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Kappa Mu Epsilon, national honorary mathematics fraternity, was founded in 1931. The object of the fraternity is four-fold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievement in the study of mathematics in the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

# THE RELATIVITY OF MATHEMATICS[1]

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An ancient sage has said, "I have lived my life among the wise, and have found nothing better for man than silence."

Despite the general acceptance of the wisdom expressed in this remark, very few organizations conduct meetings by having all those attending engage in silent meditation. I think that the reason for this apparent disregard of the sage's observation is to be found in the experimentally established fact that silent meditation proceeds best when carried on with a voice in the background. It has fallen to my lot tonight to provide that background noise. It goes without saying that for the best meditation it is essential that no attention whatever be paid to what the voice is talking about.

I would like to examine with you this evening an aspect of mathematics which is, unfortunately, hardly ever presented to the undergraduate student of the subject, so that even some members of Kappa Mu Epsilon might be unaware of its existence. Everyone knows that there is much mathematics in Relativity, but since the validity of a statement is sometimes altered by reading it backwards, it might not be clear that there is also much *relativity in mathematics.* With respect to what are the statements of mathematics relative? To answer this question we should first come to some agreement concerning what we mean by the term "mathematics." Just what is mathematics?

Interesting attempts have been made by very distinguished persons to define mathematics in a sentence or so. While the results are usually not definitions in the strict sense, they are important in telling us what their authors consider the most salient features of the subject.

The British mathematician E. W. Hobson said in 1910:

> "Perhaps the least inadequate description of the general scope of modern pure mathematics—I will

1 A banquet address given at the National Convention of Kappa Mu Epsilon, Springfield, Missouri, April 28, 1951.

> not call it a definition—would be to say that it deals with form, in a very general sense of the term; this would include algebraic form, functional relationship, the relations of order in any ordered set of entities."

The American mathematician Benjamin Peirce stressed the logical nature of mathematics and gave it an almost universal character by saying (about the year 1870):

> "Mathematics is the science which draws necessary conclusions."

As we would expect, the logician Whitehead goes still further by *identifying* mathematics with logic. According to him, "Mathematics, in its widest significance, is the development of all types of formal, deductive reasoning." This thesis finds its greatest expression in the monumental volumes of *Principia Mathematica* by Whitehead and his student Bertrand Russell. The purely formal character of mathematics is wittily expressed by Russell in his famous remark that "Mathematics is the subject in which we never know what we are talking about, or whether what we are saying is true."

Thus mathematics, or more precisely, *pure* mathematics, consists entirely of such assertions as, "If something has property $p$, then something has property $q$," and it is not pertinent to inquire whether the first something really *has* the property $p$.

It is clear that it is meaningless to ask if such statements are *true*, if one uses "true" in the ordinary sense. Whatever abstract truth may be, it plays no role in mathematics. The mathematician contents himself with aiming at the *consistent*. He regards it as his business to invent and develop consistent systems. Any such system is a branch of mathematics.

It is amusing to contrast this view of pure mathematics with one expressed by the distinguished orator Edward Everett, whose doubtless meager experience with mathematics left him with these noble thoughts:

> "The great truths with which mathematics deals are clothed with austere grandeur, far above all

> purposes of convenience or profit. It is in them that our limited understandings approach nearest to the conception of that absolute and infinite, towards which in most other things they strive in vain.
>
> "In the pure mathematics we contemplate absolute truths, which existed in the divine mind before the morning stars sang together, and which will continue to exist there, when the last of their radiant host shall have fallen from heaven."

Obviously, it is with extreme reluctance that I must reject this exalted conception of my daily activities.

Edward Everett, like most of the people of his day, thought of mathematics as being *discovered* rather than as being *invented,* and he pictures the pure mathematician as one who explores the divine mind intent upon discovering those mathematical systems and notions that exist there. Though Plato would most certainly have endorsed this view, the modern mathematician *can not* and *does not* "lay that flattering unction to his soul."

Since, then, mathematics consists in the construction and development of logical systems, it becomes immediately clear what we mean by "The Relativity of Mathematics." We are merely calling attention to the fact that *all mathematical statements or theorems are relative to the systems from which they were obtained.* But there is also a second aspect of relativity present; namely, they are relative to *the means by which they were obtained,* that is, the underlying logic that is employed in their derivations.

Just as in the physics of relativity theory, *absolute distance* and *absolute time* are meaningless, so in mathematics there are no *absolute theorems,* but only statements that are (1) provable, or (2) the negations of which are provable, or (3) undecidable, relative to a selected set of assumptions or (as the physicist would say) relative to a selected frame of reference.

Obvious as these remarks are when one realizes the real nature of mathematics, they are paradoxical without such knowledge. One may study mathematics for many years without becoming acquainted with these ideas, and

I would venture the opinion that most people regard the mathematician as one who deals in "Absolutes." Indeed, not long ago, one of my colleagues charged me with being an "Absolutist" *because* I am a mathematician.

It is easy to account for this misunderstanding of the nature of mathematics on the part of the student and intelligent layman alike. It is due to the circumstance that prior to the year 1830 mathematics *was* absolute. There was only *one* kind of geometry, *one* kind of algebra, *one* kind of calculus, and I might add at this point, *one* kind of logic for deriving theorems. Hence, the truth or falsity of a mathematical statement was absolute, for there was only *one* set of assumptions which entered into consideration, and only *one* way of operating on them to produce new statements. And even those assumptions had not been rigorously formulated despite Euclid's heroic attempt to do this for geometry and despite Aristotle's work on logic.

So mathematics was mathematics—Euclid was in his heaven and all was right with the world. If it is understandable that a development 130 years old has not yet become common knowledge, it might still be thought strange that it has not entered into the college teaching of mathematics. It seems that the only changes made in the traditional courses of college algebra, trigonometry, analytic geometry, differential and integral calculus for much longer than 130 years is the steady elimination of the more difficult parts of these subjects, and the emphasis on manipulation rather than understanding.

So the great majority of those who have contact with mathematics today are no better off, so far as understanding the nature of the subject, than those living in the year 1800, for few of the revolutionary developments that began in the first quarter of the nineteenth century have found their way into the *ordinary* undergraduate textbook. Such textbooks are for the most part designed to meet the needs of students in engineering schools, and the engineer (as well as those working in the lower levels of physics) still finds it possible to get along with the mathematics of 150 years ago. Of course, the quantum physicist needs a more modern algebra and a different logic, while the physicist working in relativity theory uses non-euclidean geometry. But such people, together with those who hope to become mathematicians, form a very small minority of the customers of a Mathematics Department, which (like many another merchant) has found it expedient to stock its shelves mostly with the goods for which there is the largest demand.

It is not part of my objective tonight to relate the engrossing story of the rise of non-euclidean geometry in which the Italian Saccheri, the German Gauss, the Russian Labatscheffsky, and the Hungarian Bolyai played leading parts. Its significance for this discussion lies in the fact that its invention radically altered men's views concerning the nature of mathematics. It would no longer be possible for any later Immanuel Kant to talk about the apodictic character of euclidean geometry. If God continues to geometrize, it is not necessarily according to Euclid's Elements. Most of the chains with which *Aristotle* had bound the thinking of mankind had been broken earlier; it was not possible to free the human mind from the still stronger bonds of *Euclid*. Though a good case could be made for the assertion that Euclid's Elements have had more influence in shaping present day life than any other product of the Greek intellect, and though modern mathematicians appreciate far better than did Euclid's contemporaries the magnitude of his achievements, it was nevertheless necessary to emancipate mathematics from euclidean bondage in order that its fullest development might be obtained.

The birth of new geometries was followed by the advent of new algebras with which the names of Peacock, de Morgan, Grassman, Peirce, and Hamilton are connected, and the stage was set for the great activity in abstract algebra that we are witnessing today.

It is time, now, to illustrate my remarks concerning the relativity of theorems to postulates by an example. The mathematical statement I have chosen is one that I am sure most of you have encountered—and perhaps even "proved" long ago. The statement is:

(*) *For every triangle* T, *the perpendicular bisectors of the sides of* T *meet in a point.*

Examining this statement, we observe that it contains the technical terms *perpendicular, bisector, side, triangle, meet, point.* Now in order for a statement to have a meaning in a given geometry, it must be formulated in terms of the primitive notions of the geometry; that is, all of the notions italicized above must be either primitive or derived notions of the system. For example the well-known statement, *The moon is made of green cheese,* has no meaning in euclidean geometry since the terms *moon, green,* and *cheese* are neither primitive nor derived notions in that system.

Many quite rigorous foundations for euclidean geometry have been constructed since Euclid's famous attempt of 300 B.C., with different choices of primitive notions and relations. But whatever foundation is chosen, it will be found that the statement *does* have meaning in that system, and the question of its "truth" or "falsity" then arises.

But what does it mean to inquire if the statement is true? Remember that all the terms and relations involved in the statement are reducible to primitive terms and relations (that is, the statement can be written so that only undefined notions and relations occur.) So we do not really know what a triangle is, or a side, or a perpendicular bisector, or a point. How then can we say whether or not such a statement is true?

What the mathematician is asking is simply whether or not the statement is derivable from the assumptions or postulates made at the outset. *How derivable?* Why, by means of logic, of course. Thus, we see that *logic is presupposed.* One final question: *Derivable by means of what logic?*

This final query is a very modern one indeed. Just as, 130 years ago, no one would have asked the question, *What geometry?* (since there was only one known, that of

Euclid), so up to a few years ago no one would have raised the query, *What logic?* (since, again, there was only one known, that of Aristotle with modern improvements.) But now the last of Aristotelian authoritarianism is on the way out, and other logics have been formulated and studied. There is, then, an amount of *arbitrariness* involved in what we select as allowable means of drawing conclusions. We are no more bound to Aristotle's criteria than we are bound to Euclid's postulates.

Hence we have in mathematics not merely one, but two degrees of relativity; relativity with respect to postulates, and relativity with respect to logics. The latter kind of relativity has already played an important part in mathematics, for there is a school of mathematicians who do not conclude that a statement is true merely because it is not false. They thus reject Aristotle's Law of the Excluded Middle (according to which for every proposition $p$, either $p$ or its negation $p'$ is true) as a device for establishing theorems.

So, if we wish to be precise, we should frame our question in the following manner: Is the statement (*) provable from a set of postulates for euclidean geometry by means of Aristotelian logic?

The answer is, of course, "Yes," and so we may say that the statement is "true" in euclidean geometry (the use of ordinary logic being pre-supposed.)

It should be pointed out, however, that a derivation of the statement by the *reductio ad absurdum* method of assuming the bisectors do not meet in a point, and showing that this leads to a contradiction, would not be acceptable to the school of mathematicians who reject the Law of the Excluded Middle. For they would rightly say that all such a procedure yields is the falsity of the falsity of the conclusion; that is, one may surely conclude that *if a figure is a triangle,* then the statement "the perpendicular bisectors of the sides of the triangle do not meet in a point" is false. But from the falsity of the falsity of the conclusion, the *truth* of the conclusion does not necessarily follow without the Law of the Excluded Middle.

Our statement may, however, be derived without the use of the *reductio ad absurdum* method, and so the school referred to above would accept it.

Is our statement then an absolute truth? Not at all, for if we substitute for the Parallel Postulate (an equivalent of which is found in any foundation of euclidean geometry) an assumption that there is more than one line through a point parallel to a given line, and keep all the other assumptions and primitive notions and relations just as they were, we obtain a geometry just as valid (that is, just as consistent) as euclidean geometry, in which the technical terms of our statement have precisely the same meanings as before, but in which *the statement is false.* In this so-called hyperbolic geometry, the negative of our statement can be proved; that is, *there exists a triangle such that the perpendicular bisectors of its sides do not meet in a point.*

Suppose, on the other hand, that we had been content merely to eliminate the parallel postulate from euclidean geometry, and had not made the *counter assumption* stated above. What is the status of our theorem in the geometry so obtained?

*The statement is certainly not true;* for if it were true (i.e., provable) in that geometry it would clearly be provable in the hyperbolic geometry obtained by adjoining the counter parallel postulate to the set of postulates—which is not the case.

*The statement is certainly not false;* for if it were, its negation would be provable in that geometry, and hence also provable in euclidean geometry—which is not the case.

So, in the new geometry, the statement is *neither true nor false* (that is, neither the statement nor its negation is provable.) We call such statements *undecidable.*

It is likely that one who meets with this circumstance for the first time feels that a geometry admitting undecidable propositions within its context falls short of what a geometry should be. If a person has this feeling, he will be shocked to learn that the very respectable subject of arithmetic contains such propositions also. This remarkable fact was established by the contemporary Austrian mathematician Kurt Godel. He has even given a method for constructing such undecidable propositions; but, alas, there is no procedure for determining whether a given proposition is or is not undecidable, despite the assertions of freshmen and sophomores that they are constantly running across such propositions in analytic geometry and calculus!

Relativity in mathematics can be illustrated by countless examples in algebra as well as in geometry. Algebra operates now with entities other than the numbers that are so useful in filing income tax returns or measuring distances, and so there are algebras whose elements behave very differently from those encountered in elementary algebra. Some of these newer systems should appeal quite strongly to the young student, for they seem bent on justifying some of his most strongly ingrained preferences for which, alas, he sometimes suffers martyrdom. In some of these domains, for example, $(a+b)^2$ actually does equal $a^2+b^2$,which countless generations of students (far ahead of their times) have stoutly maintained to be the case. But there are other systems whose behavior errs in the other direction, for we are asked to abandon the familiar and useful relations $a+b = b+a$, or $ab = ba$, or $(a+b)c = ac+bc$.

I would like to close these remarks with a quotation from a lecture given at Cambridge in 1734 by Isaac Barrow, a good mathematician of his day, who had at least one good student, Isaac Newton. After being so very abstract in our view of mathematics, it is perhaps fitting that we should pay our respects to the past by reading the very characteristic remarks of an almost forgotten day.

"These Disciplines (mathematics) serve to inure the mind to a constant Diligence in Study; to undergo the Trouble of an attentive meditation, and cheerfully to contend with such Difficulties as lie in the way. They wholly deliver us from a credulous Simplicity, most strongly fortify us against the Vanity of Skepticism, effectually restrain us from a rash Presumption, most easily incline us to a *due* Assent, perfectly subject us to the Government of right Reason, and inspire us with Resolution to wrestle against the unjust Tyranny of false Prejudices. If the Fancy be unstable and fluctuating, it is poised by this Ballast and steadied by this Anchor; if the Wit be blunt it is sharpened upon this whetstone; if luxuriant it is pared by this Knife; if headstrong it is restrained by this Bridle; and if dull, it is roused by this Spur. The Steps are guided by no Lamp more clearly through the dark mazes of Nature; by no Thread more surely through the intricate labyrinths of Philosophy; nor, lastly, is the Bottom of Trust sounded more happily by any other line. All of which I might defend by Authority, and confirm by the Suffrages of the greatest Philosophers."

"Perhaps the most surprising thing about mathematics is that it is so surprising. The rules which we make up at the beginning seem ordinary and inevitable, but it is impossible to foresee their consequences. These have only been found out by long study, extending over many centuries. Much of our knowledge is due to a comparatively few great mathematicians such as Newton, Euler, Gauss, Cauchy, or Riemann; few careers can have been more satisfying than theirs. They have contributed something to human thought even more lasting than great literature, since it is independent of language."

—E. C. Tichmarsh

# ANGLE TRISECTION[1]

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In the history of mathematics there are three famous problems that have attracted attention for over two thousand years; they are the trisection of the general angle, the duplication of the cube, and the squaring of the circle. These three problems themselves seem hardly worth more than passing attention and yet, even today, a great deal of time is spent by the uninformed trying to solve them. It is the disarming simplicity of the problems that invites amateur mathematicians to disregard all previous work and try anew to solve them by using the tools of elementary geometry, the straightedge and compasses. Today these problems have only historical interest since they have long since proven to be unsolvable. The most elementary of these three problems, and the one with the widest appeal, is the trisection of the general angle; it arose so long ago that historians can find no record of its actual beginning.

The plane geometry of the ancient Greeks was a game to be played with simple equipment and governed by a rigid set of rules. The equipment consisted of the compasses and an unmarked straightedge. The rules were postulated and permitted (1) the drawing of a straight line of indefinite length through two given distinct points, and (2) the construction of a circle with center at a given point and passing through a second given point. These rules define the conditions for the classical trisection problem. The fact that this problem could not be solved was established about 1800 by P. L. Wantzel, a French mathematician. This late date is not surprising, because it is necessary to pass beyond the scope of plane geometry in order to show that the problem cannot be solved. The proof of the impossibility of trisecting a general angle requires the help of two theorems. These theorems are generally known as the Theorem of Constructibility and the Theorem of Nonconstructibility.

1 A paper given at the National Convention of Kappa Mu Epsilon, Springfield, Missouri, April 28, 1951.

The Theorem of Constructibility states: *The necessary and sufficient condition that any analytical expression can be constructed with the straight edge and compasses is that it can be derived from the known quantities by a finite number of rational operations of addition, subtraction, multiplication, division, and extraction of real square roots.*

That the stated condition is sufficient can be shown easily. The geometrical operations of addition and subtraction need little explanation, being evident from the meanings of the words sum and difference. The operations of multiplication, division, and extraction of real square roots may be performed geometrically (that is, using compasses and an unmarked straightedge) as is

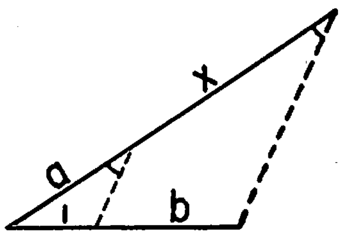


Fig. 1

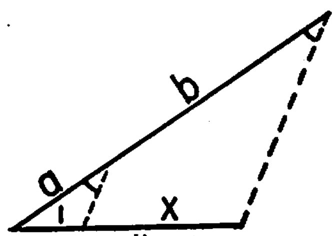


Fig. 2

shown in Figures 1, 2 and 3. In Figure 1, by similar triangles, $1:a = b:x$, whence $x = ab$. In Figure 2, $1:a = x:b$, so that $x = b/a$. In Figure 3, $1:x = x:a$, or $x = \sqrt{a}$. It follows, therefore, if an analyical expression contains only numbers $a$, $b$, etc., corresponding to line segments, and only the operations of adding, subtracting, multiplying, dividing, and the extracting of real square roots are involved, then a line corresponding to the expression can be found.

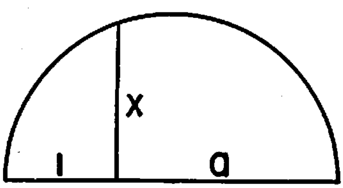


Fig. 3

To see that the condition is necessary, recall that all constructions of plane geometry are but the location of points either as the intersection of two lines, of a line and a circle, or of two circles. The algebraic interpretations of these geometrical constructions are as follows:

1. Two given lines are represented by the equations $a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$, in which the coefficients are geometrical lengths either given to start with or determined at some stage in the construction. Now $x$ and $y$ may be expressed as rational functions of these coefficients and hence are constructible because they involve no operations other than adding, subtracting, multiplying, and dividing of lengths.

2. The line $ax+by+c = 0$ meets the circle $(x-h)^2+(y-k)^2 = r^2$ at the point where $x$ and $y$ can be expressed in the form $(-B+\sqrt{B^2-4AC})/2A$ in which $A$, $B$, and $C$ are polynomials involving $a$, $b$, $h$, $k$, and $r$. Since $A$, $B$, and $C$ are rational functions, the construction of $x$ and $y$ involves the same operations as Case I with the addition of the extraction of a real square root.

3. The intersections of two circles are the same as the intersections of their common chord and one of the circles. Thus, since the coefficients in the equation of the chord are rational functions of those in the equations of the circles, this case reduces immediately to Case 2.

Thus the straightedge and compasses together are capable of making only those geometrical constructions which are algebraically equivalent to a finite number of the operations of addition, subtraction, multiplication, division, and extraction of real square roots involving the given lengths.

Due to the length and complexity of its proof, the Theorem of Nonconstructibility of irreducible equations will be postulated. The theorem states: *If the degree of an irreducible equation is not a power of two, the equation cannot be solved by a finite number of rational operations and extractions of square roots.* This theorem may be illustrated by the following equations. The degree of the irreducible equation $x^3-2 = 0$ is not a power of two, whence the roots are not constructible. The degree of the irreducible equation $x^4-4x^2+1 = 0$ is a power of two so that the roots are constructible.

A trisection equation may be formed by expressing the solution of the problem in analytical form. Given angle $AOB = 3\theta$ and line $OT$ trisecting angle $AOB$ so that angle $TOB = \theta$ (Fig. 4). Select an arbitrary length $OA$ as 1 and construct $AC$ parallel to $OT$ meeting $OB$ extended at $C$; then angle $DCO = \theta$. Construct $OD$ equal to the unit length so that triangle $AOD$ is isosceles with base angles $2\theta$. Triangle $DCO$ is also isosceles with base angles $\theta$, and $DC = DO = OA$. Let $x = OC$, $2y = AD$, and $a = ON = \cos 3\theta$. From right triangles $CMD$, $CNA$, and $CLO$, we have $x:2 = (x+a):(1+2y) = (1+y):x$, whence $x^2 = 2+2y$ and $1+2y = 2(x+a)/x$. Eliminating $y$ from the last two equations, $x^3-3x-2a = 0$; that is,

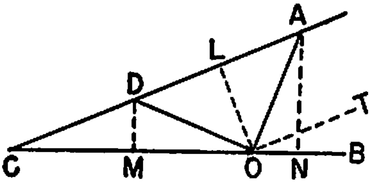


Fig. 4

$x^3-3x-2\cos 3\theta = 0$, which is known as the trisection equation.

Suppose, now, that the given angle $3\theta$ is equal to 60°. Then $a = 2\cos 3\theta = 1$ and the trisection equation becomes $x^3-3x-1 = 0$. This is clearly irreducible (the only possible rational roots are $\pm 1$, and these are not roots of the equation), and since the degree is not a power of two, the roots are not constructible. Hence, it is impossible to trisect an angle of 60° with the straightedge and compasses, from which it follows at once that it is impossible to trisect the general angle. There are a few angles, such as 0°, 45°, 72°, 90°, that may be trisected because the corresponding trisection equations have rational roots and hence are not irreducible.

Hippias of Elis, who lived in the Fifth century B.C., was perhaps one of the first men to approach the trisection problem by a method not using just the straightedge and compasses. Assuring himself that he could not do it by these means, he devised a curve called the Quadratrix which gave an exact solution to the problem. Most people of that day did not accept this solution, however, because it was not an elementary geometry construction with straightedge and compasses.

To draw Hippias' Quadratrix, rotate the radius of a circle about the center at constant velocity and move a line starting at *B* parallel to *AD* also with a constant velocity. (Fig. 5). The locus of the intersection of the radius *AE* and the line parallel to *AD* forms the Quadratrix. Let angle *DAX* be given to be trisected. Place this angle at the center of a quadrant of a circle within which a Quadratrix is drawn. Line *AX* cuts the Quadratrix at *F* and the circle at *E*. Make *FH* perpendicular to *AD*, *HK* = 2*FK*, and *KL* parallel to *AD*. Then line *AL* may be drawn and angle *DAL* is the required angle. The proof follows from the property of the Quadratrix. The arc *ED* will be to the arc *MD* as the segment *FH* is to the segment *KH*. Since the segment *KH* was constructed equal to one-third of the segment *FH*, then arc *MD* equals one-third of the arc *ED* and angle *DAL* equals one-third of angle *DAX*.

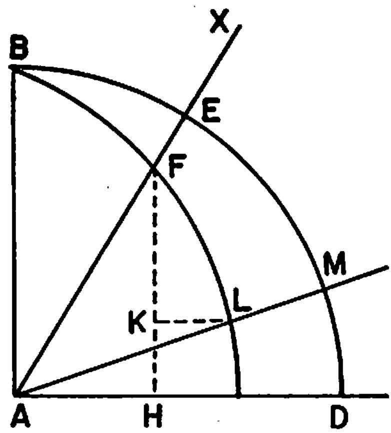

Fig. 5

A variety of mechanisms have been devised for the solution of the trisection problem. Some of these mechanisms draw the curves that aid in the solution, and others trisect the angle directly. The Tomahawk (Fig. 6) is typical of the mechanical trisectors that trisect an angle directly. It is constructed with

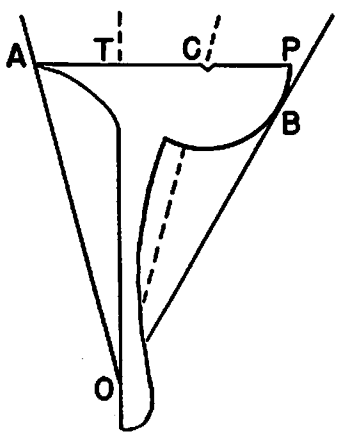

Fig. 6

a semicircle ($TBP$ with $OT$ (the handle) tangent at $T$. The point at the notch $C$ is the center of the semicircle, and $A$ is taken on $PT$ extended so that $AT$ is equal to the radius of the circle. $OT$ and $OC$ are the trisecting lines.

Another mechanical trisector is the Three Bar Apparatus (Fig. 7). In this apparatus the bars $OE$ and $OF$ are taken equal in length and jointed together at $O$. The point $E$ is attached so that $CE = OE$ and point $F$ is made to slide in a groove along $CD$. For trisection, the point $O$ is placed at the vertex of the given angle $AOB$ and $OF$ coincident with $OB$. When $C$ is brought to the produced line $OA$, angle $ACD$ is equal to one-third of angle $AOB$.

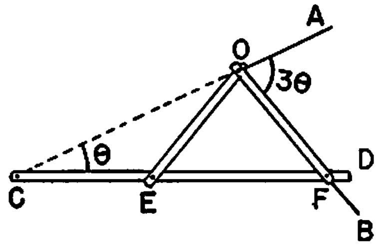

Fig. 7

The curves and mechanisms given in this paper for trisecting an angle are only a few of the many devices which have been invented. Though the interest in this problem has been dying out during this century, it must be remembered that much of the history of mathematics owes its origin to this famous problem.

# ON EXTREMIZING POLYNOMIALS WITHOUT CALCULUS

HUGH J. HAMILTON
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**1. Extremizing the Quadratic.** A standard method of extremizing the general quadratic $y=ax^2+bx+c$ $(a\neq 0)$ is to "complete the square." Thus by writing

(1.1) $\qquad y = a(x+b/2a)^2+(c-b^2/4a)$

we see that $y$ is extremized when and only when $y = -b/2a$, since a square is least when and only when it is zero. Students who are averse to the theory of differentiation can evidently use this purely algebraic device to solve a number of the maximizing and minimizing problems given in the first chapter of a calculus textbook.

These same students may inquire whether a similar algebraic device is available for extremizing more complicated functions. The purpose of this note is to answer this question and to try to inspire students to make further investigations along the lines which we shall follow. (Those who are familiar with the techniques of algebraic geometry will recognize in our procedure an adaptation of a time-honored method of finding the slopes of certain curves—the "multiple root" method. May it be none the less an entertaining exercise!)

**2. Extension to the Cubic.** Before passing to polynomials in general, we consider the typical cubic

(2.1) $\qquad y = ax^3+bx^2+cx+d \qquad (a\neq 0).$

Noting that the right-hand side of (1.1) is of the form $(x-\alpha)^2(A)+\beta$, with

(2.2) $\qquad 2a\alpha+b = 0,$

let us attempt to write (2.1) in the form

(2.3) $\qquad y = (x-\alpha)^2(Ax+B)+\beta$

and, if this is possible, to determine a condition on the $\alpha$ of (2.3) which is in some sense like (2.2). We leave aside for the moment the question of the relevance of this investigation to our extremizing problem.

Identification of (2.3) with (2.1) gives
$Ax^3+(-2\alpha A+B)x^2+(\alpha^2A-2\alpha B)x+(\alpha^2B+\beta) = ax^3 +bx^2+cx+d$, which is equivalent to the system

$$(2.4)\qquad \begin{aligned} A &= a \\ -2\alpha A+B &= b \\ \alpha^2A-2\alpha B &= c \\ \alpha^2B+\beta &= d. \end{aligned}$$

Now the first two of equations (2.4) are equivalent to the system

$$(2.5)\qquad \begin{aligned} A &= a \\ B &= 2a\alpha+b \end{aligned}$$

whence the entire system (2.4) is equivalent to the system

$$(2.6)\qquad \begin{aligned} A &= a \\ B &= 2a\alpha+b \\ 3a\alpha^2+2b\alpha+c &= 0 \\ \beta &= d-\alpha^2(2a\alpha+b), \end{aligned}$$

whose solutions consist precisely of the roots $\alpha$ of

$$(2.7)\qquad 3a\alpha^2+2b\alpha+c = 0$$

and the values of $A$, $B$, and $\beta$ subsequently determined by the first, second, and fourth equations of (2.6).

We shall now show, without calculus, that every value $x = \alpha$ which extremizes (2.1) must satisfy (2.7)—a fact which will permit us to avoid traffic with $\Delta-$ processes! (And even the most $\Delta-$ minded among us will admit that (2.7) is the same thing as $dy/dy$ at $x = \alpha$.) First: if, when $x = \alpha$, we have $y = \beta$ whether extreme or not, $x-\alpha$ is a factor of $y-\beta$ (this is guaranteed by the Factor Theorem); hence, $y-\beta = (x-\alpha)(px^2+qx+r)$. Next: since $y = \beta$ is an extreme value, $px^2+qx+r$ is zero when $x = \alpha$ (for otherwise $y-\beta$ would change sign as $x$ passes through the value $\alpha$), so that $px^2+qx+r$ is of the form $(x-\alpha)(Ax+B)$. Thus

$$y-\beta = (x-\alpha)^2(Ax+B),$$

which is the same as (2.3) above and therefore implies (2.7).

Of course there may be values $\alpha$ which satisfy (2.7) and for which (2.1) is not extremized for $x = \alpha$; these are precisely such values as satisfy $Ax+B = 0$, for $y-\beta$ changes sign as $x$ passes through the value $\alpha$ if and only if $A\alpha$

$+B = 0$. It is now clear also that $y = \beta$ is a maximum if $A\alpha+B<0$ and a minimum if $A\alpha+B>0$.

**3. Extension to the General Polynomial.** If $x = \alpha$ provides an extreme value $\beta$ of

(3.1) $$y = a_0+a_1x+a_2x^2+ \dots +a_nx^n,$$

then, by an argument like that in § 2, we must have

(3.2) $$y = (x-\alpha)^2(b_0+b_1x+b_2x^2+ \dots +b_{n-2}x^{n-2})+\beta;$$

and identification of (3.2) with (3.1) is equivalent to the system

(3.3)
$$\begin{aligned}
&b_{n-2} && -a_n = 0\\
&-2\alpha b_{n-2}+b_{n-3} && -a_{n-1} = 0\\
&\alpha^2 b_{n-2}-2\alpha b_{n-3}+b_{n-4} && -a_{n-2} = 0\\
&\alpha^2 b_{n-3}-2\alpha b_{n-4}+b_{n-5} && -a_{n-3} = 0\\
&* \quad * \quad * \quad * \quad * \quad * \quad * \quad * &&\\
& && \alpha^2 b_1-2\alpha b_0-a_1 = 0\\
& && \alpha^2 b_0+\beta-a_0 = 0.
\end{aligned}$$

The first $n-2$ of equations (3.3) are equivalent to a system which, like (2.5), expresses the $b$'s explicity in terms of the $a$'s and $\alpha$; and so the set of all $n$ of equations (3.3). is equivalent to this system along with the results of substituting its values of the $b$'s in the last two of equations (3.3). To find what the first of these last two equations becomes under these substitutions, we need only multiply equations (3.3) respectively (from the bottom up) by 0, 1, $2\alpha$, $3\alpha^2$, . . . , $n\alpha^{n-1}$ and add. We obtain

(3.4) $$a_1+2a_2\alpha+3a_3\alpha^2+ \dots +na_n\alpha^{n-1} = 0.$$

The form of $y$ required by (3.2) is thus to be achieved with and only with the roots $\alpha$ of (3.4).

Thus all of the values of $x$ which extremize $y$ are to be found among the roots $\alpha$ of (3.4). And by the sort of argument used in § 2 we find that such a value $\alpha$ does indeed extremize $y$ if and only if $\alpha$ is a root of $y-\beta = 0$ of even multiplicity $m$. Clearly, $\beta$ will be a maximum or a minimum value of $y$ according as the quotient of $y-\beta$ *by* $(x-\alpha)^m$ is negative or positive when $x$ is formally replaced by $\alpha$.

We conclude with an example: To extremize $y = 3x^4-16x^3+30x^2-24x+17$. Equation (3.4) becomes

$$12\alpha^3-48\alpha^2+60\alpha-24 = 0,$$

or
$$12(\alpha-1)^2(\alpha-2) = 0.$$
Taking $\alpha_1 = 1$, we find that $\beta_1 =$ (value of $y$ when $x = \alpha_1$) $= 10$, so that
$$y-\beta_1 = 3x^4-16x^3+30x^2-24x+7$$
which, upon being factored, becomes
$$y-\beta_1 = (x-1)^3(3x-7).$$
Since the multiplicity of the root $x = \alpha_1 = 1$ of $y-\beta_1 = 0$ is odd,, $\beta_1$ is *not* an extremum. Taking $\alpha_2 = 2$, we find $\beta_2 =$ (value of $y$ when $x = \alpha_2$) $= 9$, so that
$$y-\beta_2 = 3x^4-16x^3+30x^2-24x+8$$
which, upon being factored, becomes
$$y-\beta_2 = (x-2)^2(3x^2-4x+2).$$
Since the multiplicity of the root $x = \alpha_2 = 2$ of $y-\beta_2 = 0$ is even, $\beta_2$ *is* an extremum; and since moreover $3x^2-4x+2$ is positive when $x = \alpha_2 = 2$, $\beta_2$ is a minimum.

EDITORIAL NOTE. Professor Hamilton's paper is not intended to be exhaustive. Readers might be inspired to continue the investigation; for example, what is the situation when the first derived equation has no real roots? How does a cubic or quintic without maxima or minima appear under this plan?

"History furnishes us the example of a man who in a cell which not even a beam of light was able to pierce, without instruments, without pen and paper, inventing and building up a new branch of mathematics, which is now counted as one of its greatest achievements. I refer to Jean Victor Poncelet (1788-1867), who as an engineer took part in the Russian campaign, during which he was taken prisoner and confined at Saratoff on the Volga. It was during his imprisonment there that he worked out the ideas of the so-called projective geometry."

—I. J. SCHWATT

# A CLASS OF INTEGRAL TRIANGLES

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The topic of this paper belongs to the large field of Rational-Sided Triangles[1], the particular class of integral triangles to be considered here having one angle equal to twice another. The triangles will be non-similar, whence the sides will be relatively prime integers. It is suggested that this topic would provide an interesting club program.

Without revealing either method or details, a few observations and results are stated. They may help to stimulate and guide further study which may lead to other interesting results.

For this class of integral triangles, it can be shown

1) One side and the sum of two sides are square numbers.

2) There exist three and only three triangles in which the sum of two sides is equal to 49.

3) There is only one triangle with sides in arithmetic progression.

4) If $m$ and $n$ are relatively prime integers and the inequality $\frac{1}{2}<n/m<1$ holds, then the three numbers $n^2$, $mn$, and $m^2-n^2$ form a triangle with one angle twice another.

5) If the numbers $m$ and $n$ are chosen so that the fraction $n/m$ will equal the successive convergents, beginning with the second, of the periodic continued fraction $1/1+\ 1/2+\ 1/2+\ \ldots$ then all the triangles with sides $n^2$, $mn$, and $m^2-n^2$ will belong to this class. The series of triangles obtained will progressively approach an isosceles right triangle.

To illustrate methods which may be used in establishing these properties, let us consider the problem of finding all the non-similar integral triangles with perimeters

1 Cf. "Topics for Chapter Programs: 25. Rational-Sided Triangles. "*PENTAGON*, Vol. 9, pp. 102-104 (Spring 1950). Also, D. M. Brown, "Numerical Double-Angle Triangles." *PENTAGON*, Vol. 7, pp. 74-80 (Spring 1948).

less than 100 which have one angle equal to twice another. For the solution assume the usual notation of a triangle; i.e., $a$, $b$, $c$, and $A$, $B$, $C$ denote the sides and the angles, respectively. To satisfy the requirement, place $B = 2A$, then $C = 180°-3A$. The law of sines gives the relations

$$\sin A/a = \sin 2A/b = \sin 3A/c.$$

From the first two ratios it follows that $\cos A = b/2a$. Placing the second ratio equal to a ratio formed by combining the first and third gives the relation

$$\sin 2A/b = (\sin 3A+\sin A)/(c+a) = 2\sin 2A\cos A/(c+a),$$

whence $\cos A = (c+a)/2b$. Equating the two values for $\cos A$ gives the condition, $b^2 = a(c+a)$. From this equation, since $a$, $b$, and $c$ are assumed to be relatively prime, it is seen that $a$ and $c+a$ are also prime to each other. Therefore $a$ and $c+a$ must each be a square number. Place $a = n^2$ and $c+a = m^2$, then $c = m^2-n^2$, $b = mn$, and $\cos A = m/2n$. It is easily verified (by means of the law of cosines) that if $a$, $b$, and $c$ have these values then $\cos B = \cos 2A$. The next step is to find triads $a$, $b$, $c$ by giving suitable values to the numbers $m$ and $n$.

Since $A<B$ and $a$, $b$, and $c$ are relatively prime, it follows that $m$ and $n$ are integers prime to each other; and they must satisfy the condition that $n<m<2n$. To cover any specified range, assign to the number pair $(m,n)$ all pairs of allowable integers. In this problem where $P$, the perimeter, is to be less than 100, a further condition to be satisfied in selecting the pairs $(m,n)$ is $P = m(m+n) < 100$. The allowable number pairs are (3,2), (4,3), (5,3), (5,4), (6,5), (7,4), (7,5), and (7,6). For each set $(m,n)$, now find the corresponding triad $a = n^2$, $b = mn$, and $c = m^2-n^2$. It follows that the corresponding triads are (4,6,5), (9,12,7), (9,15,16), (16,20,9), (25,30,11), (16,28,33), (25,35,24), and (36,42,13). Thus, there are eight non-similar integral triangles, with one angle equal to twice another, whose perimeters are less than 100.

# ALGEBRA TODAY AND YESTERDAY

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"Teachers of mathematics have for some time felt that the Algebras now in use in our High Schools and Collegiate Institutes are not adapted to the wants and requirements of our present day."

"The examples (in this book) which are very numerous and varied in their character, have been tested in the class room, and proved to be suitable before being inserted. Their number is greater than the majority of students will find time for working; but as they are carefully graded it will be easy to select as many as may be desired of any required degree of difficulty."

These statements were selected from the prefaces of Books I and II, respectively, of *The High School Algebra* by Robertson and Birchard. The first book was written in 1886 (revised 1896) and the second in 1889; they were published by William Briggs, Toronto, Ontario. But the statements resemble introductory remarks by authors of current high school texts. Hence we may conclude that the effort to fit the curriculum to the needs of the students is not a modern one. It has always existed.

However, a review of the contents of these books by a classroom teacher of today tends to one of two reactions. Either it is utterly fantastic to believe that these algebras could ever meet any practical needs of the high school student or ever develop any powers of reasoning, or modern education has gone so very far in streamlining the content of modern algebras that today we are giving our high school students only a superficial knowledge of the subject. We either laugh at the old books or we are concerned with the content of the new.

Somewhere between these two reactions lies a point of view which can develop a philosophy of teaching for the teacher of mathematics in the secondary school. Compulsory education keeps in our high schools today students who would die of mental exhaustion before finishing the first of this two book series. It is right that we have modern textbooks in algebra to meet their needs. As our high school population changes, our objectives change. In 1886 only the fittest survived to study algebra. Yet, today, there are students in our high schools who become mentally lazy because they are not challenged. A few, like Oughtred, Newton, and Gauss could study books of this calibre without benefit of teacher and be challenged to do original thinking.

These books are the property of and former texts used by Professor Norman Anning of the University of Michigan and were in use in the schools in Ontario in the early twentieth century. Professor Anning says that he "suffered through these books as did hundreds of others." But he survived to become one of the finest teachers in the University. Who knows whether it was because or in spite of these textbooks!

Book I contains material which we include in three semesters of formal algebra today and in addition chapters on Symmetrical Expressions, Theory of Divisors and Complete Squares, Simultaneous Quadratics, and Theory of Quadratics. Horner's Method of Division is given very early in the book. Proofs of "theorems," a term seldom used in algebra today, are rigorous. Formulas are given to be memorized with little thought for simple, formal, analytic developments designed to give the student the meaning behind the formula and to pave the way for rerationalization in case the student forgets the rule.

Mr. Birchard and Mr. Robertson seem to have included in these texts everything they could find pertaining to each topic discussed, whereas modern authors tend to present only the rudiments. For example, we now teach elementary algebra students only the simple and commonly used methods of factoring; Birchard and Robertson include all possible types and intricate variations thereon. We find a whole chapter treating highest common factor and least common multiple involving the use of all types of factoring in the wealth of problems proposed.

Quadratic equations are solved first by factoring. Then the method by completing the square, called the Italian method, is applied to $x^2+px+q = 0$ and $ax^2+bx+c = 0$ and problems of these types involving numerical coefficients. The only place in which the method by formula is mentioned is in connection with the solution of a complex literal fractional equation which the authors point out must be reduced to the $ax^2+bx+c = 0$ form. After it is reduced, the authors say, "Now we know the roots of $ax^2+bx+c = 0$ are $-b/2a \pm \sqrt{(b^2-4ac)}/2a$ and as $a$, $b$, $c$ stand for any coefficient whatever, we can at once write down the roots of a given quadratic equation by substituting for $a$, $b$, and $c$ their particular values in the equation."

The "Hindoo Method" for solving a quadratic is also given. In this method each term is multiplied by four times the coefficient of $x^2$ and then the square is completed as previously. And as is customary in dealing with most topics in the book, "various artifices employed in quadratics, as in other equations, to lessen labor" are given in detail. Such "artifices" are common throughout the texts. To illustrate, in the chapter on addition and subtraction of fractions, we find this passage:

"In simplifying algebraical expressions two formulas are frequently of service, viz:-

(1) $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$

(2) $(a+b)^2-(a-b)^2 = 4ab.$

They should be remembered in words as well as symbols, thus:-

(1) The square of the sum of two quantities, plus the square of their difference, is equal to twice the sum of their squares.

(2) The square of the sum of two quantities, minus the square of their difference, is equal to four times their product.

The following examples (of which only one is here given) will show that a proper grouping of the fractions to be combined lessens the work required:-

Ex. 1—Simplify

$$\frac{a+b}{a-b} + \frac{a-b}{a+b} - \frac{2(a^2-b^2)}{a^2+b^2};$$

$$\frac{a+b}{a-b} + \frac{a-b}{a+b} = \frac{2(a^2+b^2)}{a^2-b^2};$$

$$\frac{2(a^2+b^2}{a^2-b^2} - \frac{2(a^2-b^2)}{a^2+b^2} = \frac{8a^2b^2}{a^4-b^4}.$$

If we had found the L.C.M. of the three denominators and combined all the fractions in one operation, the labor would have been considerably increased."

These "artifices" would be considered superficial in a modern text. Yet we cannot help but feel that they would challenge the exceptional pupils to further research and experiment because they are fun, and the real mathematician derives pleasure from such discoveries.

Signed numbers are added and subtracted by scale as we do today. This is done to illustrate the meaning of such numbers rather than to give a gradual development of the rule behind the process. The rule is stated very abruptly after four simple examples.

In discussing negative numbers the authors say, "Thus $-2$ is said to be greater than $-6$. This is only a convenient way of speaking and so long as the meaning of such an expression is clearly defined, no confusion can arise. It would, of course, be absurd to consider either a number or a quantity as really less than nothing." Mr. Anning has marked this passage by a very large exclamation point in the margin!

Every textbook author stresses the practicability of his written problems, and undoubtedly Mr. Birchard and Mr. Robertson, according to the prefaces in their books, have tried diligently to fulfill this pedagogical aim. We cannot help but recall the "work problems" which Stephen Leacock lightly ridiculed in his essay, *A, B, and C. The Human Element in Mathematics*[1]. The large number of work problems all begin with "A can do a piece of work, etc." Marginal notes written by the student tell us that the piece of work may be to "fill a tooth, get married, win

1 Stephen Leacock, *Literary Lapses*. New York, Dodd, Mead, and Co., 1927, pp. 237-245.

an election, or solve a problem." That the owner of the book possessed Mr. Leacock's humor even then is evidenced by the pert remarks which illuminate the margins in many places where such problems occur.

Here are a few of the written problems in Book I. They are typical, perhaps amusing, brain-teasers all, but are they practical?

1. A hare is 50 leaps before a greyhound and takes 4 leaps to the greyhound's 3 leaps; but 2 of the greyhound's equal 3 of the hare's. How many leaps must the greyhound take to catch the hare?

2. It is between 2 and 3 o'clock, but a person looking at his watch and mistaking the hour hand for the minute hand fancies that the time of day is 55 minutes earlier than it really is. What is the true time?

3. If $a$ men or $b$ boys can dig $m$ acres in $n$ days, find the number of boys whose assistance will be required to enable $(a-p)$ men to dig $(m+p)$ acres in $(n-p)$ days. (In case you are interested, the number of boys is $pb(ma+na+mn-mp)/ma(n-p)$.)

4. A man and his wife could drink a barrel of beer in 15 days. After drinking together 6 days, the woman alone drank the remainder in 30 days. In what time would either alone drink it?

The last problem leads the historian to remark, "Shades of the *Rhind Papyrus!*" This manuscript[2], written about 1900 B.C., a copy of a much earlier one, contains many practical problems concerned with the making of loaves of bread and the brewing of beer. Egyptian mathematics grew out of a need of the people and was therefore of a practical nature. But their beverage problems ended with the making of the beer.

Thus far, we have discussed only certain aspects of the first book of the series. It is interesting to note that neither book contains any material on graphs. The secondary school teacher today has a decided advantage over the teacher who used these books in teaching linear and

2 A. B. Chace. *The Rhind Mathematical Papyrus*. Oberlin, Ohio, Mathematical Association of America, 1927.

quadratic functions and equations. The lengthy involved analyses of these functions and equations lack the clarity which the graph lends to such discussions.

Book II in its entirety would baffle many high school teachers. Chapter I contains a discussion of zero, infinity, and vanishing fractions which reminds us of the beginning of a calculus book. Chapter II thoroughly discusses ratio in all its phases, employing such terms as duplicate ratio, triplicate ratio, subduplicate ratio, ratio of homogeneous functions, and incommensurable quantities. This is followed by a chapter on proportion and one on variation.

One chapter each is devoted to Arithmetical Progressions, Geometrical Progressions, and Harmonical Progressions. An harmonical series or an harmonical progression is defined as "a series of numbers such that of every three consecutive terms, the ratio of the first to the third equals the ratio of the difference between the first and second to the difference between the second and third, the differences always taken in the same order."

The authors tell us that this was formerly called the musical progression, give a geometric theorem involving points which form an harmonic range, and explain its relationship to the arithmetic progression.

Square root and cube root are taught in both books, not only of numbers and algebraic expressions, but of complicated examples involving surds. In treating this subject, as well as all other topics, interesting theory is added. For example, this ramification of the cube root is added:

> "When $n$ figures of the cube root of a given number have been found by the ordinary process, $2(n-1)$ more figures may be obtained by dividing the product of the last remainder and the part already found by the sum of the given number and twice the cube of the root already found."

Few teachers today teach cube root at all.

Imaginary quantities and complex numbers are treated on a college level. Although the trigonometric form of the complex number is not discussed as such, the modulus in $a+bi$ is very abruptly defined as $a^2+b^2$ and the argument (amplitude) as:

> "the angle through which the line of positive, real units must be rotated to correspond with the line denoting the complex number; its magnitude is determined by the signs of $a$ and $b$, together with their relative numerical value."

The authors tell us that $\sqrt{-1}$ has an intelligible meaning when applied to space, but is unintelligible in connection with any other kind of quantity. The discussion leads to the conclusion that, as a multiplier, the square root of a number bears the same relation to the number itself as the operation of turning a line through one right angle bears to the operation of turning it through two right angles, which is equivalent to multiplying it by $-1$. For this reason, it is convenient (and reasonable) to define $\sqrt{-1}$ to be the symbol of the operation of turning a line from its original position through one right angle.

The chapter on Scales of Notation is a very fine introduction to a course in the Theory of Numbers. Five other chapters are worthy of a college textbook on the Theory of Equations. One chapter deals with Permutations, Combinations, and Distributions, one chapter with Mathematical Induction, and two chapters with the Binomial Theorem. The book ends with twenty-five pages of theory and problems on Interest, Discount, and Annuities.

The second volume of the two book series is priceless and worthy of intense study by anyone really interested in mathematics, for here are found many fascinating ramifications of algebraic topics not found in present day texts. However, we doubt it possible to find many high school students who would care to study so intensely. But should we find such, they probably would agree in their adolescent way with the owner of these books when he wrote on the fly-leaf of his high school text:

"O Algebra is a weary thing; 'tis full of grief and pain.
It maketh e'en the poor student with sobs and tears complain,
And older people, also, it driveth clear insane."

# TOPICS FOR CHAPTER PROGRAMS—XII

## 34. THE MAGIC NUMBER NINE.

"The number Nine possesses the most remarkable properties of any of the natural numbers. Many of these properties have been known for centuries and have excited much interest among both mathematicians and ordinary scholars. So striking and peculiar are some of these properties that the number nine has been called 'the most romantic' of all the numbers."—E. BROOKS.


E. Brooks, *Philosophy of Arithmetic.* Lancaster, Pa., Normal Publishing Co., 1880.

H. D. Larsen, *An Arithmetic for Colleges.* New York, The Macmillan Co., 1950.

"Magical Number 9," *Atlantic Monthly,* Vol. 156, pp. 227-228 (August, 1935).

J. Rourke, "Properties of the Number Nine," *The Pentagon,* Vol. 7, pp. 20-21 (Fall, 1947).

H. M. Sherwood, "Properties and Peculiarities of the Number 9," *Journal of Education,* Vol. 83, pp. 486-489 (1916).

D. E. Smith and J. Ginsburg, *Numbers and Numerals.* New York, Bureau of Publications, Teachers College, Columbia University, 1937.

W. F. White, *A Scrap-Book of Elementary Mathematics.* Chicago, Open Court Publishing Co., 1910.


## 35. PLAYS.

Many clubs have discovered that the presentation of a mathematical play results in an interesting and entertaining program. The literature contains several plays which have been produced successfully. The following list contains some plays suitable for high school clubs as well as plays written for more mature audiences.


I. Adler, "Fun with Mathematics. An Assembly Program," *Mathematics Teacher,* Vol. 42, pp. 153-155 (March, 1949).

I. Adler, "Theory and Practice. An Assembly Program," *Mathematics Teacher,* Vol. 41, pp. 218-220 (May, 1948).

N. Anning, "Socrates Teaches Mathematics," *School Science and Mathematics,* Vol. 23, pp. 581-584 (June, 1923).

E. Brownell, "Mathesis," *Mathematics Teacher,* Vol. 20, pp. 459-465 (December, 1927).

D. Cohen, "Problem Play: A Musical Play in One Act and Three Scenes, "*Mathematics Teacher,* Vol. 29, pp. 78-83 (February, 1936).

E. Cowley, "A Mathematical Fantasy," *School Science and Mathematics*, Vol. 33, pp. 535-536 (May, 1933).

A. E. Crawford, "A Little Journey to the Land of Mathematics," *Mathematics Teacher*, Vol. 17, pp. 336-342 (October, 1924).

C. Denbow, "Traders and Trappers," *Mathematics Teacher*, Vol. 34, pp. 61-65 (February, 1941).

R. Fletcher, "Quarter after Ten. A Mathematical Play in One Act," *Texas Outlook*, Vol. 15, pp. 31-35 (May, 1931).

L. A. Forrest, "Euclid, Agrarian Arbitrator," *Mathematics Teacher*, Vol. 25, pp. 22-26 (January, 1932).

P. Harding (*et al.*), "A Mathematical Victory, "*School Science and Mathematics*, Vol. 17, pp. 475-482 (June, 1917).

C. Hatton and D. Smith, "Falling in Love with Plain Geometry," *Mathematics Teacher*, Vol. 20, pp. 389-402 (November, 1927).

J. Leps, "Radical Dream. A Playlet for Puppets," *School Science and Mathematics*, Vol 33, pp. 279-287 (March, 1933).

O. Leskow, "The Cubic," *Mathematics Teacher*, Vol. 36, pp. 312-316 (November, 1943).

K. McSorley, "Mock Trial of B versus A," *School Science and Mathematics*, Vol. 18, pp. 611-621 (October, 1918).

"Mathematics Playlet," *Mathematics Teacher*, Vol. 38, pp. 309-313 (November, 1945).

L. Meals, "X on the Spot," *Mathematics Teacher*, Vol. 32, pp. 296-300 (November, 1939).

F. B. Miller, "Idea that Paid," *Mathematics Teacher*, Vol. 25, pp. 470-479 (December, 1932).

F. B. Miller, "A Near Tragedy," *Mathematics Teacher*, Vol. 22, pp. 472-481 (December, 1929).

F. B. Miller, "Out of the Past," *Mathematics Teacher*, Vol. 30, pp. 366-370 (December, 1937).

K. Millay, "Flatlanders: Mathematics Play in One Act," *Scholastic*, Vol. 26, p. 94 (April, 1935); *American Mathematical Monthly*, Vol. 26 pp. 264-267 (June, 1919).

F. F. Novinger, "I Had the Craziest Dream," *Mathematics Teacher*, Vol. 40, pp. 20-23 (January, 1920).

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(Continued on page 56)

# THE PROBLEM CORNER

EDITED BY JUDSON W. FOUST

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solutions should not demand any tools beyond calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before March 1, 1952. The best solutions submitted by students will be published in the Spring 1952 number of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Dr. Judson Foust, Central Michigan College of Education, Mt. Pleasant, Michigan.

## PROBLEMS PROPOSED

41. *Proposed by the Problem Corner Editor. (From The American Mathematical Monthly,* Volume 23, page 304.)

Show that the locus of the intersections of a pair of perpendicular normals to a parabola $y^2 = 4px$ is the parabola $y^2 = p(x-3p)$.

42. *Proposed by the Pentagon Editor.* (This problem appeared in an early number of *The American Mathematical Monthly.*)

A teacher looks at his watch on leaving school at noon. When he comes back the two hands have changed places. Find both times.

43. *Proposed by the Problem Corner Editor.*

How many cards must you be dealt from a shuffled deck of cards before you can be sure of holding (a) a royal flush? (b) four of a kind? (c) a full house?

44. *Proposed by the Problem Corner Editor.*

How many parallelograms are formed when six parallel lines cross eight parallel lines?

45. *Proposed by Norman Anning, University of Michigan, Ann Arbor.* (From *Logical Nonsense* by Lewis Carroll.)

Two travelers spend from 3 o'clock until 9 in walking along a level road, up a hill, and then home again; their pace on the level being 4 miles an hour, up hill 3 and down hill 6. Find the distance walked and (within half an hour) the time of reaching the top of the hill.

## UNSOLVED PROBLEMS

33. *Selected from the tenth annual William Lowell Putnam Mathematical Competition, March* 25, 1950.

In each of $n$ houses on a straight street are one or more boys. At what point should all the boys meet so that the sum of the distances they walk is as small as possible?

34. *Proposed by Frank Mosely, State Teachers College, Florence, Alabama.*

Substantiate the assertion made by Nathan Altshiller-Court in his *College Geometry* (page 66) that a triangle may have equal external bisectors and yet not be isoceles.

37. By error a line was omitted from the statement of this problem in the Spring 1951 issue. The correct and complete statement follows:

If $a$, $b$, $c$ are three numbers in arithmetic progression, $b$, $c$, $d$ three numbers in geometric progression, $c$, $d$, $e$ three numbers in harmonic progression, show that $a$, $c$, $e$ are in geometric progression.

## SOLUTIONS

36. *Proposed by Norman Anning, University of Michigan, Ann Arbor, Michigan.*

The number $t$ is greater than 1. The sides of a triangle are $2t-1$, $2t$ and $2t+1$. Show, without using tables, that the intermediate angle is less than 60°.

*Solution by Sharon Murnick, Hofstra College, Hempstead, L. I.*

Let the angle opposite $2t$ be called angle $C$. Then by the Law of Cosines,

$(2t)^2 = (2t+1)^2+(2t-1)^2-2(2t+1(2t-1)$ Cos $C$. Thus, Cos $C = (4t^2+2)/(8t^2-2) = 1/2+3/(8t^2-2)$. Since $t>1$, $8t^2-2$ is greater than 6 and $3/8t^2-2)$ is less than 1/2. Then $1/2<\text{Cos}C<1$ and $C<60°$.

Also solved by James Kister, Wooster College, Wooster, Ohio.

38. *Proposed by H. D. Larsen, Albion College, Albion, Michigan.* (From *Journal de Mathematiques Elementaires.*)

Given the equation $mx^2-(1+8m)x+4(4m+1) = 0$, determine the two values of $m$ for which the ratio of the roots is equal to $-1/4$.

*Solution by Charles B. Grosch, Illinois State Normal University, Normal, Illinois.*

The quadratic formula gives the roots $r_1 = (4m+1)/m$ and $r_2 = 4$. If $r_1/r_2 = -1/4$, then $(4m+1)/4m = -1/4$ and $m = -1/5$. If $r_2/r_1 = -1/4$, then $4m/(4m+1) = -1/4$ and $m = -1/20$.

Also solved by Paul E. Long, Southwest Missouri College, Springfield, Mo.; Donald Lee Bruyr, Kansas State Teachers College, Pittsburg, Kansas; John Stull, Roosevelt High School, Des Moines, Iowa; James Kister, Wooster College, Wooster, Ohio; Sharon Murnick, Hofstra College, Hempstead, L.I.; Robert Blasch, Hofstra College.

40. *Proposed by Norman Anning, University of Michigan, Ann Arbor, Michigan.*

What number (or numbers) has its digits reversed when it is multiplied by 9?

*Solution by James Kister, Wooster College, Wooster, Ohio.*

In order that the number of digits do not increase, the first digit must be 1; this means the last is 9. Two and three digit numbers satisfying the given conditions are obviously impossible. Consider a four digit number with digits $A, B, C, D$. Then $9(1000A+100B+10C+D) = 1000D+100C+10B+A$. Since $A = 1$ and $D = 9$, $9000+900B+90C+81 = 9000+100C+10B+1$ and $890B+80 =$

$10C$. The only solution is $B = 0$, $C = 8$. Thus the number is 1089.

Consider a five digit number. Then $9(10000A+1000B+100C+10D+E) = 10000E+1000D+100C+10B+A$. Since $A = 1$, $E = 9$, $8990B+800C+80 = 910\ D$. $B$ must be zero for single digit solutions. Then $C = 9$, $D = 8$ and the number is 10989. Similarly all numbers of the form 10 - - - 89, with any number of 9's inserted will satisfy the condtions.

Also solved by Harvey Fiala, Forman, N. Dakota; Robert Blasch, Hofstra College, Hempstead, N.Y.; John Prussen, Hofstra College; Charles B. Grosch, Illinois State Normal University, Normal, Illinois.

## HIGH SCHOOL MATHEMATICS LETTER

A mathematical letter will be mailed twice a semester to interested high school teachers by the University of Oklahoma chapter of Pi Mu Epsilon. Each letter will contain mathematical news, a short article, and a problem section. High school students are invited to submit solutions to the problems. A list of correct solvers will be carried in the next letter. High school teachers wishing to receive these letters should send their name and school address to Professor Richard Andree, Dept. of Mathematics, University of Oklahoma, Norman, Oklahoma.

*The enchanting beauties of this sublime study are revealed in their full charm only to those who have the courage to pursue it.*

—GAUSS

= ∇ =

"I started to study algebra once, but some days the teacher would say that $x$ was three, and on other days it was five and so I quit and stopped wasting time on such foolishness."

= ∇ =

The following cryptarithm has four solutions. The $x$'s represent digits not necessarily equal.

$$
\begin{array}{r}
xxx)\,xxxxxx4\,(x4xx \\
xxx\phantom{xxxx(x4xx} \\
\hline
xx4x\phantom{xxx(x4xx} \\
xxxx\phantom{xxx(x4xx} \\
\hline
xxxx\phantom{xx(x4xx} \\
x4x\phantom{xx(x4xx} \\
\hline
xxxx\phantom{x(x4xx} \\
xxxx\phantom{x(x4xx} \\
\hline
\end{array}
$$

—MATH. GAZETTE

= ∇ =

"It is safe to say that no one will ever construct the polygon of 66,537 sides, though it is agreed that this would be possible. It is just a question of the length of human life, and the superior attractions of other activities."

—E. C. TICHMARSH

= ∇ =

An anonymous tribute to the authority of Euclid: "Q. E. D.—Quid Euklid Dixit."

= ∇ =

A circular field contains one acre. A horse is tied to a tree 20 feet from the field with a rope of sufficient length so that he can graze over one-fourth of the circular field. Show that the rope is 107.5 feet long.

—SCH. SCI. AND MATH.

$\sqrt{2}+\sqrt{3} = 3.146 = \pi$, approximately.

= ∇ =

There are many palendromic squares with an odd number of digits; for example, $121^2 = 14641$. But palendromic squares with an even number of digits are much harder to find. Here are two: $836^2 = 698896$, $798644^2 = 637832238736$.

= ∇ =

"Contrary to popular belief [Einstein] had an unusual predilection for mathematics, and because of this fact I gave him, after his promotion to the fourth grade, Spieker's textbook on geometry. I used to visit his home every week, and whenever I came he delighted in showing me his solutions of new problems which he had found in the book. Sometimes I gave him helpful advice in the solution of rather complicated propositions, thus introducing him, as it were, into the study of mathematics, and soon he had mastered the whole textbook. He then turned to higher mathematics, studying all by himself Lübsen's excellent works on the subject, which I had also recommended to him. His progress in mathematics was so rapid that very soon I was no longer a match for him in the subject."—Max Talmey in *Scripta Mathematica*, Vol. 1, p. 69 (Sept. 1932).

= ∇ =

To convert Fahrenheit to Centigrade, add 40° to the Fahrenheit temperature, multiply the sum by 5/9, and then subtract 40°. Thus, $68°F + 40° = 108°$, $108° \times 5/9 = 60°$, $60° - 40° = 20°C$.

To convert Centigrade to Fahrenheit, add 40° to the Centigrade temperature, multiply the sum by 9/5, and subtract 40°. Thus, $20°C + 40° = 60°$, $60° \times 9/5 = 108°$, $108° - 40° = 68°F$.

= ∇ =

## TRIANGULAR NUMBERS

Pythagoras and his school developed the practice of representing numbers by geometric figures. Many arithmetic discoveries were made by the use of triangular, square, pentagonal, and other figurate numbers. To illustrate, the first five triangular numbers may be represented as follows.

```
                                              o
                                  o          o o
                      o          o o        o o o
            o        o o        o o o      o o o o
  o        o o      o o o      o o o o    o o o o o
```

If $T(n)$ denotes the $n$th triangular number, one sees that

$$T(n) = 1+2+3+\ldots+n = \tfrac{1}{2}n(n+1).$$

One can conclude from the juxtaposition of the respective geometric figures that

$$T(n-1)+T(n) = n^2.$$

A table of triangular numbers is constructed easily by continuous addition.

SHORT TABLE OF $T(n)$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 3 | 6 | 10 | 15 | 21 | 28 | 36 | 45 |
| 1 | 55 | 66 | 78 | 91 | 105 | 120 | 136 | 153 | 171 | 190 |
| 2 | 210 | 231 | 253 | 276 | 300 | 325 | 351 | 378 | 406 | 435 |
| 3 | 465 | 496 | 528 | 561 | 595 | 630 | 666 | 703 | 741 | 780 |
| 4 | 820 | 861 | 903 | 946 | 990 | 1035 | 1081 | 1128 | 1176 | 1225 |
| 5 | 1275 | 1326 | 1378 | 1431 | 1485 | 1540 | 1596 | 1653 | 1711 | 1770 |
| 6 | 1830 | 1891 | 1953 | 2016 | 2080 | 2145 | 2211 | 2278 | 2346 | 2415 |
| 7 | 2485 | 2556 | 2628 | 2701 | 2775 | 2850 | 2926 | 3003 | 3081 | 3160 |
| 8 | 3240 | 3321 | 3403 | 3486 | 3570 | 3655 | 3741 | 3828 | 3916 | 4005 |
| 9 | 4095 | 4186 | 4278 | 4371 | 4465 | 4560 | 4656 | 4753 | 4851 | 4950 |

It is of interest to note that the multiplication of two numbers may be reduced to simple operations of addition and subtraction by means of triangular numbers. The method is based on the following identity:

$$a \cdot b = T(a-1)+T(b)-T(a-1-b), \qquad (a>b)$$

For example, $(86)(47) = T(85)+T(47)-T(38) = 3655 + 1128 - 741 = 4042$. Again, $(350)(.0062) = (35 \cdot 10)(62 \cdot 10^{-4}) = (35)(62)(10^{-3}) = [T(61)+T(35)-T(26)]\,10^{-3} = (1891+630-351)(10^{-3}) = 2.17$. The above table suffices for finding the product of any two-digit numbers.

$= \nabla =$

"At the age of fourteen, Pascal was admitted to the weekly meetings of Roberval, Mersenne, Mydorge, and other famous French geometricians. At sixteen he wrote an essay on conic sections, which at the time was considered superior to anything that had appeared since the time of Archimedes." —W. W. RUPERT

= ∇ =

## AN OLD EXAMINATION

We are indebted to Professor Norman Anning for a copy of a Form III Algebra examination given in 1899 by the Education Department of Ontario, Canada. Professor Anning writes: "Form III equals eleventh grade. The person who passed this and about eight similar exams could enter university, or could take his second-class teacher's certificate and enter normal school, or could enter Form IV and begin on his second book of algebra. I have both textbooks—maybe you have someone who would like to review them. A re-view after fifty years might be interesting and instructive.[1] I was in an Ontario high school in '99, but I believe I did not write on this paper. Probably on one just as stiff."

How many students of College Algebra can pass this examination?

1. Solve:—
   (a) $\frac{2}{3}(9-x)-\frac{1}{3}(13-x)-2 = 0.$
   (b) $50x+51y = 152$
   $51x+52y = 155.$
2. Solve:—
   (a) $12x^2-311x+1927 = 0.$
   (b) $2x+3y-4z = 83.$
   $8y-3z-6x = 54$
   $5z-y-3x = -83.$
3. (a) Divide $(x+y)^2-3(x+y)z+2z^2$ by $z-x-y$.
   (b) Prove the identity:—
   $(a+b+c)^2+a^2+b^2+c^2 = (b+c)^2+(c+a)^2+(a+b)^2.$

   (c) Use the identity in (b) to find four numbers such that the sum of their squares shall be equal to the sum of the squares of 5, 4, and 3.

1 Cf. p. 25 Ed.

4. Express:—

(a) $x^6-y^6$ in four factors,

(b) $(y-z)^3+(z-x)^3+(x-y)^3$ in three factors,

(c) $8b^2c^2+2c^2a^2+8a^2b^2-a^4-16b^4-c^4$ in four factors.

5. (a) Find a symmetrical rational integral homogeneous expression of two dimensions in $x$, $y$, and $z$, which is equal to 6 when $x = y = z = 1$; and which is equal to 11, when $x = y = 1$, and $z = 2$.

(b) Simplify:—

$$a^3/(a-b)(a-c)+b^3/(b-c)(b-a)+c^3/(c-a)(c-b).$$

6. (a) Solve fully the quadratic equation $px^2+qx+r = 0$.

(b) Find an expression for the sum of the roots in (a) in terms of the coefficients.

(c) Find the equation whose roots are the reciprocals of the roots of $5x^2+6x-7 = 0$.

7. (a) Find the square root of

$$a^2-2a^{3/2}b^{-1/3}+3ab^{-2/3}-2a^{1/2}b^{-1}+b^{-4/3}.$$

(b) Solve $\sqrt{x+3} + \sqrt{x} = 5$.

8. (a) Find two numbers, differing by 20, and such that five-eights of the less exceeds two-fifths of the greater by 1.

(b) The sum of two numbers is 4, and the difference of their squares is also 4. Find the numbers.

(c) A rectangular field contains 5 acres. If the field were 10 rods shorter and 4 rods broader the area would be the same. Find the length and breadth of the field.

= ∇ =

$\pi$ = *3.14159265358979*

How I want a drink, alcoholic of course, after the heavy chapters involving quantam mechanics.

—JEANS

= ∇ =

*A mathematician confided*
*That a Moebius band is one-sided,*
*And you'll get quite a laugh*
*If you cut one in half*
*For it stays in one piece when divided.*

—AUTHOR UNKNOWN

Given three bars, gold, silver, and lead, no two of which are of equal length and no two of which are of equal weight. Identify the relative length and weight of each bar:

1. The heaviest bar is not the longest bar.
2. The silver bar is not the shortest bar.
3. The silver bar is heavier than the lead bar.
4. The lightest bar is not the shortest bar.
5. The lead bar is lighter than the longest bar.
6. The shortest bar is not the one of middle weight.

—AM. MATH. MONTH.

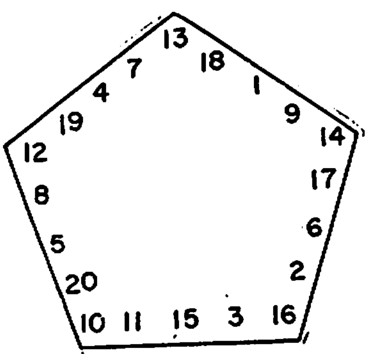
= ∇ =

"Whoever reads history with some critical judgment cannot fail to notice that the degree of civilization of a country is closely connected with the standard of mathematics in that country."

—I. J. SCHWATT

= ∇ =

From the Editor's mailbag: "Magic squares have fascinated me greatly, but of late I have tackled formations other than squares. I couldn't help thinking that a pentagon design would not be inappropriate for the PENTAGON. My foreign language teacher of nearly fifty years ago harped continually on the ability of Caesar to form hollow squares, hollow triangles, hollow pentagons. These lay dormant in my mind until quite recently. I have (constructed) them up to decagons.




"The ideal pentagon would have: five sides all equal; five numbers in each side; the hollow pentagon a multiple of five; the series a multiple of five. [Here is an example.]"

—IRA G. WILSON

*Quadratic* is from Latin *quadratus*, squared. From the same root we have squadron and quarry (a place where stone is squared).

—BOON

= ∇ =

From the Editor's mailbag: "In the Spring issue of the PENTAGON under 'Topics for Chapter Programs,' there is mentioned a problem of finding three equal rational-sided right triangles. I have found three triangles which fit the description, namely, (15, 112, 113), (24, 70, 74), and (40, 42, 58). . . . I have also found another set, but I am not revealing them at present."

—HARVEY FIALA, high school student.

= ∇ =

Some more "Freshman mathematics":

1. "In the common system of logarithms the mantissa is the same for all numbers having the same magnificent digits." — MATH. GAZ.

2. To divide 100 by 1:

```
1)1000(999     Q = 999, R = 1
   9
  —
   10
    9
   —
    10
     9
    —
     1
```

= ∇ =

The inner yellow portion of a daisy (chrysanthemum leucanthemum) exhibits a beautiful geometrical arrangement of its elements. By the mathematical principle of conformal transformations it can easily be proved that the best distribution of the elements is obtained when the lines appearing in the configuring of the flower are logarithmic spirals.

# THE BOOK SHELF

EDITED BY CARL V. FRONABARGER
*Southwest Missouri State College*

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Carl V. Fronabarger, Southwest Missouri State College, Springfield, Missouri.

*The Anatomy of Mathematics.* By R. B. Kershner and L. R. Wilcox. The Ronald Press Company (15 East 26th St., New York 10, New York), 1950. 11 + 416 pages. $6.00.

This book attempts to give to a reader, preferably with the degree of mathematical maturity represented by college courses through the calculus, a picture of the significance and scope of mathematics beyond the merely utilitarian. The authors plan "to describe the ultimate and intimate logic—in short, the anatomy—of mathematics."

To develop their plan they devote introductory chapters to a discussion of language and its limitations and difficulties, to the relation of logic and mathematics to language, to the primitive ideas of mathematics as a "science of number, of measurement, of space, and of axiomatics." These are followed by chapters devoted to the "primitive materials" of mathematics, including sets, equality, algebra of sets, ordered pairs, functions, binary operations, and other topics.

Two chapters devoted to the Postulational Method and Groups bring the reader to the point where the authors feel he has had material common to an understanding of mathematics. There follow discussions of specific bodies of subject matter, including the positive integers, inductive definition, extensions of operations, finite sets, infinite sets, isomorphism, equivalence and order relations, rational numbers, one dimensional continua and real numbers, and fields. There are a few exercises through the various chapters, with an appendix containing well-written solutions.

Altogether the book seems a successful attempt to introduce a reader to more mature concepts of mathematics and mathematical rigor. It is a kind of work needed by teachers, particularly in teacher-training institutions, for a course that will give an advanced undergraduate orientation in the meaning of mathematics. In language and in style it is readable; sometimes, when it suits the purpose, the style is informally vivid, as when the authors speak of a proof as the "provision of stepping stones across a gulf that is too broad for the mind to leap in a single effort" or when they say of a system of axioms that "consistency is almost a requisite, independence is rather nice, and categoricalness is not even particularly desirable except for certain purposes." Such informality in discussion does not prevent ample respect for formal statement in proofs.

—HOBART F. HELLER

*Mathematical Snapshots.* By Hugo Steinhaus. Oxford University Press (114 Fifth Avenue, New York 11, New York), 1950. 6 + 266 pages. $4.50.

The first edition of this book was published in Poland in 1939 and was out of print for several years. The new edition of *Mathematical Snapshots* has been revised and enlarged so as to include many more illustrations.

Professor Steinhaus brings to the reader an unusual collection of mathematical phenomena which are visualized and explained. A simple text with 295 illustrations make this a book most of which can be read by anyone with a mathematical maturity given by secondary school mathematics. Problems and tricks dealing with games, numbers, the fair division of a cake, optical illusions. honeycombs, soap-bubbles, knots, and maps are some of the many examples which make the book a wealth of supplementary material. Part of the material would be of value for high school mathematics clubs and other parts could be similarly used on the college level. The book would be very valuable for any high school or college library and also for a teacher's private library.

The book can be read straight through or the reader may skip over pages in which he is not interested. Although it lacks the convenience of an index, the book does have several pages at the end which give references to many suggestions for further reading.

—CLYDE T. MCCORMICK

*How to Study-How to Solve.* By H. M. Dadourian. Addison-Wesley Press (Cambridge 42, Mass.), 1949. 1 + 34 pages. $0.25.

This pamphlet is divided into two parts as indicated by the title. Part I contains numerous suggestions for the undergraduate's attaining desirable conditions for effective study. Systematically, the author treats Mental Attitude under the headings: Incentive, Interest, Self-Confidence, and Common Sense. Of noteworthy inclusion there is a quote from Professor W. B. Munro stating that from more than 1,000 students in Harvard Law School, all of whom were college graduates, it was determined that those "who had specialized in the exact sciences, especially in mathematics, were on the whole better equipped for the study of law and were making higher rank in it than were those who had devoted their energies to subjects more closely akin (e.g. history, government, and economics) . . ."

The author admonishes the student to acquire regularity in study, concentration, orderliness, and thoroughness in the pursuit of mathematical knowledge. Many readers will appreciate the directness and clearness as presented in the author's discussion of the function of the classroom, the function of the instructor, hints on assimilating material before examination, and the will to learn.

Part II deals specifically with general directions for solving problems. In commenting on the traditional question: "What do you care how I solve a problem so long as I get the answer?" the author says, "The main object of solving a problem is not to obtain the answer; it is rather to carry out a chain of reasoning which leads to the answer in the best possible way." The pamphlet contains many such gems of thought. The contrast between the "zigzag" and "straight-course" methods of deriving formulas is aptly illustrated in obtaining $sin(\theta+\Phi)$. An adequate set of examples is included in the exposition and affords the student a concrete means of following the author's development.

—J. HAROLD SKELTON

*The Main Stream of Mathematics.* By Edna E. Kramer. Oxford University Press (114 Fifth Avenue, New York 11, New York), 1951. 12 + 310 pages. $5.00.

The author combines in this book those essential phases of mathematics that students and the intelligent laymen should have available to them. To conserve space, Miss Kramer deliberately shies away from extended discussions, but has a wide range and good blending of topics with generally adequate treatment. The historical material with its important contributors shows the evolution of science, the kind of human beings who made their contributions, and leads one to feel that mathematics holds one of the most important keys to the future of the human race.

Some of the thought-catching and informative topics are: Thinking machines; Nim; The profundity of nothing; Natural forms; Divine proportion; Spirals; Incommensurables; Pi; Infinite summations; Electric, arithmetic, and geometric implications; Waves; Radar; Einstein mass-energy formula; A-bomb and H-bomb; Atomic arithmetic; Black magic; Trisecting the angle; Aviation and the associative law; Astronomic surveying; Laboratory or gambling den; Curve fitting; Atomic fission; Chain reaction; Annuities; The principle of jet-propulsion; Mathematics the science of form; Nature of space; and The crowning glory.

Uses are made of arithmetic, algebra, trigonometry, and some calculus, but the chief contribution of the book is that the reader is helped to understand other systems of numeration and symbolism—systems that have some factors in common with our usual mathematics, but that diverge enough to extend our knowledge in scientific application.

While nearly all of the book can be read and understood by capable students and intelligent laymen, there are places where one's knowledge and understanding is much more enriched if the reader is also fortunate in possessing a mathematical background. Nevertheless, the book should be read by college students. Students in college mathematics classes, and especially those in Kappa Mu Epsilon, will find it very much worthwhile.

—JOSEPH J. URBANCEK

*Professional Opportunities in Mathematics.* By Mina Rees, *et al.* Mathematical Association of America (University of Buffalo, Buffalo 14, N.Y.), 1951. 24 pages, $0.25.

Most students who take more than the required minimum of college mathematics do so either because they enjoy the subject or else because mathematics is a necessary tool for other professional fields in which they hope to become proficient. Too often the student has little knowledge of the diverse professional opportunities open to the trained mathematician as such, or of the types of college training needed in order to qualify for such opportunities. Because of this, the January, 1951, issue of the *American Mathematical Monthly* contains a 24-page article on "Professional Opportunities in Mathematics, a Report for Undergraduate Students of Mathematics" prepared by the following committee: H. W. Brinkman, Z. I. Mosesson, S. A. Schelkunoff, S. S. Wilks, and Mina Rees, Chairman. The article, which is amply documented with 30 excellent references for further study, has been made available in pamphlet form.

One paragraph of the Introduction states that "The report is divided into five parts. Part I is devoted to teaching in high school, college, or university. Part II explains the character of mathematical and applied statistics, and tells about career opportunities for young people who have suitable training at college to work in fields which rely heavily on statistical procedures. Part III discusses the role of the mathematician in an industrial laboratory, and Part IV tells about the work of mathematicians in government. In Part V a description is given of the work which is done by actuaries, and of the special preparation which many insurance companies make available to assist college graduates to qualify for the profession."

Although the report covers five distinct fields, each part written by an expert in his particular field, all cover essentially (1) kinds of work involved and levels attainable, (2) abilities and preparation needed, and (3) current salaries offered. Incidentally, the salary range quoted is from $2400 to $20,000 a year; while the required training naturally extends from the bachelors degree, without experience, to the doctorate with experience. As one author points out, "Salaries depend on general economic conditions and living costs in the surrounding area." This reviewer's college, for example, has been unable to satisfy the current demand by oil companies for computers at a beginning salary of about $3400 a year.

Each author of this report is to be congratulated for a scholarly work, well done in every respect. Every student, with even an incipient interest in using mathematics as a profession, should read this article. It might well be used as a guide for future training and expectations.

—EMMET HAZLEWOOD

# INSTALLATION OF NEW CHAPTERS

EDITED BY J. M. SACHS

The PENTAGON is pleased to report the installation of two more chapters of Kappa Mu Epsilon, Texas Epsilon and Louisiana Beta. There are now forty-seven active chapters on the roll.

## TEXAS EPSILON CHAPTER

*North Texas State College, Denton*

Nineteen student and faculty members were initiated as charter members of Texas Epsilon at the installation ceremony held in the Student Union Building on the campus of North Texas State College on May 31, 1951. Dr. F. W. Sparks of Texas Alpha served as installing officer. Dr. Harlan C. Miller of Texas Gamma assisted.

After the ceremony there was a brief address on the history of Kappa Mu Epsilon by Dr. Sparks and an informal party for the members and their guests.

The following officers of Texas Epsilon were installed: President, Gerald Kiel;Vice-President, Russell Bilyeu; Recording Secretary, Ruth Trammell; Treasurer, Richard Barham; Corresponding Secretary and Faculty Sponsor, J. V. Cooke.

Other charter members of Texas Epsilon are Robert Barham, C. T. Cadenhead, George Copp, Ben Cox, Lura C. Dean, Jr., Tommy Fulton, Keith Hildebrand, Gilbert Johnson, John Kostelac, Gloria Luke, William H. Parker, H. C. Parrish, Jane Pinkerton, and Mack Williams.

## LOUISIANA BETA CHAPTER

*Southwestern Louisiana Institute, Lafayette*

The installation ceremony for Louisiana Beta was held in Alumni Hall on the campus of Southwestern Louisiana Institute on May 22, 1951. Professor Z. L. Loflin acted as installing officer. Robert P. Binnings was elected acting President and Tuney Arceneaux acting Vice-President.

Other charter members of Louisiana Beta are Ray Paul Authement, Lurnice Begnaud, Geraldine Bourque, Charles Breaux, Ann S. Buchanan, Ellen Campbell, Rex Davey, Maurice P. Dossey, Luiz Figueiredo, Zachary T. Gallion, Vernon Guthrie, Walter E. Haggerty, Ida Mae Heard, Jessie Hoag, Betty Ann Hover, Rosina Landry, Clarence M. Monk, Mary J. O'Brien, Merlin M. Ohmer, James L. Peltier, Ulysse L. Pitre, Elizabeth Ann Roberts, Joe Saltamachia and Truman F. Wilbanks.

The following officers were elected at the first meeting in the academic year 1951-52: President, Tuney Arceneaux; Vice-President, Luiz Figueiredo; Recording Secretary, Elizabeth Roberts; Treasurer, Charles Breaux; Corresponding Secretary, Merlin M. Ohmer.

"One of the chief objects of scientific investigation is to find how one theory depends on another; and to express this relationship in the form of a mathematical equation—symbolic or otherwise—is the experimenter's ideal goal."

—J. W. Mellor

# J. A. G. SHIRK — IN MEMORIAM

O. J. PETERSON
*Past President, Kappa Mu Epsilon*

In the spring of 1950 we were saddened by the passing of our friend, Professor James A. G. Shirk. All of us in Kappa Mu Epsilon felt his loss keenly and personally, for somehow he had touched us all with the glow and radiance of a truly dedicated spirit.

Dr. Shirk was closely associated with Dr. Kathryn Wyant in the early development of Kappa Mu Epsilon and succeeded her as National President in 1935. Much of the early success of the Society must be attributed to his dynamic and inspiring leadership. In the high principles of Kappa Mu Epsilon Dr. Shirk found the realization of his own ideals, and through the years he labored constantly to instil an "appreciation of the beauty of mathematics" for the enrichment of the lives of students.

Dr. Shirk was a leader in promoting interest in mathematics throughout the State of Kansas. He was active in all Kansas mathematical groups, particularly the Kansas Section of the Mathematical Association of America, which he had served as chairman, the Kansas Association of Mathematics Teachers, the Kansas State Teachers Association, and the Kansas Academy of Science. His tall, white haired, dignified figure was a familiar sight at all meetings, and the groups regularly looked to him for guidance and wisdom.

Dr. Shirk was a native Kansan. He received the Bachelor's degree from McPherson (Kansas) College. His graduate work in mathematics was done at Chicago University, the University of Michigan, Stanford University, and at the University of Kansas where he received the degree of Doctor of Philosophy. After teaching mathematics at McPherson College and the University of Kansas, he went to the Kansas State Teachers College of Pittsburg as Chairman of the Department of Mathematics, where for thirty years he made important contributions to both school and community. Just before reaching the biblical age of "three score years and ten," death came as a token of work completed and a life richly lived.

Dr. Shirk has passed away; yet he is still with us. He lives in the hearts of his friends in Kappa Mu Epsilon, his former students, and his associates who all loved him and caught the spark of his inspiring enthusiasm. Through these thousands he still lives and will live for generations to come.

(Continued from page 35)

G. A. Miller, *Historical Introduction to Mathematical Literature.* New York, The Macmillan Company, 1916.

U. G. Mitchell, "Topics for Club Programs: The Number $\pi$," *American Mathematical Monthly*, Vol. 26, pp. 209-212 (May, 1919).

"Rules for Making $\pi$ Digestible," *Atlantic Monthly*, Vol. 156, pp. 118-119 (July, 1935).

W. W. Rupert, *Famous Geometrical Theorems and Problems.* Boston, D. C. Heath, 1901.

V. Sanford, *Short History of Mathematics.* New York, Houghton Mifflin Company, 1930.

H. C. Schepler, "The Cronology of Pi," *Mathematics Magazine*, Vol. 23, pp. 165-170 (January-February, 1950), 216-228 (March-April, 1950), and 270-283 (May-June, 1950).

C. Schoy, "Al-Biruni's Computation of the Value of $\pi$," *American Mathematical Monthly*, Vol. 33, pp. 323-325 (June-July, 1926).

H. Schubert, *Mathematical Essays and Recreations.* Chicago, Open Court Publishing Co., 1910.

D. E. Smith, "Historical Survey of the Attempts at the Computation and Construction of $\pi$," *American Mathematical Monthly*, Vol. 2, p. 348.

D. E. Smith, *History of Mathematics.* Boston, Ginn and Co., 1923.

H. Steinhaus, *Mathematical Snapshots.* New York, G. E. Stechert, 1938.

J. W. Wrench, Jr., "On the Derivation of Arctangent Equalities," *American Mathematical Monthly*, Vol. 45, pp. 108-109 (February, 1938).

J. W. A. Young, *Monographs on Modern Mathematics.* London, Longmans Green and Co., 1911.

# KAPPA MU EPSILON NEWS

EDITED BY LAURA Z. GREENE, *Historian*

**Hans Stetter of Colorado Alpha won honorable mention** in the William Lowell Putnam Mathematical Competition for the year 1950-1951.

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Dan Hanifan found that among the sixty-eight persons elected to **Kansas Beta** between the years 1934 and 1938 there were thirteen college professors, one college president, one research physicist, three junior college instructors, ten high school instructors, eight high school administrators, one civil engineer, one medical doctor, three businessmen, one professional army man, one meteorologist, and one deceased. There was no information available on twenty-four members.

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Anna Robben of **Kansas Gamma** was awarded the Hypatian award for her research paper on the Simson Line. Kathleen Feldausen received the underclassman award, a copy of Burington's *Mathematical Tables*. Three members had papers published during the academic year; two papers were published in the *Bulletin of the Kansas Association of Teachers of Mathematics* and one in the *Mount Mirror Magazine*. With the single exception of the host chapter, Kansas Gamma had the largest representation at the Eighth Biennial Convention of Kappa Mu Epsilon.

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Terry McAdam, the first president of **Kansas Delta**, was seriously injured in an automobile accident last May. He is now at the Kennedy Veteran's Hospital, a paraplegic center, in Memphis, Tennessee. He is on leave of absence from his position as Assistant to the President of Washburn University. Nancy Marsh received the freshman award for achievement in mathematics.

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**Michigan Alpha sponsored a Mathematics Department Open House during the annual Meet the College Day.**

Dr. Thomas H. Southard, faculty sponsor of **Michigan Gamma**, will be on leave until September, 1952. He will be at the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles 24, California. Miss Myrtle H. Keryluk, former corresponding secretary of Michigan Gamma, is now Mrs. Harold T. Slaby. Mr. Slaby was the first president of the chapter.

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Of ten members of **Missouri Alpha** who graduated in 1951, eight were graduated with honors. Five were elected to Who's Who in American Colleges and Universities.

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The faculty wives were hostesses to **Missouri Epsilon** and guests at the annual picnic.

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**Nebraska Alpha** reports the death on May 18, 1951, of Dr. Victor R. Morey, President of Nebraska State Teachers College. He was a member of Nebraska Alpha.

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**New Jersey Alpha** had a mid-term meeting with the Mathematics Club, with members of **New Jersey Beta** as guests. Dr. Howard Fehr of Columbia University was the guest speaker.

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**New York Alpha** announces that the chapter award to the best student of freshman mathematics was presented to Sue Rae Waldman. The winner is selected on the basis of grades made in freshman mathematics and a two-hour competitive examination. Jane Brandt has been awarded a Fulbright scholarship and will study in England under K. Mather and R. A. Fisher.

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On the University Honors Day, **Ohio Alpha** will present a book to the outstanding freshman in the mathematics department.

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**Oklahoma Alpha** is continuing the custom of the annual Founder's Day Banquet. This year they celebrated their twentieth anniversary. Alumni members of the chapter were special guests.

**Robert Frank Goad, President of Tennessee Alpha,** won the Engineering medal given to the graduating engineer receiving the highest grade average.

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**Wisconsin Alpha** members answer roll call with the name of a mathematician and the time that he lived, the definition of mathematics and by whom given, a quotation on mathematics, or a mathematical song title.

## PROGRAM TOPICS, SPRING SEMESTER, 1950-51

**Alabama Alpha, Athens College**
- *Cube Roots,* by Lloyd Stone
- *Application of Determinants,* by Hazel Livingston
- *Euclid's Algorithm,* by S. Hale

**Alabama Gamma, Alabama College**
- *The Calendar,* by Sarah Pepperhurst and Rae Floyd
- *Telling Time Throughout the Ages,* by Dorothy Champion and Irene Pace
- *Mathematics and Art,* by Betty Cross
- *Symbolism,* by Gay Penn, Dean Ingram, and Onita Bush

**California Alpha, Pomona College**
- *Mathematics of Finite Differences,* by Jon Mathews
- *Electronic Analog Computers,* by Rodney Weldon
- *Lattice Points,* by David Elliott
- *Eighth Biennial Convention,* by Walt Rosenow

**Colorado Alpha, Colorado A and M College**
- *Mathematical Puzzles and Paradoxes,* by Donald H. Allen
- *Number Systems,* by Dale A. Young
- *The Configurations* $(9_3)$, by Hans Stetter
- *Sir Isaac Newton,* by Roberta Cox

**Illinois Beta, Eastern Illinois State College**
- *Types of Jobs Open to College Graduates with Majors in Mathematics,* by Loren Pixley
- *Mathematics in Industrial Laboratories,* by Charles Perkins
- *Mathematical Training and the Actuary,* by Warren Farrell
- *Pythagoras,* by Patricia Tucker
- *Modern Minds and Ancient Bodies,* by Anna Bruce
- *The Slide Rule,* film
- *The Pythagorean Theorem,* film
- *The Meaning of* $\pi$, film
- *Global Concepts of Maps,* film

**Indiana Alpha, Manchester College**
- *Mathematics and Economics,* by Dr. Earl S. Garver
- *Mathematical Misconceptions,* by Lester Rouch

*Higher Mathematics in Statistical Work*, by Keith Ross
*Eighth Biennial Convention*, by Lester Rouch, Tom Dillman, and Philip Kinsey

**Iowa Alpha, Iowa State Teachers College**
*Dissection of Polygons*, by Bill Diedrichson
*Mapping*, by Miss Schurrer

**Iowa Beta, Drake University**
*Infinity*, by Christ Melis
*Men of Science*, by Ralph Moore
*Beginning of Modern Mathematics*, by Ifeanyi Osili

**Kansas Alpha, State Teachers College, Pittsburg**
*The Language of Mathematics*, by James Pike
*Spherical Trigonometry*, by William Brumbaugh
*Celestial Navigation*, by Professor R. W. Hart
*The Squirrel-Cage Slide Rule*, by Robert Green

**Kansas Beta, Teachers College, Emporia**
*How to Solve It*, by A. H. Albert
*Present Occupations of Former KME Members*, by Don Hanifan
*The Meeting of Kansas Association of Teachers of Mathematics*, by Max Woods and Richard Gregory
*The Eighth Biennial Convention*

**Kansas Gamma, Mount St. Scholastica College**
*Mathematics in the Orient*, by J. Culivan and F. Donlon
*Greek Mathematics*, by R. Link and T. Breitenback
*Hindu, Arabic, and Persian Developments*, by A. Robben and J. Sullivan
*Mathematics in the Renaissance*, by M. Acree
*Seventeenth Century Geometry*, by P. Shideler and D. McManus
*Development of the Calculus*, by M. McBride
*Eighteenth Century Mathematics*, by M. C. Brenwald
*Novelty Numbers* "Mathematical Relations"
*The Simson Line*, by A. Robben

**Kansas Delta, Washburn University**
*Report on the International Mathematics Congress*, by Dr. Eberhart
*Mathematical Puzzles*, by Clarence Grothaus
*Men in Mathematics*, by Mary Maynard, Charline Miller, Doris Baker, and Margery Gamble
*Mathematics in General Education*, by Joe Latas, Betty Moffett, and LeRoy Johnson
*Conics in Paper Folding*, by Darlene Moore
*Short Cuts in Mathematics*, by Thomas Hupp
*Models*, by Duane Reed
*Hexahexaflexagrams*, by Donna Simmons

**Michigan Alpha, Albion College**
*Cryptography*, by Milton Ehlert
*The Laplace Transform*, by Albert Foster
*Play*: *"It Can't Happen Here,"* by initiates

*The National Convention,* by William Fryer
*The Planimeter,* by William Horn

Michigan Beta, Central Michigan College
*Mathematics and Games of Chance,* by Lloyd Trinklein
*Mathematics and Religion,* by Wayne Atkins

Mississippi Gamma, Mississippi Southern College
*The Determinant,* by Mary Frances Sasser and Patsy Munn
*Hyperbolic Functions,* by William Gay
*Curve Fitting,* by Edward Thomas
*Study of Slide Rule,* by Elmer Sumergile
*Study of Slide Rule,* by Bob King
*Radius and Circle of Curvature,* by Eric Geiger

Missouri Beta, Central Missouri State College
*Geometry in Architecture,* by Robert Jones, guest speaker
*The Gamma Function,* by Keith Stumpff
*Non-Uniform Motion of a Piston,* by Charles Sigrist
*Fire Insurance Rating,* by F. W. Straulman, Mo. Inspection Bureau

Missouri Epsilon, Central College
*Pascal and His Magic Hexagon,* by Charles Berthe
*History of the Calculus,* by George Koonce
*The Abacus,* by William Cooley
*Graphing Equations by Means of Differential Calculus,* by Hector McDonald
*Elementary Curve Tracing,* by Norman Drissell
*Biography of Poincare,* by Eva Gilbertson
*The Life of Descartes,* by J. P. Karnes

New Jersey Alpha, Upsala College
*The Method of Least Squares,* by Marlin Nitzler
*Kepler's Three Laws,* by William Stachel
*Vicsor Analysis,* by Ethel Larson
*Graphical Analysis of Complex Numbers,* by Dr. Howard Fehr, Columbia University
*Cryptography,* by Lloyd Johnson
*Dynamic Geometry,* by Professor David Spolnik

New York Alpha, Hofstra College
*Mathematics of Musical Scales,* by Robert Blasch
*Probability-Geometrical Methods,* by Kenneth Feldmann
*A Report of the National Convention,* by Peter Hinrichs, Lysle Marshall, Robert Blasch, Jane Brandt, Richard Lamm, and Mary Pawelko
*The Ladder Problem,* by Alexander Basil and Lee Dunbar

Ohio Alpha, Bowling Green State University
*The Development of the Calculating Machine,* by Beverly Ferner and Marion Goodnight
*The Importance of Mathematics to Economics,* by Lewis Manhart

*Mathematics Quiz Program*, by Joanne Schisrmyer and Fama Gerhart
*Symbolic Logic*, by Dr. Frank Ogg

Ohio Beta, College of Wooster
*Careers in Mathematics*—several guest speakers in connection with college career week

Ohio Gamma, Baldwin-Wallace College
*Engineering Education in England*, by Professor Phillip Clyne
*The Mathematics of Democracy*, by Professor John Wilson
*Some Applications of the Laws of Chance*, by Jack Hafferkamp
*Fun with Figures*, by Kenneth Buser
*Professional Opportunities in Mathematics*, by Glenn Carter
*The Mathematics of Life Insurance*, by Ellsworth Strock

Oklahoma Alpha, Northeastern State College
*Law of Cosines in Plane and Spherical Trigonometry*, by Ima Jean Bittle
*Mean Value Theorem and Rolle's Theorem*, by Bill McCarty
*Various Proofs of Lim Sin $\theta = 1$*, by Allen Willyard
*Trigonometric Functions as Power Series*, by Cletis Payne
*Euler's Equations*, by Francis Lowery
*Solutions of Higher Equations*, by Thomas Wofford
*Mathematical Analysis* (20 years of K.M.E.), by L. P. Woods
*Present Values and Mathematical Induction* (Year's activities and initiation of new members), by Bob Zenor
*Probability* (The Future of K.M.E.), by Ray Carpenter
*Review of National Convention*, by Mike Reagan and Monte York

South Carolina Alpha, Coker College
*Number Bases*, by Frank Saunders
*Cryptography*
*Problems from the American Mathematical Monthly*

Tennessee Alpha, Tennessee Polytechnic Institute
*Complex Numbers*, by F. W. Harbison
*The Algebra of Matrices*, by C. L. Bradshaw
*Filmstrips dealing with Mathematics and Physics*, by R. H. Moorman

Wisconsin Alpha, Mount Mary College
*Explanation of Actuarial Work*, by T. V. Henningston
*How I Became Interested in Actuarial Work*, by Mary Hunt
*Introduction to Topology*, by Ann Sanfelippo
*Special Theorems and Ideas in Topology*, by Janet Haig
*Teaching Arithmetic in the Primary Grades*, by Marilyn Briggeman
*The Chinese Abacus*, by Adeline Madritsch
*Mathematics and Design*, by Audrey Reiff
*Numerical Test—Taken from the Fall 1947 Issue of the Pentagon*, by Joan Weller

*Calculating Devices*, by Betty Haendel
*Probability*, by Carole Sands
*Number Bases*, by Dolores Wall
*Arithmetical and Geometrical Progressions*, by Ruth Renwick
*Numerology*, by Pat Zimmer
*Report on National Convention*, by Carole Sands
*Report on the book*, "The Psychology of Invention in the Mathematical World," by Mary Ann Pichotta

## ANNOUNCEMENT

The Editor takes pleasure in announcing that arrangements have been completed for reprinting A. B. Kempe's How to Draw a Straight Line. This out-of-print classic will appear in the Spring 1952 number of THE PENTAGON.