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WHO'S WHO IN KAPPA MU EPSILON

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Kappa Mu Epsilon, national honorary mathematics fraternity, was founded in 1931. The object of the fraternity is four-fold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievement in the study of mathematics in the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

THE INFLUENCE OF MATHEMATICS ON THE PHILOSOPHY OF PLATO

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Introduction. For several years the writer has 1. been trying to study the influences of mathematics on philosophy by studying the most outstanding men who have been famous both as mathematicians and as philosophers. The present paper deals with Plato, who is more famous in philosophy than in mathematics, but who was an outstanding mathematician. Plato is difficult to study because he wrote dialogues and put most of his own views into the mouth of Socrates. It is difficult to tell whether it was Socrates or Plato speaking, since Socrates never wrote anything, or at least nothing has come down to us. Plato had seen Socrates executed and this fact made him cautious in expressing his views. Another difficulty is the fact that Plato's disciples nearly always interpreted Plato in the light of their own points of view. Plato might have exclaimed, along with Immanuel Kant. "God spare me from my disciples."

Life (c. 427 B.C. - c. 347 B.C.). Plato was of noble 2. birth and excellent education. His ancestors on both sides were of the most distinguished families of Athens. Plato studied under Socrates and spent most of his life in Athens. though he traveled extensively. About 387 B.C. he founded his Academy over the door of which he is said to have placed the inscription. "Let none but geometers enter here." Presiding over the Academy for the rest of his life, he accepted no pay for his work since he did not believe in pay for teachers. Plato wrote some fifty-five dialogues, among the most famous of which are: Republic, Laws, Politics, Timaeus, Sophista, Critias, Apology of Socrates, Theatetus, Meno, Phaedo, Philebus, Protagoras, and Euthypre. It was only in his Epistles that Plato used his own name in anything that he wrote.

3. Mathematics. Plato was not primarily a mathema-

tician and wrote no books on mathematics, but his Academy was the connecting link between the mathematics of the fifth century Pythagoreans and that of the first Alexandrian School. Plato studied mathematics under the direction of the Pythagoreans and also perhaps under Egyptian mathematicians. All of the most important mathematical work of the fourth century B.C. was done by friends or pupils of Plato. For example, Theatetus, the inventor of solid geometry, and the first men to study the conic sections were members of the Academy.

Plato made a number of specific contributions to mathematics. He introduced the analytic approach to the solution of problems in geometry and he gave a list of definitions, axioms, and postulates for plane geometry. Insisting upon accurate definitions, clear assumptions, and logical proof, he laid the foundation for the work of Euclid, who came about a century later. For example, Plato urged the examination of hypotheses in the *Republic*, declaring:

... as to the mathematical arts ... never can they behold the waking reality so long as they leave the hypotheses which they use undisturbed, and are unable to give an account of them. For when a man knows not his own principle, and when the conclusion and intermediate steps are also constructed out of he knows not what, how can he imagine that such arbitrary agreement can ever become science?¹

Plato formulated the rule that constructions should be made only with compasses and unmarked straightedge, a rule which led to the three famous problems of antiquity. He made some contributions to the theory of irrational numbers. Plato had an excellent idea of what we now call the *function concept*. In the *Meno* there is to be found a discussion of the variation of the area of a rectangle as its length and width varied. This would do justice to any twentieth century teacher trying to explain the function concept.²

4. Philosophy...Plato suggested all the problems known to philosophy even today. He was the first to give

³The Dialogues of Plato. translated into English by B. Jowett, New York: Charles Stribner's Sons, 1908, Vol. II, p. 361. ²Ibid., Vol. 1, pp. 256-259

a complete treatment of all the problems of philosophy. He esteemed philosophy, for he declared in the *Republic* that philosophy was "the noblest pursuit of all."¹

Plato had no definite system of philosophy since he did not believe in the necessity of systems of philosophy. His philosophy, however, may be described by the term *idealism*, which is merely a euphonious expression of *ideaism*. For his ultimate realities were ideas. Man was like a person chained in a cave with his back to the opening. All that he could know about the world was learned by studying the flickering shadows reflected on the back of the cave. He held that there were three worlds: The World of Ideas, the World of Phenomena, and the Mental World. Man lived in the Mental World which lay between the other two worlds, partaking of the nature of both. Plato declared that ultimate reality was in *Mind* rather than *Matter*.

The Influence of Mathematics on the Philosophy of 5. Mathematicians who philosophize usually deal with Plato. what might be called universal mathematics, which is the theoretical application of mathematics to every phase of life. Thus Plato believed that order and measure could be applied to everything in life. In the Republic he declared that "... The arts of measuring and numbering and weighing come to the rescue of the human understanding-that is the beauty of them-and the apparent greater or less, or more or heavier, no longer have the mastery over us, but give way before calculations and measure . . . "2 Plato believed that order and measure were the key to health and to the practice of medicine, stating in the Republic: "And the creation of health is the creation of natural order and government of one another in the parts of the body; and the creation of disease is the production of a state of things at variance with this natural order."3

In following out his universal mathematics, Plato was led to what might be called *number mysticism*. This was probably due to the influence of the Pythogoreans who be-

¹Ibid., Vol. II. p. 316 ²Ibid., Vol. II. pp. 433-434 ³Ibid., Vol. II. p. 271 lieved that integers held the secret of everything in life. In the *Republic* Plato gave an absurd jumble of numbers as a rule to determine the best time for the procreation of children. He declared that the gods had a period which was contained in perfect numbers, while the period of humans was very complex.¹ According to Fite, "... the Pythagoreans undertook to derive all of the universe from relations of numbers; Plato's mind works in the same direction only less crudely, more cautiously. But at times his imagination outruns his caution."²

6. The Problem of Method. Consider first the influence of mathematics on Plato's treatment of the problem of the best method to use in his philosophy. Every philosopher seeks a logical method to find the truth. Plato's logical method was influenced by the mathematical method of calculation.

... this ... is a case where present ideas serve better for a characterization of Plato than those of only a generation ago. For it is not so long since we should have thought it a sufficient account of mathematics to say that it dealt with numbers and geometrical forms. Recently a distinguished mathematician rather startled me by saying flatly, mathematics is logic. But that reminded me that on the other hand for a generation past logic has been moving in the direction of mathematics along the line of symbolic logic.³

According to Fite, in Plato's Gorgias, as in his Protagoras the logic is the logic of calculation.⁴

7. Epistemology: the Problem of Knowledge. Mathematics seems to have been related to Plato's treatment of the problem of getting correct knowledge. He thought that *order* and *measure* were the key to true and accurate knowledge. In the *Republic* Plato declared:

Then this is the sort of knowledge of which we are in search, having a double use, military and philosophical; for the man of war must learn the art of number that he may know how to array his troops, and the philosopher, also, because he has to rise out of the sea of change and lay hold of true being, \ldots ⁵

¹Ibid., Vol. II. p. 373 ²Warner Fite, The Platonic Legend, New York: Charles Scribner's Sons. 1934, p. 247. ¹Ibid., p. 232 ¹Ibid., p. 199 ³Jowett, op. cit., Vol. II, p. 352 In the Protagoras, Plato declared:

Would the art of measuring be the saving principle, or would the power of appearance? Is not the latter that deceiving art which makes us wander up and down and take the things at one time of which we repent at another, both in our actions and in our choice of things great and small? But the art of measurement is that which would do away with the effect of appearances, and showing the truth, would fain teach the soul at last to find rest in the truth, and would save our life.¹

There were two phases of Plato's epistemology which seemed to show best the influence of mathematics. The first was his "dialectic," the key to true knowledge, and the second was his "divided line."

The term *dialectic* meant knowledge by means of definitions. Plato insisted that the dialectician must have mathematical training. In the *Republic* he declared that calculation and geometry were the proper preparation for the dialectic philosopher-statesman:

The special training of the dialectician is then exclusively mathematical. It will be a graded course consisting of (1) arithmetic, (2) plane geometry, (3) solid geometry [this science, Plato explains is yet to be created], (4) . . . mathematical astronomy . . . with the eyes averted from the starry firmament above us, and (5) mathematical harmonics. And Plato finds it unfortunate that arithmetic is useful for trade. The real purpose of mathematics was to draw the soul upward.²

Consider next Plato's "divided line." He expressed many relationships in terms of proportions, and in the *Republic* he gave his entire scheme of things in terms of ratios of line segments.³ The segments of the line were divided in the same ratio as the whole line. On the basis of these line segments he gave the following proportions:

Perception : belief : : understanding : science.

Being : becoming : : pure intellect : science.

Intellect : opinion : : science : belief : : under-

standing : perception of shadows.

"Today we learn our arithmetic first, but from the beginning it seems that Greek arithmetic was closely con-

¹*lbid.*, Vol. I. p. 156 ²Fite, *op. cit.*, p. 239 ³Jowett, *op. cit.*, Vol. II, pp. 337-340

nected with geometry and that it borrowed from geometry its symbolism and its nomenclature. This helps us see why Plato put his metaphysical scheme in the form of a line."1

Plato believed in what has since been called "mental discipline." holding that mathematics could train the mind. Thus he declared in the Republic.

And have you further remarked that those who have a natural talent for calculation are generally quick at every other kind of knowledge, and even the dull, if they have had an arithmetical training, gain in quickness, if not in any other way?2

Metaphysics: the Problem of Ultimate Reality. 8. In dealing with Plato's metaphysics, we may consider the influence of mathematics on his treatment of ideas, and on his treatment of *aod*.

For Plato, ideas were superior to the objects of sense in the same way that the concept of a circle was superior to any circle that one could draw. According to Aristotle, the Platonic Ideas were nothing but numbers.³ This was brought out in the Republic when he tried to show that in dealing with phenomena, he was not dealing with reality, just as in geometry the diagram was not the reality. He declared:

And do you not know also that although they [the mathematicians] use and reason about the visible forms, they are thinking not of these, but of the ideas which they resemble; not of the figures which they draw but of the absolute squares and the absolute diameter and so on; and, while using as images these very forms which they draw or make, ... they are really seeking for the things themselves, which

can only be seen with the eye of the mind?4

Then Plato went on from mathematics to philosophy.

Consider next Plato's treatment of the problem of god. According to Felix Klein, Plato stated that "God eternally geometrizes," and it seems that mathematics was related to his concept of god. Plato thought of god as being a

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³Fite, op. cit., pp. 237-238 ³Jowett, op. cit., Vol. II, pp. 353-354 ³Edward Zelles, Plato and the Older Academy, translated by Sarah Alleyne and Alfred Goodwin, London: Longmans, Green, and Company, 1888, p. 255. ⁴R. E. Moritz, Memocabilia Mathematics New York: The Macmillan Company, 1914, pp. 269-270.

perfect circle on a perfect sphere. Fite paraphrased the New Testament to show Plato's concept: "Be ye however geometrically perfect, even as the perfect circle is perfect; or as the perfect sphere; or perfect with the infinitely subtle and complex perfection of a perfect numberset."1

Natural Philosophy: the Problem of the External 9. World. In dealing with the external world, Plato overemphasized the importance of mathematics in scientific investigation. He failed to understand the importance of empirical observation, induction, and experimentation. According to Taylor:

The result is thus that Timaeus, in the spirit of Descartes, offers us an anatomy and physiology in which the organism appears as an elaborate kinematical system; natural science is thus reduced in principle, as Descartes and Spinoza held it ought to be, to geometry. Plato is not, of course, very strictly committed by the details of speculations which he repeatedly says are provisional, but it is clear that he is in sympathy with the general attitude known today in biology as mechanistic.²

Fite made the following statement in regard to Plato's natural philosophy:

Plato's "chemistry," all of it to be found in Timasus . . . is geometry simple and pure. It is an attempt to explain the four elements: fire, air, water, earth, as so many combinations of triangles into geometrical solids, on the basis of the theorem of the "five solids" discovered either by Plato or by some other member of the academy.³

Demos described Plato's natural philosophy in the following way:

In transforming the primordial chaos into an orderly world God has recourse to the Pattern. God makes the so-called elements, fire, water, air, and earth, by arranging space according to certain geometrical figures and solids. The actual world comes about through the introduction of mathematical relations of number and proportion into the "Receptacle."4

In the Timaeus, Plato predicted mathematical physics and astronomy and advocated the endowment of research

¹Fite, op. cit., pp. 262-263 ³A. E. Taylor, Plato, The Man and His Work New York: The Dial Press, Inc. 1936. p. 454. ⁴Fite, op. cit., p. 222. ⁴Raphael Demos, The Philosophy of Plato, New York: Charles Scribner's Sons, 1939, p. 5

work. In the *Epinomis*, Plato gave his views on astronomy: The study we need to lead us to true piety, the greatest of the virtues, is thus astronomy, knowledge of the true orbits and periods of the heavenly bodies, pursued in the spirit of pure science . . But since such a study is concerned with the difficult task of the computation of the relative periods of the sun, moon, and other planets (and thus has to reckon with highly complicated arithmetical problems), it must have its foundation in a thoroughly scientific theory of numbers.¹

10. Practical Philosophy: the Problem of Ethics. Esthetics, and Politics.. Descartes thought that practical philosophy could not be treated mathematically and therefore should not be treated at all. But not Plato. He asserted the importance of mathematics in all of these fields. For example, ethics was to be a matter of calculation. From a statement of Aristotle we learn that most of those who attended Plato's lecture on the "The Good" were perplexed by hearing a lecture not on ethics but on mathematics. Nothing was said about the concrete human goods; the good was the determinate "One" as against the "Indeterminate" and "Infinite." "Good" and "Bad" should be determined by calculations. In the dialogue Euthypro, Plato made Socrates explain how disputes could be settled if everything could be dealt with in the same way that numbers are handled:

SOCRATES. And what sort of difference creates enmity and anger Suppose for example, that you and I, my good friend, differ about a number; do differences of this sort make us enemies and set us at variance with one another? Do we not go at once to calculation, and end them by a sum?

EUTHYPRO. True.

SOCRATES. Or suppose that we differ about magnitudes, do we not quickly put an end to that difference by measuring?

EUTHYPRO. That is true.

SOCRATES. But what differences are those, which because they can not be thus decided, make us angry and set up at enmity with one another? . . . this happens when the matters of difference are just and unjust, good and evil, honorable and dishonorable.²

¹Taylor, op. cit., pp. 500-501. ²Jowett, op. cit. Vol. I. pp. 290-291. Another point to be considered under ethics is Plato's "calculus of pleasure" or "hedonistic calculus." He said that one should find the "maximum" of pleasure. In the *Protagoras*, "It is the pleasure story of morals that Socrates is here defending, variously described as utilitarianism, hedonism, Epicureanism. His standard of good is pleasure and his method is that of the "hedonistic calculus!"¹

Next, consider Plato's "Moral calculus" in which can be seen the antecedent of Aristotle's "golden mean."

For Plato the moral life presupposes an eternally uniform order in the universe which is marked by the stars in their courses, an eternal mean between too fast and too slow from which we mortals are being ever and again dislodged by passion and ignorance. The task of life is then forever a matter of getting back again to the norm of reducing the sum of our human sins and aberrations to the slightest possible mean variation from the mathematical point that marks the eternal order.²

In the *Republic* Plato declared that moral degeneration was a calculable quantity, taking place in accordance with a definite law, which could be expressed by geometrical progressions. This statement of Plato bore an amazing similarity to the laws of Weber and Fechner, which have been hailed as important steps in the beginning of modern scientific psychology.

Plato's treatment of esthetics showed once again the importance of order and measure in all of his thinking. By getting proportions correct, beauty could be made absolute. In *Philebus* Plato declared:

I do not mean by the beauty of form such beauty as that of animals or pictures, which the many would suppose to be my meaning; but, says the argument, understand me to mean straight lines and circles, and the plane or solid figures which are formed out of them by turning lathes and rulers and measurers of angles; for these I affirm to be not only relatively beautiful, like other things, but they are eternally and absolutely beautiful³

Finally, consider the influence of mathematics on Plato's political philosophy. He regarded mathematics as

¹Fite, op. cit., p. 186 ²Fite, op. cit., p. 259 ³Jowett, op. cit., Vol. III, p. 191

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being very important in training for leadership in government. Thus Plato declared in the Republic:

Then this is a kind of knowledge which legislation may fitly prescribe; and we must endeavor to persuade those who are to be the principal men of our State to go and learn arithmetic, not as amateurs, but they must carry on the study until they see the nature of numbers with the mind only; nor again, like merchants or retail traders, with a view to buying or selling, but for the sake of military use, and the soul herself; and because this will be the easiest way for her to pass from becoming to being.

That is excellent, he said.

Yes, I said, and now having spoken of it, I must add how charming the science is! And in how many ways it conduces to our desired end, if pursued in the spirit of a philosopher, and not a shopkeeper.

How do you mean?

I mean, as I was saying, that arithmetic has a very great and elevating effect, compelling the soul to reason about abstract number, and rebelling against the introduction of visible or tangible objects into the argument.¹

In the *Epinomis*, according to Taylor:

We ought to give the name sophia only to studies which make a man wise and good citizen, capable of exercising or obeying righteous rule. Now there is a branch of science which, more than any other, has this tendency and may be said to be a gift of a god to man, being in fact the gift of Heaven itself. This gift is the knowledge of *number*, which brings all other good things along with itself. Without knowledge of number we should be unintelligent and unmoral. How divine a thing it is we see from the consideration that where there is nothing but confusion, formlessness, disorder. To be able to count is the prerogative which marks men off from the animals.²

There is to be found absurd number mysticism in Plato's contrast of kings and tyrants in the *Republic*:

And if you raise the power and make the plane a solid, there is no difficulty in seeing how vast is the interval by which the tyrant is parted from the king.

Yes, the arithmetician will easily do the sum.

Or if some person begins at the other end and measures the interval by which the king is parted from the tyrant "Jowett, op. cit., Vol. II, p. 353. "Taylor, op. cit., pp. 498-499. ... he will find him; when the multiplication is completed, living 729 more pleasantly and the tyrant more painfully by the same interval.¹

11. Summary and Conclusions: The following summary and conclusions seem to be valid:

1. Plato is usually thought of as being merely poetical and mystical, but he seemed to try to apply mathematical order and measure to all the problems of philosophy with which he dealt.

2. The type of analysis which Plato introduced into geometry was introduced into philosphy by Descartes.

3. The type of analysis which Descartes introduced into geometry has been used successfully in philosophy only in the twentieth century in the field of symbolic logic.

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"The student who has thus far taken the system of real numbers for granted, and worked with them, may continue to do so to the end of his life without detriment to his mathematical thought. On the other hand, most mathematicians are curious, at one time or another in their lives, to see how the system of real numbers can be evolved from the natural numbers."

-W. F. OSGOOD

"I do not know what I may appear to the world, but to myself I seem to have been only a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me." —I. NEWTON

"Every new body of discovery is mathematical in form, because there is no other guidance we can have."

-C. G. DARWIN

SOME ELEMENTARY MATHEMATICS OF SPACE FLIGHT

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With all the current interest in rockets, guided missiles, and space travel, there is considerable confusion in the popular mind between fact and fiction. It is well to distinguish between the known mathematics and astronomy involved and the still-undeveloped engineering necessary for attaining space travel. It is possible to calculate the orbits, velocities, and times required for such trips, regardless of the engineering techniques that may be used to obtain them. Hence it may be of interest to students of mathematics to see that much of this calculation involves nothing more complicated than the elementary integral calculus.

First, we need to recall Newton's laws of motion and his law of universal gravitation. The second law of motion states that the acceleration produced on a body is proportional to the force producing it. With proper choice of units this becomes, for a mass M, the familiar formula

(1) F = Ma. The law of gravitation is described by the formula

 $F = kMm/d^2$.

where F is the force of attraction between two bodies of mass M and m, respectively, separated by a distance d. The third law of motion states that to each force there is an equal force, oppositely directed. Combining this with formulas (1) and (2), we get

 $Ma = F = kMm/d^2, \text{ or }$

(2)

(4) $a = km/d^2$, i.e., the gravitational acceleration on a body is independent of its own mass. This was essentially what Galileo demonstrated by his famous experiments with falling bodies.

At or near the earth's surface, d = R = 4000 miles (approximately) and experiment shows the acceleration due to gravity to be a = g = 32.2 ft./sec.². Substituting in (4),

(5) $g = km/R^2$, gR^2 , and (4) then becomes $a = gR^2/d^2$.

From the calculus we have that the work done by a variable force F(x) exerted in a straight line from x = a to x = b is given by

work $W = \int_{a}^{b} F(x) dx$.

Applying this to the work done in projecting a unit mass vertically from the earth's surface to a distance r from the center of the earth, we get

(6) $W = \int_{R}^{r} (gR^2/r^2) dr = gR^2 (1/R-1/r).$

The work in moving the mass to "infinity" is similarly

(7) $W = \int \mathcal{B}^{\infty} (gR^2/r^2) dr = gR,$

the units, of course, being appropriately chosen. This result also ignores air resistance, as is done in all the subsequent developments. Interestingly enough, the above derivation shows that to project a body completely away from the earth requires exactly the same amount of energy as would lifting its weight a distance of 4000 miles under gravity at sea-level value. The relation (6) may be written $W = gR - gR^2/r$. For the moon, r = 240,000 miles, so that relative to energy requirements the moon is practically at "infinity." If a body is projected to "infinity" from a point distance r from the earth's center, formula (7) is replaced by

(8) $W = \int_{r}^{\infty} (gR^2/r^2) dr = gR^2/r,$

i.e., the energy required is inversely proportional to the distance of the projection point from the center of the earth. This becomes significant in consideration of projection from a "space station" located at a considerable distance from the earth.

Having developed the necessary energy requirements, let us investigate the corresponding velocity requirements. For the most efficient use of energy, it is necessary to give the projected body the necessary velocity practically at the time of projection (to avoid loss to gravitational deceleration while the body is near the earth). Then the kinetic energy of motion from the initial velocity must equal the work to be done. Hence, since $K.E. = \frac{1}{2}MV^2$, the "escape velocity" for a unit mass is derived from (7), i.e.

 $\frac{1}{2}V^2 = gR$, or

(9) $V = \sqrt{2gR} = 7$ mps or 25,000 mph (approximately). The corresponding escape velocity from a point at

distance r from the earth's center is given by (8), i.e.

(10)
$$\frac{\frac{1}{2}V_{r}^{2} = gR^{2}/r}{V_{r} = \sqrt{2gR^{2}/r}}.$$

Escape velocity is important in consideration of the atmosphere of planets and satellites. If the velocities of the molecules of the atmospheric gases have values comparable to the escape velocity of the celestial body in question, the atmosphere gradually "leaks" into space. This accounts for the complete lack of an atmosphere surrounding the moon and very little atmosphere on Mars, for example. The earth's escape velocity of 7 mps is quite adequate for holding an atmosphere. However, consideration of formula (10) shows that at great distances from the earth, gases of relatively high molecular velocities (notably hydrogen) would be lost. For the giant planet Jupiter the veolcity of escape is 37 mps, and Jupiter has a heavy blanket of atmosphere.

A body projected upward from the earth's surface at less than escape velocity would reach a certain height and then fall back. To get the relation between the initial velocity and the height reached, we use (6) to get

(11)
$$\frac{1}{2} V^2 = gR^2(1/R-1/r)$$
, from which $r = 2gR^2/(2gR-V^2)$ or

(12) $h = r - R = \frac{V^2 R}{(2gR - V^2)}.$

Note that $h = \infty$ for $V^2 = 2gR$, as is to be expected from

(9). Also an interesting fact results from calculating the value of V from (11) with r = 240,000 miles (the distance of the moon). It turns out to be about 6.9 mps, only slightly less than the velocity of escape for the earth. This is consistent with the remark just preceding formula (8).

The relation (11) also gives the impact velocity of a

body falling freely from a distance r (ignoring air resistance, of course). It should be compared with the familiar formula from elementary physics, $V^2 = 2gh$, which is valid only for distances relatively small, considered as space distances. The latter is derived on the assumption that gravitational acceleration is constant (=g), rather than varying as the inverse square. In fact, if in (6) we replace gR^2/r^2 by simply g, we get

and

$$V = \int kg dr = g(r-R) = gh_1$$

 $\frac{1}{2}V^2 = gh_1$ or $V^2 = 2gh_2$.

A very interesting consideration is that of a circular orbital motion (the moon is an example). Here the centrifugal force must exactly counteract the gravitational attraction. From elementary physics, a body having a linear velocity V_r in a circle of radius r is subject to a centrifugal acceleration of V_r^2/r . Then from (5) we will have the orbital (circular) velocity given by

$$V_r^2/r = gR^2/r^2$$
, or
 $V_r = \sqrt{gR^2/r}$.

(13)

Comparison of (13) with (10) shows that the velocity required to project a body to "infinity" from any distance ris just $\sqrt{2}$ times the circular velocity at that distance. This result is independent of the particular bodies involved. Two consequences will be mentioned here and others later. A "space station" will likely be realized long before complete escape flight, because initial velocity for the latter is about 11/2 times that for the former. Since the earth's motion about the sun is very nearly circular, at a linear speed of about 181/2 miles per second, this is the circular velocity relative to the sun at the earth's distance from the sun. The corresponding velocity of escape relative to the sun is then 18.5 $\sqrt{2} = 26$ mps (approx.). Hence any meteorite in the earth's vicinity cannot have a speed of greater than 26 mps and still remain a part of the solar system. Then the range of speeds of meteorites relative to the earth is $26 \pm 18\frac{1}{2} = 7\frac{1}{2}$ to $44\frac{1}{2}$ miles per second. This wide range of speeds is one factor in the great range in brillance of meteors seen.

From (13) we may derive a formula for the period of motion in a circular orbit:

(14) $P = 2\pi r/V_r = (2\pi r/R)\sqrt{r/g}$. Near the earth's surface, i.e., for r = R, the circular velocity turns out to be about 5 mps = 18,000 mph. Hence a satellite just outside the earth would revolve around the earth in about $1\frac{1}{2}$ hrs. A space station in such an orbit passing over the poles of the earth would view every part of the earth once every twelve hours. A calculation with formula (14) for r = 26,000 miles (i.e., 22,000 miles above the earth's surface) gives P = 24 hours. Hence a body directly above the earth's equator at this distance and revolving in the equatorial plane would remain permanently directly above the same point on the earth. The possibilities for variety in periods of circular revolution has very interesting implications relative to "space stations."

The cases considered thus far are special ones—motion either radially away from the earth or else in a circular orbit about the earth. Also all of them have been developed on the assumption of no atmospheric resistance to motion, i.e., they are strictly valid only from outside the earth's effective atmosphere.

Now let us consider the more general case of initial motion in a tangential direction from a point distance rfrom the center of the earth. If the body be given an initial velocity less than the circular velocity for that distance. it will eventually fall back to the earth, even though it may go a considerable way around the earth before doing so. If the initial velocity is greater than the circular velocity the body will move outwards along some conic curve, as first proved by Newton as an implication of the law of gravitation. The particular comic is determined by the initial velocity, the distance r, and the gravitational field (of the earth in the present case). Descriptively, this is to be expected from the fact that the curvature of the orbit will depend upon the deflection of the moving body in the direction of the attracting force and its motion in its orbit meanwhile.

It is shown in celestial mechanics¹ that for an initial

velocity V at distance r the following relation (for an elliptic orbit,) holds:

(15) $V^2 = gR^2(2/r-1/a)$, where a is the semi-major axis of the ellipse. In the case of a central body other than the earth, the factor gR^2 would be replaced with the appropriate constant, depending upon the gravitational field involved. In the case of a circular orbit, a = r, and equation (15) reduces to (18): $V^2 = gR^2/r$, i.e., circular velocity. In case the initial velocity is somewhat greater than circular velocity, the body will move out along an ellipse. As the value of V is

increased, a increases and, as will be shown presently, the eccentricity of the ellipse increases until in the limiting case $a = \infty$, (15) reduces to (10): $V^2 = 2qR^2/r$, i.e. escape velocity. The orbit is now a parabola and for this reason escape velocity is often referred to as parabolic velocity. For values of V greater than escape velocity, the orbit is hyperbolic and in formula (15) the negative sign is replaced by a positive sign. If V is increased indefinitely the path of motion approaches a straight line.



For the elliptic orbit shown in the figure we have r = PE = a(1-e)and $V^2 = gR^2(2/r-1/a)$, from which it is clear that the semi-major axis a and the eccentricity e can be determined. It can be shown that for a given distance r the particular type of conic orbit depends only upon initial velocity V and not upon the initial

¹Ste, for example, Moulton, Celestial Mechanics, Second Revised Edition, Chapter V.

direction. It can also be shown that the period in an elliptic orbit is the same as for a circle whose diameter equals the major axis of the ellipse, and hence it can be computed from (14) with r replaced by a.

As one example of the relations holding for time required for space trips, let us consider the case of a flight to the moon from near the earth. Ignore the relatively small contribution of the moon's gravitational field and suppose the initial velocity to be such that the body will just reach the moon with zero velocity. If the distance from the center of the earth to that of the moon is S, the velocity v at any point on the journey will be just that required to go from r = r to r = s. Then from (11) we get (16) $V^2 = 2gR^2 (1/r-1/S)$,

or $dr/dt = v = c\sqrt{1/r-1/S}$, where $c^2 = 2gR^2$.

To integrate, make the substitution $r = S \cos^2\theta$, whence

 $-2S\,\cos\,\theta\,\sin\,\theta\,\,d\theta/dt\,=\,\tan\,\theta/S^{1/2},$

$$2S^{3/2}d\theta/dt = -c \sec^2\theta,$$

and $c dt = -2S^{3/2}\cos^2\theta d\theta$.

When r = R, $\cos^2 \theta = R/S$ and when r = S, $\theta = 0$. Integrating and using these limits, we get

 $t = S^{3/2}/c(\theta' + \sin \theta' \cos \theta')$, where $\theta' = \arccos \sqrt{R/S}$. The initial velocity required for the body just to reach the moon, as given earlier, is about 6.9 mps, and for this value of V, we get t = 116 hours, or about 5 days. This is the maximum time required for the earth-moon journey, since a greater initial speed would reduce the time needed. In fact, an increase of one mile per second in the initial velocity would reduce the time to less than 20 hours.

For an initial velocity equal to the velocity of escape, the above development is greatly simplified. Then in (16) we replace S by ∞ and get

$$dr/dt = v = 2gR^2/r = c\sqrt{1/r},$$

 $dt = c\sqrt{r} dr,$
 $t = 2/3c(S^{3/2}-R^{3/2})$

is obtained by integration between the limits r = R and

and

r = S. Since S = 240,000 miles is so large as compared with R = 4000 miles, a very good approximation to the time is given by

$$t = 2S^{3/2}/3c = kS^{3/2}$$
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These are just a few of the mathematical relations which can be derived relative to this topic. Anyone interested in pursuing the subject further is referred to the very readable little book by Arthur C. Clarke, Interplanetary Flight, which is the source of most of the material presented here.

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"Mathematics is the science in which we never know what we are talking about nor whether what we say is true."

-BERTRAND RUSSELL

FROM GREEK TO GERSHWIN: THE MATHEMATICAL BACKGROUND OF MUSIC'

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Perhaps because of innate intellectual laziness, man often attributes something which he does not understand to that mystical quality "genius." This has proved noticeably true in the arts of music, sculpture, and painting. As music developed and some men showed skill in composing music which proved to be enjoyable to others, this ability was written off as "genius" and few musical laymen endeavored to discover the reason that this music pleased its listeners while other music did not. Those who did attempt to find a logical reason for the permanence of some music failed in their investigations by looking into the past, and overlooking the possibility of future developments.

To many modern educators and students, music and mathematics seem widely separated. This assumption is quite wrong, however, as the very basis of music and musical instruments as we know them today depends directly on developments and discoveries in the field of mathematics and its closely related science, physics.

Pythagoras, the Greek musician-mathematician, and his followers held that music was applied mathematics. Their insistence upon this point almost removed from music the pleasurable portions of playing, singing, and dancing, causing it to be used as a mere mathematical exercise. When the use of music in actual life was lessened by the Romans, they still retained the Greek theory of music as applied mathematics with its tetrochordal scale and lack of harmony. In order to make progress, musicians were forced to break away from the restrictions placed upon them by rules developed in earlier times. As the earlier developed rules did not hold true for the "modern" music written by the Christians for their church

1A paper presented to the Missouri Academy of Science at its Spring meeting in 1952.

music, musicians soon drifted away from the emphasis on the mathematical aspects which had been established earlier.

Mathematical concepts remained in the development of music as the technical developments in resonators for tonal quality, instruments, accoustical tempering in scales, and manuscript writing progressed toward their present forms. Perhaps mathematics in instrumental development is most evident in the development of organ stops, where for every doubling in length of the pipe the pitch of the organ is lowered an octave. However, actual pitch of the organ is not altogether determined by the length of the pipe; for, as the diameter of the pipe increases, the total pitch is lowered. Another outstanding example of mathematical influence in the development of musical instruments is in our present use of overtones. Most students are aware of the fact that when an open string is plucked the vibrations or sound waves which make that tone are not the only ones one hears. This same string vibrates in halves, making a tone which sounds an octave higher: vibrates in thirds, making a tone which in musical terms sounds a fifth higher than the original: and so on. with continued redivisions causing as many as sixteen or eighteen audible overtones. Fingerholes and valves have been added to make it possible for an instrument to play several fundamental tones; and by using this principle of overtones, first established by Pythagoras, man has gradually built up a system of musical instruments of varving sizes and tonal qualities which can play each half-tone of the scale; and men are today experimenting with instruments which can play the quarter-tones of the scale.

Musicians often confuse their public, and themselves, by their references to three-quarter time, four-four time, or three-eight time. These strange-sounding terms refer merely to the mathematical developments which have occurred in manuscript writing, in which one composer, in an attempt to have musicians play his music as he intended it, began a practice of rhythmical division, where a triple figure having a pulse on the first beat was designated by placing a three at the beginning of the manuscript. In order to determine the approximate speed of the composition a basic note, such as a quarter note, half note, whole note, was designated, and then the rhythmic pattern was designated by the various divisions of this basic time value. Placing the figure representing the approximate speed at the beginning of the manuscript as the denominator of a fraction, composers have gradually developed the time signature as it appears in most music today.

Gradually, as music became a more vital part of the social life of man, it was discovered that certain tones and combinations of tones were more pleasing to the ear and that many people derived greater satisfaction when they heard these tones in combinations. In attempting to construct a stringed instrument such as the piano, it was found to be impossible to use all the various shadings and pitches which could be used in vocal music; so, as the piano was developed, those tones were chosen which in combination were most pleasing to the listener. These pitches are in mathematical ratio to one another. Other instruments have been constructed to conform to the tempering of the keyboard instrument, with the result that many mathematical and physical problems must be considered in obtaining the same vibrations from instruments of varying shapes and sizes.

While developing instruments in various shapes and sizes, it was soon discovered that a different tonal quality was attained, although one instrument sounded the same pitch as another. To enlarge the tone of an instrument, a resonator must be provided. These resonators in turn give tonal color by adding more overtones, previously mentioned in the mathematical developments of instruments, or by skipping some of the tones in the series which can be obtained from the open string. As a result, one can tell the difference between a clarinet, cornet, flute, and trombone, though they all may sound in the same pitch. In the early 17th century composers began to call for the instruments which would give their music the tonal color as well as the rhythm and melodic line they desired. To preserve similarity of tonal color in an instrument it is necessary that its proportions be the same as those of the original instrument. Many times musicians have tried to play music written by earlier composers, only to find great changes made in the tonal quality of the instruments, and thus the quality of the music has been changed. Even in the natural instrument, the voice, changed mathematical ratios in the resonator make a change in the tonal quality of the individual voice.

These reasons have not explained why the ancient Greeks considered music applied mathematics other than in the mechanical phases connected with instrumental development, but the Greek musicians of centuries ago have found a modern counterpart in the person of Joseph Schillinger. Dr. Schillinger has made widespread contributions to music, mathematics, and art, and at one time lectured in classes of all three at Columbia University. After making a study of Einstein's theory of relativity, Dr. Schillinger developed the premise that all the sciences and arts are originally based on mathematics. Thus, certain mathematical ratios are found to be pleasing whether they are found in the range of visual perception or within the range of hearing. He thinks that in viewing scenic beauty there is present a certain mathematical ratio which makes a special scene particularly pleasing to the eye. Artists, in reproducing this scene on the canvas, merely present the same mathematical ratio of time and space which was found in the original. Furthermore, composers, in their use of sound. use a similarly pleasing mathematical ratio based on the same elements of time and space. With these premises as a basis Dr. Schillinger has developed a theory of harmony which not only considers music which has been written but presents a method for writing music in the future. Until Dr. Schillinger's system made its appearance, theories of harmony had been based on rules established by previous music and made no explanation for music which was to follow-music which, though pleasing to the listener and satisfactory to the composer and performer, did not conform to the previously established rules, nor did primitive music and music from the Orient. However Dr. Schillinger has given us the formulas for music written for many different cultures, including the music to which European and American man is accustomed, and yet has provided a system whereby one can develop a new formula and a new form of music, based on the mathematical premises followed by the theory of relativity of time and space.

In spite of its complex roots, this theory can be and has been taught to many musicians, who, following Dr. Schillinger's theory of musical composition, have achieved much in music. Some familiar names among these composers are those of Albino Rey, George Gershwin, perhaps the greatest of Schillinger's students, and Paul Lavalle. These men have developed new rhythmical patterns, new tonal sequences, and new harmonic progressions which have pleased and interested those who listen to these newest developments in music.

Music and mathematics, widely separated though they are in the thinking of the uninitiated person, are thus actually quite closely interlaced in development and in construction. Noted mathematicians, such as Albert Schweitzer, Albert Einstien, and Joseph Schillinger, are also well known musicians, the first as an organist, the second as a violinist, and the third as a student of harmony and composition.

And thus modern man returns to the ideas of the ancient Greeks and uses mathematical concepts to give precision to the complexities of music.

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A scientist worthy of the name, above all a mathematician, experiences in his work the same impressions as an artist; his pleasure is as great and of the same nature. —H. POINCARE

CHAINS OF INTEGRAL TRIANGLES

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This paper is about integral triangles and certain transformations by which two or more integral triangles may be associated. The integral triangles are denoted by the sets of three integers which represent their sides. A set of three relatively prime integers of which the sum of any two is greater than the third, will be named a triad. For every triad there exists a primitive integral triangle. It is known that the cosines of the angles of an integral triangle are rational numbers. The converse is also true. This means that whenever the three angles of a certain triangle have rational cosines, then there exists an integral triangle similar to it.

In this paper the word, triangle, will refer to an integral triangle unless it is otherwise stated.

The transformations are derived by considering two triangles of which one is assumed to be derived from the other. By passing from one triangle to the other certain changes in the angles are assumed. Thereupon the corresponding changes in the sides are noted. These changes —by which one triad may be said to pass into another are the transformations.

It seems natural to expect that if a transformation will change a Triangle T_0 into a triangle T_1 , then there should be a transformation that can change T_1 back into T_0 . This is the case. For each of the basic transformations there is also an inverse.

When one triangle, or its triad, has been transformed into another the two triangles so associated are linked together. In some cases a triad can be linked to two or more triads; and then a chain of triangles is formed.

Along with the applications, chains of triangles will be exhibited. Thereupon will follow transformations of right triangles and isosceles triangles. While the first group deserves notice, it is the second that merits more attention. This latter group will separate into two parts. The one will show symmetrical chains of triangles; while from the second general formulas will evolve which—for a specified range—can give all those triangles where two angles A and B are so related that $B = n \cdot A$ for n = 2, 3, 4, ... as long as $(n+1) \cdot A$ is less than 180°. Thereupon some special transformations are developed by which related triangles may be combined. Finally, it is shown that for each chain of triangles there exists a characteristic isosceles triangle which a properly selected Cevian will divide into two triangles similar to any two successive links of the chain.

The Transformations

Consider the triangle T_0 with the sides *a*, *b*, *c*, and the angles A, B, C to correspond. Assume, for the moment, that T_0 is neither equilateral nor isosceles and take A < B < C. The transformation, or the changes that are made in the angles by passing the triangle T_0 to its transform triangle T_1 , will be of three types according as the angles A, B, C of T_0 are made to correspond to the following sets of angles of the transform:

A, B+A, C-A
 A, B-A, C+A
 A+B, B, C-B

That the three angles in (1) can belong to an integral triangle will be seen by the following argument:

Since the sides of the triangle T_0 are integers it follows that the cosines of the angles A, B, C are rational numbers. Assume that the area of the triangle T_0 is \sqrt{N} , then N is a rational number. It follows that the sines of the angles A, B, C are rational multiples of \sqrt{N} . This assures that the number for $\cos (B+A)$ which is equal to $\cos B \cos A - \sin B \sin A$ is a rational number; and likewise for $\cos (C-A)$. The three angles in (1) add up to 180° and since the cosines of these angles are rational they can and do belong to one primitive integral triangle T_1 . A similar argument holds for the triangles which have the angles given in (2) and (3).

To derive the transformation that will give the tri-

angle in (1) draw a figure of the triangle T_o (a,b,c,A,B,C). Lay off from the side of CB an angle CBD equal to A so that the angle ABD is equal to B+A. Locate the point D on the side AC extended. Assume that the triangle T_o has been transformed into the integral triangle T_1 which is similar to the triangle ABD. Then the angles of T_1 will be A, B+A, and C-A. The ratios of the sides of the triangle T_1 are found from the similar triangles BCD and ABD. It turns out that

 $BD:DA:AB = ab:bc:c^2 - a^2$

When the ratios of the sides a,b,c of the triangle T_0 are replaced by the ratios of the sides a,b,c, of the triangle T_1 then the triangle T_0 is transformed into the triangle T_1 . This transformation is denoted

(1.1) $a:b:c: > ab:bc:c^2-a^2$

It gives the changes in the sides from T_0 to T_1 by which the three angles of T_0 are respectively changed to the angles given in (1).

The transformation that belongs to (2) can be found in a similar manner. The angle CBD is laid off equal to A so that the angle ABD is equal to B-A. The point D is located on AC. The transformed triangle T_{-1} will be similar to triangle ABD of which the angles are A, B-A, and C+A. From the similar triangles it will be found that

 $BD:DA:AB = ac:b^2 - a^2:bc$

The symbolic form of this transformation is

(2.1) $a:b:c \ge ac:b^2-a^2:bc$

It should be noted that the transformations (1.1) and (2.1) are inverses of each other. To illustrate: The transformation (1.1) will change the triad (6,7,8) into the triad (3,4,2); and the transformation (2.1) will return the triad (3,4,2) to (6,7,8). Also, (2.1) will transform the triad (6,7,8) into the triad (48,13,56); and (1.1) will change (48,13,56) back to (6,7,8). Two triangles of triads are linked together when a transformation will change one into the other. In this example these three triangles or triads are linked together to form the chain of three triangles

(48,13,56) , (6,7,8) , (3,4,2)

The transformation (1.1) can be reapplied to the transform when its third side is greater than the first. Similarly, the transformation (2.1) is applicable as long as the second side is greater than the first.

To derive the transformation associated with (3) again draw the triangle T_0 and make the angle CAD equal to B so that the point D falls on BC extended. The transformed triangle T' will be similar to the triangle ABD which has the angles A+B, B, and C-B. From the similar triangles ACD and ABD it will be found that

 $BD:DA:AB = ac:ab:c^2-b^2$

This gives the transformation (3.1) $a:b:c \ge ac$

 $a:b:c \ge ac:ab:c^2-b^2$

The result will be a triad of which the first number is greater than the second. This points the way to a fourth transformation by which the angles A, B, C with A>B of one certain triangle can be changed so that the angles of its transform are

(4) A-B, B, C+B By steps which are similar to those used in the preceding derivations the corresponding transformation is

(4.1) $a:b:c \ge a^2-b^2:bc:ac$

The transformations (3.1) and (4.1) are inverses of each other. The transformation (3.1) can be applied to a triad (a,b,c) when c>b, while (4.1) is applicable only when a>b. In the example previously given the transformation (3.1) applied to the triad (6,7,8) will yield the set (16,14,5). And the transformation (4.1) will send this latter back to the triad (6,7,8).

At this point the reader may choose to form the two chains which can be based on the triangle (4,5,7). Label the angles of the triangle so that A < B < C. The transformations (1.1) and (2.1) will give a chain of four triangles. This will be the chain that preserves the angle A. Next, the transformation (3.1) applied to this same basic triangle will give a chain of three links. In this chain the angle B is preserved. The result will be the two chains:

(28, 9, 35), (4, 5, 7), (20, 35, 33), (700, 1155, 689)(5) and

(4, 5, 7), (7, 5, 6), (42, 35, 11).

It may be noted that one or more of the four transformations could have been applied to the other triads of these two chains as they appear. However, it seems best to agree to limit the expansion so that from one basic triangle there will be only two chains: one, which will preserve the smallest angle; the other, which will preserve the angle second in size. Otherwise, since from each new triad another chain could be formed, there would ensue a network of triangles. This would unduly complicate the scope of this inquiry.

The Right Triangle

For the right triangle the arrangement A < B < C will make A+B = C. It follows that the transformations (1.1) and (3.1) will present two triangles which have, respectively, the angles A, B+A, C-A and A+B, B, C-B and which therefore in both cases are the basic right triangle, though its elements are in a different order. The recurrence of forms is the main feature of the transformations on right triangles.

Consider the right triangle (3, 4, 5). The transformations (1.1) and (2.1) give the chain of triangles:

(15, 7, 20), (3, 4, 5), (3, 5, 4), (15, 20, 7)

The triad (5, 4, 3) obtained by applying (3.1) to the basic triangle (3, 4, 5) will give a second chain of two links.

The Isosceles Triangle

There is ony one equilateral triangle, but an infinite number of isosceles triangles. The equilateral triangle is given by the triad (1, 1, 1) and to it none of the transformations given can be applied. The isosceles triangles fall into two groups which can be represented by the triads (a, b, b) and (a, a, c), where a is the smallest side. To the group given by (a, b, b) only the transformations (1.1)and (2.1) can be applied. The result will be one symmetric chain of triangles. Thus the triad (1, 2, 2) will give the chain:

(8, 5, 12), (2, 3, 4), (1, 2, 2), (2, 4, 3), (8, 12, 5)The triad (1, 3, 3) belongs to a chain of nine triangles. The second group represented by the triad (a, a, c)will have a < c < 2a. Only the transformations (1.1) and (3.1) are applicable. Since these two transformations will give essentially the same results use only (1.1) on the basic triangle denoted $t_1(a,a,c)$ which has the angles A, A, and C. This transformation will change t_1 into a triangle t_2 . The ratios of its sides are

(6) t_2 $a^2:ac:c^2-a^2$ Two of the angles of t_2 are A and 2A. This formula statement in (6) will—for all allowable relatively prime number pairs (a, c)—give all of that class of integral triangles t_2 which have one angle equal to twice another.

If the third number of t_2 in (6) is greater than the first then the transformation (1.1) can be repeated. The result will be a triangle t_3 . The ratios of its sides are

(7) t_3 $a^3:a(c^2-a^2):c(c^2-2a^2)$

If in t_s or (7) the third number is greater than the first the transformation (1.1) can again be applied. The result will be a formula statement which, in similarity to the preceding results, will represent the class of triangles t_4 which have one angle equal to four times another. And so on, as long as the transformation (1.1) can be applied.

To illustrate the preceding start with the isosceles triangle (4, 4, 7). Repeated use of the transformation (1.1) gives the chain of five triangles:

(4, 4, 7), (16, 28, 33), (64, 132, 119), (256, 476, 305), (1024, 1220, 231)

These five triads represent triangles which, respectively, belong to the classes t_n , n = 1, 2, 3, 4, 5, where a triangle of class t_n will have one angle equal to n times another.

From the preceding there can be many interesting developments. But to bring this paper to a close there is still one additional topic to be considered. It is the combinations of a triangle and its transforms.

When a triangle T_0 (a, b, c; A, B, C) is transformed by (1.1) into the Triangle T_1 then the angles of the latter are A, B+A, and C-A and the ratio of its sides are *ab*: *bc*: c^2-a^2 . Place these two triangles side by side with the vertices A upward, the transform T_1 on the right, to form an angle 2A. If the triangles have been lettered counterclockwise, then the sides opposite to the two angles A will be parallel. By the use of suitable factors on the triads T_0 and T_1 two triangles similar to T_0 and T_1 can be formed so that the two sides which coincide in part will coincide for their whole length. Thereby one triangle is formed of which the angles are 2A, B, and C-A. The ratios of the sides of this triangle are:

(8) $a(b^2+c^2-a^2):b^2c:c(c^2-a^2)$

For an example when the triangle (2, 3, 4) is combined with its transform (1, 2, 2) as described in the preceding paragraph, then these two triangles will combine to form the triangle (7, 6, 8). If A, B, C are the angles of (2, 3, 4), then A, B+A, C-A are the angles of (1, 2, 2)and 2A, B, and C-A are the angles of (7, 6, 8). By application of (8) to the triangle (7, 6, 8) it turns out that the triangle (119, 96, 40) will have the angles 4A, B, and C-3A.

In a similar manner the triangle T_0 can be transformed by (2.1) into a triangle T_{-1} . Then with T_{-1} placed at the left of T_0 so than an angle 2A is formed two triangles similar to T_{-1} and T_0 can be found and combined into one triangle with the angles 2A, B-A, and C. The ratios of the sides of this triangle are:

(9) $a(b^2+c^2-a^2):b(b^2-a^2):bc^2$

Also, place side by side in this order the three triangles T_{-1} , T_0 and T_1 so that an angle 3A is formed. Thereupon three triangles similar to these three can be found and combined into one triangle which will have the angles 3A, B-A, and C-A. The ratios of the three sides can then be found. For an example of this development consider the triad T_0 (2, 3, 4). The two adjoining transforms are: T_{-1} (8, 5, 12) on the left, and T_1 (1,2,2) on the right. By the use of suitable common factors these three are combined to form the triad (33, 10, 32). This will represent a triangle with the angles 3A, B-A, and C-A where A, B, C, are the angles of the triangle T_0 .

If the triad for the isoscles triangle T_0 is (a, b, b)b > a, then the ratios of the sides of the triangle which has the angles 3A, B-A, B-A will turn out to be $a(3b^2-a^2):b^3:b^3$

Thus, the isosceles triangle (1, 2, 2), where A is the odd angle, will yield the isosceles triangle (11, 8, 8) where 3A is the odd angle.

Two Characteristic Isosceles Triangles

Finally, it will be shown that for each chain of triangles there is one characteristic isosceles triangle.

Consider a chain of triangles which preserves the angle A. Represent any three successive triangles of the chain by the symbols T_0 , T_1 , and T_2 . If the sides and angles of T_1 are *a*, *b*, *c* and A, B, C to correspond, then the corresponding sides and angles of T_0 and T_2 are:

 (T_0) ac, b^2-a^2 , cb; A, B-A, C+A;

(T₂) ab, bc, c^2-a^2 ; A, B+A, C-A.

Combine the triangle T_0 and T_1 . This can be done by using the factor c so that the sides of T_1 become ac, bc, c^2 . Thereupon turn the triangle T_1 and place it adjacent to the triangle T_0 so that the angles B of T_1 and C+A of T_0 form a straight angle. The two sides which are opposite to the angles A are equal and coincide. Thereby an isosceles triangle is formed with the sides bc, bc, $b^2-a^2+c^2$ and the angles A, A, B-A+C. In a similar manner the triangles T_1 and T_2 can be combined. Since the angles C of T_1 and B+A of T_2 form a straight angle and both triangles have the angle A, the same isosceles triangle is formed. Thus any two successive triangles of the A-chain can be joined to form the A,A—isosceles triangle.

Furthermore, if a chain of triangles has been formed from a basic triangle to preserve the angle B, then any two successive links of this chain can be combined to form the isosceles triangle with the sides ac, ac, $a^2+c^2-b^2$ and the angles B, B, C+A-B. This is the B,B—isosceles triangle.

Conversely, it follows that the A,A—isosceles triangle, by a properly selected Cevian drawn from the center of the odd angle, can be separated into two triangles similar to any two successive triangles of the A-chain; and similarly, the B,B—isosceles triangle can be separated into two triangles similar to any two successive links of the B—chain.

AN ODE TO PARALLEL LINES

Geometry's a science-Pure reasoning, they say. It's pure, all right-and holy-A sacrifice each day! But reason? Well. I doubt it: Foolproof's a better word; And I got it from a speech Which, last night, I overheard. Two parallel lines were talking And discussing their complaint And telling all their troubles-Quite a picture they could paint! I'll try to tell you what I heard, As I listened to their wail: The first did all the talking: The second listened—pale. "I'm getting awful lonesome!" He shouted 'cross the plane: "I haven't got an honest friend-It causes me much pain! Man's getting to be such a boss-We lines can't meet in room or den: We meet out in infinity. A place I've never been. And when a line does come along. A new friend I could make-It's some old mean transversal. To cut me like a rake. Oh, yes, I have points up and down My back; but just the same I never get to know them well-They so often change their name.

The Pentagon

I love the funny student Who draws a line awry-So I meet a friend or brother: But all we do is cry. 'Cause in a second, one of us Will be erased away While someone says we're out of place !---And so on through the day." The second nodded knowingly And was about to speak, When a teacher called, "Stop dreaming!" And chalk began to squeak. I drew my mean transversal And cut the lines clean through; And it hurt me so I winced, as you Would wince if I cut you. The lines just lay there limp and still Upon their slate and wood: But one smiled at me faintly. He knew I understood

-JOAN DALEY Mount Mary College

Y

"For an easy way to reach the mountain top, many a traveller buys his ticket and takes the funicular. But some like a stiff climb over rocks and across streams, and such an ascent has its advantages if the heart is good and the muscles are strong!"

-W. F. OSGOOD

WHAT IS THE HEIGHT OF A SIX-FOOT MAN?

DANA R. SUDBOROUGH

Faculty, Central Michigan College of Education

Or, as a certain radio celebrity might ask, "Who is buried in Grant's Tomb?"

As elementary and simple as these questions may seem, it is still true that they ask a type of question which is frequently answered either incorrectly or, if correctly, by cumbersome methods. Examples are the following:

I. $\sqrt{7} \cdot \sqrt{7} = ?$ II. $\sqrt{-2} \cdot \sqrt{-2} = ?$ IV. $\log_{10} 10^n = ?$ V. $e^{\ln \delta} = ?$

III. $(\sqrt[3]{4})^3 = ?$ VI. $\tan(\arctan 1) = ?$

In Example I, of course, there is no real harm in the following analysis:

$$\sqrt{7}\cdot\sqrt{7}=\sqrt{49}=7.$$

But the alert student will recognize the fact that 7 is the answer by definition of $\sqrt{7}$. That is,

$$\sqrt{7}\cdot\sqrt{7}=(\sqrt{7})^2=7.$$

And, in Example II, this latter type of thinking is necessary, for it is erroneous to write

$$\sqrt{-2} \cdot \sqrt{-2} = \sqrt{4}.$$

Correct is the following:

$$\sqrt{-2} \cdot \sqrt{-2} = (\sqrt{-2})^2 = -2.$$

(We define $\sqrt{-2}$ to be a root of the equation, $x^2+2 = 0$, rather than as an ordered pair of real numbers, as in Function Theory.)

The analogy between the title of this article and the question asked in Example III should be immediately obvious. In other words, this example asks, "What is the cube of that number whose cube root is 4?"

By taking into account the definition of common logarithms, we note that Example IV asks, "To what exponent must 10 be raised to yield 10"?" Example V asks, "What is the result of raising e to that power to which e must be raised to yield 5?" And Example VI, of course, asks the question, "What is the tangent of that angle whose tangent is 1?"

Now the reader of this article, being a member of Kappa Mu Epsilon or a person particularly interested in mathematics, may have found nothing new in the above. The reason for submitting it is that the author has been surprised occasionally to find that some quite competent "mathematicians" overlook the inherent simplicity in questions of the type considered here. Also, the author would be glad to hear from any reader who knows a better analogous question than "What is the height of a six-foot man?" for helping beginners in reaching an understanding of such concepts as were considered in the examples.

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"Every word mathematicians use conveys a determinate idea and by accurate definitions they excite the same ideas in the mind of the reader that were in the mind of the writer . . . then they premise a few principles . . . and from these plain, simple principles they have raised more astonishing speculations."

-JOHN ADAMS

H. D. LARSEN

MATHEMATICIANS AND PHILATELY 43.

A description of postage stamps which carry portraits of mathematicians was published in 1949 by C. B. Boyer ("Mathematicians and Philately," Scripta Mathematica, Vol. 15, pp. 105-114, June, 1949.) A revised list of 34 mathematicians has been published recently by the writer (American Mathematical Monthly, Vol. 60, pp. 141-3, February, 1953.) An interesting series of club programs can be built around biographies of many of these mathematicians.

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(Continued next Issue)

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EDITORIAL NOTE: Professor Aude's paper may be used as a basis for club programs. The reading and study of it may suggest developments which can be carried on by students for presentation to club meetings.

THE PROBLEM CORNER

EDITED BY FRANK C. GENTRY

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before November 1, 1953. The best solutions submitted by students will be published in the Fall 1953 number of THE PENTAGON, with credit being given for other solutions received. To obtain credit a solver should affirm that he is a student and give the name of his school. Address all communications to Frank C. Gentry, Department of Mathematics, University of New Mexico, Albuquerque, New Mexico.

PROBLEMS PROPOSED

56. Proposed by C. E. Denny, Central College, Fayette, Missouri.

A student in Analytic Geometry obtained the equation $x(3-4\sqrt{3})+y(3\sqrt{3}+4)+\sqrt{3}-32 = 0$ for a certain line. His book gave the answer $x(48-25\sqrt{3})-11y-137+100\sqrt{3} = 0$. Show that the two equations represent the same line. 57. Proposed by Harold Skelton, Southwest Missouri State College, Springfield, Missouri.

If $N = n_1 + n_2 + n_3 + \ldots + n_r$ where n_i is a positive integer, show that N! is an in- $(n_1!)(n_2!)(n_3!)\ldots(n_r!)$

teger.

58. Proposed by Victor L. Osgood, Oceanport, New Jersey.

Given three concentric circles of radii a, b, and crespectively where a < b < c. If the radii do not differ too widely, it is possible to construct a fourth circle which will intersect the three concentric circles at such points that two equilateral triangles may be formed by connecting properly chosen points of intersection. In terms of a, b and c what is the distance from the common center to the center of the fourth circle and what is the radius of the fourth circle?

59. Proposed by John. R. Green, University of New Mexico, Albuquerque, New Mexico.

A man lives on a river bank one mile below a bridge. On one occasion he started upstream in his motorboat. As he passed under the bridge, his hat fell overboard and floated downstream. After continuing upstream a way he missed the hat, turned about and overtook it in five minutes at a point just opposite his house. What was the speed of the river current?

SOLUTIONS

51. Proposed by Harold Larsen, Albion College, Albion, Michigan. (From The Mathematical Gazette.)

Find the last 13 digits of 52!

Solution by Henry Beersman, Southwest Missouri State College, Springfield, Missouri.

Since there are factors of 10 in 52! some of the digits on the extreme right are zeros. The factors of 10 are 2 and 5. There are 12 factors 5 in 52!. There is at least one factor 2 in each of the even numbers just less than the multiples of 5 and at least one factor of 4 in 24 and also in 48, the even numbers just less than 25 and 50, each of which contains two factors 5. Multiplying each multiple of 5 by the even number just less than it we obtain the products: 20, 50, 210, 360, 600, 840, 1190, 1520, 1980 and 2400. It is evident then that the last twelve digits are all zeros. To determine the thirteenth digit, multiply the unit's digit of each of the numbers not used in the above products and the last non-zero digit of each of those products together, retaining only the last digit in each product. We thus find the last 13 digits of 52! to be a 4 followed by 12 zeros.

Also solved by Harvey Fiola, Forman, North Dakota.

52. Proposed by J. E. Allen, Phillips High School, Birmingham, Alabama.

In a certain corporation 20 per cent of the employees are women, 40 per cent of the unmarried employees are women, and 12¹/₂ per cent of the married employees are women. What per cent of all employees are married? What per cent of the men employees are married. What percent of the women employees are married?

Solution by Harvey Fiola, Forman, North Dakota.

Let x be the number of married employees and y the number of unmarried employees. Then x/8+2y/5 = (x+y)/5 so that y = 3x/8. Then x/(x+y) = 8/11 or 72.7%. Since 1/8 of the married employees are women, 7/8 of them are men. Hence (7/8) (8/11)/(4/5) = 35/44 or 79.5% of the men are married. Also (1/8) (8/11)/(1/5) = 5/11 or 45.4% of the women are married.

Also solved by Henry Beersman, Southwest Missouri State College, Springfield, Missouri. 52. Proposed by Judson Foust, Central Michigan College of Education, Mt. Pleasant, Michigan.

B tells C that A said, "I went to town today." A tells the truth only half the time and B tells the truth only two-thirds of the time. What is the probability that A did go to town, assuming that he made a statement with reference to going to town?

Solution by Henry Beersman, Southwest Missouri State College, Springfield, Missouri.

A could only have gone to town if both he and B had told the truth or if both of them had lied. The probability that they both told the truth is (1/2)(2/3) = 1/3. The probability that they both lied is (1/2(1/3) = 1/6. The probability that one or the other of these things took place is 1/6 + 1/3 = 1/2. This is then the probability that A went to town.

Also solved by Harvey Fiola, Forman, North Dakota.

54. Proposed by the Problem Corner Editor. (From School Science and Mathematics).

A candy dish contains 25 vanilla creams, 10 maple creams, and 10 raspberry creams. What is the least number of pieces one must take out of the bowl to be sure of having a) two with the same flavor? b) two with different flavors? c) three with different flavors? d) three with the same flavor?

Solution by Paul Hawthorne, 7th grade, California Avenue School, Hempstead, New York.

a) The first three could be all different. Any four must contain two with the same flavor. b) The first 25 could be all vanilla, any 26 must contain two with different flavors. c) The first 35 could be all vanilla and either maple or raspberry. Any 36 must contain three with different flavors. d) The first 6 could be 2 of each flavor. Any 7 must contain three with the same flavor.

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THE MATHEMATICAL SCRAPBOOK

EDITED BY H. D. LARSEN

And see how Mathematik rideth as a queen, Cheer'd on her royal progress thru' out nature's realm. —ROBERT BRIDGES

$$= \nabla =$$

"In [Choice and Chance] Whitworth defines gambling as the 'act of exchanging something small and certain for something large and uncertain.' This, of course, would include divorcing your present wife in the hope of marrying a fatter one of uncertain age and temper."—W. HOPE-JONES in the Mathematical Gazette.

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It is said that, when only six years of age, James Watt was discovered solving a geometrical problem on the hearth with a piece of chalk.

 $\pi = (3 + 1/7) (1 - .0004)$, to four decimal places.

= 7 =

PROBLEM: Given the area of a circle, to find that of another circle, which being described from a point as center on the circumference of the given circle, shall have that portion of its area outside the given circle equal to the area of the given circle.—Nature, May 2, 1878, p. 22.

= \(\nabla = \)

RULE FOR EXTRACTING THE CUBE ROOT: "Twice the nearest cube, added to the given number, is to the difference between the given number and the nearest cube, as the root of the nearest cube, to the correction; which must be *added* to the root of the nearest cube, if the nearest cube is less than the given number, otherwise *subtracted.*" — REUBEN BURROW, The Theory of Gunnery, 1779 (as quoted in the Mathematical Gazette.)

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WHY WORRY ABOUT METHOD?
Solve:
$$5/(x-6) + 4/(x-9) = 8/(x-7) + 1/(x-10)$$
.

The Pentagon

Solution: Then (x-6)/5 + (x-9)/4 = (x-7)/8 + (x-10)/1 8x-48+10x-90 = 5x-35+40x-400 297 = 27xx = 11

-MATH. TEACHER

Can you solve this cryptarithm? *TWENTY FIFTY NINE ONE EIGHTY*

-AMER. MATH. MONTH.

Gauss looked upon mathmatics as the principal means for developing human knowledge. A short time before his death he spoke to a celebrated psychologist on the possibility of putting psychology on a mathematical basis. — *Nature*, April 19, 1877.

$$= \nabla =$$

$$88^2 + 33^2 = 8833$$

$$= \nabla =$$

If the ratio of the Profit to the Cost is a:b, then the ratio of the Profit to the Selling Price is a:(a+b). If the ratio of the Profit to the Selling Price is c:d, then the ratio of the Profit to the Cost is c:(d-c).

"Another system [of complex numbers] has recently been used by Eddington in developing his theory of the universe. In Eddington's system there are no fewer than sixteen square roots of minus one. This system was used by Eddington, not all for fun, but because it appeared to be the best method of representing certain aspects of the physical world."

-E. C. TICHMARSH

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The Pentagon

"In 1627 Descartes published an epoch making treatise on analytical geometry. At one step the whole race of mathematicians strode far ahead of the Greek geometers." —E. T. BELL

 $= \nabla =$

 $= \bigtriangledown =$ The abbreviations sin, tan, sec are due to Girard (1626) and cos and cot to Oughtred, but they were forgotten till Euler revived them and brought them into general use.

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The following convenient notation has been suggested recently.

$$\begin{array}{rcl} 1 \cdot 3 \cdot 5 \cdot \ldots & (2n-1) & (2n+1) & = & (2n+1) & !! \\ 2 \cdot 4 \cdot 6 \cdot & \ldots & (2n-2) & (2n) & = & (2n) & !! \\ & & = & \bigtriangledown & = \end{array}$$

TO FACTOR A TRINOMIAL: An example will suffice to indicate the method. In $12x^2-11x-5$, note that (12)(-5) = -60. Two factors of -60 whose sum is -11 are -15 and 4. Then $12x^2-11x-5 = 12x^2-15x+$ 4x-5 = 3x(4x-5)+(4x-5) = (4x-5) (3x+1). $= \nabla =$

"The contour-lines of a column or tower, all of whose horizontal cross-sections are subject to constant specific stresses, are geometrically defined by cubic parabolas. This form results from the law of stresses under the given conditions, and may be seen in the contour of heavily supporting bridge piers, the Eiffel tower in Paris, and numerous other structures. Precisely the same problem nature has solved in building the trunks of tall trees. The famous coniferous trees of California offer the best illustration for this principle."—A. EMCH in *Popular Science Monthly*, May, 1911.

"Professor Heaviside was our Mathematical Pro. 'Old Heavy,' as he was called, was a big man, and very popular. At his lectures he used to be so engrossed in his subject that he never noticed the pranks that some of the men played, however noisy they might be; so one of them made a bet that he would drive some sheep into Heavy's lectureroom while he was lecturing, without his knowing anything about it, and he won the bet. He tipped a shepherd to let him have two or three sheep for half an hour; they were brought to the door of the lecture-room and driven in and out again, Old Heavy going on with his lecture as if nothing had happened."—SIR HASTINGS DOYLE, Tales Retaled, as quoted in the Mathematical Gazette.

TO FIND THE DAY OF THE WEEK FOR ANY GIVEN DATE. Having hit upon the following method of mentally computing the day of the week for any given date, I send it to you in the hope that it may interest some of your readers. I am not a rapid computer myself, and as I find my average time for doing any such question is about 20 seconds, I have little doubt that a rapid computer would not need 15.

Take the given date in 4 positions, viz. the number of centuries, the number of years over, the month, the day of the month.

Compute the following 4 items, adding each, when found, to the total of the previous items. When an item or total exceeds 7, divide by 7, and keep the remainder only.

The Century -Item.—For Old Style (which ended September 2, 1752) subtract from 18. For New Style (which began September 14) divide by 4, take overplus from 3, multiplying remainder by 2.

The Year-Item.—Add together the number of dozens, the overplus, and the number of 4's in the overplus.

The Month-Item.—If it begins or ends with a vowel, subtract the number, denoting its place in the year, from 10. This, plus its number of days, gives the item for the following month. The item for January is "0"; for February or March (the 3rd month) is "3"; for December (the 12th month), "12." The Day-Item is the day of the month.

The total, thus reached, must be corrected by deducting "1" (first adding 7, if the total be "0"), if the date be January or February in a Leap Year; remembering that every year, divisible by 4, is a Leap Year, excepting only the century-years, in New Style, when the number of centuries is *not* so divisible (e.g. 1800).

The final result gives the day of the week, "0" meaning Sunday, "1" Monday, and so on.

EXAMPLES

1783, September 18: 17 divided by 4 leaves "1" over; 1 from 3 gives "2"; twice 2 is "4."

83 is 6 dozen and 11, giving 17; plus 2 gives 19, *i.e.* (dividing by 7) "5." Total 9, *i.e.* "2."

The item for August is "8 from 10," *i.e.* "2"; so, for September, it is "2 plus 3," *i.e.* "5" Total 7, *i.e.* "0," which goes out.

18 gives "4." Answer, "Thursday." 1676, February 23: 16 from 18 gives "2."

76 is 6 dozen and 4, giving 10; plus 1 gives 11, *i.e.*, "4." Total "6."

The item for February is "3." Total 9, *i.e.*, "2." 23 gives "2." Total "4." Correction for Leap Year gives "3."

Answer, "Wednesday."

-LEWIS CARROL, Nature, March 31, 1887.

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HENKLE'S NAMES OF THE PERIODS IN NUMBERS

Millions (1), Billions (2), Trillions (3), Quadrillions (4), Quintillions (5), Sextillions (6), Septillions (7), Octillions (8), Nonillions (9), Decillions (10), Undecillions (11), Duodecillions (12), Tertiodecillions (13), Quarto-decillions (14), Quinto-decillions (15), Sexto-decillions (16), Octo-decillions (18), Nono-decillions (19), Vigillions (20),

Primo-vigillions (21), Secundo-vigillions (22), Tertio-vigillions (23), Quarto-vigillions (24), Quinto-vigillions (25), Sexto-vigillions (26), Septo-vigillions (27), Octo-vigillions (28), Nono-vigillions (29), Trigillions (30), Quadragillions (40), Quinquagillions (50), Sexagillions (60), Septuagillions (70), Octogillions (80), Nonagillions (90), Centillions (100), Primo-centillions (101), Decomo-centillions (110), Undecimocentilions (111), Duodecimo-centillions (112). Tertio-decimo-centillions (113), Quarto-decimocentillions (114), Vigesimo-centillions (120), Primo-vigesimo-centillions (121), Trigesimo-centillions (130), Quadragesimo-centillions (140), Quinquagesimo-centillions (150), Sexagesimo-centillions (160), Septuagesimo-centillions (170), Octogesimo-centillions (180), Nonagesimocentillions (190), Ducentillions (200), Trecentillions (300). Quadringentillions (400). Quingentillions (500), Sexcentillions (600), Septingentillions (700), Octingentillions (800), Nongentillions (900), Millillions (1000), . . . , Deci-millillions (10,000), Undeci-millillions (11,000), Duodeci-millillions (12,000), . . . , Quinqui-vici-millillions (25,000), ..., Centi-millillions (100,000), ..., Milli-millillions (1,000,000).

It should be observed that words ending in "o" represent numbers to be added, and those ending in "i" represent multipliers. When two words end in "i," the sum of the numbers indicated is to be taken as the multiplier. In each, the last word indicates the number to be increased or multiplied.—Edward Brooks, *Philosophy of Arithmetic*, 1880.

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THE BOOK SHELF

EDITED BY FRANK HAWTHORNE

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Frank Hawthorne, Hofstra College, Hempstead, New York.

Introduction to the Foundations of Mathematics. By Raymond L. Wilder. John Wiley and Sons (440 Fourth Avenue; New York 16, N. Y.), 1952. 14+305 pages. \$5.75.

The "Foundations" should appeal to those undergraduate mathematics majors, who in their senior year begin to augment their background prior to pursuing graduate work or begin a teaching career. It is a unique presentation, in that a careful, detailed treatment of the source and nature of the basic foundations of modern mathematics is brought to the undergraduate level.

It is suggested by the author that the book should normally require two semesters in classroom study. The material represents the result of more than 20 years experience in teaching these and similar topics in the Foundations of Mathematics course at the University of Michigan.

In arrangement of material the book is divided into two parts, the first including seven chapters which treat the axiomatic method, the theory of sets, foundations of analysis (linear continuum and real numbers) and the significance of group theory. Part II develops the origins and momentum which certain aspects of analysis and logic contributed to the growth of mathematics. Symbolic logic is particularly stressed. The numerous formal aspects concerning what mathematics is and existence concepts are perhaps too restrictive for mathematical applications; nevertheless such study will add greatly to the maturity of the undergraduate's appraisal of mathematics. Physically the book is of high quality and the type of notation contributes to facility in study. There is a rather extensive bibliography of source material, indices of symbols, topics and technical terms, and names included with the content.

-J. HAROLD SKELTON

An Introduction to Mathematical Thought. By E. R. Stabler. Addison-Wesley Publishing Company, Inc. (Cambridge 42, Massachusetts), 1953. 18+268 pages. \$4.50.

The author states in the preface of this stimulating book that his chief aim in the preparation of the text was to present a unified and substantial approach to the logical structure of mathematics and to develop a philosophical point of view toward mathematical knowledge. He further states that, by a suitable choice of chapters, the book is suitable as a text for the following courses:

- 1. A one-semester general education course.
- 2. A one-semester course for prospective teachers.
- 3. A one-semester course for specialists in mathematics.
- 4. A two-semester general education course.
- 5. A two-semester course for specialists in pure mathematics.

Part I, which is composed of the first five chapters, is devoted to the philosophical aspects, historical background, elementary symbolic logic, and to the use of logical reasoning in the formulation of mathematical and scientific knowledge.

In Chapter 1, the reader is introduced to the relative nature of mathematical truth by means of a discussion of modulo arithmetic, number bases, Euclidean and non-Euclidean geometries, and the historical unsolved problems of mathematics. Chapter 2 is historical and discusses the origin and influence of logical systems, with particular attention to Euclid. Chapters 3 and 4 deal with the essentials of logical reasoning and contain explanations of the rules of inference, propositional functions, general propositions, and classes. Chapter 5 considers the applications of logical reasoning to the formulation of scientific theories and illustrates the ideas by discussing the work of Newton and Einstein.

Part II, Chapters 6-11, deals with modern postulational methods, abstract postulational systems, foundations, and some foundational points of view. The method is illustrated by developing a part of elementary algebra as a logical system and by presenting an abstract system in the form of a finite geometry. Then, after a more detailed presentation of modern postulational concepts and methods, the author discusses groups, rings, Boolian algebras, relations, order systems, and lattices. The final chapter presents a survey of mathematical foundations covering geometry, algebra, natural numbers, and logic.

Throughout the book, at suitable intervals, interesting and well-chosen exercises are provided. A comprehensive bibliography will be found in the final pages of the book.

In the opinion of this reviewer, the author has indeed achieved his aim in writing the book. Futhermore, he is justified in his claim that the text is suitable for the courses mentioned above. Certainly, the content of the first five chapters should be a part of any general cultural curriculum. By all means, prospective teachers of mathematics should become acquainted with the major portion of the text. Furthermore, any student who contemplates entering the graduate field in mathematics would do well to master the entire volume.

The book is remarkably free of the tedium that is so frequently found in texts dealing with formal logic and mathematical foundations. The author avoids the use of long and involved sentences and the use of unusual and ponderous words. When a technical term is introduced, it is clearly and briefly defined and illustrated. The illustrations are interesting, pertinent, and deal with familiar situations and ideas. The author further facilitates the reading of the book by frequent repetitions of the basic notions, thus making is unnecessary for the reader to turn back in the midst of a discussion to bring himself up to date. This reviewer found the perusal of the book a pleasant and stimulating experience.

-FRED W. SPARKS

Mathematics, Its Magic and Mastery. By Aaron Bakst. Second Edition. D. Van Nostrand Company. (250 Fourth Avenue, New York 3, N. Y.), 1952. 14+790 pages. \$6.00.

This thick and handsomely bound book concerns itself with what the layman usually regards as mathematics; namely arithmetic, the algebra, geometry, and trigonometry of high school, and some elementary mechanics. Its thirtyseven chapters are written in a somewhat facetious but adult style. An appendix of almost 80 pages contains a collection of processes from algebra, facts from geometry, formulas from trigonometry, and several useful tables. The book closes with an index and a collection of answers to the 244 problems which appear at the ends of the various chapters. The typography is excellent and there are many figures and drawings accompanying the text.

The purpose of the book is to exhibit the recreational and applied phases of elementary mathematics; the magic being illustrated by many well chosen applications and tricks. It is an excellent book for the shelf of a teacher in either elementary school or high school, and it contains much material useful to a high school mathematics club. The book is not intended as a classroom text, but its gradual and detailed explanations make it easy reading for an interested student or layman.

This second edition is essentially the same as the first edition published in 1941. The effort to correct the misprints of the first edition was not entirely successful, and a few mathematical errors still remain undetected, such as that on page 381, where the author writes: "In the case of a perfectly circular object, a Flatlander may see this circular object as a straight line of the same length. On the other hand, in the case of any other round object, these lengths are not the same." As with everything else, prices of books have greatly increased over the past ten years; a copy of the first edition sold for \$3.95.

-HOWARD EVES

Numerical Solution of Differential Equations. By William Edmund Milne. John Wiley and Sons, Inc. (440 Fourth Avenue, New York 16, N.Y.), 1953. 11+275 pages. \$6.50.

The author states in the preface that his book is intended to acquaint the reader with some of the principal methods for the solution of ordinary and partial differential equations. The manner of presentation is to be elementary, with clarity and simplicity rather than completeness and rigor as primary objectives, making the book suitable as a guide for computers. Part I (Chapters 1-7) deals with ordinary and part II (Chapters 8-11) deals primarily with partial differential equations.

Chapter 1 contains a brief discussion of differential equations and their solutions. Elementary methods based on polygonal approximations together with estimates of errors are discussed in chapter 2. Methods involving numerical integration and successive approximations as well as the existence theorems on which they are based are presented in chapters 3 and 4. Kutta's and similar methods are treated in chapter 5. Systems of equations are discussed in chapter 6 and methods for two point boundary problems are presented in chapter 7. Chapter 8 treats the heat and wave equations as examples of more general partial differential equations. Matrices, their characteristic values and relaxation methods are discussed in chapter 9. Chapter 10 presents methods for solving Laplace's Poisson's and the biharmonic equation. Characteristic value problems are treated in chapter 11. A short appendix, an index and an excellent bibliography are included. The usefulness of the book is enhanced by carefully worked out examples which abound throughout.

It seems to this reviewer that the author's primary objectives have been successfully realized, and that he has produced a readable and easily understandable book. It seems to present, in reasonably good balance, well chosen representatives of the myriad of methods of existence. It might be argued, however, that partial differential equations might have been treated more fully, even at the expense of omitting a few methods for ordinary differential equations.

The elegant and useful methods for hyperbolic equations based on nets of characteristic curves, should have been given more prominence. In view of their importance in theoretical physics and engineering problems, non-linear and quasi-linear problems in partial differential equations might have been treated more fully. This reviewer realizes, however, that probably any omission would draw criticism. On the whole, it is a very useful book, which seems likely to contribute substantially to the clarification of this subject.

-H. Wolf

BOOKS RECEIVED BY THE BOOK SHELF EDITOR

- The Rational and the Superrational. By C. J. Keyser. Scripta Mathematica. (186th St. New York 33, N.Y.), 1952. 8+259 pages. \$4.25.
- A School Course in Mechanics, Part I. By A. J. Bull. Cambridge University Press (32 E. 57th St., New York 22. N. Y.), 1953. 6+156 pages. \$1.75.
- Logic for Mathematicians. By J. Barkley Rosser. Mc-Graw-Hill Book Company, Inc. (330 West 42nd St., New York 36, N. Y.), 1953. 14+530 pages. \$10.00.

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INSTALLATION OF NEW CHAPTER

EDITED BY J. M. SACHS

THE PENTAGON is pleased to report the installation of Kansas Epsilon Chapter of Kappa Mu Epsilon. With the addition of Kansas Epsilon there are now forty-seven active chapters.

KANSAS EPSILON CHAPTER Fort Hays Kansas State College, Hays

Kansas Epsilon Chapter was installed and the twentythree charter members of that chapter initiated in a morning ceremony at Fort Hays Kansas State College on December 6, 1952. Professor Charles B. Tucker, national president of Kappa Mu Epsilon, served as installing officer.

A banquet for members, guests, and faculty followed the ceremony. Douglas Sellers was toastmaster. Dr. Cunningham, President of Fort Hays Kansas State College, welcomed the group and extended to them the support of the college. Professor Charles B. Tucker talked on, "The History and Purposes of Kappa Mu Epsilon." Professor Eugene Etter of Kansas Epsilon spoke on, "The Language of Mathematics."

The following officers of Kansas Epsilon were installed. President, Douglas Sellers; Vice-President, Royce Rasmussen; Secretary-Treasurer, Kenneth Werth; Corresponding Secretary and Faculty Sponsor, Professor Eugene Etter.

We all welcome Kansas Epsilon into our fellowship. The staff of THE PENTAGON wishes to join with the national officers in extending congratulations and heartiest best wishes to our newest chapter.

KAPPA MU EPSILON NEWS

EDITED BY LAURA Z. GREENE, Historian

Alabama Beta sponsored Homecoming activities for the former members of Kappa Mu Epsilon in connection with the Homecoming for the College.

Joseph Mueller was elected president and Raymond Cowan was chosen secretary of Indiana Beta. The former president and secretary were graduated in January.

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Kansas Beta arranged mathematical demonstrations and an exhibit for the Science Open House at Kansas State Teachers College.

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Kansas Delta recently made a survey of all of the members of Kappa Mu Epsilon who have been initiated since the chapter was installed in 1947. The study revealed that the membership is divided as follows:

Teachers	.25%
Doctors	.10%
Lawyers	. 1%
Statistics	.10%
Engineering	. 5%
Business	.19%
Service	. 3%
Meteorology	. 1%
University and	
Graduate School	.16%

The members of Louisiana Beta held open house for alumni on homecoming November 1. Two members of Louisiana Beta attended an address given by Professor R. H. Bing of the University of Wisconsin at the Louisiana State University in Baton Rouge. Several members of Louisiana Beta attended the Louisiana Education Association convention in New Orleans during the Thanksgiving holidays.

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Missouri Beta published a most interesting Christmas letter which was sent to all interested alumni and friends of the college who have attended Central Missouri State College since World War II and who were either members of Kappa Mu Epsilon or majored in mathematics.

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New York Alpha members saw "Cinerama," the movie with the new dimensions, December 6. Professor Carl B. Boyer of Brooklyn College was the guest speaker at the annual initiation banquet of New York Alpha.

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Ohio Gamma members chose as their project for the year a statistical study of the results of a traffic survey for the city of Berea, Ohio. The purpose of the study was to determine the most desirable location for a new bridge. Another focal point of the year's program was the study of the research being conducted at the Lewis Flight Propulsion Laboratory of NACA.

Mr. and Mrs. L. P. Woods and Mr. and Mrs. Ray Carpenter entertained Oklahoma Alpha members and pledges of Kappa Mu Epsilon December 4, 1952. Claude Berry, a World War II and Korean veteran gave an unusually interesting program on "Codes."

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Tennessee Alpha has adopted the following requirements for new members: 3.1 quality quotient in mathematics and 2.6 in general scholarship, or 3.0 quality quotient in mathematics and 2.8 in general scholarship. This is based on 4.0 as a possible maximum. All initiates must have had at least one quarter of calculus.

PROGRAM TOPICS, SPRING SEMESTER, 1952-1953

Alabama Beta, Alabama State Teachers College Radio Quiz Program, by J. D. Clanton Requirements for Ph.D. Degree, by Dr. Ralph C. Boles Fun with Numbers, by Tom Williams Colorado Alpha, Colorado A and M College Codes and Coding Methods, by Professor Butz Basic Concepts of Mathematics, by Professor Madison Illinois Delta, College of St. Francis Astronomical Instruments, by Sister Rita Clare Astronomical Terminology, by Sister Noel Neptune and its Satellite, by Irene Regan The Planet Mars, by Geraldine Knowles and Patricia Kasak Indiana Beta, Butler University Equations Solvable with a Slide Rule, by Joseph Mueller The Universe and Dr. Einstein, by Austin Werner The Origin and Meaning of the Symbols of the KME Crest, by Donald Cassady The Philosophy of Mathematics, by Robert Simon Electronic Computers, by Chester Rector Iowa Alpha, Iowa State Teachers College Mathematical Fallacies, by Donald Licktenberg Fermat's Last Theorem, by Thomas Yoger Lumsal Arithmetic, by E. W. Hamilton Reverse Notation of Numbers, by Clyde Dilley Iowa Beta, Drake University Cardan's Solution of the Cubic, by Art England Number Systems to Various Bases, by Gary Drown Kansas Alpha, Kansas State Teachers College The Philosophy of Mathematics, by Bill England Quality Control, by John Herring Kansas Beta, Kansas State Teachers College History of K. M. E., by Charles B. Tucker Job Opportunities in Mathematics, by W. T. Stratton Mathematical Games, by Morgan Kramm Kansas Delta, Washburn Municipal University Gauss' Derivation, by Loren R. McMurray Operation Analysis, by G. Baley Price The Measurement of Molecular Diameter by the Surface Pressure Balance Method, by Harlan B. Johnson Application of Mathematics after Graduation, by Mr. Terry McAdam, Mrs. John Erdman, Mr. Milton Rubottom, and Mr. Robert Pooler. Louisiana Beta. Southwestern Louisiana Institute Some Properties of Vector Spaces, by Margaret LaSalle The Chinese Game, Nim, by Merlin M. Ohmer Dimensional Analysis, by Peter Bernays

Michigan Beta, Central Michigan College A Year's Sojourn at Columbia, by Josephine Montague Michigan Gamma, Wayne University Analog Computers, by Arvid Jacobson Missouri Beta, Central Missouri State College Pythagorean Number Triples, by Richard L. Smith Non-Euclidean Geometries, by Billy P. Mudd Denumerable and Non Denumerable Infinities, by Richard G. Laatsch Inversion, by William W. Varderan Geometric Designs by Paper Folding, by Bess Rickman Jokes and Puzzles, by Gilbert Lee Missouri Epsilon, Central College Hypercomplex Numbers, by George Koonce Relativity, by Tom Hahs Fibonacci's Series, by Carl Dulgeroff Mathematics of Poker, by Bob Zey Non-Euclidean Geometry, by Dave Morrison Pascal, by Glenn Bowmann History of Kappa Mu Epsilon, by George Koonce and Dave Morrison Mathematics of Symbolic Logic, by Dave Morrison Applications of Symbolic Logic, by George Koonce New Jersey Alpha, Upsala College Mathematical Analysis of Logic, by Gordon Fulcher Meaning and Functions of K. M. E., by Ellis Fuls Graphical Solution of Cubic Equations, by Ellis Fuls New Jersey Beta, New Jersey State Teachers College Mathematical Definitions, by John Manning What an Actuary is and the Opportunities in the Actuarial Field. by Paul Rotter The Binomial Theorem, by Dr. Howard Fehr Quality Control in Industry, by Charles Sensale New York Alpha, Hofstra College The Qualifications and Duties of an Actuary, by Kenneth Feldman Dimensions, by Geoffrey B. Charlesworth Quaternions, by Sharon Murnick The Golden Rectangle, by Walder Old The Four Color Problem, by Dr. L. F. Ollmann Axiomatic Development of Logic, by Richard Lamm North Carolina Alpha, Wake Forrest College Aims of Kappa Mu Epsilon Problems, by Evelyn Blackwell Law of Estates, by Jack Herring Foundations of Geometry, by John Ivscoe Taylor's Theorem, by Dr. I. C. Gentry Determinants, by Evelyn Blackwell

Mathematical Games of Baltaire, by Professor Medlin Square Roots, by Kenneth Byrd **Ohio Alpha, Bowling Green State University** An Economist Looks at Mathematics, by Dr. Jacob Cohen Bachst's Problem of Weights, by Miss Betty Bernhardt The Derivation of Schrodinger's Wave Equation, by Donald Bowman Euler's Totient Function, by Miles Vance Ohio Gamma, Baldwin-Wallace College Field Trip to National Advisory Committee on Aeronautics, by George Diedrich Machines used in Computing at NACA, by Nancy Hartup Traffic Survey in Berea, by Nancy Hartup Wisconsin Alpha, Mount Mary College What a Mathematics Major Can Do Besides Teach, by Sister **Mary Felice** Who Should Be Called a Mathematician?, by Sister Felice Whether or not K. M. E. Should Be an Honor Society, by Sister Felice Oklahoma Alpha, Northeastern State College Extracting Roots by Arithmetic, by R. Carpenter Short Cuts in Mathematics, by Pat Scott Trisecting an Angle, by Barbara Sloan Trisecting an Angle, by Mary Frye Proof of Impossibility of Trisecting an Angle of Plane Geometry, by Henry Adair The Laws of Chance and Applications, by David Morris Code Making, Breaking and Importance, by Claude Berry South Carolina Alpha, Coker College Uses of the Slide Rule, by M. Saunders An Irrational Uneven Problem, by Mr. Matthews An Inertia Problem, by Mr. Rice Business Machines, by Mr. Brown The Teaching of Business Mathematics, by Mr. Hobgood A Problem of Finance, by Mr. Brown Tennessee Alpha, Tennessee Polytechnic Institute Perspective Drawings, by Professor F. J. Witt Texas Epsilon, North Texas State College Mathematics in Industry, by W. J. Nemerever A Problem on Irrational Numbers, by Joe R. Ballard and Pete Reames Integer Functions, by E. H. Hanson The Role of Mathematics in the Oil Industry, by T. S. Edrington Approximate Integration, by Tom L. Gallaher Some Problems in the Mathematical Monthly, by Professor Cooke, Grace Simpson, Stanley Wilks, Charlotte Clark, and Ahelino Sanchez.