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CONTENTS

WHO'S WHO IN KAPPA MU EPSILON

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Kappa Mu Epsilon, national honorary mathematics fraternity, was founded in 1931. The object of the fraternity is four-fold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievement in the study of mathematics in the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

ON THE ROTATION OF AXES*

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In the study of Analytic Geometry, methods are developed for simplifying the general equation of the conic section,

(1) $$Ax^2+Bxy+Cy^2+Dx+Ey+F = 0,$$

by a transformation of the coordinates. Thus, the xy-term may be removed by rotating the axes through an angle θ, where $\tan 2\theta = B/(A-C)$, the equations for the rotation being

(2) $$x = x_1\cos\theta - y_1\sin\theta, \quad y = x_1\sin\theta + y_1\cos\theta.$$

Unless the given equation is "hand-picked," the algebraic manipulation in determining the new equation becomes very cumbersome. This paper presents a relatively simple formula for the new equation, its coefficients being expressed in terms of the coefficients of the original equation.

If $\tan 2\theta = B/(A-C)$, then $\cos 2\theta = (A-C)/R$, where

$$\mathrm{R} = \sqrt{(A^2+B^2+C^2-2AC)}.$$

Consequently,

$$\sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta) = (R-A+C)/2R,$$
$$\cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta) = (R+A-C)/2R.$$

Substituting the resulting values of $\sin\theta$ and $\cos\theta$ in (2) and then applying that transformation to (1), we obtain as the new equation,

$$A'x_1^2+C'y_1^2+D'x_1+E'y_1+F = 0,$$

where

$$A' = \tfrac{1}{2}\ (A+C+R)$$
$$C' = \tfrac{1}{2}\ (A+C+R)$$
$$D' = [D\sqrt{(R+A-C)}+E\sqrt{(R-A+C)}]/\sqrt{(2R)}$$
$$E' = [E\sqrt{(R+A-C)}+D\sqrt{(R-A+C)}]/\sqrt{(2R)}$$
$$R = \sqrt{(A^2+B^2+C^2-2AC)}.$$

* Condensed and adapted by the Editor from a longer paper submitted by the authors, for whom these results represent original, creative work.

THE PRYTZ PLANIMETER*

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The object of this article is to call attention to a little known but very interesting and simple mathematical instrument, the Prytz planimeter, or to give its more common name — the hatchet planimeter. It is easily made from a piece of stiff wire which is bent into the form shown in Fig. 1. One end of the wire is ground to a point and the other end is flattened into a chisel edge. The sharp point and this edge should be in the same plane. For a given area a planimeter which is longer than the longest diameter should be used.

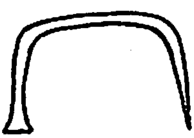

Fig. 1

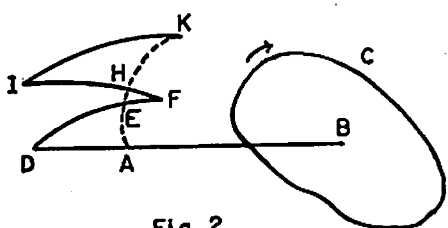

Fig. 2

To obtain the area of a closed curve with this planimeter, a straight line of indefinite length is drawn from the approximate center of gravity in any direction (Fig. 2). The pointed end of the instrument is placed on the center of gravity and the chisel edge on the straight line (at A in the figure). The legs of the planimeter must at all times be perpendicular to the plane of the area. From its position B, the tracing point is now moved along the straight line to the intersection with the boundary curve and then around the area in the direction indicated and back to B. The edged end of the instrument, upon which a slight pressure is brought to bear, traces out the curve $ADEFHIK$ and when the tracing point has returned to B, will take the position K. The product of the length AK into the length of the planimeter will be the approximate area of the given curve. (See equation (7))

* Reprinted from the *American Mathematical Monthly*, Vol. 15, pp. 55-57 (March, 1908).

This instrument was invented some fifteen years ago by Captain Prytz of the Danish Army, who published an account of his invention in the English magazine, *Engineering,* Vol. 72, page 813. A detailed analytical discussion of its theory was given by M. F. W. Hill in the *Philosophical Magazine* for 1894. In the *Bulletin de l'Académie Imperiale des Sciences de St. Petersbourg,* 1903, Professor Kriloff of the Russian Naval Academy discussed the instrument from the geometrical standpoint and gave a very elementary and simple explanation of its theory.

This theory depends on the well known theorem[1]: The total area Z swept out by a straight line AB moving in a plane is given by the formula

(1) $$z = ls + (\tfrac{1}{2}l^2 - al)(\theta_2 - \theta_1),$$

where l is the length of the moving line; s, the total normal displacement of a point P of the line; a, the distance AP; and θ_1, θ_2 are the initial and final values of θ, the angle made by the moving line with some fixed line. If, in particular, the point A moves around a closed curve C', while at the same time B makes a complete circuit of the curve C which lies entirely outside of C', then $\theta_2 = \theta_1$, and we have

(2) $$z = C - C' = ls.$$

The usual conventions as to the signs of the areas in question hold. Areas covered twice in opposite directions by the line are zero, and an area described counter clockwise is positive. If a small wheel with its axis in the moving line be attached at P, the arc through which it turns will give us the normal displacement s. We will call this wheel the measuring wheel. It should be noticed that the position of the point P does not enter into equation (2). In measuring areas with most planimeters the area of the curve C' is a constant of the instrument. In the case where the point corresponding to A moves backwards and forwards on a curve this constant is zero. In the well known Amsler polar planimeter the curve C' is a circle, and in some other instruments it is a straight line. The area C is the area to be measured.

1 See Chapter XIV, Gibson's *Calculus.*

In the Prytz planimeter there is no fixed curve C', but instead we have a curve which depends upon the curve whose area we are measuring. Referring to Figure 2, we see that as the point B moves in its path, the edge A moves in its curve of pursuit from A to K. Now turn the planimeter horizontally about the point B until it is in the initial position AB. The curve corresponding to the curve C' is now a closed curve $ADEFHIKA$. The total area swept out by the line AB is equal to the algebraic sum of the area C and the areas ADE, EFH, and HIK. The other parts of the plane swept by the line are covered twice in opposite directions and hence do not enter into the algebraic sum. Putting in the proper signs we have

(3) Total area swept out $= C - ADE + EFH - HIK$.

From equation (2) the total area swept out is measured by the product of the length l of AB into the length of arc s through which a measuring wheel at A on the line AB would have turned. During the motion of the tracing point this wheel does not turn at all, for the direction of motion is perpendicular to the edge of the wheel. But in turning the instrument about B from the position KB into AB, this wheel will turn through an arc which is equal in length to $l\Phi$ where Φ is the angle KBA. Or we have

(4) $$s = l\Phi.$$

The total area swept out by AB is ls or $l^2\Phi$. This gives the exact equation,

(5) $$l^2\Phi = C - ADE + EFH - HIK.$$

If the starting point for tracing the figure be taken as above (i.e., at the center of gravity of the area), the algebraic sum of the three areas enclosed by the curve of pursuit and the arc KA will be very nearly zero and we have the approximate equation

(6) $$C = l^2\Phi = l\cdot\text{arc}KA.$$

If the angle Φ be small, say less than 20°, the arc KA can be replaced by its chord, and we have

(7) $$C = l\cdot KA.$$

If the area whose longest diameter is four inches or less, be measured with a ten-inch planimeter, the error is very small and is about equal to the error made in finding the area of an equivalent rectangle by measuring the sides with a scale. The error due to the non-alignment of the edge and tracing point can be eliminated by tracing the curve in opposite directions and finding the mean of the two results.

An improved plainimeter of this type has a small chisel-edged wheel instead of the chisel edge.

A NOTE ON RATIONAL COSINES

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We prove the following theorem:

THEOREM. If $\cos A$ *is a rational number, then* $\cos nA$ $(n = 2, 3, \ldots)$ *also is rational.*

Proof: The symbols F_i and $f_i (i = 1, 2, 3, \ldots)$ will denote polynomial functions of $\cos A$. Write $\cos iA = F_i$ and $\sin iA = f_i \sin A$. Using this notation, the expressions for $\cos 2A$ and $\sin 2A$ can be written

$$\cos 2A = F_2 = f_1 \cos^2 A + F_1 \cos A - f_1$$
$$\sin 2A = f_2 \sin A = (f_1 \cos A + F_1) \sin A.$$

It will be seen that the corresponding expressions for $\cos 3A$ and $\sin 3A$ are

$$\cos 3A = F_3 = f_2 \cos^2 A + F_2 \cos A - f_2$$
$$\sin 3A = f_3 \sin A = (f_2 \cos A + F_2) \sin A.$$

The pattern of the successive functional dependence is now evident. Thus, we assume that $\cos kA = F_k$ and $\sin kA = f_k \sin A$. It will be seen readily that the expressions for $\cos(k+1)A$ and $\sin(k+1)A$ turn out to be

$$\cos (k+1) A = F_{k+1} = f_k \cos^2 A + F_k \cos A - f_k$$
$$\sin(k+1)A = f_{k+1} \sin A = (f_k \cos A + F_k) \sin A,$$

and the induction is complete. It follows that $\cos nA$ is a polynomial function of $\cos A$ which is rational if $\cos A$ is rational.

THE WORK OF A CENTURY —IN A FEW MINUTES*

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"How much. . .?" "How many. . .?" A surprisingly large number of important questions of modern life begin with those words. How many skilled mechanics can India rely on for building much-needed machinery? How much will a sixpence-in-the-pound increase in income tax yield? How much energy can you get from a pound of uranium 235?

Some of the most important questions of all do not, at first sight, take this form:—For instance, can India avoid a famine in 1956? But to answer it, we have to ask: How many mouths will there be to feed? How much food can India's own agriculture produce? How much of her manufactures can she export to make up for the deficiency? . . . and so back to that first question about skilled mechanics. Without "How much?" and "How many?" we should not make our modern world work.

In science, more than anything else, these two questions are of vital importance. There is scarcely any major scientific problem—from the nature of the universe to the workings of the endocrine glands, from the utilisation of atomic energy to the assessment of an adequate diet—whose solution does not depend essentially on counting and measuring, and then making calculations.

Making calculations—that is the important point. To answer "How much?" and "How many?" it is usually not good enough to measure, weigh and count. We must also do immense calculations to deduce facts about a whole population in 1956 from a few thousand answers to a sample survey in 1951, or to calculate the energy obtainable from a uranium pile on the basis of data produced from small-scale experiments with cyclotrons.

It is with this process of calculation and the inge-

* Reprinted from the *Unesco Courier*, February, 1952.

nuity that has gone into making machines to do it for us that this article is concerned.

"How much?" and "How many?" have not always been so important. They are characteristically modern questions. Until some 350 or 400 years ago they mattered rather little. The small peasant communities that made up most of the world till then could get along very well with a minimum of calculation.

Even the cities of Greece or the Empire of Rome needed less in the way of statistics than a town council does today. And in science before the 16th century, the characteristic questions were not "How much?" and "How many?" but "What sort?" and "Why?"

Calculation played so small a part in ancient life that most people were content with cumbersome systems of arithmetical notation, of which Roman numerals is only one of the worst examples. It is hard enough to add with these old systems; it is almost impossible to multiply. And so calculations were commonly done with an abacus—by moving beads on wires or counters on a marked board.

In a sense, the abacus was the first calculating machine. But it does not count for much in our story. Though it can be extremely quick, it is less efficient than calculation on paper. And the first step towards our modern arithmetical world was the development, mostly in the later Middle Ages, of our familiar methods of adding, substracting, multiplying and dividing, so that calculations could conveniently be done on paper without the help of an abacus.

Towards the end of the 16th century the modern world was fast emerging. The local, almost self-sufficient units of feudalism were being bound together to form truly national states—which needed statistics of taxability and military resources. Large-scale commerce was growing; custom or guild rules no longer served to guide the businessman, but had to be replaced by extensive and accurate calculations of cost, selling price and profit.

Surveying, mining engineering, military engineering, and a host of other techniques were coming to rely more and more on mathematical accuracy. Science was taking on its characteristically modern form with its emphasis on "How much" and "How many." Men grew very interested in saving time and trouble in their calculations. In 1585 the Dutchman Simon Stevin published his little book advocating the use of decimal fractions and so inaugurated a new era in arithmetic. And in 1614 John Napier of Scotland made logarithms known to the world.

In this atmosphere it could not long be before some genius or other would invent a calculating machine. In fact two of the really outstanding scientists of the 17th century—Blaise Pascal and G. W. Leibnitz—besides numerous minor men of science, devised such machines. Pascal's father, as a Government superintendent in the French *département* of Haute-Normandie, had to check an enormous number of accounts, and it was to aid him in this work that his 18-year old son designed the first machine for adding and substracting, and constructed it in 1642 with the aid of a Rouen blacksmith.

Several other similar machines were invented. In 1694, Leibniz produced the first machine that could do all the ordinary processes of arithmetic: add, substract, multiply and divide.

During the 18th century many more inventors tried their hands at calculating machines, and before 1800 they had created practically all the devices that go to make up a modern "general purposes" (adding, subtracting, multiplying and dividing) machine.

But none of these instruments was a practical success. Some would work well enough when handled carefully by experts, but the engineering technique was not available to turn out calculators good enough and reliable enough for day-to-day use.

However, with the gradual improvement of engineering techniques, practical machines came during the 19th century. The first was that of the Frenchman Thomas de Colmar, which appeared in 1820.

In 1892 came the best known of all calculating machines, the Brunsviga, which was such a success that some 20,000 were sold within 20 years. By now the general purpose machine had reached the end of its basic evolution—though many improvements have been added since, the most important being the electrical drive, to relieve the computer of the labour of churning away at a handle.

The machines we have so far been describing are called "digital machines," because they work with the actual digits with which numbers are written. The later 19th and especially the 20th century have also witnessed the development of a wide variety of machines of a different class—"analogue machines," as they are called.

The basic idea behind all these is very simple. The behaviour of any machine can be represented by a set of mathematical equations and their solution. Conversely, given a set of mathematical equations, one can construct a machine whose behaviour would be represented by them; if one, then, sets the machine working and observes the motions of its various parts, one has the solution of the equations. A speedometer, for example, is an analogue machine which calculates speed from the relation between distance travelled and time (mathematically, it differentiates).

A very important class of mathematical problem depends for its solution on the reverse of this process (integration.) For instance, if you know the relations between the rates at which various quantities are changing (differential equations), and their starting value, where will they get to in a given time? Or, to give an easier example: given the motorist's speed at every point, how far will he go in an hour?

As early as 1876 Lord Kelvin sketched a plan for a "differential analyser"—an analogue machine for solving problems of this type. However, he was not able to overcome the mechanical problems involved, and the development of the differential analyser had to wait till

1931, when Dr. Vannevar Bush hit on a very simple way of dealing with the difficulties. Since then differential analysers have been in constant use at dozens of computing laboratories, while many other types of analogue machines have been developed.

Analogue machines have one great advantage over the digital machines we described earlier—they will solve a complete problem at one fell swoop, whereas the digital machine will only do elementary additions, subtractions, multiplications, and divisions, and the computer must arrange to combine thousands of these elementary steps to solve the complete problem.

Against that, analogue machines have several disadvantages. Their accuracy tends to be rather low, and they are not flexible—each machine will solve one type of problem only, whereas the computer and digital machine between them can solve any problem, if given time.

Would it be possible to make a machine with the advantages of both—a machine that would solve complete problems without the minute-to-minute guidance of a computer as the analogue machines do, but which will tackle any problem and give results to any desirable accuracy? The answer is Yes.

In the last few years, remarkable machines (of the digital type) with just these powers have been built and put into use. One starts by feeding the machine with a list of instructions and a few initial numbers, and then it gets on with the calculations, adding this pair of numbers, dividing that pair, storing results in a "memory," picking out of the store the numbers it wants for the next step—and so on until the whole complex calculation is finished without any human help.

This is not quite the miracle that it seems at first sight. The duties of a computer using an ordinary digital machine to solve a typical problem can be reduced to a series of instructions which read: "At such-and-such a stage, take the two numbers that are written in such-and-such places in the first column on the paper before you, add them (or subtract, multiply or divide), and write the answer in such-and-such a place in the second column." There may be a few dozen of these instructions—and then will come the order "Repeat all this, but using the numbers in the second column," and so on. Obviously a process like this can be mechanised.

The machine will have to have units which will add, substract, multiply or divide any two numbers supplied to them; it will have to have a "memory" in which it can store the results of calculations until they are wanted again; and it will have to have some means of receiving instructions about the routine it is to follow and of transmitting these instructions to its various parts. It needs a few other elements too, which we need not bother about now. The point is that each of these units is simple in itself and the only complication arises in connecting them together to make one very complex and highly integrated organism.

In 1833 Charles Babbage, a Cambridge mathematician, proposed the construction of a machine on just these lines. His plans included in principle practically every device that is used by the modern machines, but he never succeeded in putting them into practice. It is probable that such a machine built entirely in mechanical terms—of gears, levers and the like—would have been unworkable in practice, and no other form was possible in the 19th century.

But in the 20th century we have two types of devices which alter the picture completely—electro-mechanical relays (in everyday use in automatic telephone exchanges) and electronic devices such as radio valves, photo cells and cathode ray tubes. By using these, instead of mechanical elements like gear wheels, practical problems become manageable and in addition much higher speeds are obtainable. Machines that do all that Babbage planned have within the last seven years become almost common.

The first of these giant calculators, the Automatic

Sequence Controlled Calculator at Harvard, brain-child of Professor H. H. Aiken, started work in 1944. It was an electro-magnetic relay type. By the standards that have since been reached it was a slow machine—taking about a third of a second to add two 23-figure numbers or about six seconds to multiply them—yet even at those speeds it could work about 100 times faster than a computer using an ordinary calculating machine.

The Harvard machine was of momentous importance as the first proof that the idea of a fully automatic calculating machine would work, but the future lay not with relay machines, but with those using electronic elements. The first of these, the Electronic Numerical Integrator and Calculator, designed by John W. Mauchly and J. Presper Eckert, Jr., went into action in 1946. Intended specifically for ballistic calculations, it was limited to a rather narrow range of problems, but the speed had now gone up to 5,000 additions a second.

Since then a dozen or more of these machines have been built in several countries. There is no point in describing them in detail here, and instead it will be better to note briefly what the general abilities of a fully automatic digital calculating machine are. Such a machine can work at a rate of anything from 15,000 average operations a minute upwards—that is, 10,000 or more times faster than a good computer with an ordinary calculating machine.

Given appropriate instructions at the beginning of a run, it can carry out a long series of calculations without further human intervention, and so solve in minutes or hours mathematical problems that would have needed years with earlier methods. Though most of the applications have been mathematical, these machines are by no means confined to mathematics—they can deal with any type of information that can be precisely stated, and deduce its logical consequences.

These calculating giants open up vast new possibilities for humanity. There are very many scientific problems—in such fields as aerodynamics and nuclear physics—in which the theoretician can write down a set of equations and say "Solve these equations and your scientific problem is solved." But in many cases, the process of solution would be so long that with the older methods it could not be carried out in a lifetime, and so experiments—often costly ones—had to be done instead. Now the speed of the new machines enables us to get practical results from the theory.

Again, in using X-ray crystallography to find out how the atoms are arranged in various solids, a process of trial and error is involved. If the scientist can make certain initial guesses correctly, then the data on the X-ray photographs can be used to calculate the positions of the atoms. But there are often so many choices for the first guess that the solution is in practice beyond us; the crystallographer usually confines himself to cases in which some other sort of evidence gives a strong hint on how to start.

Now, an electronic calculator could be set to try out all the possibilities one after another and to stop and give a signal when it finds the right one. In this way it could run through many thousands of guesses in a day, and give answers in cases which previously could not be tackled. In these and many other ways the new machines should help enormously to accelerate scientific advance, and particularly to facilitate the application of theory to practice.

There will probably be a comparable revolution in the handling of social and economic statistics. One of these machines (on a larger scale than the present ones) could be fed with all the available information about the economy of a country, and then in a few hours it would tell us what would be the effect of increasing a particular tax or introducing new machinery to cheapen the production of steel screws, taking into account all the complex ways in which such a simple change would react on all parts of the economy. All sorts of economic problems which are at present tackled by hit-or-miss methods would be brought within the sphere of reason.

But to see the full long-term implications of these machines we need to set them in a broader historical perspective. For some 6,000 years or so—ever since the first cart was harnessed to an ox or the first sailing ship launched—men have been developing more and more machinery for relieving them from physical drudgery. But until very recently no machine has done much to relieve us from mental drudgery—and, let us make no mistake, the "brainwork" of an office routine or even the more skilled work of a computer is just as much drudgery as the manual work of a navvy.

Now machines are beginning to take over our brainwork too—only our second-class brainwork, of course, the parts we can reduce to a routine, not the creative effort of the painter, research scientist, poet or philosopher. As this new trend develops, we can foresee a world in which all uncreative routine work, all drudgery, whether manual or mental, has been taken over by machines, and men and women are liberated to develop to the highest degree their creative faculties.

To end on a more sober note, it is necessary to say that the development of these calculating monsters does not mean that the earlier types of calculating machines will become useless. One does not use a steam-hammer to crack nuts (although it will do so). And similarly there will be plenty of work still to be done by our humbler mathematical servants, the Brunsviga, the slide-rule, the differential analyser—and even pencil and paper.

To Euler is due the notation *a*, *b*, *c*, *A*, *B*, *C* in trigonometrical formulae for the elements of a triangle.

—Boon

CHARACTERISTICS COMMON TO NUMBER SYSTEMS WITH DIFFERENT BASES

ALEXANDER LINCOLN
Lawyer, Boston

The so-called Arabic system of notation, the basis of modern arithmetic, is a place-value system with a base of ten, involving the use of nine different digits and zero. With this system and base, the processes of multiplication and division display certain elementary characteristics. Since ten is a purely fortuitous number, the selection of which is attributable to the primitive use of the ten fingers in counting, the inquiry is of interest whether, or to what extent, these characteristics are independent of the scale of notation and may be found in other place-value systems with different bases.

I. MULTIPLICATION

For the base ten, the primary multiplication combinations may be arranged in a square array as shown in Table 1. The corresponding facts for the bases nine and twelve are shown in Tables 2 and 3, respectively. A study of these three tables reveals many common characteristics. To what extent are these characteristics common to *all* such tables? In answer to this question, we enumerate a number of propositions. [1]

1. The ith column is identical to the ith row. By virtue of this fact, any proposition concerning columns is likewise true for rows.

2. The first-place numerals of numbers in the ith column, read down, are the same as the first- place numerals of corresponding numbers in the $(B-i)$th column, read up, B denoting the base.

3. Let $B=kp$, $i=kq$, where p and q are relatively prime. Then the first-place numerals of the numbers in the ith column consist of p distinct numerals arranged cy-

1 Mr. Lincoln's proofs of these propositions have been omitted in the spirit of Descartes: "But I shall not stop to explain this in more detail, because I should deprive you of the pleasure of mastering it yourself."—EDITOR.

clically. If i is prime to B, the first-place numerals in the ith column consist of a permutation of the numerals 1, 2, . . ., $(B-1)$.

4. The second-place numerals in the ith column include in order, the numerals, 1, 2, . . ., $(i-1)$.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Table 1. Base Ten.

1	2	3	4	5	6	7	8
2	4	6	8	11	13	15	17
3	6	10	13	16	20	23	26
4	8	13	17	22	26	31	35
5	11	16	22	27	33	38	44
6	13	20	26	33	40	46	53
7	15	23	31	38	46	54	62
8	17	26	35	44	53	62	71

Table 2. Base Nine.

1	2	3	4	5	6	7	8	9	t	e
2	4	6	8	t	10	12	14	16	18	1t
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	t	13	18	21	26	2e	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2e	36	41	48	53	5t	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
t	18	26	34	42	50	5t	68	76	84	92
e	1t	29	38	47	56	65	74	83	92	t1

Table 3. Base Twelve.

Let us think of the table as a square. Then the diagonal of the square running down from left to right will be called the principal diagonal line, the other being called the secondary diagonal line. Diagonals of numbers parallel to these lines will be designated as positive and negative diagonals, respectively.

5. The numbers in each negative diagonal are distributed symmetrically about the principal diagonal line.

6. If a negative diagonal has a center number, numbers on each side of it diminish according to the series, 1, 3, 5, etc.

7. If a negative diagonal has no center number, numbers on each side of the principal diagonal line diminish according to the series 2,4, 6, etc.

8. The secondary diagonal line divides each positive diagonal into halves for which numbers symmetrically situated have the same first-place numerals.

9. If a positive diagonal has a center number, the first-place numerals of numbers on each side of the center increase according to the series 1, 3, 5, etc.

10. If a positive diagonal has no center number, the first-place numerals of numbers on each side of the secondary diagonal line increase according to the series 2, 4, 6, etc.

II. DIVISION

For the base ten, there are a number of well-known tests for determining whether particular numbers are divisors of a given number N. The following tests are the more important:

10 is a divisor if the last digit is 0.

5 is a divisor if the last digit is 0 or 5.

2 is a divisor if the last digit is even.

9 is a divisor if the sum of the digits is divisible by 9.

11 is a divisor if the difference between the sum of the digits in the odd-numbered positions and the sum of those in the even positions is zero or divisible by 11.

7, 11, or 13 are divisors if they are respectively divisors of a number determined by arranging the numbers in groups of three digits beginning from the end, treating each group as a separate number, and taking the difference between the sum of the odd-numbered groups and the sum of the even-numbered groups.

Are there similar tests of divisibility in other place-value systems with different bases? The answer is yes. Indeed, in the base B, where a number N is represented by the form

$$N = p+qB+rB^2+sB^3+\ldots,$$

the following tests of divisibility can be established.

1) B is a divisor of N if $p = 0$.
2) When B is even, $\frac{1}{2}B$ is a divisor of N if $p = 0$ or $\frac{1}{2}B$.
3) When B is even, 2 is a divisor of N if p is even.
4) $B-1$ is a divisor of N if the sum of the numerals $p+q+r+ \ldots$ is divisible by $B-1$.
5) $B+1$ is a divisor of N if the difference between the sum of the numerals in the odd-numbered places and the sum of the numerals in the even-numbered places is zero or divisible by $B+1$.
6) If $(p+qB+rB^2)-(s+tB+uB^2)+\ldots$ is divisible by $B+1$ or by B^2-B+1, then these numbers are respectively divisors of N.

NOTE: For the base $10 = 3^2+1$, $1001 = 11\cdot 7\cdot 13$, that is, $B^3+1 = (B+1)\ (B^2-B+1) = (B+1)\ (B-3)\ (B+3)$. More generally, for the base $B = r^2+1$, $B^3+1 = (B+1)\ (B-r)\ (B+r)$.

From the foregoing it appears that there are several characteristics which are independent of the scale of notation and not peculiar to the scale of ten. But there is also a curiously fundamental distinction to be made between even and odd values of the base, which suggests the existence of some underlying law.

TOPICS FOR CHAPTER PROGRAMS—XIV

40. MINIATURE GEOMETRIES.

Miniature, or finite, geometries have been used to illustrate the logical structure of mathematics. In a finite geometry we have a simple abstract situation involving defined and undefined elements, a set of postulates which are demonstrably consistent and independent, and a body of theorems which follow by logical deduction from the definitions and postulates. Moreover, the resulting abstract science is categorical.

A. A. Bennett, "Modular Geometry," *American Mathematical Monthly*, Vol. 27, pp. 357-361 (October, 1920).

W. H. Bussey, "A Note on the problem of the Eight Queens," *American Mathematical Monthly*, Vol. 29, pp. 252-253 (August, 1922).

M. R. Cohen and E. Nagel, *An Introduction to Logic and Scientific Method*. New York, Harcourt, Brace and Co., 1934 (Chapter VII).

H. M. Cudy, "25-Point Geometry," *Mathematical Gazette*, Vol. 36, pp. 158-166 (September, 1952).

C. Gabriel and E. Rykowski, "A Finite Geometry of Twenty-five Points, *THE PENTAGON*, Vol. 9, pp. 21-25 (Fall, 1949).

L. R. Lieber, *The Education of T. C. Mits*. New York, W. W. Norton and Co., 1944 (pp. 153-167).

H. F. MacNeish, "Four Finite Geometries," *American Mathematical Monthly*, Vol. 49, pp. 15-23 (January, 1942).

H. H. Mitchell, "Linear Groups and Finite Geometries," *American Mathematical Monthly*, Vol. 42, pp. 592-603 (December, 1935).

U. G. Mitchell, "Finite Geometries," *American Mathematical Monthly*, Vol. 28, pp. 85-87 (February, 1921).

E. R. Ott, "Finite Projective Geometries *PG* (k,p^n)," *American Mathematical Monthly*, Vol. 44, pp. 86-92 (February, 1937).

M. Richardson, *Fundamentals of Mathematics*. New York, The Macmillan Co., 1941 (pp. 448-453).

C. E. Rickart, "The Pascal Configuration in a Finite Projejctive Plane," *American Mathematical Monthly*, Vol. 47, p. 89 (February, 1940).

O. Veblen and W. H. Bussey, "Finite Projective Geometries,"*Transactions of the American Mathematical Society*, Vol. 7, pp. 241-259 (1906).

O. Veblen and J. W. Young, *Projective Geometry*, Vol. I. Ginn and Co., 1910.

J. W. Young, *Lectures on Fundamental Concepts of Algebra and Geometry*, New York, The Macmillan Co., 1911 (pp. 38-51).

42. THE EQUIANGULAR SPIRAL.

The equiangular, or logarithmic, spiral was first discussed by Descartes. He observed that the length of the spiral from O to $P(r,\theta)$ is equal to the polar tangent, thus becoming the first to rectify a curve. James Bernoulli was delighted by many of the properties of the equiangular spiral which he discovered, and requested that the curve be engraved on his tomb. The equiangular spiral is observed in Nature in the septa of the Nautilus, the arrangement of seeds in the sunflower, the formation of pine cones, etc. It also is the basis of dynamic symmetry in art.

R. C. Archibald, "The Logarithmic Spiral,' *American Mathematical Monthly*, Vol. 25, pp. 189-193 (April, 1918).

R. C. Archibald, "Special Curves in Nature and in Practical Applications," *Scripta Mathematica*, Vol. 3, p. 366f (October, 1935).

C. Bragdon, *The Frozen Fountain.* New York, Alfred A. Knopf, 1932.

T. A. Cook, *The Curves of Life.* London, Constable, 1914.

T. A. Cook, *Spirals in Nature and Art.* London, Murray, 1903.

J. Edwards, *Elementary Treatise on the Differential Calculus.* London, Macmillan and Co., 1892 (pp. 360-361).

Encyclopedia Britannica: See "Curves, Special."

J. Hambidge, *Dynamic Symmetry.* New Haven, Yale University Press, 1920.

J. Hambidge, *The Elements of Dynamic Symmetry.* New York, Brentano's, 1926.

W. C. Risselman, "On the Equiangular Spiral," *School Science and Mathematics*, Vol. 35, pp. 55-62 (January, 1935).

D. W. Thompson, *On Growth and Form.* New York, The Macmillan Co., 1943 (Chapter XI).

W. A. Whitworth, "The Equiangular Spiral, Its Chief Properties Proved Geometrically," *Messenger of Mathematics*, Vol. 1, p. 5 (1862).

R. C. Yates, *A Handbook on Curves and their Properties.* Ann Arbor, J. W. Edwards, 1947 (pp. 206-216).

42. GAMBLING.

The very respectable theory of probability had its origin in certain problems connected with games of chance, and today we draw upon those same games (dice, cards, roulette, etc.) to illustrate fundamental propositions of probability. However, the mathematician knows the futility of gambling, for "the true moral is this, that poor men should not gamble and that millionaires should do nothing else."

"Applied Mathematics," *Time*, Vol. 50, p. 64 (December 1, 1947).

E. E. Blanche, "Is There a Foolproof Gambling System?" *Science Digest*, September, 1949, pp. 7-9.

E. E. Banche, "The Mathematics of Gambling," *School Science and Mathematics*, Vol. 46, pp. 217-227 (March, 1946).

E. E. Blanche, "A Night with Probability," *American Mathematical Monthly*, Vol. 49, pp. 54-60 (January, 1942).

E. E. Blanche, *You Can't Win*. Washington, Public Affairs Press, 1949, (This book contains a lengthy bibliography.)

B. H. Brown, "Probabilities in the Game of 'Shooting Craps'," *American Mathematical Monthly*, Vol. 26, pp. 351-352 (October, 1919).

"Calculation of Probabilities in Roulette at Monte Carlo," *Nature*, Vol. 78, pp. 147-148 (June, 1918). The same article condensed: *Current Literature*, Vol. 45, p. 339 (September, 1918).

"Craps Manual," *Time*, Vol. 43, p. 76 (March 6, 1944).

E. B. Dagobert, "Mathematical Probabilities in Games of Chance: The Game of Sevens," "*The Mathematics Teacher*, Vol. 39, pp. 155-158 (1946).

"Gambling at Monte Carlo; Systems and Why They Fail," *Scientific American Supplement*, Vol. 72, pp. 46-47 (July 75, 1911).

W. Hope-Jones, "Gambling," *Mathematical Gazette*, Vol. 15, pp. 347-358 (March, 1931).

D. Hyatt, "Poker Permutations and Combinations," "*Literary Digest*, Vol. 118, p. 33 (July 21, 1934).

H. D. Larsen, "A Note on Hedging," *American Mathematical Monthly*, Vol. 45, pp. 458-460 (August-September, 1938); also, *THE PENTAGON*, Vol. 7, pp. 94-97 (Spring, 1948).

H. C. Levinson, *Your Chance to Win*. New York, Farrar and Rinehart, 1939.

J. McDonald, "Poker, an American Game," *Fortune*, Vol. 37, pp. 128-131, 181-187 (March, 1948).

R. E. Moritz, "Some Curious Fallacies in the Study of Probability," *American Mathematical Monthly*, Vol. 30, pp. 14-18, 58.

"Now You Call It," *World's Work*, December, 1931, p. 43.

A. P. Peck, "You Can't Beat 'em," *Scientific American*, Vol. 147, pp. 350f (December, 1932).

J. S. Redding, "Playing the Numbers," *North American Review*, Vol. 238, pp. 533-542 (December, 1934).

"Scientific Argument Against Gambling," *Scientific American Supplement*, Vol. 66, p. 317 (November 14, 1908).

G. B. Snyder, "Let's Figure on Probability," *The Scholastic*, Vol. 27, pp. 17-18 (October 26, 1935).

(*Concluded on page 40*)

THE PROBLEM CORNER

EDITED BY JUDSON W. FOUST

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond the calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before March 1, 1953. The best solutions submitted by students will be published in the Spring 1953 number of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Dr. Judson Foust, Central Michigan College of Education, Mt. Pleasant, Michigan.

PROBLEMS PROPOSED

51. *Proposed by Harold Larsen, Albion College, Albion, Michigan.* (From *The Mathematical Gazette.*)

Find the last 13 digits in 52!

52. *Proposed by J. E. Allen, Phillips High School, Birmingham, Ala.*

In a certain corporation 20 per cent of the employees are women, 40 per cent of the unmarried employees are women and 12 1/2 per cent of the married employees are women. What per cent of all employees are married? What per cent of the men employees are married? What per cent of the women employees are married?

53. *Proposed by Judson Foust, Central Michigan College of Education, Mt. Pleasant, Michigan.*

B tells C that A said, "I went to town today." A tells the truth only half the time and B tells the truth only two-thirds of the time. What is the probability that A did go to town, assuming that he made a statement with reference to going to town?

54. *Proposed by the Problem Corner Editor.* (From *School Science and Mathematics.*)

A candy dish contains 25 vanilla creams, 10 maple creams, and 10 raspberry creams. What is the least number one must take out of the bowl to be sure of having a) two with the same flavor? b) two with different flavor?

c) three with different flavor? d) three with the same flavor?

55 *Proposed by Norman Anning, University of Michigan, Ann Arbor, Michigan.*

Show that a triangle given at random is three times as likely to have an obtuse angle as not to have one.

UNSOLVED PROBLEM

No solution has been received for Problem 34: Substantiate the assertion made by Nathan Altshiller-Court in his *College Geometry* (page 66) that a triangle may have equal external bisectors and yet not be isosceles.

SOLUTIONS

41. *Proposed by the Problem Corner Editor.* (From *The American Mathematical Monthly*, Volume 23, page 304.)

Show that the locus of the intersection of a pair of perpendicular normals to a parabola $y^2 = 4px$ is the parabola $y^2 = p(x-3p)$.

Solution by Dale Schlueter, Drake University, Des Moines, Iowa.

The equation of all lines tangent to the parabola is $y = mx+p/m$. Thus, the equations of any two perpendicular tangents are $y = mx+p/m$ and $y = -x/m-pm$. These intersect the parabola at $(p/m^2, 2p/m)$ and $(pm^2, -2pm)$, respectively. The equations of lines perpendicular to these tangents are $y = -x/m + k$ and $y = mx+k$, where k is determined in each case for the normal through the point of contact. That is, $2p/m = -(p/m^2)/m+k$ or $k = (p/m)(2+1/m^2)$ for the first point, and $-2pm = pm^3+k$ or $k = -pm(2+m^2)$ for the second point. The equations of the two normals therefore are $y = -x/m+p(2+1/m^2)/m$ and $y = mx-pm(m^2+2)$. Eliminating y and x in turn, we get $x = p(1+1/m)^2-p$ and $y = p(1/m-m)$. Then $y^2 = p^2(m+1/m)^2-4p^2 = px-3p^2$.

Also solved by Charles Grosch, Illinois State Normal University, Normal, Illinois.

46. *Proposed by Harold Larsen, Albion College, Albion, Michigan.*

Find all five-digit numbers N such that the cube root of N is exactly equal to the sum of the digits of N.

Solution by Donald Boothroyd, Albion College.

For convenience, set $N = n^3$, where $n = 3p+R$, and $R = 0, 1$, or 2. Then it is easily shown that $n^3 = 9q+R^3$. Since the sum of the digits of any number divisible by 9 is also divisible by 9, and 9q obviously is divisible by 9, the digit-sum of n^3 can be expressed as $9m+R^3$, where R^3 is 0, 1, or 8. Now $10{,}000 < n^3 < 100{,}000$, whence $21 < n < 47$. We list all possibilities:

$m \cdot 9+R^3 = n$	n^3	Digit-Sum
$2 \cdot 9+8 = 26$	17,576	26
$3 \cdot 9+0 = 27$	19,683	27
$+1 = 28$	21,952	19
$+8 = 35$	42,875	26
$4 \cdot 9+0 = 36$	46,656	27
$+1 = 37$	50,653	19
$+8 = 44$	85,184	26
$5 \cdot 9+0 = 45$	91,125	18
$+1 = 46$	97,336	28

Thus $N = n^3$ is 17,576 or 19,683.

Also solved by Paul Hawthorne, California Avenue School, Hempstead, New York, and Harvey E. Fiola, Forman, North Dakota.

47. *From The American Mathematical Monthly, October, 1951.*

Derive a formula for the sum of the first n terms of a progression in which the first term is a, each even-placed term is obtained from its preceding term by multiplying by the constant u, and each odd-placed term (after the first) is obtained from its preceding term by multiplying by the constant v.

Solution by Charles Grosch, Illinois State Normal University, Normal, Illinois.

Let s_1 be the sum of the odd terms, s_2 the sum of the even terms, p the number of odd terms, and q the number of even terms. Then $p+q = n$ and

$$s_1 = a+auv+a(uv)^2+ \ldots +a(uv)^{p-1} = a[(uv)^p-1]/(uv-1).$$

$$s_2 = au+au^2v+au^3v^2+ \ldots +au^qv^{q-1} = au[(uv)^q-1]/(uv-1).$$

If n is even, $p = q = \frac{1}{2}n$. If n is odd, $p-1 = q$, or $p = \frac{1}{2}(n+1)$, $q = \frac{1}{2}(n-1)$. Then if n is even, we obtain

$$S_n = s_1+s_2 = a(u+1)\ [(uv^{n/2}-1]/(uv-1)$$

and if n is odd,

$$S_n = s_1+s_2 = a(u+1)\ [(uv)^{n/2}-1]/(uv-1)$$

Also solved by Walter Old, Hofstra College, Hempstead, New York.

48 *Proposed by James P. Bradford, Laurel, Mississippi.*

A man cashed a check and found he had received twice the amount of the check plus \$3.50, or he received in dollars what the check read in cents and he received in cents what the check read in dollars. What was the actual value of the check?

Solution by Walter Old, Hofstra College, Hempstead, New York.

Let V be the value of the check in x dollars and y cents. Then $V = 100x+y$. Now $2(100+y)+350 = 100y+x$, whence $98y-199x = 350$. Since $33\cdot199-67\cdot98 = 1$, we have $98y-199x = 350(33\cdot199-67\cdot98)$ or $98(y+350\cdot67) = 199(x+350\cdot33)$. Thus $y+350\cdot67 = 199t$ and $x+350\cdot33 = 98t$; that is, $y = -67\cdot350+199t$ and $x = -33.350+98t$. A meaningful answer is obtained upon setting $t = 118$, whence $y = 32$, $x = 14$, and $V = \$14.32$.

Also solved by Charles Grosch, Illinois State Normal University, Normal, Illinois; Roger Hilleary, Pomona College, Claremont, Calif.; and Harvey Fiola, Forman, North Dakota.

49. *Proposed by the Problem Corner Editor.*

Three men — Arthur, Bernard, and Charles — with their wives — Ann, Barbara, and Cynthia — made some purchases. When their shopping was finished each found that the average cost in dollars of the articles he or she had purchased was equal to the number of his or her purchases. Arthur had bought 23 more articles than Barbara, and Bernard had bought 11 more than Ann. Each husband spent \$63 more than his wife. Who is the husband of whom?

Solution by Charles Grosch, Illinois State Normal University, Normal, Illinois.

Let m = the number of articles bought by a man and w = the number of articles bought by a woman. Then m^2 = the dollars spent by a man and w^2 = the dollars spent by a woman. Now for a married couple, $m^2-w^2 = 63$, whence $(m-w)(m+w) = 63$ with possible factors (1,63), (3,21), and (7,9). Therefore $m-w = 1, 3, 7$ and $m+w = 63, 21, 9$, respectively. Solving the three simultaneous systems, $m = 32, 12, 8$ and $w = 31, 9, 1$ respectively. Since Arthur bought 23 more articles than Barbara, he bought 32 articles and Barbara bought 9. Also, Bernard bought 11 more than Ann, so he bought 12 and Ann bought 1. This leaves Charles with 8 and Cynthia with 31. Thus the married couples are Arthur and Cynthia, Bernard and Barbara, and Charles and Ann.

Also solved by J. E. Allen, Birmingham, Alabama.

50. *Proposed by the Problem Corner Editor.*

A roll of paper tape is 6 inches in diameter and has a center core of 1 inch diameter. How many feet of tape is contained in the roll if the tape is 1/250 inch in thickness?

Solution by Harvey E. Fiola, Forman, North Dakota.

The area of the cross-section of the tape is given by $A = \pi(3^2-.5^2) = 35\pi/4$. Then $L = A \div 1/250 = 572.7$ ft.

Also solved by Donald Boothroyd, Albion College, Albion, Michigan, and Richard V. Lane, Hofstra College, Hempstead, New York.

THE MATHEMATICAL SCRAPBOOK

Tricks to show the stretch of human brain,
Mere curious pleasure or ingenious pain.

—POPE

$=\nabla=$

A shop assistant is paid on a profit-sharing basis, getting as part of his wages every year one-fifth of the total profits of the shop for the preceding year. He broke a window worth $200 in 1930. How much poorer is he now as a result of the accident?

$=\nabla=$

The mathematical term *eliminate* is derived from the Latin *e* (out) + *limen* (a threshold), and literally means to kick out of doors. The word *limit* comes from the same root.

$=\nabla=$

To integrate $\sec x$:

$$\begin{array}{lrl}
\text{Let} & \sec x &= \cosh v. \\
\text{Then} & \tan^2 x = \sec^2 x - 1 &= \cosh^2 v - 1 = \sinh^2 v, \\
\text{whence} & \tan x &= \sinh v. \\
\text{Also,} & \sec x \tan x \, dx &= \sinh v \, dv, \\
\text{so that} & \sec x \, dx &= dv. \\
\text{Therefore,} & \int \sec x \, dx &= v \\
 & &= \cosh^{-1}(\sec x) \\
 & &= \log(\sec x + \tan x).
\end{array}$$

$=\nabla=$

Euclid gave constructions for regular polygons of 3, 4, 5, 6, 8, 10, 12, and 15 sides.

$=\nabla=$

WHY WORRY ABOUT METHOD?

$$\begin{array}{lrl}
\text{Solve:} & \log(4x\text{-}1) + \log(3x+2) &= 2 \log 11. \\
\text{Answer:} & 4x - 1 + 3x + 2 &= 2 \cdot 11 \\
 & 7x + 1 &= 22 \\
 & x &= 3.
\end{array}$$

$=\nabla=$

"The ellipse was studied for centuries before it was found to be the orbit of a planet. To express astonishment at this is to mistake the nature of mathematics."

—E. C. TICHMARSH

$$M = .43429448190325182765$$

Base ten: best in practical work. Can't evaluate a logarithm? Nothing can be nicer! A constant is clearly needed first.

—W. HOPE-JONES

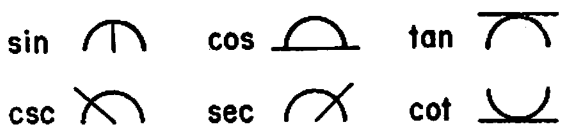
=∇=

On a scale in which the radius of the earth's orbit round the sun is represented by one inch, the *light-year* is very closely represented by one mile.

—N. M. GIBBINS

=∇=

Charles Dodgson suggested symbols to replace the expressions, sin, cos, tan, etc. Note that each of his symbols requires but two strokes, one being the same in each.



=∇=

"At nine and a half, to the amazement of all Germany, (Karl Witte) entered the University of Leipzig. In 1914, before he had passed his fourteenth birthday, he was granted the degree of Ph.D. for a thesis on the 'Conchoid of Nicomedes' a curve of the fourth degree."

—BRUCE, *Psychology and Parenthood*

$$91^2 = 8281$$
$$9901^2 = 98029801$$
$$999001^2 = 998002998001$$

=∇=

From *Arithmetic in Nine Sections* (date unknown; maybe as early as 213 B.C.): "A square city of unknown side is crossed by a street which joins the centers of the north and south sides; at a distance of 20 paces north of the north gate is a tree which is visible from a point reached by going 14 paces south of the south gate and then 1775 paces west. What is the length of each side?"

The mathematician lives long and lives young; the wings of his soul do not early drop off, nor do its pores become clogged with the earthy particles blown from the dusty highways of vulgar life.

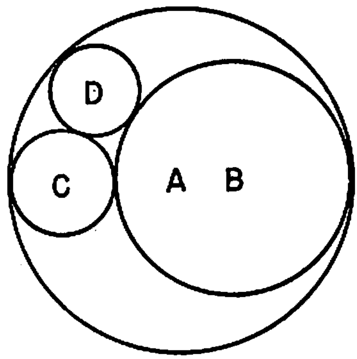
—J. J. SYLVESTER

$=\nabla=$

$$\sqrt{(i^4)} \neq i^2$$

$=\nabla=$

Circles *A, B, C,* and *D,* are mutually tangent, with radii 3 in., 2 in., 1 in., and x in., respectively. Show that x is 6/7 in.



$=\nabla=$

THE RELATIVE VALUES OF PIECES OF CHESS

"[H. M Taylor] found by a mathematical process that if a knight and king of different colors were placed on a chessboard at random, the odds against the king being in check were 11 to 1; if a bishop and a king, 31 to 5; if a rook and a king, 7 to 2; and if a queen and a king, 23 to 13. If, however, we consider only safe check (i.e. check in which the king is unable to take the piece), the odds are respectively 11 to 1, 131 to 3, 5 to 1, 107 to 37. From these numbers we can obtain a fair theoretical measure of the relative values of the pieces. Thus, if we take as our measure the chance of safe check, the values of the knight, bishop, rook, and queen are in the ratio 12, 13, 24, 37, while the values of these pieces in the same order as given by Staunton are 3.05, 3.50, 5.48, and 9.94, the value of the pawn being taken as unity. Mr. Taylor remarks that the value of a pawn depends so much on the fact that it is possible to convert it into a queen, that the method does not appear applicable to it."

—NATURE (Oct. 14, 1875)

$=\nabla=$

PROBLEM IN PROJECTILES

By Prof. Longfellow and William Walton

Swift of foot was Hiawatha;
He could shoot an arrow from him,
And run forward with such fleetness,
That the arrow fell behind him!
Strong of arm was Hiawatha;
He could shoot ten arrows upward,
Shoot them with such strength and swiftness,
That the tenth had left the bow-string
Ere the first to earth had fallen.

Supposing Hiawatha to have been able to shoot an arrow every second, and, when not shooting vertically, to have aimed so that the flight of the arrow might have the longest range, prove that it would have been safe to bet long odds on him if entered in the Derby.—*The Mathematical Monthly* (January 1859). Contributed by FRANK HAWTHORNE.

$=\nabla=$

The sum of the tenth powers of the first thousand natural numbers is

91,409,924,241,424,243,424,241,924,242,500.

James Bernoulli mentions that it took him rather less than seven and a half minutes to obtain this result.

$=\nabla=$

Let S_1, S_2, S_3, ... be the sums of successive groups of n terms of any arithmetic progression. Then

$$S_1 : S_2 : S_3 : \text{etc.} = 1 : 3 : 5 : \text{etc.}$$

$=\nabla=$

"It remains only to add a study which exemplifies reasoning in its clearest and most precise form. That study is of course mathematics, and of the mathematical studies, chiefly those that use the type of exposition that Euclid employed. In such studies the pure operation of reason is made manifest. The subject matter depends on the universal and necessary processes of human thought. It is not affected by differences in taste, disposition, or prejudices. It refutes the common answer of students, who, conformably to the temper of the times, wish to accept the principles and deny the conclusions. Correctness in thinking may be more directly and impressively taught through mathematics than in any other way. — ROBERT M. HUTCHINS, *Harper's Magazine,* (Nov. 1936).

=∇=

"Everyone knows Lewis Carroll as the creator of the immortal Alice. His nom de plume was derived by translating his given name Charles Lutwidge into Latin — Carolus Ludovicus — and then reversing and anglicizing the result. As straight-forward Charles Lutwidge Dodgson he was a mathematics teacher and a writer on mathematical subjects. His dual personality gave rise to many legends. One story, denied by Carroll, claimed that Queen Victoria delighted with *Alice in Wonderland,* ordered that Carroll's next book be delivered to her as soon as released. She was somewhat taken aback to receive *The Elements of Determinants."* — E. E. KRAMER, *The Main Stream of Mathematics.*

"I take the opportunity of giving what publicity I can to my contradiction of a silly story, which has been going the round of the papers, about my having presented certain books to her Majesty the Queen. It is so constantly repeated, and is such absolute fiction, that I think it worth while to state, once for all, that it is utterly false in every particular: nothing even resembling it has ever occurred." — Lewis Carroll in his third edition of *Symbolic Logic.*

=∇=

The first mention of determinants was made in 1693 by Leibnitz in a letter to L'Hopital. In his attempts to simplify the expressions which arose in the elimination of the unknown quantities from a set of linear equations, he used symbols nearly identical with our present determinant notation. Although he believed that the functions would develop remarkable and very important properties, he did not pursue the subject himself; and it was not until the middle of the eighteenth century that determinants were mentioned again.

In 1750 Gabriel Cramer, while working upon a particular problem on the analysis of curves, had to solve sets of linear equations. This famed geometer encountered the same functions which had attracted the attention of Leibnitz. Cramer is credited with the general rule for the solution of n simultaneous equations (non-homogeneous) containing as many unknown quantities.

Important advances have been made since Cramer's time, and with the aid of many celebrated mathematicians, a theory of determinants has evolved. A few of the most important mathematicians are Vandermonde, Gauss, Cauchy, and Jacobi — the latter two contributing most to the development of the subject. Cauchy adopted the name "determinant" from Gauss, and in 1841 Jacobi established the foundation of a treatise on the theory of determinants. —MARGARET E. MARTINSON

=∇=

"[The Royal Mathematician] pulled a long scroll of parchment out of a pocket and looked at it. 'Now let me see. I have figured out for you the distance between the horns of a dilemma, night and day, and A and Z. I have computed how far is Up, how long it takes to get Away, and what becomes of Gone. I have discovered the length of the sea serpent, the price of the priceless, and the square of the hippopotamus. I know where you are at Sixes and Sevens, how much Is you have to have to make an Are, and how many birds you can catch with the salt in the ocean — 187,798,132, if it would interest you to know.'

'There aren't that many birds,' said the King.

'I didn't say there were,' said the Royal Mathematician. 'I said if there were.' " — JAMES THURBER, *Many Moons* (Contributed by Henry Van Engen.)

THE BOOK SHELF

EDITED BY CARL V. FRONABARGER
Southwest Missouri State College

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Frank Hawthorne, Hofstra College, Hempsted, New York.

Mathematics at the Fireside. By G. L. S. Shackle. Cambridge University Press (32 East 57 Street, New York 22, N. Y.), 1952. 12+156 pages. $3.25.

This book is unique in its simplicity of approach to the mathematical ideas presented. Two very precocious children, a boy and a girl, supervised by the boy's father discover much of our school-book mathematics without the aid of any printed materials.

The entire book is made up of conversation and dialogue carried on while the three are at the home of the boy, the home of the girl, or on picnic and boating trips.

Through their intellectual recreations they delve into the fields of real and complex numbers, algebra, geometry, and calculus.

Each mathematical law is approached through the discussion of facts and situations that the children can understand and appreciate. The quadratic equation comes from the discussion of the fall of a rain drop; rational numbers by the cutting of a cake; the calculus by the secrets of a fountain and the measurement of a lawn with a curved side; mathematical induction by the building of taller and taller towers. Some fundamental law is approached in each of the twenty-four chapters of the book.

The author gives a brief table of contents at the beginning and an analytical table at the end which shows in more detail the materials covered.

To most readers the book will seem too difficult to have been developed by the thinking of this little girl and little boy who are not more than ten or eleven years of age.

It is a book that college students interested in mathematics could read with profit. They would receive new and clever ideas and unusual methods of approach. to some of the fundamental bases of mathematical knowledge. It would be a useful addition to the mathematical library of any college.

—L. E. PUMMILL

Mits, Wits, and Logic. By Hugh G. and Lillian Lieber. W. W. Norton and Company, Inc. (101 Fifth Avenue, New York 3, New York), 1947. 7+240 pages. $3.00.

That Mits (Man in the Street) and Wits (Woman in the Street) need lessons from SAM (the essence of what is best in Science, Art, Mathematics), if they are to lead a truly happy life, is Lillian Lieber's theme in this book. Her "free verse" style of writing and the illustrations by Hugh Lieber contribute much to the development of the theme.

The author has divided the material into three parts. The first is devoted to "The Emergency" in which the author introduces SAM, gives Mits and Wits his message of warning, asks them to be modern Paul Reveres in spreading the word that war must be eliminated here and now, and explains to them that if they wish to survive they must accept "modern realism." The second is given over wholly to showing what is meant by "modern realism" in science, in art, and in mathematics.

The last division is concerned chiefly with logic. The author defines logic, and then explains, in detail and with examples, what is meant by Aristotelian Logic or Traditional Logic, which she considers to be only a part of Modern Symbolic Logic. She then points out some illogical arguments, suggests being alert to logical "boners"—such as those used in advertising and in quoting references and statistics which no one ever bothers to check; and interestingly enough shows that problems which at first sight appear illogical and unsolvable because they involve "circular reasoning" are actually logical and can be solved by the use of a little elementary algebra. A study of classes, postulational thinking, Boolean Algebra, and demonstrations showing that the Algebra of Propositions is analogous to a two-valued Boolean Algebra and that Aristotelian Logic can be streamlined to five lines of Boolean Algebra closes the longest of the three divisions.

Mits, Wits, and Logic is a book that every undergraduate who is interested in mathematics and has completed a course in logic would enjoy reading. It is full of happy wit, sound mathematics, and good logic.

—SISTER MARY PETRONIA, S.S.N.D.

Nomography and Empirical Equations. By Lee H. Johnson. John Wiley and Sons, Inc. (440 Fourth Avenue, New York 16, N. Y.), 1952. 9+150 pages. $3.75.

The usefulness of nomographs in engineering, industry, and research is emphasized, and various methods of curve fitting are discussed in rather careful detail in this treatise. The author points out the time-saving feature of nomographs or alignment charts in routine, day-after-day, calculations, along with the fact that graphical solutions, where numerical data are substituted into formulae, do not require skill in mathematics by the user.

The introductory chapter gives excellent material with regard to plotting scales in clear, concise steps of procedure. The size of paper involved, the basic unit of measurement, and the range of values of the variables are all factors that are considered in the presentation.

The author's discussion of the various types of nomograph construction is fundamental, and he gives continuous attention to the actual steps involved in analyzing a problem for adaptation to a particular type of nomograph using an approach based on plane geometry and logarithms. Advantages and limitations of procedures are carefully presented and discussion of errors concerned with a specific type of nomograph is included.

Throughout the book, both the work in nomography and in empirical equations are related to the fundamental mathematical ideas involved. In curve fitting the method of selected points, the method of averages, and the method of least squares are presented in elementary fashion. Chapters are included that discuss curves of two, three, and four constants, respectively. Throughout the material, tests are suggested for determining the suitability of a particular type of equation.

The attention to detail and the emphasis on practical procedure should make this a valuable book for one dealing with experimental and operational data.

—EARLE L. CANFIELD

Geometry and the Imagination. By D. Hilbert and S. Cohn-Vossen as translated by P. Nemenyi. Chelsea Publishing Company (Washington Bridge Station, New York 33, New York), 1952. 9+357 pages. $5.00.

This book is the recent translation of the reformulation and written presentation by S. Cohn-Vossen of D. Hilbert's course of lectures, called *Anschauliche Geometric,* which he gave at Gottingen in the winter of 1920-21.

The world-famous mathematician and lecturer in this course sought to foster "a more inmmediate grasp" of geometry by developing "the tendency toward intuitive understanding" with the aid of "visual imagination." Assuming this perspective, he states, "We can illuminate the manifold facts and problems of geometry, and beyond this, it is possible in many cases to depict the geometric outline of the methods of investigation and proof, without necessarily entering into the details connected with the strict definition of concepts and with actual calculations."

"In this manner, geometry being as many-faceted as it is and being related to the most diverse branches of mathematics, we may even obtain a summarizing survey of mathematics as a whole, and a valid idea of the variety of its problems and the wealth of the ideas it contains. Thus, a presentation of geometry in large brush-strokes, so to speak, and based on the approach through visual intuition, should contribute to a more just appreciation of mathematics by a larger range of people than just the specialists."

Having thus stated his intent, he proceeds to discuss the following general topics: The Simplest Curves and Surfaces, Regular System of Points, Projective Configurations, Differential Geometry, Kinematics, and Topology.

The reviewer found that the book is expository in nature and each section is largely complete in itself. The proofs that are given are synthetic in type and are marked by a directness indicative of the thorough understanding of the logical issues at stake in each.

This book will make enjoyable reading for a mature undergraduate student of mathematics, but as the author warns, he should expect at time to find himself confronted with investigations intended for a reader with more specialized training. However, he should profit much by being lead to perceive the "Gestalt" of the subject as it is so aptly presented by the author.

—ROBERT E. HOGAN

ANNOUNCEMENT

With this number of THE PENTAGON, the present editor steps down to assume the position of Business Manager. I wish to thank all who have cooperated and assisted in making our journal such an outstanding success.

I am happy to announce that the new editor is Dr. Carl Fronabager, with his Editorial Office at Southwest Missouri State College, Springfield, Missouri. Dr. Fronabarger already is at work on the Spring 1953 number, and will appreciate the same loyal support you have given me.

—H. D. LARSEN

INSTALLATION OF NEW CHAPTER

EDITED BY J. M. SACHS

The PENTAGON is pleased to report the installation of Indiana Beta Chapter of Kappa Mu Epsilon. There are now forty-eight active chapters on the roll.

INDIANA BETA CHAPTER

Butler University, Indianapolis

Eighteen student and faculty members were initiated as charter members of Indiana Beta at the installation ceremony held at Atherton Center on the campus of Butler University on May 16, 1952. Professor H. D. Larsen of Michigan Alpha, Editor of the PENTAGON, served as installing officer. Professor J. E. Dotterer of Indiana Alpha assisted.

A banquet in the faculty dining room of Atherton Center preceded the installation. Following the ceremony Professor Larsen spoke on, "Some Famous Unsolved Problems of Mathematics."

The following officers of Indiana Beta were installed: President, Donald R. Cassady; Vice-President, Mary Alice Evans; Recording Secretary, Austin J. Werner; Treasurer, Dean H. Morrow; Corresponding Secretary, Mrs. Juna L. Beal; Faculty Sponsor, Dr. Harry E. Crull.

Other charter members of Indiana Beta are Earl L. Dickey, Joseph M. Gillaspy, George D. Goodnight, Jr., Alice A Hopkins, Alan C. Levenson, Joseph E. Mueller, Vincent M. Myer, Frank A. Rexroth, Jane A. Uhrhan, John B. Walls, Robert C. White, and David F. Woodward.

(*Concluded from page 23*)

M. Tyman, "The Dark Mystery of Race-Track Betting," *Literary Digest*, Vol. 119, p. 38 (April 27, 1935).

C. N. Williamson, "Systems and System Players at Monte Carlo," *McClure*, Vol. 40, pp. 78-91 (February, 1913).

KAPPA MU EPSILON NEWS

EDITED BY LAURA Z. GREENE, *Historian*

Alabama Beta is making plans to be co-hosts with the College for the meeting of the Alabama Teachers of College Mathematics which will be held this year at the Alabama State Teachers College. Members of the chapter who plan to teach secondary mathematics are studying ways to make mathematics more meaningful to the high school student.

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Indiana Alpha prepared an exhibit of mathematical equipment and models which was on display in the department of Mathematics on May 12-14.

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Kansas Alpha and **Missouri Alpha** have arranged an exchange of programs. Four student papers were given on April 17 at Springfield by members of Kansas Alpha. A similar program will be given by Missouri Alpha for the Kansas Alpha chapter this Fall.

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Pledges of **Kansas Gamma** sponsored an Open House, at which time the mathematics department received visitors for the entire College. Various mathematical concessions were arranged to amuse the guests. These consisted of optical illusions; brain teasers; identification of mathematical tools, figures, solids; famous men of mathematics. Prizes were awarded to the most successful contestants.

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The Vantage Press announces the publication of a textbook by Sister Helen Sullivan, former National Historian of Kappa Mu Epislon. Sister Helen's book, which interprets the physical sciences in terms of the Christian ideal, is entitled, *An Introduction to the Philosophy of Natural and Mathematical Sciences.*

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Terry McAdam, first president of **Kansas Delta,** is back in his office now after a year's leave of absence necessitated by a serious automobile accident in May,

1951. He serves as the Executive Secretary of the Alumni Association of Washburn University.

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Michigan Beta awards E. T. Bell's *Men of Mathematics* to the freshman who makes the highest score on a standardized, objective test which is open to all freshmen.

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Members of Michigan Gamma who received Mathematics Awards on the basis of a two and one-half hour examination were Richard Pauley, first place; Joseph Gerrity, second place; Duane Morrow, third place; Robert Reibel and David Morrison, honorable mention. William Shulevitz and Max Krolik were awarded student memberships in the Mathematical Association of America.

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Miss Zelia Zulauf of Missouri Beta presented her paper, *Mathematics and Music*, at the college section of the Missouri Academy of Science in St. Louis. Her paper was ranked among the three top papers given at the meeting.

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Professor M. A. Nordgaard of New Jersey Alpha, founder of the chapter, passed away on October 18.

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Miss E. Marie Hove, National Secretary of Kappa Mu Epsilon, was invited by New Jersey Beta to be their guest speaker at the annual spring banquet. She spoke on the history of Kappa Mu Epsilon.

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Morris Rosen was the winner of the award given by New York Alpha to the student ranking highest in the first year of mathematics. The winner is selected on the basis of grades earned in the courses and a two-hour examination. A book and a certificate were presented to Mr. Rosen, and his name was engraved on a permanent plaque which is kept in the mathematics department.

Members of the chapter assisted by serving as guides and helping to register the visitors on the occasion of the meeting at Hofstra College on May 3 of the Metropolitan Section of the Mathematical Association of America.

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Ruth Rickloff of Pennsylvania Alpha was awarded the Kappa Mu Epsilon prize this year. The award, a book in mathematics, is presented each year to a second-year student in mathematics on the basis of his previous academic record, in mathematics as well as other courses.

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DIRECTIONS FOR PAPERS TO BE PRESENTED AT THE KAPPA MU EPSILON CONVENTION

ST. MARY'S LAKE CAMP
April 17 & 18, 1953

Who may submit papers: One chapter may submit more than one paper for use on the convention program. Papers may be given by both graduates and undergraduates.

Subject: The material should be within the scope of understanding of undergraduates. The program committee will naturally favor papers that are within this limitation and which can be presented with reasonable completeness within the time limit prescribed.

Time limit: The average time should be 20 minutes, but the time may be extended to 30 minutes if the material warrants it.

Abstract: The abstract should be accompanied by a description of charts, models, or visual aids that are to be used in presenting the paper. A carbon capy of the complete paper may be submitted in place of an abstract if desired.

Abstract due: The abstract or a carbon copy of the paper should be submitted before February 1, 1953, at the office of the National President.

Selection of papers: A program committee will select about eight papers of the group submitted. All others will be read by title.

Prizes: Three prizes will be awarded for the best papers presented according to the judgment of a combined faculty and student committee.

Here is an opportunity for your chapter and its members to present a paper and probably have it printed in the Pentagon. Let's all try!

Charles B. Tucker,
National President

PROGRAM TOPICS, SPRING SEMESTER, 1951-52

Alabama Gamma, Alabama College

Professional Opportunities of the Statistician, by Joyce Caraway

Professional Opportunities of the Mathematician, by Elizabeth Cauley and Lillian Lindstrom

California Alpha, Pomona College

How Much Ya Wanna Bet? by Professor C. G. Jaeger

Number Mysticism, by Barbara Jobbins

The Normal Probability Curve, by Richard Hill

The Number Continuum, by Joe Seewerker

Mapping, by Ernest Kimme

Vectors, by Jon Mathews

History of Mathematical Notation, by Edward Coughran .. ..

Plausable Reasoning, by Professor Elmer B. Tolsted

Colorado Alpha, Colorado A and M College

Vibratory Motion and Waves, by Stephen Luchter

History of Mathematics, by Raoul Pettai

On the Use of Computing Machines, by Professor Harris T. Guard

Illinois Beta, Eastern Illinois State College

Adventures in Measurement, by Miss Hendrix

Historical Aspects of Mathematics, by a Student Panel

The Professionalization of Subject Matter for Teachers of Mathematics, by Dr. H. F. Fehr

Illinois Delta, College of St. Francis

A Great Physicist, R. A. Millikan, by Sister Rita Clare O.S.F.

Mathematics and Religion, by Patricia Kasak

The Scientific Method, by Jeanne Schwinn

Continued Fractions, by Doreen Loiselle

The Gamma Function, by Irene Ragan

Indiana Alpha, Manchester College

Relativity, by Philip Kensey

The History and Development of KME, by Professor J. E. Dotterer

Ethiopian Multiplication, by David E. Neuhouser

Mathematics in the High School Curriculum, by Don Bright

Rockets, by Robert Beack

Iowa Beta, Drake University

The Locus of Intersections of Normals to a Given Parabola Is a Parabola, by Raymond Schlueter

Careers in Mathematics, by Edward Oscarson

Mathematics in Education, by Rex Morrison

Evolution of Logarithms, by James Baldridge

Kansas Alpha, Kansas State Teachers College, Pittsburg

The Fundamental Theorem of Arithmetic, by Phil Doty

Mathematical Fallacies, by Richard Dale

Mathematics in Iranian Schools, by Youness Hakimi
Minimum Distance to Three Points, by Richard Slinkman
Thomas Jefferson and Mathematics, by Virginia Adams
Take a Chance, by Tom Needham
Theories on the Evolution of the Solar System, by Dick Blancett

Kansas Beta, Kansas State Teachers College
Telescopic Observation, by Dr. O. J. Peterson
History and Purposes of KME, by Dr. O. J. Peterson
Pythagorean Numbers, by Vernie Witten

Kansas Gamma, Mount St. Scholastica College
Careers for Mathematics Majors and Minors, by Jo Ann Fellin, Becky Becker, and Suzanne Swann
To Determine the Excellent Mathematical Kalkulators, by Donna Rump and Charlotte Raur
Mathematics Essential to Economics, by Professor Edward Henry
Mathematical Training Required for Positions with the American Telephone and Telegraph Company, by Miss Ruth Link, Engineering Assistant with A.T.T.

Kansas Delta, Washburn University of Topeka
Mathematical Puzzles, by Richard Fisher
Mathematics in China, by Alfred Cheng
Mathematics in College, by Margaret Moore
Mathematics of Chance, by George Ladner
Non-Euclidean Geometry, by Terry D. McAdam

Louisiana Beta, Southwestern Louisiana Institute
The Education of T. C. Mitts, by Ann Ryder
The Hydrogen Bomb, by Tuney Arceneaux

Michigan Beta, Central Michigan College
On Trisecting the Angle, by Cleon C. Richtmeyer

Michigan Gamma, Wayne University
The Game of Nim, by Dr. Bertram Eisenstadt
Inscribing a Regular Polygon in a Circle, by Max Krolik
Radar Curves, by Dr. J. D. Bell, Michigan State College

Mississippi Gamma, Mississippi Southern College
Teaching in the Demonstration School, by Eddie Miley

Missouri Beta, Central Missouri State College
Discovery Method of Teaching Mathematics, by Mary Edna Schupp
Mathematics and Music, by Zelia Zulauf
Vectors, by George Hutton
The Slide Rule, by Charles Edwards
Proofs of the Pythagorean Theorem, by Sherralyn Denning
A Minister Looks at Mathematics, by Rev. Herbert Woodruff

Missouri Epsilon, Central College
Mechanical Brains, by Wallace Jacobs
History of Fermat, by Glenn Bowman
Pythagoras, by Walter Weinard

Pythagorean Theorem, by John Maurer
Mathematical Puzzles, by Dr. W. R. Utz, University of Missouri

Nebraska Alpha, Nebraska State Teachers College
The Development of Mathematics, by Ardyce Stevens
The Mainstream of Mathematics, by Mrs. Elizabeth Wooldridge

New Jersey Beta, New Jersey State Teachers College
Einstein's Theory of Relativity, by John Loughlin
The Use of Mathematics in Astronomy, by Donald Bunger
Mathematics in Science, by Dr. D. Davis
History of Kappa Mu Epsilon, by E. Marie Hove

New York Alpha, Hofstra College
Stability and Laplace Transform, by Dr. Henry Wolf
Mathematics in Communications, by David B. Jordan
Switching Algebra, by Richard J. Jaeger, Jr.

Ohio Beta, College of Wooster
Solutions of the Cubic, by Gerald Colome

Ohio Gamma, Baldwin-Wallace College
Powdered Metalurgy, by Mr. Francis Lowey
Art and Mathematics, by Dr. Sam Greenwood
Mathematics in Music, by Dick Winter

Oklahoma Alpha, Northeastern State College
Fallacies in Mathematics, by Juanita Van Cleve
Short Cuts in Mathematics, by Joe Galey
Timely Suggestion to New Teachers, by Miss Frazee
Why Make Any Guesses (from the Paper *This is a Mathematical World*), by Dr. Wesley Deneke, Dean of Personnel
Stringed Solids in Plastic, by Mike Reagan

Pennsylvania Alpha, Westminster College
Revolving Numbers, by Jon Valentine
Numerology, by Donald Pritchard
A Proof of Euler's Formula $V \quad E+F = 2$, by Mary McKnight
History of the Regular Polyhedra and Proof That There Can Be Only Five, by William Pherson

Tennessee Alpha, Tennessee Polytechnic Institute
Magic Squares by Robert W. Glahe
A Puzzle Based on Logic, by David B. Soloff
Theory of Numbers, by Ira F. Grissom

Texas Alpha, Texas Technological College
The Hydrogen Bomb, by Dr. C. C. Schmidt

Texas Epsilon, North Texas State College
The Circle, Using Complex Numbers, by Richard Barham
A Probability Problem: Jacks or Better, by Donnie B. Alexander
Extraction of Roots, by Jane Pinkerton
The Straight Line, Using Compex Numbers, by Laroy R. Carry
Errors of Interpolation by Finite Differences, by C. T. Cadenhead
Some Mathematical Nuts, by Charlotte Clark and Carol Jenkins

The Mathematician as a Problem Solver, by Linnie Ross and Everett Bailey
An Application of Algebra, by Abelina Sanchez
Wisconsin Alpha, Mount Mary College
Mathematical Puzzles
Money at Work and *Fair Exchange,* films, by Mr. Robert J. Cunningham
It Can't Happen Here, play

THE 1953 CONVENTION

On Friday and Saturday, April 17th and 18th, we will gather at St. Mary's Lake Camp, five miles north of Battle Creek, Michigan, for our ninth Biennial Convention. Our hosts are the three Michigan chapters: Michigan Alpha, Albion College, Albion; Michigan Beta, Central Michigan College, Mount Pleasant; and Michigan Gamma, Wayne University, Detroit.

St. Mary's Lake Camp is operated by the Michigan Education Association. It boasts of good food, good single beds with innerspring mattresses, and opportunities for quiet recreation. The camp can house 140 delegates at $6.00 per person per day for board and room plus a $0.50 blanket charge. Those delegates who commute or who stay at Battle Creek may secure meals at the rate of $0.75 for breakfast, $1.25 for luncheon, and $1.50 for dinner.

Those members who attended the Eighth Annual Convention at Springfield, Missouri, will recall their most enjoyable and profitable experience. The National Council and your hosts want this convention to maintain, and, if possible, surpass the high standards and quality of our earlier conventions. They sincerely hope that every chapter will be well represented at this outstanding event in the activities of Kappa Mu Epsilon.

Local chapter officers should start making their plans to send as many delegates as possible, and to have at least one delegate on the program. Directions for preparing papers for the program have been sent to all chapter sponsors and coresponding secretaries. Please send all abstracts of papers to the office of the National President on or before February 1, 1953.

Our biennial conventions are an outstanding feature of Kappa Mu Epsilon. They provide a delightful opportunity for the faculty and student members of the various chapters to become acquainted. They provide an enjoyable stimulus and experience for both the individuals and the chapters. So start your planning now to have your chapter represented at the convention both in attendance and on the program.

— CHARLES B. TUCKER, National President