beginnings of physical science. His experimental and inductive methods were in harmony with the methods of modern science. He was also the founder of the deductive science of logic and perhaps also its perfector. It has been said that since the time of Aristotle logic has made no further progress. Throughout the Middle Ages Aristotle was the one great authority on science and philosophy.

5. **The Influence of Mathematics on the Philosophy of Aristotle.** According to Max Lerner, Aristotle, the scientist, proceeded not as a mathematician but as a biologist; hence his characteristic method was classification rather than measurement.[4] It is probably true that Aristotle did not consider the role of mathematics to extend to philosophy as immediately as many other mathematicians who have philosophized. For him the "things" of mathematics were not real, while the things dealt with by philosophy were real. In the *Politics* he declared:

> The physicist takes account not only of the attributes of physical bodies but of their matter; though he speaks of planes and lines and points, he speaks of these only as limits or extremities of a physical and movable body. The mathematician says nothing of matter; his planes, lines, and points are the result of abstraction and are considered apart altogether from physical bodies.[5]

Nevertheless, his writings were full of mathematical illustrations and

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[4] Max Lerner, **Aristotle's Politics** translated by Benjamin Jowett (New York: The Modern Library, 1943), p. 19.

[5] Thomas Heath, **Mathematics in Aristotle** (Oxford: the Clarendon Press, 1949), p. 13.

his mathematics seemed to influence his philosophy.

Aristotle, like other mathematicians who philosophize, had the concept of a *universal mathematics*. With most mathematicians this has been the theoretical application of mathematics to every phase of life, but Aristotle seems to have been more cautious in his extension of mathematics.

6. **The Problem of Method.** Aristotle realized the importance of mathematics in having a proper method of arriving at the truth, for he declared that "deliberation and calculation are identical," thus implying that all thought is mathematical thought.[6]

Aristotle invented the syllogism as a method of reaching conclusions by means of deductive reasoning. This became the method of proof used in geometry, and it was used extensively in philosophy from that time.

Aristotle's Law of the Excluded Middle, that is, that a thing must either be or not be, became the method of *reductio ad absurdum*. This was used as the basis of indirect proofs in geometry and had wide applications in philosophical problems. His Law of Contradiction, that a thing cannot at the same time both be and not be, found wide application in his philosophy.

7. **Epistemology: the Problem of Knowledge.** In his discussion of arithmetic, Aristotle gave much attention to units of measure. For him knowledge was thought of as a measure. For example, knowledge of a circle was regarded as the measure by which we know the individual circles to be circles. The same was true with sensation.[7]

In Book I of the *Metaphysics* Aristotle declared:

> For all men, as we remarked, begin by wondering that things are as they are, as they wonder at the motions of marionettes or at the solstices or at the incommensurability of the diagonal of a square (for everyone who has not yet discovered the cause seems to find it amazing that there is something that cannot be measured by even the smallest unit). But we should end in a contrary and what is proverbially a better state of mind, as men do in the cases just mentioned when they learn to understand them. For nothing would so astonish a geometrician as to find a diagonal commensurate with the side.[8]

8. **Metaphysics: the Problem of Ultimate Reality.** In considering the problem of ultimate reality, Aristotle had to find his own

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[6] J. E. C. Welldon (translator), **Aristotle—On Man in the Universe** (New York: The Classic Club, 1943), p. 171.
[7] Apostle, Op. cit., p. 86.
[8] J. H. MacMahon (translator), **Metaphysics** (New York: The Classics Club, 1943), p. 10.

answers to the fundamental questions of the causes and principles of the universe. In his *Metaphysics,* he reviewed the theories of all the leading Greek thinkers down to his own day, including those of Plato. But he rejected all of them because none was sufficiently analytical. All overlooked points that should have been considered and problems that should have been solved.

In his *Metaphysics* Aristotle declared:

> ... since the mathematician, too, uses the common axioms in a particular application, it must be the business of first philosophy to investigate the principles of mathematics also. For that when equals are subtracted from equals the remainders are equal is true of all quantities alike .... Philosophy ... does not inquire into special subjects ... but investigates each of such things so far as each of them is .... Therefore we must hold ... mathematical science to be parts of wisdom.[9]

9. **Natural Philosophy: the Problem of the External World.** Aristotle maintained that the planets moved in circular orbits because the circle is the most perfect curve. This is of course an error for there is nothing to show that perfection is a principle of control in astronomy.[10]

In *De Caelo,* Aristotle declared that "astronomy is the nearest to philosophy of all the mathematical sciences since it studies substance which is sensible but eternal; whereas the others are concerned with no kind of substance, *e.g.,* the sciences of arithmetic and geometry."[11]

In the *Metaphysics* Aristotle declared:

> We have now to consider whether it belongs to one science or to different sciences to inquire into what mathematicians call axioms and into substances. It is manifest that the inquiry into these axioms belongs to one science and that the science of the philosopher; for they hold good for all existing things, and not for some one genus in particular to the exclusion of others. Everyone makes use of them because they belong to being ... [as] being, and each genus is part of being. Men use them, however, just so far as the genus, to which the demonstrations they offer have reference, extends. Since then it is clear that they hold good for all things [as] being (for this is what they have in common), the person who knows about being ... [as] being must investigate these axioms too. This is why none of those who study the special sciences tries to enunciate anything about them, their truth or falsehood; neither the geometer, for instance, nor the mathematician does so, though it is true that some of the physicists have made the attempt, and not unnaturally seeing that they supposed that the inquiry into the

[9] Heath, Op. cit., pp. 8-9.
[10] Columbia Associates in Philosophy, **An Introduction to Reflective Thinking** (New York: Houghton Mifflin Company, 1923), p. 208.
[11] Heath, Op. cit., p. 12.

whole of nature and into being belonged to them alone. But since there is a class of inquirer above the physicist (nature being only one particular genus of being) it is for the thinker whose inquiry is universal and who investigates primary substance to inquire into these axioms as well.[12]

10. **Practical Philosophy: the Problems of Ethics, Esthetics, and Politics.** According to Aristotle, justice was based on knowledge of equality and inequality. John Calhoun recommended Aristotle as among the best writers on government:

> His concept of justice implied that equality was neither natural nor general but that men were distinguished by superior qualities, physical, intellectual, moral or political. Political justice was thus considered a function of equality, but not as an abstract and legal principle which can be mechanically and artificially applied and enforced in any given society.[13]

Aristotle said that virtue is the golden mean between two opposite vices. In the *Nicomachean Ethics,* Aristotle declared that

> It appears then that virtue is a kind of mean because it aims at the mean.
>
> On the other hand, there are many different ways of going wrong; for evil is in its nature infinite, to use the Pythagorean phrase, but good is finite and there is only one possible way of going right. So the former is easy and the latter is difficult; it is easy to miss the mark but hard to hit it and so by our reasoning excess and deficiency are characteristic of vice and the mean is a characteristic of virtue.[14]

In regard to a consideration of beauty, Aristotle declared in his *Metaphysics:*

> For the chiefest forms of the beautiful are orderly arrangement, symmetry, and definiteness, and the mathematical sciences have these characters in the highest degree. And, since these characters (such as orderly arrangement and definiteness) are the causes of many things it is clear that mathematicians could claim that this sort of cause is in a sense like the beautiful acting as cause.[15]

Aristotle disagreed with the communism advocated by Plato in his *Republic.* In regard to the problem of politics, Aristotle declared in Book V of his *Politics:*

> Now equality is of two kinds, numerical and proportional; by the first I mean sameness or equality in number or size; by the second, equality of ratios. For example, the excess of three over two is numerically equal to the excess of two over one; whereas

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[12] Ibid., p. 8.
[13] James G. Wharton, "Phi Beta Kappa's 50 years at V. U.", Vanderbilt Alumnus, XXXVII (2) (1951), p. 15.
[14] Welldon, Op. cit., pp. 108-9.
[15] Heath, Op. cit., p. 201.

four exceeds two in the same ratio in which two exceeds one, for two is the same part of four that one is of two, namely, the half. As I was saying before, men agree that justice in the abstract is proportion, but they differ in that some think that if they are equal in any respect they are equal absolutely, others that if they are unequal in any respect they should be unequal in all. Hence there are two principal forms of government, democracy and oligarchy • • •. That a state should be ordered, simply and wholly, according to either kind of equality, is not a good thing, the proof is the fact that such forms of government never last • • •. The inference is that both kinds of equality should be employed; numerical in some cases, and proportionate in others.[16]

11. **Summary and Conclusions.** Aristotle, like other mathematicians who philosophize, had the concept of a *universal mathematics.* With most mathematicians this has been the theoretical application of mathematics to every phase of life, but Aristotle seems to have been more cautious in his extension of mathematics. Aristotle invented the syllogism as a method of reaching conclusions by means of deductive reasoning. This became the method of proof used in geometry and it was used extensively in philosophy from that time. His Law of the Excluded Middle and Law of Contradiction were used extensively in mathematics and in philosophy. The influence of mathematics on his philosophy was not as great as in the case of other mathematical philosophers, because he thought the things dealt with by mathematics were not real while the things dealt with by philosophy were real.

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[16] Lerner, Op. cit., pp. 211-212.

To create a healthy philosophy you should renounce metaphysics but be a good mathematician.

—Bertrand Russell

Precocity in mathematics has often been the first flush of a glorious maturity, in spite of the persistent superstition to the contrary.

—E. T. Bell

Finding of Errors by Difference Tables*

V. R. UNRUH
Student, Kansas State Teachers College, Emporia

Since in applied mathematics a numerical result is of prime importance, it would follow that this result should be as free from errors as possible. Basically, errors in computed results may be due to one or both of two sources: errors of calculation and errors in the data. The accuracy of calculation can be controlled by rounding off or carrying to the desired degree. However, errors in the data, if found, can possibly be corrected only by redoing an experiment. This paper is primarily concerned with detecting errors in tabular data, experimental or calculated, by means of a difference table.

What are difference tables? How are they formed and used? A set of values of any function, known or unknown, with the independent variable taken at equally spaced intervals is the basis for a commonly used difference table. The differences of the first of these values from the second, the second from the third, and so on are placed in a column and are the first differences of the function. Should the first differences be subtracted in a like manner, the second differences of the function are formed. All orders of the difference table are attained by following this procedure in each of the successive differences. Table *A* indicates the orders of a difference table, whereas Table *B* shows the differences and their formation. Difference tables can be assembled in two ways—diagonally or horizontally. Preference, perhaps, is given the diagonal, since the two preceding values subtracted to give a difference can more easily be seen. However, all differences are the same, and the tables differ only in outward appearance.

There are several ways in which difference tables can be utilized. Interpolation by means of Newton's, Bessel's, Lagrange's, and Stirling's formulas all use the difference table. Numerical integration and differentiation are two more ways they may be used. Another is the detecting of errors in tabulated data with which this paper is concerned.

The basis for the detecting of errors in tabulated data rests on the following theorem, the proof of which can be found in almost any standard text on numerical mathematical analysis:

* A paper presented at the regional meeting of KME held at William Jewell College, Spring, 1955.

> *The nth differences of a polynomial of the nth degree are constant when the values of the independent variable are taken in arithmetic progression, that is, at equal intervals apart.*

and its converse,

> *If the nth differences of a tabulated function are constant when the values of the independent variable are taken in arithmetic progression, the function is a polynomial of degree n.*[1]

It is the converse that enables the approximation of any function by a polynomial if its differences of some order become constant

Table A

Diagonal Difference Table

$y = f(x)$

x y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$
x_0 y_0								
	Δy_0							
x_1 y_1		$\Delta^2 y_0$						
	Δy_1		$\Delta^3 y_0$					
x_2 y_2		$\Delta^2 y_1$		$\Delta^4 y_0 + \varepsilon$				
	Δy_2		$\Delta^3 y_1 + \varepsilon$		$\Delta^5 y_0$			
x_3 y_3		$\Delta^2 y_2 + \varepsilon$		$\Delta^4 y_1 - 4\varepsilon$		$\Delta^6 y_0$		
	$\Delta y_3 + \varepsilon$		$\Delta^3 y_2 - 3\varepsilon$		$\Delta^5 y_1$		$\Delta^7 y_0$	
x_4 y_4		$\Delta^2 y_3 - 2\varepsilon$		$\Delta^4 y_2 + 6\varepsilon$		$\Delta^6 y_1$		$\Delta^8 y_0$
	$\Delta y_4 - \varepsilon$		$\Delta^3 y_3 + 3\varepsilon$		$\Delta^5 y_2$		$\Delta^7 y_1$	
x_5 y_5		$\Delta^2 y_4 + \varepsilon$		$\Delta^4 y_3 - 4\varepsilon$		$\Delta^6 y_2$		
	Δy_5		$\Delta^3 y_4 - \varepsilon$		$\Delta^5 y_3$			
x_6 y_6		$\Delta^2 y_5$		$\Delta^4 y_4 + \varepsilon$				
	Δy_6		$\Delta^3 y_5$					
x_7 y_7		$\Delta^2 y_6$						
	Δy_7							
x_8 y_8								

[1] J. B. Scarborough, **Numerical Mathematical Analysis**, Third Edition, Baltimore: The John Hopkins Press, 1955.

or nearly so. Then, given a set of tabular values or compiling a set from experimental work, successive differences can be found; if, in some order, these values become constant or nearly so, one can be reasonably sure the original data is correct. However, should some of the values, in the order tending to be constant, vary considerably from the constant, there may exist an error in the tabular data. Table *A* shows how errors are propagated in the difference table. Should a tabular value be large by ε, this error will be enlarged in successive differences. The coefficients of the ε's are the binomial coefficients with alternating signs and the algebraic sum of the errors in any difference column is zero. Also the maximum error in the differences is in the same line as the erroneous tabular value.

Table B

Diagonal Differences

y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
y_0				
	y_1-y_0			
y_1		$y_2-2y_1+y_0$		
	y_2-y_1		$y_3-3y_2+3y_1-y_0$	
y_2		$y_3-2y_2+y_1$		$y_4-4y_3+6y_2-4y_1+y_0$
	y_3-y_2		$y_4-3y_3+3y_2-y_1$	
y_3		$y_4-2y_3+y_2$		$y_5-4y_4+6y_3-4y_2+y_1$
	y_4-y_3		$y_5-3y_4+3y_3-y_2$	
y_4		$y_5-2y_4+y_3$		$y_6-4y_5+6y_4-4y_3+y_2$
	y_5-y_4		$y_6-3y_5+3y_4-y_3$	
y_5		$y_6-2y_5+y_4$		$y_7-4y_6+6y_5-4y_4+y_3$
	y_6-y_5		$y_7-3y_6+3y_5-y_4$	
y_6		$y_7-2y_6+y_5$		$y_8-4y_7+6y_6-4y_5+y_4$
	y_7-y_6		$y_8-3y_7+3y_6-y_5$	
y_7		$y_8-2y_7+y_6$		
	y_8-y_7			
y_8				

Table C is an application using the difference table in pointing out an error in tabular values. This experimental data was obtained from a regular physics experiment. As a comparison to the experimental data, the theoretical data shows that the second differences of a polynomial of the second degree are constant when the values of

Table C

$$S = 1/2gt^2$$

	Experimental			*Theoretical*		
t	D	ΔD	$\Delta^2 D$	D	ΔD	$\Delta^2 D$
4	8.71			8.71		
		5.00			4.90	
5	13.71		1.15	13.61		1.09
		6.15			5.99	
6	19.86		1.15	19.60		1.077
		7.30			7.067	
7	27.16		1.05	26.667		1.107
		8.35			8.173	
8	35.51		1.15	34.84		1.087
		9.50			9.26	
9	45.01		1.00	44.10		1.08
		10.50			10.34	
10	55.51		$1.30+\varepsilon$	54.44		1.092
		$11.80+\varepsilon$			11.432	
11	$67.31+\varepsilon$		$.80-2\varepsilon$	65.872		1.096
		$12.60-\varepsilon$			12.528	
12	79.91		$1.30+\varepsilon$	78.40		1.082
		13.90			13.61	
13	93.81		1.10	92.01		1.09
		15.00			14.70	
14	108.81		1.05	106.71		1.09
		16.05			15.79	
15	124.86		1.10	122.50		1.09
		17.15			16.88	
16	142.01			139.38		

After Correction

45.01		
	10.50	
55.51		1.15
	11.65	
67.16		1.10
	12.75	
79.91		1.15
	13.90	
93.81		

$$-2\varepsilon = 1.1 - .8$$

$$\varepsilon = -.15$$

the independent variable are taken in arithmetic progression. In the experimental data, the constant appears to be approximately 1.1, and three values differ considerably from this. Knowing how an error in the original data is enlarged in the second differences, one can see the difference .80 would be small by 2ε. Setting $1.1 - 2\varepsilon = -.80$ and solving, $\varepsilon = .15$ or the erroneous tabular value is large by .15. By making this correction in the tabular value and taking successive differences again, the values in the second differences column will become more consistent.

Simple and purely mechanical, difference tables can be an easy and fast way in which errors in tabular data can be detected with a minimum effort. Where extended calculations or complicated experiments are involved, considerable time may be saved by the use of difference tables in checking the results.

The higher arithmetic presents us with an inexhaustible store of interesting truths—of truths, too, which are not isolated, but stand in a close internal connection, and between which, as our knowledge increases, we are continually discovering new and sometimes wholly unexpected ties. A great part of its theories derives an additional charm from the peculiarity that important propositions, with the impress of simplicity upon them, are often easily discoverable by induction, and yet are of so profound a character that we cannot find their demonstration till after many vain attempts; and even then, when we do succeed, it is often by some tedious and artificial process, while the simpler methods may long remain concealed.

—C. F. Gauss

The Problem Corner

EDITED BY FRANK C. GENTRY

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond the calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets of paper before March 1, 1957. The best solutions submitted by students will be published in the Spring, 1957, number of THE PENTAGON with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Frank C. Gentry, Department of Mathematics, University of New Mexico, Albuquerque, New Mexico.

PROBLEMS PROPOSED

97. *Proposed by Harvey Fiala, Student, North Dakota Agricultural College, Fargo, North Dakota.*

If two circles, one having a radius of 1 inch and the other a radius of 1 light year, have their circumferences increased by 6 feet, what is the difference in the increase in the radii of the two circles?

98. *Proposed by the Problem Corner Editor* (From Robinson's *Progressive Higher Arithmetic,* 1871.)

A general forming his army into a square had 284 men remaining; but increasing each side by one man he wanted 25 men to complete the square. How many men had he?

99. *Proposed by the Problem Corner Editor.*

The integers 5 and 45 satisfy the condition that their arithmetic mean, their geometric mean, and their harmonic mean are all integers. Find another pair of positive integers whose sum is less than a hundred which satisfy this condition.

100. *Proposed by the Problem Corner Editor.*

Find the smallest positive integer N such that $2N/3$ is equal to N with its digits permuted one place to the right cyclically, the units digit becoming the first digit.

SOLUTIONS

88. *Proposed by Charles Pearsall, Student, Hofstra College, Hempstead, New York.*

Suppose that with each purchase of a box of "Sogg" the buyer receives a white coupon. The coupon collection plan allows 1 blue coupon for 10 white coupons, 1 red coupon for 10 blue coupons, and 1 gold coupon for 10 red coupons. An avid collector, Mrs. A, used 233 boxes. Each of 11 of her friends used a greater number of boxes than Mrs. A but the same number as the other 10. Mrs. A received all the coupons and examining her accumulation, found that of a total of 19 coupons there was a different number of each kind and the number of white and red exceeded the number of blue and gold. Find the number of boxes of "Sogg" used by all twelve women.

Solution by Harvey Fiala, North Dakota Agricultural College, Fargo, North Dakota.

Since each coupon is worth 10 times that of another, the total number of coupons may be treated as a four digit number with the number of white coupons for the unit's digit, the number of blue coupons for the ten's digit, the number of red coupons for the hundred's digit and the number of gold coupons for the thousand's digit. Hence 233 will represent the number of coupons Mrs. A received with the boxes she bought. Since each of the other women bought the same number of boxes, a number greater than 233, the smallest number of coupons they could have contributed would be represented by the number $2574 = 11 \times 234$, giving a total of 2807. Since this would represent a total of only 17 coupons we add 11 successively until we obtain a number for which the sum of the digits is 19 and for which the sum of the unit's and hundred's digits exceeds the sum of the ten's and thousand's digits. The first such number is 2917. Hence the twelve women bought a total of 2917 boxes of "Sogg". (Note by the Editor. Mr. Fiala has tacitly assumed, as the proposer probably intended, that all coupons are converted into the fewest possible number and that the women bought the fewest possible number of boxes. An algebraic solution showing the possibility of other solutions or the uniqueness of this one would be interesting.)

89. *Proposed by C. W. Trigg, Los Angeles City College, Los Angeles, California.*

If n is even and greater than 2, then $2^n - 2$ can never be the product of two consecutive integers.

Solution by Harvey Fiala, North Dakota Agricultural College, Fargo, North Dakota.

Let $n = 2r$. Then
$$2^n - 2 = 2^{2r} - 1 - 1 = (2^r - 1)(2^r + 1) - 1.$$
This is a number of the form 1 less than the product of two integers whose difference is 2. If there were a solution it would have to be of the form $2^r(2^r - 1)$ or $2^r(2^r + 1)$. Obviously neither of these can be equal to the expression above for r greater than 1.

90. *Proposed by Frank Hawthorne, Hofstra College, Hempstead, New York.*

A merchant buys an odd number of felt hats at \$10 each and one cloth hat for a whole number of dollars less than \$10. How much does the cloth hat cost if the total amount involved is a perfect square?

Solution by Doyle G. Johnson, Nebraska State Teachers College, Wayne, Nebraska.

The number which represents the total amount of money must have its ten's digit odd. Now all perfect squares whose ten's digit is odd have 6 for their unit's digit. Hence the cloth hat cost \$6.

(Note by the Problem Corner Editor. Any perfect square may be represented by $(10a + b)^2$ where b is an integer less than 10. Expanding we obtain $100a^2 + 20ab + b^2$. Since the first two terms of this expression are even, the ten's digit can be odd only if the ten's digit of b^2 is odd. Hence b^2 is 16 or 36.)

Also solved by Kenneth Nuernberger, Nebraska State Teachers College and Harvey Fiala, North Dakota Agricultural College.

92. *Proposed by Martin Winterfield, Hofstra College, Hempstead, New York.*

Find the sum of the 9! numbers obtained by permuting the integers from 1 to 9 all at a time in all possible ways.

Solution by Harvey Fiala, North Dakota Agricultural College.

Every integer from 1 to 9 will appear the same number of times in each column. Since each column contains 9! digits and since there are only 9 different digits, each one will appear 8! times. Hence the sum of each column is $45(8!) = 1{,}814{,}400$. The solution will be the sum of $1{,}814{,}400(1 + 10 + 10^2 + 10^3 + \cdots + 10^8)$. But this is equivalent to

$$1{,}814{,}400(111{,}111{,}111) = 201{,}599{,}999{,}798{,}400.$$

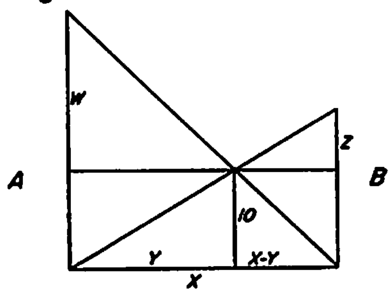
Also solved by Doyle G. Johnson and Kenneth Nuernberger, Nebraska State Teachers College; Robert A. Hoodes, Hofstra College. One solution was not signed.

93. *Proposed by Wayne Stark, Iowa Alpha Alumnus, El Paso, Texas.*

An alley is flanked on either side by buildings A and B. A 20-foot ladder extends across the alley from the foot of A to the top of B. A 30-foot ladder extends from the foot of B to the top of A. The point of intersection of the two ladders is 10 feet above the floor of the alley. How high are buildings A and B?

Solution by Robert A. Hoodes, Hofstra College, Hempstead, New York.

Utilizing the notation shown in the accompanying diagram, we have by similar triangles:



$z/10 = (x - y)/y$ and $w/10 = y/(x - y)$, from which we obtain $zw = 100$. Also, from the Pythagorean Theorem, $(w + 10)^2 + x^2 = 900$ and $(z + 10)^2 + x^2 = 400$. Elimination of x and w from the last 3 equations leads to $z^4 + 20z^3 + 500z^2 - 2000z - 10{,}000 = 0$. This equation has one positive root, $z = 5.76$, approximately. Hence $w = 17.36$ feet and $A = 27.36$ feet while $B = 15.76$ feet.

Also solved by William Dameron, Southwest Missouri State College; Pasty Renfro, Fort Hays Kansas State College; and Harvey Fiala, North Dakota Agricultural College.

94. *Proposed by L. T. Shiflett, Southwest Missouri State College, Springfield, Missouri.*

Show that n lines, no two parallel and no three concurrent, divide a plane into $1 + n + [n(n - 1)/2]$ regions.

Solution by Duane W. Dailey, Washington State College, Pullman, Washington.

A single line divides the plane into 2 regions, 2 lines divide the plane into 4 regions, 3 lines divide the plane into 7 regions and 4 lines divide the plane into 11 regions. From this it is seen that the nth line adds n regions to those determined by $n - 1$ lines. Now assume that the formula holds for n lines. Then if R_n represents the number of regions determined by n lines, we have

$$R_n = 1 + n + [n(n - 1)/2] \text{ and}$$
$$R_{n+1} = 1 + n + [n(n - 1)/2] + n + 1 =$$
$$1 + (n + 1) + [(n + 1)n/2.]$$

But the formula holds for $n = 1, 2, 3$. Therefore the induction is complete and the formula holds for all n.

95. *Proposed by the Editor* (From Ray's *Algebra*, 1852).

A traveler sets out from a certain place and travels 1 mile the first day, 2 miles the second day, 3 miles the third day, etc. In five days afterward another sets out and travels 12 miles a day. How long and how far must he travel to overtake the first?

Solution by William Dameron, Southwest Missouri State College.

If n represents the number of days traveled by the first traveler, then his total distance will be $1 + 2 + 3 + \cdots + n = n(n + 1)/2$ miles. The second traveler travels $n - 5$ days at 12 miles per day or $12(n - 5)$ miles. Hence $n(n + 1)/2 = 12(n - 5)$ or $n^2 - 23n + 120 = 0$. The solutions of this equation are $n = 8, 15$. Hence the second traveler travels 3 days or 36 miles before he overtakes the first. It is interesting to note that the first man again overtakes the second seven days later.

Also solved by Duane W. Bailey, Washington State College; Larry Williams, Nebraska State Teachers College; Harvey Fiala, North Dakota Agricultural College; Robert A. Hoodes, Hofstra College; and Kenneth Nuernberger, Nebraska State Teachers College.

96. *Proposed by Rex Depew, State Teachers College, Florence, Alabama.*

Snowfall begins and at 10:00 a.m. a snow plow begins operation. If, under maximum power, the plow moves 2 miles the first hour and 1 mile the second hour and if the speed of the plow is inversely proportional to the depth of the snow, at what time did the snowfall begin? Assume that the rate of snowfall is constant.

Solution by Harvey Fiala, North Dakota Agricultural College.

Let t be the number of hours after the snow started falling. Then the depth of the snow is kt feet and the speed of the plow is $k_1/kt = K/t = ds/dt$.

Hence $s = K \ln t + C$. When $t = t_1$, $s = 0$, $C = -K \ln t_1$.
When $t = t_1 + 1$, $s = 2 = K \ln(t_1 + 1) + C = K \ln[(t_1 + 1)/t_1]$.
When $t = t_1 + 2$, $s = 3 = K \ln(t_1 + 2) + C = K \ln[(t_1 + 2)/t_1]$.
Hence $1/K = \ln[(t_1 + 1)/t_1]/2 = \ln[(t_1 + 2)/t_1]/3$, or
$(t_1 + 1)^3 = t_1(t_1 + 2)^2$, or $t^2_1 + t_1 - 1 = 0$ and $t_1 = 0.6178$ or approximately 37 minutes. Therefore the snowfall began at approximately 9:23 a.m.

Also solved by Merlynd Nestell, Emmanuel Missionary College.

The Mathematical Scrapbook

EDITED BY J. M. SACHS

Mathematics is the gate and key of the sciences · · · Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world. And what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy.

—ROGER BACON

The quotation above, true as it was in the days of Roger Bacon, is even more applicable today. In our technological world the demand for mathematicians and for people trained in the fields allied to mathematics far outstrips the supply. We must not only work to increase the supply of experts in the field but we must also foster the idea that this great discipline must play a prominent role in general education. How can a man be educated if he be ignorant of a subject which is fundamental to technology and science in a highly technical civilization?

=△=

Johann Heinrich Lambert was born at Mülhausen in Alsace in 1728. He was the son of a poor tailor and his education was entirely the product of his own exertions, due to a systematic course of reading. It was his regular custom to spend seventeen hours per day in study and writing. At the early age of sixteen he discovered, in computations for the comet of 1744, the so-called Lambert's Theorem. During the latter part of his life, he resided in Berlin where he was much honored for his ability. It was in the application of mathematical analysis to the practical problems of life that he especially excelled. His untimely death occurred in 1777 in the 49th year of his age.

—*From General Theory of Lambert Conformal Conic Projections*, by Oscar Adams, Geodetic Computer, U. S. Coast and Geodetic Survey

It is a safe rule to apply that, when a mathematical or philosophical author writes with a misty profundity, he is talking nonsense.

—A. N. WHITEHEAD

=△=

The number systems of the Indian tribes of North America, while disclosing no use of a symbol for zero nor of the principle of local value, are of interest as exhibiting not only quinary, decimal and vigesimal systems but also ternary, quaternary and octonary systems.

—W. C. Eells
American Mathematical Monthly
Volume XX (1913)

=Δ=

Two positive integers are defined to be amicable if each is the sum of the proper divisors of the other. Such numbers are mentioned frequently in Arab mathematics and played a role in sorcery in such times as the 13th, 14th, and 15th centuries. The amicable pair 220 and 284 was known at least as early as the 14th century. The amicable pair 17,296 and 18,416 was discovered by Fermat in 1636. The pair 9,363,584 and 9,437,056 was discovered by Descartes in 1638. Euler published a list of 60 or more pairs. In 1866 a sixteen year old boy, Nicolo Paganini, found the pair 1184 and 1210 which had escaped the attention of previous investigators. At the present time there are almost 400 known amicable pairs.

Amicable pairs consist of positive integers and the divisors are always considered as positive integers. The editor of this column suggests that some of the readers might be interested in applying the ideas of amicability to the algebraic sum of proper divisors, that is applying a positive or negative sign to each divisor as you choose. Let us call a pair of integers with each equal to some algebraic sum of the divisors of the other *compatible*. As an example the integers 42 and 48 are compatible. The integer 42 has 1, 2, 3, 6, 7, 14, and 21 as proper divisors. The integer 48 has 1, 2, 3, 4, 6, 8, 12, 16, and 24 as proper divisors.

$$21 + 14 + 7 + 6 + 3 - 2 - 1 = 48.$$
$$24 + 12 + 8 + 6 + 4 + 3 + 2 - 1 - 16 = 42.$$

How many compatible pairs can you find? Some of you may wish to investigate the possibility of a construction for compatible pairs similar to the construction for amicable pairs which was known to the Arabs before the 10th Century. (For the amicable pair construction see *Number Theory and Its History*, Ore, pages 96-100.)

=Δ=

A *perfect* number is one which is equal to the sum of its proper divisors. The number $6 = 3 + 2 + 1$ is perfect. An *abundant* number is one for which the sum of the proper divisors is greater than the number. The number 12 is abundant as $1 + 2 + 3 + 4 + 6 = 16 > 12$. A *deficient* number is one for which the sum of the divisors is less than the number. The number 8 is deficient as $1 + 2 + 4 = 7 < 8$. The numbers and their divisors are all considered to be positive integers. Let us invent a new classification for numbers which are equal to some algebraic sum of proper divisors. Such numbers are not *perfect* but let us call them *admirable*. The integer 12 is admirable. The proper divisors of 12 are 1, 2, 3, 4, and 6 and $6 + 4 + 3 + 1 - 2 = 12$. Can you find other admirable numbers? (Chapter 6 of the reference given in the preceding note will provide a good background for perfect, abundant, and deficient numbers.)

For those interested in predicting the results of the presidential election, we offer the two following modern versions of ancient number magic. For Democrats only: There are six different letters in the name STEVENSON and eight different letters in the name EISENHOWER. Six is a perfect number and eight is deficient. For Republicans only: There are six different letters in the name DWIGHT and four in the name ADLAI. The number six is perfect and the number four is deficient.

This proves almost conclusively that someone will be elected president in November. (The editor confesses that his sorcery is not strong enough to predict the result should some splinter party come up with a candidate whose name contains an abundant number of letters.)

=Δ=

When Robert Recorde in *The Whetstone of Wit,* published in 1557, used our familiar symbol "=" for equality he proposed this symbolism in the following words: "And to avoide the tediouse repitition of these woordes: is equalle to: I will sette as I doe often in woorke use, a paire of parallelles, or Gemowe lines of one lengthe, thus: =, bicause noe .2. thynges can be moare equalle."

Despite this eloquent appeal to reason, the symbol consisting of a stylized combination of the letters ae from the word "aequalis" was in frequent use (See Descartes, *La Geometrie*) for most of the 17th century.

=Δ=

The symbols for "less than" and "greater than", "<" and ">", are the invention of Thomas Harriot, 1560-1621, English astronomer, mathematician, and surveyor. Harriot was the tutor of Sir Walter Raleigh who later sent him to Virginia as a surveyor with Sir Richard Grenville.

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It is from the Indians [Hindus] that there has come to us the ingenious method of expressing all numbers, in ten characters, by giving them, at the same time, an absolute and a place value; an idea fine and important, which appears indeed so simple, that for this very reason we do not sufficiently recognize its merit. But this very simplicity, and the extreme facility which this method imparts to all calculation, place our system of arithmetic in the very first rank of the useful inventions. How difficult it was to invent such a method one can infer from the fact that it escaped the genius of Archimedes and of Appolonius of Perga, two of the greatest men of antiquity.

—Laplace

=△=

If a pile of chips is counted by pairs, by threes, by fives, or by sevens, there is always one chip remaining. What is the smallest number of chips there would be in the pile?

=△=

There is a certain small plant which when it is two days old produces a new plant by budding, and then one new plant each day thereafter. Each of the new plants likewise yields a new plant when it is two days old and then one new plant each day thereafter. Starting with one new plant, how many plants will there be altogether at the end of the 13th day?

—Wilson Junior College
Mathematics Tournament

=△=

The logic of the subject (algebra), which, both educationally and scientifically speaking, is the most important part of it, is wholly neglected. The whole training consists in example grinding. What should have been merely the help to attain the end has become the end itself. The result is that algebra, as we teach it, is neither an art nor a science, but an ill-digested farrago of rules whose object is the solution of examination problems • • •. The result, so far as problems worked in examinations go, is, after all, very miserable, as the reiterated complaints of the examiners show; the effect on the examinee is a well-known enervation of mind, an almost incurable superficiality, which might be called Problem Paralysis—a disease which unfits a man to follow an argument extending beyond the length of a printed octavo page.

—George Chrystal

=△=

The proof of self-evident propositions may seem to the uninitiated, a somewhat frivolous occupation. To this we might reply that it is often by no means self-evident that one obvious proposition follows from another obvious proposition; so that we are really discovering new truths when we prove what is evident by a method is not evident. But a more interesting retort is, that since people have tried to prove obvious propositions, they have found that many of them are false. Self-evidence is often a mere will-o'-the-wisp, which is sure to lead us astray if we take it as our guide.

—B. Russell

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A jet plane flies from its field over a warm-up course at 450 miles per hour. At the end of the warm-up course it flies a test course out and back to the end of the warm-up course at 600 miles per hour. From its return to the end of the warm-up course back to its field it maintains a speed of 900 miles per hour. The entire trip takes four hours. How far does the plane fly in the four hours?

=△=

Mathematics is one of the oldest of the sciences; it is also one of the most active, for its strength is the vigour of perpetual youth.

—A. R. Forsyth

=△=

Strictly speaking, the theory of numbers has nothing to do with negative, or fractional, or irrational quantities, as such.

—G. B. Mathews

The Book Shelf

Edited by Rex D. Depew

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor R. H. Moorman, Tennessee Polytechnic Institute, Cookeville, Tennessee, who takes over as editor of The Book Shelf beginning with the next issue.

It is becoming increasingly evident that the subject of numerical analysis will have significant effects upon the mathematics curricula of colleges and universities. The burgeoning requirements of industry and of governmental agencies for personnel trained in the mathematics of computation must, the writer feels, result in the offering of courses in numerical analysis in many colleges and universities where such courses are not now or have only recently become available. In other schools it will be found desirable to increase the computational content of such courses as elementary calculus and differential equations.

For these reasons the Book Shelf in this issue presents brief discussions of five recently published books in the field of numerical analysis. It is felt that all five of these books should be available to students for reference and that at least one of the books should be suitable for use as a text in a course at almost any level.

Introduction To Numerical Analysis, F. B. Hildebrand, McGraw-Hill Book Company (330 West 42nd Street) New York, 1956, x + 511 pp., $8.50.

This book is a revised version of a set of notes prepared for classroom use by the author at the Massachusetts Institute of Technology. The volume is intended to provide "an introductory treatment of the fundamental processes of numerical analysis which is compatible with the expansion of the field brought about by the development of the modern high-speed calculating devices." Nevertheless, the author has kept in mind the fact that much computation is still being accomplished by the use of desk calculators.

The first six chapters deal with polynomial interpolation, numerical differentiation and integration, numerical summation of series, and the numerical solution of ordinary differential equations, with emphasis on error analysis and propagation of errors. Chapter 7 treats least-squares polynomial approximations over continuous and discrete ranges and develops the useful properties of several of the classical sets of orthogonal polynomials for such purposes. Chapter 8 discusses high-precision Gaussian integration formulas which have assumed especial usefulness in modern large-scale calculating devices. Chapter 9 discusses several special types of approximations, including trigonometric and exponential approximation, Chebyshev and Lanczos techniques, and continued fraction approximation by rational functions. The concluding Chapter 10, which deals with the numerical solution of equations and sets of equations, is independent of the other chapters so that its content may be inserted where it may be most useful in a particular course.

The book includes what the author describes as a "problem laboratory" in which sufficient calculation is done (presumably with the aid of desk calculators) " to establish the practical significance of the theoretical developments." There are more than 500 well-chosen problems, placed at the ends of chapters, ranging from routine calculations to analytical extensions of theory. An appendix includes a large bibliography of general references as well as collateral references to journals and texts and a comprehensive set of references to mathematical tables. The bibliography contains a total of 276 references. The "Directory of Methods", also incorporated in an appendix, provides a valuable research tool for the reader who is looking for help in a specific problem or procedural situation.

Principles of Numerical Analysis, Alston S. Householder, McGraw-Hill Book Company (330 West 42nd Street) New York, 1953, x + 274 pp., $6.00.

This book is an expansion of lecture notes for a course given by the author at Oak Ridge, Tennessee, for the University of Tennessee in 1950. As indicated on the book jacket, the volume is intended to serve as a

> senior-graduate text which develops the mathematical principles upon which many computing methods are based and in the light of which they can be assessed. Directed primarily toward digital computation, the book is designed to give a unified treatment rather than a complete catalogue of methods. Treatment is primarily theoretical. Tecniques for making estimates of errors are indicated wherever possible. Functional equations as such are not dicussed, but emphasis is placed upon the methods of solving the finite systems and performing the interpolations which are required in the digital solution of functional equations.

Topics discussed in the eight chapters include the art of computation, matrices and linear equations, nonlinear equations and systems, the proper values and vectors of a matrix, interpolation, more general methods of approximation, numerical integration and differentiation, and the Monte Carlo method. The latter method of obtaining an estimate by a random sampling process is of especial interest since it is one of the most recent developments in the field of numerical mathematics.

Dr. Householder has made a thorough and scholarly presentation of the mathematical theory underlying the aspects of numerical analysis which he has selected for exposition. As already indicated, the work is almost entirely theoretical and therefore would not prove completely satisfactory as a beginning text. However, used correlatively with a text which does specialize in the presentation of computational rules, the book should be most valuable. The very comprehensive bibliography is a most useful feature.

Numerical Analysis, Zdenek Kopal, John Wiley and Sons (440 Fourth Avenue) New York, 1955, xiv + 556 pp., $12.00.

The author of this book is Professor of Astronomy at the University of Manchester, England, and was formerly Associate Professor of Numerical Analysis at the Massachusetts Institute of Technology. The volume is based on courses given by Professor Kopal at the latter institution between 1947 and 1951. The aim of the book is "to develop, in a systematic manner, the analytical basis of such numerical processes as are necessary for an algebraization and numerical solution of a wide range of problems of the infinitesimal calculus which are encountered daily in physics or engineering but which are not amenable to solution by formal mathematical methods."

Chapter headings include Introduction (summary of present status and achievements of numerical analysis); Polynomial Interpolation; Numerical Differentiation; Integration of Ordinary Differential Equations; Boundary Value Problems: Algebraic Methods; Boundary Value Problems: Variational, Iterative, and Other Methods; Mechanical Quadratures; Numerical Solution of Integral and Integro-differential Equations. Each chapter is concluded by Bibliographical Notes, which contain lists of the principal sources and references to more detailed investigations of certain topics. The appendices include elaborations of certain theories and processes, together with auxiliary numerical data. The reviewer found the frequent historical remarks of considerable interest.

This book was intended to serve as an advanced undergraduate textbook as well as for a research handbook useful to workers in certain branches of numerical analysis. The author states that, in general, "its level is such that it can be used with profit by undergraduate students of numerical analysis from approximately the junior level of American colleges and universities; elementary analysis and some algebra being the only really necessary prerequisites." The reviewer, however, does not feel that the usual beginning course in differential and integral calculus provides an adequate basis for complete use of Dr. Kopal's text.

It should be noted that the main purposes of the book are related to the application of numerical methods to the problems of infinitesimal analysis. The author has made no attempt to consider numerical solution of algebraic equations of higher degree or of simultaneous equations. The author does state frankly that among his proposed exercises are a few problems to which he would like to know the answers himself.

The volume is probably more suitable as a valuable reference than as a basic text for a relatively elementary undergraduate course. Nevertheless, the book should certainly be on the library shelves of any institution offering a course in numerical analysis.

Methods In Numerical Analysis, Kaj L. Nielsen, The Macmillan Company (60 Fifth Avenue) New York, 1956, viii + 382 pp., $6.90.

Following a chapter devoted to the fundamentals of computation, the author discusses in succeeding chapters the topics of finite differences, interpolation, differentiation and integration, Lagrangian formulas, ordinary equations and systems, differential and difference equations, least squares and their application, and periodic and exponential functions. There are well-selected exercises at the ends of the chapters, and answers are listed in an appendix. There is also an excellent set of tables as well as a useful bibliography.

Dr. Nielsen, who is Head of the Mathematics Division of the United States Naval Ordnance Plant at Indianapolis, states that his book is intended for use as an elementary text, and that "since it is a textbook for the practical man, it does not seem appropriate to fill it with mathematical sophistication, but rather to present the methods in a simple manner." This purpose the author has achieved admirably. Of especial pedagogical value are the many specific solutions, complete with calculating forms and schematic organizations of data.

The author presents both classical procedures and newer methods which have been found effective in work with modern high-speed calculating machines. The same general procedures, of course, are useful to the worker who is making use of desk calculators.

There may be some inclination on the part of students to take too intuitive an approach to the material in the text, due to the author's not having developed much of the background theory; however, the author does carefully point out hidden assumptions, and the student should not fall into concealed traps, with proper care on the part of the instructor. This text is one of the best for use in an elementary undergraduate course.

Numerical Mathematical Analysis (Third Edition), James B. Scarborough, The Johns Hopkins Press (Homewood) Baltimore, 1955, xix + 554 pp., $6.00.

Dr. Scarborough's present book is the third edition, revised and enlarged, of a text which has been outstanding in its field since 1930. Among the subjects treated, with their several subdivisions, are accuracy of approximate calculations, interpolation, numerical differentiation and integration, numerical solution of algebraic and transcendental equations, numerical solution of differential equations, theory of errors, precision of measurements, and empirical formulas.

The Third Edition contains an extensive new chapter on the numerical solution of simultaneous linear equations, treating in detail several methods of solution and illustrating each with numerical examples. There are also discussions on errors in determinants and on the evaluation of numerical determinants of any order.

This text presents an unusually satisfactory balance between the exposition of theoretical background and the outlining of computational procedures. Each major topic is introduced by a brief general summarization of "what it is all about." This is followed by a careful presentation of the theory of the subject being considered. A development of specific processes with various checks is then presented. A variety of examples is used to illustrate the preceding discussion in detail.

The reviewer was especially impressed with the author's evident attempt to make the book valuable to a student who is conducting a self-study program. This attention to pedagogical considerations makes the book proportionately more useful in a classroom situation. In particular, a teacher who deals with approximate computations in any of his courses will find Dr. Scarborough's introductory chapter on the subject of accuracy an invaluable reference source. The later chapter on precision is a necessary complement to the study of accuracy. Another excellent feature of the book is the very lucid section devoted to the relaxation and iteration methods of solving partial difference equations with given boundary values. At the end of each chapter there are exercises to which answers are not given.

The reviewer recommends this book to any reader who wishes to acquaint himself with the fundamentals or with some of the specialized procedures of numerical analysis.

Rex D. Depew
IBM Corporation, Lexington, Massachusetts
(Formerly at Florence State Teachers College)

BOOKS RECEIVED BY THE BOOK SHELF EDITOR

The Einstein Theory of Relativity, Lillian R. Lieber, Rinehart and Company, Inc. (232 Madison Ave.) New York, 1945, x + 324 pp., $3.50.

Fundamental Concepts of Geometry, Bruce E. Meserve, Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts, 1955, ix + 321 pp., $7.50.

Calculus, G. M. Petersen and R. F. Graesser, Littlefield, Adams and Co., Ames, Iowa, 1956, x + 321 pp., $1.75.

College Algebra and Trigonometry (2nd Edition), Frederic H. Miller, John Wiley and Sons, Inc. (440 Fourth Ave.) New York, 1955, ix + 342 pp., $4.50.

Postulates and Implications, Ray H. Dotterer, Philosophical Library, New York, 1955, 509 pp., $7.50.

Analytic Geometry (5th Edition), C. E. Love and E. D. Rainville, Macmillan Co., New York, 1955, xiv + 302 pp., $4.00.

Analytic Geometry, N. H. McCoy and R. E. Johnson, Rinehart and Company, Inc. (232 Madison Ave.) New York, 1955, xiv + 301 pp., $4.00.

Basic Mathematics for Science and Engineering, P. G. Andres, H. J. Miser, and Haim Reingold, John Wiley and Sons, Inc. (440 Fourth Ave.) New York, 1955, vii + 846 pp., $6.75.

Shop Mathematics, Claude E. Stout, John Wiley and Sons, Inc. (440 Fourth Ave.) New York, 1955, xi + 282 pp., $3.70.

Mathematics Teaching Essay Contest

Kappa Mu Epsilon and the Science Teaching Improvement Program (STIP) of the American Association for the Advancement of Science (AAAS) are cooperating in the sponsorship of an essay contest on the subject, "Opportunities in Teaching Mathematics in Secondary Schools." Satisfactory essays will be published in THE PENTAGON. The prize-winning essays will also be offered for publication in *The Mathematics Student Journal.*

PURPOSE OF THE CONTEST

The Mathematics Teaching Essay Contest is planned to increase interest in the teaching of mathematics at the secondary-school level. It is intended to encourage undergraduate and graduate students in mathematics to consider the advantages of a career in secondary-school mathematics teaching. It is also hoped that the preparation of, as well as the reading of the essays, may interest good students with an aptitude for and interest in mathematics to enter the teaching profession. The importance of the ability to express oneself in writing, particularly on the part of teachers, should also be emphasized by such an essay contest.

First prize in the contest will be $50. There will be second and third prizes of $25 and $15, respectively.

CONDITIONS OF THE CONTEST

1. Essays submitted in the contest shoud reach Professor Carl V. Fronabarger, Southwest Missouri State College, Springfield, Missouri, no later than April 1, 1957.

2. The essays must be not more than 1,000 words in length and should be typed double-spaced on a good grade of paper. Four (4) copies should be submitted by each contestant.

3. The content of the essay should be as specific as possible and should be intended to point out the advantages of preparation for the teaching of mathematics at the secondary-school level. The essay may consider one or more of the special facets of the profession of mathematics teaching or cover the general area as completely as the length of the essay will permit.

4. Undergraduate and graduate students in mathematics are eligible to enter the contest.

5. Essays submitted will become the property of Kappa Mu Epsilon and STIP.

6. The essays will be judged on accuracy and objectivity of the data presented, the degree to which the essay appears to be convincing in the case presented for mathematics teaching, and composition and neatness of the essay.

7. A bibliography of source material should be included.

8. Those who submit essays should write name, address, and other items of identification on a separate sheet and clip to the manuscript.

9. The judges of the contest will be a panel of five college and secondary-school teachers of mathematics. The judges are as follows: H. G. Ayre (chairman), C. V. Fronabarger, Harold D. Larsen, John A. Brown, and Mrs. Marie S. Wilcox.

Directions for Papers to be Presented

AT THE

Kappa Mu Epsilon Convention

PITTSBURG, KANSAS

April 26 and 27, 1957

At past conventions we have had papers presented by K.M.E. members and prizes awarded. Now is the time for all chapter sponsors and members to start developing papers for our next convention. Most of the papers have been of excellent quality for the level of work represented although several people have remarked that their chapters could have done better. Here is the challenge; here is the opportunity. Start now to encourage at least one of your members to prepare a paper to submit at the Kansas Convention. Read the rules below and start to work.

Who may submit papers: Any member may submit a paper for use on the convention program. Papers may be submitted by both graduates and undergraduates; however, they will not compete against each other. Awards will be granted for the best papers presented by undergraduates. If enough papers are presented by graduates, special awards may be given for their best paper or papers.

Subject: The material should be within the scope of understanding of undergraduates, preferably the undergraduate who has completed calculus. The Selection Committee will naturally favor papers that are within this limitation and which can be presented with reasonable completeness within the time limit prescribed.

Time limit: The average time limit should be twenty minutes but may be extended to thirty minutes on recommendation of the Selection Committee.

Paper: A rather complete outline of the paper to be presented must be submitted to the Selection Committee accompanied by a description of charts, models, or other visual aids that are to be used in presenting the paper. A carbon copy of the complete paper may be submitted if desired. Each paper must indicate that the author is a member of K.M.E. and whether he is a graduate or an undergraduate student.

Date and place due: The papers must be submitted before February 1, 1957, to the office of the National President.

Selection: The Selection Committee will choose about eight papers from those submitted for presentation at the convention. All other papers will be listed on the convention program.

Prizes:

1. The authors of all papers presented will be given a two-year extension of their subscription to THE PENTAGON.
2. The two or three best papers presented by undergraduates, according to the judgment of a combined faculty and student committee, will be awarded copies of the *Mathematical Dictionary* suitably inscribed.
3. If a sufficient number of papers are submitted by graduate students and selected for presentation, then one or more similar prizes will be awarded to this group.

CLEON C. RICHTMEYER
National President

Kappa Mu Epsilon News

EDITED BY FRANK HAWTHORNE, HISTORIAN

Miss E. Marie Hove, former National Secretary and National Historian of KME, has undergone a series of four eye operations. Upon her discharge from the hospital she expects to rest at her home in Boone, Iowa.

Report of the Regional Meeting of Kappa Mu Epsilon

SALLY HORN (IOWA ALPHA), REGIONAL MEETING SECRETARY

A regional meeting of Kappa Mu Epsilon was held Saturday, April 7, 1956, with Missouri Gamma at William Jewell College, Liberty, as host. Chapters and number of representatives present included the following:

Chapter	College	City	No. Present
Kansas Alpha	Kansas State Teachers	Pittsburg	16
Kansas Beta	Kansas State Teachers	Emporia	10
Kansas Gamma	Mount St. Scholastica	Atchison	11
Kansas Delta	Washburn University	Topeka	6
Kansas Epsilon	Kansas State	Hays	6
Missouri Alpha	Southwest Missouri State	Springfield	11
Missouri Beta	Central Missouri State	Warrensburg	5
Missouri Gamma	William Jewell	Liberty	17
Missouri Epsilon	Central	Fayette	11
Iowa Alpha	Iowa State Teachers	Cedar Falls	6
Iowa Beta	Drake University	Des Moines	2
Nebraska Alpha	Nebraska State Teachers	Wayne	16
		Total	117

Following registration at 9 a.m. Saturday, the general morning session came to order at 9:30 a.m. in Marston Hall. Professor La Frenz (Missouri Gamma) greeted the convention and recognized

National Secretary Miss Greene (Kansas Delta) and former National President Mr. Charles Tucker (Kansas Beta). The meeting was then turned over to Dwain Langworthy (Missouri Epsilon), chairman of the general morning session.

The papers and the order in which they were presented during the morning session were as follows:

Edward Andalafte (Missouri Alpha), "Hyperbolic Non-Euclidean Geometry"

DeWayne Nymann (Iowa Alpha), "A Symmetric Notation for Numbers"

Joan Pawelski (Nebraska Alpha), "Infinity"

Verlyn Unruh (Kansas Beta), "Conic Sections in High School Geometry"

Virginia Breland (Kansas Gamma), "Early Interest in Mathematics"

William Martino (Kansas Alpha), "The Poincàre World"

Following the morning papers students and faculty held discussion sessions with Eddie Dixon (Missouri Gamma) as chairman of the student group and Mr. Charles Tucker (Kansas Beta) as chairman of the faculty group.

The luncheon session was held in the dining room of New Ely Hall with Mr. L. O. Jones (Missouri Gamma) in charge. The welcome was given by President Binns of William Jewell College. Miss Greene and Mr. Tucker were again recognized and Mr. Tucker announced unofficially that the national convention would be held at Kansas State Teachers College in Pittsburg next year. Also recognized were representatives of each chapter as a group. Dr. Carl Fronabarger (Missouri Alpha) spoke on "Implications of My Experience as Editor of THE PENTAGON."

The afternoon session, with Carol Law (Kansas Gamma) as chairman, began with the secretaries' reports of the discussion sessions. James E. Rush (Missouri Beta) reported on the student group. He reported the chapter activities discussed included sponsoring mathematics clubs, help sessions, picnics, banquets, and publications. Also discussed were dues and qualifications, both of which varied from chapter to chapter. The student group did not feel regional officers were necessary for the host chapter would take care of the regional convention.

Mr. L. T. Shiflett, secretary of the faculty group, reported on two points the faculty group discussed in addition to the points discussed by the students: (1) Students should be encouraged to contribute to THE PENTAGON and its "Problem Corner"; and (2) regional conventions were a good idea and their informality should be preserved, *i.e.*, organizing to the extent of electing regional officers was not necessary.

Mr. Charles Tucker extended an invitation to have the next regional convention at Emporia, Kansas. A motion was made, seconded, and carried to that effect.

The following papers were presented during the afternoon session:

Peggy Steinbeck (Missouri Epsilon) "Paradox Lost, Paradox Regained"

Robert Helmick (Iowa Beta) "A Geometric Explanation of the Expansion of a Polynomial"

Don Henderson (Missouri Gamma) "The Klein Four Group in Terms of Rotation"

Max Rumpel (Kansas Epsilon) "Number Systems with Bases Other Than Ten"

Punch and cake were served by the host chapter to conclude the regional convention.

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Alabama Beta is planning to join with the other honor clubs on their campus to form an all-honor society to be governed by a board consisting of representatives from each of the clubs. Six new members were initiated this year.

California Beta's spring field trip was a visit to the home office of the Occidental Life Insurance Company in Los Angeles. Eight students from Occidental College were employed there during the summer. Kenneth M. Hoffman, '52, received his Ph.D. in Mathematics from UCLA in June, 1956, and is now an instructor at MIT.

Students and faculty of **Illinois Beta** participated in various ways in the Eastern Sectional Conference on the Teaching of Mathematics, held at Eastern Illinois State College on April 7, 1956. The conference was under the sponsorship of the Illinois Council of Teachers of Mathematics. One chapter meeting, under the direction of Dr. W. H. Zeigel, Director of Teacher Placement, discussed

"Opportunities for Teaching in the Field of Mathematics". Twenty-four student members were initiated during the year.

Illinois Gamma of Chicago Teachers College included among its recent programs a visit to International Business Machines Corporation and enjoyed a guided tour showing the operations of some of their newer machines. During the school year, thirteen new members were initiated into the chapter.

A radio broadcast from the studios of the College of St. Francis over station WJOL was presented by the members of **Illinois Delta** in March, 1956. The program was based on the life and works of Gauss. Each year for the last three years the chapter has given similar broadcasts.

Indiana Beta sponsored a mathematical contest for freshmen. Contestants were required to take an examination in algebra. Robert Gasper won the first prize of $20. Ruth Anne Clark won the second prize of $10.

Iowa Alpha reports the initiation of eleven new members.

Iowa Beta posted a "Problem of the Month" to be solved by members and pledges before each meeting.

The Freshman Mathematics Award, sponsored annually by **Michigan Beta**, was won by Tom Corcoran in May, 1955. In May, 1956, the winner was Marilyn Burkhardt. Each prize consisted of two books, *Mathematics and the Imagination* and the *CRC Mathematical Tables,* the latter donated by the Chemical Rubber Company. In February, 1956, **Michigan Beta** sent letters of commendation to all freshmen who made the grade *A* or *B* in their mathematics courses the previous semester.

Members of **Missouri Alpha** conducted "help sections" for which they were paid the standard hourly rate for student work by the college. The money earned was placed in a special fund which enabled the chapter to charter a bus to the regional meeting at Liberty, Missouri. The chapter hopes to use a similar plan to ensure a large delegation to the convention at Pittsburg, Kansas, in 1957. John Prater won the chapter's first annual award presented to that member whose work during the year was considered most outstanding. Twenty-eight new members were initiated during the year.

In December the active members of **Missouri Beta** compiled information about alumni members and the mathematics department of the college for inclusion in a "Christmas Newsletter" sent to all former and active members.

New Mexico Alpha initiated thirty-seven new members in the spring of 1955 and forty-two during the 1955-56 school year.

New York Alpha's annual freshman award for 1955-56 was won by Barbara Stobel.

North Carolina Alpha is sponsoring tutoring classes for freshmen and transfer students. An exceptionally large number (about 125) turned out for these classes. It is hoped that these tutoring classes may be extended to involve more advanced work. Fifteen new members were initiated into the chapter.

Students of **Ohio Gamma** reported to the chapter concerning their summer experiences in jobs involving mathematics.

Oklahoma Alpha reports a most unusual program in celebration of the twenty-fifth anniversary of the founding of Kappa Mu Epsilon. Mr. L. P. Woods, one of the founders of KME, has kept a scrapbook of the local chapter. The scrapbook contains pictures and notes of national as well as local items. He showed this scrapbook, using an opaque projector and screen, as a basis for a discussion of the history of KME. The local chapter gave Mr. Woods a silver-handled penknife with "KME 1931-56" engraved on it in honor of his twenty-five years of service to the fraternity.

Virginia Alpha reports that Professors J. M. Hunter, L. S. Hunter, H. M. Linnette, R. R. McDaniel, and M. M. Walker have participated in and addressed numerous meetings on mathematics and mathematical education. Professor C. A. Taylor received a National Science Foundation fellowship which enabled him to study at the University of Wyoming during the past summer. President R. P. Daniel served on the Committee for Accreditation of Colleges and Secondary Schools and on the Educational Committee of Station WXEX(TV).

In the spring of 1955, **Wisconsin Alpha** sponsored a mathematical contest for high school seniors in which twenty schools were represented and seventy contestants participated. In the spring of 1956, they again sponsored a similar contest, this time with twenty-four schools and one hundred ten contestants.

Program Topics

(School Year 1955-56)

Illinois Delta, College of St. Francis, Joliet

The Algebra of Al-Khowarizmi by Geraldine Avsec
The Fundamental Theorem of Algebra by Virginia Libera
The Application of Some Algebraic Procedures in Elementary Mechanics by Sister Crescentia
Omar Khayyam, the Mathematician by Sister Ursuline
Boolean Algebra by Sister Claudia
Determinants by Joan Nahas

Indiana Beta, Butler University, Indianapolis

Electronic Computers by David Smith
Opportunities in Mathematics by Dr. K. L. Nielsen
Techniques in Mathematical Talks by Richard Shane
Boolean Algebra by Richard Thompson
Statistical Astronomy by Dr. Frank Edmondson
Empirical Equations and Curve Fitting by James Rogers

Iowa Alpha, Iowa State Teachers College, Cedar Falls

Pivotal Condensation Methods of Solving Simultaneous Linear Equations by Professor Fred W. Lott
Education in Mathematics in Egypt by Massouma Kazim
Metamathematics by DeWayne Nyman
Genaille's Rods by Leonard Snowden
Beyond the Ivy Towers by Harold C. Trimble
Mathematics and the Social Sciences by Robert C. Seber
Geometrical Approach to Kicking Field Goals by James Heinselman
A Symmetrical Notation for Numbers by DeWayne Nyman
Electrical Circuits and Boolean Algebra by John Newton
Constructions of Ellipses and Hyberbolas by Harold Baker and Robert Stansbury

Michigan Beta, Central Michigan College, Mt. Pleasant

The Abacus by Richard LeCronier
A Problem of Mathematical Deduction by Ellen Jones
Finding the Square Root of Two by a Number Series by Darcy Sullivan
The Russian Education System and the Lauding of Mathematics There by J. Eugene Rank
Solutions That Can't Actually Happen Mathematically by Tom Corcoran
Trisection of an Angle by Millicent Germain Osburn
Nine-the Magic Number by Theodore Brabbs
Unusual Proofs of the Pythagorean Theorem by Donald Anderson
Projective Geometry by Lionel Moyes

Missouri Alpha, Southwest Missouri State College, Springfield

Lockheed's Computer Training Program by Carolyn Cusac
Mechanical Devices for Tracing Conics by John Prater
Hyberbolic Geometry by Edward Andalafte

The Summation of i^n, by Audie DeHart
A Generalization of Synthetic Division by Patrick Knight
Boolean Algebra by Paul Trentham

Missouri Beta, Central Missouri State College, Warrensburg

Teaching Mathematics in Siam by Boonsong Wattanausum
Solution of Indeterminate Equations by Homer Hampton
Mathematics in Music by John Chronister
"Plimpton 322" by Yvonne Leach
Jet Engines by Vaughan Hopkins
The Crystal Set by Paul Hollansworth
Rockets and Space Travel by Dr. C. H. Brown

New York Alpha, Hofstra College, Hempstead

Application of Pure Mathematics in Industry by Samuel Pines
Personal Experience in Programming by Lois Chamberlain
Some Plain and Fancy Definitions by Dr. Henry Wolf
Mathematics at Hofstra by Dr. Loyal F. Ollmann
Job Opportunities by Samuel Sesskin
Some Old Friends and Relations by Dr. E. Russell Stabler
Mathematics and Research by Alexander Basil
Revolution in Freshman Mathematics by Professor Frank Hawthorne
Rigor and Reality by Samuel Sesskin
The Mathematical Actuary by Peter Hinrichs
Why Does Everyone Hate Mathematics? by Dr. M. Salvadori
Spinor Theory by Dr. W. Payne
Four Color Problem by Dr. Loyal F. Ollmann

Virginia Alpha, Virginia State College, Petersburg

A Note on Group Theory by J. L. Cooper
Nine-Point Circle by R. Edwards
Polynomial Equations by R. E. Galloway
Roots of Algebraic Equations by R. E. Galloway
Fourier Series by M. L. R. Horne
Lagrangian Functions by Professor C. A. Taylor
Linear Programming by Benjamin Williams

Wisconsin Alpha, Mount Mary College, Milwaukee

Some Fun With Numbers by Vivian Woyak
Proofs for the Pythagorean Theorem by Colette Frendreis
Trisecting an Angle by Ilene Victory
Advertising the Pentagon by Kay Cunningham
What Does "If" Mean by Mary Ann Konsinowski
Demonstration of the Chinese Abacus by Jane Caroline
Curves Can Be Fun by Marie Buban
Deciphering Codes by Kathleen Slavin
Boolean Algebra by Betty Nowlan
Explanation of the "Why" of the General Method of Finding Square Root by Marie Hagner
Demonstration of the Roman Abacus by Sister Mary Petronia
Progressions by Ilene Victory
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A MATHEMATICS MAGAZINE FOR STUDENTS

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