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Kappa Mu Epsilon, national honorary mathematics fraternity, was founded in 1931. The object of the fraternity is four-fold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievement in the study of mathematics in the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters

FOUR SQUARES

NORMAN ANNING
Faculty, University of Michigan

(Reader: Please have pencil and paper; more will be suggested than will be proved.)

Here is a problem from a textbook[1] nearly eighty years old: "The lines which join the mean points of three equilateral triangles described outwards on the three sides of any triangle form an equilateral triangle whose mean point is the same as that of the given triangle." Students still challenge one another with this problem, and every teacher has to see it. Someone may bring it to you tomorrow and I think I hear him say, "We have a senior engineer in our house; he couldn't do it either." So you prod him gently in the direction of vectors or of some other method which seems to you better. The figure of three equilateral triangles which lead rather surprisingly to a fourth has been called the figure of Torricelli. He and Cavalieri used it in 1647. It is likely that by now most of its implications and ramifications have been explored.[2]

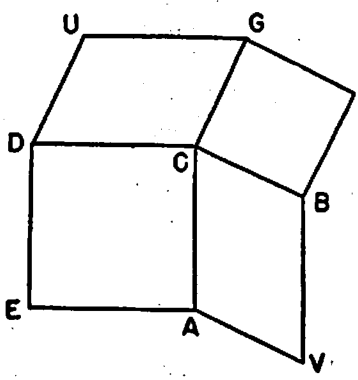
This may be the first time the student has met a situation where he starts from an *irregular* triangle and arrives by easy steps at a *regular* triangle. If he shows any sign of being thrilled by this discovery, you will do mathematics a service by directing him to the Morley theorem and to the fact that the envelope of the Simson (Wallace) lines of any triangle is a Steiner hypocycloid. To read about the intimate connection between these latter facts, see pages 345-349 of H. F. Baker's *Introduction to Plane Geometry;* this is not a text for beginners.

Is there some combination of four squares which is equally worthy of study? Yes, here is one; it is not claimed that it is the only one or the most exciting one.

1 P. Kelland and P. G. Tait, *Introductions to Quaternions*, 2nd ed. London, Macmillian and Co., 1882, pg. 42.

2 See, for example, R. Johnson, *Modern Geometry*. New York, Houghton Mifflin Company, 1929.

Euclid's *Stoicheia* is not so much the elements of geometry as the elements of mathematics treated geometrically. In proving that $(a+b)^2 = a^2+2ab+b^2$, Euclid in the fourth proposition of his second book used the same figure as we use today in illustrating the same identity. Let us keep the two squares, allowing one to be rotated about their common corner, and add a pair of congruent parallelograms as in the figure. This simple figure is a part of the one about to be described. It has interesting geometric properties in its own right. For instance, the centers of the four figures are the vertices of a square; *EVFU* is a square.

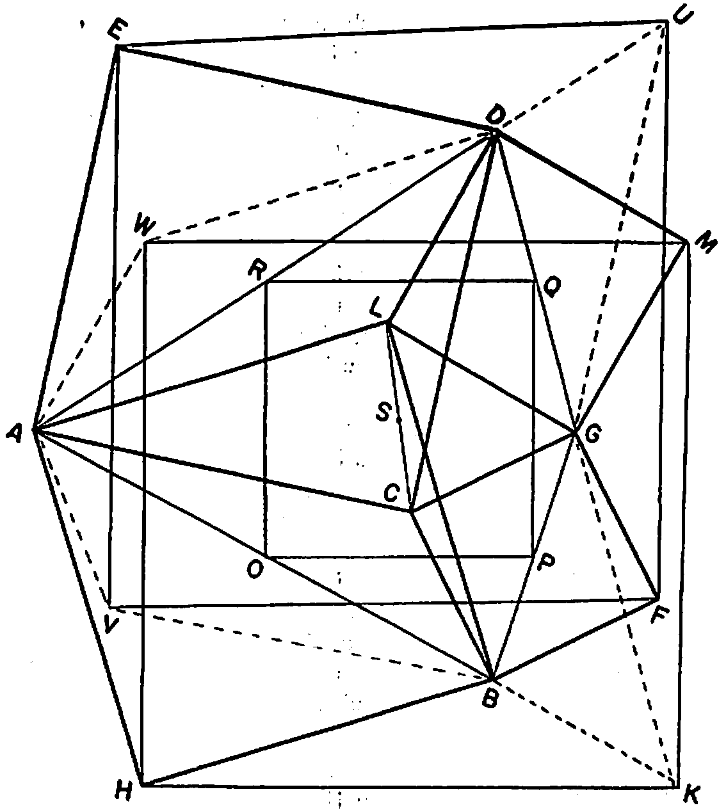


Directions will now be given for drawing the more general figure. Squares will be named by naming the corners counterclockwise, and properties will be stated without proof.

Start with any triangle *ABC* and construct the squares *ACDE* and *BFGC*. Construct also the squares *AHBK* and *DLGM*.

Then

1. *K* and *L* coincide.
2. *L* is the midpoint of *EF*.
3. *C* is the midpoint of *HM*.
4. *EF* is equal to *HM*.
5. *EF* is perpendicular to *HM*.
6. If *O, P, Q, R, S* are the midpoints of *AB, BG, GD, DA, CL*, then *OPQR* is a square whose center is *S*. This square is homothetic to *EVFU* mentioned earlier.
7. A median of triangle *ABC* is in line with an altitude of triangle *CGD*, and *vice versa*.
8. The sum of areas *ACDE* and *BFGC* is equal to the sum of areas *AHBK* and *DLGM*.
9. And so on.


E
U
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H
K

It is hard to imagine that any property of this figure can be new, but that fact need not spoil the fun of looking for properties new to us. Remember the words of Descartes[3]: "But I shall not stop to explain this in more detail, because I should deprive you of the pleasure of mastering it yourself, as well as of the advantage of training your mind by working over it, which is in my opinion the principal benefit to be derived from this science."

Suppose that $i(CB)$ means "turn CB about C through plus 90°." Observe the round trip: $i(CB) = CG, i(LG) = LD, i(CD) = CA, i(LA) = LB$. This property was used in 1908 by Netto[4] in giving a geometric representation of a group whose operators are defined by $S^4 = T^4 = STST = 1$.

As recently as 1948, a part of the figure was used by G. Gamow[5] in illustrating the usefulness of the number i. Treasure buried on an island by pirates is found after time has erased certain apparently essential evidence. But you must see for yourself. Gamow, now an American, is one of our best students of the origin, career, and destiny of stellar universes. In about three sentences he can yank you across three billion years of space-time and make you see what must have been happening in the first twenty minutes.

Do not shrink away from taking an occasional mental excursion. Francis Bacon says, "The universe is not to be narrowed down to the limits of the understanding, which has been men's practice up to now, but the understanding must be stretched and enlarged to take in the image of the universe as it is discovered."

The game is by no means ended. If you discover other interesting properties in the figure or other outcrops in the literature, please report them to the PENTAGON.

EDITOR'S NOTE—The PENTAGON will be happy to publish student reports describing further properties of the Four Squares of Professor Anning.

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3 D. E. Smith, *A Source Book in Mathematics*. New York, McGraw-Hill Book Company, 1929, pp. 400-1.
4 Netto, *Gruppen und Substitutionentheorie*.
5 G. Gamow, *One, Two, Three . . ., Infinity*. New York, Viking Press, 1947.

THE DEVELOPMENT OF CALCULATING MACHINES

JAMES D. IDOL, JR.
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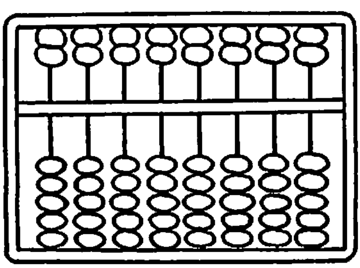
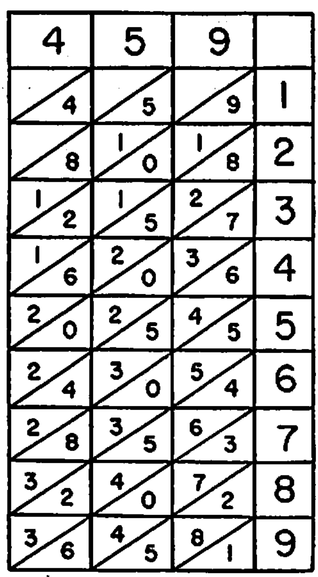
The calculating machine has surely contributed as much to the advancement of scientific research and general knowledge as any one device I can think of at this time. Yet it has been given as little credit for being one of modern society's revolutionary developments as its contributions have been great. Imagine a bank without adding machines or an engineer without his slide rule. The statement is practically a paradox in itself. Without doubt, civilization could not have advanced to its present degree without a machine which could take over where our resources are at an end. This is what the calculating machine in its various forms does, much to our good fortune, and it has had a long and honorable history in so doing.

We might imagine that ten or twelve centuries ago Fu-Yen or one of his anonymous Chinese relations ran out of fingers adding up the family grocery bill and decided that a machine which could do his adding for him wouldn't be a bad gadget to have around. This setting is, of course, fictitious, but from similar necessity came the abacus, an Oriental adding machine that still finds use in Chinese banks and business houses. It consists of a rectangular framework containing several rows of wooden balls strung on wires; the balls on each wire are divided into groups of five and two by a partition running parallel to two of the sides of the frame. To register a number on the abacus the following system is observed. The balls in each group of five represent the digits one to five when the two balls in the group of two are against the outer



frame away from the inner partition. When one of the two is moved next to the inner partition the five on the other side of the partition now assume values of six to ten. The extra ball in the group of two is also worth five. When any combination of ten is obtained on any row, that row is cleared of ten units and one unit is added to the next row, the succeeding rows denoting units, tens, hundreds, etc. Using this system of carrying tens, addition is automatic as also is subtraction if the reverse process is employed. Amazingly rapid computations can be made with the abacus by one familiar with its operation.

The next development of note was made by John Napier of Scotland. Known in the slang of the time (1617) as Napier's bones, his invention was a multiplying device which consisted of several wooden or ivory sticks (hence the name) each of which had an integer at the top with its integral multiples listed vertically downward. Where the multiple was of more than one digit, the digits were divided by a diagonal line such as we use in writing fractions. Also part of the device was a rack in which the sticks could be laid side by side. The rack had the numbers one to nine graduated regularly down one side so that each number on the frame was opposite its multiple of a number whose stick was laid in the rack. The rule for multiplying is: Select the multiplicand as the topmost integers of the required sticks and lay them in the rack in the order the integers occur in the number.



Select the single digit multiplier from the scale on the rack and add the digits in the short diagonal intervals in the row horizontal to the multiplier on the rack. The addition is from right to left. Multiplication of a poly-figured number by a poly-figured number is accomplished by summing the products of the components of the multiplier, these being first resolved into units, tens, etc.[1]

The slide rule had an humble beginning. In 1620 Edmund Gunter, an Englishman, conceived the idea of listing the numbers one to ten on a scale of suitable material, separating these numbers from each other, not according to their algebraic differences, but according to the differences of their logarithms. By the use of dividers, distances on such a scale were added and subtracted which resulted in multiplication and division. If the addition of a length corresponding to a certain value caused one to run off the scale, it could be merely added to the other end with the same effect since in that position it had the same quality as a cologarithm. A few years later, another Englishman, William Oughtred, had the thought of using two of Gunter's scales opposite each other, one sliding back and forth past its mate. This achieved the same end as the use of dividers, and it was more convenient to use. After Lieutenant Mannheim of the French Artillery had added a glass indicator to the instrument for more accuracy in reading results, we were given the slide rule in one of its present forms, known as the Mannheim type.

Newton was among the first to recognize the value of the slide rule; after making some improvements on it to suit his purpose, he used it for solving cubic equations. Many other people contributed to the development of the slide rule, and of these I must mention Peter Roget, a London physician. To Roget occurred the idea of using another scale the values of which were graduated in the proportions of logarithms of their logarithms. Now, since the other scales on the instrument are logarithmic to the first degree, and the log-log scale is logarithmic to the second degree, there is a difference in character of unit logarithmacy between the log-log scale and the other scales. Consequently, any operation between the log-log scale and any other scales on the slide rule will necessarily involve

[1] See THE PENTAGON, Spring 1949, pp. 98-100.—Ed.

powers and roots of numbers. The invention of the log-log scale is probably the most important improvement to be added to the slide rule since its invention. Equipped with the log-log scale and various others including the sine, tangent, and inverted scales, the slide rule has become an indispensable tool to physicists, chemists, engineers, and scientists in many fields. It is, in my opinion, the most important single device thus far contributed to the family of calculating machines owing to its enormously wide applicability to all types of problems and its versatility in the hands of one who understands its operation.

The first real calculating machine as we customarily picture it (some type of metal case enclosing machinery) was invented by the French physicist and mathematician, Blaise Pascal (1642). This machine was in the form of a flat, rectangular box; on the surface of the box were six dials on which were inscribed the numerals zero to nine. A pointer fixed to the case indicated the starting or zero position for each dial. Beneath the surface of the case the shaft from the dial above was terminated by a gear which was in turn geared equally in ratio to a wheel on the edges of which were numerals from zero to nine. The topmost figure on the wheel was read through a window in the case; hence, when a dial was turned through a distance corresponding to the value of, say six, the numbered wheel below was also turned so that the numeral six appeared in the window above the wheel. Each wheel was geared to the wheel immediately next on the left in a ratio of one-tenth, so that when one wheel made one revolution its next higher neighbor made one-tenth of a revolution. Thus, the carrying of tens was accomplished. The only disadvantage was that the mechanical advantage was also one-tenth with each succeeding wheel. As a result, it required one thousand times as much energy to add one to ten-thousand as it did to add one to ten, and only a superman could operate a five-place machine. This same fault was common with other machines of the time and for awhile it looked as if all calculating machines were destined to become museum pieces. The shortcoming of most of the machines was that the actual operations were performed at the same time numbers were registered into their mechanism, and both operations simply called for too much force to be supplied by the keys at one time. Charles Thomas, also a Frenchman, had this in mind when he constructed his machine a number of years later. This machine proved to be highly successful, and today's calculating machines are very similar in design to his. The actual operation of Thomas' machine is too complex to describe here; however, a very important improvement he made was the operating lever, which made the device physically much easier to operate. The small number levers of his machine, in the place which would correspond to the keyboard of our present machines, were used only for registering the quantities to be operated on with the computor. Thomas' machine would add or subtract, multiply or divide, and was actually the forerunner of our present day comptometers, although the name was patented by another man whose first invention differed considerably in principle and operation from Thomas' machine.

Thomas' invention was made about the middle of the last century and from that time on improvements and developments came rapidly. An Austrian firm, the Braunswiga Company, immediately perfected and marketed a multiplying machine which was readily adopted by many European business houses. In America, W. W. Bourroughs and D. E. Felt perfected their machine and in a short time every large business house in the country was equipped with calculating machines of some type. The common adding machine with which all of us are familiar was invented by Bourroughs and is a lever operated machine. On the keyboard of such a machine are found several rows of keys, each of which is connected underneath the keyboard to a regulating device which governs how far a number-bearing wheel will be turned when the operating lever of the machine is worked. The numbers on the wheels are viewed through small apertures and with the system of tens carrying automatic with the operation, the sum of the numbers registered in the device may be read at any time.

Some of these machines are equipped with printing attachments and are employed in book-keeping and similar occupations.

Calculating machines are becoming increasingly complex with each advancement. The "punched card type" used by the Bureau of Census is a good example. This machine is "fed" sheets containing 240 questions with an average of fifteen questions answered on each page, the answer being in the form of a hole punched in the sheet. On the operating panel of the machine are 240 keys, one corresponding to each space for an answer on the question sheet. The operator, to classify all answers of a certain type, merely punches the necessary combination of keys, and, after flipping a starting switch, is then free to finish his detective novel. Meanwhile, the faithful machine, electrically operated, sorts and classifies all the material given to it, tabulates the results into neatly printed columns, and, if told to, goes a step further and computes the number of people in a certain block in the city who own their home, and the number of houses on that block in the city in which the report was taken. The inventor reputedly guarantees the labor saver to solve any statistical problem except income tax returns (which are clearly ambiguous cases involving extraneous roots and imaginary numbers, and are thus unsolvable by logical and mathematical means.)

There recently have come onto the scene several types of calculators operating on an entirely different principle than that of any of the varieties I have thus far described. These are the relatively new electronic instruments perfected during the war at several of the larger institutions in the East. For sheer complexity of design, they probably head the list of all manufactured products in America which are produced as a unit. Some of these behemoths are over fifty feet long and others include as many as 18,000 electron tubes, although the tendency in their manufacture is away from so many small unit attachments, due, of course, to the never-ending maintenance tasks. Some almost comical names have been assigned to them, including "ENIAC" (which stands for Electronic Numerical and

Integral Calculator) and "MANIAC" (standing for Mechanical and Numerical Integrator and Calculator.)

As far as the principles of operation are concerned, there are two types of electronic calculators, the "analogue" type and the "digital" type. In the analogue type, values are designated by physical quantities such as lengths or charges of electricity. This type of calculator has the main disadvantage of possible errors from inaccuracy in measuring distances if numbers be designated by lengths, or by condenser and wire leakage if charges of electrical energy be employed in the same capacity. However, offsetting this shortcoming, the analogue computors are unbelievably rapid. In fact, they operate at the speed of light since that is approximately the speed with which electric impulses travel. The percent of error can also be reduced to a negligible amount. A mechanical monster of the analogue variety was made at Harvard during the war, and it went to work for Uncle Sam as soon as completed. For days on end it digested weighty calculus problems, dealing chiefly with projectile trajectories, supplied by the Artillery Division. Mark II, as it was known, spewed out answers on a punched tape which usually gave a nice smooth curve when plotted. One day, while working on a particularly involved problem supplied by a government office, Mark II apparently went beserk. The answers it gave were completely out of proportion with what would have been expected from a hasty review of the problem and some of them reached mountain-top proportions. The experts at once went to the task of giving their charge a mechanical aspirin, but nothing appeared to be wrong; so, the answers were turned into the war office with the advice that they should not be depended on too heavily. Some time later the first atomic bomb exploded in New Mexico and the learned professors got an idea of what their mathematical robot had been figuring.

Machines of the digital type employ a counting mechanism such as toothed wheels to register numbers. There is not the slightest possibility of error with this type as you can readily see. Its capacity is limited only by the extent of its facilities. Some of the later models of the digital type do not use the decimal system of counting, but employ a binary system in which the only two figures used are 1 and 0. This adds to speed in operation and also to simpler construction. Instruments of the ENIAC and MANIAC variety complain when they are "ill," inform the attendants when they are ready for another job, and have to be "psychoanalyzed" periodically by experts to see if they are in good "mental health", for ENIAC can blow a tube just like a twenty-five dollar radio. Incidentally, ENIACs cost around $500,000 and are so fast that they can compute the exact path of a shell while it is in flight. MANIACS (as would be expected!) work even faster than this. "In ENIAC, immediate results are stored in boxes known as accumulators. They are left there until called for. . . . Thus equipped, the electronic machine is now ready to receive orders of the following kind: 'Add the numbers in the first seven boxes and put the result away in the eighth box; add the numbers in the thirteen boxes and divide the sum by the number in box eighty-five; subtract from this quotient the product of the numbers in boxes one-twelve, one-thirteen, and one-fourteen; square the remainder, add this to the sum you left in box eight; put the result away in box 3442; and let us know when you are finished.' Chances are the machine will finish the job in one-eightieth of a second." [2]

[2] *Science Digest Magazine*, September, 1947.

"I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history."—J. W. L. Glaisher.

PYTHAGORAS AND PTOLEMY MUST HAVE LOOKED AT THE CONCLUSION

EDWIN EAGLE
Faculty, San Diego State College

"Yes, the solution seems to work, it appears to be correct, but how is it possible to invent such a solution? Yes, this experiment seems to work, this appears to be a fact; but how can people discover such facts? And how could I invent or discover such things by myself?" [1]

Such, says Dr. Polya, author of *How to Solve It,* were the questions that occurred to him as a "young and rather eager student of mathematics." No doubt every thoughtful student time and again, when presented with some neat solution of an apparently difficult problem, has asked questions such as the above. Often, after completing a problem, as we look back on the procedures used we can see how we might have proceeded in a more direct and sensible manner. Sometimes we may even say, "I should have seen in advance that this last procedure would have been more simple and direct than the others I tried." It is through looking back and analyzing, through searching for better procedures that we might have used, through detecting how we might have avoided the pitfalls that beset us that we become more efficient problem solvers. All too often this highly valuable, interesting, and rewarding activity is omitted. To gain in mathematical skill and power of analysis we must deliberately and systematically endeavor to apply the lessons of experience to make for more effective methods of attack on future problems.

It is true that in nearly all cases the concise, orderly form of the final solution of a problem is far different from the devious path with many false turnings and retractions, corrections and renewed attacks, which ultimately leads to the final form in which the solution is presented. Nevertheless, it is usually true, particularly in the "to prove" type of problems, that careful analysis and repeated

1 G. Polya, *How to Solve It*, Princeton, N. J., Princeton University Press, 1948.

attention, or continuous attention, to the *conclusion to be proved* gives definite clues regarding procedures that will lead most directly to the proof. One of Dr. Polya's frequent admonitions to his students is "Look at the conclusion."

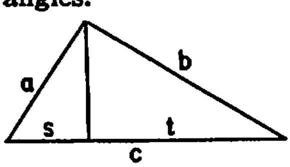
The proof of the theorem of Pythagoras by means of the similar triangles formed when the altitude to the hypotenuse is drawn is an example of a proof in which attention to the conclusion leads directly to the proof. Of the hundreds of relationships that exist it is possible to detect almost at once the ones which directly yield the proof. Of course, sufficient familiarity with relevant background material is necessary. One must have as a part of his thinking equipment the fact that two right triangles are similar if an acute angle of one is equal to an acute angle of the other, and that as a consequence of this the two triangles formed when the altitude is drawn to the hypotenuse are similar to the original triangle.

It is suggested at this point that the reader sketch a right triangle, label the hypotenuse c, the short leg a, and the long leg b, and test for himself the fact that the conclusion to be proved, namely $c^2 = a^2 + b^2$, has within it the definite clues which if carefully considered point directly to the proper steps to be taken.

If the above suggestion has been followed seriously, the reader should have gone through a thinking process somewhat as follows:

1. To get the "squared" terms which appear *in the conclusion,* mean proportions might be used.

2. In this problem the most promising way of setting up proportions would be through the use of similar triangles.



3. The need for similar triangles suggests constructing the altitude to the hypotenuse and introducing suitable notation as indicated in the figure.

4. A proportion that we could write which would yield the desired a^2 is $c/a = a/s$, this directly from "large hy-

potenuse is to small hypotenuse as large short leg is to small short leg."

5. *In the conclusion* and also in the figure, a and b appear in much the same manner. Therefore, in the steps of the proof we may expect a and b to be used in much the same way. This suggests the use of the proportion $c/b = b/t$, which can be established on the basis of similar triangles as above. From this we get $ct = b^2$.

6. The a^2+b^2 *in the conclusion* indicates the addition of the above equations, giving $cs+ct = a^2+b^2$.

7. To obtain the c^2 *in the conclusion,* the obvious procedure is to write $cs+ct$ as $c(s+t)$, and substituting c for its equal $(s+t)$ to get $c^2 = a^2 + b^2$. Q.E.D.

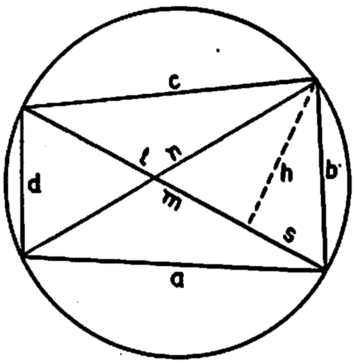
Possibly the discussion of the above systematic steps leading directly to the proof indicates more than anything else that hindsight is clearer than foresight. However, it remains true that the steps taken at each stage are probably a bit more justifiable logically than any others that might be suggested, and that concentration on the conclusion to be proved would lead to these steps. Whether or not it is reasonable to expect immediate success in arriving directly at this particular proof without first getting into a few blind alleys and trying other approaches, nevertheless greater efficiency and the elimination of much fruitless effort will result from keeping the conclusion clearly in mind.

A somewhat more difficult problem which illustrates the point equally well is the proposition: If a quadrilateral is inscribed in a circle, the product of its diagonals is equal to the sum of the products of its opposite sides. This proposition is of considerable historical interest as Ptolemy proved it about 150 A.D. as a preliminary step in developing procedures for calculating a "table of chords" which he used somewhat as trigonometry tables are used today. [2] The thought processes and the steps of a proof which can be used bear a most remarkable similarity to those discussed above in connection with the theorem of Pythag-

[2] V. Sanford, *A Short History of Mathematics*, Boston, Houghton Mifflin Company, 1930, p. 294

oras. Again it is suggested that the reader stop at this point and attempt to carry out the proof. It should be a real thrill to prove for one's self what Ptolemy proved some 1800 years ago. In a circle inscribe a quadrilateral with sides a, b, c, and d, and with diagonals m and n. Then prove that $mn = ac + bd$. The clues that have been given and again the admonition to "look at the conclusion" should enable a careful student to introduce the needed auxiliary line and proceed rather directly with the proof.

If the above suggestion has been sufficiently tested, a process somewhat as follows has probably developed. In order to prove that $mn = ac+bd$, one proportion in which a and c appear as means and another proportion in which b and d appear as means could be used. The diagonals m and n should also enter into the proportions in some manner. The use of proportions suggests the use of similar triangles. Using the most obvious pairs of similar triangles, namely the two pairs of similar triangles which include the pairs of vertical angles formed at the intersection of the diagonals, introduces a and c, or b and d, as corresponding sides; thus, they do not appear as the means of a proportion as required *in the conclusion.* It therefore seems necessary to draw an auxiliary line to introduce similar triangles where a and c appear but not as corresponding sides. Now a is a side of triangle abn. Angle na of this triangle[3] is equal to angle cm, as both of these inscribed angles have the same intercepted arc. A triangle similar to triangle abn could be formed by drawing h as indicated, dividing m into segments t and s, h being drawn so that angle ht = angle ba. The similar triangles thc and abn



[3] This unorthodox manner of indicating an angle is not ambiguous. Its use makes it unnecessary to label the intersections of the figure. Since it is the length of the line segments with which we are concerned the use of a single letter to represent each line segment has considerable psycheological advantage over the use of letters at each end of the segment.

yield the proportion $t/a = c/n$, from which $tn = ac$. A proportion is now needed in which b and d appear as means. Also we have used diagonal n and the long part of diagonal m. The short part of diagonal m must also be used. This suggests the use of triangle bhs which can readily be proved similar to triangle ncd. Using these triangles, the proportion which includes the most of the required elements and which uses b and d as means is $n/b = d/s$. This gives $ns = bd$. Carrying out the addition indicated *in the conclusion* results in $nt+ns = ac+bd$; $n(t+s) = ac+bd$; $mn = ac+bd$. Q.E.D.

In this second illustration, due to the greater complexity of the figure, there are other equally logical possibilities that could be used and which would lead to a proof just as directly. There are, of course, also many other relationships which can be expressed which do not lead to a proof. But it is true that in most cases careful attention to the *conclusion to be proved* would enable a student to reject those false leads which do not help in the proof.

To examine interesting relationships without focusing on the conclusion may be compared to manipulating the mechanisms of a rifle without keeping an eye on the target. There are times when this is most interesting and rewarding, with mathematics as well as with mechancial gadgets such as a rifle. But when we are target shooting we must focus on the target. If we have only a vague notion of its location, we may fire in the right general direction, but to score a direct hit we must draw a fine bead on the bull's eye.

Pythagoras and Ptolemy must have looked at the conclusion.

A FUTURE IN THE FIELD OF COMPUTATION

Ruth E. Mayer
International Business Machines Corporation

Through the years mathematicians have expressed formulas and equations for practical application but the arithmetic solution of these have often been approximated instead of completed because the amount of time and labor required for calculation was prohibitive. Since the turn of the century and especially since the last war, the field of computing these arithmetic solutions has grown rapidly and, therefore, the number of computers is large and continually increasing. They are employed by private industries, universities, and the national government. The computers work with several basic types of machines. This article will describe briefly these basic types and the requirements for positions using each.

The most well-known and widely used machine is the desk calculator. Most people think of these machines in connection with accounting and bookkeeping departments in offices, but since they do the basic operations of addition, subtraction, multiplication, and division, they are convenient for the computer. Those persons who operate these shall hereafter be referred to as hand computers. The hand computer's position may vary from a single to a complex one depending on where he is employed. He may be given examples like this,

$$(1) \quad 234567 \times 890197$$
$$(2) \quad (1) + 965310$$
$$(3) \quad (2) \div -335791$$

etc., where he has only to follow instructions prepared by someone else (e.g., an engineer, a research mathematician, a physicist), or he may be given a problem where he must form the equation, plan the method of numerical solution, and then carry this solution to completion. He may have to solve differential, polynomial, transcendental, or linear algebraic equations. One hand computer may carry out a single job, or if it is long and involved, more than one may work on the same job.

Most positions of hand computer require a bachelor's degree, while senior members of the staff hold advanced degrees. A major in mathematics is usually preferable, but some of the less complex positions require only a few courses in mathematics. As in other computing fields, courses in engineering, physical sciences, and statistics also are considered good background and often are required for advancement. A position as a hand computer is considered excellent training for positions involving more complicated methods of computation. The operator learns to handle numerical solutions and to check the results. If it is possible to secure such a position for a summer job, it would be rewarding to a math major interested in computing.

The invention of the punched card by Hollerith in 1890 brought about another form of computing, larger in scope than that of the desk calculator. Initial data is punched in the columns of the cards and recorded by holes, and "standard machines will automatically read these holes and perform a wide variety of operations, such as rearranging the cards in any required order, transferring data from one card to another, printing the information on the cards or on a sheet of paper, consulting tables of tabular data, and performing the arithmetical operations of addition, subtraction, multiplication, and division."[1] Pluggable connections permit the operator to direct the operations of the machines with speed and flexibility. Although these machines are widely used commercially, it can be seen how readily adaptable they are to computing. Laboratories where the standard punched card machines are used for computation have been set up by the Federal government, by private industries engaged in research, and by colleges and universities. These laboratories perform the computations either for their own use or for others who do not have their own installations. The Watson Scientific Computing Laboratory at Columbia Univer-

1 Eckert, W. J., "Punched Card Techniques & Their Applications to Scientific Problems". *Journal of Chemical Education*, XXIV, February, 1947.

sity is an important one and the following is a partial list of problems it has completed:[3]

1. *Astronomy*: integration of orbits of planets and asteroids.
2. *Geophysics*: tracing of paths of sound waves under water for various depths and directions.
3. *Optics*: calculations embodying the method of ray tracing.
4. *Chemistry*: computation of quantum mechanical resonance energies of aromatic compounds.
5. *Engineering*: building of Spring & Gear tables and computing stress calculation associated with earthquake loads.
6. *Economics*: estimates of certain coefficients in the equations of economic models, using matrix multiplication and inversion.
7. *Physics*: calculations of calcium transitional probabilities.
8. *Crystallography*: evaluation of a Fourier Transform for the structure of insulin.

The list of problems covered by industry and government would be just about the same.

Positions in these laboratories require a minimum of a bachelor's degree in mathematics or in a physical science with a mathematics minor. Most of these computers take additional courses in pure and applied mathematics regardless of whether or not they hold advanced degrees. As can be seen from the list of topics covered above, a knowledge and background of science are necessary to complete understanding of the problems required to be solved. As in the case of the hand computer, one operator usually carries a problem from its very beginning to its completion. This means securing the numerical method of solution, deciding which available machines can best be used, planning the program of the machines operations, wiring the necessary boards, punching the initial cards, running the machines, compiling the results, and checking and interpreting these

[3] Krawitz, Eleanor, "The Watson Scientific Computing Laboratory, a Center for Scientific Research Using Calculating Machines", *Columbia Engineering Quarterly*, November, 1949.

results. However, if the problem is particularly large or arduous, more than one person may be assigned to it.

The necessity of handling data manually between steps of calculations and the inherent retarded speed limit the ability of desk computers and even business machines to solve more complex problems. These limitations have led to the development of large-scale high-speed computers where the machines, directed beforehand as to what to do with intermediate results, proceed automatically until the final answer is obtained. These machines have improved man's opportunity to calculate arithmetic solutions more accurately because sequences of operations that would have required hundreds of man-years of desk computing or business machine work can now be accomplished in minutes.

There are two forms of these large-scale computers. The analog computers, first placed in service in 1930, are used to solve differential equations. This type of computer consists of apparatus in which some chosen physical quantities such as length, electrical voltage, shaft rotation, etc. are set up to vary in a manner mathematically analogous to the variation of the numbers in the problem under consideration. Variables are represented in terms of voltages and currents, can be added or subtracted by connecting equivalent voltages in series, multiplied or divided by the use of circuits, etc., The analog computer is sufficiently flexible to be adapted to a wide variety of engineering problems. It is suitable only where the usual engineering accuracy is sufficient, the accuracy being limited by the initial data and the inherent mechanical and electrical components of the computer.[3] An analog computer is handled by a staff of engineers, many of whom take advanced work in applied mathematics since an understanding of both engineering and mathematics is necessary to run such a computer.

The other form of large-scale computers is the digital calculator. The first large general-purpose, digital calculator was built by International Business Machines Corpor-

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3 Harder, E. L. & G. D. McCann, "Computer-Mathematical Merlin", *Westinghouse Engineer*, November, 1948.

ation and was presented to Harvard University in 1944. It is known as the IBM Automatic Sequence Controlled Calculator. It was followed in 1946 by the ENIAC (built by the University of Pennsylvania and now located at Aberdeen Proving Ground, Maryland), in 1948 by the IBM Selective Sequence Electronic Calculator in New York, and since then by several more, including the new SEAC at the National Bureau of Standards in Washington, D. C. The digital calculators solve complex problems requiring high accuracy and operate with discreet digits. They contain arithmetic units which add, multiply, divide, take square roots, etc., depending on the particular machine. Their high-speed memory units are made of relays, mercury delay lines, or magnetic drums, wires, or tapes. They have a slower-speed memory consisting of punched cards or tapes. Mathematical tables or functions are stored similarly. Most of these calculators operate in the binary system or a combination of the binary and decimal systems. Therefore, a knowledge of this is helpful.

The computers who operate these calculators usually work in teams and are responsible for a problem from the time that it arrives at the office of the calculator to its completion. The degree of preparation of the problem when it is brought to the calculator staff varies from the mere physical statement of the problem to a complete mathematical analysis of it. The job of the computer is to plan the instructions for the machine he operates, and to do all the necessary background work such as punching cards, tapes, and wiring boards. After these are completed he must operate the problem on the machine and produce the results for the customer in a form useful to him.

The IBM Selective Sequence Electronic Calculator, operated by a staff of twelve mathematicians and seven electronic engineers, is one of the major large-scale digital computing activities in the United States. Its large storage capacity, unusual degree of selectivity, and high speed have enabled it to successfully complete numerical problems of widely different types. A substantial portion of these problems have been of a classified nature, but the nonclassified problems have been in the following fields:

Astronomy

(1) Lunar Ephemeris—computing position of Moon according to basic theory of Professor E. W. Brown. This problem involved the evaluation of a Fourier series of 1,679 terms and required only 2,340 coded arithmetical operations.[4] In computing one position, which required about 7 minutes, 21,000 arithmetical operations were performed (including 1,870 searches of tables of sines and cosines).

(2) Planetary Orbits—computing the precise orbits in rectangular coordinates of the five major planets from 1653 to 2060 in intervals of 40 days as a six-body problem. This problem involved a set of simultaneous non-linear differential equations of the 30th order, which were integrated by a step-by-step process employing 9th differences. The final tables consisted of 1½ million digits, which were less than 1% of the total number of digits computed in the 12,000,000 arithmetical operations.

Optics

(1) Ray Tracing—tracing 586 rays of light, each of two colors, through a complex Sonar type lens system of 10 surfaces with various angles of incidence. Rays leaving the system were automatically rejected according to certain trigonometric criteria. The whole problem required about 10,000,000 arithmetical operations.

(2) Lens Design—expressing all possible aberrations up to and including the 13th order as a triple power series to obtain 1,172 equations of conditions in 112 unknowns. These equations were then reduced by "least squares" method to obtain 112 normal equations. This required about 60,000,000 arithmetical operations.

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[4] For the purposes of this description, an "arithmetical operation" is defined as an operation consisting of (1) the combination of two factors *A*, *B* by the process of addition, subtraction, multiplication or division, to yield a result C; (2) the complete transmissions of *A*, *B* and C; (c) the transmissions and complete execution of the necessary instructions.

Basic Physics

(1) Analysis of the stability of the free laminar boundary layer between parallel streams for an incompressible fluid. The solution was based on a step-by-step integration in a complex plane and required about 5,000,000 arithmetical operations.

(2) Computation of Statistical Fields for Atoms and Ions—solution of the statistical equation for an approximate field (a 2nd order non-linear ordinary differential equation) including the correction term for exchange. Investigation of about 30 of the 92 elements for the neutral, single, double and triple ionized cases required about 20,000,000 arithmetical operations.

Nuclear Physics

(1) Dynamical Analysis of Nuclear Fission—verifying the validity of the liquid drop model for explaining the asymmetrical case of nuclear fission; 1st and 2nd derivatives of the surface curvature were obtained by an eleven-point Lagrangian formula. The acceleration potential was expressed in terms of solid harmonics of degree zero through seven giving an 8×8 system of simultaneous linear equations. Investigation of the symmetrical and asymmetrical fission of Uranium and Cosmium required nearly 20,000,000 arithmetical operations.

Applied Physics

(1) Oil Field Exploitation—solution of the oil flow and oil-gas ratio in an idealized two dimensional case. This was solved as a system of simultaneous second order non-linear partial differential equations and required about 2,000,000 arithmetical operations.

Aeronautical Engineering

(1) Guided Missile (Beam Rider)—computation of 68 seconds of actual flight time. The problem involved a 14th order non-linear ordinary dif-

ferential equation, which was solved as a system of fourteen simultaneous 1st order non-linear differential equations. The solution required nearly 5,000,000 arithmetical operations.

Ballistics

(1) Computation of the reflection and refraction of a shock wave in passing from one medium to another. The formulation of the problem, involving five independent parameters, resulted in a system of complicated algebraic equations equivalent to a 12th order polynomial equation. The solution required about 10,000,000 arithmetical operations.

Naval Architecture

(1) Analysis of coupled torsional and horizontal flexural vibration for design of ship hulls. The solution was effected by the Holtzer recurrence method involving a series of matrix vector multiplications. One "problem" (i.e., class of vessels) required about 1,000,000 arithmetical operations.

This work is comparatively new and fascinating; there is always more to learn. The computers are in close contact with well known personages in the scientific field. Since more and more of these large scale calculators are being built here and abroad, interesting symposiums are held during the year to discuss new advances in the designs and operations of the calculators.

Positions on the staffs of digital computers require bachelor of arts degrees. Most of these degrees are in mathematics, with a few in engineering and the physical sciences. About one third of the computers hold advanced degrees, and most of the rest attend school either for special courses or for credit toward a higher degree. The research mathematicians hold doctors degrees in most cases. The importance of a background in science cannot be overstressed since without it there can be little understanding of the problems brought to the calculator and with it discussion with the researcher is more readily conducted. Courses in applied mathematics, including statistics, form a firm background of knowledge. When the calculator makes an error, it is necessary for the operator to determine what has happened, and therefore initiative, alertness, and diagnostic abilities are necessary for the computer.

It is difficult for the prospective computer to obtain proper training for the position, because few colleges and universities offer much work in numerical methods. Columbia University, Harvard University, the University of Pennsylvania, New York University, the University of Illinois, and Massachusetts Institute of Technology are a few of those that do offer extensive work along these lines. The Computing Laboratory for the ENIAC and the EDVAC at Aberdeen conducts classes for their computers in these methods. The best training would include the following mathematics courses:

Calculus (2½ years)
Differential Equations
Finite Differences
Function Theory
Theory of Equations
Matrix Algebra
Interpolation and Iteration Methods

Prominent computers recommend that more colleges give courses in numerical methods that could be based on the books by Milne, Scarborough, or Whittaker and Robinson.[5]

In closing, it should be emphasized that there is a severe shortage of people trained for the field of computation and that in the next decade or two this condition shall be aggravated as more and more high-speed computers are put in operation. Consequently, this is one of the best fields for intelligent young students to enter.[6]

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[5] Milne, Wm. E., *Numerical Calculcus*, Princeton, N. J., Princeton University Press, 1949.
Scarborough, James, *Numerical Mathematical Analysis*, Baltimore: John Hopkins Press, 1950.
Wittaker, E. T., and G. Robinson, *The Calculus of Observations*, Glasgow: Blackie & Son, Ltd., 1944.

[6] *Author's Note:*
The author would be pleased to send material on the IBM Selective Sequence Electronic Calculator to those who are interested in large-scale digital calculators. This information may be obtained by writing the author in care of International Business Machines Corp., Pure Science Department, 530 Madison Avenue, New York 22, N.Y.

TOPICS FOR CHAPTER PROGRAMS—XI

31. THE ABACUS.

"In Tokyo the army newspaper *The Stars and Stripes* took up the mathematical challenge of Kiyoshi Matsuzoki, an employee of the Ministry of Communications. The paper sponsored a contest between the centuries-old abacus of the Japanese clerk and a modern calculating machine run by Private Thomas N. Wood, of Deering, Missouri. Mr. Matsuzoki triumphed in addition, division, and subtraction. . . . Private Wood and his calculating machine were bettered in everything but the simple multiplication problem." — *Current History*, ns Vol. 13, p. 87 (August, 1947).

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A. Hooper, *Makers of Mathematics*. New York, Random House, 1948.

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C. G. Knott, "The Calculating Machine of the East: the Abacus," in Horsburgh, *Modern Instruments and Methods of Calculation, a Handbook of the Napier Tercentary*. London, Bell, pp. 136-154 (1911).

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V. Sanford, "Counters; Computing If You Can Count to Five," *Mathematics Teacher*, Vol. 43, pp. 368-370 (November, 1950).

D. E. Smith and J. Ginsberg, *Numbers and Numerals.* New York, Bureau of Publications, Teachers College, Columbia University, 1937.

D. E. Smith, *History of Mathematics*, Vol. II. New York, Ginn and Co., 1925.

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H. F. Spitzer, "Abacus in the Teaching of Arithmetic," *Elementary School Journal*, Vol. 42, pp. 448-451 (February, 1942).

S. Myers, "An Improved Abacus," *School Science and Mathematics*, Vol. 7, pp. 601-603 (October, 1907).

F. A. Yeldman, *The Story of Reckoning in the Middle Ages.* London, 1926.

Yen Yi-Yün, "The Chinese Abacus," *Mathematics Teacher*, Vol. 43, pp. 402-404 (December, 1950).

T. S. Young, "Abacus, One of Chinese Wonders," *China Weekly Review*, Vol. 88, p. 302 (February 12, 1938).

32. LA COURBE DU DIABLE

"Ever since the middle of the eighteenth century the equation

$$y^4-x^4-96a^2y^2+100a^2x^2 = 0$$

has been extensively employed to test the ability of students in curve tracing. The origin of the name *courbe du diable* we do not know, but in all probability the curve was summarily christened by some exasperated youth who felt strongly and expressed himself thus forcibly on the subject."

—B. H. Brown

E. Borel, "The Origin of the Name of the Devil's Curve," *American Mathematical Monthly*, Vol. 34, p. 365 (August-September, 1927).

B. H. Brown, "La Courbe du Diable," *American Mathematical Monthly*, Vol. 33, pp. 273-274 (May, 1926).

P. Frost, *Elementary Treatise on Curve Tracing*, 2nd ed. London, Macmillan and Company, 1892. (Cf. pp. 25-26 and Plate II, Fig. 15.)

A. J. Kempner, "The Devil's Curve Again," *American Mathematical Monthly*, Vol. 34, pp. 262-263 (May, 1927).

P. R. Rider, "The Devil's Curve and Abelian Integrals," *American Mathematical Monthly*, Vol. 34, pp. 199-203 (April, 1927).

33. REPEATING DECIMAL FRACTIONS.

Repeating, or circulating, decimal fractions possess many curious if not amazing properties. The study of these properties affords a delightful chapter in arithmetic.

F. E. Andrews, "Revolving Numbers," *Atlantic Monthly*, Vol. 155, pp. 208-211 (February, 1935).

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E. P. Toy, "Note on Circulating Decimals," *Nature*, Vol. 18, p. 541 (September 19, 1878).

M. O. Tripp, "Periodic Decimal Fractions," *School Science and Mathematics*, Vol. 19 pp. 110-113 (February, 1919).

W. F. White, *Scrapbook of Elementary Mathematics*. Chicago, Open Court Publishing Co., 1910.

Washington definition: "A Statistician is a man who draws a mathematical precise line from an unwarranted assumption to a foregone conclusion."

THE PROBLEM CORNER

EDITED BY JUDSON W. FOUST

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solutions should not demand any tools beyond the calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before October 1, 1951. The best solutions submitted by students will be published in the Fall 1951 number of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Dr. Judson Foust, Central Michigan College of Education, Mt. Pleasant, Michigan.

PROBLEMS PROPOSED

36. *Proposed by Norman Anning, University of Michigan, Ann Arbor, Michigan.*

The number t is greater than 1. The sides of a triangle are $2t-1, 2t$ and $2t+1$. Show, without using tables, that the intermediate angle is less than 60°.

37. *Proposed by H. D. Larsen, Albion College, Albion, Michigan.* (From *Journal de Mathematiques Elementaires.*)

If a, b, c are three numbers in arithmetic progression, c, d, e three numbers in harmonic progression, show that a, c, e are in geometric progression.

38. *Proposed by H. D. Larsen, Albion College, Albion, Michigan.* (From *Journal de Mathematiques Elementaires.*)

Given the equation $mx^2-(1+8m)x+4(4m+1) = 0$, determine the two values of m for which the ratio of the roots is equal to $-1/4$.

39. *Proposed by Leo Moser, Texas Technological College, Lubbock, Texas.*

In how many ways can a King go from the left lower corner of a chess board to the right upper corner, if the permissible moves are single steps horizontally to the right, vertically up, and diagonally up to the right?

40. *Proposed by Norman Anning, University of Michigan, Ann Arbor, Michigan.*

What number (or numbers) has its digits reversed when it is multiplied by 9?

UNSOLVED PROBLEMS

33. *Selected from the tenth annual William Lowell Putnam Mathematical Competition, March* 25, 1950.

In each of n houses on a straight street are one or more boys. At what point should all the boys meet so that the sum of the distances they walk is as small as possible?

34. *Proposed by Frank Moseley, State Teachers College, Florence, Alabama.*

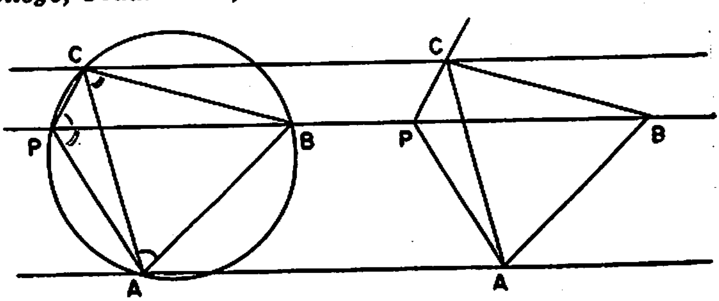
Substantiate the assertion made by Nathan Altshiller-Court in his *College Geometry* (page 66) that a triangle may have equal external bisectors and yet not be isoceles.

SOLUTIONS

31. *Proposed by William Douglas, Courtenay, British Columbia.*

Given any three parallel lines; construct an equilateral triangle with one vertex on each of the three lines.

Solution by Robert P. Robinson, Iowa State Teachers College, Cedar Falls, Iowa.



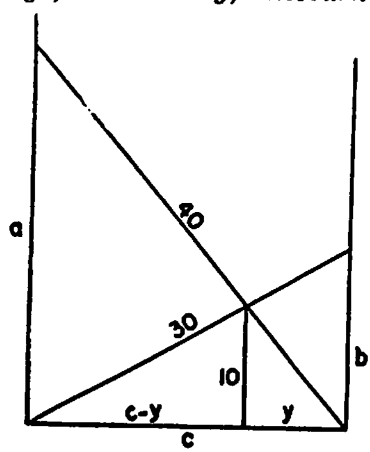
Assume the problem solved as in the figure on the left. Draw the circumcircle of triangle ABC meeting the middle parallel at B and P. Draw AP and CP. Then

<CPB = <CAB and <APB = <ACB. (Inscribed angles measured by the same arc.) Since angles *CAB* and *ACB* are each 60°, it follows that angles *CPB* and *ACB* are 60° also. In the figure at the right, choose any point *P* in the middle parallel. Construct 60° angles at *P* on each side of the middle parallel with sides meeting the other parallels at *C* and *A*. Draw *CA*. With *A* as a center and *AC* as a radius, strike an arc meeting the middle parallel at *B*. Then *ACB* is the required triangle.

32. *Proposed by the Problem Corner Editor.*

Two ladders lean in opposite directions across an alleyway between vertical buildings. The foot of each ladder is at the intersection of a building and the ground. If the ladders are thirty feet and forty feet long respectively and cross at a point ten feet above the ground, how wide is the alley?

Solution by S. T. Vaughn, Central Missouri State College, Warensburg, Missouri.



From the figure, $10/y = a/c$, $10(c-y) = b/c$. Eliminating y we get $b = 10a/(a-10)$. Again, $a^2+c^2 = 40^2$ and $c^2+b^2 = 30^2$. Eliminating c we get $a^2-b^2 = 700$. Substituting for b and simplifying, there results

$$a^4-20a^3-700a^2+14000a-70000 = 0.$$

Solving, we find, approximately, $a = 30.36$ and $c = 26.04$.

Also solved by Sharon Murnick, Hofstra College, Hempstead, L.I. Murnick called attention to a solution for the general case given in the January 1945 number of the *National Mathematics Magazine*, Vol. 19, pp. 260-207. Further reference is given to the *American Mathematical Monthly*, Vol. 48, p. 268 (April, 1941).

35. *Proposed by Dr. Alfred Moessner, Gunzenhausen, Germany-Bayern.*

Can you submit a mathematical problem without numbers, which, however, leads to a solution with definite numbers?

Solution by Harvey Fiala, High School Student, Forman, North Dakota.

Bill who has more pennies than Bob uses the number of coins he has as a factor the number of times that Bob has coins, and Bob uses the number of coins he has as a factor the number of times that Bill has coins. The results are equal. Furthermore, the number of coins Bill has is the square of the number of coins Bob has. How many coins has each?

Here $x^y = y^x$ and $y \exp 2y = y \exp y^2$. So $2y = y^2$ and $y = 2, x = 4$.

Solution by C. Stanley Ogilvy, Columbia University, New York City.

This solution was suggested by Problem E-776, *American Mathematical Monthly*, Vol. 54, p. 339 (June-July, 1947). The solution requires knowledge of the fact that New York City house numbers do not run to five digits.

"My children's parents, grandparents, and great-grand-parents are all still living," said Mr. Jones to his friend Mr. Smith, a visitor at the Jones' New York home.

"How many children have you?" asked Mr. Smith.

"More than the number of parents they have, but fewer than their great grand-parents. Also, the number of digits in our house number is a divisor of the number of children."

"Well, how many digits are there in your house number?"

"If I told you, you would then have enough information to deduce the number of children."

Mr. Smith thought about this for a moment and then he said, "I have enough information already," whereupon he correctly stated the number of children. Can you?

Solution: If the number of children is x, then $3 \leqq x \leqq 7$. The number of digits in the house number cannot be 1, 2, or 3 because knowledge of the number must uniquely determine x. But New York City house numbers do not run to five digits. Therefore the number of children is 4.

Note on Problem 29. Fenton R. Isaacson, Drake University, Des Moines, Iowa sent in an interesting variation to the solution of this problem published in the Fall 1950 number. His approach is to change the annuity into a regular one with no terms missing by discounting the 9 monthly payments and then spreading them back over the twelve months with the function Wa_9/a_{12} (at $\frac{1}{4}\%$). This then leaves a regular annuity of 48 terms to handle.

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THE MATHEMATICAL SCRAPBOOK

It is the perennial youthfulness of mathematics itself which marks it off with a disconcerting immortality from the Sciences.

—E. T. BELL

$= \nabla =$

Some more "Freshman mathematics":

1. "An ellipse is a comic section."
2. "Arctan x = arcsin x/arccos x."

$= \nabla =$

The following cryptarithm has a unique solution. The x's represent digits not necessarily equal.

$$
\begin{array}{r}
xxx)\,xx3xx\,(xx \\
\underline{x3x} \\
xxxx \\
\underline{xx3x}
\end{array}
$$

—MATH. GAZETTE

$= \nabla =$

Fermat himself published nothing, though he deserves a prominent place among those of the founders of analytic geometry, infinitesimal calculus, and the theory of probability.

$= \nabla =$

$\pi = 3.1415926535897932384626433832 79$

Now, O hero, a great advancing in method
Which you would proclaim wonderful, worketh universal;
Yet in our memories your labour is rooted;
Unto the end should'st you be amongst immortals.

—R. D. CARMICHAEL

$= \nabla =$

"Music is the pleasure the human soul experiences from counting without being aware that it is counting."

—LEIBNITZ

"Architecture is geometry made visible in the same sense that music is number made audible."

—CLAUDE BRAGDON

QUARTER SQUARES

The multiplication of two numbers may be reduced to simple operations of addition and subtraction by means of *quarter squares.* The theory is based on the identity

$$ab = \tfrac{1}{4}(a+b)^2 - \tfrac{1}{4}(a-b)^2.$$

To apply this identity it is convenient to have available a table of $\frac{1}{4}n^2$ for integral values of n. If n is even, $\frac{1}{4}n^2$ is a whole number; if n is odd, $\frac{1}{4}n^2 = (\textit{integer}) + \frac{1}{4}$. Now for integral values of a and b, either $a+b$ and $a-b$ are both odd or they are both even; if they are both odd, the fraction $\frac{1}{4}$ occurs in both $\frac{1}{4}(a+b)^2$ and $\frac{1}{4}(a-b)^2$ and disappears from their difference. Hence, we may safely ignore the fraction in constructing the table. We shall denote the tabulated values by $Q(n)$. Since $Q(2k) - Q(2k-1) = k^2-(k^2-k) = k$ and $Q(2k+1)-Q(2k) = (k^2+k)-(k^2) = k$, the table may be formed in a simple manner by adding successively the numbers 1, 1, 2, 2, 3, 3, etc. The table below listing the quarter squares from $n = 0$ to $n = 199$ permits the multiplication of all numbers of two significant figures.

Example 1. $(950)\times(890) = (95)\times(89)\times10^2$. Let $a = 95$, $b = 89$. Then $a+b = 184$, $a-b = 6$. From the table, $Q(184) = 8{,}464$ and $Q(6) = 9$, whence $(950)\times(890) = (8{,}464-9)\times10^2 = 845{,}500$.

Example 2. $(2.8)\times(0.47) = (28)\times(47)\times10^{-3}$. Let $a = 47$, $b = 28$. Then $a+b = 75$, $a-b = 19$. From the table, $Q(75) = 1{,}406$ and $Q(19) = 90$. Thus, $(2.8)\times(0.47) = (1{,}406-90)\times10^{-3} = 1.316$.

The first practical application of the method of quarter squares was made in 1817 by Antoine Voisin who published a table of $Q(n)$ for all integers from 1 to 20,000. It is interesting to note that Voisin referred to his quarter squares as "logarithms." A more extensive table was published by Joseph Blater in 1888. Blater's "Table of Quarter Squares of all Whole Numbers from 1 to 200,000" permits the multiplication of all numbers of five significant figures.

SHORT TABLE OF $Q(n)$

	0	1	2	3	4	5	6	7	8	9
0	0	0	1	2	4	6	9	12	16	20
1	25	30	36	42	49	56	64	72	81	90
2	100	110	121	132	144	156	169	182	196	210
3	225	240	256	272	289	306	324	342	361	380
4	400	420	441	462	484	506	529	552	576	600
5	625	650	676	702	729	756	784	812	841	870
6	900	930	961	992	1024	1056	1089	1122	1156	1190
7	1225	1260	1296	1332	1369	1406	1444	1482	1521	1560
8	1600	1640	1681	1722	1764	1806	1849	1892	1936	1980
9	2025	2070	2116	2162	2209	2256	2304	2352	2401	2450
10	2500	2550	2601	2652	2704	2756	2809	2862	2916	2970
11	3025	3080	3136	3192	3249	3306	3364	3422	3481	3540
12	3600	3660	3721	3782	3844	3906	3969	4032	4096	4160
13	4225	4290	4356	4422	4489	4556	4624	4692	4761	4830
14	4900	4970	5041	5112	5184	5256	5329	5402	5476	5550
15	5625	5700	5776	5852	5929	6006	6084	6162	6241	6320
16	6400	6480	6561	6642	6724	6806	6889	6972	7056	7140
17	7225	7310	7396	7482	7569	7656	7744	7832	7921	8010
18	8100	8190	8281	8372	8464	8556	8649	8742	8836	8930
19	9025	9120	9216	9312	9409	9506	9604	9702	9801	9900

= ∇ =

"One of Euler's papers contains the formula

$$\ldots + 1/x^2 + 1/x + 1 + x + x^2 + \ldots = 0.$$

This is a series infinite in both directions. The 'proof' consists of combining the formulae

$$x + x^2 + \ldots = x/(1-x)$$

and

$$1 + 1/x + 1/x^2 + \ldots = x/(x-1).\text{"}$$

—E. C. TICHMARSCH

= ∇ =

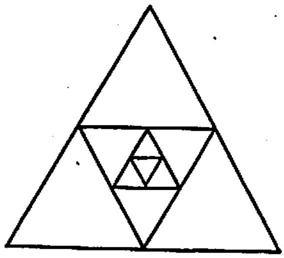
A horse is tied to a stake at the edge of a circular pond. The radius of the pond is 200 ft. and has no fence around it. How long must the rope be so that the horse may graze over one acre? (Ans. 154+ ft.)

= ∇ =

In addition to computing π to 707 places, Shanks also obtained the values of e, M, and the natural logarithms of 2, 3, 5, and 10 to 205 places.

= ∇ =

Why did Farmer Jones build his pig pen 5 feet long, 10 feet wide, and 6 feet high? (Ans. To put his pigs in.)



Here is a figure illustrating geometric progressions. For the successive equilateral triangles, the ratio of the sides is 2 and the ratio of the areas is 4.

$= \nabla =$

$1/\pi = 1/3 - 1/100 - 1/200$, approximately.

$= \nabla =$

A Quaker once, we understand,
For his three sons laid off his land,
And made three equal circles meet
So as to bound an acre neat.
Now in the center of the acres
Was found the dwelling of the Quaker;
In centers of the circles round,
A dwelling for each son was found.
Now can you tell by skill or art
How many rods they live apart?

—AM. MATH. MO. (January, 1900).

The centers of the circles three
With straight lines let united be.

. . . .

The distance, in rods, will two decimals run
In one-eighth of two hundred ninety-one.
Now we've told by skill and rhyming art
The number of rods they live apare.

—AM. MATH. MO. (April, 1900).

$= \nabla =$

One of the last publications of Charles L. Dodgson (Lewis Carroll) was a note in the Oct. 14, 1897 issue of *Nature* entitled, "Brief method of dividing a given number by 9 or 11." Dodgson considered the short methods he proposed to be more than mere curiosities for he writes, "[These two new rules] effect such a saving of time and trouble that I think they ought to be regularly taught in schools." The rules presuppose a knowledge of casting out nines and elevens, a topic which is sadly neglected in our modern schools.

Rule for dividing by 9:

1) Find the remainder by casting out nines and place it *over* the unit digit.

2) Subtract, placing the difference *over* the next digit, and continue the subtraction.

3) Mark off the remainder at the right-hand end; the other digits form the quotient.

Example.
$$\begin{array}{r} 6\ 5\ 2\ 1\ 2(8 \\ \hline 9)5\ 8\ 6\ 9\ 1\ 6 \end{array} \qquad R = 8, \quad Q = 65{,}212$$

Explanation: Cast out nines from 586,916 and obtain $R = 8$. Set this number over the right-hand digit of the dividend. Then, subtracting, $8-6 = 2$; set 2 over the next digit of the divident; etc.

Rule for dividing by 11:

1) Find the remainder by casting out elevens and place it *under* the unit digit.

2) Subtract, placing the remainder *under* the next digit, and continue the subtraction.

3) Mark off the remainder at the right-hand end; the other digits form the quotient.

Example.
$$\begin{array}{r} 11)5\ 8\ 6\ 9\ 1\ 6 \\ \hline 5\ 3\ 3\ 5\ 6(0 \end{array} \qquad R = 0, \quad Q = 53{,}356$$

Dodgson points out that these methods not only effect a saving of time and trouble but also provide an automatic test of the correctness of the computation: the last step in the subtraction necessarily gives a remainder 0. He shows further that the rules can be extended to division by $a{\cdot}10^n \pm b$. As often happens in mathematics, Dodgson was not the first to discover these short methods. In fact, the rules for dividing by 9 and 11 were given by Adolph Steen in a book published in Copenhagen in 1847.

$= \nabla =$

Herr Valentin, of Berlin, who has been working on a general mathematical bibliography for more than twenty years, estimates that the total number of different mathematical works is about 35,000 and that about 95,000 mathematical articles have appeared in the various periodicals.

—G. A. MILLER (Oct., 1908).

= ∇ =

The following mode of multiplying is an extension of the Russian Peasant Method.

39	35
117	12
351	4
1053	1

$39 \times 35 = (1053+351) - 39 = 1365.$

In column 1, multiply by 3; in column 2, divide by 3, ignoring remainders. The product is the sum of all numbers in column 1 opposite numbers of the form $3n+1$ in column 2 diminished by the sum of all numbers in column 1 opposite numbers of the form $3n-1$.

= ∇ =

"Gentlemen, that is surely true, it is absolutely paradoxical, we cannot understand it, and we don't know what it means, but we have proved it, and therefore we know it must be the truth." —BENJAMIN PEIRCE (after establishing the relation $e \exp \pi/2 = i \exp 1/i$.)

= ∇ =

From *Arithmetic in Nine Sections* (date unknown; maybe as early as 213 B.C.): "A square city of unknown side is crossed by a street which joins the centers of the north and south sides; at a distance of 20 paces north of the north gate is a tree which is visible from a point reached by going 14 paces south of the south gate and then 1775 paces west. What is the length of each side?"

THE BOOK SHELF

EDITED BY CARL V. FRONABARGER
Southwest Missouri State College

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Carl V. Fronabarger, Southwest Missouri State College, Springfield, Missouri.

Makers of Mathematics.

By Alfred Hooper. Random House (457 Madison Avenue, New York 22, New York), 1948. 9 + 402 pages. $3.75.

This is a history of mathematics which is quite different from other books on the same subject. Most histories of mathematics assume that the reader is an accomplished mathematician. Hooper assumes that the reader knows only how to add, subtract, multiply, and divide integers. In the spirit of *A Mathematics Refresher,* he traces the history of mathematics from prehistoric times to the time of Gauss. Not only does he give the history of mathematics, but he goes to great pains to explain the meaning of mathematics to the reader. In his effort to be clear to the non-mathematical reader, he calls vertices "corners," elements of a cone "edges," and progressions "series." While some mathematicians might be irritated by such things, the prospective teacher of mathematics needs to learn to explain mathematics as simply as possible and can profit greatly from seeing Hooper's methods. No other history of mathematics gives as many etymological meanings as does this book. Professors who are struggling to teach and students who are struggling to learn the meaning of increments and differentials can profit by the author's discussion of the efforts of Wallis to understand these matters.

Most histories of mathematics present mathematical facts as isolated from history in general, but Hooper does not make this mistake. The book lacks the footnotes which add scholarship and tedium to most histories of mathematics, but no book combines better the history and teaching of mathematics. Although the book is written simply, it discusses important modern concepts, as when Hooper says that number theory merges into philosophy in the same way that Archimedes' polygons would blend into a circle if the number of the sides were increased indefinitely.

—R. H. MOORMAN

Mathematics for the General Reader.

By E. C. Titchmarsh. Longmans, Green and Co., Inc. (55 Fifth Avenue, New York, New York), 1948. 4 + 156 pages. $2.00.

The author has been unusually successful, within the scope of 156 pages, in giving elementary introductions to various branches of mathematics all under the general heading of analysis, including the development of the number system, trigonometry, and the calculus. The reader's interest is maintained by rather frequent references to the history of mathematics, notable examples of which are Minoan arithmetic, page 45, and the three unsolved problems of antiquity, pages 90-93.

One characteristic of the author's writing is his apt use of similes to illustrate his ideas. In his discussion of a one dimensional world he writes, "The inhabitants would be situated like beads on a wire ...... The behavior of one's next door neighbors would be even more important than it is in ordinary life." Again, "One can think of the $f()$ as a machine into which the value of x is to be fed, and from which will then emerge the corresponding value of y." Another characteristic of his writing is his homely way of expressing himself, illustrated on one instance by: "This curious symbol, $\surd$, was once an 'r', but it has become worn down by constant use."

The final chapter entitled "Aftermath," which title may be taken facetiously or seriously, states that it is the first of a series of volumes to appear in the future among which will be one on algebra and one on geometry.

Not only will this book do much to give the serious general reader an understanding of the nature and objectives of mathematics, but college students will find it interesting and illuminating and their teachers will profit by observing how skillfully the author introduces the reader to a multitude of concepts all of which are made to appear reasonable and acceptable. It should do much to cultivate a taste for and an appreciation of mathematics, as well as whet the appetite for the remaining books of the series.

—FLOYD G. HARPER

Elementary Concepts of Mathematics.

By Burton W. Jones. The Macmillan Company (60 Fifth Avenue, New York 11, New York), 1947. 13 + 294 pages. $4.25.

In the preface to this very interesting textbook, Professor Jones states that this book was written as a result of a realization that there existed a need for a course designed for students who have had a minimum of mathematical training, who did not plan to take further courses in the field, but who wanted firmer grounding in what useful mathematics they had studied and such additional training which they, as nonmathematicians, might find useful in later life. With the needs of these students in mind, it is further stated that the aims of the book might be summed up in six statements: first, to cultivate an understanding of the material; second, to clarify certain mathematical concepts encountered in everyday life; third, to cultivate an appreciation of mathematics rather than to engender an awe of same; fourth, to emphasize the logical development of mathematics; fifth, to bring about a realization that pencil and paper are as important in developing an understanding of the subject as is the laboratory to an understanding of science; sixth, to provide much useful material for the prospective teacher of secondary school mathematics. Professor Jones further explains that the book is the cumulative effort of several of the members of the department at Cornell and the published edition is his thorough revision of the lithoprinted edition used at the university.

The nine chapters which comprise the book include such topics as Logic; The Positive Integers and Zero; Negative Integers, Rational and Irrational Numbers; Algebra; Graphs and Averages; Permutations, Combinations, and Probability; Mirror Geometry; Lorentz Geometry; and Topology.

It is the opinion of the reviewer that the purposes of the book have been achieved, and other accomplishments have been brought about as well. Modesty must have prevented Professor Jones from commenting on the entertaining manner of presentation. While pencil and paper are certainly necessary for a thorough reading of the book, the first reading is both easy and difficult. Easy because of the previously mentioned manner of writing, and difficult because of the temptation to grasp pencil and paper and investigate the stimulating exercises. One definite appeal that impressed the reviewer was the wealth of material for interesting and "different" programs for mathematics clubs. Thus, in addition to being a textbook, this volume offers the reader much material for mathematical recreation.

—L. T. SHIFLETT.

A Manual for the Slide Rule.

By Paul E. Machovina. McGraw-Hill Book Company, Inc. (330 West 42nd Street, New York 18, New York), 1950. 78 pages. $.75.

Anyone who has taught the use of the slide rule will welcome the opportunity to look over the excellent manual by Paul E. Machovina. This manual may be used to advantage both by students attending formal classes and by students learning on their own. An appropriate short history of the slide rule from the use of a pair of dividers with "Gunter's line" to the modern slide rule makes an interesting introduction. Different modern types of slide rules are described along with their uses and construction. For those not acquainted with the theory of logarithms, a brief explanation of this theory as it underlies the slide rule is given.

Students using this manual should find it easy to understand the basic ideas underlying slide-rule operations. For example, in the explanation of simple multiplication using the C and D scales, it could hardly escape the student that the slide rule is simply a most efficient device for adding logarithms. Similar explanations which are given throughout the manual should help the student to become an intelligent and efficient user of the slide rule. Of course, the author gives the student the excellent advice in the Preface that "Proficiency in operating the slide rule, like typewriting or playing a musical instrument, is gained and maintained only by practice and continued use."

The explanations given should be readily understood by the serious student. The manual covers the usual ground such as multiplying and dividing and related topics, the use of the trigonometric scales, the log, and the log log scales.

The material in the manual may be covered adequately in a short course. In addition there are six detachable problem sheets—twelve pages in all. More problems would be needed for student drill work, but the problems given cover a wide range. They can be used for tests or drill work, and they will suggest to the instructor other problem possibilities.

—LESTER V. WHITNEY

The advance and perfecting of mathematics are closely joined to the prosperity of the nation.

—NAPOLEON.

INSTALLATION OF NEW CHAPTER

EDITED BY LAURA GREENE

The PENTAGON is pleased to report the installation of another chapter of Kappa Mu Epsilon. There are now forty-five chapters on the roll.

NORTH CAROLINA ALPHA

Wake Forest College, Wake Forest

Kappa Mu Epsilon welcomes North Carolina Alpha, the first chapter from North Carolina. Twenty-five students and six members of the faculty were initiated January 12, 1951, in the Little Chapel of the Music-Religion Building on the Campus of Wake Forest College. The installation ceremony was conducted by Dr. E. R. Sleight, Past-President of Kappa Mu Epsilon.

Following the installation, the charter members and their guests attended a banquet in Raleigh, North Carolina. At that time Dr. Sleight spoke on the history of Kappa Mu Epsilon, and Harry T. Wright, Jr., president of North Carolina Alpha, reviewed the history of the Wake Forest Mathematics Club.

The following officers of North Carolina Alpha were installed: President, Harry T. Wright, Jr.; Vice-President, Bill Alexander; Secretary, Dorothy Hilburn; Treasurer, Conrad Warlick; Corresponding Secretary, Professor J. N. Bond; Faculty Sponsor, Professor R. L. Gay.

Other charter members of North Carolina Alpha are Daisy Jacquelin Beard, Francis Earl Beaudry, Jr., Loraine Bennett, J. G. Carroll, George P. Edwards, Avis Anne Elliott, Ivey C. Gentry, Walter Thomas Hall, Jr., David F. Herring, Julia Ann Higdon, David S. Humphries, Hubert A. Jones, Stan J. Najeway, LeRoy B. Martin, Jr., Mrs. Margaret E. Parker, Janice A. Parsley, John W. Person, Freddy Poston, Lee Rhodes, K. T. Raynor, Jean Scholar, Virginia Smith, Carolyn M. VonCannon, and William Young.

KAPPA MU EPSILON NEWS

EDITED BY CLEON C. RICHTMEYER, *Historian*

A panel discussion on jobs for mathematics graduates was held by **California Alpha.** Participants included three alumni, Eugenia Houg, Ruth Engvall, and Wayne Smith, now employed by Rand Corporation, and A. H. Schluefer and David Livingstone from the Naval Ordnance Training Station. The chapter members also made a field trip to California Institute of Technology to see the analog computer.

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Illinois Beta reports the loss of six mathematics majors to the Air Forces via enlistment.

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At the January initiation of **Iowa Alpha** seven new members were initiated. Wander Ponder, one of the initiates, presented a paper on Trisection of the Angle.

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Early in the Fall semester **Kansas Gamma** held a big sister-little sister party for the pledges. In December the chapter held its traditional Wassail Bowl Festivity. Last Spring, Kansas Gamma sent two delegates to Chicago to the NCTM convention. At the KME section of the convention Miss Frances Donlon read a paper on *Geometrical Constructions with Straight Edge and Compass,* and Miss Jeanne Culivan presented a paper on *The Mathematical Method in the Light of a Philosophical System.* The tenth anniversary of the founding of Kansas Gamma was celebrated by a Founders Day dinner. The Hypatian Award was granted to Jeanne Culivan and Frances Donlon. Anne Robben won the Underclassmen Scholarship Award.

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Missouri Alpha is devoting most of its time this year to preparations for the National Convention of which it is the host chapter.

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An open house meeting, to which all students were invited, was held by **Missouri Beta.** The chapter has purchased a $5.00 Tuberculosis Seal Bond. Among the fall initiates was a pre-engineering student, Kyriakos Lypirides, whose home is in Panorama, Salonica, Greece.

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Instead of the usual Christmas party, **New Jersey Alpha** used the funds for gifts to underprivileged children.

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New York Alpha is making big plans for its initiation banquet on March 2. The speaker will be Preston R. Bassett, President of Sperry Gyroscope Company, who will talk on *The Inventor Discovers Mathematics*. The chapter plans to initiate Mr. Bassett as an honorary member. Other guests for the evening will be many of the leaders of industry from Long Island. All alumni of New York Alpha and members of other KME chapters in the vicinity have been invited also.

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Past President E. R. Sleight reports a most enjoyable and inspiring ceremony at the installation of **North Carolina Alpha.**

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The meetings of **Ohio Gamma** are held in the Burrell Memorial Observatory, of which Professor Paul Annear is the director.

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As a part of their pre-initiation procedure, pledges of **Oklahoma Alpha** were given problems in Analytic Geometry and Calculus. Don Swanson, a former member of the chapter, is now in the U. S. Air Force at Warren Base in Wyoming. Mr. Swanson writes that he was afforded a great deal of pleasure by working some of the problems in the last issue of the PENTAGON.

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Dr. R. O. Hutchinson, one of the charter members of **Tennessee Alpha** and head of the Department of Mathematics at Tennessee Polytechnic Institute, died suddenly on October 22, 1950.

PROGRAM TOPICS, SPRING SEMESTER, 1950-51

Alabama Alpha, Athens College
History of Analytic Geometry, by M. F. Scoggin
Development of Modern Geometry, by F. L. Barksdale
Converse of Miquel's Theorem and Applications, by T. J. Carter
Problems in Analytic Geometry Checked by Trigonometric Formulae, by James Parks

California Alpha, Pomona College
The Four-Color Problem, by Professor Elmer Tolsted
Geometric Constructions, by Walter Rosenorv
Series and Sums, by Professor Hugh Hamilton

Colorado Alpha, Colorado A & M College
Geometrical Constructions, by Hans Stetter
Paper Folding, by Don Tucker
Mathematical Puzzles and Paradoxes, by Don Allen

Illinois Beta, Eastern Illinois State College
Interesting Problems, by Cora Coombes and George Swenford
Probabilities in Games of Chance, by L. A. Ringenberg
Group Theory Fundamentals, by L. R. VanDeventer

Illinois Gamma, Chicago Teachers College
Computational Methods, by Mr. Marvin Burack
Mathematics in Cartography, by Mr. J. M. Sachs

Indiana Alpha, Manchester College
Some Famous Unsolved Problems in Mathematics, by Prof. H. D. Larsen
Mathematical Induction as a Means of Proof, by Prof. J. E. Dotterer
A Discussion of Available Material in the Library, by Prof. J. E. Dotterer
Mathematics in European Universities, Especially the University of Latvia, by Prof. Earnest Abele
The Application of Mathematics in Economics, by Dr. Earl S. Garver

Iowa Alpha, Iowa State Teachers College
The Number System, by Jack Wilson
Transfinite Cardinal Numbers, by Mr. Lott

Iowa Beta, Drake University
Number Systems, by Waid Davidson
Great Men of Mathematics, by Bruce Workman

Kansas Alpha, State Teachers College, Pittsburg
History of Calendar Forms, by Joe Butler
Mathematical Recreations, by Tom Clark
Magic Squares, by I. G. Wilson

Kansas Beta, State Teachers College, Emporia
The 200-inch Telescope, by Brooks Becker
Short Cuts in Arithmetic, by George Crumley
Mechanical Brains, by Richard Shur
Electronic Computers, by Robert Klotz
Similar Triangles, by Elva Libben
History of Negative Numbers, by Richard McAlister
History and Development of KME, by Dr. O. J. Peterson

Galaxies and Nebulae, by Raymond Eccles
Structure of the Universe, by Billy Burgert
Famous Calculus Problems, by Don Allison
Probability, by Dale Smith

Kansas Gamma, Mount St. Scholastica College
Mathematics in the Orient, by Frances Donlon and Jeanne Culivan
Greek Mathematics, by Ruth Link and Theresita Breitenbach
Hindu Arabic and Persian Developments, by Ann Robben and Jill Sullivan
Important Mathematical Figures of the Middle Ages, by Margaret Acree
Mathematical High-Lights in Renaissance Period, by Elaine Barnes

(The following topics were presented in the Spring Semester of 1950 but not reported in the Fall issue of the Pentagon)
Korean Mathematics, by Peter Kim
Geometric Construction with Straight Edge and Compass, by Frances Donlon
Frequency Distributions, by Frances Walsh
Various Number Systems, by Victoria Fritton
Our Mathematical Heritage, by Margot Acree, Terry Breitenbach, and Dorothy Ripley

Michigan Alpha, Albion College
Arithmetic Revisited, by Dr. H. D. Larsen
Wolfgang and Johann Bolyai, by Vinod Doshi
Gottfried Wilhelm Leibniz, by Robert Hooper
Neils Hendrick Abel, by James Young
Linkworks, by Richard Burrows
A Day of Practice Teaching, by Patricia Collins

Michigan Beta, Central Michigan College
Probability Theory and Games of Chance, by Lloyd Trinklein

Michigan Gamma, Wayne University
Four Color Map Problem, by James Barry
Boolean Algebra, by Bert Eisenstat
Unique Factorization, by Dr. Harvey Cohn

Mississippi Gamma, Mississippi Southern College
The Determinant, by Mary Frances Sasser and Patsy Munn
Hyperbolic Functions, by William Gay
Curve Fitting, by Professor Edward Thomas

Missouri Alpha, Southwest Missouri State College
Mathematical Recreations, by Joe and Bob Wommack

Missouri Beta, Central Missouri State College
Proofs for $\lim (\sin x/x) = 1$, by Donald Jones
Congruences, by Leonard Molotsky
The Tower of Hanoi; Integral Values from Four Fours, by Loren W. Akers
Trigonometric Functions Expressed in Terms of Exponential Functions, by Wendell McGuire
Numbers That Mean Too Much, by Margaret Honn
Pythagorean Number Triples, by Kyriakos Lypirides
History and Methods of Computing Pi, by Martin Tempel

Missouri Epsilon, Central College
A Postulational Approach to the Rational Integers, by Dr. Floyd Helton
Boolean Algebra, by John Blattner
Non-Euclidean Geometry, by Paul Calvert
Kinematics, by Dana Chenoweth
Diophantine Equations, by Merle Cartwright
Topology, by David Morrison
History of Calculus, by George Koonce

New Jersey Alpha, Upsala College
Theory of Least Squares, by M. Netzler
The Mathematics of Astronomy, by W. Stachel

New Jersey Beta, New Jersey State Teachers College
The College and Mathematics, by Professor Mallory
The International Congress of Mathematicians, by Prof. Humphrey
Use of Visual Aids in Teaching High School Mathematics, by Prof. Kays

New York Alpha, Hofstra College
International Mathematics Congress, by Dr. Stabler
Probabilities, by Peter Hinrichs
The Wineglass Problem, by Mr. Hawthorne
Continued Fractions, by Peter Marshall
The Normal Law of Error, by Dr. Mildred Dean
The Trisection Problem, by Richard Jaeger
The LaPlace Transform, by Sam Reynolds
Paradox, by Mr. W. L. Marshall
Selective Sequence Electronic Calculator, by Ruth Mayer
Mathematics and the Field of Teaching, by Gertrude Decker

Ohio Alpha, Bowling Green State University
Harvard Congress of Mathematicians, by Dr. Ogg
Nomography, by Dr. Atkins
Calculating Machines, by Beverly Feiner and Marion Goodnight

Ohio Beta, College of Wooster
Napier and His Logarithms, by Dorothy Renzema
Mathematics in Physical Chemistry, Prof. W. F. Kieffer

Ohio Gamma, Baldwin-Wallace College
Engineering Education in England, by Prof. Phillip Clyne
The Mathematics of Democracy, by Prof. John Wilson
Some Applications of the Laws of Chance, by Mr. Jack Hofferkamp

Oklahoma Alpha, Northeastern State College
Extracting Roots by Arithmetic, by R. Carpenter
Atomic Structure, Lake Murray, by R. Zenor
De Moivre's Theorem, by Monte York
Trisection of an Angle, by Buran Woods
Annuities and Compound Interest, by Mike Reagan
Binary and Other Number Scales, by Willa Dean Place
Ratio and Proportion, by Ed Whinery
Criteria for Divisibility, by Roy Coombes
Mathematical Induction, by James Reeves

Lim $(\sin x/x) = 1$, by James Clingenpool
History, Demonstration and Discussion of Puzzles, by Ray Carpenter

Pennsylvania Alpha, Westminster College
Topography, by John Hodges
Mathematics in Building Musical Chords, by Wm. Dembaugh

South Carolina Alpha, Coker College
Highlights in the Development of Modern Mathematics and How These Developments have Influenced Physics, Chemistry, Philosophy and Other Fields, by Frank Saunders

Tennessee Alpha, Tennessee Polytechnic Institute
Report on the International Congress of Mathematicians, by R. H. Moorman
Relaxation Methods of Solving Differential Equations, by Ray Kinslow

Texas Alpha, Texas Technological College
New Methods of Puzzle Construction, by Dr. L. Moser
Lewis Carroll, by Marvin Kitten and Cynthia Armbruster
The Dual Nature of Waves and Particles, by Dr. McKinney

Wisconsin Alpha, Mount Mary College
Work of an Actuary — Requisites for Becoming an Actuary, by Mary Hunt
Women in Actuarial Work, by Mr. V. Henningston
Introduction to Topology, by Ann Sanifelippo
Special Theorems and Ideas of Topology, by Janet Haig
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"We do not listen with the best regards to the verses of the man who is only a poet, nor to his problems if only an algebraist; but if a man is at once acquainted with the geometric foundation of things and with their festal splendor, his poetry is exact, and his arithmetic musical."—Emerson

THE PENTAGON

A MATHEMATICS MAGAZINE
FOR STUDENTS

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Kansas Delta, Washburn Municipal University, Topeka. VI, 70, S47.
Mississippi Gamma, Mississippi Southern College, Hattiesburg. IX, 46-47, F49.
Missouri Gamma, William Jewell College, Liberty, VII, 44-45, F47.
Missouri Delta, University of Kansas City, Kansas City. VIII, 41, F48.
Missouri Epsilon, Central College, Fayette. IX, 46, F49.
North Carolina Alpha, Wake Forest College, Wake Forest. X, 112, S51.
Ohio Gamma, Baldwin-Wallace College, Berea. VII, 47, F47.
Pennsylvania Alpha, Westminster College, New Wilmington. X, 55-56, F50.
Texas Gamma, Texas State College for Women, Denton. VII, 45, F47.
Texas Delta, Texas Christian University, Fort Worth. VII, 46-47, F47.
Wisconsin Alpha, Mount Mary College, Milwaukee. VII, 45-46, F47.

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(Names of authors are in ordinary type; names of reviewers are in capitals.)

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