# THE PENTAGON

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Kappa Mu Epsilon, national honorary mathematics fraternity, was founded in 1931. The object of the fraternity is four-fold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievement in the study of mathematics in the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

# TEACHING MATHEMATICS IN COLLEGE

FORBES B. WILEY[1]
*Professor, Denison University*

Quite likely you are familiar with the John G. Saxe poem of the blind men who went to "see" the elephant. If so you will recall how upon their arrival at the elephant's house—shall we say, at the zoo?—each man was introduced to the animal by coming in contact with one and but one part of its anatomy: a leg, a side, a tusk, an ear, the tail, the trunk, no two men touching the same member. Following their sight-seeing trip the blind men returned home, contending loud and long among themselves on the journey as to just what the elephant was like. The first man said that the elephant was like a tree, the next, a wall, and so through the list: a spear, a fan, a rope, a snake. One can imagine them arguing far into the night and arriving at no decision. Here the poem closes, the author has made his point. I have often thought, however, that it would be interesting if he had added one more stanza in which the elephant was given an opportunity to express his views of the visit. One can picture him standing in his stall sizing up the blind men during their inspection trip. Doubtless at its close he shook himself, pulled his separate parts together, started his trunk swinging, pendulum like, and did some philosophical pondering — undoubtedly about blind men, maybe about men in general, and possibly about elephants.

And now as I turn to the subject at hand, teaching mathematics in college, do not misunderstand me. I would not for a minute infer that my readers are blind men, or that I am going to show you an animal unfamiliar to you, or that I have an elephant on my hands. My thought is a far cry from any of these. I merely wish to infer that college teaching of mathematics is as composite a subject as the blind men's elephant. Indeed, I should like to speak for each of the blind men in turn, and then I should like

[1]Professor Wiley retires at the end of the year after forty years of service at Denison University. He enjoys a wide reputation for his successful teaching of mathematics.—Ed.

to make what might be termed a brief statement on behalf of the elephant.

The first side of my composite subject is teaching. This is something which everyone, consciously or unconsciously, does. In fact it is as frequent and as universal an act as is that of eating or sleeping. Perhaps this might account for the thought, somewhat antiquated now, that if a person seems to show no promise of success elsewhere, let him turn to teaching. "Those who can, do; those who can not do, teach."

The teaching of which we would speak, however, is far above that level. It has attained the rank of a profession, and far from insignificant are the qualifications demanded for admission to its membership. This holds true in spite of the fact that the dictionary tells us that law, medicine, and theology are generally considered to be the three "learned" professions. (The quotation marks are mine, not Webster's!)

Perhaps right at this point a word of caution is needed. Far be it from me to say that a so-called professional occupation gives its members a more significant place in this world than does a non-professional one. If you ask me, "Is not the man in the pulpit doing a greater service for humanity than the man behind the grocery counter?," I think that the answer would be, "First show me the man." Is medicine a more strategic profession in which to work than law? Again I say that it is a question of the man, not of the profession.

Call it a professional or non-professional occupation, as you will, let it be dignified with the label of learned or not, teaching is what I chose to follow. And now with the experience of almost a lifetime of teaching behind me, I can say that I am glad that I made that choice and that I would make it again, were I starting life anew.

This, undoubtedly, is a confession, or a declaration, that I am somewhat of an idealist in my philosophy of life. I have come to the point where I place spiritual values above material ones. To me, contact of spirit with spirit, personality with personality gives one an opportunity to live on life's highest plane. Here we have an exceptional opportunity to experience what a college president under whom I have served termed, "life at its best."

The above statement must not be interpreted as meaning that the teacher has a monopoly on such an opportunity; surely, to a greater or less extent it is available to all. Some make the greater use of it, however. The sculptor, the artist, the poet, the architect, the musician, each endeavors to express the message which is within his soul. Each of these uses his own particular medium for conveying that message. Meaningful it is for those who can read the writing. In untold numbers lives have been made the richer for those who have read and comprehended.

The teacher in the classroom uses no such transmitting medium. He deals directly, personality with personality. In some strange unexplainable way—one of the laws of the spirit—some of his personality is implanted into the lives of his pupils and, in return, some of theirs into his. It is a reciprocal transaction, a give and a take. Thus one life lives on in the life of another, this to be passed on into the life of still another. In this way link upon link is added to the chain, a striking suggestion of life eternal even here upon earth, one of the richest rewards for a true teacher.

This characterization of teaching—ideal, I grant you, but who would dare to set the standards lower?—compels me to turn my attention more directly to the teacher, a side of my subject which is implied, if not mentioned. Humble indeed must one in this profession feel when he realizes the opportunities and the responsibilities which are his, and when he faces the criteria for excellence that these same opportunities and responsibilities demand of him.

First of all the teacher must have ability in the subject which he is to teach. Along with this ability must go an adequate background of knowledge. Neither this ability nor this knowledge can be superficial; they must be of that genuine quality which commands the respect of the student. Necessary as this ability and this background of knowledge are, they are not sufficient. They must not be static, they must be dynamic with life and growth. But even this does not say quite enough; this life and growth must have an element of enthusiasm in them, a bubbling enthusiasm which is contageous.

But ability and knowledge in one's subject, even though they be impressive, dynamic, and enthusiastic, will be largely lost unless there be the ability to know the mind of the student. He is another element in our many-sided question. The teacher must come to the level of the student's comprehension and, consciously or unconsciously, know and make use of the laws of learning. The attitude towards the student must not be a cold one; there must be the warmth of understanding, sympathy, and patience as that student, apathetic and stumbling at times, struggles forward. There must be rejoicing with him when the light breaks, especially so when there is the thrill of discovery. Most demanding of all, however, the true teacher must possess that quality of soul which ever beckons its possessor towards the highest ideals in life, and, in turn, transmits some of that same quality into the lives of his students.

Humbly we teachers of lesser magnitude bow low before the great teachers who have attained these heights. The world today stands in sore need of such as these. Even in the passing of centuries their light does not fade. Theirs are the truly great names of history.

And now having taken a look at teaching and also at the teacher, the third view of our elephant is on the mathematical side.

The statement made earlier in this paper that my choice of a profession was, and would be again, that of teaching, is inseparable in my mind from the clause which I would now add: the subject which I choose to teach is mathematics. It is not from a background of ignorance that I make this statement; I have taught other subjects, many different ones in my earlier years, and enjoyed the teaching of each one of them. As a teacher, however, no one of these possesses the appeal that is made to me by mathematics. Here I find a wealth of material which fascinates me, here I find many different possibilities in methods of teaching which challenge my ingenuity.

In speaking to my students I have often characterized mathematics as a subject which demands correctness, completeness, and conciseness of thinking and expression—the truth, the whole truth, and nothing but the truth, if you will. This however is not sufficient for the complete description. Although a mathematical truth, even when dressed rather slovenly in the garments of mathematical symbolism, is still a mathematical truth, and although a mathematical proof, even when moving in slow and cumbrous steps, is still a mathematical proof, the true mathematician is not satisfied. He is content with neither the sloven nor the cumbrous. For him mathematics is an art as well as a science; he has standards of elegance in both his thinking and his expression and will devote hours of his time to attain them.

There are also what one might term the corollaries to the teaching of mathematics. The exacting demands of mathematics upon one's ability in reasoning and in the expression of his thoughts make it insistent that logic and the correct use of English be taught in the mathematics classroom. Here too is given an excellent opportunity to train the student how to present his thoughts before his fellows in a convincing manner, and to defend his position against attack. I have thought, too, that I have never seen gestures more naturally used than when I have watched a student who in the warmth of enthusiasm is "selling" his treatment of a mathematical situation to his fellows.

It seems quite fitting at this point to speak of mathematics as a language—for its users, a universal language. The mathematician creates symbols for the representation of his ideas. With this mathematical shorthand he builds his concise language. First there are symbols for the more elementary concepts and then, as occasion arises, for the more and more involved ones. Herein lies the secret of no small amount of his mental achievement. In fact that student of mathematics who is the most adept in the invention and the use of symbols in representing his ideas is, other things being equal, the one who best conserves his time, the most successfully clarifies his thinking, and the most fully discovers the relationships existing among those ideas.

There is depth to the subject, even depths unsounded, and some mathematicians are digging downwards to study the foundations; there is height, even heights unscaled, and some are ever climbing higher into these upper regions; there is breadth, even boundaries unreached, and some are exploring these wider reaches. But the field is not alone for the explorer; there is a large demand for the expositor, there is rich food for the philosopher, there is a wide and enticing field for the teacher.

We must not overlook that side of mathematics to which the designation "applied" has been given, regions where the subject is not thought of as mathematics for mathematics sake but for the sake of advancement in some other field—mathematics as a tool. This must claim at least a small part of the attention of all who follow mathematics and a large part of the attention of many.

In fact mathematics is a subject which thoroughly permeates our civilization and our lives, conscious or unconscious as we may be of this fact. No clock ticks, no wheel turns, no engine runs, no airplane flies, no missile reaches its mark, no trumpet sounds, no drum-head vibrates, no bridge carries its load, no cathedral supports its dome, no structure rises, no shaft sinks, no industry operates, no business carries on, no radio functions, no atom is split, no planet moves in its orbit which, were each one of them able to speak, could not reveal a fascinating tale of the mathematics involved in its inception, its construction, its operation or its motion.

I have expressed earlier in this article two choices of mine, teaching and mathematics. I now add a third—in college. Here again I need not plead ignorance of other teaching areas. In fact it has been my interesting experience to teach at every level in our educational system from the one-room country school with all of the first nine grades, through the small town high school, the very large city school, the secondary school academy, the college, and on into graduate work up to the granting of masters degrees. Every level possesses its strong individual appeal.

Presumptuous indeed it would be of me in writing for readers of THE PENTAGON to dwell for long upon the college side of my subject. "Carrying coals to Newcastle" would characterize such an error. It is quite likely that many of you who have had less than four years on the campus have seen as many, if not more, sides of college life than have I with my forty-six years in college halls. You know of the advantages and the pleasures of life in a college community. However I do venture to make a comment or two from the standpoint of the teacher.

Although there is charm in the naivete and the youthful enthusiasms of the younger pupils of the secondary schools, for me this is more than offset by the fact that in college one is dealing with maturer minds. Here the teacher has the opportunity to think more deeply with the student into the beginning paths of the subject, to advance further with him along its well used highways, and to explore some of its enticing byways. Here too one is dealing with a higher percentage than he meets in high school of those who are going to use their mathematics. Among this number the teacher meets, through the years, many a choice student mind. Here arises for that teacher an obligation to furnish that mind with material that will promote growth. I have long contended that too few freshman courses throughout the land make exacting demands upon the brighter students. Many such become bored and their interest is lost. We must give greater attention than we now do to the individual student. Too much teaching is geared down to the weakest, too little geared up to the best. The latter turns elsewhere, often to extra curricular activities, to find something which will tax his talents.

But, you will say that all thus far is so idealistic, let's get down to earth; one must eat, you know. This is true enough. Fortunately one can speak with a greater degree of optimism on the realistic side of the subject than he could a few years ago. The public is becoming conscious of the fact that teachers must have advances in salaries. The result is felt both in the public schools and in the colleges. One realizes that there still is room for improvement, and we do not suggest to anyone that he go into the profession of teaching if he desires to accumulate a fortune. Another point on the realistic side is that the graduate schools are calling for more students, with fellowships and teaching assistantships for those who can qualify. Still further, the demand for well qualified teachers for the colleges is strong. I know of several positions now temporarily filled, and some not filled, which are waiting for the well qualified applicant.

And now perhaps we have viewed a sufficient number of the parts of our composite subject and it is time to let the elephant have his closing stanza. I think that he would pull himself together and remind us that he is not a mess of disjointed parts, but an integrated whole. There is something, probably the spirit of the teacher, which knits the parts of his teaching of mathematics in college into one commanding whole. I think that the elephant would say that we have placed him high on a pedestal; that he too walks the earth within the reach of humans. This should assure us that in spite of the fact that no one of us is perfect, it is still possible for us to become excellent teachers of mathematics. It is the keeping of the eyes on the ideal that saves the day. And, finally, I think that Teaching of Mathematics in College, personified by the elephant, would invite ever more of the strong students of mathematics to come and see him—and remain. For those of you who do, may your years be as many and your joys be as full as mine have been.

"The mathematical ignorance of the average educated person has always been complete and shameless."

—SIR OLIVER LODGE.

# RAMIFICATIONS IN CRYPTOGRAPHY

RAMONA H. GOLDBLATT
*Student, Chicago Teachers College*

Cryptography is a form of secret writing in which a key is used. It is often confused with another type of writing called ciphering. When communicating in cipher the message is written so that it is readable to anyone. Then it is made secret by substituting numbers for letters or by substituting certain letters for others. In cryptography the message also is originally written so that it is readable to all. It is made secret by placing the letters in a certain pattern or form, agreed upon by sender and receiver.

Examples of secret writing appear almost as early as the beginnings of language. The earliest example is found in the Bible where Jeremiah, the prophet, wishing to conceal the name "Babel" writes it as "Sheshak," replacing the second and twelfth letters of the Hebrew alphabet from the beginning (b, b, 1) by the second and twelfth letters from the end (sh, sh, k). Later the ancient Greeks used the scytale, a stick-like device for decoding messages. Interest in secret writing was displayed in the sixteenth century when Porta, a mathematician of Naples, published a book on the subject and earned himself the title, "Father of Modern Cryptology." During World Wars I and II, governments and their agents in foreign lands used secret codes to send essential information.

The kinds of cryptograms most used are of three distinct types. The first type is the transposition cryptogram. This may be defined as an alteration of the position of the letters. The simplest form is writing the message backwards, as

*NOOS TUO EB LLIW LOOHCS.*

It can be increased in difficulty by joining all the letters together and rearranging them in different forms. They can be arranged in shorter syllables or in columns of different lengths.

The second type is the grille cryptogram. In this type a number of insignificant letters, words, or numbers are added to the original words. The message is decoded by the use of a grille in the form of a rectangle with certain squares cut away. When the grille is placed over the writing the significant words are disclosed. An example of a communication and a grille with which to solve it are given in Figures 1 and 2.

| | | | | | | |
|---|---|---|---|---|---|---|
| *MAT* | *235* | *OIL* | *586* | *MAI* | *L95* | *684* |
| *PLM* | *ONE* | *591* | *TWO* | *971* | *HIS* | *OUR* |
| *LMN* | *ESS* | *CAT* | *FIV* | *9LL* | *PUT* | *ALL* |
| *158* | *321* | *LOW* | *IOU* | *ENT* | *915* | *IAL* |

Fig. 1

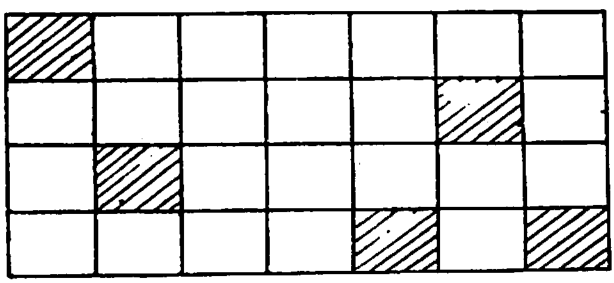

Fig. 2

Longer and more detailed communications can be decoded by writing them in such a manner that the grille can be placed over them in as many as four different positions to reveal a separate message each time.

The third type is the scytale cryptogram. The scytale is a small staff of wood around which a long narrow strip of parchment is wound in spiral form so that the edges of the parchment meet. The message is written while the paper is wound in this manner. When the writing is removed from the scytale only a series of unrelated letters is seen. To decode the communication the receiver must have a scytale identical with that of the sender.

Cryptography is an interesting field and one that can be a source of great diversion. Anyone can make up cryptograms. Solving them is a challenge to the imagination. Let's have fun with a few of our own. I have conceived a series of my own cryptograms of the transposition type. These make use of matrices. Suppose the key to solve these indicates that the first number determines the width of the matrix and the second the length. You receive the message,

*MAS67ZA ATTR IVT TIHEOI IHC5 E692E S7CFL OM61 OCIN.*

Using the key you put the letters in the form of a six-by-seven matrix (Fig. 3). By reading one set of diagonals of

| | | | | | |
|---|---|---|---|---|---|
| *M* | *A* | *S* | *6* | *7* | *Z* |
| *A* | *A* | *T* | *T* | *R* | *I* |
| *V* | *T* | *T* | *I* | *H* | *E* |
| *O* | *I* | *I* | *H* | *C* | *5* |
| *E* | *6* | *9* | *2* | *E* | *S* |
| *7* | *C* | *F* | *L* | *O* | *M* |
| *6* | *I* | *O* | *C* | *I* | *N* |

Fig. 3

the matrix, the message is revealed to be,

*MATHEMATICS IS THE CORE OF CIVILIZATION.*

The other numbers in the code are insignificant as the first two numbers are the key and reveal the matrix form. Solving a secret message is fun, isn't it?

Now let us take a message and put it in cryptic form. Suppose that we wish to write,

*THE CONVENTION IS A GREAT SUCCESS.*

The number of letters in the message must be counted. We find that there are twenty-eight. We can make the matrix in a form of a square six by six by adding eight more digits. Our message will then look something like this:

*THE CO5NVE3NT6ION IS A G4REA6T SU6CC9ESS6.*

Before we can send our message we must place it in the form that we are using following our key. The result is something completely unintelligible like this:

*TN6 A6C9H VIG TSEE EO4 RUSC3 NIE6 SON TSA C65.*

When the writing is sent, the decoder knows from the key which he has that the first two numerals encountered give him the form of the matrix. Putting the coded message in

| | | | | | |
|---|---|---|---|---|---|
| *T* | *N* | *6* | *A* | *6* | *C* |
| *9* | *H* | *V* | *I* | *G* | *T* |
| *S* | *E* | *E* | *E* | *O* | *4* |
| *R* | *U* | *S* | *C* | *3* | *N* |
| *I* | *E* | *6* | *S* | *O* | *N* |
| *T* | *S* | *A* | *C* | *6* | *5* |

Fig. 4

that matrix form (Fig. 4) and solving he gets the original message.

Now that we are no longer tyros, I have a message for you:

*G3T R2P5 LOCHT OUY OAEOC KU5RD.*

Attempting a solution should challenge you. The difficult part is to discover the key. After that it is easy. If you find the new key you are a

*F9T5 PICOG H4R YRE NEPAR,*

which uses still another key for its solution.

"Mathematicians are often asked why they spend their lives trying to solve such curious problems. What good is it to know that every number is the sum of four squares? Why do you want to know about prime-pairs? What does it matter that $\pi$ is rational or irrational?

"A Mathematician faced with these questions is in much the same position as a composer of music being questioned by someone with no ear for music. Why do you select some sets of notes and have them repeated by musicians, and reject others as worthless? It is difficult to answer except to say that there are harmonies in these things which we find that we can enjoy."

—E. C. Tichmarsh.

# ELEMENTS OF LEWIS CARROLL

DOROTHEA REIFFEL
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An amazing aspect of the writings of Lewis Carroll is the different and delightful interpretation that they present to almost every type of reader. Children, for example, are enchanted by the fantastic "Wonderland" discovered by Alice; adults, on the other hand, are intrigued by the superb satire and the unique logic that Carroll loved to insert into all of his work. A scene between Alice and the Cheshire Cat illustrates a typical characteristic of this work. The cat is trying to prove that he is mad, so he argues:

> To begin with . . . a dog's not mad. You grant that? . . . You see a dog growls when it's angry and wags its tail when it's pleased. Now I growl when I'm pleased and wag my tail when I'm angry. Therefore I'm mad.[1]

From a logical standpoint, of course, this argument is fallacious; but this is certainly a delightful illustration of the false logic.

The acme of Carrollan satire appears in a letter to the Senior Censor of Christ Church in burlesque of one written by the Professor of Physics enumerating the needs of his department at Oxford. Mr. Carroll found that he needed such things as:

> A piece of open ground for keeping Roots and practicing their extraction: it would be advisable to keep Square Roots by themselves, as their corners are apt to damage others.
>
> A narrow strip of ground . . . for investigating the properties of Asymptotes, and testing practically whether Parallel Lines meet or not.[2]

Thus, even using so serious a subject as mathematics, Lewis Carroll wrote in his own inimitable fashion, making

[1]Lewis Carroll. *Alice in Wonderland, Through the Looking Glass, and Other Comic Pieces.* Toronto, L. M. Dent and Sons, 1930, p. 55.
[2]Stuart Dodgson Collingwood. *The Life and Letters of Lewis Carroll.* New York, Century Co., 1898, p. 162.

fun of anything that occurred to him. He addressed everyone. A student struggling to learn the Calculus would be wise to relax his studies for a few minutes to read about that very complex subject, *The Dynamics of a Particle.* He would discover, for example, that particles of the order genius are called enlightened. "Moment is the product of mass into velocity (and) . . . no moment is ever really lost by fully enlightened Particles."[1] If the student were having trouble with differentiation, he would only have to follow Carroll's procedure. For example.

> If L.S. = Leader in the Saturday (a valueless particle), the first differential = L.S.D.
> = pounds, shillings, pence (an enlightened particle.)
>
> With succeeding differentials, the enlightenment decreases, and if a C is added, it vanishes completely.[2]

Because he realized how too many people feel about "Ambition, Distraction, Uglification, and Derision"[3], Lewis Carroll was constantly inventing intriguing problems to interest his readers and invite solutions. His questions include:

> (For students): A takes in 10 books in the Final Examination and gets a 3rd Class: B takes in the Examiners and gets a 2nd. Find the value of the Examiners in terms of books. Find also their value in terms in which no Examination is held.
> (For betters): Find the total Loss incurred by three men (who bet on three losing horses) *a*) in money, *b*) in temper.[4]
>
> (For anyone): A and B began the year with only £1000 apiece. They borrowed nought; they stole nought. On the next New Year's Day they had £60,000 between them. How did they do it? Solution: They went that day to the Bank of England.

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[1]Lewis Carroll, *The Complete Works of Lewis Carroll.* New York, Random House, n.d., p. 1134.
[2]*Ibid.*, p. 1135.
[3]Lewis Carroll, *Alice in Wonderland*, etc., p. 80.
[4]Lewis Carroll, *The Complete Works*, etc., p. 1136.

A stood in front of it, while B went around and stood behind it.[1]

Lewis Carroll is also credited with the invention of Double-Talk in his famous poem, "Jabberwocky" from *Through the Looking Glass.* Alice is completely confused when she first hears the poem, but Humpty-Dumpty later explains some of the words. *Mimsy,* for example, is a combination of *flimsy* and *miserable;* to *gyre* means to go round and round like a gyroscope; *brillig* is simply the time of the afternoon when the broiling of the dinner is begun.[2] Carroll continued in this manner to invent appropriate words for the story that he was telling.

Although these illustrations are obviously trivial, there are many similar parts in Carroll's writings that resemble the paradoxes of Zeno in their implications. For instance, consider the question of the clock that is right once a year and the one that is right twice a day. You would probably prefer the latter. Think again, though; a clock that loses a minute a day is right once a year, while one that doesn't go at all is right twice a day. Now which would you prefer?[3]

Another of the unanswered questions posed by Carroll is, "Where does the day begin?"[4] He was able to solve few of the problems that occurred to him, but his cogitating about them is well worth reading. In one essay, Achilles is found testing Euclid's first Proposition on the Tortoise:

(A) Things that are equal to the same are equal to each other.

(B) The two sides of this Triangle are things that are equal to the same.

(Z) The two sides of this Triangle are equal to each other.

. . .Z follows logically from A and B, so that anyone who accepts A and B as true must accept Z as true.[5]

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[1] *Ibid.,* p. 1046.
[2] Lewis Carroll, *Alice in Wonderland, etc.,* p. 160.
[3] Lewis Carroll, *The Russian Journal and Other Selections.* J. F. McDermott, ed. New York, E. P. Dutton and Co., 1935, p. 43.
[4] *Ibid.,* p. 41.
[5] Lewis Carroll, *The Complete Works, etc.,* p. 1226.

In the delightful farce that follows (in which the list of propositions increases infinitely) Carroll manages to shed some light on the difference between a principle of inference and a theorem about implication. After a weary argument, the "warriors" reach no conclusion but to rename each other "Taught-Us" and "A Kill Ease."[1]

In a short play, *Euclid and His Modern Rivals*, which Carroll considered his most important work, he tried to show through the Ghost of Euclid that the original *Elements*, in form and sequence, should be retained in the teaching of Elementary Geometry. Such a topic might easily form the basis of a very dull debate; not so with Lewis Carroll. This was one of the subjects he felt would be unharmed under light treatment.[2] As the play proceeds, the principal geometry texts since Euclid's are each examined and discarded. For example, after studying the work of Henrici, one of the characters of the play concludes:

> I can't resist giving you just one more tit-bit—the definition of a square at page 123: "A quadrilateral which is a kite, a symmetrical trapezium, and a parallelogram is a square!" And now farewell, Henrici! Euclid, with all thy faults, I love thee still.[3]

Another character of the play, Professor Minos, becomes thoroughly enraged when he reads Wilson's *Syllabus*. He says:

> On page 17 I find a "Question." State the fact that "all geese have two legs" in the form of a theorem. This I would not mind attempting; but, when I read the additional request to "write down its converse theorem," it is so powerfully borne in upon me that the writer of the Question is probably himself a biped, that I feel I must, however reluctantly, decline the task.[4]

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[1] *Ibid.*, pp. 1225-1230.
[2] Lewis Carroll, *Euclid and His Modern Rivals*. London, Macmillan and Co., 1885, p. x.
[3] *Ibid.*, p. 96.
[4] *Ibid.*, pp. 199-200.

Until now, I have purposely avoided mentioning Charles Lutwidge Dodgson (Lewis Carroll's real name) because he represents, from a standpoint of mathematics, a good but not outstanding mathematician who did not even use the Calculus in his investigations. As a matter of fact, there is a rather surprising entry in his diary of December 19, 1897, in which he says that he was unable to find more than two right triangles with rational sides; he found those with sides 20, 21, 29 and 12, 35, 37. Dodgson does, however, represent a superb logician who made extensive contributions to his field. He slyly inserted lessons of logic throughout his writings so that his readers were hardly aware that they were being taught. We find such examples of logical fallacies as a conversation between Alice and the Cheshire Cat in which the cat is "begging the question" when he says:

> We're all mad here. I'm mad. You're mad . . . you must be . . . or you wouldn't have come here.[1]

An excellent, but ridiculous, illustration of "nonsequitur" reasoning is the conversation in *Alice in Wonderland* between the March Hare and the Hatter. The March Hare has been accused of ruining the Hatter's watch by oiling it with butter. The Hare defends himself by saying, "It was the *best* butter, you know."[2]

The summing up of Dodgson's work in Logic appears in *Symbolic Logic,* an extensive group of problems in reasoning to be solved by the reader. After a very serious introduction by Dodgson in which the reader is given directions to follow, Lewis Carroll assumes control and presents the problems. A typical example is the following:

1) Babies are illogical;
2) Nobody is despised who can manage a crocodile;
3) Illogical persons are despised.[3]

These logical puzzles become increasingly complex, but never tedious. They present further illustration of Lewis Carroll's amazing faculty for creating problems that

[1]Lewis Carroll, *The Complete Works*, etc., p. 72.
[2]Lewis Carroll, *Alice in Wonderland*, etc., p. 59.
[3]Lewis Carroll, *The Complete Works*, etc., p. 1242.

are both stimulating and whimsical, a result very difficult to achieve.

Lewis Carroll was, then, a man who had a touch of genius that can never be thoroughly analyzed, but can only be illustrated. Although he is not numbered among the great mathematicians, he left his mark in our very complex mathematics heritage that provides a welcome sidelight for anyone who is fortunate enough to have read his writings.

"There is no mathematical process, either elementary or advanced, which is not the product of mathematical research of some bygone age."

—G. A. Bliss.

"Mathematics is like an endless game against a skilled opponent. If we can think of the right move, we win. Once we have made the right move, we gain some definite piece of knowledge which is never afterwards in doubt. How to think of the right move is another question. It is largely a matter of experience. Mathematical technique consists of the accumulated bright ideas of thousands of years."

—E. C. Titchmarsh.

# A FINITE GEOMETRY OF TWENTY-FIVE POINTS

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In Euclidean geometry a plane, or even a line, has an infinite number of points. Now in the geometry which we are going to discuss there are only twenty-five points and therefore it is called a finite geometry. The twenty-five points are letters of the alphabet arranged in three blocks which were evolved by the aid of higher number theory. The arrangement of the letters is as follows:

| (1) | (2) | (3) |
|---|---|---|
| *A B C D E* | *A I L T W* | *A X Q O H* |
| *F G H I J* | *S V E H K* | *R K I B Y* |
| *K L M N O* | *G O R U D* | *J C U S L* |
| *P Q R S T* | *Y C F N Q* | *V T M F D* |
| *U V W X Y* | *M P X B J* | *N G E W P* |

This arrangement and a few assumptions were introduced by L. R. Lieber in *The Education of T. C. Mits.*[1] By formulating additional assumptions and definitions, many theorems corresponding to those of Euclidean geometry can be proved.

It is essential that the basic assumptions do not contradict each other. In fact, we are free to select any basic assumptions that we please, as long as they are not contradictory, and build a geometry on these assumptions. This will be illustrated by this finite geometry of twenty-five points.

**Assumptions and Definitions.**

1. A straight line shall mean any row or column in any of the three blocks and is determined by any two of its points.

2. A point-pair shall be congruent to another point-pair when both pairs occur in rows or columns and if the number of steps between the points is the same in both pairs. (Note that congruence in this finite geometry does

[1]L. R. Lieber. *The Education of T. C. Mits.* New York. W. W. Norton & Co., 1944, pp. 153-167.

not have the same meaning as in Euclidean geometry since we are concerned only with the number of steps between points and their position.)

3. Two straight lines are parallel if they have no point in common.

4. If two lines have a single point in common, that point is called the point of intersection.

5. If two or more lines intersect in a point they are said to be concurrent.

6. A triangle is formed by a triple of points whose sides are straight lines determined by taking the triple of points in pairs.

7. Two lines are perpendicular if one of them is a row and the other a column in the same block.

8. If there exists a triple of points on the same line such that the number of steps from the first point to the second point is the same as the number of steps from the second point to the third point, then the second point is the bisector of the line segment.

9. A circle is a set of points such that any one of them taken with the center is a point-pair which is congruent to every other such point-pair.

10. A tangent is a straight line that has one and only one point in common with a circle.

11. A quadrilateral is a quadruple of points whose sides are straight lines determined by taking the four points in pairs of first and second, third and fourth, first and third, and second and fourth.

12. A rhombus is a quadrilateral whose sides are parallel in pairs and all of whose sides are congruent.

**Illustrations of Assumptions and Definitions.**

1. $AE$ in block (1), $ER$ in block (2), and $TF$ in block (3) are straight lines.

2. $KO$ in block (1) is congruent to $MJ$ in block (2); $OK$ in block (1) is congruent to $BY$ in Block (3).

3. $AC$ in block (1) is parallel to $PT$ in block (1).

4. $S$ is the point of intersection of $PT$ in block (1) and $AM$ in block (2).

5. $PT$ in block (1), $AM$ in block (2), and $JL$ in block (3) are concurrent. All have only $S$ in common.

6. $EL$ in block (2), $LK$ in block (1), and $EK$ in block (2) define the triangle $ELK$.

7. $LE$ is perpendicular to $SH$ in block (2).

8. $N$ in block (2) is the bisector of $UNB$; $B$ in block (2) is the bisector of $NT$.

9. $AC$ and $DA$ in block (1), $AL$ and $TA$ in block (2), and $AQ$ and $OA$ in block (3) define a circle with center $A$.

10. $CW$ in block (1) is the only line in any of the blocks that has only one point in common with the circle just described and therefore is a tangent to that circle.

11. The four points $GJPS$ define a quadrilateral with sides $GJ$ and $PS$ in block (1) and sides $GP$ and $JS$ in block (3).

12. The quadrilateral $GJPS$ is a rhombus since the four congruent sides are parallel in pairs.

We will now illustrate for our finite geometry a number of theorems which correspond to Euclidean theorems.

*Theorem I. The altitudes of a triangle are concurrent.*

Consider triangle $HAR$ whose sides are $HA$, $AR$, and $HR$. By assumptions 7 and 4, $ARJVN$ in block (3) is perpendicular to $HA$ through $R$, $AXQOH$ in block (3) is perpendicular to $AR$ through $H$, and $ABCDE$ in block (1) is perpendicular to $HR$ through $A$. The only point in common to these three lines is $A$. Thus by assumptions 4 and 5, the altitudes of triangle $HAR$ are concurrent.

*Theorem II. The medians of a triangle are concurrent and are divided by the point of concurrency in the ratio of 2:1.*

Consider triangle $HAR$. By assumption 8, the midpoint of $HA$ is $Q$ in block (3). By assumption 4, the points $Q$ and $R$ determine the median $PQRST$ in block (1). Similarly, the points $V$ and $H$ determine the median $SVEHK$ in block (2), and the points $M$ and $A$ determine the median $ASGYM$ in block (2). The only point in common is $S$. Thus by assumptions 4 and 5, the medians of triangle $HAR$ are concurrent.

In block (1), from the midpoint $Q$ of side $HA$ to the point of concurrency $S$ there are two steps, and from the point of concurrency $S$ to the vertex $R$ there are four steps. In block (2), from the point of concurrency $S$ to the midpoint $V$ of side $AR$ there is one step, from the vertex $H$ to the point $S$ there are two steps, from the midpoint $M$ of side $HR$ to the point $S$ there are two steps, and from the point $S$ to the vertex $A$ there are four steps. Thus, the point $S$ divides each median of triangle $HAR$ in a ratio of 2:1.

*Theorem III. The perpendicular bisectors of the sides of a triangle are concurrent.*

Consider triangle $HAR$. In block (3) the midpoint of $HA$ is $Q$ whence its perpendicular bisector is $QIUME$. Similarly, $VTMFD$ is the perpendicular bisector of $AR$ in block (3), and $KLMNO$ is the perpendicular bisector of $HR$ in block (1). The only point in common to these three lines is $M$. Hence, the perpendicular bisectors of the sides of triangle $HAR$ are concurrent.

*Theorem IV. The points of concurrency of the altitudes, medians, and perpendicular bisectors all lie on a straight line corresponding to Euler's line in Euclidean geometry.*

Consider triangle $HAR$. In the previous theorems we saw that $A$ is the point of concurrency of the altitudes, $S$ is the point of concurrency of the medians, and $M$ is the point of concurrency of the perpendicular bisectors. These points lie on the straight line $ASGYM$ in block (2).

*Theorem V. There exists one and only one tangent line to a circle at a designated point on the circle.*

Consider the circle with center at $A$ and passing through the points $C$, $D$, $L$, $T$, $Q$, and $O$. Selecting at random the point $L$ on the circle, the only lines containing $L$ are $KLMNO$ and $BGLQV$ in block (1), $AILTW$ and $LERFX$ in block (2), and $JCUSL$ and $HYLDP$ in block (3). Obviously, $LERFX$ is the tangent line since all the other lines contain more than one of the points of the circle.

*Theorem VI. The diagonals of a rhombus are perpendicular and bisect each other.*

Consider the rhombus $GJPS$ whose sides are $GJ$, $PS$, $GP$, and $JS$ and whose diagonals are $GS$ and $JP$. By assumptions 3 and 2, $GJ$ and $PS$ in block (1) are parallel and congruent, and so also are $GP$ and $JS$ in block (3). Therefore $GJPS$ determine a rhombus. In block (2), $M$ is common to $JP$ and $GS$. According to assumptions 4 and 7, $JP$ and $GS$ are perpendicular. In block (2), from $J$ to $M$ is one step and from $M$ to $P$, the vertex diagonally opposite the vertex $J$, is one step. Hence, $M$ bisects $JP$. Similarly, from vertex $G$ to $M$ is two steps and from $M$ to vertex $S$ is two steps, so that $M$ bisects $GS$. Thus, the diagonals of the rhombus $GJPS$ are perpendicular and bisect each other.

Many other propositions may be verified for our finite geometry. The reader may find it of interest to obtain examples which illustrate the following selected theorems:

1. Two straight lines each parallel to a third straight line are parallel to each other.

2. Perpendiculars to the same straight line are parallel.

3. The medians to equal sides of an isosceles triangle are equal.

4. The diagonals of a parallelogram bisect each other.

5. A tangent to a circle is perpendicular to the radius drawn to the point of contact.

6. The perpendicular at the middle point of a chord of a circle passes through the center.

"Anyone who has experienced the esthetic thrill which mathematics can induce . . . knows that mathematics not only has beauty, but that it is a pure beauty of relation and structure, condensed and unified in its expression, unsuspectedly rich in its implications and universal in its significance."

—W. WEAVER.

## THE MATH STUDENT BLUES

(Tune: *I've Been Working on the Railroad.*)

I've been working on a problem
All the livelong night,
I've been working on a problem
To hit that quiz just right.
Don't you hear the teacher raving,
"Get up early in the morn,
Study hard or you'll be flunking
Sure as you were born."

I've been wondering about that formula
And how it's ever got,
I've been wondering about that answer
And all that tom-e-rot,
Cause I hear the teacher raving,
"Get up early in the morn,
Study hard or you'll be flunking
Sure as you were born."

I've been working all this winter,
All this livelong term;
I've been listening to professors
And for rest my heart does yearn.
I've been goin' to lab'ratory,
To gym and math class too—
If I go to many other things,
I don't know what I'll do.

—NEW YORK ALPHA CHAPTER.

# ECCENTRICITY AND SLOPE[1]

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**Introduction.** The material for this paper was suggested by the observation that since the slope of a line determines one of the important properties of the line, namely, its direction, and the eccentricity of a conic one of its important properties, namely, its type, there might be a possibility of establishing a correspondence between lines of varying slope $m$ and conic sections of varying eccentricity $e$. The fact that the slope of a line may take on all real values, positive and negative, while the value of the eccentricity is always positive, must be taken into account in establishing the correspondence. The lines considered are those through the origin and the conics considered have their centers or vertices (in the case of the parabolas) at the origin and their axes on the coordinate axes.

**The Correspondence.** Every line $y = mx$ is made to correspond to a family of conics of eccentricity $e = m$ as follows.

| | |
|---|---|
| $m = 0$: | Circles, $x^2 + y^2 = c^2$ |
| $0 < m < 1$: | Ellipses, $e^2x^2/c^2 + e^2y^2/c^2(1 - e^2) = 1$ |
| $m = 1$: | Parabolas, $y^2 = \pm 4cx$ |
| $m > 1$: | Hyperbolas, $e^2x^2/c^2 + e^2y^2/c^2(1 - e^2) = 1$ |

The *Y-axis*, for which the slope becomes infinite, is associated with a family of hyperbolas whose eccentricity becomes infinite. The asymptotes coincide with the *Y-axis*.

Lines of negative slope are associated in a similar manner with conics whose foci are on the *Y-axis*. With this in mind, it will not be necessary to discuss them further.

As a result of this correspondence, a particular line is associated with a family of conics. A pair of points on the line at a fixed distance $c$ from the origin is used to determine a particular member of the family—that one whose foci are at the point $(\pm c, 0)$. Thus a correspondence is established

[1] Presented to the Upper New York State Section of the Mathematical Association of America at Rochester, New York, on May 10, 1947.

between pairs of points on a line and particular members of the family of conics associated with the line. For example, the line $y = x/2$ is associated with the family of ellipses $x^2/4c^2 + y^2/3c^2 = 1$, while the pair of points on this line at a distance equal to 1 from the origin correspond to the particular ellipse $x^2/4 + y^2/3 = 1$.

**The Director Circle.** The circle $x^2 + y^2 = c^2$, for a fixed value of c, passes through all the pairs of points on all the lines $y = mx$ which are at a distance equal to $c$ from the origin. Each pair of points on each line, for $m > 0$, determines a central conic or a pair of parabolas whose foci are at the point $(\pm c, 0)$. These are the same points in which the circle cuts the $X$-axis. Each circle can for this reason be thought of as a director circle for the pair of points and their corresponding conics.

**Relationships for Reciprocal Slopes.** The points of intersection of the different conics for a fixed value of $c$ are related in a special way if the values of the eccentricity of the ellipse and hyperbola are taken as reciprocals. Let the eccentricity of the ellipse be $e_1$, that of the hyperbola be equal to $1/e_1$. Because of symmetry it is sufficient to consider only one point of intersection. The $x$-coordinate of the point of intersection of the ellipse and hyperbola, designated by $x_{EH}$, is equal to $c$. The $x$-coordinate of the intersection of the parabola and the ellipse is $x_{PE} = (c/e_1)(1-e_1)/(1+e_1)$. Similarly, $x_{PH} = (ce_1)(1+e_1)/(1-e_1)$. It is seen that $x_{EH}$ is the geometric mean between $x_{PE}$ and $x_{PH}$.

If $c$ is alowed to vary, the point $(PE)$, whose $x$-coordinate was just given, moves along a line of slope $(2\sqrt{e_1})\sqrt{(1+e_1)/(1-e_1)}$. The point $(PH)$ moves along a line of slope $(2/\sqrt{e_1})\sqrt{(1-e_1)/(1+e_1)}$. The slope of the line $y = 2x$ is the geometric mean of these two slopes. This line $y = 2x$ meets each parabola $y^2 = 4cx$ at the point where $x = c$, that is, this line meets the parabolas in points directly above the point $(EH)$ which was shown to have an $x$-coordinate equal to the geometric mean of the $x$-coordinates of $(PE)$ and $(PH)$.

**Conics Determined by the Line $x = k$.** The line $x = k$ intersects each line of the family $y = mx$ in a point whose $y$-coordinate $h$ is given by $h = km$. This point and its symmetric point with respect to the origin are associated with the conic of eccentricity $e = m$ and with $c = \sqrt{(k^2+h^2)} = k\sqrt{(1+e^2)}$. For a fixed value of $k$ and for $m = e$ between 0 and 1 the corresponding conics are ellipses with semi-axes $a$ and $b$ equal to $(k/e)\sqrt{(1+e^2)}$ and $(k/e)\sqrt{(1-e^4)}$, respectively.

As $e \to 0$, $c \to k$, $a \to \infty$, $b \to \infty$, the foci of the ellipses approach limiting positions at $(\pm k, 0)$ and the axes of the ellipses become infinite.

As $e \to 1$, $c \to k\sqrt{2}$, $a \to k/2$, $b \to 0$, the foci and vertices of the ellipses approach the same limiting positions $(\pm k\sqrt{2}, 0)$, whereas the minor axis approaches zero. The ellipse degenerates into the $X$-axis taken twice.

For $m > 1$, the corresponding conics are hyperbolas with semi-transverse and semi-conjugate axes $a$ and $b$ equal to $(k/e)\sqrt{(1+e^2)}$ and $(k/e)\sqrt{(e^4-1)}$, respectively.

As $e \to 1$, $c \to k\sqrt{2}$, $a \to k\sqrt{2}$, $b \to 0$, the foci and vertices of the hyperbolas approach the same limiting positions $(\pm k\sqrt{2}, 0)$ whereas the conjugate axis approaches zero. The asymptotes approach coincidence with the $X$-axis.

As $e \to \infty$, $c \to \infty$, $a \to k$, $b \to \infty$, the foci move out along the X-axis, the vertices approach $(\pm k, 0)$ and the conjugate axis becomes infinite. The asymptotes approach coincidence with the $Y$-axis.

**Conclusion.** The method of establishing the correspondence described in this paper is purely arbitrary. Although some other method might remove the discontinuities encountered, many of the properties resulting from the chosen method would be sacrificed. One might compare the results obtained by different methods. There is also the possibility of an analogous correspondence in three dimensions.

"Let us grant that the pursuit of mathematics is a divine madness of the human spirit, a refuge from the goading urgency of contingent happenings."

—WHITEHEAD.

# SOME CURIOUS IDENTITIES

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$$133 + 134 + 10 + 11 = 59 + 158 + (-14) + 85$$

$$133^2 + 134^2 + 10^2 + 11^2 = 59^2 + 158^2 + (-14)^2 + 85^2$$

$$133^3 + 134^3 + 10^3 + 11^3 = 59^3 + 158^3 + (-14)^3 + 85^3$$

$$133^4 + 134^4 = 59^4 + 158^4$$

$$133 + 134 - 5^2 = 59 + 158 + 5^2$$

$$133^2 + 134^2 - 60^2 = 59^2 + 158^2 + 60^2$$

$$133 + 134 = 44 + 45 + 178$$

$$133^2 + 134^2 = 44^2 + 45^2 + 178^2$$

$$(-59) + 158 = (-92) + 125 + 66$$

$$(-59)^2 + 158^2 = (-92)^2 + 125^2 + 66^2$$

$$59 + 158 = (-96) + 3 + 5(62)$$

$$59^2 + 158^2 = (-96)^2 + 3^2 + 5(62)^2$$

$$59^2 + 158^2 = (-13586) + 7897 + 3(11378)$$

$$59^4 + 158^4 = (-13586)^2 + 7897^2 + 3(11378)^2$$

# TOPICS FOR CHAPTER PROGRAMS—VIII

## 23. NON-EUCLIDEAN GEOMETRY.

Among the postulates upon which Euclid built his geometry there appears the rather artificial *fifth postulate*: "If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles." This is equivalent to assuming that through a given point one and only one line can be drawn parallel to another line. There is evidence that Euclid endeavored to prove the statement before setting it down as a postulate, and that he finally did so only when he found he could neither prove it nor proceed without it. A proof of the fifth postulate was attempted by Ptolemy in the second century A.D., and he was followed by a host of other mathematicians who vainly attacked the problem. The whole question was answered early in the nineteenth century by Lobachevsky and Johann Bolyai, working independently, who showed that the denial of the fifth postulate while all the others are retained leads to a geometry as consistent as Euclid's own.


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## 24. FERMAT'S LAST THEOREM.

Every student knows that the equation $x^2 + y^2 = z^2$ has integral solutions; for example, $3^2 + 4^2 = 5^2$. Fermat (1601?-1665) wrote in the margin of his copy of Diophantus, "However, it is impossible to write a cube as the sum of two cubes, a fourth power as the sum of two fourth powers, and in general any power beyond the second as the sum of two similar powers. For this I have discovered a truly wonderful proof, but the margin is too small to contain it." Great mathematicians such as Abel, Cauchy, Dirichlet, Dixon, Euler, Frobenius, Gauss, Kummer, and Legendre worked on the problem, not to mention an army of amateurs who have offered "proofs." Fermat's theorem has been proved for all powers at least up to 619, but no one has as yet shown that the theorem is true for *all* whole numbers.

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"Mathematics and Music, the most sharply contrasted fields of scientific activity, are yet so related as to reveal the secret connection binding together all the activities of our mind."

—HEMHLOTZ.

# THE PROBLEM CORNER

EDITED BY JUDSON W. FOUST
*Central Michigan College of Education*

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solutions should not demand any tools beyond the calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before March 1, 1950. The best solutions submitted by students will be published in the Spring 1950 number of THE PENTAGON. Credit will be given for all correct solutions received. Address all communications to Dr. Judson W. Foust, Central Michigan College of Education, Mount Pleasant, Michigan.

## PROBLEMS PROPOSED

Solutions for Problems 1, 2, 5, 7, 8, and 16 have not been received. The student presenting the best solution of problem 1 or 2 will be given a one-year subscription to THE PENTAGON.

19. *Proposed by Dr. C. B. Read, University of Wichita, Wichita, Kansas.*

A problem frequently found in algebra books a generation ago was: At what time after a specified hour will the hour and minute hands of a clock be together? The modern electric clock often has an hour, a minute, and a second hand; at what time after twelve o'clock will the three hands again be together?

20. *Proposed by the Problem Corner Editor. (A problem of historic interest due to John Bernoulli.)*

Find the numerical value of $i^i$.

21. *Proposed by the Problem Corner Editor. (From Taylor and Bartoo,* Introduction to College Geometry, *The Macmillan Company, 1949. Exercise 2, page 43.)*

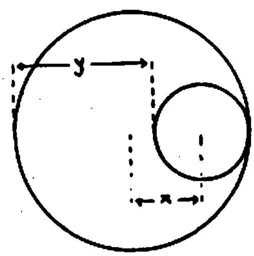
Through a given point within a given angle to draw a line which will form with the given angle a triangle having a given perimeter.

22. *Proposed by the Problem Corner Editor. (From Christofferson,* Geometry Professionalized for Teachers, *Banta Publishing Company, 1933. Exercise 38, page 117.)*

Construct a square so that each side shall pass through a given point.

23. *Proposed by the Problem Corner Editor. (From Christman,* Shop Mathematics, *The Macmillan Company, 1926, page 26.)*

Find $y$ if $x$ is 0.312 inches.




24. *Proposed by the Problem Corner Editor. (From Christman,* Shop Mathematics, *The Macmillan Company, 1926, page 87.)*

Prove that in any right triangle the sum of the two legs is equal to the sum of the hypotenuse and the diameter of the inscribed circle.

## SOLUTIONS

Credit should have been given in the last issue to Don E. Barbarick of Southwest State College, Springfield, Missouri, for solving problem 10. Since his solution is somewhat different from that published, it is given here.

10. *Proposed by the Problem Corner Editor. Seen in a newspaper in 1924.*

An automobile is twice as old as its tires were when it was as old as its tires are now. When the age of the tires equals the present age of the car the sum of their ages will be $2\frac{1}{4}$ years. How old is the car and how old are the tires?

*Solution by Don E. Barbarick.*

Let $x$ = age of tires when the car was as old as the tires are now. Then

$2x =$ age of car now,

$3x/2 =$ age of tires now and age of car when tires were age $x$ (since the ages of both car and tires have increased by the same amount),

and

$5x/2 =$ age of car when tires are as old as the car is now (reasoning as before.)

Given that $2x + 5x/2 = 2\frac{1}{4}$, it follows that $x = \frac{1}{2}$ yr. Hence, age of the car is 1 year, and age of the tires is $\frac{3}{4}$ year.

15. *Proposed by Geoffrey B. Charlesworth, Hofstra College, Hempstead, N.Y.* (*From the* American Mathematical Monthly, *March, 1946.*)

You are given twelve coins identical in appearance and a pair of scales without weights. Eleven of the coins are the same weight and the twelfth is different in weight from the others. How could you identify the coin of different weight and tell whether it is heavier or lighter than the others in three weighings? A weighing consists of placing a certain number of coins in each pan of the scales and discovering which set of coins is heavier or lighter than the other; no removals are allowed during each operation. (As a warm-up exercise for this, see if you can discover a lightweight coin from among eight coins if only two weighings are allowed.)

*Solution by Charles Canfield, Southern Methodist University, Dallas, Texas.*

Divide the twelve coins into three groups of four coins each, designating them as $A_1$, $A_2$, $A_3$, $A_4$, $B_1$, $B_2$, $B_3$, $B_4$, $C_1$, $C_2$, $C_3$, $C_4$. Place groups $A$ and $B$ in the pans of the scales. If they balance, the bad coin is in group $C$. Then, place coins $C_1$ and $C_2$ in one pan and coin $C_3$ with one good coin (from $A$ or $B$) in the other pan. This leaves coin $C_4$ out. If the pan with $C_3$ rises, that coin is light, or $C_1$ or $C_2$ is heavy. Remove all coins and place $C_1$ and $C_2$ in opposite pans. If they balance, $C_3$ is the bad coin. If they do not balance, the lower pan holds the bad coin. If originally the scales had balanced with $C_1$ and $C_2$ on one side, and $C_3$ and a coin from $A$ or $B$ on the other side, the bad coin would have been $C_4$.

Then placing it in one pan and a good coin in the other pan you could determine whether $C_4$ was light or heavy.

If groups $A$ and $B$ had not balanced (suppose $A$ coins had risen), group $C$ would have been made up of good coins. Leave $A_1$, $A_2$, and $A_3$ in their original pan and move $A_4$ to the opposite pan. Take $B_1$, $B_2$, and $B_3$ out and set them aside. Move $B_4$ to the opposite pan. Now place three good coins in the pan with $A_4$. If the pan with $A_1$, $A_2$, and $A_3$ rises, the bad coin must be one of these and must be light. Remove all coins and place $A_1$ and $A_2$ in opposite pans. The side that goes up holds the bad coin, or if they balance, $A_3$ is the bad coin. If in the original situation, the pan with $A_1$, $A_2$, and $A_3$ had lowered, the bad coin would have been either $A_4$ or $B_4$ ($A_4$ light or $B_4$ heavy.) Then remove all coins and place $A_4$ and a good coin in opposite pans. If $A_4$ rises, it is the bad coin; if the two coins balance, $B_4$ is the bad coin. If in the original situation the scales had balanced, the bad coin would have been $B_1$ or $B_2$ or $B_3$, and heavy. By the method used on $A_1$, $A_2$, and $A_3$, the bad coin could be detected.

*Solution by Norman Cox, Southwest Missouri State College Springfield, Missouri.*

The solution is much as above except as follows: If the four $A$ coins are heavier than the four $B$ coins, place two of the possible heavy coins and one of the possible light ones in each pan, such as $A_1$, $A_2$, $B_1$ against $A_3$, $A_4$, $B_2$. If these six balance, the odd coin is either $B_3$ or $B_4$ and is light. A third weighing selects the bad coin. If say $A_1$, $A_2$, $B_1$ is heavier than $A_3$, $A_4$, $B_2$, then either $B_2$ is a bad coin or $A_3$ or $A_4$ is bad. So next weigh $A_3$ against $A_4$. If these balance, then the odd coin is $B_2$; if they don't balance, the heavier one is the bad coin.

The proposer offers another variation in the solution. If $A_1$, $A_2$, $A_3$, $A_4$ is heavier than $B_1$, $B_2$, $B_3$, $B_4$ then $C_1$, $C_2$, $C_3$, and $C_4$ are all equal in weight. The second weighing is $A_1$, $A_2$, $B_1$ against $A_3$, $B_2$, $C_1$. If the weights are equal, either $A_4$ is heavier or $B_3$ or $B_4$ is lighter than the rest. The third weighing of $B_3$ against $B_4$ will determine which is true. If $A_1$, $A_2$, $B_1$ is heavier than $A_3$, $B_2$, $C_1$, then either $A_1$ or $A_2$ is heavier or $B_2$ is lighter. The third weighing of $A_1$ against $A_2$ will determine which is true, etc.

A solution by E. D. Schell in the January 1947 number of the *American Mathematical Monthly* uses the following process: If $A_1$, $A_2$, $A_3$, $A_4$ is heavier or lighter than $B_1$, $B_2$, $B_3$, $B_4$, weigh $A_1$, $A_2$, $B_2$ against $A_3$, $A_4$, $B_1$ and proceed as in the solution of Cox. If $A_1$, $A_2$, $A_3$, $A_4$ balance $B_1$, $B_2$, $B_3$, $B_4$, then weigh $A_1$, $A_2$, $A_3$ against $C_2$, $C_3$, $C_4$. If they balance, weigh $A_1$ against $C_1$. If they do not balance, weigh $C_2$ against $C_3$.

A much different solution by Joseph Rosenbaum is given in the January 1947 number of the *American Mathematical Monthly*. Label the coins, 1, 2, . . . , 12, and make the following three weighings:

(1) 1, 2, 3, 4 against 5, 6, 7, 8
(2) 1, 2, 3, 5, against 4, 9, 10, 11
(3) 1, 6, 9, 12, against 2, 5, 7, 10.

It will now be shown how the counterfeit coin can be detected, and whether light or heavy, from the results of these weighings. Designate the relationships "is lighter than," 'is the same as," "is heavier than" by $L$, $S$, $H$, and denote by $x$, $y$, $z$, respectively, the observed relationships of the left to the right in (1), (2), (3). It is now easy to verify that out of the $27 = 3^3$ permutations with repetitions of $L$, $S$, $H$, the three $(x, y, z) = (S, S, S)$, $(L, H, H)$, and $(H; L, L)$ cannot happen. For the remaining 24 permutations the following key may be verified:

*Light coin.*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *LLL* | *LLH* | *LLS* | *LHS* | *HLH* | *HSL* | *HSH* | *HSS* | *SHL* | *SHH* | *SHS* | *SSL* |

*Heavy coin.*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *HHH* | *HHL* | *HHS* | *HLS* | *LHL* | *LSH* | *LST* | *LSS* | *SLH* | *SLL* | *SLS* | *SSH* |

Also solved by John Wood. The problem of eight coins was solved by Vernon Vickland and Howard O. Mielke.

17. *Proposed by the Problem Corner Editor.*

A farmer has a calf, a goat, a colt, a pony, a sheep, and a pig to pasture in two fields. How many different ways can he divide them so that there is at least one animal in each field?

*Solution by James D. Louck, Alabama Polytechnic Institute, Auburn, Alabama.*

At any particular division, part of the animals are chosen for one certain field and the remainder go to the second field. The animals may be chosen one, two, three, four, or five at a time to go to the first field. Thus, the total number of ways they may be divided is $C(6,1) + C(6,2) + C(6,3) + C(6,4) + C(6,5) = 6 + 15 + 20 + 15 + 6 = 62$. For $n$ animals, we have $C(n,1) + C(n,2) + \ldots + C(n,n-1) = (1 + 1)^n - C(n,0) - C(n,n) = 2^n - 2$.

18. *Proposed by the Problem Corner Editor.*

$B$ is a place in a city five blocks east and four blocks north of $A$. How many different routes, nine blocks in length, are there from $A$ to $B$ by following the streets?

*Solution by Judson Foust, Central Michigan College, Mt. Pleasant, Michigan.*

If $R$ stands for a movement one space to the right and $U$ for a movement one space up, typical trips from $A$ to $B$ could be described as *RRRRRUUUU*, *RRURURRUU*, etc. It is seen that the description of all such trips consists of the permutations of nine things, four of which are alike of one kind and five of which are alike of another kind. Thus, the number of different routes is $9!/5!4! = 126$.

Also solved by James Louck and Frank Mosely. Mr. Louck suggests a similar problem for a solid figure: Point $A$ is at the corner of a large cube, which is composed of eight smaller cubes, diagonally opposite from $B$. How many routes are there, six blocks in length, from $A$ to $B$ by following the edges of the cubes? He derives the formula, $N = 3C(3a-1,a)C(2a-1,a)$, where $a$ is the length of the edge of the large cube. For $a = 2$, $N = 3C(5,2)C(3,2) = 90$.

Using the scheme of permutations suggested for the original problem where $R$ stands for a movement of one square to the right, $U$ a movement one unit up, and $B$ a movement one unit to the back, a typical path could be written *RUBBUR*, or *RRUUBB*, etc. Hence, the number of paths is $6!/2!2!2! = 90$.

*Circles to square and cubes to double would give a man excessive trouble.*

—MATTHEW PRIOR.

=∇=

Benjamin Peirce is said once at a meeting of the National Academy of Sciences to have spent an hour filling the blackboard with equations, and then to have remarked, "There is only one member of the Academy who can understand my work and he is in South America."

=∇=

The number $N$ = 1900045846159977680727771663l is a perfect cube and the twenty-eight numbers which are formed by cyclic permutations of its digits are all divisible by the cube root of $N$.

—VICTOR THEBAULT.

=∇=

Two players sit at solitaire, each with a pack of 52 cards. Each deals himself a hand of four cards. A, showing his hand, remarks, "I have the ace of hearts." B, doing likewise, replies, "I also have an ace." The probability that both players hold another ace is 1/35.

—SCH. SCI. AND MATH.

=∇=

$\sqrt{2}$ = 1.41421,35623,73095,04880,16887,24209,69807,
85696,71875,37694,80731,76679,73799,07324,
78462,10703,88503,87534,32764,15727,35013,
84623

—DE MORGAN.

=∇=

"[The mile], as Americans understand it, is 1760 yards; but the Swedes know a mile as 11,700 yards, the Russians as 1165 yards, the Italians as 1614, the Scotch as 1984, the Irish as 2240, and the Germans as 5285 or 8805 yards, according to whether it be understood as a 'short mile' or a 'long mile.' "

—W. M. BUTTERFIELD.

## WAS HE AN IDIOT?

"In the quiet little town of Hampton Falls, New Hampshire, there has lately died a man whose life appears to the writer to present a psychological study of marked interest. Nature, in what are called her freaks, or abnormal products, oft-times gives us hints of powers altogether beyond the ordinary, but destined, it may be, through the development of the race, to become possessions of mankind. This man furnishes a case in point.

". . . His education, if so we may call it, was limited to learning the letters of the alphabet, so as to know them singly at sight, but he was unable to combine them into syllables or words. He could count as far as five or six, but beyond that he became confused . . . .

"The mathematical powers of this man were really wonderful in certain directions. Without a moment's seeming thought he would tell the dominical letter for any year past or future that might be named. There seemed no limit to his power in this one line. He appeared to go through no process of calculation, but at once saw or grasped the result as by some more inward or subtile power of apprehension. His brother again and again proved the correctness of his answers, although the mathematical result that the brother obtained by a half-hour's 'figuring' this seeming idiot attained in a moment. Strangers coming to the house would oft-times tell him their age, the day and month of their birth. He would immediately tell them the day of the week they were born, also the day of the week their birthday would fall upon in any year to come. The day of the week that Christmas or Fourth of July would come in any year they would mention, he would tell without a moment's apparent calculation, and yet he could not count, or reckon in the ordinary way, more than a child of three years old! His particular literary preference seemed to be for almanacs, often having three or four about him, which he apparently studied and compared. When it came near the end of the year, he was anxious and urgent to get the new year's almanac . . . ." —Rev. W. A. Cram, *Pop. Sci. Mo.*, May, 1884.

Two sons of Carl Friederick Gauss came to America and settled in Missouri. Eugene lived on a farm a short distance from Columbia where he died in 1896 and William lived in St. Louis where he died in 1879.

=∇=

$\pi$ = *3.1415926535897*

How I wish I could recollect of circle round
The exact relation Archimede unwound.

=∇=

A square contains just 2 acres of ground. A cow fastened to a stake in the middle of one side is allowed to graze one acre inside the square. Show that the rope must be 172 feet long.

—SCH. SCI. AND MATH.

=∇=

Before Napier invented the name logarithm, he referred to logarithms as "artificial numbers."

=∇=

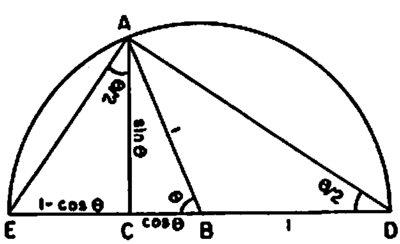
"Mathematics is the music of Reason. The musician *feels* Mathematics, the mathematician *thinks* Music."

—J. J. SYLVESTER.

=∇=

Proof of the half-angle formulas, $\theta < 180°$:

$$\tan \theta/2 = AC/CD = \sin\theta/(1+\cos\theta)$$
$$= EC/AC = (1-\cos\theta)/\sin\theta$$




The other half-angle formulas are easily obtained from these.

"Physics is today in the process of becoming almost purely mathematical in structure."

—WARREN WEAVER.

=∇=

In the following cryptic multiplication, the $x$'s denote missing digits, not necessarily equal. The solution is unique.

$$\begin{array}{r} x\ x\ x \\ x\ x \\ \hline x\ x\ 3\ x \\ x\ 3\ x\phantom{\ x} \\ \hline x\ x\ 3\ x\ x \end{array}$$

—MATH. GAZ.

=∇=

Benjamin Gompertz suggested the following notation:

$$.(5)718 = .00000718$$
$$718(6) = 718000000$$

—NATURE.

=∇=

What is the probability of a tin can tied to a dog's tail? One! (It is bound to occur.)

=∇=

If $r$ be nearly the $n$th root of $N$, a still nearer root is

$$\frac{(n+1)N + (n-1)r^n}{(n-1)N + (n+1)r^n} \times r.$$

—HUTTON.

=∇=

"It now appears that the ultimate standard of length has been found in a wave length of radiation emitted by mercury 198, an isotope transmuted from gold by neutron bombardment . . . . The homogeneity, reproductibility, and convenience of this standard cannot be surpassed by any other. It is, therefore, inevitably the ultimate standard of length, basic for the definition of all other units, including the meter."—W. F. Meggers, *Sci. Mo.*, Jan. 1949.

=∇=

$$11^3 + 15^3 + 27^3 = 29^3$$

Start with any four numbers. Obtain their numerical differences in cyclic order. Continue this differencing as long as necessary. A point is reached eventually where the four numbers are equal. For example:

| | | | |
|---|---|---|---|
| 59, | 1, | 31, | 27 |
| 58, | 30, | 4, | 32 |
| 28, | 26, | 28, | 26 |
| 2, | 2, | 2, | 2 |

=∇=

William Shanks who carried the computation of $\pi$ to 707 places of decimals did nothing but computing. He prepared a table of prime numbers up to 60,000.

=∇=

By increasing your speed from: 12 mph to 15 mph, you save 1 min. on each mile; 15 mph to 20 mph you save 1 min. on each mile; 20 mph to 30 mph you save 1 min. on each mile. If you are traveling at the rate of 60 mph, how much must you increase your speed in order to save another minute on each mile?

=∇=

"The man of science sings a song as full of joy to him who is able to appreciate it as does the diva an aria, while he responds pleasurably to a lyric like the theorem of Pythagoras and to an epic like Newton's *Principia.*"

—R. L. LAGEMANN.

=∇=

A phonograph record has a total diameter of 12 inches. The recording itself leaves an outer margin of an inch; the diameter of the unused center of the record is four inches. There are an average of 90 grooves to the inch. How far does the needle travel when the record is played?

—SCIENCE DIGEST.

=∇=

*Beware lest you lose the substance by grasping at the shadow.*

—AESOP.

# INSTALLATION OF NEW CHAPTERS

Edited by LAURA GREENE

The PENTAGON is pleased to report the installation of two new chapters of Kappa Mu Epsilon. There are now forty-two chapters on the roll.

## MISSOURI EPSILON

*Central College, Fayette*

Fifteen student and faculty members were initiated as charter members of the Missouri Epsilon Chapter at the installation ceremony on May 18, 1949. The installation was conducted by Dr. Claude H. Brown of Missouri Beta. He was assisted by James Green, Robert Boothe, and Donna Lee Chitty, all of Missouri Beta. Following the installation the group viewed Saturn through the twelve-inch telescope.

The following officers of Missouri Epsilon were installed: President, Mark Quayle Barton; Vice-President, Lew Wallace Jacobs III; Secretary, Mary Frances Cronbaugh; Treasurer, Robert A. Christian; Corresponding Secretary, Professor Finis L. Barrow; Faculty Sponsor, Professor Floyd F. Helton. Other charter members are David Eugene Allison, John William Blattner, Paul L. Calvert, Dana Ann Chenoweth, William E. Cooley, Professor Clifton E. Denny, Mary Julia Groce, Alvin Rae Lowe, and Niels C. Nielson.

## MISSISSIPPI GAMMA

*Mississippi Southern College, Hattiesburg*

The Mississippi Gamma Chapter of Kappa Mu Epsilon with twenty-two charter members was installed at Mississippi Southern College on May 21, 1949. Professor S. B. Murray of Mississippi Beta served as the installing officer. He was assisted by Dr. Arthur Ollivier and Alton Grimes of Mississippi Beta.

Immediately following the installation ceremony, a dinner was given in the dining room of the Hattiesburg Community Center. Professor Murray spoke briefly on Chapter Activities and the Seventh National Convention.

The following officers were installed: President, Lewis Webb; Vice-President, James Parker; Secretary, Addie Coleman; Treasurer, Myra Jean Townsend; Corresponding Secretary, Miss Virginia Felder; Faculty Sponsor, Professor Orval L. Phillips. Other charter members include James O. Batte, Alva Dave Boykins, Ermon Bryant, Wright Cross, Dr. Olin B. Ader, John Jones, Charles E. Lane III, Arthur McCarey, Eva Nell Pickle, Lester R. Rogers, Philip Slade, Gaston Smith, George Stewart, Shirley Vick, Richard Ward, and Oscar Wilkerson. Dr. Ader was not initiated as he is a member of the New Mexico Alpha Chapter.

The national officers and the fraternity welcome these new chapters of Kappa Mu Epsilon.

The term logarithm came into use before the theory of exponents was well developed. In Napier's first system of logarithms no base is explicitly recognized.

—G. A. MILLER.

Pure mathematics is on the whole distinctly more useful than applied. . . . For what is useful above all is *technique,* and mathematical technique is taught mainly through pure mathematics.

—G. H. HARDY.

What science can there be more noble, more excellent, more useful for men, more admirably high and demonstrative than mathematics?

—BENJAMIN FRANKLIN.

# KAPPA MU EPSILON NEWS

EDITED BY CLEON C. RICHTMEYER, *Historian*

At their banquet on May 13, members of **Alabama Alpha** heard reports from the delegates to the national convention at Topeka.

—+—

**California Alpha** sponsored a field trip to the California Institute of Technology to see their differential analog and electron microscope.

—+—

National President Dr. H. Van Engen addressed the annual spring banquet of **Illinois Alpha** on the topic, "Mathematics as an Element in the History of Thought." The members of this chapter publish a news letter. The May issue contains interesting sketches of members of the faculty.

—+—

Fifty-seven members and guests were present at the annual spring banquet of **Illinois Gamma** held at the Normandy House.

—+—

Miss *Jane Rourke* was elected by **Illinois Delta** to represent that chapter at a mathematics symposium sponsored by the Catholic colleges of the Chicago area and held at DePaul University. Miss Rourke reviewed the timely book by Weiner entitled *Cybernetics.*

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Each pledge at **Iowa Alpha** is required to write a paper as part of his initiation requirements.

—+—

The executive committee of **Iowa Beta** planned a series of summer meetings to work on the by-laws and to outline a program for the coming year.

—+—

**Kansas Beta** sponsored a Mathematics Department "Open House" on March 25-26. Demonstrations were given on astronomy, probability, telescopes, computers, and linkages. Astronomy movies also were shown.

**Kansas Gamma** presented a school assembly centering around the applications of mathematics in the life of the ordinary student. *Mary Alice Weir* was given the Hypation award for her outstanding and original research on nomography.

—+—

**Kansas Delta** was host to the national convention of KME on April 10-12. During the year a number of juniors and seniors tutored patients at Winter General Hospital.

—+—

At the March initiation of **Michigan Alpha**, each initiate was required to present the solution of two problems from the problem corner of the PENTAGON.

—+—

**Michigan Beta** has recently changed its scholarship requirements for admission to membership. The new standards require a B average in mathematics courses and a C+ average in all college work.

—+—

*Edward Rykowski* of **Missouri Alpha** had the distinction of presenting the opening student paper at the national convention. His paper was entitled, "A Finite Geometry of Twenty-Five Points."

—+—

Members of the **Missouri Beta** Chapter journeyed to Fayette, Missouri, on May 18 to install the **Missouri Epsilon** chapter at Central College. A picnic dinner followed the installation and guests had the opportunity of viewing Saturn through the Central College twelve-inch refracting telescope.

Except for Kansas chapters, Missouri Beta had the largest representation at the Biennial Convention.

—+—

Three faculty and four student members of the **Nebraska Alpha** attended the April meeting of the Nebraska section of the National Council of Teachers of Mathematics held at Grand Island.

—+—

The general theme "Famous Problems" was selected for the second semester program at **New Jersey Alpha**. The discussions included old and new methods, approximations, and practical applications.

Two alumni of Upsala College, *Robert Wornken* and *Betty Rudeback*, are now instructors at their Alma Mater.

—+—

**New York Alpha** has voted an annual award to the outstanding freshman mathematics student, selection to be made by competitive examination. The award this year was won by *George Henson.*

Members of New York Alpha were guests of the mathematics club of Adelphi College to hear Dr. Howard Fehr speak on the topic "The Number *e* and the Law of Continuous Growth."

Dr. C. V. Newsom was guest speaker at the annual banquet of New York Alpha.

—+—

The annual banquet of **Ohio Gamma** was addressed by Dr. R. Wagner of Oberlin College on "Probability and the Law of Averages."

—+—

**South Carolina Alpha** devoted several meetings to solution of mathematical puzzles and discussion of problems proposed in the *Pentagon* and the *American Mathematical Monthly.*

—+—

Each pledge of **Wisconsin Alpha** is required to give a short talk on a mathematical subject.

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PROGRAM TOPICS, SPRING SEMESTER, 1948-49

**Alabama Gamma, Alabama College**

*Non-Euclidean Geometry,* by Mamie Braswell

**California Alpha, Pomona College**

*Differential Analyzer,* by Frank Baum
*Double-Angled Triangle,* by Don Benson
*Measurements of the Earth,* by Bob Johnson
*History of the Binomial Theorem,* by Ruth Ann Engval
*Four-Color Problem,* by Dr. Elmer Tolstead

**Colorado Alpha, Colorado A & M College**

*Short Cuts in Arithmetic*, by Donald Strauss
*Straight Line Motion*, by Andrew J. Clark
*On the Mechanical Solutions of Differential Equations*, by James Staley
*The Trisection Problem*, by David Hughes
*Paper Folding*, by Leo Jarzomb

**Illinois Alpha, Illinois State Normal University**

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*Descartes,* by Dorothy Vande Walle

*Gallileo,* by Kathleen Hanley

*John Napier,* by Mary Suehr

The mathematician is fascinated with the marvelous beauty of the forms he constructs, and in their beauty he finds everlasting truth.

—J. B. SHAW.

# CHAPTERS OF KAPPA MU EPSILON

ALABAMA ALPHA, Athens College, Athens.
ALABAMA BETA, Alabama State Teachers College, Florence.
ALABAMA GAMMA, Alabama College, Montevallo.
CALIFORNIA ALPHA, Pomona College, Claremont.
COLORADO ALPHA, Colorado A & M College, Fort Collins.
ILLINOIS ALPHA, Illinois State Normal University, Normal.
ILLINOIS BETA, Eastern Illinois State College, Charleston.
ILLINOIS GAMMA, Chicago Teachers College, Chicago.
ILLINOIS DELTA, College of St. Francis, Joliet.
IOWA ALPHA, Iowa State Teachers College, Cedar Falls.
IOWA BETA, Drake University, Des Moines.
KANSAS ALPHA, Kansas State Teachers College, Pittsburg.
KANSAS BETA, Kansas State Teachers College, Emporia.
KANSAS GAMMA, Mount St. Scholastica College, Atchison.
KANSAS DELTA, Washburn Municipal University, Topeka.
MICHIGAN ALPHA, Albion College, Albion.
MICHIGAN BETA, Central Michigan College, Mount Pleasant.
MICHIGAN GAMMA, Wayne University, Detroit.
MISSISSIPPI ALPHA, State College for Women, Columbus.
MISSISSIPPI BETA, Mississippi State College, State College.
MISSISSIPPI GAMMA, Mississippi Southern College, Hattiesburg.
MISSOURI ALPHA, Southwest Missouri State College, Springfield.
MISSOURI BETA, Central Missouri State College, Warrensburg.
MISSOURI GAMMA, William Jewell College, Liberty.
MISSOURI DELTA, University of Kansas City, Kansas City.
MISSOURI EPSILON, Central College, Fayette.
NEBRASKA ALPHA, Nebraska State Teachers College, Wayne.
NEW JERSEY ALPHA, Upsala College, East Orange.
NEW JERSEY BETA, New Jersey State Teachers College, Montclair.
NEW MEXICO ALPHA, University of New Mexico, Albuquerque.
NEW YORK ALPHA, Hofstra College, Hempstead.
OHIO ALPHA, Bowling Green State University, Bowling Green.
OHIO BETA, College of Wooster, Wooster.
OHIO GAMMA, Baldwin-Wallace College, Berea.
OKLAHOMA ALPHA, Northeastern State College, Tahlequah.
SOUTH CAROLINA ALPHA, Coker College, Hartsville.
TENNESSEE ALPHA, Tennessee Polytechnic Institute, Cookeville.
TEXAS ALPHA, Texas Technological College, Lubbock.
TEXAS BETA, Southern Methodist University, Dallas.
TEXAS GAMMA, Texas State College for Women, Denton.
TEXAS DELTA, Texas Christian University, Fort Worth.
WISCONSIN ALPHA, Mount Mary College, Milwaukee.