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WHO'S WHO IN KAPPA MU EPSILON

Kappa Mu Epsilon, national honorary mathematics fraternity, was founded in 1931. The object of the fraternity is four-fold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievement in the study of mathematics in the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

A SIMPLE AND UNBREAKABLE CODE

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Is there, as some people may have carelessly assumed, no such a thing as an unbreakable code? Well, it depends upon the definition, of course. But if we *define* an unbreakable code as one which all the experts on earth, with all time from now on at their disposal, could not possibly decipher without the key, then we can safely brand this claim as nonsense. And now, having put ourselves out on a controversial limb, we're handing the reader a saw in the form of just such a code.

Consider, for example, the array

(1) 5, 8, 13, 2, 9, 6, 4, 11, 17, 17, 17, 6, 3, 3,

in which it is freely admitted that each number stands for a letter (the simpleton's code). To come to the point at once, if this set of numbers conveys any fourteen-letter message we care to send, depending upon the key we agree upon in advance, then there is of course no possibility of deciphering it from internal evidence. It is simply any fourteen-letter message whatever, and that is that.

To illustrate, let's make it say "Come home at once." Our key is then

(2) 3, 20, 1, 24, 2, 18, 18, 7, 17, 24, 3, 19, 1, 25;

and the simple scheme by which the first array becomes the desired message in the light of this key is seen in the following indicated table. For speedy translation and repeated use the reader should write it out in full.

	a	*b*	*c*	*d*		*x*	*y*	*z*
	1	2	3	4	. . . . .	24	25	26
(3)	2	3	4	5	. . . . .	25	26	1
	3	4	5	6	. . . . .	26	1	2
	. .	. .	. .	. .	. . . . .	. .	. .	. .
	25	26	1	2	. . . . .	22	23	24
	26	1	2	3	. . . . .	23	24	25

Evidently the letter "c" is represented by the message symbol 5 in line 3, so that 3 is the first number in our key. The letter "o" is represented by 8 in line 20, giving 20 as the second key number, and so on.

Now let us translate the mesage (1) given the key (2). The first number in our key is 3. In line 3 the first message number 5 stands for "c." The next key number is 20; in line 20 the message number 8 represents the letter "o." The rest is detail.

Observe that knowledge of the array (3) does not help the decipherer, nor does the remaining part of a message of which the first part is already translated, except in the matter of an upper limit on the number of letters involved.

Of course, if the collaborators are dumb enough to use a key like 1, 2, 3, 4, or any other detectable sequence, the message would be breakable. But an unbreakable key would be any set of random numbers obtained by a device such as that of drawing from a hat cards numbered from 1 to 26, restoring the cards and mixing them thoroughly after each draw.

My final message to you is 1, 2, 3, 4, and if you think you can translate it without my key, you're a 1, 2, 3, 4. (You're wrong—my key is not 22, 14, 15, 19. Of course, it *could* be 24, 14, 18, 11, or even 2, 2, 16, 6.)

If we compare a mathematical problem with an immense rock, whose interior we wish to penetrate, then the work of the Greek mathematicians appears to us like that of a robust stonecutter, who, with indefatigable perseverance, attempts to demolish the rock gradually from the outside by means of hammer and chisel; but the modern mathematician resembles an expert miner, who first constructs a few passages through the rock and then explodes it with a single blast, bringing to light its inner treasures.

—HANKEL.

SOME PROPERTIES OF PRIME NUMBERS

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In this article, the words *number* and *divisor* shall signify positive integers, unless otherwise stated.

A *prime number*, or prime, is defined as being a number, distinct from 1, which has no divisors other than 1 and itself. In contrast, a *composite number* is one that can be written as the product of two divisors or factors each of which is greater than 1. From this property of composite numbers, we can write composite number a as $a = bc$, where $b,c > 1$. Let $b \leqq c$ so that $b \leqq \sqrt{a}$. If b itself is composite, it can be factored as above, and its factors will be divisors of a. Since a is a finite number, a finite number of factorizations will yield a prime d which is a divisor of b and of a such that $d \leqq \sqrt{a}$. Therefore, at least one of the divisors of a composite number will be a prime not exceeding the square root of the number.

From this factorization, a simple test of primality follows. If a number is not divisible by any prime less than or equal to its square root, then the number itself is prime. For example, the square root of 91 is less than 10; testing divisibility by the primes 2, 3, 5, and 7 (which are the primes less than 10), we find that $91 = 7 \cdot 13$ which is a composite number. In like manner, 97 is found to be prime. This test is quite satisfactory unless the numbers tested are large.

Around 200 B. C., the Greek mathematician, Eratosthenes, devised a method of sifting primes from a set of integers not exceeding a prescribed limit. This process, known as the sieve of Eratosthenes, is simple in nature but tedious in practice. If the primes not exceeding a number N are known, the primes between N and N^2 can be obtained by striking out all multiples of the known primes in the set to be sifted. Those numbers remaining will be the required primes as none are divisible by a prime less than their square root.

The ancient Greeks knew that there is no greatest prime. A proof to this effect is found in the *Elements* of Euclid. In essence, this proof is as follows. Let p be the greatest known prime in the sequence $2, 3, 5, \cdots, p$ of primes in their natural order, and consider the number $P = (2 \cdot 3 \cdot 5 \cdots p) + 1$. If P is prime it is not in the set for it is larger than any known prime. If P is composite it has a prime divisor π which in turn is not one of the known primes. For P is not divisible by any of the known primes since all of the known primes divide the first term of P, but none divide the other term, 1. Since π is a prime divisor of P not in the known set, it is greater than p. Therefore, for any set of primes, another prime can be found greater than any prime in the set; that is, there is an infinite number of primes.

This brings up a problem. If we know a prime, what is the next largest prime? How can we find it? What limitations can be placed upon it? For convenience and uniformity, a standard notation will be adopted. Let $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$, $p_5 = 11$, etc., and, in general, p_n be the nth prime in natural order. In this notation, $P = (p_1 p_2 p_3 \cdots p_n) + 1$. By the proof above, P is either itself a prime or has a prime factor which is greater than p_n. Since the next largest prime is p_{n+1}, we have $p_{n+1} \leqq (p_1 p_2 p_3 \cdots p_n) + 1$. Consider now the number $Q = (p_1 p_2 p_3 \cdots p_n) - 1$. By modifying Euclid's proof slightly, we obtain $p_{n+1} \leqq Q$ for $n > 1$, which shows a slight improvement in the limitation of p_{n+1}.

In 1851 the Russian mathematician, Tshebysheff, improved on this limitation when he was able to prove that $p_{n+1} < 2p_n$. No one, however, has been able to derive a general formula for obtaining primes, and the above limitation of p_{n+1} by Tshebysheff has not been improved.

Casual observation of a table of primes shows that their distribution in a set of integers is extremely irregular. In the prime sequence, there are found many consecutive primes with difference 2. These are called "twin primes" and are such as 3 and 5, 41 and 43, 809 and 811. On the other hand, consecutive primes have large differences. Consider the $n - 1$ consecutive numbers, $n! + 2$, $n! + 3$,

$\cdots, n! + n$. The first is divisible by 2, the second by 3, and so on with the last being divisible by n. Thus they are all composite and it follows that there are two consecutive primes differing by more than an arbitrary number.

In 1837 Dirichlet proved analytically that every arithmetic progression whose first term and common difference are relatively prime (that is, they have no common factors), contains infinitely many primes. Or, in other words, there are infinitely many primes of the form $ax + b$, where a and b are relatively prime. Thus there is an infinite number of primes in the progressions of form $4x - 1$, $4x + 1$, $6x - 1$, $6x + 1$. Although there have been no arithmetic proofs advanced for the general proposition, there are arithmetic proofs for special cases. For example, it can be shown that there is an infinite number of primes of the form $4x - 1$. Thus, consider $P_0 = 4(3 \cdot 7 \cdot 11 \cdots p) - 1$, where p is the largest known prime of the required form. If P_0 is prime, it is a prime of the required form which is larger than those in the known set. If P_0 is composite, the following argument may be used. Since P_0 is odd, its prime factors must be odd. All odd numbers are of the form $4x - 1$ or $4x + 1$. The product of the factors of P_0 must be of the form $4x - 1$. If the factors are all of the form $4x + 1$, their product will also be of that form which is unsatisfactory. Therefore at least one of the factors of P_0 must be of the form $4x - 1$. This prime factor will be a prime greater than those in the set, using an argument similar to that contained in Euclid's proof. Hence, another prime of the required form can always be found greater than any in the known set; or, there are infinitely many primes of the form $4x - 1$.

There are many unsolved problems concerning primes. It has always seemed easier to propose problems on the subject of primes than it has been to solve them. The unsolved problems include the following. Are there infinitely many twin primes? Is there always at least one prime between any two consecutive squares? Are there infinitely many primes of the form $x^2 + 1$, such as 2, 5, 17, and 37?

Through the years many people have tried to derive a formula that represents primes only. Such functions as

$f(x) = 5 \cos 2\pi x$ are satisfactory but trivial, since $f(x) = 5$ for all integral values of x. The function $f(x) = x^2 + x + 17$ represents a prime for values of x through 15, but substitution shows that the function represents a composite number when $x = 16$ or $x = 17$. In fact, no non-constant polynomial with integral coefficients can represent primes for all integral values of the unknown. The proof of this statement is as follows.

Let $f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_n$ be any polynomial, with $a_0, a_1, \cdots, a_{n-1}$ not all zero. Let u be a prime given by the function when $x = x_0$, that is, $f(x_0) = u$. Next consider the value v of the function when $x = x_0 + ku$, where k is any integral constant. Then

$$v = a_0(x_0 + ku)^n + a_1(x_0 + ku)^{n-1} + \cdots + a_n.$$

Writing $(x + b)^m$ as $x^m + b \cdot g(x)$, where $g(x)$ is a polynomial of the $(m\text{-}1)$st degree, it follows after slight simplification that $v = u\,[1 + k \cdot G(x_0)]$, which is a composite number. Therefore, no non-constant polynomial can represent primes for all values of the unknown.

The ancient Chinese thought that a prime represented a fertile union of a male, or odd, number and a female, or even, number. Be this as it may, primes have afforded much fertile thought and experimentation, and who knows but that the Pythagorean "Everything is number" may in the future be modified to "Everything is prime."

Mathematics, too, has its triumphs of the creative imagination, its beautiful theorems, its proofs and processes whose perfection of form has made them classic. He must be a "practical" man who can see no poetry in mathematics.

—W. F. WHITE.

TRANSFINITE NUMBERS

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All have at times come into contact with large numbers, perhaps with some even larger than those which are mentioned here. We come into contact with the amount of the national debt (a staggering figure), with astronomical distances (very great figures indeed), and so forth, but these numbers are dwarfed in comparison to some we could mention. For example, the very innocent appearing number A (given below) consisting of three digits, would, if expanded, have 370,000,000 digits. Written five digits to the inch, this number would be 1170 miles long and would take one person 50 years to write if he wrote at the rate of 60 digits a minute, 40 hours a week, 52 weeks a year. An example of a large number occurring in nature is $136 \cdot 2^{256}$, which according to theories propounded by Sir Arthur Eddington is the number of protons in the universe. The total number of moves in a chess game is the huge number B given below [*1*, p. 32]*. Skewe's number which gives information concerning the distribution of primes is expressed by the number C below, a number that dwarfs even the aforementioned ones [*1*, p. 32].

$$A = 9^{9^{9}} \qquad B = 10^{10^{50}} \qquad C = 10^{10^{10^{34}}}$$

As large as these numbers are, still each is finite. Any part of a number is less than the whole number and it can be handled by the ordinary rules of arithmetic. But what of the number of integers 1, 2, 3, 4, and so on? We have identified the quantities in the preceding paragraph by assigning to each an integer. It is evident that to handle or even think of such a quantity as the totality of the integers we must adopt new concepts and definitions. Consider a concert which is well attended; in fact, every seat is occupied and no one is standing. How many seats there are is not known definitely since they have never been

*Numbers in brackets refer to the books cited at the end of this paper.

counted, nor is it known how many people are listening. But it is known that there are precisely the same number of people as seats as they have been placed in a one-to-one correspondence; that is, to each person there corresponds a seat and to each seat there corresponds a person. In general we say that two sets have the same number of objects if the respective objects can be placed in a one-to-one correspondence. The first use of this notion of one-to-one correspondence has been attributed to Galileo. With this notion we can begin our investigation into the set of integers. For example, there are as many odd integers as all the integers, both odd and even.

Set A: $1, 2, 3, 4, 5, 6, \ldots, n, \ldots$

Set B: $1, 3, 5, 7, 9, 11, \ldots, 2n - 1, \ldots$

The two sets can be put in a one-to-one correspondence and therefore must be equal in number. Thus we come to the definition of an infinite set. A set is infinite if it is equivalent to a proper part of itself. If a set is not infinite it is finite. The cardinal number of an infinite set is called a transfinite number. A cardinal number of a finite set is called a finite cardinal number.

The cardinality of the integers is our first measuring rod for the transfinites. If the elements of a set can be put into a one-to-one correspondence with the integers, the set is said to be denumerable. Georg Cantor (1845-1918) was the first to make a systematic study of transfinite numbers [*3*, p. 409]. He suggested that the cardinality of the integers be denoted by A_0 (read "aleph-null")[1], and that transfinite numbers of larger order be denoted by A_1, A_2, A_3, etc.

Let us investigate for just a moment some of the sets that are equivalent to the set of integers. We have already seen that there are the same number of odd integers as all integers; that is, $A_0 = \frac{1}{2} A_0$, or $A_0 + A_0 = A_0$. In like manner it can be shown that there is an equal number of even integers. Again, consider the following sets:

Set A: $1, 2, 3, 4, 5, \ldots, n, \ldots$

Set C: $11, 12, 13, 14, 15 \ldots, n\text{-}10, \ldots$

Clearly A and C are equal in number; that is, $A_0 = A_0 - 10$.

[1]Cantor used the Hebrew letter *aleph*. Since this character is not available at our press, we depart from custom and use a substitute letter.—Ed.

The positive fractions can be arranged as follows:

$$\begin{array}{cccccc}
\frac{1}{1} & \frac{2}{1} & \frac{3}{1} & \frac{4}{1} & \frac{5}{1} & \cdots \\[1ex]
\frac{1}{2} & \frac{2}{2} & \frac{3}{2} & \frac{4}{2} & \frac{5}{2} & \cdots \\[1ex]
\frac{1}{3} & \frac{2}{3} & \frac{3}{3} & \frac{4}{3} & \frac{5}{3} & \cdots \\[1ex]
\frac{1}{4} & \frac{2}{4} & \frac{3}{4} & \frac{4}{4} & \frac{5}{4} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots &
\end{array}$$

If the fractions are taken in the order indicated by the arrows, we have the denumerable set E:

Set A: 1, 2, 3, 4, 5, . . .

Set E: 1/1, 1/2, 2/1, 3/1, 2/2, . . .

Hence, there are as many positive fractions as integers; that is $A_0^2 = A_0$.

All of which brings us to some of the transfinite arithmetic. We have shown that $A_0 + 10 = A_0$. Similarly, we could go further and show that $A_0 + n = A_0$, where n is any finite number; in fact we have shown $A_0 + A_0 = A_0$. The multiplication table would be:

$$\begin{aligned}
1 \times A_0 &= A_0 \\
2 \times A_0 &= A_0 \\
&\cdots \\
n \times A_0 &= A_0 \\
A_0 \times A_0 &= A_0 \\
(A_0)^n &= A_0
\end{aligned}$$

As yet we have confined our discussion to denumerable sets, sets whose cardinality is A_0. Other transfinite numbers, listed in order of size, could be called $A_1, A_2, A_3, \ldots$.

However A_0 is the only member of this sequence which has been identified. Certainly there do exist transfinite numbers whose cardinality is not A_0. One such higher transfinite number has received considerable study and is conjectured to be A_1, though a proof is lacking. It is the set of real numbers which we shall denote as C, for continuum. The points on a straight line can be put into a one-to-one correspondence with this set and therefore has the same cardinality as C.

For something of the properties of C, consider two line segments, AB and CD, where $AB = 2CD$ (Fig. 1). For every point P on AB there is point P' on CD, and for every

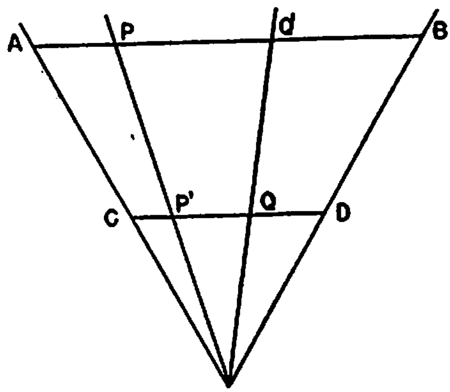

Fig. 1

point Q on CD there is a corresponding point Q' on AB. Thus the points of AB are in one-to-one correspondence with the points of CD. Also consider figure 2. The points on CD can be placed in one-to-one correspondence with the points of AM, or half of AB. Many other interesting com-

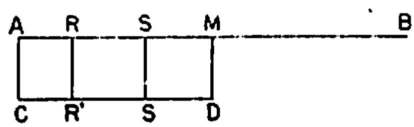

Fig. 2

parisons can be devised involving C. Sets that can be identified with the contiuum include the points in space, real numbers, instants of time, colors of the spectrum, and positions of an object in motion. Such sets have the property of being everywhere dense. The points on a line are everywhere dense, for example, since it is impossible to find the point next to a given point as between any two points there is a non-denumerable infinitude of other points.

Returning to transfinite arithmetic, we purposely omitted A_0 exp A_0 in the preceding discussion. Cantor found this to be equal to C [*1*, p. 61]. A few illustrations will suffice to show the arithmetic of C:

$$C + n = C \qquad C \times A_0 = C$$
$$C + A_0 = C \qquad C \times C = C$$
$$C \times n = C \qquad C \exp A_0 = C$$

But we find that C^C is not equal to C. This quantity is sometimes denoted by F but it is not known just what it represents.

The subject of infinite numbers has been a mystery to many in the past and will probably continue to be a mystery to many in the future. However, some progress is being made in this field of study now that the transfinite numbers have been defined and some of their properties found.

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"Pure mathematics is on the whole distinctly more useful than applied. . . . For what is useful above all is *technique,* and mathematical technique is taught mainly through pure mathematics."

—G. H. HARDY.

FINDING EXTREMES BY ALGEBRAIC MEANS

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1. Introduction. The fact that the arithmetic mean of n positive quantities exceeds the geometric mean except when the quantities are all equal is an important principle with many useful consequences. In this article we wish to show how a large number of maximum and minimum problems can be solved rather easily by this principle without resorting to the use of the calculus.

In section 2 we give a proof of the inequality between means, the proof involving only elementary algebra. In section 3 we prove four theorems of which these are the first two:

THEOREM I. If n positive variables $x_1, x_2, \ldots, x_n$ have a given sum S, then their product is a maximum when the n quantities are each equal to S/n.

THEOREM II. If n positive variables $x_1, x_2, \ldots x_n$ have a given product P, then their sum is a minimum when the n quantities are each equal to $P^{1/n}$.

In section 4 we illustrate the solution of maximum and minimum problems by the use of these two theorems and two others derived from them. Few if any are the maximum and minimum problems given in a first semester course in the calculus which cannot be solved without calculus by this method.

2. Arithmetic and Geometric Means. The arithmetic mean A of n variables $x_1, x_2, \ldots, x_n$, is a common value for all which will yield the same sum S. This value is the nth part of their sum:

(1) $$A = (x_1 + x_2 + \ldots + x_n)/n = S/n.$$

The geometric mean G of n positive variables $x_1, x_2, \ldots, x_n$ is a common value for all which will yield the same product P. This value is the nth root of their product:

(2) $$G = (x_1 x_2 \ldots x_n)^{1/n} = P^{1/n}$$

For any two positive quantities x and y we have the relation

(3) $A^2 - G^2 = [(x + y)/2]^2 - xy = [(x - y)/2]^2$

which is easily verified by elementary algebra. The right hand member of (3) is the square of a real quantity, so it is positive unless that quantity is zero. Hence either $A^2 - G^2$ is positive and A is greater than G, or else $x = y = A = G$.

We now show that this inequality holds for the means of any n positive quantities $x_1, x_2, \ldots, x_n$.

LEMMA. If A and G are defined by (1) and (2), then

(4) $A > G$ unless $x_1 = x_2 = \ldots = x_n = A = G$.

Proof: Consider all sets of n positive quantities having the same sum as the given numbers $x_1, x_2, \ldots, x_n$, and let $a_1, a_2, \ldots, a_n$ be the set having the greatest product. If two quantities x_i and x_j in such a set are unequal, then by (3) we can obtain a new set with the same sum and a larger product by replacing x_i and x_j each by $(x_i + x_j)/2$ and keeping all the other x's unchanged. Since the product $a_1a_2 \ldots a_n$ is the largest possible for the given sum S, this product can not be further increased. Hence no two of the a's can be unequal, and we have

(5) $a_1 = a_2 = \ldots = a_n = S/n = A.$

On the other hand if the set $x_1, x_2, \ldots, x_n$ contains two unequal x's the product could be increased without changing the sum, so this product is less than the largest possible product. Thus

(6) $a_1a_2 \ldots a_n = A^n > x_1x_2 \ldots x_n$ unless $x_1 = x_2 = \ldots = x_n$.

From (6) and (2) the required inequality (4) is obtained by extracting the nth root.

3. Theorems about Maxima and Minima. Theorems I and II, which were stated in section 1, are immediate consequences of the inequality (4) in section 2. First, if the sum S of the n positive quantities $x_1, x_2, \ldots, x_n$ is given, then the arithmetic mean $A = S/n$ is determined, so the product $P = x^n$ cannot exceed A^n. This maximum value occurs only when each of the n quantities equals A. Similarly, if the product P of the n positive quantities is given, then their geometric mean $G = P^{1/n}$ is determined, so the sum cannot be less than nG. This minimum value occurs only when each of the n quantities equals G.

It is to be observed that when any function of the n variables such as P (or S) is kept constant, then a function such as kP^m (or kS^m) is also constant if k and m are constants. Furthermore, if certain so-called *critical values* of the variables make P a maximum they also make kP^m a maximum, $-kP^m$ a minimum, kP^{-m} a minimum, and $-kP^{-m}$ a maximum, if k and m are positive constants.

We are now ready to generalize the sum S and product P in a way that will be useful for certain applications. For any positive variables x and y, positive constants a, b, c, k, and positive integers l, m, we define a sum T and a product Q as follows:

$$(7) \qquad T = ax^c + by^c, \quad Q = kx^ly^m$$

THEOREM III. If x and y are positive variables such that the weighted sum T in (7) is constant, then the power product Q is a maximum if and only if x and y satisfy equation (8).

$$(8) \qquad ax^c/l = by^c/m.$$

THEOREM IV. If x and y are positive variables such that the expression Q in (7) is constant, then T is a minimum if and only if x and y satisfy equation (8).

The condition (8) is most easily stated by saying that *the terms whose sum is* T *are proportional to the exponents in* Q.

Proof of Theorems III and IV: We define $n = l + m$ variables $x_1, x_2, \ldots, x_n$ so that the first l are equal to ax^c/l and the last m are equal to by^c/m. Then their sum S is given by $S = T$, and their product is $P = (a/l)^l(b/m)^mk^{-c}Q^c$. Hence by Theorems I and II, equation (8) is the required condition that Q be maximum for given T, or that T be minimum for given Q.

4. Applications to Problems.

Problem 1. To find the dimensions of the rectangle of smallest perimeter having a given area K.

Solution: Let the sides of the rectangle be x_1 and x_2. Then their product is the given area and their sum is half the perimeter. For the minimum sum we have $x_1 = x_2 = \sqrt{K}$ by Theorem II. The rectangle is a square.

Problem 2. A farmer has 100 feet of chicken wire with which to enclose a rectangular plot and subdivide it into three equal rectangular enclosures. What width w and breadth b should each of the three enclosures have for largest area?

Solution: If the large plot has length $3w$ and breadth b, then the total length of fence required is

$$T = 2(3w) + 4(b) = 100 \text{ feet.}$$

The area to be maximized is bw. Hence by Theorem III,

$$6w = 4b = 50 \text{ ft.}; \; w = 25/3 \text{ ft.}, \; b = 25/2 \text{ ft.}$$

Problem 3. Find the height h of the right circular cone of maximum volume which can be inscribed in a given sphere of radius a.

Solution: If the cone is inscribed in the sphere, then the radius r of its base wll be a mean proportional between h and $2a-h$. That is, $r^2 = h(2a-h)$. The volume V is given by

$$V = \tfrac{1}{3}\pi h^2(2a-h).$$

Since h and $2a-h$ have a constant sum, the volume is a maximum, by Theorem III, when these quantities are proportional to the exponents 2 and 1. Hence

$$h/2 = 2a-h, \text{ or } \; h = 4a/3.$$

Problem 4. A closed cylindrical tin can is to contain a given volume V and have a minimum total surface area T. What should be its proportions?

Solution: Let the radius of the base be r and the height be h. Then

$$T = 2\pi r^2 + 2\pi rh, \quad V = \pi r^2 h.$$

In Theorem IV let $x = r^2$ and $y = rh$. Then $xy^2 = V^2/\pi^2$ is constant, and $2\pi x + 2\pi y$ is a minimum. Hence

$$2\pi x/1 = 2\pi y/2 \; \textit{or} \; 2\pi^2/1 = 2rh/2; h = 2r.$$

The height must equal the diameter of the base.

Problem 5. The corner A of a rectangular strip of paper $ABNM$ of width $AB = a$ and length $AM > 1.2a$ is folded over to meet the side BN in the point A', forming a crease CD which meets the short side AB in C and the long side AM in D. a) Find the length L of the shortest crease CD. b) Find the area K of the smallest triangle ACD which can be so formed.

Solution: Let $AC = A'C = z$. Then $AB = a$, $CB = a - z$, $A'B = \sqrt{(2az - a^2)}$, $DA' = a(A'C/A'B) = az/\sqrt{(2az-a^2)}$, and

$$L^2 = (DA')^2 + (A'C)^2 = 2z^3/(2z-a)$$
$$4K^2 = (DA')^2(A'C)^2 = az^4/(2z-a).$$

The trick now is to introduce two new variables x and y whose sum is constant and in terms of which L and K may be expressed as power products. Set

$$x = (2z-a)/z,\ y = a/z,\ x + y = 2.$$

Then

$$L^2 = 2a^2/xy^2,\ K^2 = a^4/4xy^3,$$

and L and K will have their minimum values when xy^2 and xy^3 attain their maximum values, respectively. Hence L is a minimum when

$$x/1 = y/2 = 2/3;\ z = 3a/4;\ L^2 = 27a^2/16.$$

K is a minimum when

$$x/1 = y/3 = 2/4;\ z = 2a/3;\ K^2 = 4a^4/27.$$

"He was an arithmetician rather than a mathematician. None of the humor, the music or the mysticism of higher mathematics ever entered his head. Men might vary in height or weight or color, just as 6 is different from 8, but there was little other difference."

—JOHN STEINBECK, *The Moon is Down.*

TOPICS FOR CHAPTER PROGRAMS—VI

17. THE CONSTRUCTION OF SUNDIALS

The calibration of a sundial affords an interesting application of trigonometry. The theory is quite elementary, requiring little or no knowledge of astronomy.

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"Please Tell Me How to Set up a Sundial," *House Beautiful*, Vol. 71, p. 212 (March, 1932).
R. W. Porter, "Sun Dials and Sun Dialing," *Scientific American*, Vol. 139, pp. 150-152 (August, 1928).
R. J. Robinson, "Making a Sundial," *Country Life*, Vol. 31, p. 88 (February, 1917).
"Something New Under the Sun," *Arts and Decoration*, Vol. 52, p. 27 (August, 1940).
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C. F. Talman, "Calendars and Almanacs," *The Mentor*, January 1, 1919.
The Story of Our Calendar. Telling Time Throughout the Ages. Washington, D. C., American Council on Education, 1933.
C. F. Whitney, "Story of a Sundial; School Project," *School Arts Magazine*, Vol. 24, pp. 490-494 (April, 1925)..
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18. FIBONACCI SERIES

Leonardo of Pisa, otherwise known as Fibonacci, was the outstanding mathematician of the thirteenth century. His *Liber Abaci* written in 1202 is a "storehouse from which for centuries authors got material for works on arithmetic and algebra." In this work is found the problem: How many pairs of rabbits can be produced from a single pair in a year if (a) every month each pair begets a new pair which, from the second month on, becomes productive, and (b) deaths do not occur? The problem leads to the famous series 1, 1, 2, 3, 5, 8, 13, 21, . . . , in which each term after the second is equal to the sum of the two preceding terms. This series has been the subject of several interesting studies.

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19. TRISECTION OF AN ANGLE

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M. F. Daniels, "The Trisector of Amadori," *Mathematics Teacher*, Vol. 33, pp. 80-81 (February, 1940).

L. E. Dickson, *Elementary Theory of Equations.* New York, John Wiley and Sons, 1914.

L. E. Dickson, "The Trisection of an Angle, etc." *American Mathematical Monthly,* Vol. 21, pp. 259-262 (October, 1914).

L. E. Dickson, "Why It Is Impossible to Trisect an Angle," *Mathematics Teacher,* Vol. 14, pp. 217-223 (May, 1921).

A. Dresden, *An Invitation to Mathematics.* New York, Henry Holt and Co., 1936.

D. F. Ferguson, "Geometrical Construction of an Angle to any Required Degree of Accuracy," *Mathematical Gazette,* Vol. 9, p. 373 (1919).

C. S. Floyd, "To Trisect an Acute Angle," *School Science and Mathematics,* Vol. 6, pp. 358-359 (May, 1906).

R. Garver, "Bieberbach's Trisection Method," *Scripta Mathematica,* Vol. 3, pp. 251-255 (July, 1935).

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R. E. Moritz, "Another Trisection Fallacy," *School Science and Mathematics,* Vol. 6, pp. 621-622 (October, 1906).

R. K. Morely, "A Trisection," *American Mathematical Monthly,* Vol. 39, pp. 230-231 (April, 1932).

T. W. Nicolson, "The Multisection of Angles," *The Analyst,* Vol. 10, pp. 41-43 (1883).

C. Ohlendorf, "The Trisection of an Angle by Means of a Graduated Ruler and Compasses," *School Science and Mathematics,* Vol. 13, p. 546 (June, 1913).

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R. C. Yates, "A Rose Linkage, Trisection, and the Regular Heptagon," *School Science and Mathematics,* Vol. 39, pp. 870-872 (December, 1939).

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J. W. A. Young, editor, *Monographs on Topics of Modern Mathematics.* New York, Longmans Green and Co., 1911.

"In the pure mathematics we contemplate absolute truths, which existed in the Divine Mind before the morning stars sang together, and which will continue to exist there, when the last of their radiant host shall have fallen from Heaven."

—E. T. BELL.

PRELIMINARY ACTUARIAL EXAMINATIONS

Professional status as an actuary is attained by passing the series of examinations[1] jointly sponsored by the Actuarial Society of America and the American Institute of Actuaries. On May 13 and 14, 1949, the first three of these examinations, the Preliminary Actuarial Examinations, will be given to undergraduate students of mathematics who may be interested in going into the actuarial profession.

These examinations have been prepared under the direction of a joint committee of actuaries and mathematicians. They will be administered by the Educational Testing Service at centers throughout the United States and Canada.

The Actuarial Society of America and the American Institute of Actuaries will jointly award one $200 and eight $100 prizes to the nine successful undergraduates who rank highest in the General Mathematics Examination and who concurrently pass, or previously have passed, the Language Aptitude Examination.

Detailed information regarding these examinations, as well as examples of the type of questions that are asked, is contained in a pamphlet entitled "Preliminary Actuarial Examinations." This pamphlet and applications for taking the examinations may be obtained from either of the following organizations:

THE ACTUARIAL SOCIETY OF AMERICA
393 Seventh Avenue
New York 1, New York

AMERICAN INSTITUTE OF ACTUARIES
135 South La Salle Street
Chicago 3, Illinois

[1]For a description of these examinations, see "A Career As Actuary," the PENTAGON, Spring 1948, pp. 69-73.

THE MATHEMATICIAN IN CIVIL SERVICE

(Editorial Note: Prior to 1940 there was little outlet for the employment of mathematicians except in the field of education. Persons trained in mathematics found opportunities in the actuarial and statistical professions, but in such cases they lost their identities as mathematicians. "It was reliably estimated that in 1940 there were only 150 mathematicians employed by industry exclusive of actuaries and statisticians."[1] The recent war opened new channels for the employment of mathematicians in industry and government. Thus the government found it expedient to create the civil service classification of *mathematician.* The following description of the qualifications and duties of a government mathematician is extracted from an announcement recently issued by the U. S. Civil Service Commission.[2] A subsequent article in the PENTAGON will describe the new opportunities in industry.)

Mathematicians appointed to [government positions, grades P-2 to P-5] will plan, direct, perform or assist in performing (a) research in basic mathematical theory, related theoretical analytic or evaluation studies; or (b) mathematical calculations and computations incident to investigative, developmental, and research work in the scientific fields, such as engineering, physics, astronomy, etc., or in actuarial mathematics. The duties include mathematical and statistical analyses of observational data, computation of scientific tables, preparation of graphs and charts, and the writing of scientific reports, all involving a thorough knowledge of basic mathematics equivalent to that acquired by the completion of a curriculum of study in mathematics at an accredited college or university and in most cases involving a familiarity with the physical sciences or with engineering practices.

Examples of typical duties are as follows:

1. Applying methods of mathematical analysis to problems relating to geophysics, including least square reduc-

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[1] I. S. Sokolnikoff, "Opportunities for Mathematically Trained College Graduates," *Mathematics Magazine*, Vol. 21, pp. 102-105 (Nov.-Dec., 1947).
[2] Announcement No. 123, issued September 28, 1948.

tion of magnetic and electrical data; solving equations relating to heat conduction, electrical circuits, and geomagnetic anomalies; making power and Fourier series analyses of curves of total magnetic intensity to determine the depth of origin of anomalies; computing magnetic fields of models to obtain fits with observed fields.

2. Planning and initiating research in theory of pure and applied mathematics aimed primarily at developing methods of analysis which will permit the most efficient and general use of high speed automatic electronic computing machinery. Studying and formulating requirements for "intelligence" and internal organization of such machinery; developing performance specifications for such machinery. Reviewing, analyzing, and assisting in the mathematical formulation of physical problems arising in other government agencies or in industry.

3. Making necessary computations in reduction of data from tide and current records and correlation of resulting data; computing tidal and current differences and constants, and harmonic constants for prediction.

4. Planning and initiating fundamental theoretical investigations in trajectory theory, aerodynamics and hydrodynamics of missiles and aircraft, the effect of control surfaces on the motion of long-range and high-velocity guided missiles and aircraft, and advanced theories of optics and radar as applied to ballistic instrumentation.

5. Computing, adjusting by the method of least squares, and making analyses of geodetic observations consisting of those for triangulation, leveling and astronomy; computing and making analyses of observations made for the determination of the intensity of gravity and of the isostatic reduction of gravity.

6. Performing difficult, complex, and comprehensive actuarial calculations, tabulations, and studies in (a) mortality and disability, (b) analysis of insurance experience, (c) valuation for liability determinations, and apportionment of dividends.

Education or Experience Requirements

All applicants must meet the basic requirements specified in A, B, or C below:

A. Completion of a full 4-year course in an accredited college or university, leading to a bachelor's degree in mathematics. This study must have included courses in mathematics consisting of lectures and recitations totaling at least 24 semester hours, and courses in the physical sciences (engineering, physics, geology, astronomy, chemistry, etc.) totaling 12 semester hours.

The above courses in mathematics must have included analytic geometry, differential calculus, integral calculus, and, in addition, any 3 of the following: (a) Theory of equations, (b) vector analysis, (c) higher algebra (beyond elementary college algebra), (d) differential equations, (e) advanced differential calculus, (f) advanced integral calculus. All of these courses must have been acceptable for credit toward the completion of a standard 4-year professional curriculum leading to a bachelor's degree in mathematics at an accredited college or university.

The successful completion of college work in non-accredited institutions will be accepted on the same basis as indicated immediately above, provided that such institutions give instruction of definitely collegiate level and that the State University of the State in which the institution is located accepts courses and gives advanced credit for them. (In those States where there is no State University, the evaluation and acceptance of college credit as made by the State Department of Education will be accepted.)

B. Four years of successful, progressive technical experience in mathematics. This experience must show an intimate knowledge of the theory and applications of the principles of higher algebra, analytic geometry, differential and integral calculus, differential equations, etc. The experience must also show that the applicant possesses an understanding of the field of higher mathematics comparable to that which would have been acquired through successful completion of a standard mathematical curriculum in an accredited college or university. In addition, the experience must show a knowledge of the principles underlying at least one of the physical sciences such as engineering, physics, geology, astronomy, chemistry, etc.

C. Any time equivalent combination of A and B. In combining education and experience, the applicant must show for each year of education for which credit is claimed an average of at least 6 semester hours of study in mathematics.

In addition to meeting the above requirements, applicants must show experience as follows:

For P-2: One year of professional experience in mathematics involving the use of the principles of theoretical or applied mathematics in the solution of scientific problems.

For P-3: Two years of progressive professional experience in mathematics involving the use of the principles of theoretical or applied mathematics in the solution of scientific problems. This experience must have included 1 year of difficult and important work in mathematics.

For P-4: Three years of responsible, progressive professional experience in mathematics involving the use of the principles of theoretical or applied mathematics in the solution of scientific problems. This experience must have included 2 years of difficult and important work in mathematics and must have demonstrated the applicant's initiative, resourcefulness, and ability to perform work at the professional level under only general supervision.

For P-5: Four years of broad, progressive, and responsible professional experience in mathematics involving the use of the principles of theoretical or applied mathematics in the solution of scientific problems. This experience must have included 2 years of very difficult and important work in mathematics and must have demonstrated either (a) the applicant's ability to organize, direct, and coordinate research or similar difficult work of an important character in mathematics, or (b) marked capacity for original research in mathematics.

For any grade the required amount of experience will not in itself be accepted as proof of qualification for a position. The applicant's record of experience and training must show that he has the ability to perform fully the duties at the level for which he applies.

Part-Time or Unpaid Experience.—Credit will be given for all valuable experience of the type required, regardless of whether compensation was received or whether the experience was gained in part-time or full-time occupation. Part-time or unpaid experience will be credited on the basis of time actually spent in appropriate activities. Applicants wishing to receive credit for such experience must indicate clearly the nature of their duties and responsibilities in each position and the number of hours a week spent in such employment.

Substitution of Graduate Study for Experience.—Graduate study in mathematics or related fields such as mathematical studies in engineering, etc., successfully completed in an accredited college or university may be substituted for the professional experience prescribed for these positions up to a maximum of 2 years of experience. Applicants who have successfully completed study fully equivalent to the requirements for a Master's degree in mathematics may substitute such study for 1 year of general experience. In order to substitute graduate study for 2 years of experience, applicants must have received the Ph.D. degree, which will be accepted as meeting in full the experience requirements for the P-3 grade. For the P-4 grade, the Ph.D. degree may be substituted for the general experience and for 1 year of the prescribed 2 years of difficult and important work in mathematics. For the P-5 grade, the Ph. D. degree may be substituted for the 2 years of general experience but not for any part of the prescribed 2 years of very difficult and important work in mathematics. Graduate study alone will not qualify one above the P-3 grade.

College Teaching.—When accompanied by a reasonable amount of scientific research in mathematics or related fields, college teaching of mathematics with the rank of instructor or higher will be considered acceptable mathematical experience, provided that it is shown that all other requirements for the grade under consideration have been met.

Salary and Workweek

Salary is based on the standard Federal workweek of 40 hours. Additional compensation is provided for any authorized overtime worked in excess of the 40-hour week. The salary range for each grade of these positions is given below. For employees whose services meet prescribed standards of efficiency, the entrance salary is increased by the amount shown in the table, following the completion of each 12 months of service (18 months for grades P-4 and above), until the maximum rate for the grade is reached.

Grade of Position	*Basic Entrance Salary*	*Periodic Increase*	*Maximum Basic Salary*
P-2	$3,727.20	$125.40	$4,479.60
P-3	4,479.60	125.40	5,232.00
P-4	5,232.00	250.80	6,235.20
P-5	6,235.20	239.40	7,192.80

A Federal employee serving in a position in the competitive civil service at a salary above the basic entrance salary for the position in which he is appointed or classified from this examination, may continue to be paid at his current salary rate if it is not beyond the maximum salary for the position in which he is so appointed or classified. All basic salaries are subject to a deduction of 6 percent for retirement benefits.

"No one who has studied the works of such men as Euler, Lagrange, Cauchy, Riemann, Sophus Lie, and Weierstrass, can doubt that a great mathematician is a great artist."

—E. W. Hobson.

THE PROBLEM CORNER

Edited by JUDSON W. FOUST
Central Michigan College of Education

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solutions should not demand any tools beyond the calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before March 1, 1949. The best solutions submitted by students will be published in the Spring number of THE PENTAGON. Credit will be given for all correct solutions received. Address all communications to Dr. Judson W. Foust, Central Michigan College of Education, Mount Pleasant, Michigan.

PROBLEMS PROPOSED

(Solutions are invited for Problems 1, 2, 5, 7, and 8 proposed in previous numbers of THE PENTAGON.)

9. *Selected from the third Stanford University Mathematics Examination, April 10, 1948.*

Three numbers are in arithmetic progression, three other numbers in geometric progression. Adding the corresponding terms of these two progressions successively we obtain 85, 76, and 84, respectively; adding all three terms of the arithmetic progression we obtain 126. Find the terms of both progressions.

10. *Proposed by the Problem Corner Editor. Seen in a newspaper in 1924.*

An automobile is twice as old as its tires were when it was as old as its tires are now. When the age of the tires equals the present age of the car the sum of their ages will be $2\frac{1}{4}$ years. How old is the car and how old are the tires?

11. *Proposed by Dr. C. C. Richtmeyer, Central Michigan College of Education.*

A city with a circular wall has two gates, one at each end of the north and south diameter. From the north gate a road leads directly north and from the south gate a road leads directly east. What is the diameter of the city if from a point 3 miles north of the north gate it is just possible to see past the wall to a point nine miles east of the south gate?

12. *Proposed by Lester Serier, Central Michigan College of Education.*

The graph of a traffic count past a certain point is found to resemble a sine curve with a minimum of 30 at midnight and a maximum of 900 at noon. Write an equation which will give the traffic count at any time of the day. Also find how many cars passed the point between 11 a.m. and noon.

13. *Proposed by the Problem Corner Editor.*

An iron band which just fits around the earth at its largest circumference is cut and after one foot is inserted it is adjusted so that it stands out from the earth equally at all points. Assuming the radius of the earth to be 4000 miles, how far out from the earth will the band be after the foot is inserted?

Write a formula for the distance the enlarged band will stand out when one foot is inserted in any circle.

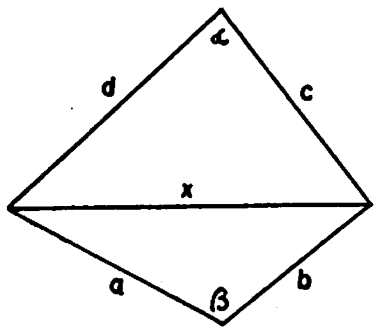
14. *Proposed by the Problem Corner Editor.*

A fish pole 10 feet long weighing 40 ounces is conical in shape and tapers uniformly. It balances at a point 30 inches from the large end. When a certain fish was caught it was noticed that with the weight of the fish on the small end the pole balanced at its midpoint. Find the weight of the fish.

SOLUTIONS

6. *From the American Mathematical Monthly, Vol. 23.*

Determine the greatest quadrilateral which can be



found with four given sides, a, b, c, d, taken in order. (Consider only convex quadrilaterals.)

Solution by Robert Pruitt, Southwest State College, Springfield, Missouri.

1) Let the diagonal of the quadrilateral be called x, and let the angles opposite x be called α and β as shown in the figure.

2) By the formula for the area of a triangle,

$$A = \tfrac{1}{2}ab \sin \beta + \tfrac{1}{2}cd \sin \alpha$$

By the law of cosines,

$$\cos \beta = (a^2+b^2-x^2)/2ab, \quad \cos \alpha = (c^2+d^2-x^2)/2cd,$$

whence, eliminating x,

$$\cos \alpha = [c^2+d^2 - (a^2+b^2 - 2ab \cos \beta)]/2cd.$$

Differentiating with respect to β,

$$\sin \alpha \, d\alpha/d\beta = ab \sin \beta/cd$$

or $$d\alpha/d\beta = ab \sin \beta/cd \sin \alpha.$$

3) For maximum area, $dA/d\beta=0$. Thus,

$$dA/d\beta = \tfrac{1}{2}(ab \cos \beta + cd \cos \alpha \, d\alpha/d\beta)$$
$$= \tfrac{1}{2}ab(\cos \beta \sin \alpha + \sin \beta \cos \alpha)/\sin\alpha = 0.$$

But $$\cos \beta \sin \alpha + \sin \beta \cos \alpha = \sin(\alpha+\beta).$$

Hence, $$\sin(\alpha+\beta) = 0,$$

so that $$(\alpha+\beta) = n\pi.$$

By inspection, π gives a maximum value, the convex case prohibiting values greater than π.

4) Since α and β are supplementary, it follows from plane geometry that the quadrilateral may be inscribed in a circle.

Conclusion: Given four sides a, b, c, d, a maximum quadrilateral exists when it can be inscribed in a circle.

5) Now $\cos\alpha = \cos(\pi-\beta) = -\cos\beta$. Substituting and solving for $\cos\beta$ (step 2), there results

$$\cos\beta = (a^2+b^2-c^2-d^2)/2(ab+cd).$$

This gives a means of determining the angles for the case of maximum area.

"The man of science sings a song as full of joy to him who is able to appreciate it as does the diva an aria, while he responds pleasurably to a lyric like the theorem of Pythagoras and to an epic like Newton's *Principia*."

—ROBERT L. LAGEMANN.

"Let us grant that the pursuit of mathematics is a divine madness of the human spirit, a refuge from the goading urgency of contingent happenings."

—WHITEHEAD.

"If a man is at once acquainted with the geometric foundation of things and with their festal splendor, his poetry is exact and his arithmetic musical."

—EMERSON.

I believe we shall never know exactly why a thing is beautiful.

—ANATOLE FRANCE.

$=\nabla=$

Writing in the year 1864, De Morgan said he was x years old in the year x^2. When was he born?

$=\nabla=$

$1/2 + 1/3 + 1/4 + \cdots + 1/n$ is not equal to an integer for any value of n.

$=\nabla=$

"During the latter part of the nineteenth century there was much talk about the possible inhabitants of Mars. The desire arose to find a way of getting in contact with those hypothetical creatures through light signals. Some mathematicians came forward with the suggestion that the signal should be a gigantic right triangle built somewhere in the Sahara Desert, the assumption being that if there were intelligent inhabitants on Mars, such a signal will convey to them the idea that there are intelligent inhabitants on the earth, since the Pythagorean theorem is a universal truth."

—N. A. COURT.

$=\nabla=$

$$4^5 + 5^5 + 6^5 + 7^5 + 9^5 + 11^5 = 12^5$$

$=\nabla=$

Decode the following problem in cryptarithmetic:

$$FORTY + TEN + TEN = SIXTY.$$

$=\nabla=$

"As a boy I knew the logarithms of thirty or fifty numbers."

—NAPOLEON.

A MNEMONIC FOR HYPERBOLIC FORMULAE: In any trigonometrical formula for θ, 2θ, 3θ or θ and Φ, after changing *sin* to *sinh*, *cos* to *cosh*, etc., change the sign of any term that contains (or implies) a product of *sinhs* (e.g. $\tanh\theta \tanh\Phi$ implies a product of *sinhs*.) This rule would fail for terms of the 4th degree, but it covers everything that is likely to be required, and is very convenient for teaching purposes.

—G. OSBORNE in MATH. GAZETTE.

=∇=

Newton became so absorbed in making discoveries that he had to be reminded to tell anyone about them, and the printing and publishing was always done for him by someone else.

=∇=

$1/\pi = .318310$

Can I discover the reciprocal?

=∇=

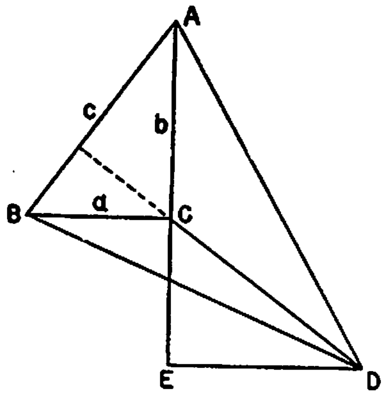
222	333	444		999
222	333	444		999
2	3	4		9
222	333	444		999
22222	33333	44444	etc.	99999
24642	36963	49284		110889
2	3	4		9
49284	110889	197136		998001

=∇=

Show that if 6^{592} is divided by 11 the remainder is 3.

=Δ=

Let N = any odd number greater than 1, and set $N^2 = 2x + 1$. Then $N^2 + x^2 = (x + 1)^2$ is the condition that N, x, and $x + 1$ are sides of a right triangle. This formula is useful for furnishing right triangles with integral sides.



Turn right triangle ABC through a right angle around the center of the square described on BC from the position ABC into the position DCE. Then the quadrilateral $ADBC$ consists of the triangles DBC and DAC having a common base of length c and of total altitude c. The area of the quadrilateral is therefore $\frac{1}{2}c^2$. But the two triangles are respectively of areas $\frac{1}{2}a^2$ and $\frac{1}{2}b^2$. Hence, $c^2 = a^2 + b^2$.

W. J. DOBBS.

$=\nabla=$

There are 12 stop lights on a highway. If it takes 8 minutes to pass 8 of them, how long will it take to pass all of them?

$=\nabla=$

A convenient method for finding the factors of a comparatively small number consists in finding a square such that when the given number is subtracted from it a square is left. The sum and difference of the two square roots are the factors of the given number.

—NATURE (Feb. 28, 1889).

$=\Delta=$

In 1874 an English mathematician named William Shanks calculated the value of π to 707 places of decimals, using the Machin formula, $\pi/4 = 4 \arctan 1/5 - \arctan 1/239$. In 1946, D. F. Ferguson of the University of Manchester calculated the value of π to 810 places of decimals and found that Shank's value was incorrect from the 527th place on. Ferguson's result was verified by J. W. Wrench, Jr., of Washington, D. C., and L. B. Smith of Talbolton, Georgia. The correct value of π was published in "Mathematical Tables and other Aids to Computation" in April, 1947:

$$
\begin{aligned}
\pi = 3.14159\ &26535\ 89793\ 23846\ 26433\ 83279\ 50288\ 41971\ 69399\\
37510\ &58209\ 74944\ 59230\ 78164\ 06286\ 20899\ 86280\ 34825\\
34211\ &70679\ 82148\ 08651\ 32823\ 06647\ 09384\ 46095\ 50582\\
23172\ &53594\ 08128\ 48111\ 74502\ 84102\ 70193\ 85211\ 05559\\
64462\ &29489\ 54930\ 38196\ 44288\ 10975\ 66593\ 34461\ 28475\\
64823\ &37867\ 83165\ 27120\ 19091\ 45648\ 56692\ 34603\ 48610\\
45432\ &66482\ 13393\ 60726\ 02491\ 41273\ 72458\ 70066\ 06315\\
58817\ &48815\ 20920\ 96282\ 92540\ 91715\ 36436\ 78925\ 90360\\
01133\ &05305\ 48820\ 46652\ 13841\ 46951\ 94151\ 16094\ 33057\\
27036\ &57595\ 91953\ 09218\ 61173\ 81932\ 61179\ 31051\ 18548\\
07446\ &23799\ 62749\ 56735\ 18857\ 52724\ 89122\ 79381\ 83011\\
94912\ &98336\ 73362\ 44065\ 66430\ 86021\ 39494\ 63952\ 24737\\
19070\ &21798\ 60943\ 70277\ 05392\ 17176\ 29317\ 67523\ 84674\\
81846\ &76694\ 05132\ 00056\ 81271\ 45263\ 56082\ 77857\ 71342\\
75778\ &96091\ 73637\ 17872\ 14684\ 40901\ 22495\ 34301\ 46549\\
58537\ &10507\ 92279\ 68925\ 89235\ 42019\ 95611\ 21290\ 21960\\
86403\ &44181\ 59813\ 62977\ 47713\ 09960\ 51870\ 72113\ 49999\\
99837\ &29780\ 49951\ 05973\ 17328\ 16096\ 31859\ 50244\ 594(55)
\end{aligned}
$$

$=\nabla=$

In the middle ages the Pythagorean Theorem went by the name of "Magister Matheseos."

$=\nabla=$

Legendre used binary numeration for calculating high powers.

$=\nabla=$

In a certain bank there were eleven distinct positions; namely, in decreasing rank, President, First Vice-President, Second Vice-President, Third Vice-President, Cashier, Teller, Assistant Teller, Bookkeeper, First Stenographer, Second Stenographer, and Janitor. These eleven positions are occupied by the following here listed alphabetically: Mr. Adams, Mrs. Brown, Mr. Camp, Miss Dale, Mr. Evans, Mrs. Ford, Mr. Grant, Miss Hill, Mr. Jones, Mrs. Kane, Mr. Long. Concerning them the following facts are known:

1. The Third Vice-President is the pampered grandson of the President, but is disliked by both Mrs. Brown and the Assistant Teller.

2. The Assistant Teller and the Second Stenographer shared equally in their father's estate.

3. The Second Vice-President and the Assistant Teller wear the same style of hats.

4. Mr. Grant told Miss Hill to send him a stenographer at once.

5. The President's nearest neighbors are Mrs. Kane, Mr. Grant, and Mr. Long.

6. The First Vice-President and the Cashier live at the exclusive Bachelor's Club.

7. The janitor has occupied the same garret rooms since boyhood.

8. Mr. Adams and the Second Stenographer are leaders in the social life of the younger set.

9. The Second Vice-President and the Bookkeeper were once engaged to be married to each other.

10. The fashionable Teller is son-in-law of the First Stenographer.

11. Mr. Jones regularly gives Mr. Evans his discarded clothing to wear, without the elderly Bookkeeper knowing about it.

Show how to match correctly the eleven names against the eleven positions occupied.

—AMER. MATH. MONTH., April, 1937.

=∇=

If, a, b, c, d form a geometrical progression, show that

$$(a-d)^2 = (b-c)^2 + (c-a)^2 + (d-b)^2$$

=∇=

"Perhaps the considering and answering such (trifling and useless) questions may not be altogether useless if it produces by practice an habitual readiness and exactness in mathematical disquisitions, which readiness may on many occasions be of real use."

—BENJAMIN FRANKLIN.

INSTALLATION OF NEW CHAPTERS

The PENTAGON is pleased to report the installation of three new chapters of Kappa Mu Epsilon. There are now forty chapters on the roll.

COLORADO ALPHA

Thirty-three student and faculty members of the Euclidean Club of Colorado A & M College were installed as charter members of the Colorado Alpha Chapter at the installation ceremony on May 16, 1948. The installing officer was Dr. H. Van Engen, National President of Kappa Mu Epsilon. Immediately following the installation ceremony, a dinner was given in the dining room of the Northern Hotel, Fort Collins. The following program was arranged by members of the Euclidean Club, with Fred J. Clark, Jr., acting as toastmaster.

"Welcome," by Professor A. G. Clark, head of the Mathematics Department of Colorado A & M College.

"Colorado A & M," by J. C. Clevenger, Dean of Men, Colorado A & M College.

"The Euclidean Club," by Walter C. Brown, President, Euclidean Club.

"Mathematics in 18th Century Thought," Installation Address by Dr. H. Van Engen.

The following officers of Colorado Alpha were installed: President, Fred J. Clark, Jr.; Vice-President, Robert K. Butz; Secretary, Eldon L. Dunn; Treasurer, Oswald E. Bartram; Corresponding Secretary, Professor M. L. Madison.

The interest shown in the activities of Kappa Mu Epsilon by the new members of Colorado Alpha augers well for an active chapter at Fort Collins, Colorado. The National officers and the fraternity wish to congratulate the new chapter on obtaining a running start. We are sure that Colorado Alpha will, in the future, make a considerable contribution to the National Organization of Kappa Mu Epsilon.

MISSOURI DELTA

The Missouri Delta Chapter of Kappa Mu Epsilon with twelve charter members was installed at the University of Kansas City on May 26, 1948. Dr. Reid Hemphill of the Missouri Beta Chapter acted as installing officer. He was assisted by Professor Loren Akers, Dr. C. H. Brown, Dorothy Fight, Samuel Herndon, Gerhardt Jaeger, Vonda Langford, Virginia Moore, Harold Peabody, Patricia Stewart, and Victor Wilson, all of Missouri Beta.

As the installation occurred during the last week of school and both students and faculty had been participating in term examinations, an elaborate ceremony was not arranged. Tables in the browsing room of the Administration Building were simply but attractively decorated with the colors of the Fraternity. Refreshments were served following the installation and a pleasant social hour was enjoyed by all present.

The following officers were installed: President, Karl Eaton; Vice-President, Charles Allendoerfer; Secretary, Elizabeth Sullivan; Treasurer, Charles Mullis; Faculty Sponsor and Corresponding Secretary, Dr. Marie Castellani. Other charter members are Willard Bosler, Bebe Bruto, Sullivan Campbell, Dr. J. S. Rosen, Patricia Saunders, Robert Smoot, and Michael Wells.

CALIFORNIA ALPHA

The California Alpha Chapter of Kappa Mu Epsilon was installed on June 6, 1948 at Pomona College, Claremont, California. Professor Fred W. Sparks of the Texas Alpha Chapter, Texas Technological College, served as the installing officer. The installation ceremony was followed by a dinner in the Frary Dining Hall at which talks were made by President E. Wilson Lyon of Pomona College, and by the installing officer.

The following officers were installed: President, Charles A. Halberg, Jr.; Vice-President, Wayne E. Smith; Secretary-Treasurer, Adele K. Kahn; Corresponding Secretary, Dr. Hugh J. Hamilton; Faculty Sponsor, Dr. Chester G. Jaeger. Other charter members are Armen G. Albarian, Frank Baum, Donald C. Benson, Edward L. Chapin, Carol Clarke, Ruth Ann Engvall, Robert E. Fortney, Walter I. Futterman, Lewis L. Grimm, Vernon E. Howes, Chester E. Jaeger, Thomas Jenkins, Robert H. Johnson, David C. Line, Gilbert Madden, Quentin C. McKenna, John C. McMillan, Kenneth S. Patterson, Bernard W. Porter, Robert C. Rempel, Aubrey H. Schluerter, Dr. Elmer Tolsted, Richard E. Utman, James E. Vedder, Ralph Vernon, Don Walter, E. Thomas Wiggins, and Laughorue P. Withers.

"The advance and perfecting of mathematics are closely joined to the prosperity of the nation."

—NAPOLEON.

"What science can there be more noble, more excellent, more useful for men, more admirably high and demonstrative than mathematics?"

—BENJAMIN FRANKLIN.

KAPPA MU EPSILON NEWS

EDITED BY CLEON C. RICHTMEYER, *Historian*

At their annual banquet on April 24, **Alabama Alpha** made a special effort to bring back former members. A pleasant time was had by all.

—+—

As a "pledge duty" the pledges at **Illinois Beta** made a set of 20 posters, each containing a pencil sketch of a famous mathematician and a paragraph concerning his life work. These were put on display and created considerable interest among the faculty and student body.

—+—

In connection with "Teachers Day," **Illinois Gamma** prepared a display of charts, posters, special books, and models related to mathematics.

—+—

Miss Victoria Fritten of **Kansas Gamma** was presented by that Chapter with the first Hypatian Award. It was given in recognition of her achievement in contributing a new chapter song. This award is a bronze plaque on which will be engraved the names of those receiving the honor.

—+—

Professor E. R. Sleight, **Michigan Alpha,** has retired after 41 years of service at Albion College. The chapter meeting on April 27 was dedicated to Dr. Sleight who was presented with a portable typewriter in honor of the occasion. Many former students were present to honor their beloved teacher.

Professor H. D. Larsen has been appointed head of the mathematics department to succeed Dr. Sleight.

—+—

Michigan Gamma requires each pledge to give a short talk on some aspect of mathematics during the semester before his formal initiation.

—+—

At the annual banquet of **New Jersey Alpha,** Dr. Nathan Lazar of Teachers College, Columbia University, spoke on "Development of the Abacus." In April the Chapter took a trip to the Hayden Planetarium.

Howard F. Fehr, formerly corresponding secretary of New Jersey Beta, is leaving Montclair to accept a position as Professor of Mathematics at Teachers College, Columbia University.

—+—

New York Alpha has recently inaugurated the plan of granting a yearly reward to the outstanding freshman students in mathematics to be chosen through a competitive examination. Fred Bloschies was the winner this Spring and was awarded a copy of the book, *What is Mathematics?*, by Courant and Robbins.

—+—

Oklahoma Alpha held a Founders Banquet at which time they were entertained by Ross Anderson and Thomas Summers in "Moments of Magic."

—+—

Tennessee Alpha for the first time held regular meetings during the 1948 summer session.

—+—

Texas Beta requires each prospective member to give a talk before he can be initiated. The initiation is held early in the afternoon and is followed by a picnic.

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PROGRAM TOPICS, SPRING SEMESTER, 1947-48

Alabama Alpha, Athens College

Exhibit of Ray's Arithmetic Used by Great Grandfather, by Robert Sibley

Methods for Dividing Triangles into Equal Area, by Prof. Garrett

Alabama Gamma, Alabama College

Waves and Vibrations, by Frances Jones

Mathematics and the Physical Sciences, by Virginia Havens

Illinois Gamma, Chicago Teachers College

Mathematics in Business, by Robert E. Olson

History of Weights and Measures, by William J. Coyne

Movie: *The Teaching of Number*

Much Ado about Nothing (The Many Meanings of Zero), by William Coyne and John Kelly

Probabilities of Winning in Gambling, by Joseph J. Urbancek

Budan's vs. Sturm's Method, by Joseph J. Urbancek

Iowa Beta, Drake University

Opportunities for Students Majoring in Mathematics, by Walter F. Potts, Jr.

The Field of Engineering, by Everett Gilman

History and Construction of Magic Squares, by William Chappell

Kansas Beta, State Teachers College, Emporia

Astronomy, by Dr. O. J. Peterson

Kansas Gamma, Mount St. Scholastica College

True Value of Science in the Liberal Arts Program, (theme for the year), by Sister Helen Sullivan

The Contribution of the Particular Sciences to Learning, by Elaine Carson and Carrie Nelle Bremmer

Constructions with Ruler and Compass—Three Classical Problems, by Gloria Jaskowiak

Panel Review of Book, "Human Destiny", by Joanne Shuey and others

Michigan Alpha, Albion College

How the Middle Ages Counted, by Charles Bishop

4000 *Years for Numerals,* by William Doddrill

Evolution of Our Exponential Notation, by Ray Gillespie

The Zero, by Lucy Richardson

Fallacies in Mathematics, by Dorothy Manley

E. R. Sleight, by Faye Engstrom

History of the Mathematics Club at Albion, by Dr. E. E. Ingalls

Michigan Beta, Central Michigan College

Mathematical Paradoxes, by James Louck

Michigan Gamma, Wayne University

A Method for Summing a Finite Series of Polynomials, by Arthur Vuylsteki

The Alignment Nomogram, by Dr. T. H. Southard

Aspects of the Theory of Relativity, by Philip Chase

Rotation of Axes in Analytical Geometry, by Arthur Vuylsteki

Problems in Foundations of Mathematics, by Dr. Churchman

Structure, by Professor Norman Anning

Missouri Alpha, Southwest Misouri State College

Golden Section, by James Cheek

Solution and Generalization of Problem 3, "The Pentagon", by Robert E. Hogan

Missouri Beta, Central Missouri State College

Mathematics in the Biological Sciences, by Dr. Sam P. Hewitt

Nebraska Alpha, State Teachers College

Mathematical Preparation for Scientific Work, by Richard Campbell

Calendar Reform, by Howard Prouse

Agricultural and Meteorological Statistics, by Robert Dale

New Jersey Alpha, Upsala College

Teaching of Mathematics in Shrivenham University and other English Colleges, by Dr. M. A. Nordgaard

Relationship of Mathematics to Philosophy, by June Davidson

Theory of Probability, by James Gill and Martin Monroe

Development of the Abacus, by Dr. Nathan Lazar

New Jersey Beta, State Teachers College

Trends of Education in England, by Dr. D. R. Davis

Non-Euclidean Geometry, by Gloria Senapole

Mathematical Tricks, by Dr. I. Brune

New Mexico Alpha, University of New Mexico

Mathematics and Engineering, by Hugh Munn

Series and Figured Numbers, by Professor H. P. Rogers

Precise Surveying, by Professor Elvin C. May

Experiments in Physics, by Dr. Victor C. Regener

New York Alpha, Hofstra College

Four Color Problem, by Dr. L. F. Ollmann

Our Nearest Star, by Dr. Francis Wilson

A New Series and Some Useless Limits, by Geoffrey Charlesworth

Some Important Properties of Prime Numbers, by Lysle Marshall

History of KME as a National Organization, by E. Marie Hove

A Comparison of the Oxford System with the American System, by Dr. Banesh Hoffman

Stephen Leacock's "A B and C", by Eleanor Blodgett

Ohio Alpha, Bowling Green State University

Mathematics in Chemistry, by Dr. Lewis Miller

Teaching Mathematics, by Martha Gesling

Beauty of Mathematics, by Dr. F. C. Ogg

Ohio Gamma, Baldwin Wallace College

Proofs of the Pythagorean Theorem, by LaMoyne Holley

John Napier and Logarithms, by Jim Harmon

Mechanical Mathematical Devices, by Fred Rakowsky

Demonstration of Napier's Rods, by Howard Mielke

Oklahoma Alpha, Northeastern State College

Determinants, by Charles Brown and Thomas Summers

Exponential Function, by J. B. Willis

Applied Mechanics

Tennessee Alpha, Tennessee Polytechnic Institute

Alignment Charts, by Ray Kinslow

Applications of Abstract Mathematics in Everyday Life, by R. O. Hutchinson

Mathematics as Related to Growth of Civilization, by C. C. Kelly

History of the Calendar, by W. H. Mikel

Opportunities for Mathematical Work in Industry, by H. E. Mayer

Mathematical Wrinkles, by L. J. Stulce

Mathematical Nuts, by B. F. Mullins
Mathematical Tricks, by F. E. Eastes

Texas Alpha, Texas Technological College

The Application of Mathematics to Radar, by Professor Byron E. Bennett

Applications of Calculus to Engineering, by William Adair and Allen Orr

Texas Beta, Southern Methodist University

Concepts of Infinity, by Joseph Rice
Hyper-Spacial Tit-Tat-Toe, by Grace Mitchell
A Definition of i, by Gene Archer

Wisconsin Alpha, Mount Mary College

Infinity, by Norma Harding

"Architecture is geometry made visible in the same sense that music is number made audible."

—CLAUDE BRAGDON.