THE PENTAGON

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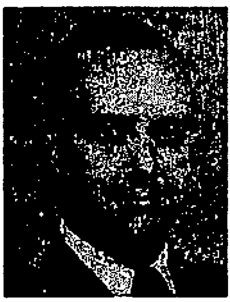

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Kappa Mu Epsilon, national honorary mathematics society, was founded in 1931. The object of the fraternity is fourfold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievements in the study of mathematics at the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

National Officers – Twenty-five Years

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Laura Z. Greene	1951 - 1955
Frank Hawthorne	1955 -

Greetings to KME on Its Silver Anniversary

The Editor feels that the occasion of the Silver Anniversary of Kappa Mu Epsilon deserves special recognition. As part of the observance he has invited several persons, each of whom has an especially significant relationship to the Fraternity, to send greetings to the Fraternity through the pages of THE PENTAGON. Illness has prevented Miss E. Marie Hove, Secretary of the Fraternity for the major part of its existence, from responding. An invitation was addressed to the first secretary, Mrs. Walter Harvey, but no answer has come, and it is believed that the communication was not delivered.

On the page following the greetings to KME are pictures of the old Administration Building at Northeastern State College, Tahlequah, Oklahoma, and the First National Convention. The Administration Building is the birthplace of KME. Below the picture of The First National Convention appear the names of the sponsors that have been identified. Any assistance from the readers for further identification will be appreciated. Dean L. P. Woods of Tahlequah furnished the pictures as well as much of the filler that has been used in this issue.

Greetings from Professor Carroll V. Newsom, the living ex-president of earliest service; Professor Cleon C. Richtmeyer, the incumbent president; Professor L. P. Woods, sponsor of the oldest chapter; and Professor Reuben R. McDaniel, sponsor of the youngest chapter appear below.

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It is a pleasure indeed to be granted this opportunity of writing a note of greeting to the many thousand members of Kappa Mu Epsilon who this year are celebrating the Society's Silver Anniversary. When some twenty-five years ago a few students and professors discussed the need for an honorary mathematics society that would be concerned especially with the recognition of undergraduate students, little did they realize that they were creating an organization that would have profound effect upon American academic life. It is impossible to measure the significance of Kappa Mu Epsilon in advancing the cause of mathematics, but there is little doubt that it has been substantial.

My association with the Society in its earliest days has provided me many of my happiest memories. I only wish that I could continue my active service in its behalf. However, I have the satisfaction of observing the society's continuing progress and its growing prestige; this is ample testimony to the present leadership and to the loyal devotion of the members.

My best wishes to all the members of Kappa Mu Epsilon and to the nearly fifty chapters.

CARROLL V. NEWSOM
Executive Vice Chancellor
New York University

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On the occasion of the Silver Anniversary of Kappa Mu Epsilon, it affords me a great deal of pleasure to bring greetings to members of the Fraternity from the Office of the National President.

It is not likely that the founders of the Fraternity envisaged the great growth and spread of influence that have taken place in the organization during the past twenty-five years. The present flourishing state of the Fraternity is due of course to the faithful work of the many dedicated men and women who have served as national officers, as corresponding secretaries, and in many other capacities.

Our present state of well-being should not be allowed to lull us into complacency. We are just on the threshold of many new developments in the field of science and the accompanying need for more well-trained people in mathematics. The next twenty-five years surely present marvelous opportunities for the Fraternity to rise to new achievements and to extend its influence in an ever-widening circle.

The present officers, corresponding secretaries, committee members, and all faculty and student members have a responsibility to share in fulfilling our purpose to "unfold the glory of Mathematics."

CLEON C. RICHTMEYER
National President
Kappa Mu Epsilon

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As a member of the first chapter of Kappa Mu Epsilon and as one of the original faculty sponsors, I wish to express my appreciation to all of those who have contributed to the development of Kappa Mu Epsilon.

When Dr. Wyant and I were writing the original ritual we saw the need for such an organization, but we did not anticipate its rapid spread into every part of the United States. It is with much gratitude that I have watched its growth and have observed its contribution to the undergraduate student.

If Kathryn Wyant were living today, there is no doubt but that her interest and devotion would be at least partially repaid by the satisfaction she would get from observing the activities of Kappa Mu Epsilon in its broad field of service.

Much work was done by the other twenty-two members of the original chapter, and their contribution cannot be overestimated. I wish to take this opportunity to express to them my appreciation for their untiring efforts in working out the details of the original chapter.

L. P. Woods
Sponsor, Oklahoma Alpha Chapter

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On the Twenty-Fifth Anniversary of the founding of Kappa Mu Epsilon it is an honor to extend greetings. It is also a signal honor to be associated with this newest chapter in the organization.

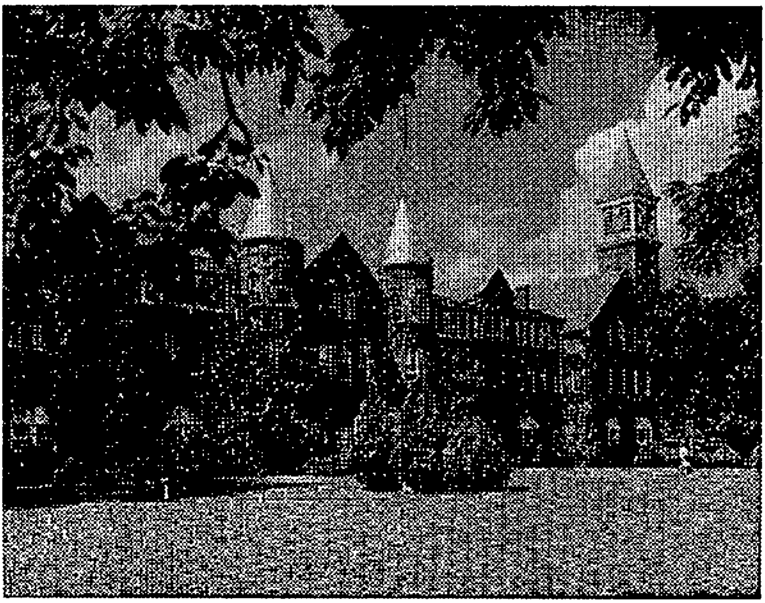
One is cognizant of the far-reaching influence that Kappa Mu Epsilon has had on the mathematical development of college students whose interests and talents have labeled them as future mathematicians of great promise. No less influence has been exerted upon the teachers who, through this organization, have been inspired to give added guidance and direction to these capable students of mathematics.

The members of Alpha Chapter of Virginia wish on this Silver Anniversary of Kappa Mu Epsilon to extend best wishes and greetings to all of the officers and members of every chapter who enjoy with them the privilege of the associations in this organization.

Reuben R. McDaniel
Sponsor, Virginia Alpha Chapter

FIRST NATIONAL CONVENTION

Second row (R to L): number one, Dr. Ira S. Condit; number two, Dr. Kathryn Wyant. Third row (R to L): number one, L. P. Woods; number three, Marie Hove; number six, Iva King. Fourth row (R to L): number one, Dr. J. A. G. Shirk; number eight, Paul Lewis.



BIRTHPLACE OF KME

Old Administration Building, Northeastern State College, Tahlequah, Oklahoma.

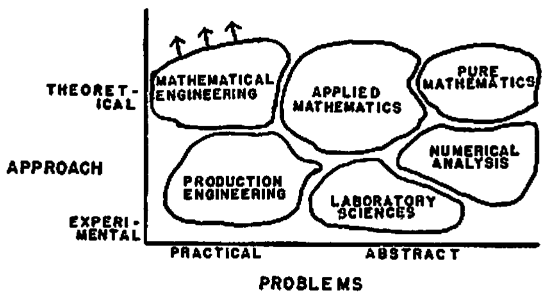
Mathematical Engineering

A. B. Clymer

Coordinator, Bituminous Coal Research, Inc., Columbus, Ohio

1. Introduction.

To introduce the field of Mathematical Engineering, we will locate it on a rather abstract map of the "Technical World." The coordinates of this map are more qualitative than quantitative. The horizontal scale is the degree of abstractness or concreteness of the problems to which technical men address themselves. At the left are the most practical, tangible immediate problems of industrial society; at the right are the most abstract, the most purely scientific; in the middle are the fundamental applied problems of engineering science. The vertical scale is the type of approach which technical men use. At the top is the purely theoretical or mathematical approach; at the bottom is the purely empirical or experimental approach; in the middle is a 50-50 mixture of the theoretical and the experimental approach.



THE TECHNICAL WORLD

Thus the coordinates of this map are problems and approaches or, if you prefer, jobs and tools. Accordingly, any locus on this map represents the application of some type of approach to some type of technical problem. To the extent that an individual scientist may be characterized by the approaches he uses and the problems he chooses both of which are limited in range for most men—it is possible to locate scientists of various types within distinct areas on this map.

This paper is a condensation of a public lecture, sponsored by Ohio Alpha Chapter of KME, and given at Bowling Green State University. Mr. Harold Tinnappel, corresponding secretary of Ohio Alpha, prepared this paper for its publication in THE PENTAGON.

Men who are products of the same narrow culture are very likely to be plotted close together on this map. This fact arises from the restraints imposed by the scientific philosophy and methodology which are currently dominant in that culture. An example would be the ancient Greeks who should be plotted near the upper right-hand corner of this map. The followers of Francis Bacon would be plotted at the bottom. The philosophy of science current in today's Western World, together with the industrialization of the Western World, makes it possible for our contemporaries in the Technical World to scatter all over this map for the fisrt time in human history.

Before describing the land of Mathematical Engineering, we shall first study the other countries in the Technical World. South and a little west of it lies the hot, dirty, noisy, and dangerous country of Production Engineering. Its natives are usually seen carrying wrenches, and they have oily rags hanging out of the hip pockets of their coveralls—the national dress. Their task is to keep the power in harness, to keep the eternal flows moving. Every one of their problems is an emergency. Their method is trial and error; they ask no help from the rest of the Technical World except when they have encountered the same problem many times and have been unable to cope with it. These men serve on the firing line of the industrial battle.

Just east of Production Engineering lie The Laboratories in which routine tests, measurements, and empirical studies are conducted in all sorts of fields: Engineering, Chemistry, Physics, Bacteriology, etc. Thomas Edison lived a little north and east of The Laboratories, but never very far away. So did Kepler, who operated by trial and error, fitting curves to Brahe's observational data for the planets. Brahe stayed in his observatory, an exclusive estate lying at the east end of the land of The Laboratories.

Farther to the east we find the numerical analysts using digital computers to study the density of the prime numbers and other purely empirical problems in the mathematical realm.

We leave the southern half-plane, where observation and experiment reign and travel to the northern half-plane where we find the paper-and-pencil boys. In the extreme northeast corner are the Pure Mathematicians who study problems of their own imagining. They invent games and all the rules to go with them. The more abstract and general the framework, the better are they pleased. Some of the high priests in the land of Pure Mathematics even insist that their handiwork is corrupted and despoiled when it is applied to any of the purposes of the rest of the Technical World to the west. Because of their efforts, the Technical World is continually expanding to the east.

The clearings abandoned by the pioneering Pure Mathematicians are eventually settled permanently by their neighbors to the west, the Applied Mathematicians. The boundary between these two countries is always changing as a result of the aggressions and exploitations by the Applied Mathematicians. In many places the Applied Mathematicians may barge in before the Pure Mathematicians are quite through.

Whereas the stiff-necked Pure Mathematicians face the east exclusively, the Applied Mathematicians have swivel necks. They look to the west for the problems, to the south for data, and to the east for new tools. Thus they are a powerful integrating influence in the Technical World in spite of their cruel foreign policies with respect to their neighbors to the east.

Applied Mathematics is by no means a unified nation made up of stereotyped individuals. At the eastern end of the country we find the Theoretical Physicists, who concern themselves with the problems at the Abstract Frontier—the entire Universe and the tiny universe within the atom. The Theoretical Chemists, and several other breeds of Theoretical Scientists live not far to the west. The western end of Applied Mathematics is populated by Engineering Scientists, men who devote themselves to very general and difficult theoretical problems arising in the west.

The vast land of Applied Mathematics is so broad—in terms of degree of abstraction of subject matter—that very few men have ever traversed it completely. The most notable of these men was the Marco Polo of Mathematics, the immortal Euler. His lifework took place all the way from Pure Mathematics to Mathematical Engineering. His intellectual travels have never been equaled. Today it is difficult to make contributions in more than one small area in the broad country of Applied Mathematics.

To the west of Applied Mathematics lies the country of Mathematical Engineering, the national effort of whose natives is to push their frontier to the northwest, as the arrows suggest. They strive to overcome the technical problems in industry by encircling their northern flank. In this battle they take advantage of the experience of the Production Engineers to the south, and they use weapons forged by the Engineering Scientists to the east. Some Mathematical Engineers stay at the northwestern frontier; these are the elite warriors who achieve the big gains. Most Mathematical Engineers stay behind the firing lines and mop up the little problems previously encircled and bypassed. This is an essential and valuable task, although not so trying to the intellect as life on the frontier.

2. The Place of Mathematical Engineering in Industry.

All Mathematical Engineers have similar goals, drives, interests, and habits of work. Like all other Applied Mathematicians, they are essentially problem solvers, but Mathematical Engineers like best the problems which come up in industry. They would be happiest if they could solve all of their problems by means of mathematics alone, but they are resigned to the necessity of experimenting to some extent in connection with solving most of their problems.

Contrary to what one might expect, the number and caliber of Mathematical Engineers working in a given industry vary inversely somehow with the age of the industry. A century ago there were no engineers of any kind in industry. There were only millwrights, machinists, inventors, and other clever men who solved all their problems by the aid of trial-and-error, experience, and a little reasoning. These men were the predecessors of today's Production Engineers. Mathematics had little or no part in the development of industry. As the nineteenth and twentieth centuries rolled on, the old industries stayed pretty much the same and still are slow to call upon Mathematical Engineers. The first call is upon other engineers, physicists, and chemists. A demand upon an old industry to mass-produce a new product to meet the specifications of a new industry may create a need for Mathematical Engineers. This happened to the glass industry when it suddenly had to produce millions of large glass envelopes to close tolerances for the picture tubes in TV sets.

On the other hand, various inventions in the nineteenth and twentieth centuries led rapidly to the establishment of brand new industries. The communications industry, for example, grew out of the invention of the telegraph. The electrical manufacturing industry and the electric power industry grew out of the invention of the electric light. The invention of the vacuum tube was the starting point of the electronic industry. From our standpoint the most important feature of all of these inventions was the comparative simplicity of their mathematical representations. Telegraph lines, electric motors, and generators, power lines with resistance loads, and vacuum tube amplifiers, can all be represented, experimented with, and even brought to optimum design by the use of mathematics. This is true also of the flight of aircraft, of the flow of oil through sand, and in general of the essential components and physical phenomena involved in all recently-born industries. Because of this ready susceptibility to mathematical analysis, the new industries have mathematics as their life blood.

The first inroads made upon an industry by mathematics are usually simple cases of the basic general problems which characterize the industry. For example, in the oil industry the earliest significant applications of mathematics were simple cases of seismic prospecting and of the flow of a viscous fluid through a porous solid. In the glass industry, simple cases of the flow of hot viscous glass and the stresses in glass products were the earliest problems studied. The early analyses, which gradually increase in scope and rigor, lead to the discovery and formulation of the basic principles and the optimum values of the parameters which should govern the design of products and the equipment for manufacturing them. In most industries, these principles will have been only partially known and understood before the mathematical approaches have been pushed to a successful conclusion.

Out of the basic original analyses come also a set of new mathematical tools for manipulating the problems of the industry. For example, in the electrical industry the formulation of AC network theorems, the invention of notation to denote phase and amplitude, and so on, made it possible to analyze any new case systematically and readily.

The next phase in the use of mathematics by a new industry is the application of the mathematical approach to all problems arising in any part of engineering work, such as the guidance of experiment design, the analysis of experimental data, the determination of specifications for manufactured parts and for materials, the synthesis of radically new and improved designs, the study of production costs, and other analyses.

Mathematical Engineers characteristically find important practical problems for themselves and then find the solutions either entirely or principally by use of paper and pencil. The finding or recognizing of a practical problem which is susceptible to the mathematical approach is often half the battle. Only a mathematically-minded person who has been exposed to the problem would ever recognize it as an opportunity for mathematical analysis.

The Mathematical Engineer after recognizing the problem must properly formulate the problem in mathematical terms. In many cases it is expedient to set up the equations in one mathematical language and then transform them into another for the sake of convenience in subsequent steps. If, as is usually the case, the equations are too complicated he must simplify them within the prescribed margin of error. If the system is tractable analytically he may feed it into a computing machine of some kind. He may approximate and obtain a numerical solution by Monte Carlo methods or other card games invented for this purpose. After he has obtained his first result he will often find that he does not have sufficient data to confirm or disapprove it. Therefore he might find it necessary to design and perhaps conduct a set of crucial experiments, on either a small or a large scale or possibly both, in order to test his solution and all of the assumptions underlying it. In many cases the theory will be shown to be inadequate in some respect, and the data will suggest the location of the weak assumption in the theory.

After the results have been confirmed by experiment, they must be put into forms which will be most widely and easily useful, such as tables, charts, formulas, rules, design specifications and principles, etc.

Some classic analyses of engineering components and systems have been Hardy Cross' numerical techniques for the analysis of stresses and moments in structures, the Schwarz-Christoffel transformation for study of airfoil performance, Heaviside's calculus for transient network analysis, Shannon's application of Boolean algebra to relay networks, the Bell Laboratories work on feedback amplifier analysis, Fry's analyses of telephone traffic, Childe's derivation of the current-voltage law for a vacuum tube, Mayers' analysis of combustion in fuel beds, Steinman's analysis of the lateral wind-induced vibrations of suspension bridges, the development of servo theory at

Massachusetts Institute of Technology, Kron's tensor analysis of rotating electrical machinery, and Lanchester's equations for the flight of aircraft.

3. Employment Opportunities.

Some of the employment opportunities arise in fields other than "industry" in the narrow sense. Employment for Mathematical Engineers exists in organizations which conduct research or engineering work toward the ultimate production of some product. Among these are non-profit research institutions, firms which undertake engineering or research for profit, engineering foundations, branches of the federal government that are responsible for weapon or defense equipment development, and organizations within universities which are under contract to such branches of the federal government.

In industry proper there are relatively few administrative posts for supervision of work which is primarily mathematical, even in new industries. However, there are a great number of administrative positions in the federal government for men trained in mathematics. Such positions occur in the National Science Foundation, the Bureau of Standards, the Office of Naval Research, the Army Office of Ordnance, the Air Force Office of Scientific Research, the Weapons System Evaluation Group, and many others.

Work of a statistical nature is most plentiful in civil service employment. Computational work for and by computing machines is now done on a variety of subjects including missile and aircraft flight simulation, weather forecasting, mathematical programming of procurement and of production operations, and many others. Most giant computers either are government-owned, or are used primarily for government problems. In addition, Mathematical Engineers may find employment in telephone system work, production and purchasing, quality control work, mathematical management work, design, and analysis of experiments. To cite but one example of the scope of this work, the scientific establishment of fair rates for public utilities by means of vast and thorough statistical research studies is now beginning to replace the old method of merely trying to reconcile the conflicting pressures exerted by interested parties.

4. Qualifications and Training of the Mathematical Engineer.

The qualifications generally accepted as necessary, or at least highly desirable, of a Mathematical Engineer are:

1. A substantial knowledge of the physical sciences and at least one branch of engineering.
2. A drive to do applied mathematical research, a curiosity, a wide range of interests easily aroused.
3. A practical sense, a willingness to approximate and to adapt the attack to the problem.
4. The personality of a good team worker, with clarity and flexibility of expression.
5. An excellent working knowledge of analysis, differential equations, mathematical statistics, and preferably much more mathematics.

As far as the author knows there are no schools with departments of Mathematical Engineering. A student must settle for a department bearing another name but with an overlapping purpose, such as Applied Mathematics or Engineering Mechanics. He will probably have to elect courses in many departments. Especially helpful would be the graduate engineering courses in which theoretical physics is applied to the analysis of specialized engineering systems. He will profit greatly from courses or part-time work in which he can participate in research team projects.

The growth and development of Kappa Mu Epsilon has come through four easy steps: the need recognized simultaneously in many colleges, the building of the chassis, the first PENTAGON, and the devotion of national officers and chapter sponsors.

Mathematics and Religion

J. D. Haggard

Faculty, Kansas State Teachers College, Pittsburg

Recently a student was heard to say, "If only I could establish the truth of a proposition in religion like I can in mathematics, I would be quite satisfied." His understanding is that the two areas are completely different not only in their methods of gaining truth but also in the validity of the obtained results; that in religion one is compelled to base the discourse on tenuous assumptions that must be taken on faith, while in mathematics the logic is infallible, the postulates absolute, and all the results certain.

A university president tells of a student in his school who was summoned to the dean's office and asked why she was enrolled in college algebra for the fourth time when she had in fact made a passing grade in each of the three previous enrollments. The young lady replied that she had a distaste for areas of study in which the validity of a proposition was open to discussion but in mathematics facts were absolute, thereby leaving no room for alternatives.

Unfortunately these expressions are fairly typical of the undergraduate student, even though his major work has been in mathematics. Far too many students view mathematics as an area where nature has dictated with a free hand and where uniformity and agreement of all who labor without error are inevitable. "It's as certain as two and two make four," is a trite statement to which the uninitiated is all too ready to nod assent. In our intense desire to have the student learn as much mathematics as possible we often neglect his understanding of the nature and structure of mathematics itself. We don't see the forest for the trees.

Students often see mathematics as an expression of nature with its inflexibility in structure and in the results it produces; perhaps this misunderstanding can be minimized even in elementary college classes by looking at alternative methods of carrying out the operations of mathematics and examining the many assumptions on which conclusions are based. The four fundamental operations of arithmetic can be carried out by several methods, each yielding the same result. We are not compelled always to multiply or to add numbers in the usual way. It might be refreshing for the student to examine some of these procedures or to work with an arithmetic where 2 plus 2 does not give 4.

There is nothing absolute about the symbol 243. Just what it represents depends on the assumptions. It could symbolize two times eight squared plus four times eight plus three, instead of the usual two hundred forty-three. The area of a plane figure is customarily represented in terms of square units, but circle units, the area of a circle with unit radius, or perhaps triangular units, with a triangular unit being the area of a unit equilateral triangle could be used. Thus a rectangle two by three would have $6/\pi$ circle units or $8\sqrt{3}$ triangular units in its area. What is obtained for the area depends on the assumptions made.

High school geometry provides the first—perhaps the last—opportunity for the student to come to grips with the nature and structure of a mathematical system. Here he begins to see that the proof of a first proposition is impossible, just as to give the definition of a first word is impossible. Postulates or assumptions are *deliberately selected,* not because they are obvious or true in the natural world about us but as a basis from which to begin the development of a subsequent theory. Perhaps it was unfortunate that Euclid selected for his five postulates certain ones that did seem to be verifiable in his surroundings. For over two thousand years people thought that since the postulates of Euclidean geometry correlated with nature that no other geometry could be possible. "If God geometrizes," a pre-nineteenth century writer said, "He surely does so with Euclid's geometry." He saw mathematics as a revelation of God. While this is no longer an understanding held by mathematicians, all too often curriculums are constructed as if it were.

That the "truth" of mathematics should be an absolute that people have held to as their god is not at all surprising when one examines the history of intellectual thought since Euclid. *A priori* thinking in mathematics was hardly questioned for over two centuries. Such a man as Isaac Newton, who contributed so much to the field of mathematics, as late as the eighteenth century certainly seemed to consider the postulates of Euclidean geometry as self-evident. In all fairness to the list of really great mathematicians who contributed to the development of the non-Euclidean geometries, it can be said that their efforts were directed toward trying to construct a valid proof of Euclid's famous fifth postulate, whereas out of it all eventually arose the elliptic and hyperbolic geometries. In their efforts to show that the fifth postulate was really intrinsically contained in the first four, they were amazed to learn that in fact the famous assumption could be entirely replaced by another inconsistent with it, without producing a geometry of queer proportions. This discovery was one of the most liberating ones in science since the day of Galileo. Despite its importance it is, in the words of one mathematician, "an aspect of mathematics which is, unfortunately, hardly ever presented to the undergraduate student of the subject so that some members of Kappa Mu Epsilon might be unaware of its existence.[1]

What kind of a world do we live in? Is it not of necessity Euclidean? How could it be that the sum of the angles of a triangle differ from 180°? One of the most enlightening and liberating examples pertaining to these questions has come to be called the "Poincare World." Here we shall attempt to be objective by separating ourselves from the Poincare model in order to examine its characteristics and then later to relate them to what we understand as reality.

Imagine a world made up of the points inside a circle of radius a, with the distance of any object from the center designated by r. Suppose, moreover, that the temperature in this world is influenced only by the distance from the center and that the formula is, $t = c(a^2 - r^2)$, where c is a constant. Thus at the center of the circle the temperature would be a maximum, while it approaches zero as we move toward the periphery. Now suppose that all objects, including the inhabitants, are subject to change in size with length being directly proportional to the temperature and this due to change in distance from the center. That is, the length and width of an object grow smaller as its distance from the center increases, and this adjustment of size to temperature is continuous.

What would be some of the intuitive observations of an inhabitant of this world? What kind of geometry would he likely develop, and what sort of reality would he experience? In the first place his world would seem infinite in extent, for as he moves toward the periphery of the circle he, his steps, and his measuring tools, all grow smaller. Thus all objects would, as far as he could see, remain fixed in size and his world boundless.

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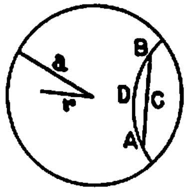
[1]Leonard M. Blumenthal, "The Relativity of Mathematics" The Pentagon, Fall, 1951.



What about the shortest distance between two points in this world? Consider two points A and B in the figure. In going from A to B it is quite conceivable that by moving toward the center a path such as ADB might be shorter than ACB. That is, in walking from A to B it might take fewer steps to go by ADB than ACB. Indeed the "shortest distance" between two points in this world is along the arc of the circle through the two points and orthogonal to the larger circle. Recall too that there is one and only one circle through two points inside a given circle and orthogonal to it, so that in this world two points determine a unique line.

Now if in this world light rays travel along "shortest paths," then points A, D, and B would appear to the observer to be "lined up." He could sight along them just as we could sight along three points in a line, and rays from the headlights of an automobile would light the path in front of the car assuming it is traveling by way of a "shortest path."

This imaginary excursion through the Poincare world is a rather liberating experience in at least two ways: First, we see that being limited to a small region of a "universe" severely limits one's objectivity. It is quite possible for one to have experiences of which he has no conscious awareness. As we stand off and look on the inhabitants of the Poincare world we see how terribly deluded they are. They actually live in a finite region, but their measurements and observations lead them to consider it to be infinite. Light would travel in a curved path which they would "see" as straight; all objects would actually change in size as they change their distance from the center but would appear fixed in size to the inhabitants. Secondly, the imaginary world just described satisfies, with appropriate definitions, all the postulates of one of the non-Euclidean geometries. These people would have experiences much like our own and yet they reside in a hyperbolic world. If their life were confined to a small region of their space as ours is so confined, they could likely be persuaded that Euclidean geometry more nearly represented their reality, whereas travel over a larger area would reveal the non-Euclidean features of their space. Perhaps if we could roam about our universe, we might change our present conception of reality. As to whether the universe is Euclidean or non-Euclidean is a question not amenable to the methods of pure mathematics, but can only be approximated by measurement and calculations, and even these methods hold little promise of an accurate answer. The modern engineer or physicist makes use of the geometry that seems to serve his purpose best. On a given occasion elliptic geometry might best explain the existing phenomenon, as was the case with some of Professor Einstein's work. However this position is a modern one indeed, as expressed by Professor Schilling: "You will find that Newton started with Euclidean assumptions about space, and assumes these to be *a priori*, self-evident and absolutely necessary for describing the world. This he seemed to feel is embedded both in our thinking and in the universe. This has been the point of view of physics for most of the time since Newton. The really big revelation in physics that brought about modern physics came with the realization that the old *a priori* foundation had disappeared—or had never been there."[2]

Pure mathematics does not profess to establish the truth of physical phenomena or any relationships therein. These are accessible only through the methods of experimentation: weighing, measuring, and observing. Similarly in religion and theology our truths are established by the experiences of people addressing themselves to the problem and not by logical discourse however arduous and meticulously constructed. A purely logical proof of the existence of God is but folly.

In a yet unmentioned way mathematics is relative to the assumptions one is using, and this is the area of logical thinking. In most areas of conventional mathematics we employ what is called "Aristotelian logic" wherein the so-called "law of the excluded middle" prevails. That is, either a proposition is true or else it is false. There is no middle ground. To show it is false that a certain proposition is false, is sufficient proof that it is true. This is the indirect method so useful to the student of mathematics but which would not be allowed in non-Aristotelian logic.

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[2]H. K. Schilling, Unpublished Seminar Lecture Notes. Pennsylvania State University, August 6, 1954, p. 554.

As to the certainty of mathematics, compared to the faith of religion, the work of the contemporary Austrian mathematician Godel speaks very directly. The one characteristic which every mathematician seeks to incorporate in the system at hand is consistency. He must be assured of its being impossible to prove a proposition both true and false. He strives for internal consistency and thus validity. But the prodigious work of Godel has shown that this will always remain only a dream. It can be shown that hyperbolic geometry is just as consistent as Euclidean, Euclidean just as consistent as elliptic, or even Euclidean geometry just as consistent as ordinary arithmetic, but never will a logical proof of the consistency of the entire framework of mathematics be constructed. Whether or not contradictory theorems may someday be discovered is unanswered. Here we get a glimpse of faith. All of mathematics must, if it is used, be accepted without absolute proof.

Each "fact" in mathematics can be put in the following form: If A is true, then B is true. However, even knowing this statement to be true is not sufficient reason to assert the truth of B. The detachment principle demands that the *statement* and A both be true before B is known to be true. But an investigation of the "truth" or "falsity" of the antecedent A is outside the scope of pure mathematics. Thus the truth of B is likewise not an appropriate question but only the truth of the relationship between A and B expressed by the proposition.

Mathematics seems to relate itself to religion in yet another and completely different way from that discussed above; namely, in offering a vehicle by which the student can discipline his habits and his personality. He must develop an attitude of open-mindedness and objectivity toward alternatives that confront him and be prepared always to follow the truth wherever it may lead. A sort of self-transcendence is possible whereby one is able, at least partially, to stand apart from his problem and view it with a rather cool impartiality. There is hardly a substitute for honesty in handling results and reporting solutions of problems. Precision in definition of terms and exact use of language are constant imperatives to the logical thinking and critical analysis which form the very touchstones of mathematics. And while these attitudes are important here they are likewise to be reckoned with in other aspects of the student's life. If transfer of training is still a reality, as most modern educational psychologists will attest, perhaps the study of mathematics and the work of other disciplines will complement each other.

Too, there seems to be a relationship between these two fields in the area of creative expression. Mathematics certainly provides the student with abundant opportunity to express himself—to be creative. He is continually exhorted to formulate novel solutions to problems, to find new ways of expressing old relationships, and to construct new and interesting proofs of standard theorems. It may be that our adherence to traditional courses in the college curriculum has somewhat limited the freedom of the student in this important area. The young lady in the college algebra course for the fourth time certainly had not yet caught the spirit of modern mathematics. In fact she was there because of its lack of freedom, so she thought.

As teachers of mathematics, we have really oversold the student on the nature and boundaries of mathematics. Religion and theology suffer by comparison. To that student who complained of the great chasm between the methods available to the mathematician and those of the theologian we submit that if in the latter, terms, defined and undefined, are carefully set forth, and basic assumptions and postulates recognized, then a proof equally as valid as any in mathematics can surely be constructed. Propositions in religion could be formulated and a logical proof given for them.

In transforming the mathematics club, the Pentagon, into Kappa Mu Epsilon, the following quotation was used, "Who hath seen heights and depths shall not again know peace; not as the calm heart knows low ivied walls, a garden close, or the sweet enchantment of a rose. And though he tred the humble ways of men, he shall not speak the common tongue again."

An Introduction to the Algebra of Vectors

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1. Introduction.

There are some physical quantities, such as length, time, and mass, that may be expressed by a single real number. These quantities are called *scalar quantities*. We also have some physical quantities that have a direction specified as well as a magnitude. Examples of such quantities are velocity, electric and magnetic field intensities, and acceleration. We call these *vector quantities*.

DEFINITION: *A vector is a quantity having both magnitude and direction.*

Vector quantities may be represented by a directed line segment. A vector directed from a point A to a point B can be represented by the symbol $\overrightarrow{AB}$. The point A will be called the "tail" of the vector and the point B its "head". The symbol $\overrightarrow{BA}$ would then mean a vector of the same length as $\overrightarrow{AB}$ but directed from B to A.

DEFINITION: *Two nonzero vectors are equal if and only if they have the same magnitude and direction.*

2. Addition and Subtraction of Vectors. In the addition of vectors let us regard, momentarily at least, the vectors representing rectilinear displacements in space. If a particle be given two such displacements, one from A to B and another from B to C, the result would be the same as though the particle had been given a single displacement $\overrightarrow{AC}$ as in Figure 1.

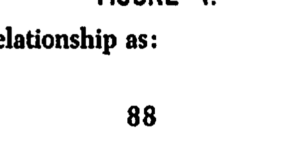


FIGURE 1.

We can express this relationship as:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}, \text{ or letting } \overrightarrow{AB} = \mathbf{U} \text{ and } \overrightarrow{BC} = \mathbf{V},$$

2.1 $$\mathbf{U} + \mathbf{V} = \overrightarrow{AC}$$

From the parallelogram in Figure 1 we note, using the definition of equal vectors, that $\overrightarrow{BC} = \overrightarrow{AD} = \mathbf{V}$, $\overrightarrow{AB} = \overrightarrow{DC} = \mathbf{U}$. By the addition of vectors,

$$\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC},$$

2.2 $$\mathbf{V} + \mathbf{U} = \overrightarrow{AC}.$$

From 2.1 and 2.2 we obtain

2.3 $$\mathbf{U} + \mathbf{V} = \mathbf{V} + \mathbf{U}$$

From 2.3 we may say that vector addition obeys the commutative law for addition.

Figure 2 represents any three displacements, **U**, **V**, **W**, of a particle.

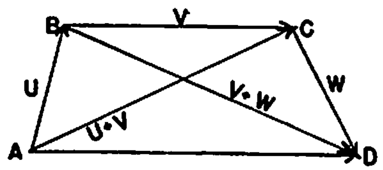


FIGURE 2.

From the figure it is easily seen that

2.4 $$\overrightarrow{AD} = (\overrightarrow{AB} + \overrightarrow{BC}) + \overrightarrow{CD} = (\mathbf{U} + \mathbf{V}) + \mathbf{W} \text{ and}$$

2.5 $$\overrightarrow{AD} = \overrightarrow{AB} + (\overrightarrow{BC} + \overrightarrow{CD}) = \mathbf{U} + (\mathbf{V} + \mathbf{W}).$$

These relationships show that vectors obey the associative law for addition.

From the fact that the commutative and associative laws hold we can conclude that: *The sum of any number of vectors is independent of the order in which taken and of their grouping for partial sums.*

DEFINITION: *A zero vector is one having zero magnitude.*

Following our notation $\overrightarrow{AA}$ or $\overrightarrow{BB}$ in Figure 2 would be a zero vector. We can then extend our original concept and write

2.6 $$\overrightarrow{AB} + \overrightarrow{BB} = \overrightarrow{AB} \text{ or } \overrightarrow{AA} + \overrightarrow{AB} = \overrightarrow{AB}$$
or $$\mathbf{U} + \mathbf{0} = \mathbf{U} \text{ and } \mathbf{0} + \mathbf{U} = \mathbf{U}$$

We thus have an additive identity, namely, the zero vector.

THEOREM: *The sum of two nonzero vectors is a zero vector if and only if they have the same length but opposite directions.*

Proof: Using the notation previously introduced

2.7 $$\overrightarrow{AB} + \overrightarrow{BA} = \overrightarrow{AA} = \mathbf{0}$$
Hence the sum of two vectors equal in length but oppositely directed is zero. Now consider two vectors whose sum is zero; i.e.,

$$\overrightarrow{AB} + \overrightarrow{BC} = \mathbf{0}. \text{ Then } \overrightarrow{AC} = \mathbf{0} = \overrightarrow{AA}.$$

Hence C coincides with A and $\overrightarrow{BC} = \overrightarrow{BA}$; thus, if the sum of two vectors is zero, the vectors must be equal in length but oppositely directed. This completes the proof of the theorem.

We agree as in the geometry of directed line segments that $\overrightarrow{BA} = -\overrightarrow{AB}$.

Then

2.8 $$\overrightarrow{AB} + (-\overrightarrow{AB}) = \overrightarrow{AB} + \overrightarrow{BA} = \overrightarrow{AA} = \mathbf{0}$$
or $$\mathbf{U} + (-\mathbf{U}) = \mathbf{0}.$$
Thus the additive inverse of $\mathbf{U}$ is $-\mathbf{U}$.

We define the difference of $\mathbf{U}$ and $\mathbf{V}$ as the vector $\mathbf{W}$ which added to $\mathbf{V}$ gives $\mathbf{U}$. That is, if $\mathbf{V} + \mathbf{W} = \mathbf{U}$ then $\mathbf{W} = \mathbf{U} - \mathbf{V}$.

THEOREM: *The difference of* $\mathbf{U}$ *and* $\mathbf{V}$ *is found by adding the negative of* $\mathbf{V}$ *to* $\mathbf{U}$, *that is,* $\mathbf{U} - \mathbf{V} = \mathbf{U} + (-\mathbf{V})$.

Proof: According to our definition of the difference of two vectors it will be sufficient to show that $\mathbf{V} + [\mathbf{U} + (-\mathbf{V})] = \mathbf{U}$.

$$\begin{aligned} \mathbf{V} + [\mathbf{U} + (-\mathbf{V})] &= \mathbf{V} + [(-\mathbf{V}) + \mathbf{U}] && \text{by the commutative law,} \\ &= [\mathbf{V} + (-\mathbf{V})] + \mathbf{U} && \text{by the associative law,} \\ &= \mathbf{0} + \mathbf{U} && \text{by (2.8),} \\ &= \mathbf{U} && \text{by (2.6),} \end{aligned}$$
and the proof is complete.

3. Scalars. If we take a as any real number and $\mathbf{U}$ as any vector we define the symbol $a\mathbf{U}$ or $\mathbf{U}a$ as a vector having the same direction as $\mathbf{U}$ but of magnitude a times that of the vector $\mathbf{U}$. In this way $0\mathbf{U}$ would be a zero vector and $(-1)\mathbf{U}$ would be a vector having the same length as $\mathbf{U}$ but oppositely directed; *i.e.*, $(-1)\mathbf{U} = -\mathbf{U}$. The real number a is called a *scalar*.

4. Components of Vectors. Let $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ denote a right-hand system of mutually perpendicular unit vectors as in Figure 3.

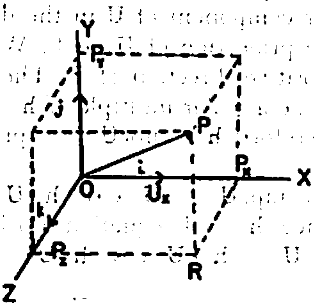


FIGURE 3.

From an origin O we can draw the coordinate axes with positive directions given by $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. Any vector $\mathbf{U}$ is then

4.1 $$\mathbf{U} = U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}$$

where if $\overrightarrow{OP}$ equals $\mathbf{U}$, U_x equals the length OP_x which is positive or negative according as OP_x has the direction $\mathbf{i}$ or $-\mathbf{i}$ and similarly for U_y and U_z.

If the length or absolute value of a vector $\mathbf{U}$ is represented by the symbol $|\mathbf{U}|$ and $(\mathbf{i}, \mathbf{U})$, $(\mathbf{j}, \mathbf{U})$, $(\mathbf{k}, \mathbf{U})$ represents the angle from the positive direction of the axes to the vector $\mathbf{U}$, then

4.2 $$U_x = |\mathbf{U}| \cos (\mathbf{i}, \mathbf{U}).$$
$$U_y = |\mathbf{U}| \cos (\mathbf{j}, \mathbf{U})$$
$$\text{and } U_z = |\mathbf{U}| \cos (\mathbf{k}, \mathbf{U}).$$

From Figure 3

4.3 $$(OP)^2 = (OR)^2 + (RP)^2 = (OP_x)^2 + (OP_y)^2 + (OP_z)^2$$
$$\text{or } |\mathbf{U}|^2 = |U_x|^2 + |U_y|^2 + |U_z|^2$$

Substituting 4.2 in 4.3 we have

$|\mathbf{U}|^2 = |\mathbf{U}|^2 [\cos^2 (\mathbf{i}, \mathbf{U}) + \cos^2 (\mathbf{j}, \mathbf{U}) + \cos^2 (\mathbf{k}, \mathbf{U})]$ and thus

4.4 $$\cos^2 (\mathbf{i}, \mathbf{U}) + \cos^2 (\mathbf{j}, \mathbf{U}) + \cos^2 (\mathbf{k}, \mathbf{U}) = 1$$

5. The Scalar or Dot Product.

DEFINITION: *The scalar or dot product of two vectors* **U** *and* **V**, *is defined as the product of their absolute values and the cosine of their included angle.*

5.1 $$\mathbf{U}\cdot\mathbf{V} = |\mathbf{U}|\,|\mathbf{V}| \cos(\mathbf{U}, \mathbf{V}).$$

From 5.1 we see that if $\mathbf{U}\cdot\mathbf{V} = 0$ and $\mathbf{U}, \mathbf{V} \neq 0$ then **U** is perpendicular to **V**; also if $|\mathbf{U}\cdot\mathbf{V}| = |\mathbf{U}|\,|\mathbf{V}|$ then **U** is parallel to **V**.

Consider an arbitrary directed line L. We define the symbol $\text{comp}_L\mathbf{U}$ as the vector component of **U** in the direction of L and the symbol $\text{proj}_L\mathbf{U}$ as the projection of **U** on L. We shall choose **h** as a unit vector in the positive direction of L. Then the projection of a vector **U** on L would be a scalar multiple of **h**.

5.2 Therefore $$\mathbf{h}(\text{comp}_L\mathbf{U}) = \text{proj}_L\mathbf{U}$$

As before

5.3 $$\text{comp}_L\mathbf{U} = |\mathbf{U}| \cos(\mathbf{h}, \mathbf{U})$$

and since $|\mathbf{h}| = 1$, equation 5.3 becomes

5.4 $$\text{comp}_L\mathbf{U} = |\mathbf{h}|\,|\mathbf{U}| \cos(\mathbf{h}, \mathbf{U}) = \mathbf{h}\cdot\mathbf{U}.$$

To find the projection of a vector $\overrightarrow{PT}$ on a line m we pass planes M and N perpendicular to m through P and T respectively. Let M and N intersect the line m in P_1 and T_1 respectively as in Figure 4.

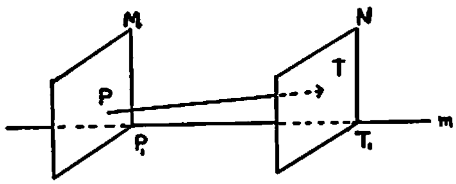


FIGURE 4.

Then $\overrightarrow{P_1T_1}$ is the projection of $\overrightarrow{PT}$ on m. Now let $\overrightarrow{PT}$ and $\overrightarrow{TR}$ be vectors with the sum $\overrightarrow{PR}$. Then the sum of their projections on m will be

5.5 $$\overrightarrow{P_1T_1} + \overrightarrow{T_1R_1} = \overrightarrow{P_1R_1}.$$

This equation implies that the projection of a vector sum on a given line may be found by adding the projections of the vectors or by projecting the sum of the vectors.

Using this idea we may write

5.6 $$\text{Proj}_L\ (\mathbf{U} + \mathbf{V}) = \text{proj}_L\mathbf{U} + \text{proj}_L\mathbf{V}$$

5.7 $$\text{comp}_L\ (\mathbf{U} + \mathbf{V}) = \text{comp}_L\mathbf{U} + \text{comp}_L\mathbf{V}.$$

From 5.1 and 5.3 we get

5.8 $$\mathbf{U}\cdot\mathbf{V} = |\mathbf{U}|\ \text{comp}_u\mathbf{V} = |\mathbf{V}|\ \text{comp}_v\mathbf{U}.$$

From 5.8 we write

5.9 $$\mathbf{W}\cdot(\mathbf{U} + \mathbf{V}) = |\mathbf{W}|\ \text{comp}_W\ (\mathbf{U} + \mathbf{V}),$$
$$\mathbf{W}\cdot\mathbf{U} = |\mathbf{W}|\ \text{comp}_W\mathbf{U},$$
and $$\mathbf{W}\cdot\mathbf{V} = |\mathbf{W}|\ \text{comp}_W\mathbf{V}.$$

But from 5.7
$$\text{comp}_W\ (\mathbf{U} + \mathbf{V}) = \text{comp}_W\mathbf{U} + \text{comp}_W\mathbf{V}.$$
and, since $|\mathbf{W}|$ is a scalar, we have

5.10 $$\mathbf{W}\cdot(\mathbf{U} + \mathbf{V}) = \mathbf{W}\cdot\mathbf{U} + \mathbf{W}\cdot\mathbf{V}.$$

Therefore a scalar product is distributive. Also from 5.8 it is seen that

5.11 $$\mathbf{U}\cdot\mathbf{V} = \mathbf{V}\cdot\mathbf{U}.$$

Hence a scalar product is commutative.

From similar triangles, as in Figure 5, it is easily seen that if a is a real number then

5.12 $$a(\mathbf{U} + \mathbf{V}) = a\mathbf{U} + a\mathbf{V}$$

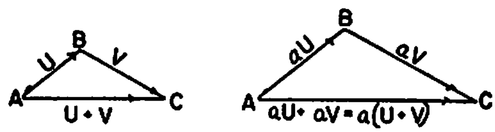


FIGURE 5.

If a is a real number, then
$$a|\mathbf{U}|\ |\mathbf{V}| \cos(\mathbf{U}, \mathbf{V}) = |a\mathbf{U}|\ |\mathbf{V}| \cos(a\mathbf{U}, \mathbf{V})$$
$$= |\mathbf{U}|\ |a\mathbf{V}| \cos(\mathbf{U}, a\mathbf{V}).$$

Since these expressions are equivalent to
$a(\mathbf{U}\cdot\mathbf{V})$, $a\mathbf{U}\cdot\mathbf{V}$, and $\mathbf{U}\cdot a\mathbf{V}$ respectively we have
$$a(\mathbf{U}\cdot\mathbf{V}) = a\mathbf{U}\cdot\mathbf{V} = \mathbf{U}\cdot a\mathbf{V}$$

If the associative law is to hold then

5.13 $$\mathbf{U}\cdot(\mathbf{V}\cdot\mathbf{W}) = (\mathbf{U}\cdot\mathbf{V})\cdot\mathbf{W}.$$

Performing the indicated operation we have

$$\mathbf{U}\cdot[|\mathbf{V}|\ |\mathbf{W}| \cos (\mathbf{V}, \mathbf{W})] = [|\mathbf{U}|\ |\mathbf{V}| \cos (\mathbf{U}, \mathbf{V})]\cdot\mathbf{W}$$

but $|\mathbf{V}|\ |\mathbf{W}| \cos (\mathbf{V}, \mathbf{W})$ and $|\mathbf{U}|\ |\mathbf{V}| \cos (\mathbf{U}, \mathbf{V})$ are scalars and the dot product of a scalar by a vector has no meaning. Therefore the associative law *does not* apply to the scalar product.

From the definition of **i**, **j**, **k** in Section 4 and Equation 5.1 it is seen that

$$\mathbf{i}\cdot\mathbf{i} = 1 \qquad \mathbf{j}\cdot\mathbf{j} = 1 \qquad \mathbf{k}\cdot\mathbf{k} = 1$$
$$\mathbf{i}\cdot\mathbf{j} = 0 \qquad \mathbf{i}\cdot\mathbf{k} = 0 \qquad \mathbf{j}\cdot\mathbf{k} = 0$$

6. Vector or Cross Product.

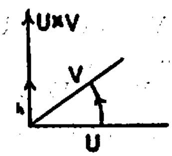


FIGURE 6.

DEFINITION: *The vector product or cross product of two vectors* **U** *and* **V** *is defined as the vector*

6.1 $$\mathbf{U} \times \mathbf{V} = |\mathbf{U}|\ |\mathbf{V}|\ \mathbf{h} \sin (\mathbf{U}, \mathbf{V})$$

where **h** *is a unit vector perpendicular to both* **U** *and* **V** *and so directed that an observer standing at the intersection of* **U** *and* **V** *and with his head in the direction of vector* **h** *will see the angle from* **U** *to* **V** *as a positive angle.* (See Figure 6)

Then, if $\mathbf{U} \times \mathbf{V} = 0$ and $\mathbf{U}, \mathbf{V} \neq 0$, **U** is parallel to **V**.

In our definition of the cross product we have placed the unit vector **h** in such a manner as to read the angle $(\mathbf{U}, \mathbf{V})$ as a positive angle. If **U** and **V** are interchanged with **h** remaining fixed then the angle $(\mathbf{U}, \mathbf{V})$ would be negative.

Hence

6.2 $$\mathbf{U} \times \mathbf{V} = |\mathbf{U}|\ |\mathbf{V}|\ \mathbf{h} \sin (\mathbf{U}, \mathbf{V})$$
$$\mathbf{V} \times \mathbf{U} = |\mathbf{V}|\ |\mathbf{U}|\ \mathbf{h} \sin [-(\mathbf{U}, \mathbf{V})]$$

But $\sin(-\theta) = -\sin \theta$. Therefore

6.3 $$\mathbf{U} \times \mathbf{V} = -(\mathbf{V} \times \mathbf{U}),$$

which shows that vector products *are not* commutative.

If vector products are associative then

$$\mathbf{U} \times (\mathbf{V} \times \mathbf{W}) = (\mathbf{U} \times \mathbf{V}) \times \mathbf{W}, \text{ or}$$

$$\mathbf{U} \times [|\mathbf{V}|\,|\mathbf{W}|\,\mathbf{h} \sin (\mathbf{V}, \mathbf{W})] = [|\mathbf{U}|\,|\mathbf{V}|\,\mathbf{h} \sin (\mathbf{U}, \mathbf{V})] \times \mathbf{W}.$$

Suppose **U**, **V**, and **W** are vectors with their "tails" at a common point. $\mathbf{V} \times \mathbf{W}$ and $\mathbf{U} \times \mathbf{V}$ are vectors themselves and are perpendicular to the planes determined by **V** and **W** and **U** and **V** respectively. Then the vector $\mathbf{U} \times (\mathbf{V} \times \mathbf{W})$ must lie in the plane of **V** and **W** and $(\mathbf{U} \times \mathbf{V}) \times \mathbf{W}$ must lie in the plane of **U** and **V**. Hence, in general, for vector products the associative law *does not* hold.

If the distributive law is to hold for a vector product then

6.4 $$\mathbf{W}\times(\mathbf{U} + \mathbf{V}) = \mathbf{W}\times\mathbf{U} + \mathbf{W}\times\mathbf{V}.$$

We will make use of the fact that the projection of any closed surface on any plane is zero if the sign of each projection is determined by the cosine of the angle between the outward normal to the surface and a fixed normal to the plane. In Figure 7, **U**, **V**, and **W** are three general vectors and *OABCDE* a triangular prism. Since the area of any triangle is equal to $1/2ab\sin(a,b)$ we can prove 6.4 in the following manner.

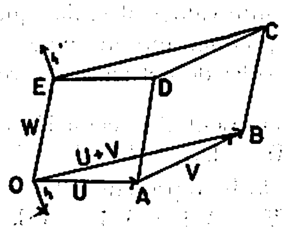


FIGURE 7.

The number of linear units in the vector $1/2(\mathbf{U} + \mathbf{V})\times\mathbf{U}$ is equal to the number of units of area of *OAB*. This vector is directed outward because to read the angle $(\mathbf{U} + \mathbf{V}, \mathbf{U})$ as positive the unit vector **h** would be directed downward and away from the prism. The number of linear units in the vector $1/2\ \mathbf{U}\times(\mathbf{U} + \mathbf{V})$ is equal to the number of units of area of *CDE* and the unit vector **h'** is directed outward. In like manner $\mathbf{U}\times\mathbf{W}$ is equal to the area of *OADE* and $\mathbf{V}\times\mathbf{W}$ is equal to the area of *ABCD* and $\mathbf{W}\times(\mathbf{U} + \mathbf{V})$ is equal to the area of *OBCE*. When the sum of the projections of these polygons on a given plane is zero, the projection of the corresponding vectors on a normal to the given plane must be zero. Hence the sum of these vectors is zero.

$$1/2(\mathbf{U} + \mathbf{V})\times\mathbf{U} + 1/2\mathbf{U}\times(\mathbf{U} + \mathbf{V}) + \mathbf{V}\times\mathbf{W} + \mathbf{W}\times(\mathbf{U} + \mathbf{V}) + \mathbf{U}\times\mathbf{W} = \mathbf{0}$$

Using 6.3 we simplify the above to

$-\mathbf{W}\times\mathbf{V} + \mathbf{W}\times(\mathbf{U} + \mathbf{V}) - \mathbf{W}\times\mathbf{U} = \mathbf{0}$ or

$\mathbf{W}\times(\mathbf{U} + \mathbf{V}) = \mathbf{W}\times\mathbf{U} + \mathbf{W}\times\mathbf{V}$ and the proof of 6.4 is complete.

7. The Cross Product in Determinant Form. From Section 4 and the definition of a vector product we have

7.1 $$\mathbf{i}\times\mathbf{i} = 0; \quad \mathbf{j}\times\mathbf{j} = 0; \quad \mathbf{k}\times\mathbf{k} = 0$$
$$\mathbf{i}\times\mathbf{j} = -\mathbf{j}\times\mathbf{i} = |\mathbf{i}|\ |\mathbf{j}|\ \mathbf{k} \sin(\mathbf{i},\mathbf{j}) = \mathbf{k} \sin \pi/2 = \mathbf{k}$$

Similarly $\mathbf{j}\times\mathbf{k} = -\mathbf{k}\times\mathbf{j} = \mathbf{i}$ and $\mathbf{k}\times\mathbf{i} = -\mathbf{i}\times\mathbf{k} = \mathbf{j}$.

It is easily shown that

$$(a\mathbf{U})\times(b\mathbf{V}) = ab(\mathbf{U}\times\mathbf{V}).$$

To show this we expand this expression and obtain

$\mathbf{h}\ |a\mathbf{U}|\ |b\mathbf{V}| \sin(a\mathbf{U},b\mathbf{V}) = \mathbf{h}\ |ab|\ |\mathbf{U}|\ |\mathbf{V}| \sin(\mathbf{U},\mathbf{V})$ since the product of the absolute values of scalar quantities is equal to the absolute value of their product and an angle is unchanged so long as its initial and terminal sides do not change directions.

In Section 4 we represented a vector $\mathbf{U}$ as

$$\mathbf{U} = U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}.$$

Any other vector $\mathbf{V}$ could be represented by

$$\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}.$$

Now $\mathbf{U}\times\mathbf{V}$, by 6.4 is

$$\mathbf{U}\times\mathbf{V} = (U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k})\times(V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k})$$

or
$$\begin{aligned}\mathbf{U}\times\mathbf{V} = {} & (U_x\mathbf{i})\times(V_x\mathbf{i}) + (U_y\mathbf{j})\times(V_x\mathbf{i}) + (U_z\mathbf{k})\times(V_x\mathbf{i}) \\ & (U_x\mathbf{i})\times(V_y\mathbf{j}) + (U_y\mathbf{j})\times(V_y\mathbf{j}) + (U_z\mathbf{k})\times(V_y\mathbf{j}) \\ & (U_x\mathbf{i})\times(V_z\mathbf{k}) + (U_y\mathbf{j})\times(V_z\mathbf{k}) + (U_z\mathbf{k})\times(V_z\mathbf{k}).\end{aligned}$$

Using the relations in 7.1 this reduces to

$$\mathbf{U}\times\mathbf{V} = (U_xV_y - U_yV_x)\mathbf{k} + (U_zV_x - U_xV_z)\mathbf{j} - (U_zV_y - U_yV_z)\mathbf{i}$$

or
$$\mathbf{U}\times\mathbf{V} = (U_yV_z - U_zV_y)\mathbf{i} - (U_xV_z - U_zV_x)\mathbf{j} + (U_xV_y - U_yV_x)\mathbf{k}$$

which is recognized as

7.2
$$\mathbf{U}\times\mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ U_x & U_y & U_z \\ V_x & V_y & V_z \end{vmatrix}$$

This determinant form of the vector product can be used to prove the following identities.

7.3 $\quad (\mathbf{U}\times\mathbf{V})\times\mathbf{W} = (\mathbf{W}\cdot\mathbf{U})\mathbf{V} - (\mathbf{W}\cdot\mathbf{V})\mathbf{U}$

7.4 $\quad \mathbf{U}\times(\mathbf{V}\times\mathbf{W}) = (\mathbf{U}\cdot\mathbf{W})\mathbf{V} - (\mathbf{U}\cdot\mathbf{V})\mathbf{W}.$

From 7.2 we may write 7.3 as

$$(\mathbf{U}\times\mathbf{V})\times\mathbf{W} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \begin{vmatrix} U_y & U_z \\ V_y & V_z \end{vmatrix} & \begin{vmatrix} U_z & U_x \\ V_z & V_x \end{vmatrix} & \begin{vmatrix} U_x & U_y \\ V_x & V_y \end{vmatrix} \\ W_x & W_y & W_z \end{vmatrix}$$

$$= [(U_zV_x - U_xV_z)W_z - (U_xV_y - U_yV_x)W_y]\mathbf{i}$$
$$+ [(U_zV_y - U_yV_z)W_z + (U_xV_y - U_yV_x)W_x]\mathbf{j}$$
$$+ [(U_yV_z - U_zV_y)W_y + (U_xV_z - U_zV_x)W_x]\mathbf{k}$$

Expanding the right-hand side of 7.3, using the results of Section 5, we have

$$(\mathbf{W}\cdot\mathbf{U})\mathbf{V} = [(W_x\mathbf{i} + W_y\mathbf{j} + W_z\mathbf{k})\cdot(U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k})]$$
$$[(V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k})]$$
$$= [W_xU_x\mathbf{i}\cdot\mathbf{i} + W_xU_y\mathbf{i}\cdot\mathbf{j} + W_xU_z\mathbf{i}\cdot\mathbf{k} + W_yU_x\mathbf{i}\cdot\mathbf{j}$$
$$+ W_yU_y\mathbf{j}\cdot\mathbf{j} + W_yU_z\mathbf{j}\cdot\mathbf{k}$$
$$+ W_zU_x\mathbf{i}\cdot\mathbf{k} + W_zU_y\mathbf{k}\cdot\mathbf{j} + W_zU_z\mathbf{k}\cdot\mathbf{k}]$$
$$[V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}]$$
$$= [W_xU_x + W_yU_y + W_zU_z][V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}].$$

Similarly, $(\mathbf{W}\cdot\mathbf{V})\mathbf{U} = [W_xV_x + W_yV_y + W_zV_z]$
$$[U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}].$$

Then $\quad (\mathbf{W}\cdot\mathbf{U})\mathbf{V} - (\mathbf{W}\cdot\mathbf{V})\mathbf{U} = [-(U_xV_z - U_zV_x)W_z$
$$- (U_xV_y - U_yV_x)W_y]\mathbf{i}$$
$$+ [-(U_yV_z - U_zV_y)W_z + (U_xV_y$$
$$- U_yV_x)W_x]\mathbf{j}$$
$$+ [(U_yV_z - U_zV_y)W_y + (U_xV_z$$
$$- U_zV_x)W_x]\mathbf{k}$$

which is identically equal to $(\mathbf{U}\times\mathbf{V})\times\mathbf{W}$. This proves 7.3. A similar proof holds for 7.4.

8. Vector Identities. Two more identities that are of interest are

8.1 $\quad a(\mathbf{U}\times\mathbf{V}) = a\mathbf{U}\times\mathbf{V} = \mathbf{U}\times a\mathbf{V}$ and

8.2 $\quad \mathbf{U}\cdot(\mathbf{V}\times\mathbf{W}) = \mathbf{V}\cdot(\mathbf{W}\times\mathbf{U}) = \mathbf{W}\cdot(\mathbf{U}\times\mathbf{V}).$

Equation 8.1 is easily proved if we remember that multiplying a vector by a positive scalar does not change its direction, but multiplying by a negative scalar reverses its direction.
From 6.1

$$a(\mathbf{U}\times\mathbf{V}) = (a|\mathbf{U}|\ |\mathbf{V}|)h \sin(\mathbf{U},\mathbf{V}),$$
$$a\mathbf{U}\times\mathbf{V} = (|a\mathbf{U}|\ |\mathbf{V}|)\mathbf{h} \sin(a\mathbf{U},\mathbf{V}) \text{ and}$$
$$\mathbf{U}\times a\mathbf{V} = (|\mathbf{U}|\ |a\mathbf{V}|)\mathbf{h} \sin(\mathbf{U},a\mathbf{V}).$$

The reader should verify that the right-hand members of these three equations are equal when a is any real number. It follows immediately that 8.1 is true.

To prove 8.2 let us consider Figure 8.

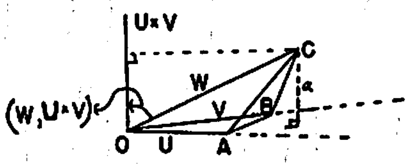


FIGURE 8.

From the figure and 5.1

$$\mathbf{W}\cdot(\mathbf{U}\times\mathbf{V}) = |\mathbf{W}|\ |\mathbf{U}\times\mathbf{V}| \cos(\mathbf{W},\mathbf{U}\times\mathbf{V}),$$

but $\mathbf{U}\times\mathbf{V}$ is perpendicular to the plane determined by $\mathbf{U}$ and $\mathbf{V}$. Also $|\mathbf{W}| \cos(\mathbf{W},\mathbf{U}\times\mathbf{V})$ equals the altitude a of the pyramid determined by the vectors $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{W}$. But $(\mathbf{U}\times\mathbf{V})$ is twice the area of OAB. Therefore, since the volume of a pyramid is one-third the area of a base times the altitude to that base, $\mathbf{W}\cdot(\mathbf{U}\times\mathbf{V})$ is six times the volume of the pyramid $OABC$.

In like manner $\mathbf{V}\cdot(\mathbf{W}\times\mathbf{U})$ and $\mathbf{U}\cdot(\mathbf{V}\times\mathbf{W})$ are each equal to six times the volume of the pyramid $OABC$. Hence they are equal to each other.

"I have known no one more devoted to mathematics and the students of mathematics than Kathryn Wyant."

—L. P. Woods

On the Improvement of Mathematics and Science Teaching, 1956

JOHN R. MAYOR

Director, Science Teaching Improvement Program, AAAS

The action program of the American Association for the Advancement of Science (AAAS) is planned to contribute to the improvement of science and mathematics teaching in the secondary schools and to increase the supply of college graduates qualified to teach science and mathematics. The program, known as the Science Teaching Improvement Program (STIP), was reported in detail in the July 22, 1955, number of SCIENCE and the September, 1955, issue of THE SCIENTIFIC MONTHLY.

Statistics on the teacher shortage in general, and in science and mathematics in particular, have been given wide publicity. There is no evidence at present of a rise in enrollments in colleges and universities of those preparing to teach science and mathematics. Increased attention by radio, television, and newspapers to the problem of the shortage of scientists and engineers, and especially of teachers, has been heartening. It is hoped that more specific programs through national communication media can be directed to the desirability of the study of science and mathematics and the need for teachers in these areas.

STIP is addressed first to scientists. Much of the activity in the program to date has been in conferences with scientists on college and university campuses and in state and regional groups. Similar conferences are planned for 1956 in all parts of the country. Discussions will be devoted to the consideration of ways and means by which scientists and science departments can better accept their responsibility for teacher education, develop closer working relationships with secondary-school teachers, and acquire more intimate knowledge of secondary-school science and mathematics programs and teaching problems. A report on STIP was given an important place at meetings of Academies of Science in Oklahoma, Texas, Alabama, North Carolina, South Carolina, West Virginia, Virginia, Pennsylvania, Ohio, and Iowa. Regional conferences of scientists have been held in Ann Arbor, Michigan; Chicago, Illinois; Lawrence, Kansas, and plans are being developed for such meetings in the far west for May and June. It is generally recognized that scientists have a responsibility in teacher education and the development of school programs in science and mathematics which they have not adequately met in the past 25 years.

Cooperation in the project of the National Academy of Sciences-National Research Council in Arlington County has provided good opportunities for direct work with scientists. This project promises to become an unusually fine example of what can be accomplished in the improvement of science teaching at the local level when scientists, community groups, PTA's, school administrators and teachers, and college and university staffs unite to carry out a carefully developed program.

A part of the activities of STIP is in cooperation with national professional education organizations and their state branches. Such groups as the National Education Association, National Science Teachers Association, National Association of Secondary-School Principals, American Association of Colleges for Teacher Education, Cooperative Council on Teacher Education, and Association for Supervision and Curriculum Development have carried announcements on STIP in their publications and invited representatives of STIP to appear on the programs of their national meetings. It is also hoped that representatives of STIP and AAAS will be asked to serve with committees of these groups in their study of problems of common concern. In December, 1955, a Conference on Science Teacher Education was held in Washington under the sponsorship of AAAS, National Science Foundation, and the U. S. Office of Education. This Conference provided an excellent opportunity for scientists and educators to discuss problems of importance to both groups which too infrequently in the past have been attacked jointly by vitally interested groups on a national basis or on university campuses. It is now proposed that a special cooperative project of considerable magnitude be developed by AAAS and AACTE.

Because scientists have been disturbed about teacher-certification requirements, a plan to obtain representation from science on teacher-certification commissions and councils in all states is being discussed with the National Association of State Directors of Teacher Education and Certification. Assistance of state academies of science at the local level is in process of organization. The Cooperative Committee on the Teaching of Science and Mathematics (AAAS) will be asked to prepare recommendations for teacher-certification requirements in science to be referred to the various state agencies.

STIP will sponsor a pilot study on the use of science and mathematics counselors in four centers during the school year 1956-57. The University of Nebraska, University of Oregon, Pennsylvania State University, and University of Texas have been designated as centers. The study purposes to develop a method for "up-grading" the science instruction of teachers, many of whom are relatively inexperienced and may be lacking in several aspects of desirable preparation. Many teachers of science would profit from improved laboratory and demonstration techniques and greater knowledge of subject matter with stress on most recent developments in science. It is hoped that this pilot study will be completed before the scarcity of science teachers reaches its peak and that the results will point to a method for alleviating the situation which may merit support by public funds. It is believed that a science counselor with the proper breadth and depth of training in the sciences and mathematics, with a background of outstanding accomplishments in teaching, a natural ability to work with people, and practical knowledge of the learning process, can stimulate and direct the work of a small group of less-well prepared and less-experienced teachers to the end that the outcome of instruction will be on a level which is far higher than could be attained without the assistance of the counselor.

Concern with the adjustment of teachers salaries at the secondary-school level, as well as for the improvement of the working conditions of teachers, arose from the realization that inadequate salaries, together with heavy workloads and unfavorable working conditions, are important factors which have caused teachers to leave the teaching profession and others to be reluctant to enter it. The plan of STIP is to work with appropriate national agencies in their efforts to improve the salary situation. Experimental studies of particular aspects of the problem of working conditions will be started soon.

A special committee has been established to review the entire question of interesting secondary-school students in science-teaching careers. When funds become available, a plan for awards for outstanding achievement in teaching secondary-school science and mathematics will be inaugurated. These awards will be given by AAAS in national competition.

The development of correspondence courses in science and mathematics for in-service teachers is under consideration. In this connection the advice of the extension divisions in leading universities will be sought.

In a meeting of scientists, concern is always expressed regarding the teaching of science and mathematics in elementary schools. Although STIP was originally planned for operation at the secondary-school level, there appears to be an obligation to consider possibilities for contribution to the improvement of science teaching in the elementary schools.

MATH FRATERNITY INITIATES PLEDGES AT FOUNDER'S FEAST

Local Mathematics Fraternity Undergoes Transformation to National Society

Kappa Mu Epsilon had its Founders' Day banquet last Saturday evening, April 18, at the Hotel Thompson. The banquet room was decorated in pink and white. Wild flowers and redbuds were in the corners of the room. Tall white candles and floor lamps gave a soft light. Roast nut cups and hand-painted place cards added to the color of the room. The menu, written inside the place cards, told of paraboloids, ruled surfaces, and even parallel lines that were to be eaten at the mathematical table.

Paul Lewis was the Radical Axis (toastmaster) of the evening. The program consisted of the fraternity song and the following talks: Parabolas (parables) by "Bus" Layton; Comic (conic) Sections by Dr. Kathryn Wyant; Lipstick (elliptic) Conditions by Clara Green; and Transformations by Dean L. P. Woods.

The formal transformation from THE PENTAGON into Kappa Mu Epsilon was directed by Mr. Woods.

—*Muskogee Phoenix*
April 20, 1931

A Note on Tetrahedrons

HARVEY FIALA

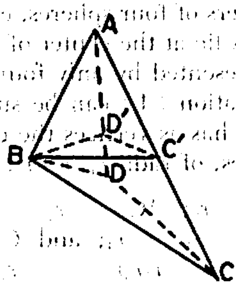
Student, North Dakota Agricultural College

What is the altitude of a tetrahedron? Also, what is its volume, the angle between two of its faces, or the radius of its inscribed sphere? There are many important problems concerning tetrahedrons and spheres where an answer to one of these questions leads to the answers of the others.

It is the purpose of this paper to derive several of the properties of a tetrahedron in terms of its sides.

1. Volume

Given: Tetrahedron $A-BCD$. Let h be the altitude of the tetrahedron; V its volume; A_a the area of $\triangle BCD$, (the face opposite vertex A); A_b the area of $\triangle ACD$; A_c the area of $\triangle ABD$; and A_d the area of $\triangle ABC$.



Form a second tetrahedron A-$BC'D'$ by choosing points C' and D' on the edges AC and AD, respectively, so that $AC' = AD' = AB$. Let O be the circumcenter of $\triangle BC'D'$ and BO the circumradius. Let $V' =$ volume A-$BC'D'$, h' its altitude, and A_a' the area of its base, $\triangle BC'D'$. Then $V/V' = [(AB)(AC)(AD)]/[(AB)(AC')(AD')] = [(AC)(AD)]/(AB)^2$, but $V' = [h'(\text{Area } \triangle BC'D')]/3 = (h'A_a')/3$. Therefore

$$V = [(AC)(AD)h'A_a']/[3(AB)^2] \text{ but}$$

$$h' = [(AB)^2 - (BO)^2]^{1/2}, \text{ and}$$

$$BO = [(BC')(BD')(C'D')]/4A_a', \text{ so}$$

$$h' = [16(A_a')^2(AB)^2 - (BC')^2(BD')^2(C'D')^2]^{1/2}/(4A_a').$$

Thus,

(1) $$V = (AC)(AD)[16(A_a')^2(AB)^2 - (BC')^2(BD')^2(C'D')^2]^{1/2}/[12(AB)^2].$$ But

$$(BC')^2 = (AB)^2 + (AB)^2 - 2(AB)^2\cos\angle BAC,$$ where $\cos\angle BAC = [(AB)^2 + (AC)^2 - (BC)^2]/2(AB)(AC)$ by the cosine law, so

$$(BC')^2 = 2(AB)^2(1\text{-}\cos\angle BAC)$$
$$= (AB)^2[(BC)^2 - (AB) - AC)^2]/(AB)(AC).$$

Similarly,

$$(BD')^2 = (AB)^2[(BD)^2 - (AB - AD)^2]/(AB)(AD),$$ and
$$(CD')^2 = (AB)^2[(CD)^2 - (AC - AD)^2]/(AC)(AD).$$

A_a', the area of $\triangle BC'D'$, is expressed by the relationship
$16(A_a')^2 = 4(BC')^2(C'D')^2 - [(BC')^2 + (C'D')^2 - (BD')^2]^2$
derived from Hero's formula. Now the values of A_a', AB, BC', BD', and $C'D'$ can be substituted in Equation (1) to find the volume.

Consider the special case of a tetrahedron whose sides are the lines connecting the centers of four spheres, each tangent to the other three, and whose vertices lie at the center of the four spheres. These tetrahedrons can be represented by only four elements (four radii), instead of six sides. Equation (1) can be simplified greatly. Under these conditions, *A-BCD* has as vertices the centers of four mutually externally tangent spheres, of radii r_1, r_2, r_3, and r_4.

Then $AB = r_1 + r_2$; $AC = r_1 + r_3$; $AD = r_1 + r_4$; $BC = r_2 + r_3$; $BD = r_2 + r_4$; and $CD = r_3 + r_4$. Let $g = (r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4)$; $j = r_1r_2r_3r_4$;
$$k = (r_1r_2 + r_1r_3 + r_1r_4 + r_2r_3 + r_2r_4 + r_3r_4);$$
$$n = r_1^2r_2^2r_3^2 + r_1^2r_2^2r_4^2 + r_1^2r_3^2r_4^2 + r_2^2r_3^2r_4^2).$$ Then
$$g^2 = n + 2jk.$$

Substituting these values in the expressions for A_a', AB, BC', BD', and $C'D'$ Equation (1) then becomes

$$V = [2r_1r_2r_3r_4(r_1r_2 + r_1r_3 + r_1r_4 + r_2r_3 + r_2r_4 + r_3r_4) - (r_1^2r_2^2r_3^2 + r_1^2r_2^2r_4^2 + r_1^2r_3^2r_4^2 + r_2^2r_3^2r_4^2)]^{1/2}/3$$ or
$$V = [2jk - n]^{1/2}/3 = [g^2 - 2n]^{1/2}/3.$$

2. Dihedral angle and circumradius.

Let the dihedral angle between two faces, $\triangle DAB$ and $\triangle CAB$, of a tetrahedron be represented by the symbol $\angle D\text{-}AB\text{-}C$ where AB is the edge common to the two faces. Then $\sin\angle D\text{-}AB\text{-}C = h_o/h_d$

where h_o is the altitude of the tetrahedron, drawn to the base $\triangle ABC$, and h_d is the altitude of triangle $\triangle ABD$ drawn to the side AB from the vertex D. The area of $\triangle ABD$ equals $h_d(AB)/2$ so $h_d = 2A_c/(AB)$, and $h_o = 3V/A_d$, so $\sin\angle D\text{-}AB\text{-}C = h_o/h_d$

$$= 3(AB)V/(2A_dA_c) \text{ or } \angle D\text{-}AB\text{-}C = \sin^{-1}(h_o/h_d)$$
$$= \sin^{-1}[3(AB)V/(2A_dA_c)].$$

To find the radius R of an inscribed sphere, divide $ABCD$ into four tetrahedrons whose bases are the triangular sides and whose common vertex is at the center of the inscribed sphere; then the altitudes of each of these four tetrahedrons equals the radius of the inscribed sphere. Thus,

$$V = RA_a/3 + RA_b/3 + RA_c/3 + RA_d/3$$
$$= R(A_a + A_b + A_c + A_d)/3 = RS/3,$$

where S is the total area of the surface of the tetrahedron.

$$R = 3V/S = h_oA_d/S.$$

In the case of the special tetrahedrons whose vertices are the centers of four mutually externally tangent spheres,

$$\angle D\text{-}AB\text{-}C = \sin^{-1}[(AB)(g^2 - 2n)^{1/2}/(2A_dA_c)] \text{ and}$$
$$R = (g^2 - 2n)^{1/2}/S.$$

The next National Convention of KME, Spring, 1957, will be held at Kansas State Teachers College, Pittsburg. Full details will appear in the Fall, 1956, issue of THE PENTAGON.

Problem Corner

EDITED BY FRANK C. GENTRY

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond the calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before October 1, 1956. The best solutions submitted by students will be published in the Fall, 1956, issue of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Frank C. Gentry, Department of Mathematics and Astronomy, University of New Mexico, Albuquerque, New Mexico.

PROBLEMS PROPOSED

88. *Proposed by Charles Pearsall, Student, Hofstra College, Hempstead, New York.*

Suppose that with each purchase of a box of "Sogg" the buyer receives a white coupon. The coupon collection plan allows 1 blue coupon for 10 white coupons, 1 red coupon for 10 blue coupons and 1 gold coupon for 10 red coupons. An avid collector, Mrs. A, used 233 boxes. Each of 11 of her friends used a greater number of boxes than Mrs. A but the same number as the other 10. Mrs. A received all the coupons and, examining her accumulation, found that of a total of 19 coupons there was a different number of each kind and the number of white and red exceeded the number of blue and gold. Find the number of boxes of "Sogg" used by all twelve women.

89. *Proposed by C. W. Trigg, Los Angeles City College, Los Angeles, California.*

If n is even and greater than 2, then $2^n - 2$ can never be the product of two consecutive integers.

90. *Proposed by Frank Hawthorne, Hofstra College, Hempstead, New York.*

A merchant buys an odd number of felt hats at \$10 each and one cloth hat for a whole number of dollars less than \$10. How much does the cloth hat cost if the total amount of money involved is a perfect square?

91. *Proposed by Leo Moser, University of Alberta, Edmonton, Alberta, Canada.*

In a cartesian coordinate system, find 6 points with integral coordinates which form the vertices of a regular hexagon.

92. *Proposed by Martin Winterfield, Hofstra College, Hempstead, New York.*

Find the sum of the 9! numbers obtained by permuting the integers from 1 to 9 all at a time in all possible ways.

93. *Proposed by R. Wayne Stark, Iowa Alpha Alumnus, El Paso, Texas.*

An alley is flanked on either side by buildings A and B. A twenty foot ladder extends across the alley from the foot of A to the top of B. A thirty foot ladder extends from the foot of B to the top of A. The point of intersection of the two ladders is 10 feet above the floor of the alley. How high are buildings A and B. (Note by the Editor. This is almost the same as Problem 32, Fall, 1950. It is published because it asks for the heights of the buildings rather than the width of the alley.)

94. *Proposed by L. T. Shiflett, Southwest Missouri State College, Springfield, Missouri.* (From Struik, *Analytic and Projective Geometry*, 1953.)

Show that n lines, no two parallel and no three concurrent, divide the plane into $1 + n + n(n - 1)/2$ regions.

95. *Proposed by the Editor.* (From Ray's *Algebra*, 1852.)

A traveler sets out from a certain place and travels 1 mile the first day, 2 miles the second day, 3 miles the third day, etc. In five days afterward another sets out and travels 12 miles a day. How long and how far must he travel to overtake the first?

96. *Proposed by Rex Depew, State Teachers College, Florence, Alabama.*

Snowfall begins and at 10:00 a.m. a snow plow begins operation. If, under maximum power, the plow moves 2 miles the first hour and 1 mile the second hour and if the speed of the plow is inversely proportional to the depth of the snow, at what time did the snowfall begin? Assume that the rate of snowfall is constant.

SOLUTIONS

81. *Proposed by the Editor.* (From the First William Lowell Putnam Mathematical Competition, April, 1938).

A can buoy is to be made of three pieces; namely, a cylinder and two equal cones, the altitude of each cone being equal to the altitude of the cylinder. For a given area of surface, what shape will have the greatest volume?

Solution by B. W. Blair, University of New Mexico, Albuquerque, New Mexico.

The volume $V = 5\pi r^2 h/3$ is to be a maximum. Hence $D_r V = 5\pi r(2h + r\,D_r h)/3 = 0$ or $D_r h = -2h/r$. Also total area $A = 2\pi r[h + (h^2 + r^2)^{1/2}]$ is to be constant. Hence

$$D_r A = 2\pi[h + r\,D_r h + (r^2 + h^2)^{1/2} + r^2(r^2 + h^2)^{-1/2} + h\,D_r h\,(r^2 + h^2)^{-1/2}] = 0, \text{ or}$$

$D_r h = [h(h^2 + r^2)^{1/2} + h^2 + 2r^2]/[r(h^2 + r^2)^{1/2} + rh]$. Equating the two expressions for $D_r h$ and simplifying leads to $2r = h\sqrt{5}$.

Also solved by Sam H. Sesskin, Hofstra College.

82. *Proposed by Sam H. Sesskin, Student, Hofstra College, Hempstead, New York.*

A box in the shape of a cube 4 feet on an edge is placed with one face against a wall. A ladder 12 feet long is leaned over the box and against the wall so as to touch one edge of the box. How high up the wall does the ladder reach, and how far from the wall is its foot?

Solution by Patsy Renfro, Hays State College, Hays, Kansas.

Let $x + 4$ be the distance the ladder reaches up the wall and $y + 4$ be the distance from the wall to its foot. From similar triangles $x/4 = 4/y$ or $x = 16/y$. From the Pythagorean Theorem, $(x + 4)^2 + (y + 4)^2 = 144$. Substituting for x and simplifying this becomes $y^4 + 8y^3 - 112\,y^2 + 128y + 256 = 0$. Approximate values of the positive roots are 2.68 ft. and 5.97 ft. Hence the ladder reaches 6.68 ft. up the wall when its foot is 9.97 ft. from the wall or vice versa.

Also solved by Gib Leiter, William Jewell College and Marie Smith, University of New Mexico.

83. *Proposed by Marie Smith, Student, University of New Mexico, Albuquerque, New Mexico.*

Nine men: Brown, White, Adams, Miller, Green, Hunter, Knight, Jones, and Smith play the several positions on a baseball team. Determine from the following data the position played by each: 1) Smith and Brown each won $10 playing poker with the pitcher; 2) Hunter is taller than Knight and shorter than White, but each of these weighs more than the first baseman; 3) the third baseman lives across the corridor from Jones in the same apartment house; 4) Miller and the outfielders play bridge in their spare time; 5) White, Miller, Brown, the right fielder, and the center fielder are bachelors and the rest are married; 6) of Adams and Knight one plays an outfield position; 7) the right fielder is shorter than the center fielder; 8) the third baseman is brother to the pitcher's wife; 9) Green is taller than the infielders and the battery, except for Jones, Smith, and Adams; 10) the second baseman beat Jones, Brown, Hunter, and the catcher at cards; 11) the third baseman, the shortstop, and Hunter made $150 each speculating in U. S. Steel; 12) the second baseman is engaged to Miller's sister; 13) Adams lives in the same house as his own sister but dislikes the catcher; 14) Adams, Brown, and the shortstop lost $200 each speculating in copper; 15) the catcher has three daughters, the third baseman has two sons, but Green is being sued for divorce. (From Bennett and Baylis, *Formal Logic.*)

Solution by Sidney Peacocke, Central College, Fayette, Missouri.

Green must play the outfield (9) and he is married (15) but both the right and center fielders are bachelors (5). *Green plays left field.* The third baseman and catcher are married (15); the pitcher is married (8). White, Miller, and Brown must play first base, second base, and shortstop (5). White or brown must play second base (12). *White is the second baseman* (10). *Miller is the shortstop* (14). *Brown is first baseman.* Jones, Smith, and Adams must catch, pitch, and play third base (9). Jones or Smith must be the catcher (12). *Smith is the catcher* (10). *Adams is the third baseman* (3). *Jones is the pitcher.* Knight and Hunter must play right field and center field since they are the only two men and positions left. *Hunter is the center fielder* (7) and *Knight is the right fielder.*

Also solved by Shirley T. Leeven, Central Missouri State College and the Proposer.

The Mathematical Scrapbook

EDITED BY J. M. SACHS

Mathematics is, in many ways, the most precious response that the human spirit has made to the call of the infinite and eternal.

—C. J. KEYSER

=△=

The science of Pure Mathematics, in its modern developments, may claim to be the most original creation of the human spirit.

—A. N. WHITEHEAD

=△=

A great part of mathematics consists of attempts to replace difficult problems by easier problems having the same answers.

=△=

"What is the time, Professor Rackbrane?" asked a friend. The professor answered, "If you add one quarter of the time from noon till now to one half the time from now to noon tomorrow, you will get the time exactly." What was the time of day when the professor spoke?

—WILSON JUNIOR COLLEGE
Mathematics Tournament

=△=

Pure mathematics consists of such assertations as that, if such and such a proposition is true of that thing then such and such proposition is true of that thing. It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is of which it is supposed to be true. • • • If our hypothesis is about anything and not some one or more particular things, then our deductions constitute mathematics. Thus mathematics may be defined as the subject in which we never know what we are talking about nor whether what we say is true.

—B. RUSSELL

=△=

A mathematician wishing to write a paper needs to prove a particular point. He works at it continuously for 48 hours and finally proves it. In his paper he casually refers to this point with the remark, "It is easily seen that • • •."

Another mathematician wishing to write a paper needs to prove a particular point. He also works without pause for 48 hours but is unable to prove it. He wishes to write the paper anyway so he casually refers to this point with the remark, "It is obvious that • • •."

$=\Delta=$

Moreover, distrust of speculation often serves as a cover for loose thinking.

—A. S. EDDINGTON

$=\Delta=$

The effort of the economist is to see, to picture the inter-play of economic elements. The more clearly cut these elements appear in his vision, the better; the more elements he can grasp and hold in his mind at once, the better. The economic world is a misty region. The first explorers used unaided vision. Mathematics is the lantern by which what before was dimly visible now looms up in firm, bold outlines. The old phantasmagoria disappear. We see better. We also see further.

—I. FISHER (1892)

$=\Delta=$

$$\rho = 1 - \cos\theta$$

A musing mathematician sat
In an analytical daze
And watched a topological hat
Through a geometric haze.

The hat was perched on an angular lass
Who jangled his central nerves.
The girl was a calculated mass
Of osculatable curves.

He rose and he said, "My dear, I deduce
That your topological hat
Has set my cardiod running loose.
Could we have a logical chat?"

"Do you think that we could coordinate
Our differentiable parts?"
Said she, "If you are my lord and mate,
We two will co-sign our hearts.

"I'll want a small rectangular hut,
And three acute children I'll need.
And it's evident you as their daddy can cut
A beautiful figure, indeed."

The man and the maid have used their math.
Now their isomorphic souls
Are integrated along a path
With infinitesimal shoals.

—NORMAN GOLDSMITH
Chicago Teachers College

$=\Delta=$

Before you enter on the study of law a sufficient ground work must be laid • • •. Mathematics and natural philosophy are so useful in the most familiar occurrences of life and are so engaging and delightful as would induce everyone to wish an acquaintance with them. Besides this, the faculties of the mind, like the members of a body, are strengthened and improved by exercise. Mathematical reasoning and deductions are, therefore, a fine preparation for investigating the abstruse speculations of the law.

—T. JEFFERSON

$=\Delta=$

In Mathematicks he was greater
Than Tycho Brahe, or Erra Pater;
For he, by Geometrick scale,
Could take the size of Pots of Ale;
Resolve by Signs and Tangents streight,
If Bread or Butter wanted weight;
And wisely tell what hour o' th' day
The Clock doth strike, by Algebra.

—S. BUTLER

$=\Delta=$

The following were suggested by Joseph J. Urbancek.

A man without eyes
Saw plums on a tree.
He neither took plums nor left plums.
So, how can it be?
(A man with *one* eye saw two plums and took *one*.)

A man left three quarters of a section of land to his four sons. The land was composed of three square quarter-sections arranged in the form of the letter "L". How could the land be divided into four congruent pieces similar to the original piece? (Are there any additional solutions to this problem if the condition of similarity to the original is removed?)

=△=

In speaking of arithmetic (algebra, analysis) as a part of logic I mean to imply that I consider the number concept entirely independent of the notions or intuitions of space and time, that I consider it an immediate result from the laws of thought • • •. Numbers are the free creations of the human mind; they serve as a means of apprehending more easily and more sharply the difference of things. It is only through the purely logical processes of building up the science of numbers and by thus acquiring the continuous number-domain that we are prepared accurately to investigate our notions of space and time by bringing them into relation with this number-domain created in our mind. If we scrutinize closely what is done in counting an aggregate or number of things, we are led to consider the ability of the mind to relate things to things, to let a thing correspond to a thing, or to represent a thing by a thing, an ability without which no thinking is possible. Upon this unique and therefore indispensable foundation must, in my judgment, the whole science of numbers be established.

—R. Dedekind

=△=

Philo Vance solved a murder case recently in which the murderer was making social use of a simple, but little known, fact in arithmetic. Suppose we try it. Write down any three-place number. Reverse it. Subtract one of your numbers from the other. Give me the first digit, and I'll tell you the rest. (The middle digit is 9. The other two digits add up to 9.)

Here is another from the same source. Think of a card below 10. Double the number and add 1. Multiply by 5. Add in the number of the suit, 1 for spades, 2 for hearts, 3 for diamonds, four for clubs. Tell me the answer and I'll name the card. (The tens place has the number of the card. The units place is 5 greater than the identifying number for the suit.)

=△=

There are only two ways open to man for attaining a certain knowledge of truth: clear intuition and necessary deduction.

—R. DESCARTES

=△=

A man who could give a convincing account of mathematical reality would have solved very many of the most difficult problems of metaphysics. If he could include physical reality in his account, he would have solved them all.

—C. H. HARDY

=△=

Three cubes have edges which are consecutive integral numbers of inches. The largest cube is placed on a table. The second cube is placed on top of the largest cube. The smallest cube is placed on top of the second cube. Find the lengths of the edges of the cube if the total exposed area is 1064 square inches.

—WILSON JUNIOR COLLEGE
Mathematics Tournament

=△=

A quadratic function ambitious,
Said it's not only wrong but it's vicious,
It's surely no sin
To have both max and min,
To limit me so is malicious.

With this rather sad beginning the editor of the Scrap Book asks the readers of THE PENTAGON to send him mathematical limericks as well as other material.

=△=

Dr. Wyant often told her classes that mathematics was like a rose. You must get close to it to see its real beauty.

The Book Shelf

EDITED BY REX D. DEPEW

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Rex D. Depew, State Teachers College, Florence, Alabama.

How to Lie With Statistics, Darrell Huff, W. W. Norton and Company, Inc. (55 Fifth Avenue) New York, 1954, 142 pp., $2.25.

Figures don't lie, but liars will figure. This is the theme which runs throughout the book. In his introduction, the author intimates that the book is "a sort of primer in ways to use statistics to deceive." But the reader quickly and happily discovers that the author is not really a villain and that he has not written a manual or handbook for swindlers but has, instead, prepared a delightful guide for the unwary as a guard against statistical fallacies and chicanery.

Examples of statistical distortions and atrocities are drawn from such sometimes reliable sources as *Fortune Magazine, The New Yorker, Collier's, New York Times, Newsweek,* and many others, and packages of Grape-Nuts Flakes. The author cites many cases of "rigged" or tricky statistics, including use of the sample with built-in bias and the use of samples of inadequate size. The heavily biased sample is illustrated by the never-to-be-forgotten *Literary Digest* poll, and the inadequate sample by the advertising displays of Dr. Cornish's Tooth Powder, the impressive superiority of which was "established" by dubious testing of no more than six selected subjects.

The content of some of the chapters is suggested by the chapter titles: *The Well-Chosen Average, Much Ado About Practically Nothing, Post Hoc Rides Again, How to Statisticulate,* and *How to Talk Back to a Statistic.* Artemus Ward once declared, "It ain't so much the things we don't know that get us in trouble; it's the things we know that ain't so."

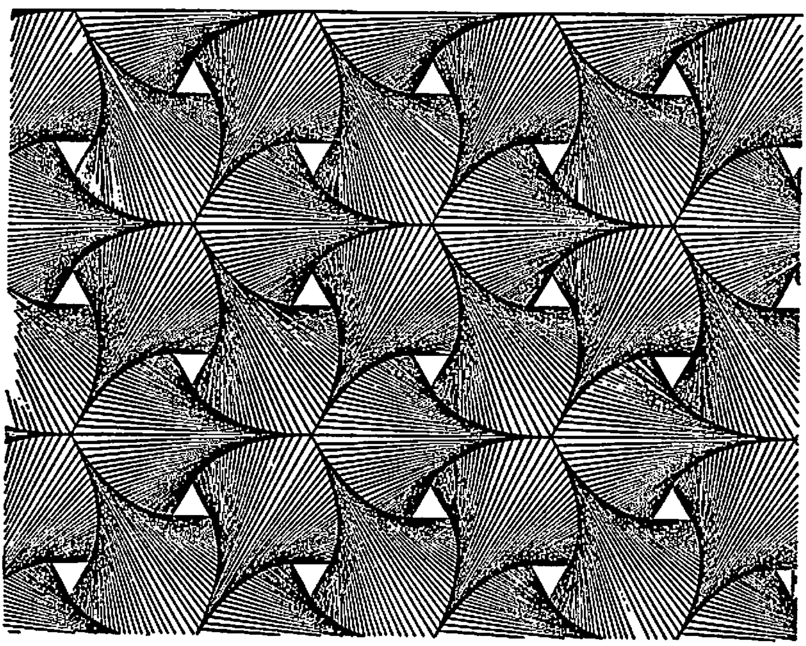
The book is refreshingly written, and the illustrations by Irving

Geis are delightful. The reader will be continually intrigued by the author's examples of tricky statistics, and he will find himself adding examples drawn from his own experience and observations—examples of statistical nonsense and intentional or unintentional statistical deception.

—OSCAR J. PETERSON
Kansas State Teachers College

Illustrated Mathematical Postal Cards, Pictorial Mathematics, c/o Scripta Mathematica (186th Street and Amsterdam Avenue) New York 33, New York: Series A (10 cards)—Portraits of Great Mathematicians, $0.50; Series B (32 cards)—Beautiful Geometrical Forms and Designs, $1.60.

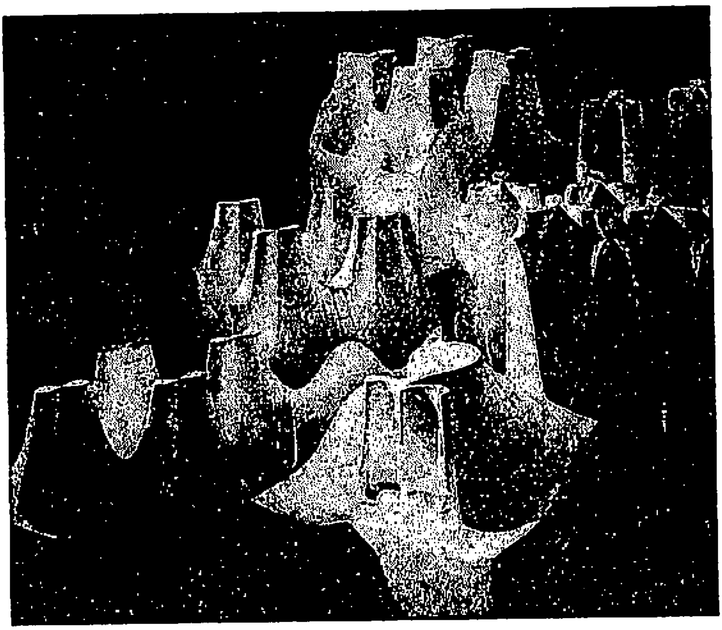
Mathematical Themes in Design
By Rutherford Boyd



A figure built exclusively of triangles and hexagons, but completely dominated by the spade-shaped curvilinear figures.

This set of beautifully reproduced postal cards provides an excellent source of bulletin board materials or the basis for a most interesting mathematical exhibit. The series of designs should stimulate interested students to attempt creative applications of mathematical themes in art.

Series A includes portraits of Plato and Aristotle, Pythagoras, Descartes, Newton, Pascal, Lagrange, Fermat, Euler, Kepler, and Galileo. Series B presents photographic reproductions of such interesting subjects as abstract electronic designs, mathematical models, and the splash of a falling drop of milk; this series also includes many line drawings of dynamic geometric forms, mathematical themes in design, and a great variety of beautiful mathematical curves.



MATHEMATICAL MODELS
ARRANGED BY RUTHERFORD BOYD.

The reviewer feels that the postal cards may be useful in many ways to students and teachers of mathematics. The two cuts shown

(Series B, Nos. 4 and 26, respectively) are full sized, although the cards are slightly larger (3½" x 5½").

REX DEPEW
State Teachers College
Florence, Alabama

The Science of Chance, Horace C. Levinson, Rinehart and Company, Inc. (232 Madison Avenue) New York, 1950, vii + 348 pp., $3.00.

Levinson discusses in this book the concepts of elementary probability theory and their role in everyday affairs in a concise, simple, but forceful language. His discussions stem from an evident acquaintance with the broad utilization of these concepts in physical, biological, and socal sciences, government, agriculture, advertising, and business. The reader is aware of the fact that the author is leading him gently through the theory of probability into the field of statistics, where it finds its most fruitful application.

In order to accomplish this, the author has drawn on a fairly extensive list of games of chance where the laws of chance appear in their simplest form and out of which grew the theory of probability. Feeling that the theory loses nothing by the simplicity of examples used, he devotes considerable time to the analysis of problems associated with dice, cards, roulette, and other games, in order to establish the basic concepts of probability which must be mastered "before mathematical statistics can take a hand."

After devoting the first half of his book to the laws of chance, Levinson does everything in his power to cause the reader to see clearly the intimate relation that exists between statistics and the theory of chance. He expends considerable effort in attempting to make clear the nature of the modern science of statistics. He discusses and illustrates many of its basic ideas. One quite notable and worthwhile achievement is his emphasis on "misuses that lead to fallacious statistics," a subject too often neglected by authors.

Particular emphasis is placed on the use of statistics in the fields of advertising and business. "By its very nature, advertising . . . inevitably involves problems in which statistical thinking is called for." This is a fact "overlooked by many elaborate books devoted to advertising." Attention is called to the fertility of the field of modern business with its extensive complications for the science of statistics. The author stresses the need for trained statisticians who are familiar with theory and procedures and who are not merely manipulators of numbers—men with scientific training and knowledge who know the difference between statistics and arithmetic.

A point of considerable interest is that the author insists throughout his book that "intelligently conducted statistics is not a substitute for judgment." He insists that it is an important aid to sounder and keener judgment and must be used as such.

The Science of Chance may well be used as supplementary reading material in any course where the treatment of elementary statistics is of a technical nature. The non-mathematician can read the book with understanding and the advanced student will find in it a valuable text for reference purposes.

Frank C. German
Kansas State Teachers College

An Introduction to the History of Mathematics, Howard Eves, Rinehart and Company, Inc. (232 Madison Ave.) New York, xv + 422 pp., $6.00.

This book is designed to be used as a textbook for an undergraduate course in the history of mathematics and is the result of such a course developed by the author while (1948) at Oregon State College. As is usual in such texts, the treatment is restricted to mathematics through the beginning of calculus.

A distinguishing feature of this text is the inclusion of a set of problem studies at the end of each chapter. Each problem study contains a number of related problems and questions. According to the author's introduction: "It is felt that by discussing a number of these Problem Studies in class, and working others as home assignments, the course will become more concrete and meaningful for the student, and the student's grasp of a number of historically important concepts will become crystallized. Appreciation and understanding of number systems can be given in no better way than by actually working with these systems. And rather than just tell a student that the ancient Greeks solved quadratic equations geometrically, let him solve some by the Greek method; in so doing he will not only achieve thorough understanding of the Greek method, but he will obtain a deeper appreciation of Greek mathematical accomplishment. Thus it is hoped that the student will learn much of his history, as well as some interesting mathematical sidelights, from these Problem Studies. Some of the Problem Studies concern themselves with historically important problems and procedures, others furnish valuable material for the future teacher of either high school or college mathematics, and still others are purely recreational." The problem studies are well worked out and the number is large enough to provide for selection according to needs; suggestions for the solutions and answers to some of the problems are given in an appendix.

The book begins with a consideration of number systems, of Babylonian and Egyptian mathematics, and of Pythagorean mathematics. These are followed by chapters on the three famous problems of Greek geometry, on Euclid's Elements, and on Greek mathematics after Euclid. A chapter on Hindu and Arabic mathematics and one on European mathematics (500 to 1600 A.D.) complete Part One of the text. Part Two is entitled "Later European Mathematics" with especial emphasis on the seventeenth century and contains chapters: *The Dawn of Modern Mathematics, Analytic Geometry and Other Pre-Calculus Developments, The Calculus and Related Concepts,* and *Transition to the Twentieth Century.*

Your reviewer has used this book as a classroom text and considers it the most satisfactory one available. The author has done a good job of relating his materials to the development of the subject. The problem studies are very worthwhile. The book is well written and well printed. Line drawings only have been used for illustrations; there are no portraits (the reviewer regards this as a weakness.) Each chapter has a well-selected bibliography and there is a general bibliography at the end of the work. In addition to its usefulness as a textbook, Eves' book can be recommended to the person of moderate mathematical background who wishes an up-to-date book on the history of mathematics for reference or self study.

Dean L. Robb
Baldwin-Wallace College

Infinity, Lillian R. Lieber, Rinehart and Company, Inc. (232 Madison Ave.) New York, 1953, 359 pp., $5.00.

One of the striking features of Lillian R. Lieber's volume entitled *Infinity* is its unique style of presentation. The terse lines, along with careful simplicity of expression, make for easy, interesting reading, as well as clarity of understanding and point of view. It is largely this unusual style that would encourage one to read the book just for fun.

The writer delves frequently into the philosophical realm suggested not only by the actual topic being discussed, but by connotations remotely related. The first chapter takes the word "*Sam,*" derived from abbreviations for Science, Art, and Mathematics and makes it embody, in an easy-going manner, the broad connotations of "reality," "intuition," and "inference." This "*Sam,*" as it is discussed throughout the book, then becomes a part of the personality of each thinking individual. Although these concepts are dealt with categorically, one is not left without the idea of what the author means to convey about their relationship to infinity in particular and mathematics in general.

The second chapter continues with real concepts of largeness hardly conceivable to the average mind. Several examples are drawn from the physical sciences, illustrating the idea of largeness of numbers, as in the examples of the number of molecules in one cubic centimeter of a gas under specified conditions. Yet, as is pointed out, such numbers are not infinite, for they can be written as integers in our number system. The idea is extended in the philosophical sense to include man's yearning to know more about the unlimited, the unbounded, and the infinite.

There are distinct treatments of the term "potential infinity" and the term "actual infinity." In the treatment of potential infinity, many illustrations from elementary arithmetic, algebra, and geometry are drawn. Hence, one is cautiously led into more and more mathematics on a higher level, soon finding oneself in the midst of some properties of conic sections, projective geometry, and non-Euclidean geometry.

In describing concepts of actual infinity, the writer takes the opportunity to introduce and describe some basic concepts of Cantor's Theory of Sets. Just enough is given to define "actual" infinity and at the same time to arouse the interest of the curious reader who may not be too well acquainted with the theory. There is more than a mere introduction to the theory. The treatment of transfinites and their operations is more than intuitively introduced. It is well described and illustrated.

The discussion of the infinite in the calculus of Newton and Leibnitz gives the author an opportunity to introduce not only elementary calculus *per se*, but also some of the properties of functions well known to that area of the calculus. The treatment of the Riemann integral in the usual classical manner follows a chapter on very elementary areas. However, the author does not omit at this point the opportunity to describe in not too technical language some basic ideas of the Theory of Measure. Not much more than mere mention could be made of the contributions of Borel to point-set theory, and of the integrals due to Lebesgue, Stieltjes, and Denjoy; but enough is said to place these contributions in their proper perspective in modern mathematics.

Although there is much mathematics to be learned in the text by one not already familiar with the concepts presented, one great contribution of the book is its unique presentation, which makes the reading of this mathematics text a pleasure rather than a chore.

REUBEN R. MCDANIEL
Virginia State College

" 'The King is dead, long live the King.' This may be applied to the Mathematics Club of Northeastern. As The Pentagon, the club is dead; as Kappa Mu Epsilon, it lives."

—Kathryn Wyant

Kappa Mu Epsilon News

EDITED BY FRANK HAWTHORNE, HISTORIAN

On this twenty-fifth anniversary of the founding of Kappa Mu Epsilon, it is fitting that recognition be given to some of the outstanding alumni members of the fraternity. In one sense all members of the fraterntiy are "outstanding" or they would not have been elected to membership; however, such a complete listing would be pointless (and spacewise, impossible). Each of the corresponding secretaries was asked to submit short notes concerning a few of the alumni of his chapter whom he considered especially worthy of inclusion. The choice which the secretaries were asked to make was not an easy one, and the basis for selection was necessarily subjective.

It is recognized that some of our chapters are much older than others, and this report is not to be construed as a comparison between chapters.

Alabama Beta reports that the following members have earned doctorate degrees: Frances Kennedy Millican '36; Kenneth Wann '38; T. A. Ban Croft '39; Cecil Harbin '42; William H. Blackburn (M.D.) '43; William Huckaba (D.D.S.) '43; James Calloway (M.D.) '46; Leonard Trapp '46; J. H. Banks '46; Robert L. Ford, '47. Other outstanding alumni include: John Finley, Jr. '35; Associate Professor of Education, Florence State Teachers College; Hermine Wilson '41, outstanding in civic affairs; L. T. Shiflett '46, member of Mathematics Department, Southwest Missouri State College; Carl Prince '48, Section Head, Computation Laboratory, Redstone Arsenal, Huntsville, Alabama; Rex D. Depew '48, instructor and mathematician, IBM Applied Science Division, Lincoln Laboratory, Lexington, Mass.; Jean Parker '49, supervisor of student teachers of mathematics; George Walden '55, graduate assistant, University of Mississippi.

California Alpha chapter is proud of: Charles J. A. Halberg, Jr., charter member, who earned his doctorate in mathematics from U.C.L.A. and is now teaching at the University of California at Riverside; Donald Benson, who has a Ph.D. from Stanford University and is now an assistant professor at Carnegie Tech; Ernest Kimme, who has completed a Ph.D. in mathematics at Minnesota and is teaching at Oregon State College.

The following **Iowa Alpha** alumni have earned Ph.D. Degrees. Following the name of each individual is the name of the university from which the degree was received and his present position:

Albert D. Bailey, University of Illinois, 1954, Professor of Electrical Engineering, University of Illinois; Robert D. Huntoon, University of Iowa, 1938, Associate Director, Physics, National Bureau of Standards; Richard G. Kadesch, University of Chicago, 1941, Director of Research, Emery Industries; Tom A. Lamke, University of Wisconsin, 1951, Coordinator of Research and Assistant to the Dean of Instruction, Iowa State Teachers College and recently appointed Editor of the *Review of Educational Research,* official publication of the American Educational Research Association; Leonard O. Olsen, University of Iowa, 1937, Professor of Physics, Case Institute of Technology; Florence V. Rohde, University of Kentucky, 1950, Associate Professor of Mathematics, University of Florida; John H. Smith, University of Chicago, 1941, Senior Statistician, Bureau of Labor Statistics; Milton Moon, University of Iowa, 1951, now at Johns Hopkins University; Warren J. Thompson, University of Iowa, 1952, now at Johns Hopkins University; John S. Wahl, University of Iowa, 1952, Physics Research, Los Alamos Laboratories.

A member of **Illinois Alpha,** Roger Hufford, has been chosen to receive a Rotary Foundation Fellowship. The grant will permit Mr. Hufford to study liberal arts during the 1956-57 school year at one of the major universities in the British Isles. His study will be aimed at preparing for a career as a college mathematics teacher. The grant is for approximately $2,500.

Three alumni members of **Iowa Beta** are Fellows of the Society of Actuaries. They are Floyd T. Beasley '40; Robert S. Yoder '50; and Daniel G. Stewart '52. The following are Associates in the Society of Actuaries: Floyd A. Bash, Jr. '40; William Lones '47; Jack Morgan '47; Dean Williams '47; John Coons '49; Waid Davidson '49; Lewis Workman '49; James Curtis '51; and Jack Wood '52.

Walter Bishop, Jr. '47 is a Fellow of the Life Office Management Institute.

Kansas Beta lists the following alumni who have completed doctor's degrees: Aldro Bryan, Galen Bull, Ramon Charles, Ralph Dunham, Daryl Errett, Frank Faulkner, Charles Girod, Edison Greer, Herbert Jackson, John S. Malik, Harold McFarland, Francis McGowan, Otho Rasmussen, Charles Richart, Worth Seagondollar, Virgil Stout, James Smith, Howard Tempero, and John Zimmerman.

Graduates who are doing full time college teaching include: Harold Bechtoldt, University of Iowa; George Blair, University of

Pennsylvania; David Cropp, Kansas State Teachers College, Emporia; Orville Etter, Fort Hays Kansas State College; Ray Hanna, Wichita University; Donald Hanifan, University of California; Lawrence Huntly, Wichita University; Lester Laird, K.S.T.C., Emporia; Alvin Morris, K.S.T.C., Emporia; Alfred Philips, Gray's Harbor College, Aberdeen, Washington; Raymon Schobe, Knoxville, Tenn.; Lt. Vernon Switzer, Naval Air Station Officers School, Glenview, Ill.; Masanobu Yonaha,, University of Ryukyus, Okinawa.

James R. Bower, President of **Michigan Beta** in 1954, who gave the address of welcome at the Ninth Biennial Convention, was awarded the State College Scholarship to the University of Michigan. He received his M.S. in mathematics in 1955 and is continuing study toward the doctorate.

Malcolm McPhee '52, won the State College Scholarship to the University of Michigan and received the M.S. degree in mathematics in June, 1953. He now teaches in Bay City.

Harold T. Slaby '46, President of **Michigan Gamma** during his senior year, was granted the Ph.D. in mathematics by the University of Wisconsin in 1954. He is at present an instructor at Wayne University. Walter Hoffman '48, is now an instructor at Wayne, and the present sponsor of **Michigan Gamma.** Marilyn Hamilton '50, received her Master's Degree from Wayne University in 1951 and was awarded a scholarship for advanced study at the University of Michigan. David Morrison '53, served in the computation laboratory at Aberdeen Proving Ground. He is now at the Wayne University Computation Laboratory. Kathleen Hamlin '55, is doing programming in the computation laboratory of Lockheed Aircraft. Richard Pavely left the university for military service at Aberdeen computation laboratory in 1952. He returned and completed his undergraduate work and is now working in the Wayne Computation Laboratory.

Missouri Alpha reports that the following members have reflected credit upon Kappa Mu Epsilon in their achievements: Inks Franklin, Editor of *School and Community*; Edwin Martin, Ph.D., President-elect, Trenton State Teachers College, Trenton, N. J.; Charles K. Martin, Ph.D., President, Radford College, Radford, Virginia; Byron Calloway, Ph.D., Professor of Education, University of Georgia; Marion Emerson, Ph.D., Mathematics Department, Harpur College; Myrle Johnson, Ph.D., Research Chemist, Eastman Kodak Company; Mrs. Dorothy Martin Simon, Ph.D., Chemist, Avco Manufacturing Company; Ivan D. Calton, Ph.D., Professor of Business Administration, Southwest Missouri State College; Frederic

Holt, Manager, General Electric Plant, Morrison, Ill.; Louise Beasley, Mathematics Department, Lindenwood College; John M. Teem, Ph.D., research in atomic physics and teaching, California Tech; William R. Foster, Ph.D., Research Chemist, Sun Oil Company; William Cheek, M.D., Medical Research, University of Houston; Milford Holt, Ph.D., Assistant Professor of Secondary Education, Central Michigan State College; James Reavis, Ph.D., University of Utah; David O. Ellis, Ph.D., Mathematician, National Cash Register Company; Earl E. Bilyeu, Mathematics Department, Southwest Missouri State College; Joseph Guida, Mathematics Department, Kansas State University; Gaylen Bradley, Ph.D., post-doctoral research in bacteriogenetics, University of Wisconsin; Dale Sparks, M.D.; Dan Holmes, M.D.; C. Frank Knox, M.D.

Missouri Beta reports that the following alumni have earned Ph.D. degrees: Harvey Donley, Nylen Edwards, John Burges, Irvin L. Sparks, and Robert Granger. Other outstanding alumni are: Olen Nance, Professor of Physics, Louisiana State College; Charles E. Kelley, Central Missouri State College; Richard Baile, Head, Empire Geophysical Exploration Co.; Harold Banich, Robert Goetz, Central Missouri State College; Charles R. Bonnell, Director of Research, Cedar Engineering Company, Minneapolis; J. Dale Boyd, Boeing Aircraft; Howard Bryant, Oak Ridge, Tenn.; Norman Hoover, Fresno State College; James Remley, General Electric, Schenectady, New York; L. P. Curtis, Westinghouse, Pittsburg, Pa.; Kenneth Waugh, Indianapolis, Indiana; John Ferguson, University of Missouri; Harold Peabody, I.B.M., Atlanta, Ga.; Irl Gladfeiter, Central Missouri State College; James Green, Empire Geophysical Co.; Norton Jones, McDonnell Aircraft, St. Louis, Mo.; Barbara W. Haverty, University of Kansas; William Vardeman, U.S.N., China Lake, Calif.; William Klingenbery, Business Men's Assurance Co., Kansas City, Mo.

Missouri Epsilon reports that Mark Barton received the Ph.D. in Physics from the University of Illinois in 1955 and is now at the Brookhaven National Laboratory. Niels C. Nielsen received a Ph.D. in Chemistry from the University of Illinois in 1953. He is now Assistant Professor at the University of Missouri. William E. Cooley received a Ph.D. in Chemistry from the University of Illinois in 1954. John Blattner should soon receive his Ph.D. in mathematics from the University of California at Los Angeles. He has also done graduate work at the University of Wisconsin and the University of Chicago.

John D. Garwood, an alumni member of **Nebraska Alpha**, completed his Ph.D. in Economics from the University of California in 1951. He is presently at Fort Hays Kansas State College. He has had some 20 papers on economic subjects published in the last five years. Jack F. Morgan is at present Chief Chemist at General Aniline and Film Corp., Rensselaer, N. Y. His publications include some 10 patents as well as numerous articles in the field of dyestuffs. Jack R. Davenport is Assistant Regional Manager for Scott, Foresman and Company, supervising sales in seven western states, Hawaii, Alaska, and the Pacific territories. John R. Banister received his Ph.D. from Iowa State College in 1953. He is now a physicist with Sandia Corp., Albuquerque. Ruth A. Wagner has an M.A. from Colorado State College of Education at Greeley. She has been particularly active with the Norfolk (Nebr.) Youth Council. Van W. Bearinger has a Ph.D. in Physics from Iowa State College and is now with Minneapolis-Honeywell.

Other outstanding alumni of **Nebraska Alpha** include: John J. Jones, Marian S. Petersen, Carroll C. Petersen, David Garwood, and Beulah Bornhoft.

Outstanding alumni of **New Jersey Beta** include: Lawrence Campbell, Assistant Professor of Mathematics, Navy Training School, Denver; Betty Wright, Assistant Statistician for NJEA, Trenton; Earnest Yeager, Ph.D. in Chemistry, Associate Professor of Chemistry, Western Reserve.

New Mexico Alpha's Walter Biddle '35, is a member of the State Board of Technical Registration of Arizona. Chester Russell '35, is chairman of the Department of Electrical Engineering at Clarkson College. Robert Ronney is Vice-President of the Electronic Engineering Co. of Los Angeles. Kenneth Bullington '35, received the Morris Liebmann Memorial Prize in 1955. Steve Reynolds '37, is State Engineer for the State of New Mexico. H. W. Benischek '39, is Chairman of the School of Petroleum Engineering at the University of Oklahoma. Clifford O. Firestone '39, Ph.D. (Mathematics) Cornell, 1946, is a staff member at the John Hopkins Applied Physics Laboratory, Silver Springs, Maryland. Frank Lane '40, is employed at Sandia in Albuquerque. John Coy '41, received a Ph.D. in Mathematics from Michigan and is now at White Sands Proving Ground, New Mexico. Esther Barnhart '41, is completing a five year term as Lutheran missionary in Japan.

The following alumni of **New York Alpha** have completed Doctorates: Robert Ackerson, Alabama Polytechnic Institute; Louis Bauer, Brown; Robert Beyer, Cornell; Gerald Endres, (Chemistry) Polytechnic Institute of Brooklyn; Harold Game, Alabama; Mario Juncosa, Cornell; Robert Mulford, Brown; Wanda Scala Walsh, New York University.

Ohio Alpha reports that the following alumni have completed Doctorates: Lawrence Ringenberg, (Mathematics) Ohio State University; Darwin Mayfield, (Chemistry) University of Chicago; Winifred Cole, (Chemistry) Ohio State University; Clayton McDole, (Physics) Pittsburgh University; Kenneth Smith, (Mathematics) University of Wisconsin; Phyllis Blosser, (Chemistry) Ohio State University; Frank Ogg, (Mathematics) Johns Hopkins. Other outstanding alumni include: Franklin E. Sheidler, C. E. Britt, David Shu-i Nee, Roger Lemelin, L. H. Archer, and David Slough.

Oklahoma Alpha reports that Dr. Paul Lewis (charter member No. 13) did outstanding work in the organization of KME and was very active in the installation of other chapters. He received the Doctor of Philosophy degree from the University of Illinois and has taught at Oklahoma A & M and the University of North Carolina. In 1955, Dr. Lewis joined the Convair Aircraft Corporation in Los Angeles as Director of the Budget.

Lt. Col. Charles Nussbaum (charter member No. 16) graduated at Northeastern State College in 1932. He taught mathematics in Tulsa, Oklahoma and was Director of Athletics in Central High School, Tulsa. After several years in the army, he returned to Tulsa, but soon resigned and accepted an army appointment as director of the army budget in the Carribean area. At present, he is in the Pentagon associated with the Armed Services Command.

Mr. Philip Bohart, graduate of Northeastern in 1936, is director of government employees in the Pacific area. After leaving Northeastern Mr. Bohart received his Master's Degree from Oklahoma University. His present headquarters is in Honolulu.

Mr. Noble Bryan, Jr., has been with Boeing Aircraft since graduation from Northeastern State College in 1946. At the present time he is Supervisory Design Engineer. The Bryans live at Seattle, Washington.