THE PENTAGON

Volume X **FALL, 1951** **Number 1**

CONTENTS

WHO'S WHO IN KAPPA MU EPSILON

HENRY VAN ENGEN.........................*President*
Iowa State Teachers College, Cedar Falls, Iowa

HAROLD D. LARSEN.......................*Vice-President*
Albion College, Albion, Michigan

E. MARIE HOVE.............................*Secretary*
Hofstra College, Hempstead, L. I., New York

LOYAL F. OLLMANN.........................*Treasurer*
Hofstra College, Hempstead, L.I., New York

C. C. RICHTMEYER...........................*Historian*
Central Michigan College of Education,
Mt. Pleasant, Michigan

E. R. SLEIGHT..........................*Past President*
University of Richmond, Richmond, Virginia

HAROLD D. LARSEN....................*Pentagon Editor*
Albion College, Albion, Michigan

L. G. BALFOUR COMPANY........................*Jeweler*
Attleboro, Massachusetts

Kappa Mu Epsilon, national honorary mathematics fraternity, was founded in 1931. The object of the fraternity is four-fold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievement in the study of mathematics in the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

GEOMETRIC INVERSION*

DOROTHY KARNER
Student, Mount Mary College, Milwaukee

The method of inversion given in modern geometry books is based on the theory of inverse points and their one-to-one correspondence with respect to a given circle called the circle of inversion. By definition, two points colinear with the center of a circle are inverse points if the product of their distances from the center is equal to the square of the radius of the circle. Thus, if P and P' are two points colinear with the center O of the circle of inversion whose radius r is greater than zero and $OP \cdot OP' = r^2$, then each of the points P and P' is the inverse of the other with respect to the circle of inversion. The radius of the circle of inversion is known as the radius of inversion, while its center is recognized as the center of inversion.

With respect to a given circle of inversion the following facts are quite obvious:

(a) Each point in the plane except the center has a unique inverse.

(b) A point on the circle of inversion is its own inverse.

(c) Of two distinct inverse points, one lies inside and the other lies outside the circle of inversion.

It is also quite obvious that when $OP \cdot OP'$ is positive the two inverse points P and P' lie on the same side of the center of inversion. If $OP \cdot OP'$ is negative the two inverse points lie on opposite sides of the center of inversion and there would be no points coinciding with their inverse points because in this case the circle of inversion is imaginary.

Another unique factor regarding inverse points is that the closer the point P is to the center O the farther away is P' from O. Or, as the distance OP decreases indefinitely

*A paper presented on April 14, 1950 at the Chicago meeting of the National Council of Teachers of Mathematics. A section of this meeting, sponsored by the Illinois Gamma Chapter of Kappa Mu Epsilon, was devoted to student papers.

the distance OP' increases indefinitely. For this reason we say that the inverse point of the center of inversion is a point at infinity on a line through the center.

If the point P traces a figure (f) the point P' will also trace a figure (f'). These two figures are each the inverse of the other with respect to the given center of inversion. A curve inside the circle of inversion has its inverse curve outside of the circle. When the inner curve passes through the center O the outer passes through infinity. When a curve lies partly outside and partly inside of the circle of inversion, the inverted curve will also be partly inside and partly outside. It can thus be observed that if two curves intersect in the point P, their inverses intersect in the point P', the inverse point of P. It follows that if two curves are tangent to each other at the point P their inverses are tangent to each other at the point P'. If a curve intersects the circle of inversion its inverse curve intersects the circle of inversion in the same point or points, because every point on the circle of inversion is its own inverse.

The point transformation of a curve into its inverse curve has been shown to be both the simplest and most interesting of transformations after the linear ones. If the coordinates (x', y') of the point P are given, we can easily arrive at the coordinates (x'', y'') of its inverse point with respect to the circle $x^2+y^2=r^2$. Using polar coordinates, we have $x'=\rho'\cos\theta$, $y'=\rho'\sin\theta'$, $x''=\rho''\cos\theta'$, $y''=\rho''\sin\theta'$, and by the condition of inversion, $\rho'\rho''=r^2$. Hence, $x''/x'=\rho''/\rho'=\rho'\rho''/\rho'^2=r^2/(x'^2+y'^2)$ and $x''=r^2x'/(x'^2+y'^2)$. Similarly $y''=r^2y'/(x'^2+y'^2)$. Thus, if one has the analytic equation of a curve he can determine the analytic equation of its inverse curve by use of these equations of transformation.

An inverse point can be determined in another way than by use of the above equations of transformation; it may be determined by construction. The inverse point P of the point P' outside the circle of inversion is found by drawing tangents to the circle from the point P' and joining the points of tangency. The inverse point P will then be the intersection of this chord with the radius vector of P' (Fig. 1).

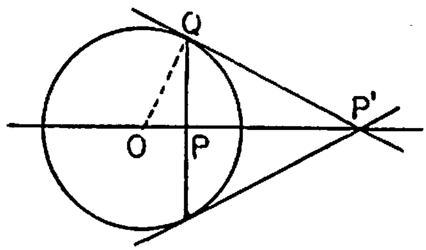


Fig. 1

The inverse point P' of point P within the circle is determined by first drawing the radius vector of the point P and prolonging it. At P a chord is constructed perpendicular to the radius vector and tangents are drawn from the extremities of the chord to find P' on OP prolonged.

A simple geometric proof of this is as follows:

1. $\angle OPQ = \angle OQP'$	1. Construction.
2. $\angle QOP = \angle QOP'$	2. Identity.
3. $\angle OP'Q = \angle OQP$	3. If two angles of one triangle are equal, respectively, to two angles of another triangle, the third angles are equal.
4. $\triangle OPQ$ is similar to $\triangle OQP'$	4. Corresponding angles are equal
5. $OP/OQ = OQ/OP'$ or $OP \cdot OP' = OQ^2$	5. Corresponding sides of similar triangles are proportional.
6. Hence P and P' are inverse points.	6. By definition of inverse points.

It is difficult to determine just who was the first person to develop the inversion transformation. Geometers themselves attribute the origin to various people. Julian Lowell Coolidge says the transformation is usually accredited to Julius Plucker. But he does not say precisely where in Plucker's works the method is explicitly mentioned. Coolidge himself is not sure because, in a later work of his, he says that the transformation was first mentioned by Pappus who knew that it carried a line or circle into a line or circle. Other authors attribute the origin of the theory to Jacob Steiner. But it is difficult to find any evidence of this in Steiner's published works. If he had originated the method it would be difficult to say just how much actual use he had for it, for Steiner dearly loved to cover up his tricks.

Still other authors merely mention the practical use to which the method has been put. They mention Sir William Thomson who used the inversion transformation to obtain geometrical proofs of some of the most difficult propositions in the mathematical theory of electricity. John Clerk Maxwell used the theory in connection with the method of electrical images. Others went on to show the applicability of the method of inversion to the solution of problems in optics. These men were trying to show that inversion is not only a beautiful mathematical concept but also a useful tool for the study of the laws of nature.

John Casey and H. V. Baravelle have given credit for the inversion transformation to Stubbs and Ingram. They believe that these two men were co-discoverers of the method about 1842. Even if the principle involved was known before, evidence seems to show that Stubbs and Ingram were at least the first to make any extended application of the principle and to develop the properties of the transformation.

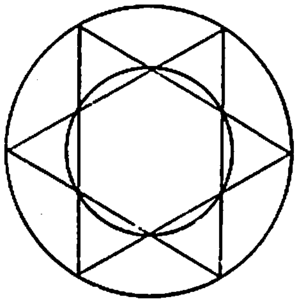
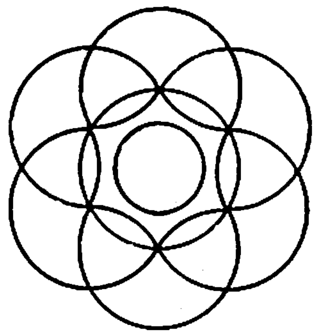
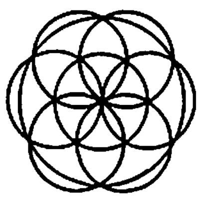
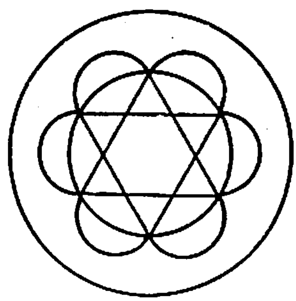
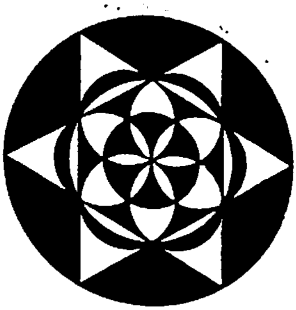
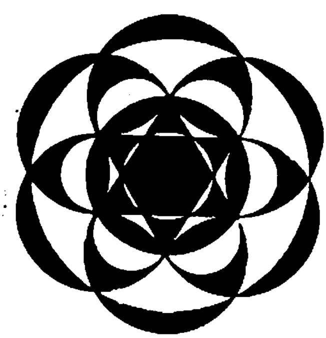
Of particular interest are the inverses of the conic sections. The straight line not through the center of inversion is a circle through the center. A straight line through the center of inversion is its own inverse. A circle not through the center of inversion inverts into another circle. If the focus of a parabola is at the center of inversion the inverse figure is a cardioid with its cusp at the center. If the center of inversion is at a principal vertex the parabola inverts into a cissoid of Diocles.

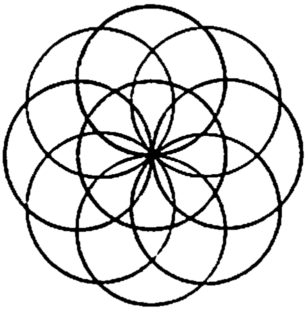
The inverse of an equilateral hyperbola is a lemniscate. This is true when the center of the hyperbola is in coincidence with the center of inversion. Taking the focus of an equilateral hyperbola as the center of inversion the curve inverts into a limacon. Since projectively the ellipse and the hyperbola are the same curves the inverses of the ellipse are the same as those of the hyperbola.

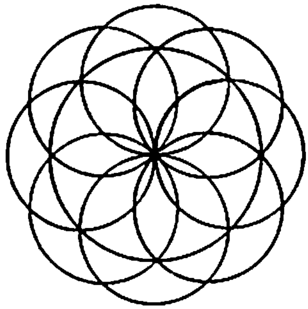
of Curves by Inversion."* In a single figure he has drawn a circle, two ellipses, a parabola, two hyperbolas, and two coincident straight lines. The center of the circle is at the common focus of these sections and its radius equals the distance between the focus and the common vertex. Taking the circle of the family of curves as the circle of inversion and inverting the curves he states, "These inverted curves are all limacons including, as a special case, the cardioid which is the inverse of the parabola." Thus, all the inverse curves of the conic sections are considered conchoids of the circle.

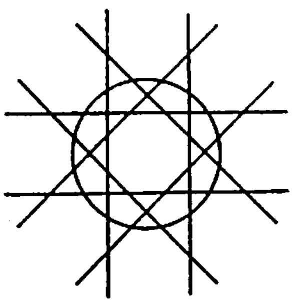
Another thing that Mr. Baravalle has done in his paper is to apply the method of inversion to a few figures to form geometric designs which are both beautiful and intriguing. He has inverted the inscribed and circumscribed pentagrams and the 12X12 checker board. I have applied the same idea to various figures of my own, a few of which are given here. For each case there is given the original figure and its inverse, and the final design that results when these two figures are superimposed upon each other.

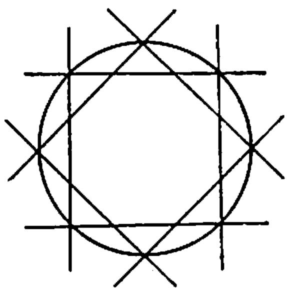
**Scripta Mathematica*, Vol. 14, pp. 113-125 (June, 1948), pp. 266-272 (September-December, 1948).

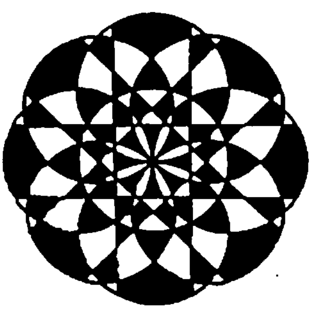







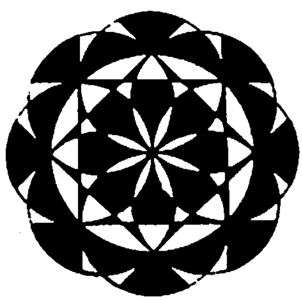


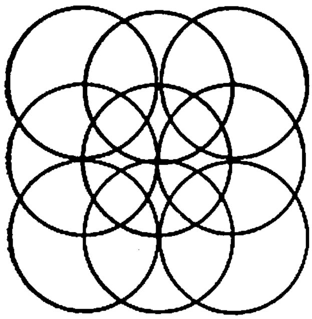


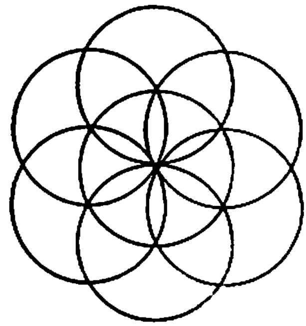


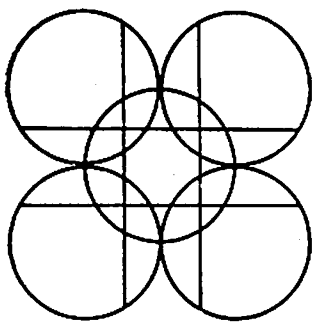


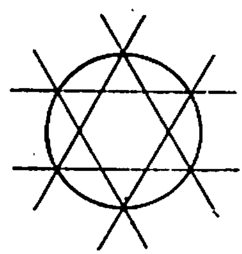


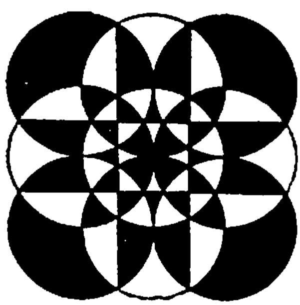


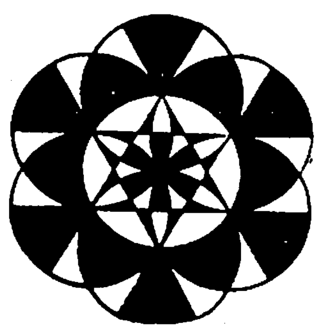












A STEP FORWARD*

MARY LOU HODOR AND NAN HUTCHINGS
Students, College of St. Francis

Because we are members of Kappa Mu Epsilon we have pledged ourselves to develop an appreciation for the beauty of mathematics. The beauty of mathematics supplies the life that is needed to elevate mathematics from only a skill subject into a study both fascinating and beautiful. The mathematics teacher who uses bits of historical information in her class sets off the spark of enthusiasm which makes her teaching a real success. Because we are prospective teachers eagerly looking forward to our career, and because we know that today we are speaking to other future teachers, we would like to acquaint you with a few geometric theorems and problems that somehow or other, despite their historical importance and mathematical simplicity, have not found their way into the secondary school text book. They make ideal supplementary material.

Our first problem can be found in the Latin work of Johann Felden, *Ars Geometria,* published in 1690 at Jena. It is stated in the original text as *Lineam ovi Formam referentem ducere,* which means construct an egg. The construction itself is not difficult but must be done very accurately in order to obtain a perfect figure. Students will be interested in following the many steps it takes to complete the oval in Figure 1.

(1) Draw line AB, and divide it into 10 equal parts.
(2) Draw circle ML.
(3) Draw arcs ANC and BLC.
(4) Draw line CDM.
(5) Bisect arc LD at H, and arc ND at I.
(6) Draw line EDF parallel to AB.
(7) Draw arcs HFK and IEK.
(8) Bisect DK, obtaining G.
(9) Draw circle GK.

*A paper presented on April 14, 1950 at the Chicago meeting of the National Council of Teachers of Mathematics. A section of this meeting, sponsored by the Illinois Gamma Chapter of Kappa Mu Epsilon, was devoted to student papers.

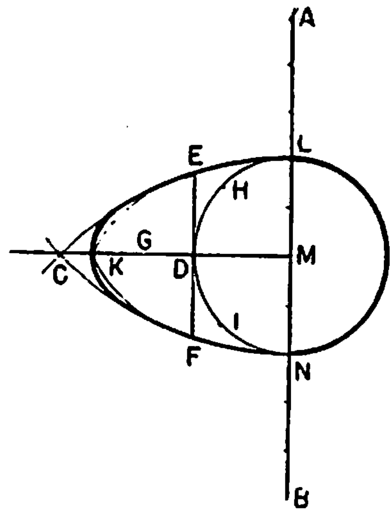


Fig. 1

The impossible problem "to square the circle," that is, "to find a square or some other rectilinear figure exactly equal in area to a given circle" was developed to some regree by Hippocrates of Chios.[1] Various stories are told of Hippocrates. One of these states that he was an unsuccessful merchant who became a Pythagorean philosopher with a special interest in mathematics.[2] While visiting Athens he spent his leisure time in attending lectures; finally he himself became a teacher of geometry. As early as 430 B.C. the problem of squaring the circle had been forwarded by the Sophist School. The attention of several mathematicians was stimulated but history attributes to Hippocrates the discovery of the first example of a curvilinear area which admits of exact quadrature.[3] He described a semicircle on a straight line *AC,* inscribed in this semicircle an isosceles triangle *ABC,* and drew semicircles on *AB* and *BC* (Fig. 2). Hippocrates proved that the lunar areas thus formed are

An interesting feature of the inversion of conic sections is developed in Mr. Baravalle's paper, "Transformation

[1]F. Cajori, *A History of Mathematics.* New York, Macmillan, 1926, p. 22.
[2]D. E. Smith, *History of Mathematics.* New York, Ginn and Co., 1925, Vol. 2, p. 82.
[3]J. Gow, *History of Greek Mathematics.* New York, G. E. Stechert and Co., 1923, p. 165.

equal to the area of the triangle *ABC*, thereby "squaring the lunes."

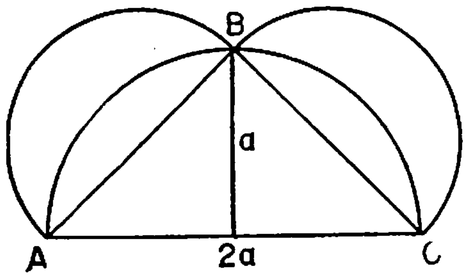


Fig. 2

Continuing with this idea, Hippocrates inscribed a half hexagon in a semicircle (Fig. 3). On the three sides *AB, BC, CD,* he described semicircles. The total area of the lunes thus formed equals the area of the half hexagon minus the area of a semicircle drawn on one side of the hexagon.

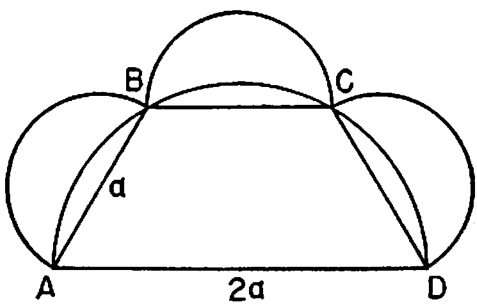


Fig. 3

A third problem of historical interest is one given us by Archimedes. Archimedes, called the greatest mathematician of antiquity, was born in Syracuse about 287 B.C. Plutarch says he was a relation of King Hieron, while Cicero states that he was of low birth. Diodorus says he visited Egypt, and, since he was a great friend of Conon and Eratosthenes, probably studied in Alexandria. Archimedes had thorough acquaintance with all the work done in mathematics up to his time. In Syracuse he made himself useful to his admiring friend and patron, King Hieron, by applying his extraordinary inventive genius to the construction of various war-engines which were used to inflict much loss on the Romans during the siege of Marcellus. The story is probably a fiction that Archimedes used mirrors reflecting the sun's rays to set on fire the Roman ships when they came within bow-shot of the walls.[4] However, Archimede's theorem of the Broken Chord suggests a capacity and ingenuity that is far from fictitious. This theorem attracted the attention of Arabic writers. Strange to say it has not found its way into English texts, not even in mathematical histories.[5]

If in any circle (Fig. 4) there be drawn two chords AC and CD forming a broken line of unequal lengths, and if from the midpoint B of the arc of the broken chord a perpendicular BH is drawn to the larger chord, then $AH = CH + CD$. By drawing the dotted lines with $CB = BE$, congruent triangles are formed and the conclusion follows without difficulty.

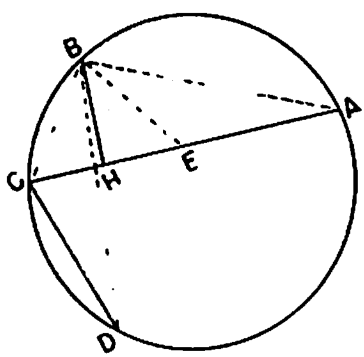


Fig. 4

Ptolemy, born about 150 A.D., wrote an exceedingly influential work called the *Syntaxis*. This book was used as the culmination of the study of astronomy at Alexandria and was given the title of "megiste" or the "greatest book." Translated into Arabic, it was called by this nickname with the prefix of the article "al" and thus became known as the *Almagest*. The work contains a study of heavenly bodies and tables of chords of certain arcs.[6] An important

[4] J. Cajori, *op. cit.*, p. 34.
[5] See for instance: Sister Mary Claudia Zeller, O. S. F., *The Development of Trigonometry from Regiomontanus to Pitiscus*. Ann Arbor, Edward Brothers, 1946, p. 5.
[6] V. Sanford, *A Short History of Mathematics*. Chicago, Houghton-Mifflin Co., 1930, p. 293.

if not significant property of chords that is suitable for supplementary work in a high school geometry class is contained in a theorem given in the *Almagest*. This theorem of Ptolemy states that the product of the diagonals of a cyclic quadrilateral is equal to the sum of the products of the opposite sides. That is, for the cyclic quadrilateral *ABC* (Fig. 5),

$$AC \cdot BD = AB \cdot CD + AD \cdot BC.$$

By adding the line *AE* such that $<DAE = <BAE$, similar figures are determined leading to proportions from which the conclusion follows.

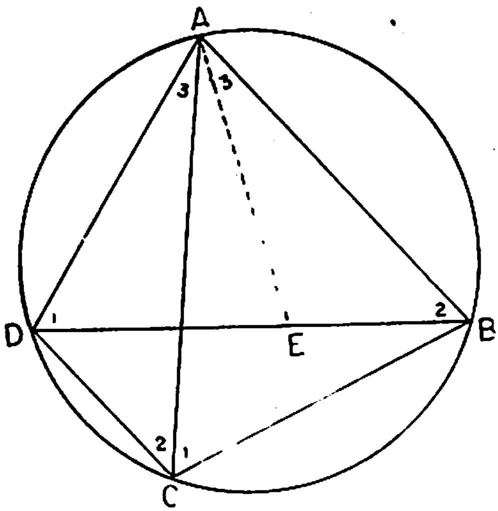


Fig. 5

Many important properties of geometric figures depend on the concurrence of lines and the collinearity of points. An important theorem which is useful in establishing such properties is one bearing the name of Menelaus. This theorem, elegant because of its power and simplicity, appeared in a work written by Menelaus near the close of the first century A.D.[7] The theorem states that if a straight line (Fig. 6) intersects the sides *BC, CA,* and *AB* of a triangle in the points *L, M, N,* respectively, then

$$(AL/LB) \cdot (BN/NC) \cdot (CM/MA) = -1.$$

[7]L. S. Shively, *An Introduction to Modern Geometry.* New York, John Wiley, 1939, p. 22.

By letting *AP, BQ,* and *CR* be the perpendiculars from *A, B, C,* respectively, similar figures are determined, proportions may be set up, and the conclusion reached.

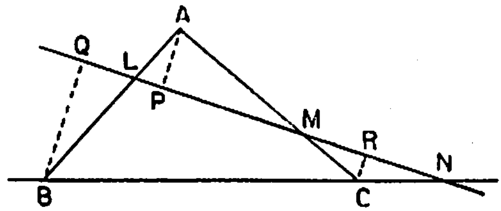


Fig. 6

The theorems and problems that we have just discussed are only a few out of many that might be presented. Mathematical history can supply work for the classroom and the Mathematics Club. Mathematical history can help the teacher emphasize that mathematics is old, it is active, and it is strong with the vigor of perpetual youth.

THE VISIBLE HORIZON

"A point of some scientific interest has just been argued in the High Court of Justice. It was contended by the Solicitor-General that the three miles' limit of territorial waters was of modern origin, and by Sir R. Phillimore that it was due to that being the distance a cannon ball would reach from the shore. There can, however, be no doubt that the limit was recognized long before the invention of gunpowder.

"Three miles is the distance of the *offing* or visible horizon to a person six feet in height standing on the shore. It is natural to suppose that the early maritime peoples of Europe would lay claim to the sea as far as the eye could reach. This distance they would find by experience was just *three miles,* and it can be proved mathematically to be correct . . ."

—NATURE, May 18, 1876.

ETHIOPEAN MULTIPLICATION

C. STANLEY OGILVY

Col. L. B. Roberts, now with the Merritt-Chapman & Scott Co. in New York, and formerly a Colonel of the U.S. Army Engineers, gives an amusing account of what he calls "Mathematics a la Ethiopea." During an expedition which he once made into the interior of that country for the purpose of finding the headwaters of the Blue Nile, his party had occasion to purchase eight bulls.

"This we attempted to do," he writes, "at the first market place we came to, but although there were bulls for sale there, neither the owner of the stock nor my headman knew how many Maria Theresa dollars should change hands. As neither could do simple arithmetic, they just stood and yelled at each other, getting nowhere. Finally a call was put in for the local priest, as he was the only one who could handle questions like this.

"The priest and his boy helper arrived and began to dig a series of holes in the ground, each about the size of a teacup. These holes were ranged in two parallel columns; my interpreter said they were called houses. What they were about to do covered the entire range of mathematics necessary to transact business in this area, and the only requirement was ability to count, and to multiply by and divide by two.

"The priest's boy had a bag full of little pebbles. Into the first cup of the first column he put eight stones (one for each bull), and eleven into the first cup of the second column, since each bull was to cost (\$*MT*) 11. It was explained to me that in this way of doing business, the first column of houses is used for multiplying by two; that is, twice the number of pebbles in the first cup are placed in the second, then twice that number in the third, and so on. The second column is for dividing by two: half the number of pebbles in the first cup are placed in the second, and so on down until there is one pebble in the last cup. Fractions are discarded.

"The division column is then examined for odd or even numbers of pebbles in the cups." (The colonel makes no comment on the fact that the comparatively subtle notion of parity is evidently within the grasp of these primitive mathematicians.) "All even houses are considered to be evil ones, all odd houses good. Whenever an evil house is discovered, the pebbles are thrown out and not counted. All pebbles left in the remaining cups of the multiplication column are then counted. The sum of them is the answer."

On paper, the problem of the bulls looks like this:

Multiplication Column	Division Column
8	11
16	5
(*32*)	2 (Evil)
64	1
$88 = 8 \times 11$	

The colonel marvels at the fact that even though half of five is not two (he calls this an "error") the correct result is always obtained by this system of multiplication.

The process if analyzed in Kasner and Newman, "Mathematics and the Imagination" (pp. 167-8) and an example is worked out. The authors call it the Russian Peasant method of multiplication. It is also treated in some texts on elementary number theory. Uspensky and Heaslet, for instance, cite an example (page 22) and give the clue: a representation of the numbers in the binary scale reveals the true nature of the operation and shows the rule to be general.

It is interesting to discover that this process is not just an artificial text-book exercise but is actually in use in remote corners of the world.

TYPES OF JOBS OPEN TO COLLEGE GRADUATES WITH MAJORS IN MATHEMATICS[1]

BY THE DEPARTMENT OF MATHEMATICS
Oklahoma A. and M. College

Introduction

There is a large variety of jobs available for college graduates with majors in mathematics. This is especially true if the student has received a certain amount of specialized training in mathematics applied to specific fields needed by business, industry, and government service. It is being recognized that workers trained as engineers, physicists, accountants, etc., are not wholly qualified to solve the increasingly difficult mathematical problems which occur in industry and business. Hence, there is justification in using the term Applied Mathematician to cover a wide field of mathematical workers who range from the highly skilled mathematical consultant to the mathematical worker who uses the principles of statistics, the mathematics of finance, etc., in many of our smaller business concerns and offices. This article proposes to discuss some of the broad fields which offer employment opportunities to mathematics graduates and to set up tentative curricula which we think will prepare students for specific jobs.

Enviable Reputations

Mathematics graduates have enviable reputations with prospective employers. This is doubtless due to the fact that success in college work in such an exact science is almost impossible without the possession of more than average intellectual ability. The training required for these jobs mentioned above should contain the fundamentals of elementary mathematics. Beyond that it may be hard to predict just what will be needed, though this can be more easily done in some fields than in others. For example, we

[1]Selected with revisions from "Mathematics—Its Vocational Aspects," *Bulletin Oklahoma A. and M. College*, May, 1945, by permission of Professor James H. Zant, Department of Mathematics.

shall find that the actuarial profession has very definite subject matter prescribed for those who wish to enter the field. An oil company, on the other hand, has so many types of work, and emphasis changes so often that you cannot be sure that the proper courses are recommended. However, if a student has a good background in fundamental mathematics and uses his minor and electives to get a broad background in related fields of science, engineering, commerce, etc., he should be able to make adjustments as conditions change. Hence, students majoring in mathematics should take mathematics through the calculus and advanced algebra, and should choose their minors and electives from courses and fields which seem closely related to the type of work which the individual student expects to do. These electives should furnish the applications of mathematics to this particular field as well as a broad scientific background in subjects which are related to it.

The general training required by the School of Arts and Sciences is a distinct advantage to a worker in business and industry. Though the technical nature of his work is fundamental and occupies the major portion of his time, especially during the early years, this should not be overemphasized. The employee is essentially a business man (or woman) and the higher he gets in his company the more contacts he has with the public. He must interview people and explain the work of his company to representatives of larger firms. This matter of human relationships is stressed more and more by employment managers who are seeking young people to enter their business. Therefore, the employee's cultural background and his ability to work with and meet people is quite as important as his technical knowledge and training.

1. Arithmetical Clerks, Computers, etc.

Business offices, industry and civil service have many jobs with some such designation as the above. Workers of this sort use comparatively little mathematics, and probably no advanced mathematics. However, employment managers and personnel men often insist on hiring graduates in mathematics since they are usually thoroughly dependable for this sort of work.

Duties

The duties required naturally vary with the employer and type of business, but appointees should expect to be called upon to perform clerical work involving arithmetic computations and verifications, compiling of reports, comparing or posting figures, making simple statistical computations, verifying tax returns, checking accounts and other related duties. The difficulty of the work, the degree of supervision to which the employee is subject and the amount of responsibility assumed will usually be determined by the grade of the position.

Training Required

Jobs of this sort are often filled by persons who have not finished college or who have had little or no college mathematics. Hence, it should probably be considered the lowest form of mathematical employment. A graduate accepting this sort of a position should be able to look forward to improving his position in the office or company when better opportunities present themselves. Hence, the student's training should be for a better type of job, similar to that discussed in No. 2. This should include courses in mathematics of finance, probability and statistics after completing the basic fundamental mathematics courses. A minor and electives may be chosen from related fields in commerce, business statistics, accounting, business practice, and secretarial work.

2. Junior Mathematician, Junior Engineer, Mathematical Clerk, Junior Statistician, etc.

Workers of the type designated above use considerably more mathematics than those discussed under No. 1. They must have some specialized skills of elementary mathematics as well as applications to mechanics, statistics, astronomy and the like. To qualify for this type of work the student must be thoroughly familiar with elementary college mathematics as well as with some of its applications.

Duties

The type of work performed will vary widely with the employer but the student should expect to be called upon to do professional and semi-professional work in engineering, statistics, astronomy, etc., under the supervision of more experienced workers.

Training Required

Jobs of this type require a sound training in basic fundamental mathematics and courses in the application of mathematics to the field or fields in which the student expects to work. Since this varies widely there must be several sequences from which to choose. For example, one who wishes to become a junior engineer should probably study the applications of mathematics to mechanical problems as well as some engineering, physics and mechanics courses. On the other hand, a student who expects to become a junior statistician would take courses in mathematical statistics, probability, actuarial mathematics, business statistics, accounting, etc.

3. Statistician and Research Statistician

The use of statistics is becoming increasingly important in business, industry and government service. Statistical work varies widely. Some of the more elementary parts are required in several of the classifications described above. However, there is a growing field for the statistical expert.

Duties

Such workers should expect to perform professional statistical work involved in the collection, compilation and analysis of data for written reports and special studies on unemployment compensation, employment, economic conditions, business trends, etc. He might also be expected to organize and direct statistical research activities regarding unemployment compensations, fire losses, employment service units and the like. Actuaries and actuarial workers also use statistics, but this is such highly specialized work that it will be discussed in No. 4.

Training Required

Training for jobs of this sort must be of a very high type and rather highly specialized. The background in mathematics must be broad and thorough. Since the work is highly specialized, new employees seldom attain these classifications until they have several years of professional mathematical experience. Hence, there will be a chance for a continued improvement in mathematical knowledge either through private study or part-time class attendance. Beyond the basic fundamental mathematics the student should devote most of his time including the minor and electives to specialized courses in mathematical statistics, probability, the mathematics of finance, business statistics, accounting and business practice. If such study can be continued to the Master's degree, the student will be much better prepared to assume the responsibilities of this sort of work.

4. ACTUARY, PRINCIPAL ACTUARIAL MATHEMATICIAN, ACTUARIAL MATHEMATICIAN, ASSISTANT ACTUARIAL MATHEMATICIAN, ETC.

"The task of obtaining satisfactory material for actuarial student employment is one of the most vital problems facing the actuarial profession at the present time. Not only do the insurance companies require actuaries, and there are more than 300 insurance companies in the country, but so do the State Insurance Departments, the Federal Government and various firms of consulting actuaries."

The above statement was made by an official of one of the large insurance companies in this country a few years ago. While the intellectual and mathematical abilities needed to succeed in this type of work are of very high caliber, many of our mathematics majors could easily qualify if they decide in their sophomore or junior year to take special training along these lines. However, the training in college must be specialized. The same official quoted above also said that 70% of the college mathematics majors generally fail the entrance test given prospective actuarial employees of that particular company. It is true that it may take several years and a great deal of study to attain the full status of an actuary. However, the reward in both position and salary is very high.

Duties

The duties of an actuarial worker are rather definite, since it is such a highly specialized profession. They include devising methods of actuarial valuations of liabilities and costs, involving the development of difficult actuarial formulas, assisting in preparing actuarial tables and scales, conducting actuarial research, reviewing periodic statements for accuracy, the assembling and analysis of special reports, giving actuarial advice and the like. In the earlier years of employment much of this work would be done under the supervision of older men. However, if the employee continues to improve himself, the advancement available to him will be limited only by his own progress.

Training Required

On the undergraduate level it seems reasonable to suppose that a student may obtain the basic mathematics needed and also be able to qualify for one or more of the examinations given by the Society of Actuaries. These examinations will be listed below. Such a student should take a major in applied mathematics consisting of the basic work through two semesters of calculus, higher algebra, mathematics of finance, calculus of finite differences, probability and statistics. It would be well for such a student to minor in commerce and secure some knowledge of business statistics and life insurance problems.

On the graduate level, this study may be easily continued to include the mathematics of life insurance, life contingencies, mathematical statistics, mortality studies, life insurance accounting, and the like. Many of these courses would help in qualifying the student for some of the more advanced examinations of the Society of Actuaries.

1951 Actuarial Examinations

The 1951 Preliminary Actuarial Examinations will be prepared by the Educational Testing Service and will be administered by the Society of Actuaries at centers throughout the United States and Canada on May 18, 1951. The closing date for applications is March 15, 1951.

Detailed information concerning the Examinations can be obtained from: The Society of Actuaries, 208 South LaSalle Street, Chicago 4, Illinois.

The Preliminary Actuarial Examinations consist of the following three examinations:

Part 1. Language Aptitude Examination. (Reading comprehension, meaning of words and word relationships, antonyms, and verbal reasoning.)

Part 2. General Mathematics Examination. (Algebra, trigonometry, coordinate geometry, differential and integral calculus.)

Part 3. Special Mathematics Examination. (Finite differences, probability and statistics.)

A pamphlet containing an outline of the subject matter to be covered by the examinations and illustrative questions can be secured by applying to the Society of Actuaries. This pamphlet also contains a list of the subjects covered by the fourth and fifth examinations required for Associate membership and the three additional examinations which the student must pass in order to become a full member or Fellow.

5. Mathematical Consultants in Mathematics and Theoretical Mechanics, Research Mathematicians, Astronomers, etc.

While the qualifications of these types of workers are very high, the number needed is not large. However, wages paid are relatively high and the demand is steadily increasing as industry and business become more complex and technical. This is the type of work to which the best and most industrious students of mathematics can look forward. It demands knowledge and training that is beyond most of the engineers and mathematicians both in colleges and in industry. As such, these jobs call for a high degree of technical skill, resourcefulness and individual initiative.

Duties

Mathematical and Applied Mathematical consultants are called in to solve problems and suggest methods which are beyond the experience of the staff of a business or industry. Research mathematicians are found mostly in the larger colleges and universities as well as in the employ of large companies and in government service. Their work is varied and usually little restriction is placed on their research activities. Those employed by large companies like the Bell Telephone Company have, of course, been taken because of their proved interest in problems related to the work of that particular company. The same is true of those in government offices. Almost complete freedom as to choice of problems of research is allowed by those who are on the staffs of the large universities. Astronomers are employed as university professors as well as workers in the large observatories including the Naval Observatory and in other types of government service. Astronomy applied to advanced navigation problems is becoming important as long range flying increases.

Training Required

As stated above the qualifications for these jobs are very high. The mathematical training required is very rigorous and must be continued to the Ph.D. if the student expects to obtain a desirable position. Most people who attain these positions do so after several years spent in teaching or as assistant research workers and the like. Hence, this should also be considered in selecting college courses. Since much of the work which these men will be called upon to do will deal with some field of engineering or the physical sciences, minors and electives should be chosen from these fields.

6. TEACHERS OF SECONDARY AND COLLEGE MATHEMATICS ENGINEERING MATHEMATICS AND APPLIED MATHEMATICS

Teaching will remain one of the most fruitful fields for the employment of mathematics students. Well-trained young people who continue their studies can look forward to advancement in this field. While most will probably begin as secondary teachers and many will remain in this field, advancement to junior college and college teaching is possible for those who can qualify themselves.

Duties

The duties of a teacher of mathematics are well known. However, if the jobs outlined in this chapter are actually available, there will be need for teachers of specialized courses in mathematics whose training must also be specialized. Furthermore, teachers of students who expect to enter business or industry should have some practical work experience themselves. Hence, the teacher's duties will involve not only teaching classes, but also building the specialized mathematical curricula for courses and advising the student as to his opportunities and training.

Training Required

The amount of training required depends on the level at which the student expects to teach. For secondary school teaching, the B.S. is probably sufficient from the standpoint of subject matter. However, it is becoming almost necessary to have a Master's degree in order to get the desired advancement. The student who looks upon secondary teaching as a possible stepping stone to junior college or college teaching must be more careful in his basic preparation. Advancement in college teaching almost requires the Ph.D. Hence the student should work with this in mind. This requires a rigorous background in mathematics and little specialization will be possible in the undergraduate period. If a student knows he wishes to teach in some applied mathematics field, he may take his undergraduate degree in Applied Mathematics and perhaps get some work experience in that field during vacations or at other times. Minors and electives should include courses in psychology and education as well as other fields which relate to his special interest.

TOPICS FOR CHAPTER PROGRAMS—X

28. MAGIC SQUARES

A magic square consists of n^2 distinct positive integers arranged in a square so that the sum of the n numbers in every horizontal, vertical, and diagonal line is the same. The square is said to be of the nth order if the integers are the consecutive numbers from 1 to n^2. The formation of magic squares is a very old pastime, some simple cases being known to the Chinese as early as 2200 B.C. Amulets containing magic squares were worn in the Middle Ages to ward off the plague, and similar good-luck charms are worn today by various Oriental peoples. Many mathematicians have been attracted to the study of magic squares. Among the many interesting problems which are presented, these two emerge as fundamental: 1) to determine completely general methods of constructing magic squares of all orders, and 2) to determine the number of magic squares of all orders.

E. G. Allen, "Pan-Magic Squares of the Fourth Order," *American Mathematical Monthly*, Vol. 53, pp. 450-451 (October, 1946).

F. J. Anderson, "The 34 Supermagic Square," *Scientific American Supplement*, Vol. 87, pp. 44-45 (January 18, 1919).

W. S. Andrews, *Magic Squares and Cubes*. Chicago, Open Court Publishing Company, 1917.

W. S. Andrews and A. L. Baker, "Magic Squares," *Monist*, Vol. 15, pp. 429, 355.

W. S. Andrews and P. Carus, "Franklin's Magic Squares," *Monist*, Vol. 16, p. 597.

A. S. Anema, "Franklin Magic Squares," *Scripta Mathematica*, Vol. 11, pp. 88-96 (March, 1945).

W. W. R. Ball and H. S. M. Coxeter, *Mathematical Recreations and Essays*. New York, Macmillan, 1947.

E. Bergholt, "The Magic Square of Sixteen Cells," *Nature*, Vol. 83, pp. 368-369 (May 26, 1910).

C. Bragdon, "The Franklin 16×16 Magic Square," *Scripta Mathematica*, April, 1936.

C. Bragdon, "Ornament from Magic Squares," *Architectural Record*, Vol. 60, pp. 506-516 (December, 1926).

C. Bragdon, "More Ornament from Magic Squares," *Architectural Record*, Vol. 62, pp. 473-480 (December, 1927).

C. A. Browne, "Magic Squares and Pythagorean Numbers," *Monist*, Vol. 16, p. 422.

J. C. Burnett, "Bordered Squares of Fifth Order and Their Magic Derivatives," *Nature*, Vol. 116, pp. 573-574 (October 17, 1925), Vol. 127, p. 443 (March 23, 1931).

J. C. Burnett, *Easy Methods for the Construction of Magic Squares.* London, Rider and Company, 1936.

J. C. Burnett, "Magic Square of the Fifth Order," *Nature*, Vol. 125, p. 17 (January 4, 1930).

J. C. Burnett, "Subsidiary Rectangles as Applied to the Formation of Magic Squares," *Nature*, Vol. 121, pp. 57, 172, 985 (January 14, February 4, June 23, 1928).

A. L. Candy, *Construction, Classification, and Census of Magic Squares of Even Order.* Ann Arbor, Edwards Brothers, 1937.

A. L. Candy, *Pandiagonal Magic Squares of Composite Order.* Lincoln, Nebr., the author, 1941.

A. L. Candy, *Pandiagonal Magic Squares of Prime Order.* Lincoln, Nebr., the author, 1940.

A. L. Candy, *Supplement to Pandiagonal Magic Squares of Prime Order.* Lincoln, Nebr., the author, 1942.

P. Carus, "Magic Squares," *Monist*, Vol. 16, p. 123.

J. Chernick, "Solution of the General Magic Square," *American Mathematical Monthly*, Vol. 45, pp. 172-175 (March, 1938).

"Derivation of New Magic Squares," *Scientific American Supplement*, Vol. 88, p. 191 (September 20, 1919).

H E. Dudeney, *Amusements in Mathematics.* London, T. Nelson and Sons, 1917.

"Dr. Franklin's Magic Square of Squares," *Knowledge*, Vol. 4, p. 233.

A. H. Frost, *Quarterly Journal of Mathematics*, London, Vol. 15, pp. 34-49 (1878).

Encyclopedia Americana: "Magic Squares."

Encyclopaedia Britannica: "Magic Squares."

A. Gloden, "Magic Squares and Multigrade Chains," *Scripta Mathematica*, Vol. 12, pp. 225-226 (September, 1946).

S. Guttman, "New Magic in Old Magic Squares," *Scripta Mathematica*, Vol. 14, pp. 284-286 (September-December, 1948).

S. Guttman, "Universal Magic Squares and Multigrade Equations," *Scripta Mathematica*, Vol. 13, pp. 187-202 (September, 1947).

R. V. Heath, "A Curious Magic Square," *Scripta Mathematica*, July, 1935.

R. V. Heath, "A Magic Circle," *Scripta Mathematica*, October, 1935.

R. V. Heath, "A Magic Cube with $6n^3$ Cells," *American Mathematical Monthly*, Vol. 50, pp. 288-291 (May, 1943).

R. V. Heath, "A Panelled Magic Square," *Scripta Mathematica*, April, 1936.

A. R. Kennedy, "Magic Squares," *Scientific American Supplement*, Vol. 78, pp. 223-224 (October 3, 1914).

M. Kraitchik, *Mathematical Recreations.* New York, W. W. Norton, 1942.

F. Lane, "Magic Squares," *Pentagon*, Vol. 6, pp. 10-16 (Fall, 1946).

H. B. Loomis, "Pandiagonal Magic Squares and Their Relatives," *School Science and Mathematics*, Vol. 44, pp. 831-838 (December, 1944).

H. B. Loomis, "Pandiagonal Magic Squares on Square Bases and Their Transformations," *School Science and Mathematics*, Vol. 45, pp. 315-322 (April, 1945).

E. McClintock, *American Journal of Mathematics*, Vol. 19, pp. 99-120 (1897).

J. C. McCoy, "The Anatomy of Magic Squares," *Scripta Mathematica*: 1. Vol. 5, pp. 137-141 (April, 1938); 2. Vol. 5, pp. 203-207 (July, 1938); 3. Vol. 6, pp. 114-118 (June, 1939); 4. Vol. 6, pp. 175-178 (October, 1939); 5,6,7. Vol. 7, pp. 143-153 (1940); 8,9. Vol. 8, pp. 49-55 (March, 1941); 10,11,12. Vol. 8, pp. 122-123 (June, 1941); 13. Vol. 8, pp. 183-187 (September, 1941); 14. Vol. 8, pp. 257-261 (December, 1941); 15. Vol. 9, pp. 278-284 (December, 1943); (with Norman Stewart, Jr.) 16. Vol. 11, pp. 85-88 (March, 1945).

J. C. McCoy, "The Magic Knight's Tour," *Scripta Mathematica*, Vol. 12, pp. 79-86 (March, 1946).

H. P. McLaughlin, "Algebraic Magic Squares," *Mathematics Teacher*, Vol. 14, pp. 71-77 (February, 1921).

P. A. MacMahon, "Magic Squares and Other Problems Upon a Chess-Board," *Nature*, Vol. 65, pp. 447-452 (March 13, 1902).

"Manuel Moschopoulo's Treatise on Magic Squares" (tr. by J. C. McCoy), *Scripta Mathematica*, Vol. 8, pp. 15-26 (March, 1941).

"Magic Squares That Are Zero Determinants," *American Mathematical Monthly*, Vol. 53, pp. 98-99 (February, 1946), pp. 394-395 (August-September, 1946).

"Magic Squares," *Knowledge*, Vol. 4, pp. 125, 234.

"Magic Squares," *Saturday Review of Literature*, Vol. 10, pp. 203, 235, 253 (October 21, November 4, November 11, 1933).

A. Moessner, "Curious Magic Square," *Scripta Mathematica*, Vol. 13, p. 234 (September, 1947).

A. Moessner, "A Magic Multiplication Square," "*Scripta Mathematica*, Vol. 13, p. 231 (September, 1947).

C. Planck, "General Rule for Constructing Ornate Magic Squares of Orders $\equiv 0$ (mod. 4)," *Monist*, Vol. 26, pp. 463-470 (July, 1916).

C. Planck, "Magic Squares of the 5th Order," *Monist*, Vol. 26, pp. 470-476 (July, 1916).

C. Plank, "Ornate Magic Squares of Composite Odd Orders," *Monist*, Vol. 26, pp. 470-476 (July, 1916).

C. Plank, "Pandiagonal Magics of Orders 6 and 10 with Minimal Numbers," *Monist*, Vol. 29, pp. 307-316 (April, 1919).

L. R. Posey, "A General Formula for Magic Squares of Various Orders Beginning with Numbers Different from Unity," *School Science and Mathematics*, Vol. 40, pp. 315-329 (April, 1940).

A. Rosenfeld, "Another Magic Multiplication Square," *Scripta Mathematica*, Vol. 14, pp. 287-288 (September-December, 1948).

J. B. Rosser and R. J. Walker, "On the Transformation Groups for Diabolic Magic Squares of Order Four," *Bulletin of the American Mathematical Society*, Vol. 44, pp. 416-420 (June, 1938).

V. Sanford, "Magic Circles," *Mathematics Teacher*, Vol. 16, pp. 348-349 (October, 1923).

H. A. Sayles, "Even Order Magic Squares with Prime Numbers," *Monist*, Vol. 26, pp. 137-144 (January, 1916).

H. A. Sayles, "General Notes on the Construction of Magic Squares and Cubes with Prime Numbers," *Monist*, Vol. 28, pp. 141-158 (January, 1918).

H. A. Sayles, "Pandiagonal Concentric Magic Squares of Orders 4m," *Monist*, Vol. 26, pp. 476-480 (July, 1916).

H. S. Schubert, "The Magic Square," *Monist*, Vol. 2, pp. 487-511 (July, 1892).

H. S. Schubert, *Mathematical Essays and Recreations*. Chicago, Open Court Publishing Company, 1898.

D. E. Smith and Y. Mikami, *A History of Japanese Mathematics*. Chicago, Open Court Publishing Company, 1914.

E. M. Smith, "Puzzle Corner Turns a Corner," *Christian Science Monitor Magazine*, pp. 4-5 (September 2, 1936).

E. Stern, "General Formulas for the Number of Magic Squares Belonging to Certain Classes," tr. by W. R. Transue, *American Mathematical Monthly*, Vol. 46, pp. 555-581 (November, 1939).

W. G. Swart, "Magic Squares," *Scientific American Supplement*, Vol. 78, p. 406 (December 26, 1914).

"Tricks with Figures," *Popular Science*, Vol. 143, p. 81 (October, 1943).

C. W. Trigg, "Determinants of Fourth Order Magic Squares," *American Mathematical Monthly*, Vol. 55, pp. 558-561 (November, 1948).

O. Veblen, "On Magic Squares," *Messenger of Mathematics*, Vol. 37, pp. 116-118 (1908).

F. A. Woodruff, "Four-Ply Pandiagonal Associated Magic Squares," *Monist*, Vol. 26, pp. 315-316 (April, 1916).

29. DUPLICATION OF THE CUBE

One of the three "classical geometrical problems of antiquity" was that of duplicating the cube, that is, of constructing the side of a cube whose volume is double that of a given cube. It can be shown that the problem is insoluble if the only tools permitted are the compass and straightedge. However, the construction can be effected by the use of other tools such as conic sections; a number of solutions of this nature were presented by early Greek geometers.

G. J. Allman, *Greek Geometry from Thales to Euclid.* Dublin, 1889.

W. W. R. Ball and H. S. M. Coxeter, *Mathematical Recreations and Essays.* New York, The Macmillan Company, 1947.

W. W. R. Ball, *A Short Account of the History of Mathematics.* London, Macmillan and Company, 1888.

F. Cajori, *A History of Mathematics.* New York, The Macmillan Company, 1926.

L. E. Dickson, "Constructions with Ruler and Compasses," *Monographs on Topics of Modern Mathematics,*" ed. by J. W. A. Young. New York, Longmans, Green, and Company, 1915.

A. A. Dmitrovsky, "An Approximate Solution of the Problem of the Duplication of the Cube," *School Science and Mathematics,* Vol. 13, pp. 311-312 (April, 1913).

A. Dresden, *An Invitation to Mathematics.* New York, Henry Holt and Company, 1936.

"Duplicating the Cube, Almost," *Scientific American Monthly,* Vol. 3, p. 364 (April, 1921).

Encyclopaedia Britannica: "Cube."

J. Gow, *A Short History of Greek Mathematics.* New York, G. E. Stechert, 1923.

T. L. Heath, *A History of Greek Mathematics,* Vol. I. London, Clarendon Press, 1921.

T. L. Heath, *A Manual of Greek Mathematics.* London, Clarendon Press, 1931.

E. Kasner and J. Newman, *Mathematics and the Imagination.* New York, Simon and Schuster, 1940.

F. Klein, *Famous Problems of Elementary Geometry,* tr. by W. W. Beman and D. E. Smith. Boston, Ginn and Company, 1897. Second edition revised and enlarged by R. C. Archibald: New York, G. E. Stechert, 1930.

G. M. Merriman, *To Discover Mathematics.* New York, John Wiley and Sons, 1942.

H. Roeser, "The Derivation and Applications of the Conchoid of Nicomedes and the Cissoid of Diocles," *School Science and Mathematics,* Vol. 14, pp. 790-796 (December, 1914).

W. W. Rupert, *Famous Geometrical Theorems and Problems.* Boston, D. C. Heath and Company, 1900.

V. Sanford, *A Short History of Mathematics.* New York, Houghton Mifflin Company, 1930.

D. E. Smith, *History of Mathematics.* Boston, Ginn and Company, 1925.

J. H. Weaver, "The Duplication Problem," *American Mathematical Monthly,* Vol. 23, pp. 106-113 (April, 1916).

J. H. Weaver, "Pappus's Solution of the Duplication Problem," *School Science and Mathematics,* Vol. 15, pp. 216-217 (March, 1915).

30. HISTORY OF MATHEMATICS IN THE UNITED STATES

An interesting and important chapter in the history of the United States is the story of the development of its mathematical productivity from nothing in Colonial days to world leadership at the present time. The first three centuries of our history produced little of value in the domain of mathematics. After 1875, however, a revolutionary change took place and our mathematicians made increasingly important contributions to the development of mathematics.

R. C. Archibald, *A Semicentennial History of the American Mathematical Society, 1888-1938.* Semicentennial Publications of the American Mathematical Society, Vol. 1, 1938.

G. D. Birkhoff, *Fifty Years of American Mathematics.* Semicentennial Publications of the American Mathematical Society, Vol. 2, 1938.

G. D. Birkhoff, "Fifty Years of American Mathematics," *Science,* ns Vol. 88, pp. 461-467 (Nov. 18, 1938).

A. D. Bradley, "Pennsylvania German Arithmetical Books," *Scripta Mathematica,* Vol. 5, pp. 45-51 (January, 1938).

F. Cajori, "American Contributions to Mathematical Symbolism," *American Mathematical Monthly,* Vol. 32, pp. 414-416 (October, 1925).

F. Cajori, "The Earliest Arithmetic Published in America," *Isis,* December, 1927.

F. Cajori, "The Rise of Mathematics in the United States," *Education,* Vol. 12, pp. 170-175 (November, 1891).

F. Cajori, "The Teaching and History of Mathematics in the United States," Bureau of Education Circular No. 3, 1891.

R. D. Carmichael, "Mathematics in the United States Today," *American Mercury,* Vol. 2, pp. 79-83 (May, 1924).

J. L. Coolidge, "Robert Adrain, and the Beginnings of American Mathematics," *American Mathematical Monthly,* Vol. 33, pp. 61-76 (February, 1926).

J. L. Coolidge, "The Story of Mathematics at Harvard," *Harvard Alumni Bulletin,* Vol. 26, p. 376 (1924).

J. L. Coolidge, "Three Hundred Years of Mathematics at Harvard," *American Mathematical Monthly,* Vol. 50, pp. 347-356 (June-July, 1943).

B. F. Finkel, "The Human Aspect in the Early History of the American Mathematical Monthly," *American Mathematical Monthly,* Vol. 38, pp. 305-320 (June-July, 1931).

T. S. Fiske, "Mathematical Progress in America," *Science,* ns Vol. 21, pp. 209-215 (February 10, 1905).

C. D. Hellman, "Jefferson's Efforts Toward the Decimalization of United States Weights and Measures," *Isis*, November, 1931.

L. C. Karpinski, "The Elusive George Fisher 'Accomptant'—Writer or Editor of Three Popular Arithmetics," *Scripta Mathematica*, (October, 1935).

L. C. Karpinski, *History of Arithmetic*. Chicago, 1925.

O. D. Kellogg, "Decade of American Mathematics," *Science*, ns Vol. 53, pp. 541-548 (June 17, 1921).

C. J. Keyser, "Mathematical Productivity in the United States," *Educational Review*, Vol. 24, pp. 346-357 (November, 1902).

G. A. Miller, "American Mathematics," *Popular Science*, Vol. 79, pp. 459-463 (November, 1911).

G. A. Miller, "American Mathematics During Three Quarters of a Century," *Science*, ns Vol. 59, pp. 1-7 (January 4, 1924).

G. A. Miller, "First Thousand Mathematical Works Printed in America," *Science*, ns Vol. 92, pp. 216-217 (September 6, 1940).

H. R. Phalen, "The First Professorship of Mathematics in the Colonies," *American Mathematical Monthly*, Vol. 53, pp. 579-582 (December, 1946).

L. G. Simons, "Algebra at Harvard College in 1730," *American Mathematical Monthly*, Vol. 32, pp. 63-70 (February, 1925).

L. G. Simons, "The German-American Algebra of 1837," *Scripta Mathematica*, September, 1932.

L. G. Simons, "The Influence of French Mathematicians at the End of the Eighteenth Century Upon the Teaching of Mathematics in American Colleges," *Isis*, Vol. 15, p. 104.

L. G. Simons, *Introduction of Algebra Into American Schools in the Eighteenth Century*. Washington, 1924.

H. E. Slaught, "Retrospect and Prospect for Mathematics in America," *American Mathematical Monthly*, Vol. 27, pp. 443-451 (December, 1920).

D. E. Smith, "The First Work on Mathematics Printed in the New World," *American Mathematical Monthly*, Vol. 28, pp. 10-15 (January, 1921).

D. E. Smith, "A Glimpse at Early Colonial Algebra," *School and Society*, Vol. 7, pp. 8-11 (January 5, 1918).

D. E. Smith and J. Ginsburg, *A History of Mathematics in America Before 1900*. Carus Mathematical Monograph No. 5. Chicago, Open Court Publishing Company, 1934.

D. E. Smith, "Thomas Jefferson and Mathematics," *Scripta Mathematica*, Vol. 1, pp. 3-14, 87-90 (1932).

W. D. Wood, "An Historical Outline of the Development of Mathematics in the United States during the Last Fifty Years," THE PENTAGON, Vol. 7, pp. 52-68 (Spring, 1948).

R. C. Yates, "Sylvester at the University of Virginia," *American Mathematical Monthly*, Vol. 44, pp. 194-201 (April, 1937).

THE PROBLEM CORNER

EDITED BY JUDSON W. FOUST

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solutions should not demand any tools beyond the calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before March 1, 1951. The best solutions submitted by students will be published in the Spring 1951 number of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Dr. Judson Foust, Central Michigan College of Education, Mt. Pleasant, Michigan.

PROBLEMS PROPOSED

31. *Proposed by William Douglas, Courtenay, British Columbia, Canada.*

Given any three parallel lines, construct an equilateral triangle with one vertex on each of the three lines.

32. *Proposed by the Problem Corner Editor.*

Two ladders lean in opposite directions across an alleyway between vertical buildings. The foot of each ladder is at the intersection of a building and the ground. If the ladders are thirty feet and forty feet long respectively and cross at a point ten feet above the ground, how wide is the alley?

33. *Selected from the tenth annual William Lowell Putman Mathematical Competition, March 25, 1950.*

In each of n houses on a straight street are one or more boys. At what point should all the boys meet so that the sum of the distances they walk is as small as possible?

34. *Proposed by Frank Moseley, State Teachers College Florence, Alabama.*

Substantiate the assertion made by Nathan Altshiller-Court in his *College Geometry* (page 66) that a triangle may have two equal external bisectors and yet not be isosceles.

35. *Proposed by Dr. Alfred Moessner, Gunzenhausen, Germany-Bayern.*

Can you submit a mathematical problem without numbers, which, however, leads to a solution with definite numbers? They are easy to solve, but they are peculiar because so few of them can be made. Here is an example: Walter has in his coin purse just as many dollars as William has cents. If one multiplies the number of Walter's dollars with itself, then the number is obtained which gives in cents the combined possession of Walter and William. How much money has Walter and how much money has William? Solution: If we assume that William has H cents then Walter has the value of $100H$ cents. We obtain the equation $H^2=(100+1)H$, therefore $H=101$. Therefore Walter has 101 dollars and William 101 cents.

SOLUTIONS

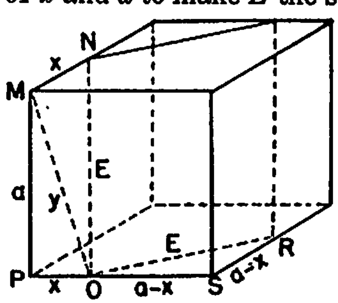
The Editor of the Problem Corner regrets that credit cannot be given for some correct solutions received since no names were signed to them.

8. *Proposed by the Problem Corner Editor.*

A cube with edge a has a hole cut in it so a larger cube can be passed through. Find the edge of the cube of maximum size which can be passed through.

Solution by J. E. Allen, Birmingham, Alabama.

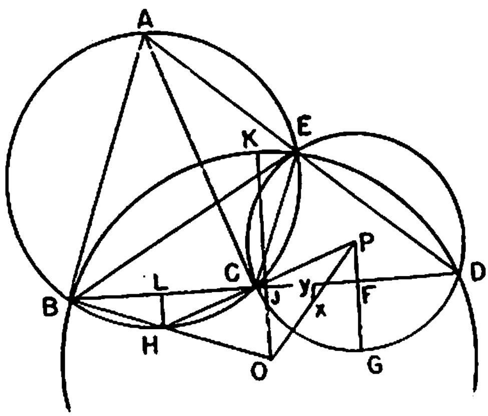
Let the edge of the large cube be E and find the relation of x and a to make E the side of a square as indicated in the figure. In $\triangle MNO$, $E^2=x^2+y^2$. In $\triangle MPO$, $y^2=a^2+x^2$. Therefore, $E^2=a^2+2x^2$. In $\triangle ORS$, $E^2=(a-x)^2+(a-x)^2=2a^2-4ax+2x^2$. Therefore $a^2+2x^2=2a^2-4ax+2x^2$ whence $x=\frac{1}{4}a$ and $E^2=9a^2/8$, or $E=1.06a$, approximately.



16. *Proposed by Geoffrey B. Charlesworth, Hofstra College, Hempstead, N.Y.*

Let ABC be an isosceles triangle with $AB=AC$, D any point on BC orBC produced, and E the intersection of AD with the circumcircle of triangle ABC. Let O and P be the centers of circles BDE and CDE. Find the locus of the mid-point of OP.

Solution by Earl T. Boone, Wayne University, Detroit, Michigan.



1. Let H be the point of intersection of OB with circle ABC and H' the intersection of PC with circle ABC. Let X be the midpoint of OP. Construct LH, KO, YX, and PG perpendicular to BC. Let D lie on BC produced. (Similar proof may be obtained if D is taken on BC.)
2. $<BOK$ is measured by arc$BK=\frac{1}{2}$ arcBKD. $<BED$ is measured by one-half major arcBD. Therefore angles BOK and BED are supplementary. Angles BED and BEA are supplementary. Therefore $<BOK=<BEA=<ACB$.
3. $<CPG$ is measured by arc$CG=\frac{1}{2}$ arcCGD. $<CED$ is measured by $\frac{1}{2}$ arcCGD. Therefore $<CPG = <CED$. $<AEC$ is measured by $\frac{1}{2}$ arcABC. $<ABC$ is measured by $\frac{1}{2}$arcAEC. Therefore angles AEC and ABC are supplementary. Angles AEC and CED are supplementary. Therefore $<CED=<CPG=<ABC$.

4. $<FCP$ and $<CPG$ are complementary. Therefore $<BCH'$ and $<ACB$ are complementary. Therefore arcABH' $=180° =$arcACH'. Similarly, $<BOK$ and $<JBO$ are complementary, and $<HBC$ and $<CBA$ are complementary. Therefore arc$ACH=180° =$arcACH'. Hence, H and H' are identical.

5. $JO/LH=JB/LB$. $PF/LH=CF/LC$. $XY=\frac{1}{2}(JO-PF)$ $=\frac{1}{2}LH \cdot (JB/LB-CF/LC)$. $LB=LC$. $JB-CF=\frac{1}{2}(2JB-2CF) = \frac{1}{2}(BC - CD) = LB$. Therefore $XY = \frac{1}{2}LH \cdot (LB/LB) = \frac{1}{2}LH$.

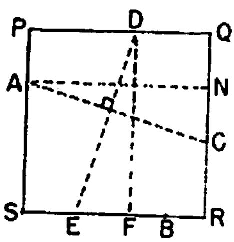
The desired locus may be defined as a line parallel to the base of the triangle and at a distance below it equal to one half of the distance from the orthocenter to the base.

Also solved by Otto C. Juelich, Forest Hill, N.Y.

22. *Proposed by the Problem Corner Editor. (From Christofferson,* Geometry Professionalized for Teachers, *Banta Publishing Company, 1933. Exercise 38, page 117.)*

Construct a square so that each side shall pass through a given point.

Solution by Lotta Stallma, Patterson, N.J.



Given points A, B, C, and D. Draw AC. Draw DE perpendicular to AC equal to AC. Draw EB. From A and C, drop perpendiculars to BE, intersecting at S and R. Draw a line parallel to BE through D, intersecting AS and CR at P and Q. $PQRS$ is the required square.

Proof: Construct DF perpendicular to SR and AN perpendicular to QR. In right triangles DEF and ANC, $DE=AC$ and $<CAN=<EDF$ since their corresponding sides are perpendicular. The triangles are therefore congruent and $DF=AN$. By construction, the angles at P, Q, R,

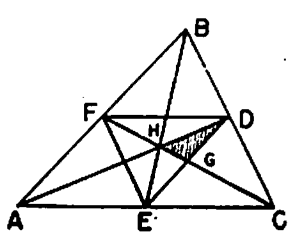
and S are right angles and the adjacent sides of the figure, which are equal to the perpendiculars DF and AN, are also equal. Therefore the figure is a square.

Also solved by William Douglas, Courtenay, British Columbia. Our attention is called to the fact that a complete discussion of this problem is to be found in Davis's *Modern College Geometry*, pages 71-72. Davis shows the possibility of twenty-four solutions for a given set of four points since they may be selected in pairs in six ways and to each of these six lines a perpendicular may be dropped from either of two points and along this perpendicular a given distance may be measured in either of two directions.

25. *Proposed by Frank Hawthorne, Hofstra College, Hempstead, N.Y.*

Triangle ABC has medians AD and CF meeting at H; E is the midpoint of AC; ED meets CF in G. Show that the area of triangle DGH is 1/24 of the area of triangle ABC.

Solution by Stanley Petersen, Wayne State Teachers College, Wayne, Nebraska.



$\triangle ABC$ is similar to triangles AFE, FBD, EDC, EFD and four times the area of each since the sides of $\triangle ABC$ are double the corresponding sides of these triangles. $\triangle FDE$ is divided into six triangles all of equal area one of which is $\triangle HDG$. (Triangles with equal altitudes are to each other as their base and triangles with equal bases are to each other as their altitudes.) Since $\triangle HDG$ is 1/6 of $\triangle EFD$ and $\triangle EFD$ is 1/4 of $\triangle ABC$, it follows that $\triangle HDG$ is 1/24 of $\triangle ABC$.

Also solved by Malcolm Humphrey, Mount Pleasant, Michigan; Robert Blasch, Hofstra College; Harry Tymkiw, Albion College; Harvey Fiala, Forman, North Dakota; Christ Louis Melis, Lynbrook, Iowa; Beta Chapter, Drake University; J. E. Allen, Birmingham, Alabama; Charles Standley, Kansas State Teachers College, Pittsburg.

26. *Proposed by Stanley Ogilvy, New York, N.Y.*

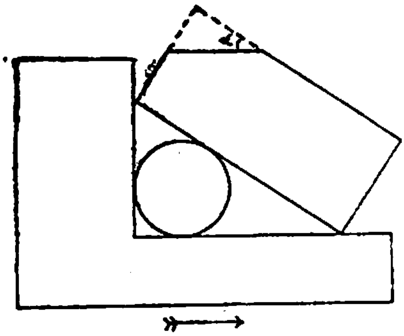
Four mothers, each with one daughter, went out to buy ribbons. Each bought twice as many yards as her daughter. Each person bought as many yards of ribbon as the number of cents paid per yard. No stores sold any ribbon in fractions of cents per yard. Mrs. Jones spent 76¢ more than Mrs. White. Nora bought three yards less than Mrs. Brown. Gladys bought two yards more than Hilda, who spent 48¢ less than Mrs. Smith. What is the name of Mary's mother?

Solution by Eva Joan Gilbertson, Central College, Fayette, Missouri.

Mrs. Jones, Mrs. White, Mrs. Brown, Mrs. Smith, Nora, Gladys, Hilda, and Mary bought respectively a yards at a¢ per yard, b at b¢, c at c¢, d at d¢, w at w¢, x at x¢, y at y¢, z at z¢. Then $a^2-76=b^2$ or $(a-b)(a+b)=76$ with possible factors (2,38), (1,76), and (4,19). If $a-b=1$ and $a+b=76$, we find that the values of a and b are not whole numbers as required. Similarly, (4,19) is eliminated. If $a-b=2$ and $a+b=38$, then $a=20$ and $b=18$. In like manner $y^2+48=d^2$ gives $y=4$ and $d=8$. Since $x-2=y=4$, then $x=6$ and one of the values of a, b, c, or d must be 12. Since all but c have been determined, $c=12$. But $w+3=c$, so $w=9$. One girl must have bought 10 yards since $a=20$, and inasmuch as x, y, and w have been determined, $z=10$. Therefore Mary's mother is Mrs. Jones.

Also solved by Robert P. Robinson, Iowa State Teachers College; Schuyler D. Hales, Drake University; J. E. Allen, Birmingham, Alabama; Harvey Fiala, Forman, North Dakota; Ignacio Tinoco, Jr., University of New Mexico; James Andrews, Hofstra College; Earl Eidson, Fresno, California; Robert Blasch, Hofstra College; Bernice Eidson, Santa Cruz, California.

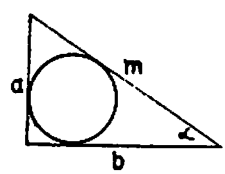
27. ***Proposed by Cleon C. Richtmeyer, Central Michigan College of Education, Mount Pleasant, Michigan.***



In order to saw a rectangular block at an angle, the block is laid on the moving table against a cylinder as indicated in the figure. If the block is m inches long, find a formula for the diameter of the cylinder necessary to cut the block at an angle α.

Solution by J. E. Allen, Birmingham, Alabama.

From Problem 24 of the Problem Corner: In any right triangle the sum of the two legs is equal to the sum of the hypotenuse and the diameter of the inscribed circle. Hence, diameter $=a+b-m$. But $a=m\sin\alpha$, $b=m\cos\alpha$. Therefore diameter $=m(\sin\alpha+\cos\alpha-1)$.



Also solved by Frederick Wendland, Wayne University; Ignacio Tinoco, Jr., University of New Mexico.

28. *Proposed by Norman Anning, University of Michigan, Ann Arbor.*

If 70 per cent have lost an eye, 75 per cent have lost an ear, 80 per cent an arm, 85 per cent a leg, what per cent, at least, must have lost all four?

Solution by Robert J. Wood, San Antonio, Texas.

If 70 per cent have lost an eye and 75 per cent have lost an ear there must be at least 45 per cent overlapping counting these percents from opposite ends of the group. Similarly, if 45 percent have lost both an eye and an ear and 80 per cent have lost an arm, there must be at least

25 per cent overlapping. If 25 percent have lost an eye, ear, and arm and 85 per cent have lost a leg, there must be at least 10 per cent overlapping or at least 10 per cent have lost all four.

Also solved by Stanley Peterson, Wayne State Teachers College; Harvey Fiala, Forman, North Dakota; and Ignacio Tinoco, Jr., University of New Mexico.

29. *Proposed by Cleon C. Richtmeyer, Central Michigan College of Education, Mount Pleasant, Michigan.*

An interesting variation of a familiar problem in the mathematics of finance may be stated as follows: A father sets aside \$4,000 for his son when he starts to college, to provide him with a fixed monthly income while he is in school. He is to receive equal payments at the end of each of the nine months of each of the four school years. If the \$4,000 is invested at 3% compounded monthly one month before he is to receive the first payment, how large will the monthly payment be?

Solution by Richard Campbell, University of Colorado.

Let B_i be the balance in the fund at the end of the ith month. Then $B_9 = A(1+i)^9 - WS_9$, $\quad S_9 = [(1+i)^9 - 1]/i$.

$$B_{12} = B_9(1+i)^3 = A(1+i)^{12} - Ws_9(1+i)^3$$

etc.

$$B_{45} = A(1+i)^{45} - WS_9[(1+i)^{36} + (1+i)^{24} + (1+i)^{12} + 1] = 0.$$

Solving for W and inserting $A = 4000$, $i = \frac{1}{4}\%$, we have

$$W = 4000(1.0025)^{45} \cdot 1/s_9 \cdot [(1.0025)^{12} - 1]/[(1.0025)^{48} - 1]$$

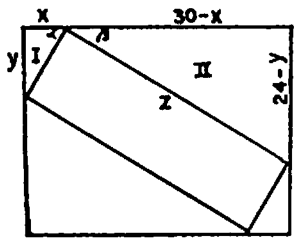
or

$$W = \$117.61 \text{ per month.}$$

30. *Proposed by John K. Osborn, Central Michigan College of Education, Mount Pleasant, Michigan.*

Find the length of the largest runner of carpet two feet wide that can be placed diagonally in a room 24 feet by 30 feet so that each of the four corners of the runner touch a wall of the room.

Solution by Robert Blasch, Hofstra College, Hempstead, N.Y.



In the figure (not drawn to scale), triangle I is similar to triangle II since they both are right triangles with α of I the complement of β of II. Therefore $x/(30-x) = y/(24-y)$ and $x^2=4-y^2$. Solving these simultaneously, $x^2=100/41$, $y^2=64/41$. But $z^2=(30-x)^2+(24-y)^2 = 1480-984/\sqrt{41}$, and $z=$ 36.4 ft., approximately.

Also solved by S. J. Vaughn, Warrensburg, Missouri; Robert J. Wood, San Antonio, Texas.

"It can be of no practical use to know that π is irrational, but if we can know, it surely would be intolerable not to know."

—E. C. TICHMARSH.

=∇=

"All the effects of nature are but mathematical results of a small number of immutable laws."

—LAPLACE.

=∇=

"I contend that each natural science is a real science in so far as it is mathematics."

—KANT.

Do not then train boys to learning by force and harshness; but direct them to it by what amuses their minds.

—PLATO.

=∇=

In 1726, arithmetic and geometry were studied during the senior year at Harvard College.

=∇=

From a point outside a square the distances to the three nearest vertices of the square are 20, 30, and 40. Find the side of the square. (Ans. 20.073)

—SCH. SCI. AND MATH.

=∇=

If there should be another flood,
Hither for refuge fly,
Were the whole world to be submerged,
This book would still be dry.

—A verse often found in a scholboy's Euclid.

=∇=

To square numbers between 75 and 125 we may use the identity,

$$N^2 = 100\ [100 + 2(N-100)] + (N-100)^2.$$

Thus, we find $N-100$, double it, add 100, annex two zeros, and finally add $(N-100)^2$. To illustrate, for 112^2 we find in order $112-110=12$, $2\times12=24$, $24+100=124$, $124\times100=12{,}400$, $12{,}400+12^2=12{,}544$.

=∇=

"Perhaps I may without immodesty lay claim to the apellation of the Mathematical Adam, as I believe that I have given more names (passed into general circulation) to the creatures of the mathematical reason than all the other mathematicians of the age combined."

—J. J. SYLVESTER.

=∇=

"Pa, how do you find the greatest common divisor?"
"Great scot! Ain't that been found yet?"

PROSTHAPHAERESIS

Logarithms owe their importance in arithmetic to the fact that they permit replacing certain operations by simpler ones; multiplication is replaced by addition, division is replaced by subtraction, etc. But logarithms are not the only quantities which permit such simplifications. Indeed, a table of natural sines can be used to replace a multiplication by a subtraction.[1] Such a method was devised by Wittich of Breslau. For a short time Wittich was an assistant to Tycho Brahe and they used the method in their calculations in 1582. The method disappeared upon the invention of logarithms in 1614.

The method of Wittich was called *prosthaphaeresis* and was based on the identity

$$\sin \alpha \sin \beta = \tfrac{1}{2}[\sin[90^\circ - (\alpha-\beta)] - \sin[90^\circ - (\alpha+\beta)]].$$

Four entries in a table of natural sines are required to compute a product with the aid of this formula. The following example in which a five-place table was used will illustrate the method.

Example: (0.8695 × (0.3170).

$$\begin{array}{ll} \text{Here, } \alpha = \text{arc}\sin(0.8695) = 60^\circ\ 24.1' & \\ \qquad \beta = \text{arc}\sin(0.3170) = 18^\circ\ 28.9' & \\ 90^\circ - (\alpha-\beta) = 48^\circ\ 4.8' & \sin 48^\circ\ 4.8' = 0.74408 \\ 90^\circ - (\alpha+\beta) = 11^\circ\ 7.0' & \sin 11^\circ\ 7.0' = 0.19281 \\ & 2\overline{)0.55127} \\ & \overline{0.27563} \end{array}$$

Hence, (0.8695) × (0.3170) = 0.27563.

Obviously, in applying this method, each factor must be reduced to a number between 0 and 1 before a table of sines can be used. Thus, $(869.5) \times (3.170) = (0.8695 \times 10^3) \times (0.3170 \times 10^1) = (0.27563) \times 10^4 = 2756.3$. We see here an analogy to the characteristics of common logarithms. It has been speculated that *prosthaphaeresis* may have given Napier the suggestion for his logarinthms.

=∇=

$\pi = 3 + 1/7 - 1/800$ (accurate to 5 significant figures)

[1]J. W. L. Glaisher, *Nature*, Oct. 10, 1889.

"If mathematical physics be annexed as a province of mathematics, a detailed professional mastery of the whole domain of modern mathematics would demand the lifelong toil of twenty or more richly gifted men."

—E. T. BELL.

=∇=

The following cryptarithm has two solutions. The x's indicate missing digits, not necessarily equal.

$$\begin{array}{r} xxx \\ xx \\ \hline xxx \\ xx4 \\ \hline xxx17 \end{array}$$

—AM. MATH. MONTH.

=∇=

"Freshman Mathematics": To show $P_1(2,3)$,$P_2(4,6)$, and $P_3(6,9)$ are collinear, we have $P_1P_2=P_2P_3=\sqrt{}(4+9)=2+3=5$, $P_1P_3=\sqrt{}\ (16+36)=4+6=10$, whence $P_1P_2+P_2P_3=P_1P_3$. Q.E.D.

=∇=

"The differential calculus is such a necessary part of mathematics that we may suppose that sooner or later it would have been discovered by someone."

—E. C. TICHMARSH.

=∇=

"The nineteenth and early twentieth centuries stored up an enormous potential of theoretical mathematics. All over the world, in every civilized nation, they were at work. Poincaré, Dedekind, Riemann, Lobachevski, Whitehead, Bernstein, Gibbs, Lorentz—these men carried mathematics far in advance of our capacity to make use of it at the time. By 1905, they had already equipped physics with the framework for the Curies' discovery of radioactivity, for Einstein's relativity, and for Planck's constant. But the big event of that year was the installation of the first steam turbine in a commercial power plant."

—GERALD PIEL.

Descartes spent nearly all his life in bed merely because he liked a bed better than a table as a place for work.

=∇=

3.14159265358979323846264338327 9

Now I, even I, would celebrate
In rhymes inapt, the great
Immortal Syracusan, rivall'd nevermore,
Who in his wondrous lore,
Passed on before,
Left men his guidance how to circles
mensurate.

—A. C. ORR, *Literary Digest*, 1906.

=∇=

Which would you prefer, a clock that is right but once a year or one that is right twice a day?

(A clock that loses a minute a day is right once a year, but a clock that doesn't go at all is right twice a day.)

=∇=

On the day before yesterday the weather man said: "Today's weather is different from yesterday's. If the weather is the same tomorrow as it was yesterday, the day after tomorrow will have the same weather as the day before yesterday. But if the weather is the same tomorrow as it is today, the day after tomorrow will have the same weather as yesterday." It is raining today, and it rained on the day before yesterday. What was the weather like yesterday?

(Note: The prediction was correct.)

=∇=

If $\sqrt{2}$ be expressed as a continued fraction its convergents are 1/1, 3/2, 7/5, 17/12, 41/29, 99/70, 239/169, etc.

1) The numerators are all atomic weights of radioactive isotopes.

2) Take any odd convergent. The denominator gives the hypotenuse of a triangle; the numerator split into two consecutive integers gives the other two sides. Thus: (3,4,5), (29,20,21), (169,119,120).

—MATH. GAZ.

THE BOOK SHELF

EDITED BY CARL V. FRONABARGER
Southwest Missouri State College

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Carl V. Fronabarger, Southwest Missouri State College, Springfield, Missouri.

Geometrical Tools: A Mathematical Sketch and Model Book. By Robert C. Yates. St. Louis Educational Publishers, Incorporated (122 N. 7th Street, St. Louis, Missouri), 1949. 194 pages. $3.50.

In the preface the author says, "This book has been designed especially for college students who are prospective teachers of mathematics." It is admirably suited for the purpose for which the author has designed the book. However, mathematics clubs and chapters of Kappa Mu Epsilon will find it a veritable gold mine of interesting ideas for club programs and club projects. The suggested projects are organized in such a way that topics of interest can readily be located and the book is written in such a way that it leads the student on to interesting ideas without "spoonfeeding" by "telling outright."

The author makes it clear that this is not a book on Modern Geometry or Synthetic Projective Geometry. However, some of the more interesting theorems, and the more readily adaptable theorems for the purposes of this book, have been included from these two geometries. In addition, however, the reader will find such topics as (1) The Geometry of Mascheroni, (2) The Geometry of Paper Folding, (3) How to Draw a Straight Line (linkages), (4) The Geometry of Poncelet-Steiner, (5) Constructions with the Parallel and Angle Ruler, (6) The Assisted Straightedge, and (7) Higher Tools and Quartic System. Topics such as these can provide many pleasant hours with mathematics.

Each section of the book is organized around a number of plates which facilitate the construction work suggested in the exercises. Brief introductions to each section orient the student to the problem under consideration and provide an abundance of reference material for further study.

The book has a rather attractive design—a design which invites one to look further. It is suprisingly free of typographical errors. This fact alone would indicate that the author took great pains in preparing the book. The approach to each idea has been carefully worked out so that a student should have a minimum of difficulty in mastering any section of the book. This is true even though each section of the book is quite independent of all other sections.

Geometrical Tools should prove to be of value, not only to prospective teachers, but to the teachers of undergraduate mathematics who are looking for something different with which to stimulate interest in the subject of mathematics, as well as provide for that inner satisfaction which comes to a student who has found something which affords a wealth of pleasure.

—H. VAN ENGEN.

The Concepts of the Calculus. By Carl B. Boyer. Hafner Publishing Company (31 E. 10th Street, New York, New York), 1949. 18 + 346 pages. $5.50.

This is a formidable but withal an attractive book. Sometimes pat and trenchant in style, sometimes ponderous (page 5: "Intuition, or the putative immediate cognition of an element of experience which ostensibly fails to adequate expression—"), it is always suggestive and thought-provoking. It could have been written only by a scholar long immersed in his subject and with the scholastic digestive power of a middle-sized university. Though in the preface the author disavows the intention to "display a meticulously precise erudition," yet the erudition is not successfully concealed. And "erudition" is exactly the word for it unless the author knows a bigger one, as is probable. The froth of this learning rises to the surface in the form of mouthfuls such as "autochthonous," "apodictic," "hylozoistic," "aporia," "categorematic," "indagation," "phoronomic," and so on; but pretty soon the reader gets to feel that these linguistic behemoths (now he's got me doing it!) are not rolled across the path in malice or ostentation, but simply in pure innocence about the limitations of human beings. In spite of this the reading is easy and clear if one has a big enough dictionary.

The general theme of the book is that the concepts of the calculus cannot in justice be ascribed to any one or two or even ten mathematicians. They were evolved through the long centuries by hundreds of individuals, including the obscure and forgotten as well as the brilliant figures of mathematical history. It is not correct, says author Boyer, that Newton or Leibniz, or Newton *and* Leibniz, invented calculus. They merely channeled the pertinent speculations and discoveries of two millenia or so into rules of procedure which speeded up the operational machinery. Of course, if we *must* pin the badge of "Discoverer" upon each of a few selected individuals, then these two unquestionably should have that honor. But before them came a galaxy of Greeks from Plato to Archimedes (with Euclid curiously excluded because his thinking was too precise to deal with the vague concepts and paradoxes not yet under mathematical control); a scattering of medieval scholars who revived the study of the classics and made contributions of their own; a brilliant group of precursors who collectively "almost did it,"—Stevin, Wallis, Kepler, Cavalieri, Descartes, to mention a few; and finally, following Newton and Leibniz, the theoreticians who bolstered the foundations. Of these Cauchy (says Boyer) did most of all in putting the subject in its present form; while the great triumvirate consisting of Weierstrass, Dedekind, and Cantor contributed much to the final structure.

To read this book is an intellectual experience. While the presentation would have been improved, in this writer's opinion, by the collaboration of someone with a less extensive vocabulary (to insure reconsideration, at least, of the linguistic startlers) and with perhaps a little more liking for a humorous touch now and then, yet with these minor reservations it is well and clearly written. It left this reader with a better understanding of the long and winding path leading up to differentials and integrals. Certainly anyone who attempts to explain the mysteries of calculus, either by teaching or writing, will gain much from the study of this important and stimulating book.

—R. S. UNDERWOOD.

Jacob Steiner's Geometrical Constructions With a Ruler. Trans. by M. E. Stark; ed. with an introduction and notes by R. C. Archibald. Scripta Mathematica (Yeshiva University, New York), 1950. 3 + 88 pages. $2.00.

It is unfortunate that over a hundred years elapsed between the time of publication of this book and its translation into English. Any student who has completed a course in either college or projective geometry will find this a quite simple and an extremely interesting book. However, it is so written that an interested person who has completed high school plane geometry can enjoy it.

The primary aim of the book is to demonstrate that any construction possible with ruler and compass can be made with ruler alone provided there is given, in the plane, some fixed auxiliary circle with its center.

In the first chapter harmonic ranges and pencils are defined metrically and several constructions with the ruler alone are delineated. In the second chapter the theory of harmonic ranges and pencils with regard to circles is developed sufficiently to permit a standard definition of poles and polars of circles and to examine their relation to tangents to a circle from a given external point. This chapter also deals extensively with centers of similitude of two circles and the locus of points with equal powers with respect to circles.

The third and climactic chapter uses many of the ideas developed heretofore to accomplish the primary aim of the book. This is done by a detailed presentation of the solutions to eight problems. In the appendix twenty-two additional problems are given that are also solvable under the same conditions. They are for the most part projective and not metric, and the constructions are given for most of them without proof. Many are duals so that the principle of duality is emphasized.

The notes by the translators are pertinent and most helpful as is their introduction which is mainly historical. This book should be in the library of every teacher of geometry.

—M. L. MADISON.

The Mathematics of Great Amateurs. By Julian Lowell Coolidge. Oxford University Press (114 Fifth Ave., New York, N.Y.), 1949. 8 + 211 pages. $6.00.

Professor Coolidge has devoted a chapter of this book to each of sixteen men who made noteworthy contributions to mathematics, but to whom this subject was not of primary interest. He discusses the mathematical problems and theory worked out by such men as Plato, Omar Khayyam, John Napier, and Buffon, and relates their work to the body of mathematical knowledge which existed in their time. In some instances he finds evidence that an idea whose origination is popularly attributed to some one man, is the product of other minds, and he does not hesitate to revaluate his achievements in this light.

Since the subject of Professor Coolidge's discussion are people of a number of countries, it is not surprising that our author has included quotations from their works in the original languages. This feature, however, makes the book difficult reading for all but those who are familiar with French, German, Italian, and Latin.

This book is an excellent reference for teachers and students of the history of Mathematics. In it there are many complete "first" solutions of now familiar problems, and reproductions of figures used by these men in publishing their work. It also contains a list of books and articles by the amateurs themselves, to which manuscripts the students' interest might well be directed upon reading Professor Coolidge's account.

—MARGARET OWCHAR.

Measuring Our Universe. By Oliver Justin Lee. The Ronald Press Company (15 E. 26th. St., New York 10, N.Y.), 1950. 10 + 170 pages. $3.00.

Written by a professor of astronomy and director, emeritus, of the Dearborn Observatory of Northwestern University, this little book is a volume of the Humanizing Science Series, published under the editorship of Jaques Cattell.

As the subtitle—From the Inner Atom to Outer Space —indicates, the book is devoted to a step by step account of the evolution of the methods of measurement of length from the crude "yardsticks" growing out of the practical experience of men for thousands of years to the millimicrons and megaparsecs of modern science.

Starting with a brief historical account of the development of our units of distances, the author describes precision measuring machines, the use of light waves as standards of length, parallax, distance to the sun and the dimensions within the Solar System, spectra as criteria of distance, interferometer methods, and the study of the cepheid variables.

The book ends with a chapter on the Palomar reflector and a conclusion entitled "Thinking it over."

—ALEXANDER W. BOLDYREFF.

PUBLICATIONS RECEIVED BY THE BOOK SHELF EDITOR

Analytic Geometry. By Raymond D. Douglass and Samuel D. Zelden. McGraw Hill Book Company (330 West 42nd Street, New York 18, New York), 1950. 9 + 216 pages. $2.75.

Plane Trigonometry With Tables. By Gordon Fuller. McGraw Hill Book Company (330 West 42nd Street, New York 18, N.Y.), 1950. 11 + 270 pages. $2.75.

Elements of Ordinary Differential Equations. By Michael Golomb and Merrill Shanks. McGraw Hill Book Company (330 West 42nd Street, New York 18, N. Y.), 1950. 9 + 356 pages. $3.50.

Plane Trigonometry (Alternate Edition) With Tables. By E. Richard Heineman. McGraw Hill Book Company (330 West 42nd Street, New York 18, N.Y.), 1950. 14 + 259 pages. $2.50.

Calculus and Analytic Geometry. By Cecil Thomas Holmes. McGraw Hill Book Company (330 West 42nd Street, New York 18, New York), 1950. 10 + 416 pages. $4.75.

Arithmetic for Colleges. By Harold D. Larsen. The MacMillan Company (60 5th Avenue, New York, N.Y.), 1950. 11 + 275 pages. $3.75.

Fundamentals of the Calculus. By Donald E. Richmond. McGraw Hill Book Company (330 West 42nd Street, New York 18, N.Y.), 1950. 11 + 233 pages. $3.00.

PLEASE!

If your are changing your address, please notify us, giving your old as well as new address. Otherwise, please leave instructions and postage with your postmaster for forwarding your copy. Unlike first-class matter, THE PENTAGON requires additional postage when remailed to a different address.

THE PENTAGON
310 Burr Oak St.
Albion, Michigan

INSTALLATION OF NEW CHAPTERS

EDITED BY LAURA GREENE

The PENTAGON is pleased to report the installation of two new chapters of Kappa Mu Epsilon. There are now forty-four chapters on the roll.

INDIANA ALPHA CHAPTER

Manchester College, North Manchester

Twenty-one student and faculty members were initiated as charter members of the Indiana Alpha Chapter at the installation ceremony on May 18, 1950. Dr. Harold D. Larsen, Vice-President of Kappa Mu Epsilon, served as the installing officer. Dr. Larsen was assisted by Everett Sauders and Thelma Fair.

The installation ceremony was preceded by a banquet in Elizabeth Hall at Manchester College. Gary Deveal furnished dinner music. After the installation Dr. Larsen gave a very interesting address entitled, "Some Famous Unsolved Problems of Mathematics."

The following officers of Indiana Alpha were installed: President, Everett Sauders; Vice-President, Ted Harman; Secretary, Thelma Fair; Treasurer, Dale Hill; Corresponding Secretary and Sponsor, J. E. Dotterer. Other charter members are Phillip Barnhart, William Bird, Max Boyer, Warren Garner, Richard Hagans, Lavon Hart, Robert Holcomb, Paul Hong, Robert Mertz, Donald Miller, Prant Pfeiffer, James Rowe, Howard Royer, Tom Summe, Denver Ulrey, and Robert Weimer.

PENNSYLVANIA ALPHA CHAPTER

Westminster College, New Wilmington

The Pennsylvania Alpha Chapter of Kappa Mu Epsilon was installed May 17, 1950 at Westminster College. Dr. Loyal F. Ollmann, National Treasurer, served as the installing officer. He was assisted by Paul E. Brown, George W. Conway, Jr., and John Williamson.

Following the installation ceremony an informal reception was held for the members and their guests.

The following officers of Pennsylvania Alpha were installed: President, John Williamson; Vice-President, John S. Kratz, Jr., Secretary, Barbara Williams; Treasurer, Henry L. Barnhart; Sponsor, George W. Conway, Jr.; Corresponding Secretary, Paul E. Brown. Other charter members are Herbert B. Bolt, Jr., Frederick W. Cassell, William E. Dembaugh, Jr., William W. Duff, Robert E. Gunnett, William H. Hazlett, John H. Hodges, William A. Margraf, Charles W. McGary, Jr., Thomas R. Petrecca, Samuel R. Salaga, Milford J. Shimer, Howard F. Snyder, Jr., Robert J. Thomas, and Charles Tod.

"The celebrated mathematician Alexis Claude Clairault (now Clairaut) . . . was born in Paris in 1713 and died there in 1765. His treatment on curves of double curvature (printed in 1731) received the approbation of the Academy of Sciences, August 23, 1729. Fontenelle, in his certificate of this, calls the author sixteen years of age, and does not strive to exaggerate the wonder, as he might have done, by reminding his readers that this work, of original and sustained mathematical investigation, must have been coming from the pen at the ages of fourteen and fifteen. The truth was, as attested by DeMolieres, Clairaut had given public proofs of his powers at twelve years old."

—DEMORGAN.

KAPPA MU EPSILON NEWS

EDITED BY CLEON C. RICHTMEYER, *Historian*

At their spring initiation, **Illinois Beta** inducted thirteen new members. The ceremony was followed by a banquet.

— + —

Illinois Gamma devoted one of the fall meetings to a social party for entering freshmen. The president of the chapter told about KME and questions were answered about the nature of the organization. Other meetings featured the annual banquet and a post exam picnic. Illinois Gamma also sponsored a meeting for student papers at the Chicago convention of the National Council of Teachers of Mathematics in April.

— + —

Illinois Delta and **Iowa Alpha** were among the chapters sending delegations to the N.C.T.M. meeting in Chicago. Bob Allender of Iowa Alpha presented a paper.

— + —

Kansas Delta held a very successful high school guest night. Several parents called in to say how much they appreciated having something truly "school-like" rather than some over-worked "extracurricular" activities. The program included talks on topics of general interest such as Zeno's paradoxes, trisection of an angle, Mobius rings, etc. Refreshments were served following the program, and then the group went to the observatory to look through the telescope. There was a short talk explaining what they had seen.

— + —

Raymond Gillespie of **Michigan Alpha** presented a paper "Solutions of the Quadratic Equation" before the Undergraduate Mathematics Conference held in conjunction with the March meeting of the Mathematical Association of America at Ann Arbor. The Conference was sponsored by **Michigan Gamma.** Richard Little of **Michigan Beta** also presented a paper at this meeting entitled "Mechanical Brains."

Ann Lucile Lewis of **Mississippi Alpha** is one of 85 outstanding graduate students from 24 countries to be awarded a Rotary Foundation Fellowship for advanced study abroad during the 1950-51 school year. She will study mathematics at the University of Strasbourg, France.

—+—

Missouri Alpha cordially invites all members of Kappa Mu Epsilon to the campus of Southwest Missouri State College for the biennial convention to be held April 27-28, 1951. Twenty-four people were initiated into the Missouri Alpha chapter during the past year.

An invited speaker at the annual banquet of **Missouri Beta** was Mr. Ray Watson, market and sales analyst for Hallmark Brothers of Kansas City. Mr. Watson presented a paper, "Market Potentials—Their Use and Instructions," embodying results of his own research and investigations.

—+—

Missouri Epsilon held a picnic dinner at the home of Dr. Floyd Helton to close the year's activities. A guest speaker for the April meeting was Dr. Paul Burcham, chairman of the department of mathematics, University of Missouri, who spoke on the topic "Methods of Summability of Infinite Series."

—+—

The **New Jersey Alpha** Chapter was invited to a special program at **New Jersey Beta** on February 15. Professor Howard Fehr of Teachers College spoke on special problems in mathematics.

—+—

Each year **New York Alpha** presents an award to the best student of freshman mathematics. The winner is selected on the basis of his grades in freshman mathematics and a two-hour competitive examination. The winner this year was John Richard Prussen. During the past semester the chapter has sponsored student help sections. KME members gave their time to help those who came in at the assigned periods.

—+—

At the annual banquet twelve persons were initiated into **Tennessee Alpha.**

Texas Alpha has recently raised the standards and requirements for admission to KME.

—+—

Dorothy Karner of Wisconsin Alpha presented a Paper on "Inversion" at the Chicago meeting of the National Council of Teachers of Mathematics.

—+—+—+

PROGRAM TOPICS, SPRING SEMESTER, 1949-50

Alabama Alpha, Athens College

A Problem in Indeterminate Equations, by Glenn Hymer
Infinite Products, by Rodney Hughes
Separation of Triangles and Quadrilaterals into Equal Areas by Means of Straight Lines, by Johnnie Tuten
The Spherical Glass—Conical Glass Problem, by Rebecca Davenport
Scales of Notation, by Glenn Brock

Alabama Gamma, Alabama College

The Number System, by Annie Laura Falls
The Calendar, by Lyda Gay Donald and Hortense Barnes

California Alpha, Pomona College

Diophantine Equations, by William Paxton
Polygonal Numbers, by Dr. Jean Walton
Existence Proof for Transcendental Numbers, by Richard Edelstein
Boolean Algebra, by Rodney Weldon
The Numbers Racket, by Dr. Aubrey J. Kempner
Rigidity of the Earth, by Dr. Walter Whitney

Illinois Beta, Eastern Illinois State College

Mathematics Through Sound Films
Some Sources of Inspiration, by Dr. E. H. Taylor
Automorphic Numbers, by Dr. J. A. Ringenberg

Illinios Gamma, Chicago Teachers College

Concepts of Infinity and Infinite Sets, by Norman Stein
Discriminants of Polynomials, by Howard James
Relativity (Films)

Illinios Delta, College of St. Francis

Einstein, by Mary L. Hodor
Mathematical Viewpoint of a Philosopher, by Pat Kane
Omar Khayam, by Rita Grogan
Journal Reports, by B. Arsenau and Lois Gilgen

Iowa Alpha, Iowa State Teachers College

Problem of Apollonius, by Eddie Sage
Boolean Algebra, by Robert Robinson

Iowa Beta, Drake University

Application of Boolean Algebra to Actuarial Science, by Howard Hill
Boolean Algebra, by Jack Matsui

Kansas Beta, State Teachers College, Emporia

Thought Problems, by Varvel and Strauch
Binary Numbers, by Lloyd Wilkie
Geometric Paradoxes, by Walter Woods
Dimensional Analysis, by Alvin Rusk
Algebraic Paradoxes, by Lester Wilson
Logic, by Lorna Holle
The Number of the Beast, by Robert Burch

Kansas Delta, Washburn Municipal University

On the Hausdorff Summability of Derived and Conjugate Derived Fourier Series, by Dr. Paul Eberhart
Zeno's Paradoxes, by Kenneth Lake
Trisection of an Angle, by Nancy Martin
Topics in Algebra, by Betty Moffett
Mobius Rings, by Dale Long

Michigan Alpha, Albion College

Solutions of the Quadratic Equation, by Raymond Gillespie
Tesselations, by Dr. H. D. Larsen

Michigan Beta, Central Michigan College

Congruences, by Darold Comstock
Mechanical Brains, by Dick Little
Applications of Mathematics to Forces, by Richard Hyman

Mississippi Gamma, Mississippi Southern College

Modern Developments in Mathematics, by Dr. H. L. Smith

Missouri Alpha, Southwest Missouri State Callege

Mathematics Quiz Program, by Joe Guida
Some Geometric Interpretations of the Binomial Theorem, by Earl Bilyeu
Diophantine Problems, by J. H. Skelton
Puzzle Problem Solutions, by James Jakobsen
Given, by Joe Guida
Archimedes, by Jessie Belvea
Apollonius, by Evelyn Ruark
Euclid, by Ernest Fontheim
Pythagoras, by George Nash
Eudoxus, by Patricia Maddux
Thales, by Earl Phillips
The Algebra of Sets, by James Jakobsen

Missouri Beta, Central Missouri State College

Cryptography, by Wilfred Poese
The Maneuvering Board, by Charles Sigrist
The Expression of Cos θ as $f(n, \cos\theta)$ by Charles Warden
Prime Numbers, by Michael Stratton
A Minimum Body of Mathematical Concepts and Information for High School Graduates, by Mrs. Margaret M. Shrake
Volumes and Surface Areas of Regular Polyhedra, by Charles Warden
Market Potentials—Their Use and Instructions, by Ray Watson

Missouri Epsilon, Central College
The Star Gazer, (Galileo), by Eva Gilbertson
The Duodecimal System of Counting, by John P. Karnes
Methods of Summability of Infinite Series, by Dr. Paul Burcham
Evariste Galois, by Mary Julia Groce
Calculating Prodigies, by Robert Christian
Peaucellier's Cell, by Mary Cronbaugh
New Jersey Alpha, Upsala College
Foundation Studies for Algebra, by Dr. Norgaard
Mathematics of the Arabs, by James Christakos
Foundation Study of Analytic Geometry, by Victor Valentino
Axioms and Postulates, by William Stachel
Type Theorems from College Algebra, by Edward Alquist
Hyperbolic Functions, by Ethel Larsen
Previews of Graduate School Topics, by Professor Robert Reed and Professor Donald Lindtreit
The Exponential, by Robert Rupprecht
New York Alpha, Hofstra College
The Nature of Applied Mathematics, by Prof. J. E. Eaton
Lissajous Figures, by Otto Juelich
Biometry, by Jane Brandt
Ohio Alpha, Bowling Green State University
Ceramic Problems, by Dr. A. J. Hammer
Ohio Beta, College of Wooster
The Calculus of Variations, by Professor M. P. Fobes
Cooperative Engineering Plan at the University of Cincinnati, by Professor J. H. Kindle
Trisection of the Angle, by Charles Sauder
Ohio Gamma, Baldwin-Wallace College
Centrifugal Force in Industry, by Mr. Walter H. Craig
The Mathematics of Sound, by Dr. E. C. Unnewehr
The Science and Philosophy of Mathematics, by George Smolensky and Robert Miller
Exponential Functions and Their Applications to Science, by Robert Noblit
The History of Mathematics, by Theron Schwegler
Mathematics at NACA, by Betty Hostetler
Quality Control, by Dr. Fred Leone
Tennessee Alpha, Tennessee Polytechnic Institute
The Calculus of Finite Differences, by R. H. Moorman
Mathematical Puzzles, by C. R. Barlow
Short Cuts in Mathematics and *Odds on a Parley Card*, by S. D. Srite
The Age of Methuselah, by D. S. Marion
Applications of Mathematics, by R. O. Hutchinson
Texas Alpha, Texas Technological College
Fourier Series (illustrated by electronic devices), by John Craig
Trisecting an Angle, Squaring of Circle, Duplication of Cube, by Dorothy Miles

Mathematical Fallacies, by Don Medlock
Utilizing Education, by Dr. E. N. Jones

Wisconsin Alpha, Mount Mary College

History of the Calculus, by Mary Hunt
Relativity, by Kathleen Hanley
Number Systems, by Wanda Kropp
The Slide Rule, by Janet Haig
Planetariums, by Betty Prossen

THE 1953 CONVENTION

The Ninth Biennial Convention of Kappa Mu Epsilon will be held at St. Mary's Lake Camp, Battle Creek, Michigan on April 17 and 18, 1953. The hosts will be the three Michigan chapters.

St. Mary's Lake Camp, operated by the Michigan Education Association, has facilities that are particularly convenient for large conferences. It boasts of its good food, fine single beds with inner spring mattresses, and opportunities for quiet recreation, all at reasonable prices. The Camp is located about four miles north of Battle Creek, home of Kellogg and Post Cereals. Battle Creek is conveniently located on the Michigan Central Railroad and main bus lines, and is readily accessible by auto. The popularity of the Camp as a meeting place is such that it is necessary to make reservations well in advance.

The National Council of Kappa Mu Epsilon deems it to be for the best interests of the fraternity to accept the invitation of the Michigan chapters to hold the Ninth Biennial Convention at St. Mary's Lake Camp. The activity of the Michigan chapters in Kappa Mu Epsilon affairs and their uniform excellence in performance assures the fraternity that another successful convention will be enjoyed by the delegates and members of Kappa Mu Epsilon attending the Ninth Biennial Convention.

H. VAN ENGEN,
National President

CHAPTERS OF KAPPA MU EPSILON

ALABAMA ALPHA, Athens College, Athens.
ALABAMA BETA, Alabama State Teachers College, Florence.
ALABAMA GAMMA, Alabama College, Montevallo.
CALIFORNIA ALPHA, Pomona College, Claremont.
COLORADO ALPHA, Colorado A & M College, Fort Collins.
ILLINOIS ALPHA, Illinois State Normal University, Normal.
ILLINOIS BETA, Eastern Illinois State College, Charleston.
ILLINOIS GAMMA, Chicago Teachers College, Chicago.
ILLINOIS DELTA, College of St. Francis, Joliet.
INDIANA ALPHA, Manchester College, North Manchester.
IOWA ALPHA, Iowa State Teachers College, Cedar Falls.
IOWA BETA, Drake University, Des Moines.
KANSAS ALPHA, Kansas State Teachers College, Pittsburg.
KANSAS BETA, Kansas State Teachers College, Emporia.
KANSAS GAMMA, Mount St. Scholastica College, Atchison.
KANSAS DELTA, Washburn Municipal University, Topeka.
MICHIGAN ALPHA, Albion College, Albion.
MICHIGAN BETA, Central Michigan College, Mount Pleasant.
MICHIGAN GAMMA, Wayne University, Detroit.
MISSISSIPPI ALPHA, State College for Women, Columbus.
MISSISSIPPI BETA, Mississippi State College, State College.
MISSISSIPPI GAMMA, Mississippi Southern College, Hattiesburg.
MISSOURI ALPHA, Southwest Missouri State College, Springfield.
MISSOURI BETA, Central Missouri State College, Warrensburg.
MISSOURI GAMMA, William Jewell College, Liberty.
MISSOURI DELTA, University of Kansas City, Kansas City.
MISSOURI EPSILON, Central College, Fayette.
NEBRASKA ALPHA, Nebraska State Teachers College, Wayne.
NEW JERSEY ALPHA, Upsala College, East Orange.
NEW JERSEY BETA, New Jersey State Teachers College, Montclair.
NEW MEXICO ALPHA, University of New Mexico, Albuquerque.
NEW YORK ALPHA, Hofstra College, Hempstead.
OHIO ALPHA, Bowling Green State University, Bowling Green.
OHIO BETA, College of Wooster, Wooster.
OHIO GAMMA, Baldwin-Wallace College, Berea.
OKLAHOMA ALPHA, Northeastern State College, Tahlequah.
PENNSYLVANIA ALPHA, Westminster College, New Wilmington.
SOUTH CAROLINA ALPHA, Coker College, Hartsville.
TENNESSEE ALPHA, Tennessee Polytechnic Institute, Cookeville.
TEXAS ALPHA, Texas Technological College, Lubbock.
TEXAS BETA, Southern Methodist University, Dallas.
TEXAS GAMMA, Texas State College for Women, Denton.
TEXAS DELTA, Texas Christian University, Fort Worth.
WISCONSIN ALPHA, Mount Mary College, Milwaukee.