THE PENTAGON

Volume XIII **FALL, 1953** **Number 1**

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WHO'S WHO IN KAPPA MU EPSILON

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Kappa Mu Epsilon, national honorary mathematics fraternity, was founded in 1931. The object of the fraternity is four-fold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievement in the study of mathematics in the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

A DEVICE FOR DRAWING N-FOCUS PLANE CURVES*

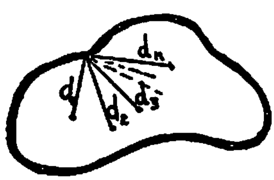
RAOUL PETTAI
Student, Colorado A & M College

In the development of our basic mathematics we are often introduced to the ellipse as a member of a class of curves called conic sections. The ellipse is defined to be the locus of a point moving so that the ratio of its distance from a fixed point (the focus) to its distance from a fixed line is a constant less than 1.

There exist other definitions for the ellipse. For example, in drawing the ellipse, use is made of the fact that the ellipse is also the locus of a point the sum of whose distances from two fixed points is constant. Since this latter type of definition cannot generally be applied to all conic sections, it is ordinarily not used when conic sections are discussed.

From a certain idea, implicit in this definition, I became interested in a class of plane curves which I will call n-focus curves and define as follows:

An n-focus curve is the locus of a point the sum of whose distances from n fixed points, which I shall call the foci, is constant.



$$\sum_{1}^{n} d_i = K$$

Figure 1

With respect to this definition the ellipse now becomes a two-focus curve since the sum of the distances from the

*A student paper presented at the 1953 National Convention of Kappa Mu Epsilon and awarded first place by the Awards Committee. This paper represents original, creative work on the part of Mr. Pettai. At the time of the convention Mr. Pettai did not know of the work of Clerk Maxwell on this type of problem. In "The Scientific Papers of James Clerk Maxwell", Dover, an article taken from the *Proceedings of the Royal Society of Edinburg*, Vol. II, April 1846, there is a paper entitled "On the Description of Oval Curves and Those Having a Plurality of Foci."

two foci to any point on the ellipse is constant, and the circle can be considered a one-focus curve since the distance from the center of the circle to any point on the circle is always constant.

Next let us consider a three-focus curve which, as defined, is the locus of a point the sum of whose distances from three fixed points is constant.

In general these points can be placed in any desired manner with respect to each other; but I am going to confine myself, throughout the entire discussion, to the special case where the points all lie on the same straight line and the distances between adjacent points are equal. A device which can be used to draw a three-focus curve is illustrated in Figure 2. The string begins at one of the points and its other end is fastened to the drawing instrument.

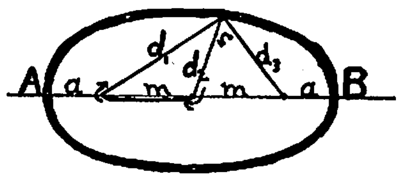


Figure 2

The proof that the curve thus obtained is a three-focus curve follows:

The length of the string is obviously constant, C. Hence

$$d_1+d_2+d_3+m = C.$$

Since m is constant, it can be combined with the constant C, giving

$$d_1+d_2+d_3 = K.$$

For the purpose of investigating the curve, let us call the distance AB the major axis of the curve and designate its length with the letter L. Now we can write (Figure 2)

(1) $$AB = L = 2m+2a.$$

Since the point B is a point of the curve, it must be true that

(2) $K = d_1+d_2+d_3 = (2m+a)+(m+a)+a = 3m+3a$
where the distances are expressed in terms of m and a.

Combining equations (1) and (2) and eliminating the quantity a

(3) $$L = (2/3)K.$$

Thus the length of the major axis of this special three-focus curve is equal to 2/3 of the constant.

For a four-focus curve the reasoning will be quite similar to that of a three-focus curve. In drawing the curve, the same device can be used. This time, however, the string is run so that its other end is fastened to one of the points rather than to the drawing instrument (Figure 3). This is always true when there is an *even* number of points; for an *odd* number of points the string terminates as shown for the three-focus curve.

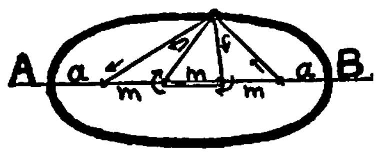


Figure 3

The analysis used in finding the length of the major axis of the three-focus curve, also applies to the four-focus curve. The result is that the length of the major axis of a four-focus curve is one half the constant.

(4) $$L = (\tfrac{1}{2})K$$

According to the definition which was used before, an n-focus curve is the locus of a point the sum of whose distances from n fixed points is constant. Applying the same symbols of designation and the same reasoning as before, we can find the length of the major axis of an n-focus curve as follows.

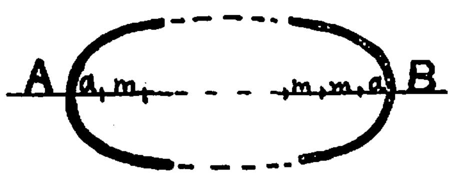


Figure 4

(5) $$AB = L = 2a + (n-1)m$$

At the point B,

$$K = a + (a+m) + (a+2m) + \cdots + [a+(n-1)m]$$

which can be written

$$K = na + m[1+2+3+ \cdots + (n-1)].$$

Using the summation formula for an arithmetic progression,

(6) $$K = na + m[n(n-1)/2]$$

Solving equation (6) for a and substituting in (5) yields

(7) $$L = (2/n)K$$

where n is the number of foci.

A comparison of this expression with (3) and (4) shows that those two equations are merely special cases of equation (7). Equation (7) holds also for the ellipse and for the circle, which can be shown if $n = 2$ and $n = 1$ are substituted into (7). It should be emphasized that if the points are placed irregularly as in Figure 1, equation (7) no longer holds. However, the device illustrated before can still be used to draw any regular or irregular n-focus curve, provided that the general rules concerning the termination of the string are followed.

As a student of engineering I have been interested primarily in the geometric side of this problem; as yet I have not worked out the algebraic theory of these curves which appears to be fairly complicated.

"No Roman ever lost his life because he was absorbed in the contemplation of a mathematical diagram."

—E. T. Bell

SOME MATHEMATICAL CONSIDERATIONS OF DYNAMIC SYMMETRY UNDERLYING VISUAL ART*

BERNADINE LAW
Student, Mount St. Scholastica College

Recently, on one of our college campuses, a controversy arose as to whether or not mathematics can ever be justly regarded as an art. An anthropologist cinched the argument by stating, "It is a common error to suppose that a work of art must have *human* reference. Actually all that is necessary is that it have *great* form. And mathematics certainly has that." I feel that in a select group—where the common interest that draws us together is mathematics — agreement with the foregoing statement would be readily and unanimously given. Since my status is that of the student who is still striving for an elementary knowledge of this great structure—or form—of mathematics, I can do little more at this time than point out some of the general, basic principles of mathematics which constitute the foundation of visual art. I would like also to give you some illustrations which I hope will prove interesting.

At the basis of great art is proportion and measurement, that is, the use of numbers. It is always present in the sensitive mind and in the skillful hand of the artist. Principles of geometry are applied more or less ingenuously, unaffectedly, and simply in art.

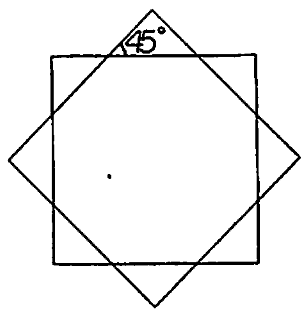
Though visual art is concerned with symmetry, balance and proportion, I shall restrict my discussion to the fundamental notions of one type of symmetry, namely dynamic symmetry as it has been rediscovered by Mr. Jay Hambidge of Yale. I say rediscovered because dynamic symmetry was employed to its fullest extent in the classic art of the ancient Egyptians and Greeks. At the decline of Greek civilization, however, dynamic symmetry was lost and remained completely unknown until the early part of the 20th century. Area relationships which constitute the backbone of dy-

*A paper presented at the 1953 National Convention of K.M.E. and awarded second place by the Awards Committee.

namic symmetry were entirely forgotten until Jay Hambidge restored the idea through his study of Greek vases.

Perhaps, before proceeding to the heart of the matter, a few definitions of terms would be in order. Symmetry for the artist means the arrangement of elements around an axis which passes through the center of the composition. The symmetric effect thus produced may be static or it may be dynamic. To achieve static symmetry, one employs designs of definite, fixed arithmetic lengths.

Thus, concentric circles, where the radii constantly increase by fixed amounts, are symmetric in a static sense. As the name implies an effect of stillness and quiet is produced. See Figures 1 and 2, plate iv. Another familiar design of static symmetry is that obtained by superimposing two squares at 45° angles as seen in the figure below.



Dynamic symmetry in opposition to static symmetry makes use of proportions which cannot be measured in length, but which are commensurable in areas. Some ratios commonly used are 1:root—2, 1:root—3, and 1: root—5. We know that it is not possible directly to obtain a length equal to root—2 (1.4142 . . .) whereas it is possible to obtain the dimension root—2 by merely transferring the diagonal length of a unit square. Dynamic means growth, power, and movement. It gives animation and life to an artist's work. This movement evolves from the designs of dynamic symmetry which employ areas rather than lines. Examples will be shown as we progress further with the technique of dynamic designs. The techniques which I propose to treat and illustrate are: (1) root rectangles according to the proportions previously given, and (2) whirling square rectangles. I will attempt to show the various methods of construction, the unique qualities of each, and the best examples of their use in visual art.

To state the matter of root-rectangles very simply one might proceed as follows: draw a square and its diagonal. Use the side of the square and its diagonal. The result is a root–2 rectangle. Using the side of the square and the diagonal of the root–2 rectangle as our dimensions, we obtain the root–3 rectangle. By continuing in this manner, we construct the root–4 rectangle which is merely two of the original figure. From its diagonal we obtain the length of the root–5 rectangle. These constructions are shown in Figure 1.

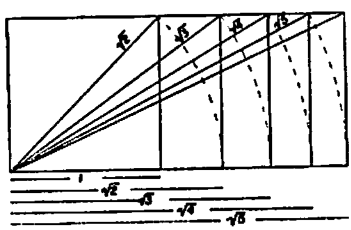


Figure 1

These same root rectangles may be constructed within a square by a simple geometric method. This method is shown in Figure 2.

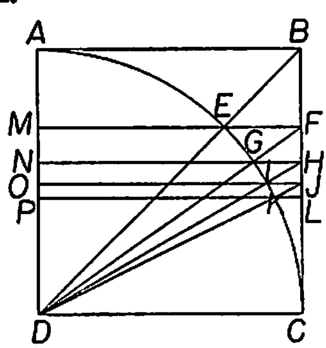


Figure 2

In the square, ABCD, the quadrant arc is drawn, AC. Where the diagonal of the square, BD, cuts the arc, E, marks the side of a root–2 rectangle, MFCD. (With FC as unity, MF is $\sqrt{2}$.

The point where the diagonal of the root–2 rectangle, DF, cuts the arc, G, marks the side of a root–3 rectangle, NHCD.

In the same manner the root–4 rectangle, OJCD, and the root–5 rectangle, PLCD, are obtained.

I have now indicated two methods of drawing the root–2, root–3, root–4, and root–5 rectangles. You will note that in the first case we started with a square and built our rectangles in successive steps on the outside. In the second case our procedure was reversed; we started with a square and constructed the series of rectangles on the inside of the given square.

Perhaps at this point you are wondering: what is the precise relationship of all these figures to visual art? Please bear with me while I explain the mathematical structures; that being done, I shall point out where and how these have been used by the great masters.

Before I can show any of the unusual characteristics of these root rectangles, I must first explain the *reciprocal* and the *gnomon* of a rectangle.

The reciprocal of a rectangle is a division of the original rectangle which has the same shape as the whole figure. The gnomon is that part of the rectangle which is left over after the reciprocal has been cut off. The reciprocal of *any* rectangle is obtained by constructing first its diagonal and second, a line drawn from one of the other corners at right angles to the diagonal. The place where this second line—the perpendicular—crosses the horizontal line of the rectangle will mark the side of the vertical rectangle. This vertical rectangle is the *reciprocal,* and as can be seen it has the same shape as the whole figure. The *gnomon* is the shape remaining. This is shown in Figure 3.

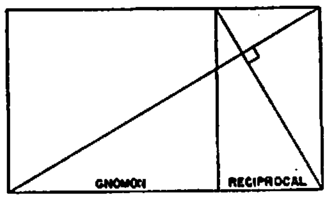


Figure 3

The root—2 rectangle is unique in that its reciprocal and its gnomon are equal, and each has the same shape as the whole root—2 rectangle. Thus a root—2 rectangle divided in half would result in two root—2 rectangles. This will be seen later in the analysis of a Greek vase. Figure 4 shows that the reciprocal and gnomon of the root—2 rectangle may be divided indefinitely into figures of the same proportion. Root—3 and root—5 rectangles as shown in figures 4a and 4b also possess unique characteristics similar to this.

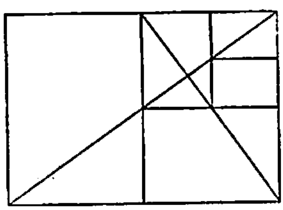


Figure 4

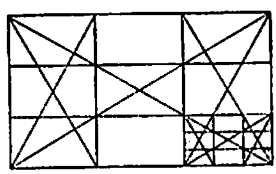


Figure 4a

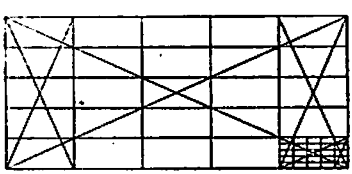


Figure 4b

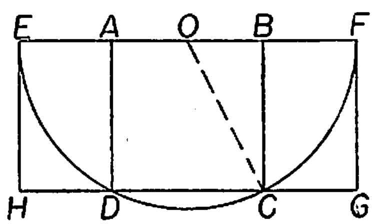


Figure 5

The root—5 rectangle as shown in Figure 5 above, may be obtained by Euclidean construction: draw the square ABCD; from O (midpoint on AB) use OD or OC as a radius to describe a semi-circle. Continue the lines AB and CD. Use as the dimension the line cut off by the arc of the semi-circle. Designate these points E and F. Complete the rectangle EFGH. This is a root—5 rectangle, the arithmetic derivation of which is shown below:

BC = AB = 1
BO = .5
CO = $\sqrt{(BC)^2+(BO)^2} = \sqrt{1^2+.5^2} = \sqrt{1.25} = \sqrt{5}/2$
EO = OF = $\sqrt{5}/2$
EF = 2(EO) = $\sqrt{5}$
EFGH is a root-5 rectangle

The root—5 rectangle is especially significant because it contains what is known as the golden sector rectangle. This rectangle AFGD is named the golden sector rectangle because its dimensions are those of the famous golden section proportion. This proportion which the Greeks developed is that which we call the extreme and mean ratio. In this ratio the shorter dimension is to the longer as the longer segment is to the sum of the two. In Figure 5, AFGD and also EBCH are rectangles of the golden sector proportion. The dimensions form the proportion:

FG:GD::GD:(FG + GD).

The significance of the golden sector proportion resides in the uniqueness of its members, neither of which is accountable in terms of the other. The longer dimension in relation to the shorter has the greater possible difference. This, however, neither approaches twice the length of the shorter nor suggests the shorter plus a recognizable increment such as ¼ or ½. Numerically the ratio of the proportion approximates 1:1.618. This can be seen from Figure 6 and the calculation following it.

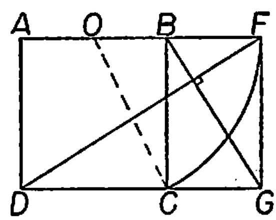


Figure 6

OF = 1.118 (approximately), AO = .5
AF = 1.618 which approximates the golden sector proportion:
$1:1.618::1.618:(1 + 1.618)$ or
$FG:GD::GD:(FG + GD)$

The golden sector rectangle in Figure 6 is more often called the *whirling square* rectangle. This name is given it because the gnomon cut off each time by the reciprocal is a square. We shall draw a diagonal and a perpendicular, a line perpendicular to the diagonal from an opposite corner, which meets the opposite horizontal line. The gnomon is a square, and the reciprocal is a second whirling square rectangle. Successive divisions of this reciprocal will cut off a succession of squares which whirl inward as shown in Figure 7.

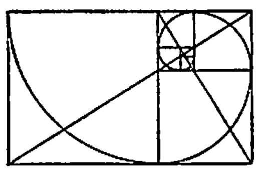


Figure 7

The involute of this figure seems to have a natural significance since it appears in natural forms of growth such as the conch-shell. See Figures 3 and 4, plate iv.

Since there are two diagonals and each diagonal may have two perpendiculars, there are in the whirling square rectangle four possible eyes or poles around which the squares whirl. These are shown in Figure 8.

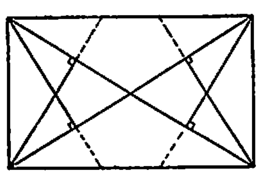


Figure 8

I have now explained the methods of construction of root rectangles and the whirling square rectangle and have shown a few of the unique characteristics of each. The remainder of my discussion will consist of examples of the principles which I have explained.

An excellent example of the root−2 theme is furnished by the handsome red-figured amphora of the Nolan type in the Fogg Museum at Harvard. A photograph of this amphora, reproduced through the courtesy of the Fogg Art Museum, Harvard University, is shown in Figure 5, plate iv. For an analysis of the amphora let us examine the figure below.

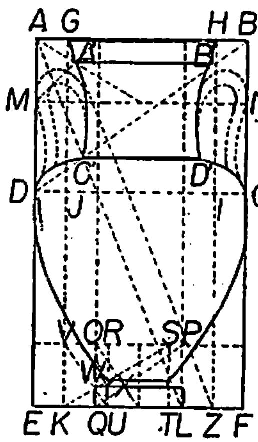


Figure 9

This amphora is contained within the area of a root-2 rectangle, ABCD, plus a square, DCFE, on its side. This was determined by dividing its greatest width into its height. It produces the ratio of 1 : 1.7071, thus showing that it is composed of a square plus the reciprocal of a root-2 rectangle. The reciprocal of a root-2 rectangle is in itself a root-2 rectangle.

The width of the lip, GH, in relation to the overall form, shows that it is a side of a square comprehended in the center of the root-2 rectangle. This square, GHIJ, is drawn, and the projection of the sides of this square through the major square produces in the center of that square a root-2 rectangle, JIZK, so that the shape as defined by the lip is a square plus a root-2 rectangle, but the square is on the end of the rectangle instead of on the side as it is in the major shape.

The side, AD, of the root-2 rectangle is equal to half the diagonal of the square, DCFE.

A root-2 rectangle, MNFE, is cut off within the major shape, its side being made equal to the diagonal of the major square. The area left over after this area has been applied is composed of two squares and one root-2 rectangle. The same construction is used, working from the other end of the major shape.

Through the centers of the small squares on each corner, lines are drawn parallel to the sides of the major figure. These lines determine the secondary square, GHIJ, and the root-2 rectangle, JIZK. A diagonal, GZ, to this secondary shape determines the angle pitch of the lip, its thickness, the width of its base, A′B′, and the width of the neck, C′D′.

The foot of the amphora is proportioned by the small root-2 figure, OPLQ, and two squares at the base. A square, RSTU, is placed in the center of the root-2 rectangle, OPLQ. The width of the ring above the foot is a dimension of this square. VRUK is a derived root-2 rectangle, and its diagonal is cut at W by a line through the point, X. This determines the width of the top of the foot. The thickness of the foot and its width at the bottom are determined by the diagonal and perpendicular of the root-2 shape, OPLQ.

The thickness of the ring above the foot is established by the line, KS, a diagonal to the square RSTU plus the root-2 rectangle VRUK, intersecting the side of the square at Y.

Another example of root rectangles is the Greek Kantharos shown in Figure 6, plate iv. Its height divided in half yields two root—5 rectangles. It is called a theme in double root—5.

An exceptional illustration of the use of the whirling square rectangle is to be found in Botticelli's *Birth of Venus*.

See Figure 7, plate iv. The over-all dimensions approximate those of the golden sector rectangle. From the analysis of this work of art, we see that the order of its symmetry is that of the whirling squares.

The rectangle is of the ratio of the golden sector. The diagonals are drawn, and the perpendiculars are produced from the lower corners to meet the upper side. The figures of action are contained in the reciprocals on either end which in themselves are whirling squares. Their movements are in the directions of the succession of squares which whirl toward the center. The heads of these figures are slightly above the poles of the whirling square rectangle, but the arms which are the instruments of action are situated almost directly on the poles. Venus, the center of interest and receiver of the action, stands in the area that remains after the two reciprocals have been cut off. This space in itself is composed of a square and a whirling square rectangle. Botticelli used the extreme and mean proportion to mark both the top of the shell and the height of the horizon.

Whether he purposely employed the whirling square rectangle as such is undetermined, but the analysis of his work shows the presence of this figure.

These, the root rectangles and the golden sector proportion or whirling square rectangle, are only a few of the mathematical calculations that are related to artistic productions. It is fairly simple to understand the necessity of mathematics as a foundation for visual art.

However, it is not possible to produce a work of art solely by means of extreme applications of proportions or measurements. A skillful product is never perfected from the assembling of already constructed parts. The artist must have at his command a knowledge of geometric proportions and a sense of the relations of spaces. Without them, he has as great a disadvantage as if he were unskilled in the use of his tools. He should never give undue emphasis either to his knowledge or to his skill, for the product will then be abnormal and perverse. An undue stress placed upon proportions and measurement will cause the product to appear mechanical and lifeless. Too much emphasis on technique, such as over-refinement of lines, will destroy the relationships between the parts of the work.

For these reasons we reject factory-made products in favor of handwork. The ever present blemishes in handwork certainly do not beautify it, but they give it originality. This uncopied product is more pleasing because we know that it is inspired by the personality of the artist. Whenever his personality is evident in his product and it reveals a sensitive intelligence, the work of the artist may indeed prove to be "a joy forever." It is better to see the artist in the work of art rather than a group of ill-assorted rectangles. As Whistler has said, "A picture is finished when all trace of the means used to bring about the end has disappeared." We don't want to *see* the shapes employed, but we want to *feel* that they are present.

The Greek artist did not replace originality with geometry but coordinated the two into his masterpieces. Never superfluous in his designs, he employed simple and logical rules as foundations for his art.

Modern art, on the other hand, tends toward "an overstress of personality and [consequently] a loss of vigor."

Nature's structural form is perfect, and the Greeks realized this when using nature in their design. They also saw that the material manifestation of this structural form became marred in animal and vegetable growth and development and was thus seldom, if ever, perfect. In their creations they eliminated the imperfect and adopted only nature's ideal. See Figure 8, plate iv.

"Mathematics is the science in which we never know what we are talking about nor whether what we say is true."

—BERTRAND RUSSELL

SOLVING A DIFFERENTIAL EQUATION ON A DIFFERENTIAL ANALYZER*

JOSEPH WEIZENBAUM
Student, Wayne University

Electron wave functions and energy levels for the mercury atom have been computed by Hartree using non-relativistic wave equations. The Wayne University Computation Laboratory undertook to compute these functions and energy levels using relativistic wave equations. This project was carried on in the laboratory's Differential Analyzer and was directed jointly by Professor A. W. Jacobson, the laboratory's director, and Professor Keith Symon of the University's Physics Department.

The differential equations considered in this paper are somewhat simplified over the equations actually put on the differential analyzer.

They are—

(1) $$dx/dr = x/r + f'(\varepsilon,r)y$$

(2) $$dy/dr = y/r + g'(\varepsilon,r)x$$

where r is the independent variable and ε the "eigen value" to be determined.

In order to put this system of two simultaneous differential equations on the differential analyzer, they must first be integrated. Performing the indicated operation, we obtain:

(3) $$x = \int x\, d(\ln r) + \int y\, df(\varepsilon,r)$$

(4) $$y = \int y\, d(\ln r) + \int x\, dg(\varepsilon,r)$$

where

(5) $$f = \int f'\, dr$$

(6) $$g = \int g'\, dr$$

The equations (3) and (4) are then the ones actually put on the differential analyzer.

*A student paper presented at the 1953 National Convention of Kappa Mu Epsilon and awarded third place by the Awards Committee.

Functional Description of an Analogue Machine

An analogue computing machine performs its mathematical operations by measuring certain physical components or certain changes in certain components. This is as opposed to a digital machine which performs its mathematical operations by counting discrete events, like the passage through a circuit of so many electronic pulses. The Wayne University differential analyzer—which is the original instrument built by Vannevar Bush of the Massachusetts Institute of Technology—measures rotations of shafts. It is entirely mechanical.

Such a machine must at least be able to add, subtract, and integrate if it is to be useful in solving ordinary differential equations. In addition it is highly desirable that some provision be made to make it possible for the machine to receive arbitrary functions; i.e., functions the analytic expression of which is either very complex or entirely unknown as, for example, in the case of many empirical data functions encountered in engineering practice. This provision exists on the Wayne differential analyzer and is called an input table.

The operations addition, subtraction, and integration are all binary operations; i.e., in each of them two quantities are operated upon to produce a third. Hence any device which performs these operations must have two inputs and one output. Since the device which performs additions also performs subtractions, we may represent it by a single symbol and call it an adder. It is symbolically represented as in Figure 1.

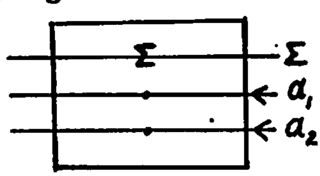


Figure 1

The devices which perform integrations will be called integrators. These are symbolically represented as in Figure 2.

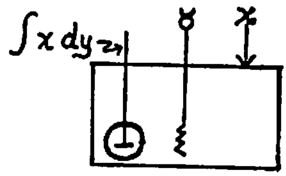


Figure 2

An input table has only one input and one output. Thus if the function $f(x)$ is to be produced from x, x will be the input and $f(x)$ the output. Figure 3 shows a schematic representation of an input table.

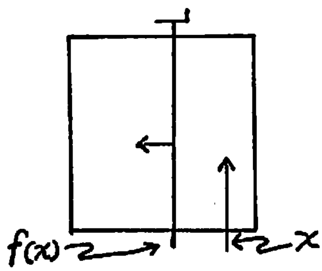


Figure 3

Inspection of the differential equations to be solved now makes possible the design of a machine layout.

We see that, given the independent variable r, three special functions of it must be obtained: *ln* r, $f(\varepsilon r)$, $g(\varepsilon, r)$. For any particular run on the machine ε is a constant. In practice the functions f and g are obtained by integrations and additions. But here we may think of them as coming from input tables. Hence we have—

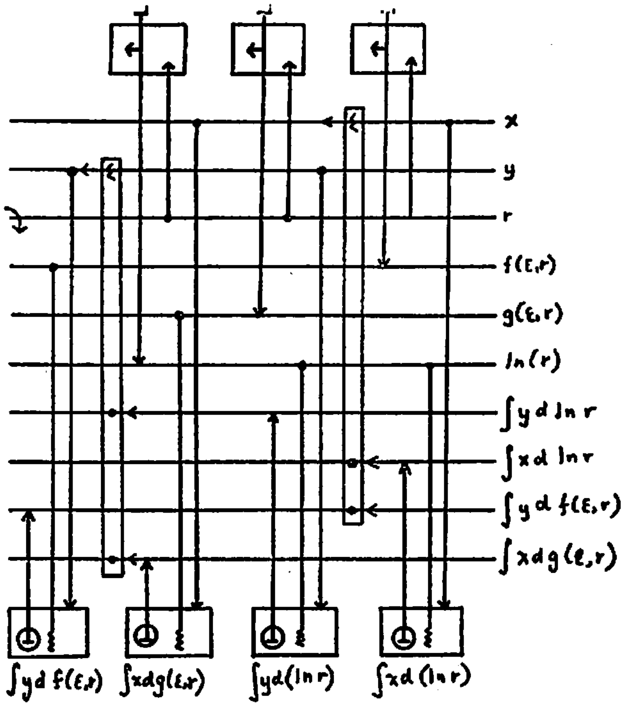


Figure 4

The outputs of the input tables are all differential inputs to integrators, while x and y are both used as integrand inputs on integrators. This is indeed what the equations (3) and (4) require. The integrators are then paired and their outputs become inputs to two separate adders. Again, this is a realistic representation of the equations (3) and (4). One of the sums so produced is x and the other y. These are then *fed back* into the system. This process of *feed-back* is of central importance to the entire field of analogue computation. Unfortunately it is a large topic beyond the scope of this paper.

The Wayne University differential analyzer has an attachment called an output table which makes it possible to obtain a continuous graphical representation of any two variables plotted against a third while the problem is being run. In this case we are interested in plotting x and y against r. The result will then look something like Figure 5.

Figure 5

In principle, if the correct ε has been put into the equations, the x and y portions of the curve become asymptotic to the r-axis. But machine computation is at best approximate. Hence the asymptotic behavior desired cannot be obtained even given the best possible ε. This means that either the x portion or the y portion of the curve will touch the r-axis eventually. Physical considerations are such that when this happens, the component which has touched the axis will cross it, and both components will then diverge in the direction of the side on which they then both find themselves. Furthermore, if the ε chosen is too small, x will cross, if ε is too large y will cross. Figure 5 shows the picture for ε too large.

This of course makes it possible to tell at the end of every solution whether or not the ε chosen was too large or too small. Hence it becomes possible to bracket the correct ε after a few runs. Having two epsilons which bracket the correct ε, a simple interpolation yields a third and better one. This operation is carried out by the staff by ordinary desk calculator methods. A corect ε is presumed to have been obtained when the same ε causes the solution to diverge first in one direction and then the other. The functions so produced very nearly exhibit the asymptotic behavior mentioned above. They are considered the desired wave functions. In practice they are computed beyond the point of divergence by an extension of methods described here.

The computing laboratory has run this same problem on a large scale, high speed electronic digital computer.

Here the problem becomes that of developing the equations (3) and (4) in some numerical way suitable to calculation in discrete steps. The method used in this case, which will not be further discussed here, was the so-called modified Euler method for approximate solution of differential equations. The point of interest here is that when the problem is approached this way, the interpolation for a new ε can be done automatically. Hence it is possible to program the problem for the digital computer, put it on the computer, and wait for the computer to deliver only the best possible (within its own limitations) wave function and eigen value.

"It is easier to square the circle than to get round a mathematician."

—Augustus De Morgan

"If a man's wit be wandering, let him study mathematics; for in demonstrations, if his wit be called away never so little, he must begin again."

—Lord Bacon

BOOLEAN ALGEBRA*

DONALD G. PARRISH
Student, Baldwin-Wallace College

The distinguishing characteristic of the modern approach to mathematics is the recognition of its postulational nature. The characteristic structure of a mathematical system, an algebra for example, is like a game, dependent upon the rules agreed upon at the beginning. There are many curious algebras which result from changing the conventional rules of the game of algebra. One of these is found to be a valuable aid to reasoning, not about quantity but quality. This is Boolean Algebra, named after George Boole, who was the first to develop the analogy between mathematics and logic.

George Boole was born in 1815. By dint of almost incredible struggle he taught himself Latin, Greek, French, German, Italian—and mathematics.[1] Compelled to work as an elementary teacher in order to support his parents, Boole's opportunity finally came when in 1849 he was appointed Professor of Mathematics in Queen's College, Cork, Ireland. Here in 1854 he published his masterpiece: *An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities.*[2]

Boolean Algebra was developed and improved by C. S. Peirce (U.S.A., 1839-1914), Ernst Schroeder (Germany), and others. Its field was widened to include what is now called Symbolic Logic. These developments led to the work of G. Peano (Italy, 1858-1932), and culminated in the monumental *Principia Mathematica* of Whitehead and Russell in 1910. In this work the bases of the whole structure of mathematics were analyzed, beginning with the logical axioms of set theory and proceeding to the theorems of mathematics.

Bertrand Russell stated that "Pure mathematics was discovered by Boole in a work which he called 'The Laws of

*A paper presented at the National Convention of Kappa Mu Epsilon in 1949 at Topeka, Kansas.

Thought'—His work was concerned with formal logic, and this is the same thing as mathematics."

Begun as mathematics, Symbolic Logic is now recognized to have a much wider application. It can replace the older Aristotelian logic and do a better job.

Hans Reichenbach, in his *Elements of Symbolic Logic*, writes that modern logic can analyze structures which traditional logic has never understood, and it can solve problems the very existence of which may have been unrecognized. He believes that the study of symbolic logic is the best initiation to a scientific philosophy. It can be taught to those without special mathematical training.

This movement has fostered the recognition that every mathematical theory is based upon a small number of fundamental hypotheses or postulates from which all other propositions of the theory can be deduced. Therefore, mathematicians have invented new systems of hypotheses. Any set would serve providing there was no internal contradiction. Non-Euclidian geometries, artificial algebras, and the Theory of Abstract Groups have been accepted in the domain of mathematics. Mathematics may be considered to be a collection of hypothetical deductive systems distinguished by the assumptions adopted as bases.

Also the structure and foundations of logic have been investigated. Now we have a methodology which permits the derivation of all possible information about one term contained in a group of complex premises.

One prime reason for the need of symbolic logic is the lack of clarity and other difficulties of language. These deficiencies make a paramount deterrent to the science of logic or any science. In his book, *How to Think Straight*, R. H. Thouless stated: "The growth of the exact thinking of modern science has been very largely the result of its getting rid of all terms suggesting emotional attitudes and using only those which unemotionally indicate objective facts."

Symbolic Logic circumvents the pitfalls of language by using strictly defined symbols and rules of operation. Boolean Algebra or the Algebra of Classes may use the operational symbols of ordinary algebra but with different meanings. This will give us some insight into Boole's approach to the analogy. Other symbols have been devised but we will not consider them. The fundamental notions of this system follow: *1* indicates the universe of our discourse. It might be the universe of thinkable objects, all human beings, or all points on a blackboard. The symbol *0* indicates the null class, the class with no elements. Some of the non-existent elements of this class might be men fifteen feet tall, cats with eight tails, and so on. Notice that there is only one null class.

The letters of the alphabet may be used to designate the classes of things in our universe. The letters may designate a substantive or an adjective. For instance, x might stand for the class of things called horned, or the class of things with horns. And y might stand for the class of things called sheep. xy indicates the logical product of x and y and designates the class of horned sheep. Or x might designate the class of professors and y the class of automobile drivers. Then xy would designate the group of professors who were also drivers.

We use the notation $x + y$ to designate the logical sum of classes x and y. In the last mentioned case it would include all who were either professors or drivers, including professors who do not drive, professors who drive, and non-professors who drive. This can be shown by Euler circle diagrams.

The idea of subtraction can be introduced. Thus the class of non-drivers would be $1 - x$, the universe except the elements of class x. The class of professors who were not drivers would be $x - y$. Notice, however, that this does not follow the usual definition of subtraction. If we designate the class of non-driving professors by z, then z equals $x - y$. However z plus y does not equal x.

Another operation eliminates the need of subtraction. The symbol $'$ is a unary operator such that x' designates the class of things not x. Thus $x' = 1 - x$. And xy' means the class of things which are x and not y. Notice that $(x')' = x$.

Another symbol from algebra is used. $x < y$ means x is included in y or all the elements of x are elements of y. Now if $x < y$ and $y < x$ then $x = y$. This means that all the elements of x are elements of y. To write $x \neq y$ would mean that x and y differ by at least one element.

We can define the binary operations of logical product and sum by the operation of inclusion. Thus: $a < a + b$ and $ab < a$ and, if $a < y$ and $b < y$, then $a + b < y$. Also if $x < a$ and $x < b$, then $x < ab$. Again, if $a < b$, then $a + b = b$ and $ab = a$. The other notions can be defined:

$$aa' = 0 \qquad a + a' = 1 \qquad a + 0 = a$$
$$a0 = 0 \qquad a + 1 = 1 \qquad a1 = a$$

The basic laws of Boolean Algebra are in many cases exactly the same as those of regular algebra if the symbols are given the conventional meanings. One exception is that we can have two sets which are not empty and yet their product may be empty. In symbols:

$$a \neq 0 \qquad b \neq 0 \qquad \text{and} \qquad ab = 0$$

However, we saw before that $ab < a$. Because of this it is agreed the null class is contained in all classes.

Other exceptions are:

$$aa = a \qquad \text{and} \qquad a + a = a.$$

Therefore, there are no multiples, no fractions, no powers or roots and no numerical coefficients in the system.

It is interesting to note that if x and +, 0 and 1, $<$ and $>$, are all interchanged in any correct formula, we obtain another correct formula.

John Venn used the clarification and simplification of three club rules as an illustration of this algebra.[3] The rules are:

(1) The financial committee shall be chosen from among the general committee.

(2) No one shall be a member of both the general and library committees unless he is also on the financial committee.

(3) No member of the library committee shall be on the financial committee.

Let F signify the members of the financial committee, G for the general committee and L for the library. We can now condense Rule (1) to: All F are G, or $F < G$ or $FG' = 0$. Rule (2) becomes: All that are both G and L are F, or there are no members G and L and not F. Written $GL < F$ or $GLF' = 0$. Rule (3) becomes: The class of members both L and F is empty, or $LF = 0$.

Adding:
$$FG' + F'GL + FL = 0$$
$$FG' + F'GL + FL(G + G') = 0$$
(Permitted because $G + G' = 1$)
$$FG' + F'GL + FGL + FG'L = 0$$
$$FG'(1 + L) + GL(F + F') = 0$$
$$FG' + GL = 0 \qquad \text{(Since } 1 + L = 1 \text{ and } F + F' = 1\text{)}$$

If there are no elements in the logical sum of these classes there can be no element in either. Therefore:
$$FG' = 0 \text{ and } GL = 0$$

This means that all F are G and that there are no members both G and L. Therefore the three rules may be simplified to two:

(1) The financial committee shall be chosen from among the general committee.

(2) No member of the general committee shall be on the library committee.

For another example, if we should want to combine the operations of this system with conventional algebra it would be necessary to distinguish between the symbols. Therefore we will designate the operator for the logical sum by $\cup$, and the operator for the logical product will be $\cap$. The symbols $\cup$ and $\cap$ are read "cup" and "cap" respectively. Now the class consisting of things a or b or both will be given by $a \cup b$. And the class of things which are both a and b will be given by $a \cap b$. All the statements previously made can be rewritten with these symbols. For example, $a(b+c) = ab+ac$ becomes $a \cap (b \cup c) = (a \cap b) \cup (b \cap c)$. We will use this identity in the following discussion.

Suppose we had a list of data concerning various employees in a given organization. We might like to check the accuracy of the data. Such a list might be as follows:[4]

1000 total employees
525 colored
312 male
470 married
42 colored male
147 colored married
86 married male
25 married colored male

There are three classes of employees mentioned. Let C stand for colored, M for male, and W for married. Then let N(A) stand for the number in any class A. We have a basic theorem on the value of $N(A \cup B)$:

$$N(A \cup B) = N(A) + N(B) - N(A \cap B)$$

The truth of this theorem is easily verified from the following considerations: If we add the number of individuals in class A to the number of individuals in class B we have counted twice those individuals in both A and B: therefore subtract the number in both A and B to obtain the correct number. This theorem is easily extended to three or more classes:

$$\begin{aligned}
N(A \cup B \cup C) &= N[A \cup (B \cup C)] \\
&= N(A) + N(B \cup C) - N[A \cap (B \cup C)] \\
&= N(A) + N(B) + N(C) - N(B \cap C) \\
&\quad - N[(A \cap B) \cup (A \cap C)] \\
&= N(A) + N(B) + N(C) - N(B \cap C) \\
&\qquad - [N(A \cap B) + N(A \cap C) \\
&\qquad - N(A \cap B \cap C)] \\
&= N(A) + N(B) + N(C) - N(B \cap C) \\
&\qquad - N(A \cap B) - N(A \cap C) \\
&\qquad + N(A \cap B \cap C)
\end{aligned}$$

Substituting the number of employees in each category mentioned before:

$$N(C \cup M \cup W) = 525 + 312 + 470 - 86 - 42 - 147 + 25 = 1057$$

Therefore the data must be erroneous because the total number of employees was given as 1000.

In conclusion, there are three branches of Symbolic

Logic. The one we have just surveyed is the Logic of Classes or Concepts. The second is the Logic of Propositions. These two comprise the Algebra of Logic. They have the same postulates and theorems although they give different interpretations. The third major part is the Logic of Relations, founded by DeMorgan. This branch considers such real-life relationships as creditor-debtor, parent-child, etc. The methodology of Symbolic Logic has applications in mathematics, in language, in electrical engineering, in logic, in actuarial work, and even in biology, sociology, and psychology.

Notes

[1]See his life in E. T. Bell: "Men of Mathematics."

[2]This work has recently been reprinted. E. T. Bell ("Development of Mathematics") says: "The contents of the 'Laws of Thought' are gorgeously heterogeneous, ranging from 'The fundamental principles of symbolical reasoning, and of the expansion or development of expressions involving logical symbols' through an 'Analysis of a portion of Dr. Samuel Clarke's "Demonstration of the Being and Attributes of God" and of a portion of the "Ethica ordine geometrico demonstrata" of Spinoza' to a constitution of the intellect.' "

[3]J. Venn: "Symbolic Logic."

[4]This problem is from the "Joint Associateship Examination for Actuaries, 1935," Part 5, question 9B.

"Mathematics is the most exact science, and its conclusions are capable of absolute proof. But this is so only because mathematics does not attempt to draw absolute conclusions. All mathematical truths are relative, conditional."

—Charles P. Steinmetz

TOPICS FOR CHAPTER PROGRAMS—XV

H. D. LARSEN

43. MATHEMATICIANS AND PHILATELY
(Continued from the Spring 1953 Issue)

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"The spirit of mathematics is eternal youth."

—E. T. BELL

THE PROBLEM CORNER

EDITED BY FRANK C. GENTRY

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond the calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before March 1, 1954. The best solutions submitted by students will be published in the Spring 1954 number of THE PENTAGON, with credit being given for other solutions received. To obtain credit a solver should affirm that he is a student and give the name of his school. Address all communications to Frank C. Gentry, Department of Mathematics, University of New Mexico, Albuquerque, New Mexico.

PROBLEMS PROPOSED

60. *Proposed by Frank Hawthorne, Hofstra College, Hempstead, New York.*

Two men play a game with a deck of 45 cards. Fifteen of the cards have the letter "A" on both sides, fifteen have the letter "B" on both sides and fifteen have "A" on one side and "B" on the other. After thorough shuffling, including turning cards over, the first man cuts the pack exposing one side of one card. The second man then tries to guess the letter on the under side of the exposed card, winning if he does so and losing if he doesn't. If one man always cuts and the other always guesses, do they have equal chances of winning?

61. *Proposed by Charles Pearsall, Student, Hofstra College, Hempstead, New York.*

A disk L_0 of radius $R_1 = 1''$ is placed in the exact center of a circular floor and six other identical disks are placed around it so that each is tangent to L_0 and to one another, thus forming a ring L_1. Around this ring another ring L_2 of 6 disks of greater radius R_2 are placed so that each disk is tangent to two disks in L_2 and to two disks in L_1. Successive rings L_3, L_4, L_5, L_6, each of 6 disks of increasing radii $R_3 < R_4 < R_5 < R_6$, are placed in the same way. If the disks of L_6 are tangent internally to the edge of the floor, what is the diameter of the floor?

62. *Proposed by C. W. Trigg, Los Angeles City College, Los Angeles, California.*

Decipher the following anagrams of the names of 19 mathematicians: 1) NO CAB, 2) NO CART, 3) UNCLE EVAN, 4) HIS PAL LOU, 5) RAN MINE, 6) SHE CAN RUN THIS, 7) HOT ROD USE, 8) U. S. GAS, 9) TANK, 10) WE START, 11) CRUISE UP, 12) VINE ST., 13) CUT A RAIL, 14) IN SLOW, 15) ZONE, 16) LOST A TIRE, 17) NOT NEW, 18) NO RAVING, 19) CHASE ON RIM.

63. *Proposed by David T. Benedetti, University of New Mexico, Albuquerque, New Mexico.*

A golf pro wishes to arrange a tournament for the sixteen members of his club. They are to play in foursomes and each member is to play with every other member once and only once. Show how the rounds are to be arranged.

64. *Proposed by the Editor.*

There are four fractions of the form a/b, $a < b < 10$, such that $N \cdot a/b$, where N is a two-digit integer is equal to N with its digits interchanged. Find the fractions and the corresponding values of N.

SOLUTIONS

34. *Proposed by Frank Moseley, State Teacher's College, Florence, Alabama.*

Substantiate the statement made by Altschiller-Court in his College Geometry (Page 66) that a triangle may have equal external bisectors and not be isosceles.

Solution by Sharon Murnick, Hofstra College, Hempstead, New York.

Let A, B, C be the vertices and a, b, c the lengths of the opposite sides of the triangle. Let t_a be the length of the external bisector of angle A and let d be the length of the external segment cut off the side BC by t_a. Let t_a meet BC in D. Then from triangles ABC and ACD by the Law of Cosines we have

$c^2 = a^2 + b^2 - 2ab \cos BCA$ and $t_a^2 = d^2 + b^2 + 2db \cos BCA$. Hence $t_a^2 = d^2 + b^2 + d(a^2 + b^2 - c^2)/a$. Also, since

t_a is an external bisector, $(a + d)/d = c/b$ or $d = ab/(c-b)$. By substitution we obtain

1) $t_a^2 = bc(2bc - b^2 - c^2 + a^2)/(c - b)^2$.

From symmetry

2) $t_b^2 = ac(2ac - a^2 + b^2 - c^2)/(c - a)^2$.

Set $t_a^2 = t_b^2$ and, with no loss of generality take $a = 1$. Then, since $b \neq 1$, the resulting equation becomes

3) $c^4 + c^3(-2b - 2) + c^2(b^2 + 5b + 1) + c(-4b^2 - 4b) + (b^3 + 2b^2 + b) = 0$

Now, if we let $c = b + x$, then since $c < b + a$, $x < 1$ and since $b < c + a$, $x > -1$. Substituting $c = b + x$ in 3) and noting that $x - 1 \neq 0$ we obtain

4) $b^2(x + 1) + b(2x^2 + x - 1) + x^3 - x^2 = 0$

From 4)

$$b = \frac{-2x^2 - x + 1 \pm \sqrt{(4x^3 + x^2 - 2x + 1)}}{2x + 2}$$

Now take $b > a$. This may be done without loss of generality. Then the expression for b is greater than 1 and since $2x + 2$ is positive for $-1 < x < 1$, we have

5) $\sqrt{(4x^3 + x^2 - 2x + 1)} > 2x^2 + 3x + 1$

where the positive sign must be chosen on the radical when $x \geqq -\frac{1}{2}$.

The inequality 5) then reduces to

$-x(x + 1)(x^2 + x + 2) > 0$, so that $x < 0$.

Choosing $x = -\frac{1}{2}$ we find $a = 1$, $b = (2 + \sqrt{7})/2$, $c = (1 + \sqrt{7})/2$ and $t_a^2 = t_b^2 = 9(3 + \sqrt{7})/4$.

Also solved by George Ladner, Washburn University, Topeka, Kansas.

56. *Proposed by C. E. Denny, Central College, Fayette, Missouri.*

A student in Analytic Geometry obtained the equation $x(3 - 4\sqrt{3}) + y(3\sqrt{3} + 4) + \sqrt{3} - 32 = 0$ for a certain line. His book gave the answer $x(48 - 25\sqrt{3}) - 11y - 137 + 100\sqrt{3} = 0$. Show that the two equations represent the same line.

Solution by Charles Pearsall, Hofstra College, Hempstead, New York.

Multiplying both sides of the first equation by $4 - 3\sqrt{3}$ reduces it to the second equation.

Also solved by John Manias, Jr., University of New Mexico; Sharon Murnick, Hofstra College; Herbert D. Kivligha, Hofstra College; Carl Dulgaroff, Central College; and Walt Braiserd, Roosevelt High School, Des Moines, Iowa.

57. *Proposed by Harold Skelton, Southwest Missouri State College Springfield, Missouri.*

If $N = n_1 + n_2 + n_3 + \cdots + n_r$, where n_i is a positive integer, show that $\dfrac{N!}{(n_1!)\,(n_2!)\,(n_3!)\cdots(n_r!)}$ is an integer.

Solution by Sharon F. Murnick, Hofstra College, Hempstead, New York.

Since the formula given represents the number of permutations of N things taken altogether where n_1 are alike, n_2 are alike $\cdots$ n_r are alike; the expression is obviously an integer.

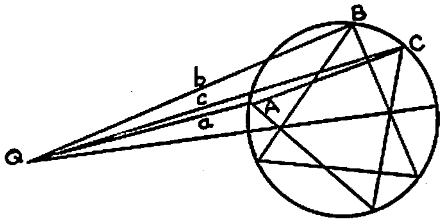
Also solved by Charles Pearsall, Hofstra College.

58. *Proposed by Victor L. Osgood, Oceanport, New Jersey.*

Given three concentric circles of radii a, b, and c respectively, where $a < b < c$. If the radii do not differ too widely, it is possible to construct a fourth circle which will intersect the three concentric circles at such points that two equilateral triangles may be formed by connecting properly chosen points of intersection. In terms of a, b, and c what is the distance from the common center to the center of the fourth circle and what is the radius of the fourth circle?

Solution by Charles Pearsall, Hofstra College, Hempstead, New. York.

Let O be the center of the fourth circle and inscribe any two equilateral triangles therein. The center Q of the concentric circles lies on one of the diameters of the circle which is a line of symmetry for the two triangles.



Let A, B, and C be the vertices of the triangles which lie on one side of the line of centers and let a, b, and c be the radii of the concentric circles. Then if angle QOA $= \alpha$, angles QOB and QOC are $2\pi/3 - \alpha$ and $2\pi/3 + \alpha$ respectively. From the Law of Cosines, we have, if $h = QO$

$$a^2 = h^2 + R^2 - 2hR\cos\alpha$$

$$b^2 = h^2 + R^2 - 2hR\cos(2\pi/3 - \alpha)$$

$$= h^2 + R^2 + hR(\cos\alpha - \sqrt{3}\sin\alpha)$$

$$c^2 = h^2 + R^2 - 2hR\cos(2\pi/3 - \alpha)$$

$$= h^2 + R^2 + hR(\cos\alpha + \sqrt{3}\sin\alpha)$$

Adding these expressions gives

1) $a^2 + b^2 + c^2 = 3(h^2 + R^2)$. Also,

$c^2 - b^2 = hR(2\sqrt{3}\sin\alpha)$, $c^2 - a^2 = hR(3\cos\alpha + \sqrt{3}\sin\alpha)$

$b^2 - a^2 = hR(3\cos\alpha - \sqrt{3}\sin\alpha)$. From these expressions it follows that

2)
$$9h^2R^2 = \frac{(c^2 - a^2)^2 + (b^2 - a^2)^2 + (c^2 - b^2)^2}{2}$$

$$= a^4 + b^4 + c^4 - a^2b^2 - a^2c^2 - b^2c^2 = K.$$

Solving equations 1) and 2) simultaneously for h^2 and R^2 we obtain

$$h^2 = [a^2 + b^2 + c^2 + \sqrt{(a^2 + b^2 + c^2)^2 - 4K}]/6$$

and for R^2, the same expression with a negative sign on the radical.

59. *Proposed by John R. Green, University of New Mexico, Albuquerque, New Mexico.*

A man lives on a river bank one mile below a bridge. On one occasion he started upstream in his motorboat. As he passed under the bridge his hat fell overboard and floated downstream. After continuing upstream a way he missed the hat, turned about and overtook it in five minutes at a point just opposite his house. What was the speed of the river current?

Solution by Paul Hawthorne, Eighth Grade, Northern Parkway School, Uniondale, New York.

The hat represented a stationary point in the water. Hence, the time spent by the boat going to and from the hat was the same. The hat was in the water 10 minutes, during which time the stream moved one mile. Therefore, the speed of the river current was six miles per hour.

Also solved by Sharon Murnick and Charles Pearsall, both of Hofstra College.

WHAT COLOR WAS THE BEAR?

Frank Hawthorne

Faculty, Hofstra College

Among the perennial problems of elementary mathematics there is one "oldie" which refuses to die. Within the year it and its alleged solution were given new vigor by publication in a national magazine. It seems unlikely that this note will put an end to it, but the author hopes for its early demise.

As usually stated, the problem is, "A hunter set out from his camp, walked ten miles South, then ten miles East, then ten miles North and thus arrived at his camp again. He shot a bear. What color was the bear?"

The standard solution is, "Since the North Pole is the only possible point on the earth at which his camp could have been located, the bear was white."

This is very neat. It is so neat that it has frequently been cited by the unwary as an outstanding example of applied logic. One of the best selling popularizations of mathematics includes it. The only trouble is that it just "doesn't follow".

The conditions of the problem simply do not require the camp to be at the North Pole. A little reflection will cause one to realize that there are possible campsites in the Antarctic which also satisfy the conditions. The conditions do define a very interesting locus first discussed (as far as the author has been able to determine) by E. J. Moulton.[1]

In his words, "He must have started at some point of a locus S which consists of the North Pole and the circles which are ten miles north of the parallels of latitude whose circumferences are 10/n miles, where n ranges over the positive integers . . . It consists of an isolated point and an infinite set of circles which have as a limit the parallel of latitude which is ten miles from the South Pole; this limit circle is not a part of the locus."

Now, the color of bears (if any) indigenous to points on this locus seems to the author to be undetermined. It is recommended that the problem of finding the locus of possible campsites be retained and extended by replacing 10 by x and considering S as a function of x for positive x less than the semi-circumference of the earth. This results in an extremely interesting discontinuous functional relationship. But let us have no complications concerning bears, or their color.

By the way, how far north does Ursus Maritimus range? And is white a color?

[1]E. J. Moulton. *A Speed Test Question... A Problem in Geography.* American Mathematically Monthly. Vol. 51, Pages 216 and 220, April 1944.

THE MATHEMATICAL SCRAPBOOK

EDITED BY H. D. LARSEN

These things surely lie on the knees of the gods.

—HOMER

= ∇ =

"We are told that when a lad who had just begun geometry asked, 'What do I gain by learning all this stuff?' Euclid made his slave give the boy some coppers, 'Since,' said he, 'he must make a profit out of what he learns.' "

—W. W. RUPERT

= ∇ =

A CURIOUS PUZZLE. Given a square box having an area of 27 square inches on its floor and having vertical sides, and filled with water to a depth of 2 inches, it is required to find the size of a heavy cube which, when resting on the bottom of the box, will have its upper surface high and dry above the surface of the water.—THOMAS ALEXANDER in *Nature*, June 5, 1902.

= ∇ =

$\pi = 3(1 + 1/20)(1 - 1/400 - 1/6000)$, approximately.

= ∇ =

A "PROOF" OF WILSON'S THEOREM: To show that

$$(p - 1)! + 1 = M\cdot p.$$

This is equivalent to

$$p! - 1! + 1 = p! = M\cdot p.$$

—BOLETIN MATEMATICO, 1929

= ∇ =

"The student of mathematics who in the course of a single introductory lecture on the calculus completes the differentiation of the function x^n would be a good deal soothed to know that he has covered in an hour a problem which took the generation of Barrow, Newton, and Leibniz about forty years to clear up."—L. HOGBEN, *The Realist*, Dec., 1929.

Multiplication without carrying:

$$\begin{array}{r} 67432 \\ \times 34 \\ \hline 48628 \\ 22110 \\ 81296 \\ 12100 \\ \hline 2292688 \end{array}$$

= ∇ =

"The next world might be four-dimensional, and the spirits of the dead who inhabit it could easily enter confined three-dimensional space. That was Caley's ingenious theory of Ghosts."—W. J. LOCKE, *The House of Baltazar.*

= ∇ =

This cryptarithm has one solution, $Y \neq 0$.

$$\begin{array}{l} LMN)RSTUN(UX \\ RTYX \\ \hline TYYN \\ TYYJ \\ \hline Y \end{array}$$

—AM. MATH. MONTH.

= ∇ =

"The purpose of mathematical teaching in the secondary school should be to increase and *widen* the range of the scholar's mathematical knowledge, so that he can deal more intelligently with the quantities and spatial features of the world about him. This implies not only the learning of a considerable body of exact knowledge covering a wide field of mathematics, but also a realization on the scholar's part that although mathematics can be made to look dismayingly abstract, it nevertheless has its roots in the real world and has risen from the needs of man to measure land, capacity, time, motion, etc." — B. EVANS, *Math. Gazette*, December, 1949.

Which of these columns will give the larger sum?

1 2 3 4 5 6 7 8 9	1
1 2 3 4 5 6 7 8	2 1
1 2 3 4 5 6 7	3 2 1
1 2 3 4 5 6	4 3 2 1
1 2 3 4 5	5 4 3 2 1
1 2 3 4	6 5 4 3 2 1
1 2 3	7 6 5 4 3 2 1
1 2	8 7 6 5 4 3 2 1
1	9 8 7 6 5 4 3 2 1

= ∇ =

It may not be generally known that Professor E. T. Bell, the eminent author of *The Development of Mathematics, Men of Mathematics, Queen of the Sciences, The Handmaiden of the Sciences, The Search for Truth,* and other distinguished mathematical works, has written popular fiction under the pseudonym of John Taine. Some of Taine's novels are:

The Purple Sapphire (Dutton, 1924)
The Golden Tooth (Dutton, 1927)
Quayle's Invention (Dutton, 1927)
Green Fire (Dutton, 1928)
The Greatest Adventure (Dutton, 1929)
The Iron Star (Dutton, 1930)
White Lily (*Amazing Stories Quarterly,* Jan., 1930)
Seeds of Life (*Amazing Stories Quarterly,* 1931)
Before the Dawn (Williams and Wilkins, 1934)

= ∇ =

"Why the excitement of intellectual activity pleases is not here the question; but that it does so is a general and acknowledged law of the human mind. We grow attached to the mathematics only from finding out their truth; and their utility chiefly consists (at present) in the contemplative pleasure they afford to the student. Lines, points, angles, squares, and circles are not interesting in themselves; they become so by the power of the mind exerted in comprehending their properties and relations."

—W. Hazlitt

John Wallis (1616-1703), the Savilian Professor at Oxford, in middle life developed, for his own amusement, his powers of mental arithmetic. As an illustration of his achievements: when in bed one day, he occupied himself in finding mentally the integral part of the square root of 3×10^{40}; several hours afterward he wrote down the result from memory.

= ∇ =

$\pi = 3{\cdot}14159265358979323846264$

Wie, o dies π macht ernstlich so vielen viele Muh?
Lernt immerhin, Jünglinge, leichte Verselein
Wie so zum Beispiel dies mochte zu merken sein.

—Zacharias Dahse

= ∇ =

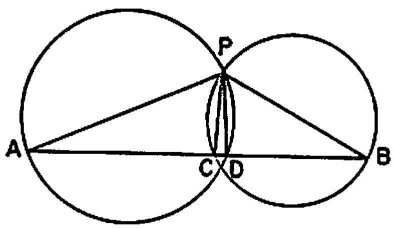


Figure 2

Consider two intersecting circles having diameters PA and PB. Draw AB cutting the circles at C and and D. Then angle PDA is a right angle since it is inscribed in a semicircle; therefore PD is perpendicular to AB. Similarly angle PCB is a right angle and PC is perpendicular to AB. Thus it is possible to drop two perpendiculars from an external point to a line!

= ∇ =

"An ambassador from the Netherlands remarked to King Henry VI that France did not have any geometrician who was able to solve a problem propounded in 1593 by his countryman, Adrian Romanus, to all the mathematicians of the world, and which required the solution of an equation of the 45th degree. Whereupon the king summoned Vieta and informed him of the challenge. Vieta was able to furnish two solutions in a few minutes, as the problem was related to some work done earlier by Vieta on the expression of sin $n\theta$ in terms of sin θ and cos θ."

—G. A. Miller

The haversine function,

$$\text{hav } A = \tfrac{1}{2}(1 - \cos A) = \sin^2 \tfrac{1}{2}A,$$

generally is omitted in the study of plane trigonometry. However, haversines may be applied to the solution of plane triangles. Thus, if b and c are given in the right triangle ABC, then

$$\text{hav } A = (c - b)/2c.$$

Again, given the three sides of a general triangle,

$$\text{hav } A = (s - b)\ (s - c)/bc.$$

If two sides and the included angle are given, then

$$a^2 = (b - c)^2 + 4bc \text{ hav } A.$$

= ∇ =

"One of Gauss's last acts was, a little while before his death, to have engraved at the foot of his portrait, as giving the best summary of his views and labors, the lines from Shakespeare's 'King Lear':

Thou, Nature, art my goddess; to thy laws
My services are bound."

—Popular Sci. Mo., 1888

= ∇ =

A NOTE ON KIRKMAN'S SCHOOLGIRL PROBLEM: — "Prof. Sylvester, F.R.S., gave an account of results arrived at in his communication 'on the fifteen young ladies problem and a general mathematical theory of pure syntax.' The problem, which was first considered by Mr. Sylvester more than twenty-five years ago, was not at that time published by him: it was then discussed by Prof. Cayley, next proposed by Rev. T. P. Kirkman in the 'Lady's and Gentleman's Diary' for 1850: solutions were given in the 'Diary' for 1851; but it was not until the year 1862 that an elaborate solution was given by Mr. W. S. B. Woolhouse in the volume for that year. The problem may be enunciated as follows:—'In a school of fifteen girls, a rule has been laid down that they shall walk out every day in rows of threes, but that the same two girls shall never come together twice in the same row. The rule is supposed to have been carried out correctly during the six working days of the week, but when the time comes for their going to church together on Sunday it is found to be absolutely impossible to continue it any further. Can the rule have been carried out correctly during the six previous days?"

—Nature, Nov. 18, 1875, p. 57. Report of meeting of London Math. Soc. on Nov. 11, 1875.

THE BOOK SHELF

EDITED BY FRANK HAWTHORNE

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Frank Hawthorne, Hofstra College, Hempstead, New York.

Mathematical Recreations, by Maurice Kraitchik, Second Revised Edition, Dover Publications Inc., (1780 Broadway), New York, 321 pp. Paper $1.60.

Occasionally there appears on the market, at a reasonable price, a book which catches and holds the attention of all who come into contact with it. This inexpensive reprint is such a book.

"Mathematics is applied logic in its simplest and purest form," says Professor Kraitchik, and he illustrates his statement with several thought provoking problems. In this class also belong the problems of the queen, the knight, and the rooks of the chessboard. The terminology of these chapters is such that the problems are readily understood by readers not familiar with chess. The chessmen form a question of objects in cells on rectangular portions of planes, and since previous chapters are concerned with magic squares—to which the author gives considerable space—and permutational problems, solutions are easily comprehended by anyone who follows the explanations.

One chapter is devoted to a number of interesting problems having algebraic solutions. Geometry enters via the Pythagorean Theorem with elementary trigonometric relations included. In a later section emphasis is placed on the dissection and combination of polygons. The discussion of the laws of chance is quite extensive with tables showing the probabilities of the results of play of dice and cards. Calendars and various games are discussed in separate chapters to complete the book.

Mathematical Recreations is a collection of the most varied types of mathematical entertainment. Each is treated specifically and individually. The problems are broken down and the solutions are given in direct and simple terms. It is fascinating relaxation for anyone.

The paper-bound edition is adequate for personal use, but it will not stand abuse and cannot be rebound easily. If a more durable book is required, a cloth edition is available at $3.50.

—BERNADINE LAW

An Introduction to the History of Mathematics, by Howard Eves, Rinehart and Company, (232 Madison Ave.) New York, 422 pp. $6.00.

This interesting book discusses in a clear and nonponderous style the more important men, ideas, and trends in the history of mathematics from its beginnings in "primative counting" to the end of the seventeenth century. A final chapter contains an admittedly sketchy treatment of the more remarkable achievements of the next two centuries.

The whole book has been written with what the reviewer considers admirable restraint. This is particularly evident in the treatment of the Tartaglia, Cardan story.

The really unusual contribution of this book is the inclusion of quite extensive "Problem Studies" at the end of each chapter. These contain carefully chosen exercises whose solution is intended to give greater insight into the types of thinking discussed in the chapter. These problems are mostly of a nonroutine nature. The selection of problems seems particularly good and reflects the unusual experience of the author in solving, proposing, and editing problems. Any undergraduate student will find the careful solution of these problems a profitable experience.

At the end of the book there is a collection of hints, suggestions, and answers for the problem studies. These are doubtless intended to prevent the student from getting too far off the track. They should prove very helpful.

Each chapter is followed by a short bibliography. The items included are, for the most part, pertinent standard books in print and not too specialized. A general bibliography, a chronological table, and an unusually complete index are also attached.

This book should prove useful in many ways. As a textbook for undergraduate courses in "History of Mathematics," it may soon become the most common choice. As a source of material for club programs, it will prove fruitful. The secondary teacher who is in search for enrichment material for use with his more capable students should look here.

No typographical errors were noted, but no special search for them was undertaken.

—F. HAWTHORNE

Logic for Mathematicians, by J. Barkley Rosser, McGraw Hill, (330 West 42nd St.,) New York, 1953. 14+530 pp. $10.00.

There are many books devoted to the study of logic as mathematics, the art, but they do not indicate its uses in applied mathematics. Professor Rosser attempts to give a reader with mathematical maturity, "a precise knowledge of the logical principles which he uses in daily mathematics."

To fulfill this endeavor, the author begins with a general discussion of symbolic logic, its purpose, and its terminology. After an explanation of symbolism, he presents an axiomatic treatment of the statement calculus and a development of the restricted predicate calculus with illustrations of its applications. This is followed by chapters devoted to the fundamental concepts of mathematics, including equality, description, class membership, relations and functions, cardinal numbers, ordinal numbers, and counting. Each of these chapters contains a section on the applications of the topic in "daily mathematics."

A full chapter is devoted to the axiom of choice. Three popular views of the axiom are considered together with a discussion of its relative necessity in mathematics. The chapter ends with an investigation of the denumerable axiom of choice. The last chapter contains a general view of the development with an indication of how it may be continued into the more familiar fields of mathematics.

The object of the book—to present a logical system for application—seems to be achieved. Professor Rosser's illustrations of logic used in various fields of mathematics clarify this achievement. In style and language the book is readable although one will have some difficulty until the symbolism is mastered. Since numerous exercises have been included in the book, it may be used as a text for classroom instruction. In this case, the student should have mathematical maturity represented by college courses beyond the calculus which stress mathematical reasoning.

—RICHARD LAMM

Principles of Mathematical Analysis, by Walter Rudin, New York, McGraw-Hill. (330 West 42nd Street, New York 26, N.Y.) 1953. 227 pp. $5.00.

It is the author's intention that this book be used as a text for an advanced undergraduate or a first year graduate course in analysis. Recent trends in mathematical research have created a need for a modern text to serve as a bridge between undergraduate and graduate courses in analysis. It is this reviewer's opinion that in many respects this book fills that need.

The first chapter gives a development of the real and complex number systems. Chapter 2 treats the rudiments of set theory. Chapters 3, 4, and 5 are devoted to material usually included in a course on advanced calculus, treating numerical sequences, continuity, and differentiation, respectively. A well-handled, compact treatment of the Riemann-Stieltjes integral is the subject material of Chapter 6. Chapters 7, 8, and 9 are again concerned with material found in advanced calculus courses; sequences and series of functions, and functions of several variables. (A notable exception to this is the inclusion of the Stone-Weierstrass theorem in chapter 7). Chapter 10 begins with a short but clear treatment of the theory of Lebesque measure for n-dimensional Euclidean space and then proceeds to the more abstract concept of measure spaces, in which the theory of Lebesque integration is developed. The chapter ends with a comparison of Lebesque and Riemann integration and an introduction to the functions of class L^2.

This reviewer feels that a slightly more comprehensive introduction to the subject matter of topology and vector space theory would have been advantageous. For example, the usual topological definition of continuity in terms of open sets or neighborhoods might have been employed.

In general, the author's exposition is exceptionally clear, if somewhat brief. Many cumbersome and tedious details of proofs are left to the student—a procedure which in some respects is desirable, since many beginning graduate students "can't see the theorem for the epsilons." A wealth of provocative exercises adds to the usefulness of the book.

This book should be highly appreciated by teachers and serve as a pleasant introduction to analysis for many students.

—CHARLES J. HALBERG, JR.

ALSO RECEIVED BY THE BOOK SHELF EDITOR

Beginning Algebra for College Students, Loyd L. Lowenstein, John Wiley and Sons Inc. (440 Fourth Avenue) New York, 9+279 pp., $3.50.

Calculus, C. R. Wylie Jr., McGraw Hill Book Company, (330 West 42nd St.) New York, 10+565 pp. $6.00.

Engineering Statistics and Quality Control, Irving W. Burr, McGraw Hill Book Company, (330 West 42nd St.) New York, 7+442 pp., $7.00.

Flatland, Edwin A. Abbott, Dover Publications Inc., (1780 Broadway) New York, 109 pp. $1.00.

An Introduction to Statistics, Charles E. Clark, John Wiley and Sons Inc., (440 Fourth Ave.) New York, 10+266 pp., $4.25.

Introduction to the Theory of Functions of a Complex Variable, Wolfgang J. Thron, John Wiley and Sons Inc., (440 Fourth Ave.) New York, 9+230 pp., $6.50.

Sampling Techniques, William G. Cochran, John Wiley and Sons Inc., (440 Fourth Ave.) New York, 14+330 pp. $6.50.

Stochastic Processes, J. L. Doob, John Wiley and Sons Inc., (440 Fourth Ave.) New York, 7+654 pp., $10.00.

Survey of Modern Algebra, Revised Edition, Garrett Birkhoff and Saunders MacLane, The Macmillan Company, (60 Fifth Ave.) New York, 11+472 pp., $6.50.

A Brief Survey of Modern Algebra, Garrett Birkhoff and Saunders MacLane, The Macmillan Company, (60 Fifth Ave.) New York, 8+276 pp., $4.75.

Trigonometry, John F. Randolph, The Macmillan Company, (60 Fifth Ave.) New York, 10+220 pp., $3.00.

"A scientist worthy of the name, above all a mathematician, experiences in his work the same impression as an artist; his pleasure is as great and of the same nature."

—HENRI POINCARE

INSTALLATION OF NEW CHAPTER

EDITED BY J. M. SACHS

THE PENTAGON is pleased to report the installation of Pennsylvania Beta Chapter of Kappa Mu Epsilon.

PENNSYLVANIA BETA CHAPTER
La Salle College, Philadelphia

Pennsylvania Beta Chapter was installed and seventeen charter members of that chapter initiated in an afternoon ceremony at La Salle College on May 19, 1953. Professor L. F. Ollman of Hofstra College served as installing officer.

Professor Ollman addressed the group before the formal initiation. Following the initiation there was a banquet for members and guests.

Brother G. Paul, F.S.C., President of LaSalle College was among the initiates. The faculty sponsor of Pennsylvania Beta is Brother Damian Connelly, F.S.C.

We welcome Pennsylvania Beta into our fellowship. The staff of THE PENTAGON wishes to join the national officers in extending congratulations and heartiest best wishes to our newest chapter.

NOTICE

Professor Dana R. Sudborough, Central Michigan College of Education, Mount Pleasant, Michigan has been appointed Business Manager of THE PENTAGON. Correspondence regarding subscriptions, back numbers, reprints, and advertisements should be addressed to Professor Sudborough.

THE NINTH BIENNIAL CONVENTION

The Ninth Biennial Convention of Kappa Mu Epsilon was held at St. Mary's Lake Camp, Battle Creek, Michigan with Michigan Alpha of Albion College, Michigan Beta of Central Michigan College, and Michigan Gamma of Wayne University as hosts.

One Hundred fifty-eight official delegates and members were registered as follows:

	Students	Faculty		Students	Faculty
Colorado Alpha	1	1	Missouri Alpha	3	1
Illinois Alpha	5	1	Missouri Beta	2	1
Illinois Beta	1		Missouri Epsilon	4	1
Illinois Gamma	1	1	Nebraska Alpha	8	2
Indiana Alpha	3	1	New Jersey Alpha	1	
Indiana Beta	3		New Jersey Beta	4	1
Iowa Alpha	4	2	New York Alpha	8	4
Iowa Beta	2	1	North Carolina Alpha	5	1
Kansas Alpha	1		Ohio Alpha	4	2
Kansas Beta	7	3	Ohio Gamma	2	2
Kansas Gamma	5		Oklahoma Alpha	3	2
Kansas Delta	4	2	Pennsylvania Alpha	1	
Kansas Epsilon	1	1	Tennessee Alpha	4	1
Louisiana Beta	1		Texas Gamma	2	
Michigan Alpha	10	3	Texas Epsilon	2	
Michigan Beta	5	2	Wisconsin Alpha	6	2
Michigan Gamma	4	3			

Eight student papers were read at the convention and eight were presented by title. Those read were:

1. *Naperian Logarithms,* Gary Drown, Iowa Beta
2. *Some Fundamental Mathematical Princiles Underlying Visual Art,* Bernadine Law, Kansas Gamma
3. *Certain Geometrical Properties of the Tesseract,* Henry Beersman, Missouri Alpha
4. *A Device for Drawing the Locus of Points,* Raoul Pettai, Colorado Alpha
5. *Mathematics, Pure or Otherwise,* Nancy Marsh, Kansas Delta
6. *The Line, Circle, and Parabola Using Complex Numbers,* Joe E. Ballard, Texas Epsilon
7. *Solving a Differential Equation on a Differential Analyzer,* Joseph Weizenbaum, Michigan Gamma

8. *A* 1953 *Bridge Problem*
   Nancy Hartup, Ohio Gamma

   Supplementary Papers:

9. *On the Euler Totient Function,*
   Miles E. Vance, Ohio Alpha
10. *Flexible Cables,*
    Eugene Nadig, Iowa Beta
11. *Calculus in Electricity and Magnetism,*
    Bert Helmick, Iowa Beta
12. *The Significance of Convergence,*
    Ruth Renwick, Wisconsin Alpha
13. *Measure of an N-Dimensional Frustrum of a Pyramid,* Bill Northrip, Missouri Alpha
14. *A Checkerboard Problem,*
    Norma L. Jones, Missouri Alpha
15. *The Theory of Relativity of Applied Mathematics,*
    William Charles Spengler, Illinois Alpha
16. *A Reverse Notation for Numbers,*
    Clyde Dilley, Iowa Alpha

The national officers were elected during the second business session to serve for the biennium 1953-1955. The results of the election were:

President	Charles B. Tucker, Kansas Beta
Vice-President	C. C. Richtmeyer, Michigan Beta
Secretary	E. Marie Hove, New York Alpha
Treasurer	M. Leslie Madison, Colorado Alpha
Historian	Laura Z Greene, Kansas Delta

One petition, that of La Salle College, Philadelphia, Pennsylvania, was presented to the convention for consideration. The petition was approved by the national council and the convention delegates.

Professor M. Leslie Madison of Colorado Alpha, Chairman of the Committee on the Revision of the Constitution, Article II, presented the results of the study made by the committee. The committee recommended that Article II, Section 2 of the constitution of Kappa Mu Epsilon be allowed to remain in its present form. The delegates of the convention voted to accept the recommendation of the committee.

Report of the President

I have attended seven of the nine biennial conventions, but this is the first time I have had the privilege of presenting the national president's report—perhaps it is because at the last biennial convention you granted me the privilege of being your national president for the past two years.

Somehow this convention seems different from the preceding ones in other than the physical aspects—again, perhaps because I am looking at it from a different point of view. In fact, you seem to represent fifty problems all tied into one. We have fifty chapters now with several more in immediate prospect. Three of the fifty chapters are inactive; these present special problems in themselves. Some of our chapters are located in relatively small colleges, some in large colleges, some are in denominational colleges, some in state colleges, some in privately endowed institutions. Some of the colleges are in the Midwest, some in the East, and some in the West. Some chapters are large; some are small. Some chapters have felt the effect of decreased enrollments and economic pressure.

Each chapter has its own personality, its own individuality, and its own characteristics. I am sure that is the way it should be. We must not attempt to mold each chapter into a standard form. In so far as possible we must preserve the autonomy of the individual chapters. A chapter's characteristics will change with its membership and especially with its faculty advisors. But education thrives under freedom, and true education dies under dictatorship.

Kappa Mu Epsilon is an Honorary Mathematical Society. We have set ourselves up to be the judge of what is "honorary." Our constitution sets up *minimum* standards and conditions so that we may deserve the term honorary and be a society. Our standards must be fulfilled by each chapter or we are no longer honorary; our reports and records must be filed with the national or we are no longer a society. Our enthusiasm and inspiration must be maintained at all times or we have no reason for existence. So each chapter under its autonomy must uphold its standards and provide the enthusiasm and inspiration needed.

Some members of Kappa Mu Epsilon get quite concerned because other chapters are not holding their standards up as high as they are. Others get concerned because their local programs are not as good as others. To them I would like to say this: "You must realize that things are not as bad as they seem, nor are they as good as they appear." Isn't that a silly statement? Yet, it says exactly what I mean. If we could analyze the difference between several chapters either quantitatively or qualitatively, we would find most of the difference purely in our imagination. Inspiration and enthusiasm, credit, and honor do not yield readily to quantitative and qualitative analysis. Minimum standards must be maintained by all chapters, but the national organization itself must not set these standards so high that they will penalize, yes even destroy, any chapter.

The colleges in general have been operating through a period of lower enrollments. The decrease in graduating classes will continue until 1955. It will not be until 1960 that our graduating classes will equal the 1952 level of 325,000. Estimating that the average Kappa Mu Epsilon member is initiated at the end of his second or beginning of his third year, that means that we are at the lowest point of this cycle. The next biennium should show an improvement in the number of initiates per chapter on the average.

In spite of these conditions, Kappa Mu Epsilon has been growing in number of chapters at the usual or normal rate of about two chapters per year. At the convention in 1951, we approved two new chapters—Louisiana Beta at Southwestern Louisiana Institute at Lafayette and Texas Epsilon at North Texas State College at Denton. During 1952 we installed Indiana Beta at Butler University at Indianapolis, and Kansas Epsilon at Fort Hays College at Hays. I feel that the next biennium should show an even better growth both in the number of chapters and in the size of the present chapters. The future of Kappa Mu Epsilon appears to me to be very bright in these respects.

Newton once observed that if he had seen further than most men, it was because he had stood on the shoulders of giants. Analogously, I would like to observe that if we have been successful during the past biennium, it is because we had the support of some very fine national officers.

Miss Hove has given graciously of time and energy in order to keep our national records in order. It is essential that every chapter keep the national secretary informed accurately and completely of initiations and other reports. I wish to express publicly my appreciation for the fine job she has done.

Dr. Ollman has taken care of our finances and paid our bills—not out of his own pocket of course. These are essential and important tasks in our organization. Dr. Richtmeyer as vice-president, Miss Greene as historian, and Dr. Van Engen as past president have all given me their support and I am deeply appreciative of their fine and loyal support of Kappa Mu Epsilon.

Dr. Larsen has done an exceptionally fine job on our publication, THE PENTAGON. It was with deep regret that I accepted his resignation as editor. However, he has consented to continue as business manager, and I know that he will continue his fine work for Kappa Mu Epsilon in that capacity. Dr. Fronabarger has taken over the editorship of THE PENTAGON, and I feel sure that he will continue the high quality and fine work that Dr. Larsen has set as a precedent.

Still others have served on various committees throughout the biennium. To all of these, I should like to express my thanks and appreciation.

As for the future of Kappa Mu Epsilon, I have prophesied continued and more rapid growth both in the size of the present chapters and in the number of chapters. In fact, by 1960 I believe that it is possible that the size of Kappa Mu Epsilon may have doubled—of course depending on the desires and functioning both of the national officers and the individual chapters. Problems, we will always have with us. The problem of standards, the problem of finances, the changes that appear needed in the constitution, and many other details especially due to the increased size of this organization. However, we are mathematicians and I am sure that we will solve these problems where the assignment is made, and by the time the assignment is due.

I have one recommendation to make both to the national and to the local organizations. I believe that each chapter individually, and the national itself should set as an objective a capitalization of about $100 per chapter. Then during the biennium, the treasuries should attempt to build up a reserve above this capitalization to defray the convention expense. The national is getting very close to this objective. This is the status of the Kansas Beta treasury, and it seems to give us an economic stability entirely satisfactory for our needs. I believe there is no value to capitalizing above this figure although some of the larger chapters may want a larger amount.

In closing, I'd like to express to the membership my appreciation for the opportunity to serve you during the past two years. It has meant more than I had expected, but I have enjoyed it and I hope that I have served you well. There has been only one essential thought on my mind and that is "the welfare of Kappa Mu Epsilon for its undergraduate members."

CHARLES B. TUCKER
National President

Summary of the Report of the Vice-President

In the spirit of the tradition of brevity of vice-presidential reports, Professor Richtmeyer said, "The duties of the vice-president are not onerous. In the main they consist of assisting the president in any way he requests, and in standing ready to take over the duties of the president in case he is incapacitated. President Tucker has been very capable and efficient and has not had to call upon me very much for assistance. In addition he has been disgustingly healthy and as a result I have had practically nothing to do. This I have done with great diligence."

Summary of the Report of the National Secretary

A survey of some of the statistics for the biennium shows that four new chapters have been installed.

Louisiana Beta, Southwest Louisiana Institute, Lafayette, May 22, 1951

Texas Epsilon, North Texas State College, Denton, May 31, 1951

Indiana Beta, Butler University, Indianapolis May 15, 1952

Kansas Epsilon, Fort Hays State College, Fort Hays, December 6, 1952

Two chapters, Alabama and Missouri Delta, have become inactive during the biennium. Since 1931, fifty chapters have been installed in twenty-three states; three are now inactive.

During the past biennium 1119 persons were initiated into the society, making the total membership 9,725. There were 144 separate initiation ceremonies reported by the chapters.

In addition to keeping the records of the chapters, the secretary's office is one of supply. The following supplies may be obtained from the secretary: small membership certificates, copies of the booklet, "Some Pertinent Suggestions Concerning Kappa Mu Epsilon," permanent record cards, Pentagon cards, initiation blanks, jewelry order forms, grade report sheets, and engraved invitation and acceptance cards.

The records of your society are of little value if they are not accurate, and it is only with the help of the chapters that Kappa Mu Epsilon can have accurate records. It is important that each chapter have a very capable corresponding secretary, a faculty member, who will make the reports promptly and accurately.

Summary of the Report of the National Historian

During the biennium, 1951-1953, four issues of THE PENTAGON have been prepared. In each issue there have been news items from the chapters and a list of the papers given at the chapter programs. These were compiled from the reports of the corresponding secretaries of the chapters. The average number of chapters sending news items for each issue of THE PENTAGON was 12, while the number sending program topics was 27. Only two chapters have reported both news and program topics for each of the four issues of THE PENTAGON.

FINANCIAL REPORT OF THE NATIONAL TREASURER

April 3, 1951—April 3, 1953

Cash on hand April 3, 1951......			$2270.72	
Receipts from chapters				
Initiates 289 at $3.50......	$1011.50			
844 at $5.00....	4220.00			
Miscellaneous..............	190.79			
		$5422.29		
Miscellaneous receipts				
Interest on bonds..........	120.00			
Balfour Co. (Commissions)..	115.50	235.50		
Receipts for period..........			5657.79	
			5657.79	
Total receipts plus cash on hand..				$7928.51
Expenditures (1951-1953)				
National Convention 1951				
Paid to chapter delegates..	$1161.20			
Officer's expenses........	577.82			
Speakers, prizes, printing and miscellaneous expense	111.45			
		1850.47		

L. G. Balfour Co. (Membership certificates, charters, etc.)	494.93		
Pentagon (Printing, Mailing 3 issues)........	1918.47		
Installations (3 chapters)..	61.67		
Incorporation expense......	40.00		
National officers expense....	174.76		
Total expenditure..............		4540.30	
Cash balance on hand April 3, 1953			$3388.21
Bonds on hand April 3, 1953....			2775.00
Total Assets as of April 3, 1953			$6163.21
Net gain for the period (Item 13—Item 1)		$1117.49	

LOYAL F. OLLMAN
National Treasurer

"Euclid is the only man to whom there ever came, or can ever come again, the glory of having successfully incorporated in his own writings all the essential parts of the accumulated mathematical knowledge of his time."

—D. E. SMITH

BIOGRAPHICAL SKETCHES

H. Van Engen, Head of the Department of Mathematics at Iowa State Teachers College, received his Ph.D. degree from the University of Michigan.

Dr. Van Engen has given very generously of his time to Kappa Mu Epsilon. His first national office in Kappa Mu Epsilon was that of treasurer. During the four years, 1947-1951, he served as national president and since that time he has been on the national council as the immediate past president.

+ + +

Professor **Charles B. Tucker** of Kansas State Teachers College, Emporia, Kansas, is serving his second term as national president of Kappa Mu Epsilon. Professor Tucker did his graduate work at Brown University. During World War II Mr. Tucker spent some time in government research.

+ + +

Professor **Cleon C. Richtmeyer**, Director of Instruction at Central Michigan College of Education, Mt. Pleasant, Michigan, has served on the national council of Kappa Mu Epsilon since 1947. He served as national historian for four years and is now beginning his second term as national vice-president.

Dr. Richtmeyer received his bachelor's degree from Albion College, his master's from George Peabody, and his Ph.D. from Colorado State College of Education.

+ + +

M. Leslie Madison, a native of Colorado, received his B. S. degree from Colorado A and M College and his M. S. from Colorado University. He did additional graduate work at the University of California and has been on the staff of Colorado A and M since 1934 with the exception of five years, which he spent on leave with the army, during World War II.

Professor Madison is serving his first term as national treasurer of Kappa Mu Epsilon.

Miss E. Marie Hove, Assistant Professor of Mathematics at Hofstra College, Hempstead, New York has served on the national council of Kappa Mu Epsilon since 1933. The first four years she held the office of national historian and since 1937 she has served as national secretary of Kappa Mu Epsilon.

She received her bachelor's degree at St. Olaf College and her master's degree from the University of Iowa. She has done additional graduate work at the University of California, University of Iowa, and Iowa State College.

+ + +

Laura Z. Greene, National Historian, is a member of the Department of Mathematics of Washburn University of Topeka. She did her undergraduate work at Washburn and received the S. M. degree at the University of Chicago.

+ + +

Dr. Harold D. Larsen, a native of Pennsylvania, started his career at the University of Michigan where he received both his bachelor's and master's degree. He received his Ph.D. degree at the University of Wisconsin.

Professor Larsen was editor of THE PENTAGON from 1943-1952. He has served as business manager of THE PENTAGON since 1943. He was national vice-president of Kappa Mu Epsilon from 1949-1951. From 1951-1953, he was associate editor of the *American Mathematical Monthly.* He has recently assumed the editorship of a new publication, *The American Mathematics Student.*

+ + +

Professor Carl V. Fronabarger of Southwest Missouri State College, Springfield, Missouri, received his bachelor's degree from Southeast Missouri State Teachers College, his master's degree from George Peabody College, and his Ph.D. from the University of Missouri.

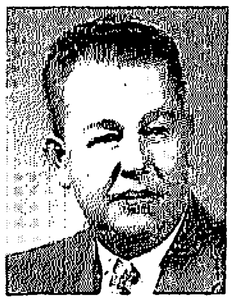
Dr. Fronabarger served as book shelf editor of THE PENTAGON from 1950 to 1952. He was appointed editor of THE PENTAGON by the National Council of Kappa Mu Epsilon in the fall of 1952.

Henry Van Engen
Past National President

Charles B. Tucker
National President

Cleon C. Richtmoyer
National Vice-President

E. Marie Hove
National Secretary



M. Leslie Madison
National Treasurer

Laura Z. Greene
National Historian

Carl V. Fronabarger
Pentagon Editor

Harold D. Larsen
Retiring Bus. Manager of the Pentagon

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Plate I

1953 Convention

Plate II

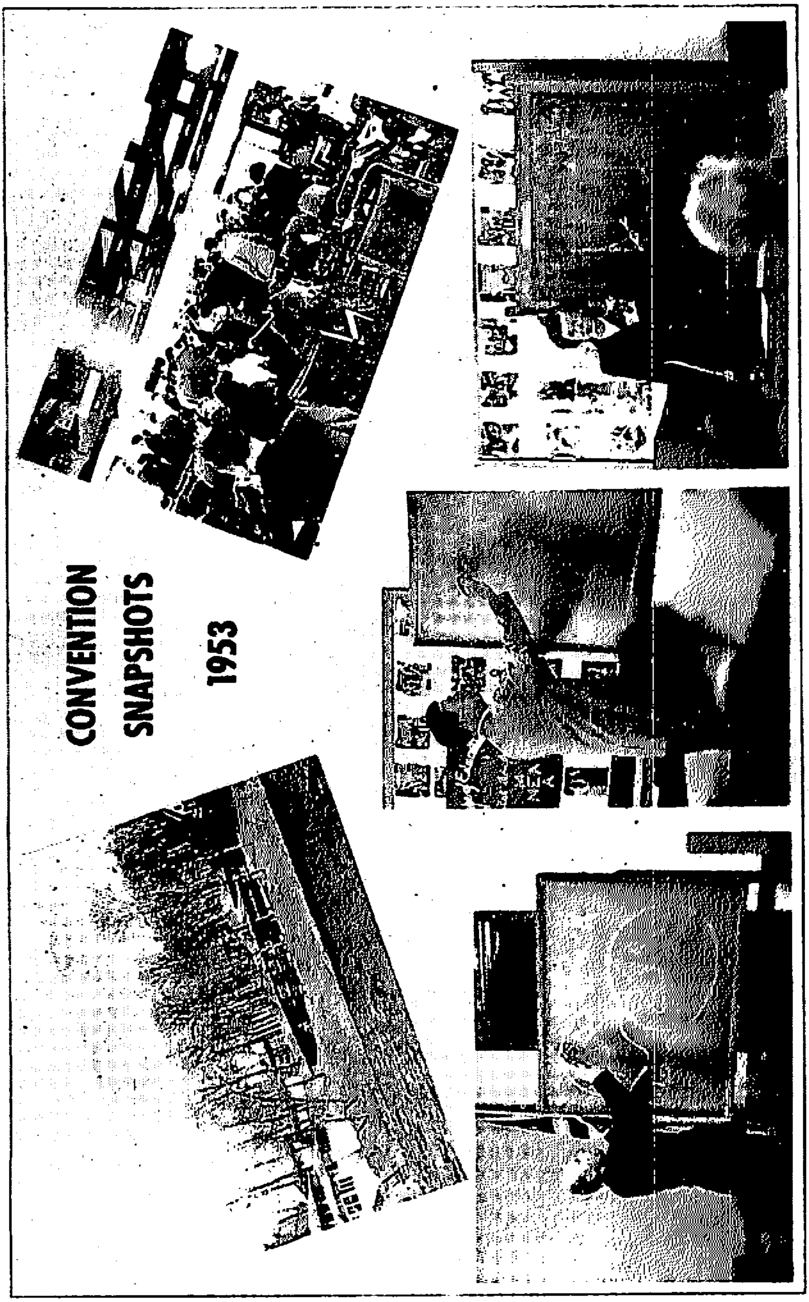

Plate III

Figure 1.

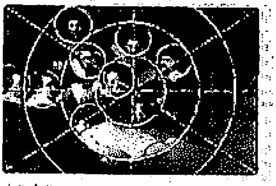

Figure 2.

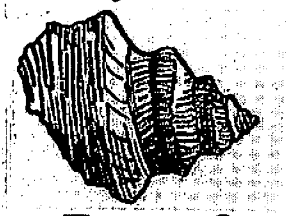

Figure 3.

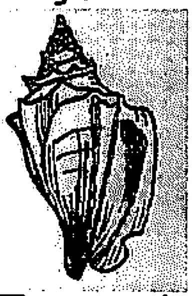

Figure 4.

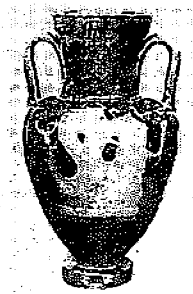

Figure 5.

Figure 6

Figure 7

Figure 8.

Plate IV