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Kappa Mu Epsilon, national honorary mathematics fraternity, was founded in 1931. The object of the fraternity is four-fold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievement in the study of mathematics in the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

IRA SHIELDS CONDIT

Ira Shields Condit, Professor of Mathematics and former head of the Department of Mathematics at Iowa State Teachers College, died November 7, 1947. He was born November 18, 1886, near Washington, Iowa, where he received his early education in rural schools of that community. In September, 1898, Professor Condit came to Iowa State Teachers College, at that time known as the Iowa State Normal School, as assistant professor of mathematics. In 1909, he was promoted to the head of the department and served in this capacity until he retired in 1936.

Professor Condit participated in the preliminary negotiations for the founding of Kappa Mu Epsilon, National Honorary Fraternity. His files contained voluminous correspondence with Dr. Katherine Wyant, the first president of Kappa Mu Epsilon. As a result of all this correspondence with Dr. Wyant the first chapter of Kappa Mu Epsilon was installed at Tahlequah, Oklahoma on April 18, 1931. The second chapter to be installed was Iowa Alpha, May 27, 1931. The records of Iowa Alpha show that Mr. Condit was greatly interested in the welfare of Kappa Mu Epsilon and served as its first sponsor.

Mr. Condit served as the first National Vice-President of Kappa Mu Epsilon, at that time known as Vice-President Euclid. As Vice-President Euclid, he participated actively in formulating the policies of the new organization and in setting up the first Kappa Mu Epsilon conventions at Tahlequah, Oklahoma, in 1933, Pittsburg, Kansas, in 1935, and State College, Mississippi, in 1937.

All members of Iowa Alpha and alumni of Iowa Alpha feel that we have lost a friend. However, those who did know him have felt themselves richer because he breathed the spirit of good will toward his fellowmen.

H. Van Engen
Iowa State Teachers College

AN HISTORICAL OUTLINE OF THE DEVELOPMENT OF MATHEMATICS IN THE UNITED STATES DURING THE LAST FIFTY YEARS

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INTRODUCTION

The development of mathematics in the United States, lethargic until the latter part of the nineteenth century, has been so remarkable that to attempt to catalogue all the important contributions would require a great deal more space than can be devoted here. The author, therefore, has endeavored to outline a few of the highlights which have marked the growth of the science to its mature state today. In treating the subject it seems appropriate to trace briefly a little of the background which led to its condition at the turn of the century.

Practically no original work in mathematics was attempted in the Colonies or in the United States before 1800. The schools were content to pass on from the work of scientists in the Old World enough elementary mathematics so that a person seeking it would have a knowledge of simple astronomy and navigation, and a little surveying. After 1700, the American colleges began to copy the work done in the two great universities of Britain. The quality was definitely inferior, but it was possible to take courses in algebra, Euclid, trigonometry, calculus, conic sections, astronomy, and natural philosophy or physics.

In the early nineteenth century, however, America began to show some interest in advancing the subject by original research, and, although mathematics was still closely connected with astronomy and natural philosophy, some attention was given to research in pure mathematics. Interest in the field grew until, in the last quarter of the century, mathematics was no longer a minor topic with applications in one of the other sciences as its prime objective. Young Americans began to go abroad to study mathematics, but instead of remaining there, came home to do research. Their work inspired such intense interest in others that the growth of mathematics as a science in America since that time has been phenomenal.

The trend toward research in pure mathematics was given its first notable impetus by the work of Benjamin Pierce. Predominately an algebraist, Pierce was also well informed about most of the significant mathematical developments of his day, and was probably the most influential American scientist of the nineteenth century. Two of the other men important in the change of temper of this period and typical of leaders in the field were J. W. Gibbs and G. W. Hill. Gibbs accomplished more in the fields of physics and chemistry, but his contributions to vector analysis and the study of Fourier's series were sufficient to gain him belated recognition in the field of mathematics. Hill, an unusual scientist, was one of the world's most outstanding men in the field of mathematical and theoretical astronomy. Thus it can be seen that, even in this period of transformation where mathematics gains recognition as a science in itself, American mathematicians can lay undisputed claim to only one of perhaps the three most important contributors.

The founding of the New York Mathematical Society by T. S. Fiske in 1888 was certainly one of the most important factors in advancing research in this country. Its inception brought the opportunity for mathematicians to gather together, present papers on their work, and in general to spur interest and speed progress in mathematical research. *The Bulletin,* first published by the Society in 1891, became the second important American mathematical periodical, having been preceded by the *American Journal of Mathematics* in 1878. The Society, which became the American Mathematical Society in 1894, and its *Bulletin* have continued to play leading roles in the advancement of mathematical research throughout the intervening period between 1888 and the present time.

The founding of the Johns Hopkins University in 1876 marked the first step toward a high level of training in mathematics by the colleges and universities of this country. It was sixteen years later, in 1892, that the University of Chicago suddenly came into existence with a department of mathematics headed by a trio of whom we are to hear much more, E. H. Moore, Oskar Bolza and Heinrich Maschke. At about the same time every effort was being made to advance the standard at Harvard by Maxime Bôcher and W. F. Osgood.

Thus, as the turn of the century approached, mathematics was growing up in the New World. Strong beginnings were being made in the abstract and algebraic side of mathematics at Chicago, while progress was under way at Harvard in the vast field of analysis. To draw a line through the year 1900, however, and to say that everything worth noting was accomplished after that date and anything done previously was merely introductory would not only seem incredulous, but would be contrary to fact. Let us say, then, that the real period of accomplishment was under way as the twentieth century appeared on the horizon.

GENERAL HISTORY

During the period from 1892 through 1908 the University of Chicago was unsurpassed in America as an institution for the study of higher mathematics. In this space of sixteen years many of America's most outstanding mathematicians had their training at Chicago. L. E. Dickson, G. D. Birkhoff, Oswald Veblen, H. E. Slaught, G. A. Bliss, R. L. Moore and T. H. Hildebrandt are a few of these. In 1908, Maschke died as the result of an emergency operation, and a few years later Bolza returned to Germany. Bolza had accepted the position at Chicago on the condition that Maschke would be appointed simultaneously, and with his colleague's death he felt he could no longer carry on. The loss of these two men was irreparable, but Chicago, nevertheless, has continued to be one of our leading centers of mathematical activity.

At Harvard, where the first rays of the dawn of American mathematics had been seen in the work of Pierce and W. E. Byerly, another important research center was developing. Bôcher, with his work on the theory of ordinary linear differential equations and his investigation of boundary problems, and Osgood, whose work in the calculus of variations as well as other fields of analysis was outstanding, were the early leaders here. Harvard has had almost a steady procession of outstanding mathematicians on its staff for the past fifty years. Among some of the more important of whom we shall see more later are G. D. Birkhoff, E. V. Huntington, Marston Morse, and Garrett Birkhoff.

Soon after the turn of the century H. B. Fine, head of the department at Princeton University, in order to strengthen his department, called L. P. Eisenhart, Veblen, and J. H. M. Wedderburn to his staff. Since their appointment in 1905, Princeton has been a leading center in the field of geometry, especially topology. In 1930, the Institute for Advanced Study, at first devoted primarily to mathematics and theoretical physics, was founded. Through the leadership of men like Albert Einstein, Veblen, and Hermann Weyl the Institute remains the outstanding institution for advanced study beyond the doctorate.

The three institutions just mentioned were the pioneers in advancing the study of higher mathematics. However, many others have followed similar paths in intervening years until today there are more than thirty colleges and universities where an advanced student may advantageously go for work on the doctorate.

Obviously the progress of a science depends to a great degree upon its professional societies and publications. The tremendous importance of the American Mathematical Society and its *Bulletin* have already been mentioned as has the *American Journal of Mathematics,* published at Johns Hopkins University. It was in January of 1900 that the *Transactions of the American Mathematical Society* was first published. This periodical was intended to serve as a medium for publication of the leading American research papers, and has been regarded as such since its founding.

In 1916, the Mathematical Association of America was founded, largely through the efforts of Slaught, and a year later *The American Mathematical Monthly* was made its official organ. This organization had as its purpose the advancement of mathematics at the collegiate level, and today numbers a membership of almost 3000. There are numerous localized societies to say nothing of the many organizations of secondary school teachers of mathematics which, through their meetings and publications, have done their part to promote research and improve teaching methods.

An institution for the advancement of mathematics the world over was the first International Mathematical Congress held in connection with the Chicago World's Fair of 1893. American mathematicians took a leading role in organizing the Congress, which was the forerunner of many similar meetings, and seven countries participated actively. The Congress was followed by a colloquium presided over by the outstanding German mathematician, Felix Klein. A similar meeting was the Congress of Arts and Sciences in 1904 at St. Louis where the Louisiana Purchase Exposition was being held. Simon Newcomb was president of the Congress and the mathematics section was headed by H. S. White. In 1924 the seventh International Congress was held in Toronto where 387 persons representing 25 countries gathered. The staggering total of 235 papers was read before the various sections of the Congress.

In attempting to review progress of mathematics in the past fifty years, it seems inadvisable to follow a purely chronological development. Therefore, we shall investigate some of the significant advances in the fields of algebra, analysis, geometry and logic. There are, however, several men who have exerted a great deal of influence in more than a single field. Two of the greatest mathematicians in American history who can be classed as universalists are E. H. Moore and G. D. Birkhoff. They have made such extensive contributions to mathematics that it seems advantageous to focus attention on their individual accomplishments first. It is appropriate that these two men should have been the leaders at the two institutions which have perhaps been our most outstanding mathematical centers.

The "foremost leader in freeing American mathematicians from dependence on foreign universities and in building up a vigorous American school" was E. H. Moore. It was at Yale, where Moore received his formal undergraduate and graduate training, that he first developed his interest in research. This interest was furthered during the year he spent at the University of Berlin, where Weierstrass and Kronecker were lecturing. Called as acting head of the mathematics department at the opening of the University of Chicago in 1892, he was appointed head of the department four years later, a position he held until a year before his death in 1932.

His research work can be divided into four groups: geometry; groups, numbers, and algebra; theory of functions; integral equations. His studies in algebra and the theory of groups fell in the period of his greatest activity as a writer while integral equations and his general analysis were his absorbing interests during later life when he published least. His greatest enthusiasm was for the general analysis in which he never lost interest.

Moore's research in geometry was largely in algebraic geometry and in postulational foundations where he was inspired by the work of Pasch and Peano and especially by the publication in 1899 of the celebrated book, *Foundations of Geometry* by David Hilbert, the leading mathematician of the day. He incorporated into his work to a great extent the theory of linear systems of curves. Moore's chief accomplishment in the field of geometry was the formulation of an axiomatic system for n-dimensional geometry using points as the only undefined elements instead of the points, lines and planes used by Hilbert in the three-dimensional case.

In 1893, Moore presented a paper at the Chicago Congress in the field of algebra which contained a generalization of the modular group, the first statement and proof of the theorem which states that every finite field is a Galois field, and a discussion of a doubly infinite system of simple groups, most of which had not been known before. In nearly all of his algebraic research there was the essence of generalization and abstractness. In a paper of 1896 he generalized Fermat's theorem.

His great power of analysis was first shown in an exceptionally clearly written paper on transcendentally transcendental functions which also demonstrated his ingenuity in mathematical generalizations. The space filling curves of Peano and Hilbert are discussed in one of his best known papers, *On certain crinkly curves,* published in the first volume of the *Transactions,* of which he was an editor.

Of all subjects upon which Moore did research work, the theories of integral equations and general analysis were the ones which most captured his interest. He saw the equations studied by Fredholm and Hilbert in their fundamental papers must be special instances of a much more general linear equation and he set about evolving a general theory which would include them all. In colloquium lectures of 1906 and papers published in 1911 and 1913, he outlined his first theory of general analysis and his generalization of the preceding theories of linear equations. He was particularly original in that his postulational attack applied to classes of functions rather than to individual cases. In attempting to complete the theory he encountered such complexities that about 1915 he turned to a second, more constructive theory which was similar but had a somewhat simpler basis. The resulting development of the theory has been published posthumously with the cooperation of R. W. Barnard.

G. D. Birkhoff was fortunate enough to have studied at the two leading institutions and under all the leading mathematicians of the day. His first contact with the field of higher mathematics was at Chicago, but after a year he transferred to Harvard for his A.B. and A.M. before returning to Chicago to work on the doctorate under Moore. After instructing in mathematics at Wisconsin and Princeton he was appointed to the faculty at Harvard in 1912 and remained there the rest of his life. Birkhoff admired his teachers Moore and Bôcher, but, except for the fact that he followed somewhat the pattern of Poincaré in his work on analysis, his research was marked by individualism. He regarded his European contemporaries as colleagues rather than as teachers as was the custom at that time. As a matter of fact he was inclined to treat lightly the work of many of the leading European mathematicians. A few exceptions were Riemann, who drew Birkhoff's respect, and his friends, Levi-Civita and Hadamard.

An overall picture of Birkhoff's work is usually highlighted by the proof of Poincaré's last theorem and his proof of the ergodic theorem, but it is doubtful if these are any more outstanding than some other phases of his work, such as his contributions to problems of difference equations and to the theory of the generalized Riemann problem, or his formal theory of stability in dynamics and its consequent geometrical theory. All of these problems involve structures upon which much could be built. Altogether, it can be said that his wide assortment of research topics included problems mainly in differential and difference equations, the calculus of variations, dynamical systems and stability, Poincaré's geometric theorem, and relativity.

Birkhoff published his first paper in 1904 with H. S. Vandiver while both were still students. The paper, *On the intergral divisors of* a^n-b^n, contained the basis for the proof that there are an infinity of primes congruent to 1 mod n. He also worked on general mean value and remainder theorems with regard to their applications to differentiation and quadrature. In 1908, he first introduced in a general manner the conception of adjoint pairs of linear systems, each consisting of an homogeneous linear differential equation of the nth order and of n homogeneous linear boundary conditions. This was later applied to the theory of linear boundary problems by Bôcher.

With his interest aroused by Veblen's seminar on topology, Birkhoff published two papers in 1912 and 1913 on the four-color problem where he introduced the quantitative approach by means of his "chromatic polynomials." This approach was not as successful as the qualitative method, however. One of Birkhoff's more important contributions was his investigation with O. D. Kellogg of fixed points in function space. In 1932, Birkhoff wrote a set of postulates for plane geometry based on scale and protractor. This project was topped by publication of a secondary school textbook, *Basic Geometry*, by Birkhoff and Ralph Beatley in 1940. The purpose of the book is to do away with "self evident truths" and to emphasize the necessity for undefined terms and assumptions at the high school level.

Birkhoff's varied interests were brought into even sharper focus in his book, *Aesthetic Measure,* which was the climax of a series of studies in the mathematical analysis of art and music.

His major interests in the last years of his life was in the study of relativity and gravitation. As the final phase of his work he proposed a linear gravitational theory in 1943 shortly before his sudden and premature death. It can be said that no mathematician of Birkhoff's day possessed greater facilities for combining formal algebraic methods with penetrating numerical analysis. During the major portion of his life he was the recognized leader of American mathematics.

ALGEBRA

The story of the rapid growth of algebra in this country has been one of change from the viewpoint that everything was special and detailed to the highly abstracted and general outlook of today's modern higher algebra. Through the long neglected contributions of Pierce and the work of the English algebraist, J. J. Sylvester, at Johns Hopkins, algebra took its first faltering steps in the New World.

One of the earliest, influential advocates of the modern algebra was F. N. Cole, who exerted his influence in the theory of finite groups through his own work and through that of his many pupils. We have already discussed some of the vital contributions of Moore, but in the field of algebra it should be mentioned that it was one of his students, L. E. Dickson, who was to become the leading algebraist of the twentieth century. The influence of these two men and, indeed, of the Chicago group itself, was the first strong impulse toward the abstractness found in algebra today.

Let us look, first, at the contributions of Dickson which, alone, could be said to give a general outline of the progress in the three subdivisions of algebra, linear algebra, finite groups, and the theory of numbers. An extremely prolific research mathematician, Dickson has published more than 280 papers and eighteen books of which at least fifteen have been of a highly important nature. His monumental *History of the Theory of Numbers* in three volumes is perhaps the most widely known. In his *Algebras and Their Arithmetics* he devised a new theory to determine what should be the subject matter of algebras and arithmetics. This theory was used to construct a highly developed science of arithmetics which resulted in great advances in the theory of algebraic numbers and in the generalization of Hurwitz's integral quaternions. Dickson developed the fundamental cyclic algebras and was the founder of the theory of modular invariants.

In the theory of numbers Dickson proved that Fermat's equation

$x^p+y^p+z^p=0$ .where p is an odd prime

has no solutions in integers prime to p for $p<7000$. He had an outstanding solution of Waring's problem, that every integer is the sum of a finite fixed number of nth powers, where $9\leqq n\leqq 400$. It has been said that much of Dickson's work in this field indicates an outlook similar to that of Gauss, but apparently Gauss never approached the problem of generality in algebraic solubility.

Almost all of the total output in the theory of groups during the last fifty years can be traced to G. A. Miller (a student of Cole), Dickson, and to mathematicians who took their advanced training under these men. Miller is regarded as perhaps the outstanding authority on special finite groups.

The modern postulational method, an outgrowth from geometry, was first adapted to algebra by Moore, Dickson, and E. V. Huntington. Many of the earlier applications of this method were to Boolean algebra, which will be discussed later in connection with symbolic logic. A considerable period of time elapsed between the inception of the postulational method in America for application to groups early in the twentieth century and the next specific use of the method in regard to special groups by R. Garver, who gave a remarkable set of three postulates in 1935.

Work on the theory of algebraic invariants was almost neglected between 1900 and 1930, but has since shown some signs of resurrection partly because of its application to atomic physics. The study of modular invariants has had both a rise and decline apparently due to a lack of extensions. A. B. Coble has contributed to the theory of algebraic invariants and to the modular groups associated with the abelian θ-functions.

The theory of equations, which at one time included almost all of what was known as algebra, is no longer a popular research topic, but still attracts some interest most of which is absorbed in college textbooks. The Galois theory, introduced in this country by Bolza and later treated by Dickson, seems to be too abstract to invite much research in spite of a new approach by A. A. Albert in 1936. What research has been done in this field is largely devoted to methods of obtaining roots or discussions of equations of degree greater than four. Some investigation has also been made in determinants and matrices.

Great progress has been made in the general theory of linear algebras in the past fifty years, much of it contributed by Dickson, Wedderburn, and Albert. This progress took its first notable steps about 1905, both Dickson and Wedderburn entering the field, the latter with a proof that a Galois field is the only algebra with a finite number of elements that is a linear associative division algebra. The next year Dickson made two contributions to the field of division algebras, roughly consisting of a method of constructing an algebra of mk units from an algebra of m units when m equals two and an investigation of commutative linear algebras in $2n$ units, n of the units defining a field subalgebra. It is impossible even to attempt a complete discussion of the contributions to linear algebras at this time, but let it suffice to say that linear algebra is a field of much more activity than it was fifty years ago.

The latest phase of algebra is the modern abstract algebra, which has been attracting interest of any degree in this country for only fifteen to twenty years. Albert has made several contributions to cyclic systems. His book, *Structures of Algebras*, published in 1939, is a general approach to modern algebra, and forms an excellent supplement to his *Modern Higher Algebra* of 1937. So far the most striking advances in this field are the similar theories of O. Ore and Garrett Birkhoff, son of G. D. Birkhoff. Ore has worked on the foundation of abstract algebra through his theory of "structures," while Birkhoff's theory of "lattices" has found many applications. This latter theory is similar in many respects to group theory. For example, it is based on a single operation, that of inclusion, and on a set of three postulates which result in a "partial ordering." Applications in abstract algebra and especially in projective geometry are numerous.

ANALYSIS

The vast advances in the field of analysis over this half-century began with the work of Moore, Osgood, Bôcher and E. B. Van Vleck. The work of Moore has already been discussed. Contributions of Osgood have been largely in the theory of functions of a single complex variable; however, his fundamental theorem of the calculus of variations was established in 1901 and has become a classical theorem in the study of that subject. His work has been continued and extended by J. L. Walsh, whose contributions to the theory of the approximation to analytic and harmonic functions by polynomials and by rational functions have been important. A great deal of Bôcher's work concerns the potential equation and similar related topics, such as boundary value problems in one dimension. Van Vleck did considerable work on continued fractions as defined by linear difference equations of the second order in n, with coefficients dependent on x, and continued his work on linear difference equations largely through lectures rather than published papers.

Although important contributions have been made to functional analysis in the general sense, as originated by Moore, through the work of T. H. Hildebrandt, L. M. Graves, and W. L. Hart, a later trend seems to limit attention to a few significant spaces. The newer form has found interested followers in von Neumann, W. Mayer, T. Y. Thomas and G. Birkhoff. The approach to functional analysis by existence theorems initiated by G. D. Birkhoff and Kellogg in 1922 has proved to be somewhat more effective than direct abstraction. The general attack on functions of a complex variable has been carried on by S. Lefschetz and J. W. Alexander.

The calculus of variations, given a good beginning by Bolza, has continued to find support at Chicago through Bliss and his students, M. R. Hestenes and W. T. Reid. Application of the calculus of variations to dynamical systems was made by G. D. Birkhoff as early as 1918. His work was generalized by Marston Morse, whose colloquium lectures on the subject, published in 1934, were particularly important. Abstractly, "the calculus of variations may be regarded as a critical point problem for a real function defined over a certain kind of function space." By taking this viewpoint, Morse has produced important results.

J. Douglas and T. Rado, by different methods, solved the famous problem of Plateau to find the surface of minimum area bounded by one or more given closed curves. Several important advances have been made within the past twenty-five years in potential theory, especially through the work of Kellogg and Weiner. Much of the work on Fourier's series and integrals has been done by Dunham Jackson and Weiner, while contributions to series in general have been made by C. N. Moore and R. P. Agnew.

In the field of ordinary differential equations Osgood was an early leader. In 1898, he showed in a discussion of the equation of the first order, $y'=f(x,y)$, that the usual uniqueness theorem did not hold. Bliss and Mason subsequently made separate studies of existence theorems, the latter in his Yale Colloquium Lectures of 1906 giving a convenient method of obtaining the unique solution of a linear equation. Of considerable importance also is the work of Ritt regarding polynomial differential equations satisfied by transcendental functions.

Advances beyond those of Bôcher in boundary value problems have been made by Tamarkin, Stone, and R. E. Langer. Most of the important work in linear differential equations has been discussed in connection with Birkhoff. R. D. Carmichael was the first American to work on linear difference equations, and with Birkhoff has continued to make important progress. I. M. Sheffer and W. B. Ford are others who have contributed to this field.

GEOMETRY

Geometry in this country got a somewhat later start than some of the other branches of mathematics, the first real advances in this field being noted about 1904 when Veblen published a set of postulates for Euclidean geometry using the concept of "order" rather than Hilbert's notion of "betweenness." In 1910, Coolidge's *The Geometry of the Circle and the Sphere* was of considerable value to elementary geometry. At about the same time Veblen and Young's *Projective Geometry* brought the axiomatic approach to geometry into sharper focus. E. V. Huntington demonstrated the unlimited possibilities of the postulational approach, in 1913, when he organized Euclidean geometry using a sphere as the undefined term.

In differential geometry the work of Eisenhart has been outstanding. In the closely associated realm of projective differential geometry many important contributions have been made by E. J. Wilczynski. In 1918, Weyl used the idea of parallel displacement, developed by Levi Civita a year earlier, as a basis for his projective theory of affine synmetric connection. A similar non-Riemannian geometry was derived in 1922 by Eisenhart and Veblen using a different approach.

Although it is often considered to be a separate branch of mathematics, especially by those who work in the field, analysis situs or topology is closely related to geometry and will be treated here. The beginnings of topology in this country may conceivably be traced to E. H. Moore, although he made no specific contribution to the subject as such, because of the tremendous importance, in its rapid development, of two of his students, R. L. Moore and Veblen.

Notable activity in the field of topology was initiated by Veblen's Cambridge colloquium lectures of 1916, which were published in 1921. Veblen had been interested in the subject as early as 1911 and, with Alexander, has given strong support to its development since that time. These two early leaders were later joined by a third when Lefschetz began to investigate questions in combinatorial topology. He has contributed important work on fixed point theorems for *n*-dimensional manifolds.

It is important to note that the previously mentioned men were members of the Princeton group and that their studies in topology did not include the point-set-theoretical side of the subject. The possibilities of this phase were noted by R. L. Moore over forty years ago, and it has been Moore and his students who have led in its progress. Some of the men who have done outstanding work are W. L. Ayres, H. M. Gehman, J. R. Kline, G. T. Whyburn and R. L. Wilder. The inter-relation of topology and dynamics has been remarkable. It so happens that many point-set-theory problems stem from theoretical dynamics, but it has been by some sort of intuition that mathematicians have formulated many of the questions of most interest to dynamics. Of course some mention should be made of the four-color problem which has been such an outstanding curiosity in the field of topology. An interesting comparison can be made of the two schools of topology through two books published in 1942, G. T. Whyburn's *Analytical Topology* and S. Lefschetz' *Algebraic Topology.*

SYMBOLIC LOGIC

Definite work on symbolic logic began in America in 1904 when E. V. Huntington published a set of postulates in which he showed that one operation could be substituted for the two established by the English mathematician, George Boole, who had opened up the field with his algebra of logic in 1847. C. S. Pierce, son of Benjamin Pierce, had worked in Boolean algebra earlier, but had contributed few outstanding results. In 1913, H. M. Sheffer set up a system of only three postulates by using a single operation. M. H. Stone contributed to Boolean algebra by introducing, in 1937, the idea of a "ring" of numbers with reference to certain operations. He established Boolean algebra as a special algebra in the ordinary sense, and consequently was able to use modern algebraic methods in attacks on the subject.

The importance of logic has grown to the point where the general idea now prevalent is that mathematics is a game to be played with symbols, among which are 1, 2, a, x, and —, according to specific rules which are essentially those systematized in Bertrand Russell and A. N. Whitehead's *Principia Mathematica,* of 1910. In this book they sought to establish a theory of logical propositions which would govern all mathematics. Logic and mathematics are so closely connected that Russell has set forth the theory that mathematics can be completely developed from logic, while on opposing school led by L. E. J. Brouwer argues that the basic properties of mathematics are found in nature and that logic is merely an offshoot of the main tree consisting of the rules to be followed. The general feeling seems to lie somewhere between the two.

Any discussion of symbolic logic must necessarily include reference to the influence of the axiomatic system. The author would like to propose an hypothesis that the axiomatic system has been one of the most important factors, if not the most important factor, in the growth of mathematics to the mature science we know today. The unprecedented activity in axiomatics in the United States during the twentieth century came as a direct consequence of Hilbert's book on the foundations of geometry, and although geometry has perhaps been influenced to the greatest extent, logical formalism in all mathematics has resulted.

The idea that mathematics is not necessarily real and tangible, an idea developed during the period of non-Euclidean geometries, has been expanded in the twentieth century in the fields of algebra and analysis as well as geometry. It is a prime necessity in symbolic logic.

One of the latest and perhaps most important developments in axiomatics is the theory of lattices discussed earlier in connection with algebra. It may be that more will be accomplished in the unification of mathematics by the theory of lattices than has been accomplished by Moore's general analysis.

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A CAREER AS ACTUARY*

CHARACTER OF ACTUARIAL WORK

Inasmuch as there is now and probably will be for many years to come a shortage of young men with the mathematical ability, knowledge, and qualities of business leadership required in the actuarial profession, this pamphlet has been prepared to furnish college students with a description of the character of actuarial work and an outline of the opportunities.

It is the actuary who is responsible for calculating the premiums a life insurance company must charge and who prepares the tables of death rates upon which such calculations are based. In actual practice his duties cover a much wider field than such technical responsibilities. They include the decision as to what benefits shall be contained in life insurance policies and how much money must be set aside from year to year to guarantee the payment of such benefits many years in the future. The actuary must analyze the sources of earnings under policy contracts so that he may determine proper rates of dividends. He investigates the effect on mortality of various physical impairments, hazardous occupations, and other unusual risks, and in collaboration with the medical officer determines the basis for accepting or rejecting applicants for insurance. Because of his broad fundamental training the actuary of a life insurance company usually has an important part in developing the general executive policies of the company. Although he cannot operate without a thorough knowledge of the mathematical basis of life insurance, essentially he is a business man rather than a mathematician. Not the least of his duties is to explain complicated problems to other business men and to policyholders in language clear enough to be readily understood.

The following statement made about sixty years ago before the Institute of Actuaries of Great Britain is still true today:

*Reprinted from a pamphlet prepared by the Actuarial Society of America and the American Institute of Actuaries, 1946.

"An Actuary should be a man of general culture with a knowledge of both books and men and the more he has of both the better."

A prominent contemporary actuary has described the qualities required for success as "competent statistical and mathematical capacity, adequate economic and financial knowledge, and wide social information," and the successful actuary has been further characterized as a person "with a determined, lively, and ingenious mind and a broad outlook." Althugh the actuarial profession is not restricted to the single field of life insurance, the majority of actuaries are to be found in life insurance companies. Some actuaries are employed by the Federal and state governments, and a few are established as independent consultants.

QUALIFICATION AS AN ACTUARY

Qualification is attained in the process of passing a number of examinations required for membership in a recognized actuarial body. In the life insurance field in the United States and Canada, these bodies are the Actuarial Society of America and the American Institute of Actuaries. Both of these Societies are private organizations run entirely by their members and have been organized for the purpose of promoting the theory and practice of actuarial science. Through their publications and the regular meetings of the members, there is an interchange and development of congenial friendships.

Admission to membership is by a series of examinations given jointly by the two Societies. On passing a series of five examinations the candidate becomes an Associate member, and on passing three more examinations a full member or Fellow. Ordinarily the later examinations are taken by the candidate while he is employed by a life insurance company or in some other actuarial capacity, since practical experience is very helpful, in fact almost essential, in preparing for these examinations. It is a great advantage to the applicant, however, to be able to take the earlier examinations while he is still in college, as they cover subjects which are ordinarily taught in colleges. Another advantage is that if the student starts his examinations while sitll in college he can measure his aptitudes and probability of passing and thus determine before he graduates whether to pursue an actuarial career or to enter some other field.

PRELIMINARY EXAMINATIONS

There are three examinations, given usually in June of each year, which it is well for the candidate to take while he is in college.

Language Aptitude Test. This consists of a series of questions and exercises to test the student's knowledge of words and their shades of meaning, his ability to read the English language and to interpret it logically, and his ability to express himself clearly and concisely.

General Mathematics. This examination covers such mathematical subjects as are usually taught during the first two years in colleges and universities in North America, with special emphasis on college algebra and differential and integral calculus.

Finite Differences, Probabilities and Elementary Statistics. If these subjects are not covered in the student's college course, he may be able to do some independent studying for them while in college, or he may take this examination after he has begun his employment following graduation.

These three examinations are conducted by the College Entrance Examination Board and a pamphlet giving the dates of the examinations, an outline of the subject matter to be covered and illustrative questions can be secured by applying to the Actuarial Society of America[1] or the American Institute of Actuaries[2]. This pamphlet also contains a list of the subjects covered by the fourth and fifth examinations required for Associate membership and the three additional examinations which the student must pass in order to become a full member or Fellow.

SUBJECTS TO BE TAKEN IN COLLEGE

The student should prepare for the different mathematical subjects on which the early examinations are given;

1—The Acturial Society of America, Room 912, 393 Seventh Avenue, New York 1, N.Y.
2—American Institute of Actuaries, 135 South LaSalle Street, Chicago 3, Illinois.

usually his major will be mathematics as a matter of course. As in all occupations requiring judgment and a broad cultural background, a thorough grounding in English composition is essential. The student should study general business accounting, if courses are available in that subject, as it is an important aid in understanding life insurance accounts, both for the later actuarial examinations and for actuarial practice. If he has the opportunity the student should also take at least one full year course in economics. Aside from these particular subjects, it is most important for the candidate to study subjects which will give him a broad cultural foundation, as his most important responsibility will be the exercise of sound judgment.

EMPLOYMENT

Employment by life insurance companies, or other employers, of persons who wish to enter the actuarial profession depends upon the individual choice of the employer in each case. The first requirement is that the candidate give evidence that he will probably be able to pass the examinations. While a very good college record in mathematics is usually considered satisfactory evidence to this effect, the candidate who has received credit for some of the preliminary actuarial examinations taken while in college will have an advantage in this respect. After the candidate has given satisfactory evidence of his mathematical competence, the employing company generally has him interviewed by several indiivduals who attempt to assess his abilities on the basis of the non-mathematical qualities required of an actuary, and even though there may be no question as to his technical mathematical ability, he is employed only if the result of these interviews is favorable.

Information as to the location of the life insurance companies and other organizations seeking actuarial students may be obtained in the year of graduation from college by applying to the Secretary of the Actuarial Society or the American Institute of Actuaries.

OPPORTUNITIES

The field is not crowded. There are more than 350 life insurance companies in the United States and Canada. Most of these companies are growing in size and, as is the case with other activities in our civilization, their business is continually becoming more complex. As a result of the wide scope of actuarial training the actuary is particularly well fitted to secure a proper perspective of the interrelated problems of a life insurance company and his advice is much sought after on questions of general company policy.

All the life insurance companies need the services of actuaries and the large companies, particularly those that are engaged in writing Industrial insurance and Group insurance, need large staffs of actuaries. Moreover, there is a growing tendency for men initially trained as actuaries to move on to other spheres of activity in the insurance offices, particularly in the investment, administrative, underwriting, and accounting fields, and a considerable number of actuaries fill high executive positions.

There are many needs for actuaries outside of the life insurance companies. The increasing trend towards pension and retirement plans in business and industry provides many opportunities in the various firms of consulting actuaries. There are actuarial positions in many of the state insurance departments, which supervise and regulate the insurance business. The Federal government uses actuaries in several of its departments, as for example in connection with the administration of the Social Security act and in the Census bureau. There are occasionally calls from Latin America and other foreign countries for American-trained actuaries.

An actuarial career has the advantage that adequate salaries are paid even during the training period. Later on, the actuary's salary naturally depends upon the extent of the responsibilities he is capable of assuming. The average salary earned by qualified actuaries is large, because they are highly trained specialists in considerable demand.

There are only about 600 persons who are Fellows of the Actuarial Society of America or the American Institute of Actuaries. Many more are needed. Anyone with the talents necessary for success in this field should seriously consider entering the actuarial profession.

NUMERICAL DOUBLE-ANGLE TRIANGLES

PROFESSOR DONALD MEEKER BROWN

Central Michigan College of Education

Introduction. Write any proper rational fraction, add 1 to it, and express the sum as an improper fraction reduced to lowest terms. Let the square of the denominator of the result be s, the product of the numerator and denominator be d, and the square of the numerator minus the square of the denominator be t. Then it will be shown that the triangle with sides s, d, and t (all integers) is such that the angle opposite d is double the angle opposite s.

Definitions:

1. A double-angle triangle is one in which one angle is double another angle.

2. A numerical triangle is one each side of which is an integer.

3. A primitive triangle is a numerical triangle the sides of which have no common divisor other than 1.

4. A Pythagorean triangle is a numerical right triangle.

5. Two integers are said to be relatively prime if they have no common divisor other than 1.

It is the purpose of this article to show that the triangle with sides s, d and t obtained by the above procedure is always a primitive double-angle triangle, and that any numerical double-angle triangle is similar to a triangle formed in that way.

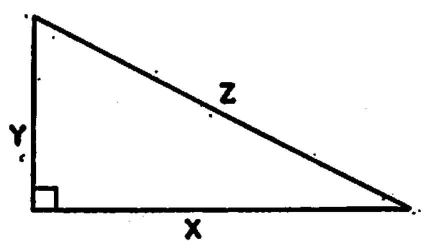

Fig. 1

Unless otherwise noted, all symbols in the discussion will denote positive integers, with the exception of those letters used to denote angles or points of a triangle.

Statement of Relationships. It is well known that a triangle is a Pythagorean triangle (Fig. 1) if, and only if, its sides x, y and z satisfy the following relations [1]:

(1.1) $b < a$
(1.2) $a + b$ is an odd integer
(1.3) a is relatively prime to b
(1.4) $x = (2ab)f$
(1.5) $y = (a^2 - b^2)f$
(1.6) $z = (a^2 + b^2)f$
(1.7) The triangle is primitive if, and only if, $f = 1$.

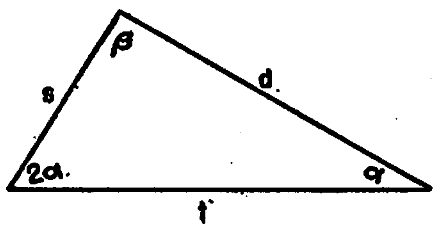

Fig. 2

Let Fig. 2 represent a double-angle triangle, and let s be the side opposite the single angle, d the side opposite the double angle, and t the side opposite the third angle. Then it will be shown that this triangle is a numerical double-angle triangle if, and only if, its sides s, d, and t satisfy the following relations, somewhat analogous to relations (1):

(2.1) $b < a < 2b$
(2.2) a is relatively prime to b
(2.3) $s = (b^2)f$
(2.4) $d = (ab)f$
(2.5) $t = (a^2 - b^2)f$
(2.6) The triangle is primitive if, and only if, $f = 1$.

In Fig. 1 the fundamental relationship is the remarkable Pythagorean theorem,

1—R. D. Carmichael. *Theory of Numbers*. New York, John Wiley & Sons, 1914, pp. 85-90.

(3) $$x^2 + y^2 = z^2.$$

Fig. 1 can be constructed for *any* x, y, and z satisfying (3) irrespective of whether the sides are integral, rational, or irrational. If the restriction that a and b be integers is removed in relations (1), a right triangle will still be produced but it may not be a numerical triangle. An equally fundamental and remarkable relationship holds for double-angle triangles, namely,

(4.1) $$s^2 + st = d^2$$

provided that

(4.2) $$s < d < 2s.$$

It will be shown that Fig. 2 can be constructed if, and only if, the sides s, d, and t satisfy (4.1) and (4.2) regardless of whether they are integral, rational, or irrational. If the restriction that a and b be integers is removed in relations (2), then a double-angle triangle will still be produced but it may not be numerical.

Proofs of Relationships. The necessity of (4.1) and (4.2) will be established first. That is, it will be shown that if a triangle is a double-angle triangle, labeled as in Fig. 2, then it is necessary that (4.1) and (4.2) be true.

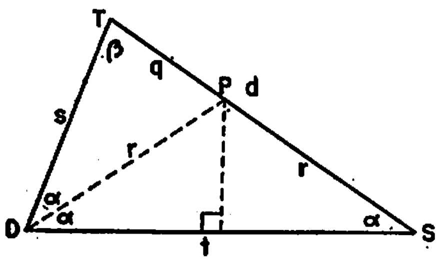

Fig. 3

Fig. 3 is constructed from Fig. 2 in an evident manner. Since $\triangle\, DPT$ is similar to $\triangle\, SDT$, one has

(5) $$s/q = d/s = t/r = 2\cos\alpha,$$

whence $s^2 = dq$ and $dr = st$. Now $q = d-r$ so that $s^2 = d(d - r) = d^2-st$. Since this is equivalent to (4.1), its necessity has been established. From Fig. 3, $3\alpha + \beta = 180°$ so that $0° < 3\alpha < 180°$. Dividing each member of this inequality by 3, one has $0° < \alpha < 60°$. Thus, $2\cos 60° < 2\cos\alpha < 2\cos 0°$ or $1 < 2\cos\alpha < 2$. From (5), $2\cos\alpha = d/s$, whence $1 < d/s < 2$. Multiplication of each member by s yields $s < d < 2s$ which is (4.2). Thus both (4.1) and (4.2) are necessary conditions that a triangle be a double-angle triangle. The only other restrictions on s, d, and t are that they be real positive numbers.

Next it will be proved that for any real positive numbers s, d, and t which satisfy (4.1) and (4.2), a triangle with these numbers as sides can always be constructed such that the angle opposite side d will be double the angle opposite side s. In other words, it will be proved that (4.1) and (4.2) are sufficient conditions for a double-angle triangle.

If (4.2) holds, division of each member by $2s$ gives $1/2 < d/2s < 1$. From this it follows that there exists an angle α such that $\cos\alpha = d/2s$ and $0° < \alpha < 60°$. Therefore, $\alpha + 2\alpha < 180°$ and it is possible to construct a triangle with one angle α and another angle 2α. Let such a triangle be constructed with the included side t (Fig. 4). It is clear that Fig. 4 is identical in form with Fig. 3. It needs

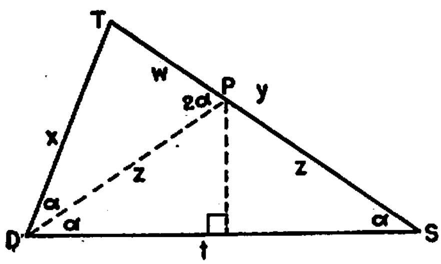

Fig. 4

only to be shown that $x = s$ and $y = d$ to complete the proof of the sufficiency of (4.1) and (4.2).

Applying (5) to Fig. 4, one has $x/w = y/x = t/z = 2\cos\alpha$. Replacing w by its equivalent $y - z$ and utilizing $\cos\alpha = d/2s$, it follows that

(6) $\qquad x/(y-z) = y/x = t/z = d/s = 2\cos\alpha.$

Setting each of the first three members of (6) equal to the fourth member yields the simultaneous equations, $sx - dy + dz = 0$, $dx - sy = 0$, and $dz = st$. Solving for x and y, there results

(7) $$x = s^2t/(d^2 - s^2), \quad y = std/(d^2-s^2).$$

But if (4.1) holds, $d^2-s^2 = st$, and substitution in (7) gives $x = s$, $y = d$. Thus, (4.1) and (4.2) are sufficient conditions for a double-angle triangle.

Le it be required now that s, d, and t be integers. It will be shown that relations (2) result. Multiplication of both members of (4.1) by 4 and the addition of t^2 to both members gives $(2s + t)^2 = 4d^2 + t^2$, whence

(8) $$2s + t = \sqrt{4d^2 + t^2}.$$

Now the left number of (8) is an integer, and so also must be the right member. Let this integer be k. Then

(9) $$k^2 = (2d)^2 + t^2$$

which is of the form (3) with either $x = 2d$ and $y = t$ or $x = t$ and $y = 2d$.

If $x = 2d$, then by (1), (3), and (9), it is possible to find integers a and b such that

(10.1) $b < a$
(10.2) $a + b$ is an odd integer
(10.3) a is relatively prime to b
(10.4) $d = (ab)f$
(10.5) $t = (a^2 - b^2)f$
(10.6) $k = (a^2 + b^2)f$
(10.7) $s = (k - t)/2 = b^2f$
(10.8) s, d, and t have no common factor if, and only if, $f = 1$.

If $x = t$, then by (1), (3), and (9) there exists integers A and B such that

(11.1) $B < A$
(11.2) $A + B$ is an odd integer
(11.3) A is relatively prime to B
(11.4) $t = (2AB)F$
(11.5) $2d = (A^2 - B^2)F$
(11.6) $k = (A^2 + B^2)F$
(11.7) $s = (k - t)/2 = (A - B)^2F/2$

If $A - B$ is even, then $A - B + 2B = A + B$ must also be even which contradicts (11.2). Hence, $A - B$ is odd. From (11.7) it follows that F must be an even integer if s is to be an integer. Let $F = 2F_1$. Then

(11.8) s, d and t have no common factor if, and only if, $F_1 = 1$.

Now let $A + B = p$ and $A - B = q$, so that

(12) $$A = (p + q)/2, \quad B = (p - q)/2.$$

Then

(13.1) $$q < p.$$

Since $p + q = (A + B) + (A - B) = 2A$,

(13.2) $p + q$ is an even integer.

By (11.3), it follows that

(13.3) p is relatively prime to q.

Substitution of (12) in (11.5), (11.4), and (11.7) yields, after simplification,

(13.4) $$d = (pq)F_1$$

(13.5) $$t = (p^2 - q^2)F_1$$

(13.6) $$s = q^2F_1$$

As yet we have not used (4.2). Substitution of (10.4) and (10.7) in (4.2) gives $b^2f < abf < 2b^2f$, whence

(14) $$b < a < 2b.$$

Similarly, substitution of (13.4) and (13.6) in (4.2) gives

(15) $$q < p < 2q.$$

Now it can be seen that the following pairs of statements are identical in form: (14) and (15); (10.1) and (13.1); (10.3) and (13.3); (10.4) and (13.4); (10.5) and (13.5); (10.8) and (11.8). It can also be seen that (10.2) complements (13.2). Thus all of these statements may be combined into the form of relations (2), which was one of our objectives. That is, it has been established that if a triangle is a numerical double-angle triangle, its sides must have the form given in relations (2).

Consider now the converse, namely, if the sides s, d, and t of a triangle have the form of relations (2), then the triangle is a numerical double-angle triangle. Note that $(abf)^2 = a^2b^2f^2 + b^4f^2 - b^4f^2$ or $(abf)^2 = (b^2f)^2 + (b^2f)\ (a^2-b^2)f$. Comparing this with relations (2), one sees that $d^2 = s^2 + st$ which is (4.1). Next note that by multiplying each member of (2.1) by bf, one obtains $b^2f < abf < 2b^2f$. By use of relations (2), this becomes $s < d < 2s$ which is (4.1). Thus the triangle is numerical and a double-angle triangle if the sides are of the form (2).

Next the validity of the procedure given in the opening set of instructions in the introduction will be demonstrated. Let the proper fraction be $m/(m + n)$, where, clearly, $0 < m/(m + n) < 1$. Adding 1 to the fraction results in $(2m + n)/(m + n)$ in which $1 < (2m + n)/(m + n) < 2$. Let this fraction be a/b when reduced to lowest terms, so that $1 < a/b < 2$ or, multiplying by b, $b < a < 2b$. Hence, (2.1) and (2.2) hold. Continuing to follow the directions, $s = b^2$, d $= ab$, and $t = a^2 - b^2$. But these are equivalent to (2.3), (2.4), and (2.5), respectively, with $f = 1$ as in (2.6). Hence, s, d, and t form a primitive numerical double-angle triangle as in Fig. 2, with the angle opposite side d double the angle opposite side s. By multiplying each side of such a triangle by an integer f, a similar triangle is obtained. On the other hand, given any numerical double-angle triangle, there exist a and b which will produce a primitive triangle similar to the original one. Starting with the proper fraction $(a - b)/b$ and following the given directions, one obtains the primitive triangle under consideration. Furthermore, $\cos\alpha = d/2s = a/2b$.

Problems.

1. Show that the triangle with sides 4, 5, 6 is the simplest primitive double-angle triangle.

2. Find all numerical double-angle triangles whose sides are in arithmetic progression.

3. Prove that no numerical double-angle triangle exists whose sides form a geometric progression or a harmonic progression.

4. Find numerical double-angle triangles which have an angle in common with a Pythagorean triangle.

5. Find numerical double-angle triangles such that two sides are perfect squares.

6. Prove that there is no Pythagorean double-angle triangle.

TOPICS FOR CHAPTER PROGRAMS—V

14. CODES AND CIPHERS

The importance of cryptography in time of war is obvious. The subject has been studied for centuries, investigators being attracted by the implied challenge of a piece of secret writing to find its key. The peculiar talents which enable a mathematician to solve his problems are essentially those which enable a cryptologist to solve a code or cipher. Indeed, many mathematicians have aided in the development of cryptography, and mathematical analysis has played a role. Among others, John Wallis and Francois Viete became deeply interested in cryptography.

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15. LINKAGES

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16. APPORTIONMENT IN CONGRESS

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THE PROBLEM CORNER

Edited by JUDSON W. FOUST

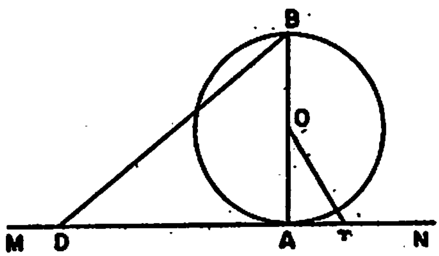
Central Michigan College of Education

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solutions should not demand any tools beyond the calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before November 1, 1948. The best solutions submitted by students will be published in the Fall number of THE PENTAGON. Credit will be given for all correct solutions received. Address all communications to Dr. Judson W. Foust, Central Michigan College of Education, Mount Pleasant, Michigan.

PROBLEMS PROPOSED

5. *Proposed by the Problem Corner Editor.*

The following approximate construction of π was presented in 1685 by Kochansky, a Polish mathematician. Let O be the center of a circle with a radius of one unit.



Let A be the point of tangency of a tangent MN. At O construct an angle AOT equal to 30°, T being the intersection of the side of the angle with the tangent. From T, on MN, in the direction of A, lay off TD equal to 3 units. Then, if B is the other end of the diameter through A, BD

is approximately equal to π. Find to the fifth significant figure the error in this approximation.

6. *From the American Mathematical Monthly, Vol. 23.*

Determine the greatest quadrilateral which can be formed with four given sides a, b, c, d taken in order. (Consider only convex quadrilaterals.)

7. Reference is made in *There Is Fun in Geometry* by Kasper to Huyghen's approximation to the length of an arc. The Problem Corner Editor has sought without success to locate more definite reference to this. Show that, approximately, $L = (8c - C)/3$ in which L is the length of the arc, c is the chord of half the arc, and C is the chord of the arc.

8. *Rupert's Problem*[1].

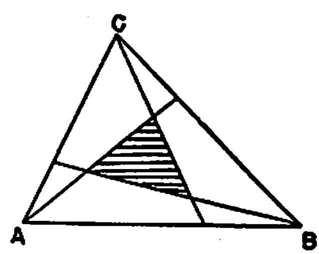
A cube with edge a has an hole cut in it so a larger cube can be passed through. Find the edge of the cube of maximum size which can be passed through.

SOLUTIONS

3. *Proposed by the Editor.*

Each side of triangle ABC is trisected. Show that the area of the shaded part of the figure is exactly one-seventh of the whole triangle.

Solution by Robert E. Hogan, Missouri Alpha Chapter, Southwest State College, Springfield, Missouri.



1) Let the points of trisection of the sides AC, AB, and BC be called D, E, and F, respectively. Place A at the origin and AB along the positive X-axis. The coordinates of A

1—Rupert served in Napoleon's army.

are (O,O). Let B and C have the coordinates $(3a,0)$ and $(3b,3c)$, respectively. Then the coordinates of the points of trisection are $D(b,c)$, $E(2a,0)$, and $F(2b + a,2c)$.

2) By the two-point formula, the equation of AB is $y = 2cx/(2b + a)$, of BD is $y = c(x - 3a)/(b - 3a)$, and of CE is $y = 3c(x - 2a)/(3b - 2a)$.

3) Solving these equations in pairs gives the coordinates of the vertices of the shaded triangle as follows:

$[3(2b + a)/7, 6c/7]$, $[3(4a + b)/7, 3c/7]$, and $[6(2b + a)/7, 12c/7]$.

4) The area of the shaded triangle in terms of the coordinates of the vertices becomes $9ac/14$.

5) Similarly, the area of the large triangle is $9ac/2$.

6) Area of the small triangle divided by the area of the large triangle is one-seventh.

Mr. Hogan suggests the following generalization of the problem: "Each side of a triangle ABC is divided into m equal divisions and a shaded area formed as above. Show that the area of the shaded part is $(m - 2)^2/(m^2 - m + 1)$ of the whole triangle."

The coordinates of A, B, C are respectively (O,O), $(ma,0)$, (mb, mc). The coordinates of D, E, F are respectively (b,c), $[(m - 1)a,0]$, $[(m - 1)b + a, (m - 1)c]$. Carrying through similar steps of finding the equations of AB, BD, and CE and their intersections in pairs, the area of the shaded triangle is found to be $acm^2 \text{A}m - 2)^2/2(m^2 - m + 1)$. Since the area of the large triangle is $acm^2/2$, their ratio is the required amount.

Observe that $m = 3$ gives one-seventh for the special case first discussed. Also note that the ratio of the areas is zero when $m = 2$. This means that the shaded area is therefore zero, thus proving that the midians of the triangle meet in a point. Furthermore, it is obvious that as m increases the ratio of the two areas approaches unity.

4. *Proposed by the Editor.*

Into a full conical wineglass of depth a and generating angle α there is carefully dropped a sphere of such size as to cause the greatest overflow. Show that the radius of the sphere is $a \sin\alpha/(\sin\alpha + \cos 2\alpha)$.

Solution by Stanley Poley, New York Alpha Chapter, Hofstra College, Hempstead, New York.

1) The volume of the overflow is $V = \pi(rh^2 - h^3/3)$, where h is the altitude of the spherical segment immersed in the wine.

2) The distance the center of the sphere is above the bottom of the glass is $r \csc\alpha = r + a - h$. Therefore, $r = (a - h)/(\csc\alpha - 1)$.

3) Substituting the value of r from (2) in (1) gives

$$V = \pi[(a - h)h^2/(\csc\alpha - 1) - h^3/3].$$

For maximum overflow, $dV/dh = 0$, whence

$$dV/dh = \pi[(2ah - 3h^2)/(\csc\alpha - 1) - h^2] = 0.$$

Therefore,

$$h = 2a/(\csc\alpha + 2).$$

4) Substituting this value of h in (2), gives

$$r = a \csc\alpha/(\csc^2\alpha + \csc\alpha - 2)$$

which reduces to

$$r = a \sin\alpha/(\sin\alpha + \cos 2\alpha).$$

The domain of mathematics is the sole domain of certainty. There and there alone prevail the standards by which every hypothesis respecting the external universe and all observation and all experiment must be finally judged. It is the realm to which all speculation and thought must repair for chastening and sanitation, the court of last resort, I say it reverently, for all intellection whatsoever, whether of demon, or man, or deity. It is there that mind as mind attains its highest estate.

—C. J. KEYSER.

No pleasure endures unseasoned by variety.

—PUBILIUS SYRUS.

=∇=

If you walked around the earth, your head would travel about 38 feet farther than your feet.

=∇=

. Lord Kelvin, unable to meet his classes one day, posted the following notice on the door of his lecture room: "Professor Thompson (Lord Kelvin) will not meet his classes today."

The disappointed class decided to play a joke on the professor. Erasing the 'c' they left the legend to read, "Professor Thompson will not meet his lasses today."

When the class assembled next day in anticipation of the effect of their joke,they were chagrined to find the legend now read, "Professor Thompson will meet his asses today."

=∇=

44	55	66		99
44	55	66		99
16	25	36		81
1616	2525	3636	etc.	8181
16	25	36		81
1936	3025	4356		9801

=∇=

If $\sin A$, $\sin B$, and $\sin C$ are in harmonic progression, so also will be $1 - \cos A$, $1 - \cos B$, and $1 - \cos C$; $A + B + C = 180°$.

=∇=

The phrase "harmony of the spheres" sprang from the notion of Pythagoras that the distances of the stars from the earth formed a musical progression.

A man and a boy agree to dig a patch of potatoes for ten dollars. The man can dig as fast as the boy can pull tops, and he can pull tops twice as fast as the boy can dig. A fair division of the ten dollars is \$4.14 for the boy and \$5.86 for the man.

—SCH. SCI. & MATH.

=∇=

What is the shape of a kiss? Why, a lip tickle, of course.

=∇=

If AC is the bisector of angle A of the triangle BAD, then $AB \cdot AD = AC^2 + BC \cdot CD$. If the triangle BAD is isoceles, then $AB^2 = AC^2 + BC^2$.

=∇=

For each of the three conics, $x^2/a^2 + y^2/b^2 = 1$, $x^2/a^2 - y^2/b^2 = 1$, and $y^2 = 4\,px$, consider the following set of points: F, the focus, taking, for definiteness, the focus with positive abscissa; A, the corresponding intersection of the conic with the x-axis; P, a corresponding intersection of the conic with its latus rectum; T, the foot of the tangent to the conic at P; N, the foot of the normal to the conic at P; C, the center of curvature of the conic for the point A. Then for the central conics the abscissas of T, A, F, C, and N are in geometric progression with $r = e$, and for the parabola the abscissas of T, A, F, C, and N are in arithmetic progression of $d = p$.

—L. S. JOHNSTON.

=∇=

How much does a cork ball six feet in diameter weigh, if cork is known to be one fourth as heavy as water? Estimate the weight, then compute it.

=∇=

If a number is written as a polynomial in 1000 (e.g. $1{,}484{,}365 = p(x) = x^2 + 484\,x + 365$) then $p(-1)$ is the remainder after division by $1001 = 7 \cdot 11 \cdot 13$. Hence a necessary and sufficient condition for the divisibility of the original number by any one of these factors is the divisibility of p (-1) by that factor.

=∇=

The notation arc sin x was introduced by Euler in 1737.

=∇=

In a deck of ordinary playing cards, two of the Jacks are one-eyed, and the other jacks have two eyes. What is the total number of eyes on the four Jack cards?

—DR. I. Q.

=∇=

$$312 \times 221 = 68952$$
$$213 \times 122 = 25986$$

=∇=

Illustration for $x^0 = 1$: A fairly large sheet of uncreased paper is folded repeatedly so that each fold divides the surface into two equal rectangles. One fold gives 2^1 compartments, 2 folds give 2^2 compartments, . . ., n folds give 2^n compartments. It only remains to realize that for the unfolded sheet $n = 0$ and there is 1 compartment.

MATH. GAZETTE.

=∇=

The period is used in mathematical notation both for a decimal point and to indicate multiplication. When it is used to represent either of these ideas its position is not fixed. The English generally write it before the middle of a figure to represent a decimal point and near the bottom to represent multiplication, while in America the reverse is the practice.

=∇=

ABCD is a rectangular barn of dimensions a and b. A cow is tied at A with a rope c feet long. Over how much ground can the cow graze?

3.14159265358979323846264338832795

Now I read a rhyme, expressed in verses clear,
The ratio circular expanding (science inerudite):
And if old Ludolph's very number do appear,
Then 'tis but Egyptian art, or ancient
Syracusan might.

—A. A. BENNETT.

=∇=

Among the sporting fraternity, a "smart" gambler is one who hedges his bets on a sporting event so that at the very worst he will break even. As a matter of fact, under certain conditions he can eliminate the element of luck entirely; that is to say, it is sometimes possible to place wagers in such a manner that the same profit is assured regardless of the outcome of the event. To determine the conditions when this is possible, consider a horse race in which n horses are entered. Suppose the odds quoted on the ith horse to be a_i to b_i. This means that a bettor who wagers b_i dollars on the ith horse stands to gain a_i dollars if that horse wins; if he wagers W dollars, he stands to gain $(a_i/b_i)W$ dollars. The estimated probability that the ith horse will win is $p_i = b_i/(a_i + b_i)$. Let

s_i = the sum wagered on the ith horse,
g_i = the net gain if the ith horse wins,
$S = s_1 + s_2 + \cdots + s_n$,
$P = p_1 + p_2 + \cdots + p_n$.

Now if the ith horse wins the race, the bettor will be returned his wager on that horse together with his winnings of $(a_i/b_i)s_i$. Since his total wager on the race is S dollars, the bettor's net gain will be

$$g_i = s_i + (a_i/b_i)s_i - S = s_i/p_i - S.$$

Hence,

$$s_i/p_i = g_i + S.$$

If it is desired to have the same gain regardless of which horse wins the race, then g_i and consequently $g_i + S$ must be a constant for all values of i. It follows that

$$s_1/p_1 = s_2/p_2 = \cdots = s_n/p_n = g + S.$$

In other words, to be assured of a constant gain it is necessary to distribute the total wager over all the horses in proportion to their estimated probabilities of winning.

Is the condition just stated a sufficient condition to assure a constant gain? Suppose the bets are placed on the horses in proportion to the estimated probabilities; that is, $s_i/p_i = g + S$. Then $s_i = p_i(g + S)$, and $g_i = s_i/p_i - S = (g + S) - S = g$. Thus, the gain on each horse is a constant and the following theorem has been established.

THEOREM. In betting on horse races, a necessary and sufficient condition for a constant gain is that wagers be placed on all the entries in proportion to their estimated probabilities of winning.

Now the gain g that we have been discussing may be positive or negative; that is, it may represent either a profit or a loss. Let us next investigate the conditions that assure an actual profit. Consider the continued proportion,

$$s_1/p_1 = s_2/p_2 = \cdots = s_n/p_n = g + S.$$

By a well-known property of proportions,

$$s_1/p_1 = s_2/p_2 = \cdots = s_n/p_n = (s_1 + s_2 + \cdots + s_n)/(p_1 + p_2 + \cdots + p_n),$$

so that

$$S/P = g + S,$$

whence

$$g = S/P - S = S(1 - P)/P.$$

An examination of this result leads at once to the following theorem:

THEOREM: If a bettor has placed a wager on each horse proportionate to its probability of winning, then he will have a constant loss if P > 1, *will break even if* P = 1, *and will have a constant profit if* P < 1.

The accompanying table exhibits the track odds on the horses entered in the 1945 Kentucky Derby. The third column gives the estimated probabilities as determined by the quoted odds. The sum of these probabilities, computed with the aid of a table of reciprocals, gives $P = 1.47185$. Hence, if a person placed a bet on each horse in proportion to its estimated probability, he was assured of a constant loss. In this case, only a bookmaker who in effects bets *against* each horse could have assured himself of a profit. (Since in practically all horse races $P > 1$, we have demonstrated the futility of betting *on* the horses.) Now suppose a bookmaker at the 1945 Derby limited his "book" to \$1,000, and accepted total bets on each horse proportionate

Horse	*Odds*	p_i	s_i
Air Sailor	10-1	1/11	\$ 62
Alexis	12-1	1/13	52
Bert G.	30-1	1/31	22
Burning Dream	10-1	1/11	62
Bymeabannd	30-1	1/31	22
Darby Dieppe	6-1	1/7	96
Fighting Step	8-1	1/9	76
Foreign Agent	20-1	1/21	32
Hoop Jr.	5-2	2/7	194
Jacobe	30-1	1/31	22
Jeep	4-1	1/5	136
Kenilworth Lad	30-1	1/31	22
Misweet	30-1	1/31	22
Pot O'Luck	4-1	1/5	136
Sea Swallow	30-1	1/31	22
Tiger Rebel	30-1	1/31	22
TOTAL		1.47185	\$1,000

to the estimated probabilities. Then $S = \$1{,}000$ and $g = S(1 - P)/P = -\$320.58$. Thus, $g + S = \$679.42$ and $s_i = (679.42)p_i$. The values of s_i are given in the final column of the table. These values have been rounded off to the nearest \$2 since it is customary to accept no smaller bet than that. No matter which horse won the Derby, the bookmaker was sure to win about \$320. The actual profit could be slightly more or less than this amount since the values of s_i are rounded off.

The Derby was actually won by Hoop, Jr. How did our bookmaker fare? To the holders of the winning tickets he paid out $7/2 \times \$194 = \$679$. Since he took in \$1,000, his profit was \$321. Suppose a long shot, say Tiger

Rose, had won the race. Then he would have paid out $31 \times \$22$, or \$682. The profit in this case would have been \$318. The reader can assume other outcomes of the Derby and verify the fact that in every case the profit would have been about \$320.

An interesting application of the above theory is found in the case of a contest in which there are but two entries such as a boxing match or a football game. It often happens that different odds are quoted on such a contest. For example, a betting commissioner posted the following odds for the 1940 bowl game between Stanford and Nebraska: against Stanford, 1 to 2; against Nebraska, 8 to 5. Here, $P = 1/3 + 8/13 = 37/39$ and the conditions assure him a constant profit. In order to win the same profit no matter which team wins the game, he should distribute his wagers in the proportion $1/3 : 8/13 = 13:24$. Suppose he places \$130 against Stanford and \$240 against Nebraska. Then if Nebraska wins the game, he gains \$260 on the first bet and loses \$240 on the second bet. If Stanford wins, he loses \$130 on the first bet and wins \$150 on the second bet. In either case his net profit is \$20.

$=\nabla=$

Decode the following problem in addition. (Two solutions.)

$$\begin{array}{r} SEVEN \\ THREE \\ TWO \\ \hline TWELVE \end{array}$$

—AM. MATH. MONTH.

$=\nabla=$

It is better, of course, to know useless things than to know nothing.

—SENECA.

A REPORT FROM THE PRESIDENT OF KAPPA MU EPSILON

H. VAN ENGEN

Iowa State Teachers College

The National Council of Kappa Mu Epsilon met at Albion College, Albion, Michigan, on March 12 and 13, 1948. All members of the Council were present. Our Past-President, Dr. E. R. Sleight, and the Pentagon Editor, Dr. H. D. Larsen, made all the arrangements for the meeting and entertained the Council in a most hospitable manner.

The Council spent considerable time reading and rewording the Initiation Ritual. Prior to the 1947 meeting of the Council at Normal, Illinois, the Council was aware of a feeling on the part of those interested in the organization that the ritual did not express the depth of purpose that many Kappa Mu Epsilon members thought it should. Neither was it as dignified an instrument as many desired. Accordingly, at the Council meeting held at Normal, Illinois, in 1947, the Council voted to have the ritual revised. In accordance with this motion the President of the Council asked Dr. C. V. Newsom, Oberlin College, Oberlin, Ohio, to prepare a draft of a revised ritual for the consideration of the Council.

Using Dr. Newsom's draft of the ritual as a basis, the Council agreed on a revised form of the ritual which is better than the initiation ritual now in use. This revised ritual will be available for distribution in the near future. Those chapters wishing to try out the new ritual should ask the National Secretary for a copy. After you have tried the new form of the ritual, the Council would appreciate having your criticisms of it.

The Council gave considerable thought to ways and means to lighten the load of the National Secretary. Better than 800 initiations are being reported annually. The amount of work required to process these initiations is considerable. It would be a great help to Miss Hove, National Secretary of Kappa Mu Epsilon, if the chapters would report initiations as outlined in the pamphlet, "Some Pertinent Suggestions Concerning Kappa Mu Epsilon." Copies of this pamphlet have been sent to all chapters. In case these copies have been mislaid, Miss Hove will gladly send additional copies, upon request.

The Corresponding Secretary of the local chapters is a key member of the organization. No organization can function without an efficient secretary. An efficient corresponding secretary can do much to keep the local chapter in contact wtih the National Organization and with other chapters. For this reason local chapters should provide for the continuance in office of the corresponding secretary, particularly if the corresponding secretary has been at all conscientious in doing the necessary work. The Council urges the local chapters to consider the problem of maintaining close contact with the National Organization through its corresponding secretary. Too frequently vital communications from the National Officers do not find their way to the chapters and, conversely, vital information from the chapters to the National Officers, for example initiations, are not sent promptly to the proper National Officer.

The Council is laying plans for a national convention in the spring of 1949. The time and place of meeting have not been set, hence, there is an opportunity for local chapters to send in suggestions for a better and bigger convention. With a growing Kappa Mu Epsilon it should be possible to hold conventions which are progressively better than any previous convention. It is the Council's desire to serve the best interests of Kappa Mu Epsilon, and to this purpose, we pledge our efforts.

THE CONSTITUTION OF KAPPA MU EPSILON

Article I.—The Fraternity and Its Objectives

Section 1. The name of this organization shall be Kappa Mu Epsilon, Mathematics Fraternity for accredited colleges. Its motto (Greek) shall mean "Develop an appreciation for the beauty of mathematics."

Section 2. The object of the fraternity shall be

A. to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program;

B. to help the undergraduate realize the important role that mathematics has played in the development of the western civilization;

C. to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought;

D. to provide a society for the recognition of outstanding achievement in the study of mathematics in the undergraduate level.

Article II.—Membership

Section 1. The active members shall consist of those members who pay their current dues.

Section 2. Qualifications for membership. A member

A. must be or have been a faculty member or a regularly enrolled student of an accredited four-year college;

B. must have completed at least eight semester hours or the equivalent in college mathematics, which *must* include the basic ideas of Cartesian Geometry;

C. must be above the average in his institution in mathematics and in general scholarship.

Section 3. Members of chapters upon moving to the locality of another chapter may become affiliated there-with by presenting credentials of membership in good standing from the secretary of the initiating chapter.

Section 4. During each biennium, a chapter may initiate two honorary members. The qualifications for membership listed in Section 2 do not apply to honorary members. The chapter shall pay the regular initiation fees for such honorary members.

Article III.—Insignia

Section 1. The badge of this organization shall be a pentagon with the center slightly concave; on the upper half shall be a five-leaf rose, and on the lower half the letters K. M. E. The badge may be worn as a pin, key, or pendant.

Section 2. The seal shall be a five-pointed star enclosing a five-leaf rose, and being encircled by the legend "Kappa Mu Epsilon, founded 1931."

Section 3. The crest shall be a shield enclosing the five-pointed star; in the star shall be the rose rho $= a \sin 5\theta$, the symbol of pure mathematics. Around the star shall be the symbols for the sciences which apply mathematics: at the upper left a book of knowledge, for students and teachers; in the lower left a shamrock beneath a slide rule, for the engineers; at the upper right a conventionalized butterfly, for the biological sciences; at the lower right a moon and three stars, for the physical sciences; at the bottom the symbol "s angle n", for the business world. Above the shield proper is the design of the badge of the fraternity, and below it is a streamer upon which is printed the Greek motto.

Section 4. The colors shall be rose-pink and silver; the flower shall be the wild rose.

Article IV.—Chapters

Section 1. An organized group of at least ten members from an accredited four-year college may petition for a chapter.

Section 2. A request for the establishment of a chapter shall be referred to the National Council by the

National President. This request shall include such information concerning the school and the mathematics faculty as required by the petition. If the Council endorses the request, the National President shall then direct the National Secretary to notify the chapters of this application, giving only the pertinent details. If, within 60 days, one-fourth of the chapters have not filed disapproval with the National Secretary, the National President shall, with the approval of the National Council, grant a charter to the petitioning group.

Section 3. Chapters may also be approved by the voting delegates at the convention, as provided in Article VI, Section 1.

Section 4. Each chapter shall be governed by chapter by-laws which shall not be in variance with the national constitution. Copies of chapter by-laws shall be filed with the National President and the National Secretary.

Section 5. Chapters shall be designated by state and Greek letters. The Greek letter to each chapter shall be the Greek alphabetical order in which the chapter was granted, i.e., Kansas Alpha, Kansas Beta, Kansas Gamma, etc.

Section 6. If a chapter fails to comply with the constitution or receives an indictment from two other chapters for failure to fulfill the purpose of the fraternity, its charter may be withdrawn by a two-thirds vote of the chapters upon a report of the National Council including the charge against the chapter and the chapter's defense.

Section 7. (a) A chapter upon its request presented to the National Council, may become inactive.

(b) A chapter which is delinquent in paying its dues, making required reports, or answering letters, may be suspended by the National Council. Failure to report on any matter where the thirty or sixty days' silence is equivalent to an approval, shall be considered as delinquency in respect to chapter reports, except when the National Council finds the failure to report is unavoidable.

(c) A chapter shall be expelled by the methods of Section 6, Article IV. A chapter which is inactive or suspended shall not be permitted to have membership cards or pins, nor shall it be allowed to participate in any of the benefits of the national fraternity such as the defraying of the expenses of a delegate to the national convention or any other expenditures of money. In any matter requiring approval of two-thirds or three-fourths of the chapters, the number required for approval shall be based on the chapters in good standing; i.e., chapters not inactive or suspended. Upon evidence of willingness to abide by the constitution presented to the National Council and upon payment of a reinstatement fee of ten dollars ($10) and an additional amount to be determined by the National Council, an inactive or suspended chapter may be reinstated.

Article V.—National Officers

Section 1. The national officers shall be President, Vice-President, Secretary, Treasurer, Historian, immediate Past President, and shall be members of Kappa Mu Epsilon. These shall be elected at a national convention. If, for any reason, a national convention is not held, the National Council shall present a nomination for each national officer to the local chapters for their approval or disapproval. Failure to reply within thirty days shall be considered an approval. Each nominee shall be considered elected when two-thirds of the chapters give their approval of his election.

Section 2. The term of office of these officers shall be for two years, except that of the Secretary and Treasurer who may be reelected as often as the chapters deem it advisable. Except for the offices of Secretary and Treasurer, no office shall be filled by the same chapter for more than two successive terms.

Section 3. The duties of the President shall be to preside at the national convention, to provide for the installation of new chapters, and to perform such duties as usually pertain to this office.

Section 4. The duties of the Vice-President shall be to assist the President.

Section 5. The Secretary shall take charge of the records of the fraternity, as provided, and shall perform those duties that usually pertain to this office.

Section 6. The duties of the Treasurer shall be those that usually pertain to this office. The President shall approve all bills before they are paid by the Treasurer.

Section 7. The Historian shall be the official historian of Kappa Mu Epsilon. He shall be assistant-editor of the official publication of Kappa Mu Epsilon and shall keep a scrap book of interesting news notes about chapters or members.

Section 8. The immediate Past President shall act as an advisory officer, aiding any and all of the other officers.

Section 9. The above officers shall constitute the National Council. The duties of the National Council shall be to investigate petitioning groups and their institutions as a field of expansion, to fill any vacancies for the unexpired term, and to serve as a general executive committee in the interim between conventions. Any four members of the Council shall constitute a quorum for the transaction of business of the fraternity.

Article VI.—Conventions

Section 1. The national convention shall consist of members of the National Council and delegates elected by each chapter. Each chapter may send as many delegates as it desires and each delegate shall be entitled to participate in all discussions, and to vote individually on all motions except on motions to amend the constitution or by-laws, or in the election of national officers, or on motions relating to the establishment of new chapters. On all motions relating to the exceptions mentioned above, each chapter shall be entitled to two votes. These votes shall be cast by two special voting delegates whose names and chapter membership must be certified to, in writing, by the chapter represented. A three-fourths vote of the voting delegates shall be required to approve all motions to establish a new chapter.

Section 2. The national convention shall be held every two years, the time and place of which shall be decided by the National Council, and the chapters shall be notified of their action at least six months prior to the date which is set for the convention. (The National Organization pays the first-class railroad fare of one delegate from each chapter provided such a sum does not exceed the biennial fees paid in by the chapter.)

Section 3. Biennial reports of all national officers shall be made at each national convention. A written copy of each report must be presented to the National Secretary for permanent filing.

Article VII.—Publications

Section 1. The official journal of Kappa Mu Epsilon shall be known as *The Pentagon.*

Section 2. The National Council shall appoint the editor of the official journal. The National Historian shall be assistant editor.

Section 3. The official journal shall contain material which will tend to achieve the objectives of Kappa Mu Epsilon and which will tend to establish fraternal ties between the chapters.

Section 4. The official journal shall be published at least once a year and as more often as the editor and the National Council shall deem advisable.

Section 5. Each initiate shall receive the official journal for a period of two years. The yearly dues paid by the faculty members shall include a subscription to *The Pentagon.*

Article VIII.—Amendments

Section 1. This constitution may be amended by two-thirds vote of the national convention provided that a copy of the proposed amendments shall have been sent to every chapter by the National Council at least one month before the convention meets.

Section 2. Amendments may also be proposed at the national convention, and if approved by a two-thirds vote, be submitted to each chapter. Such amend-

amendments become effective upon ratification by two-thirds of the chapters. Failure of any chapter to report within sixty days shall be considered equivalent to approval.

BY LAWS

Article I.—Finance

Section 1. Each new chapter shall pay in advance to the National Treasurer three and one-half dollars per capita. Expenses incurred by the national organization incidental to the establishment of a chapter shall be paid by the chapter; however, in any case, this sum shall not be less than twenty-five dollars.

Section 2. Each chapter shall pay the national organization an initial fee of three and one-half dollars for each subsequent member.

Section 3. All faculty members shall pay the three and one-half dollars initiation fee when initiated. This initiation fee shall cover dues for a period of two years, and thereafter each faculty member shall pay annually one dollar national dues.

Section 4. An auditing committee of three members shall be appointed by the National President to audit the accounts of the National Treasurer prior to the convention.

Article II.—Ritual

Section 1. Each chapter shall be provided with a copy of the initiation ritual, to be used at all initiations and installations of the fraternity.

Article III.—Chapter Officers

Section 1. The chapter officers shall be President, Vice-President, Recording Secretary, Treasurer, Corresponding Secretary, and Faculty Sponsor.

Section 2. The Corresponding Secretary shall have charge of the official correspondence with the Naments shall become effective upon ratification by two-thirds of the chapters.

Section 3. Amendments may be proposed at any time by any local chapter or by the National Council to each local chapter for its approval or disapproval. Such tional Fraternity. He shall be a faculty member.

Article IV.—Amendments

Section 1. These by-laws may be amended in the same manner as prescribed for the amendment of the national constitution.

Article V.—Printed Forms

Section 1. Petitions for the establishment of a chapter shall be of the form approved by the National Council.

Section 2. The charter of any chapter shall be of the form approved by the National Council.

Section 3. Permanent record cards of all chapter members shall be sent by each Corresponding Secretary to the National Secretary for permanent filing. These cards shall be provided by the National Secretary and be of the form approved by the National Council.

Section 4. The National Secretary shall, upon the request of the Corresponding Secretary, and upon receipt of the necessary fees and records, issue membership cards of the forms approved by the National Council.

Section 5. All requests for pins or keys must be made on the forms provided for that purpose and must bear the approval of the National Secretary.

There is an astonishing imagination, even in the science of mathematics. . . . We repeat, there was far more imagination in the head of Archimedes than in that of Homer.

—VOLTAIRE.

KAPPA MU EPSILON NEWS

Edited by CLEON C. RICHTMEYER, *Historian*

A joint meeting of **Kansas Alpha, Beta, Gamma,** and **Delta,** was held in conjunction with the meetings of the Kansas Section of the Mathematical Association of America and the Kansas Association of Teachers of Mathematics at Mount St. Scholastica College, Atchison, Kansas, on April 10. Professor C. V. Newsom, past president of Kappa Mu Epsilon, was the principal speaker.

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The annual pledge program of **Kansas Gamma** was presented on March 8 in the Little Theater. The pledges cast their program in the form of a mathematical hour broadcast over radio station KME. The chief feature of the broadcast was a mock meeting of five great women mathematicians of past centuries, Hypatia, Agnesi, Sommerville, Germain, and Kovalesky, who told of their achievements and their struggles to gain recognition.

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Michigan Alpha bids farewell this year to Dr. E. R. Sleight who retires after more than forty years at Albion College. The chapter has enjoyed many successful years under his able and inspiring leadership. A special program in his honor is being planned for the chapter meeting on April 27.

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Michigan Beta voted to purchase 250 reprints of the "Guidance Report of the Commission on Post-War Plans" (*Mathematics Teacher*, November, 1947) and to distribute these to the high schools of the area.

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Missouri Alpha initiated 23 new members on January 21. This was the largest class ever to be initiated by this chapter.

Missouri Beta sponsored the organization of a new science club at Missouri State Teachers College. Each new member of the chapter is required to present a paper during the year.

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Nebraska Alpha sponsored a homecoming float depicting "The Tree of Knowledge." A tree was mounted on the float and various boxes hung from it.

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New Jersey Alpha joined with the mathematics club to hear Dr. David R. Davis of Montclair State Teachers College speak on the topic "Mathematics in Shrivenham American University and in English Colleges."

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At the regular October meeting of **New York Alpha**, Dr. Francis Wilson of the department of Physics spoke on "Our Nearest Star." This talk anticipated the November meeting which included a trip to the Hayden Planetarium. There the members enjoyed viewing the collection of meteorites, sundials, astrolabes, telescopes, etc., before attending the lecture and parade of stars.

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Ohio Beta occasionally opens its meetings to the entire college, when the topic is of general interest.

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Oklahoma Alpha set up a program for helping veterans and other students in mathematics. Sessions were scheduled two evenings a week and K.M.E. members rotated in the work with at least two assisting each night.

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Professor R. S. Underwood of **Texas Alpha** has been elected Traveling Lecturer by the Southwestern Section of the Mathematical Association of America. Professor

Underwood will present a popular lecture on mathematics to the students of colleges in Arizona, New Mexico, and western Texas.

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PROGRAM TOPICS, FALL SEMESTER, 1948-49

Alabama Alpha, Athens College

Some Seeming Absurdities in Mathematics, by Professor Thomas J. Carter

History and Aims of K. M. E., by Lloyd O. Stone

Alabama Gamma, Alabama College, Montevallo

Pythagoras and the Pythagoreans, by Lida True

Some Topics from the Theory of Numbers, by Rosa Lea Jackson

Regular Polygons and How They Grow, by Margaret O'Gwynn

Waves and Vibrations, by Frances Jones

Mathematics and the Physical Sciences, by Virginia Harens

Illinois Delta, College of St. Francis, Joliet

Review of *The Schoolmaster's Assistant*, by B. Freiburg

Fermagoric Triangles, by Sr. M. Claudia

Mathematics for the Million, by Sr. M. Hillary

Plato's Republic, by R. Grogan

Calculating Prodigies, by M. J. LaFond

Review of *Attacks on Mathematics and How to Meet Them*, by Sr. M. Elizabeth.

A Young Lady to Her Lover, by M. Dreska

Mascheroni's Constructions, by Sr. M. Rita Clave

Cryptographs and Ciphers, by P. Young

Arithmetical Recreations; Three Fallacies, by M. Crawley

History and Transcendence of Pi, by B. Lanoue

Astronomy and the Calendar, by M. Hutchings

Illinois Gamma, Chicago Teachers College

Movies: *Origin of Mathematics*, *Celestial Navigation*, and *The Earth*

Magic Squares, by Edna Boedeker

Movie: *The Slide Rule*

Mathematical Tidbits

Quadratic Curves and Applications, by Joseph J. Urbancek

Mathematics and Our Knowledge of Nature, by Dr. Arturo Fallico

Iowa Alpha, State Teachers College, Cedar Falls

Demonstrating the Use of a Planimeter, by Margaret Burnett

Euler's Formula, by George Mack

Berekley's *The Analyst or a Discourse Addressed to an Infidel Mathematician*, by Royce Merting

Kansas Beta, State Teachers College, Emporia

The Part Played by Mathematics in the Construction of Model Airplanes, by Eldon E. Breazier

Kansas Gamma, Mount St. Scholastica College, Atchison

Student Obligation to Evaluate the Role of Science, by Frances Knightley.

Is Full Academic Life Possible Without a Correct Understanding of the Hierachy of the Branches of Learning?, by Victoria Fritton

Mathematics as a Logical System of Thought with a Unique Position in the Hierarchy of Knowledge, by Jean Moran

Non-Euclidean Geometries, Their Foundations and Developments, by Sr. Marietta Lueken

Kansas Delta, Washburn Municipal University, Topeka

Limits, by Paul Eberhart, Beverly Brown, and John Nipps

Matrices, by William Willard

Michigan Alpha, Albion College

Arithmetic Revisited, by Professor H. D. Larsen

Michigan Beta, Central Michigan College, Mount Pleasant

Star Solids, by Gertrude V. Pratt

Michigan Gamma, Wayne University, Detroit

Magic Squares, by Bruce Drew

Mathematical Fallacies, by Meyer Fleishman

Tests for Primality of Numbers, by Ted Slaby

The Theory of Aggregates, by James Thompson

Missouri Gamma, William Jewell College, Liberty

Value of Honorary Clubs, by President Walter Pope Binns

Mathematics Before 1600 B.C., by Wiley Crawford.

Biography of Leibnitz, by Walter Binns

Biography of J. W. Gibbs, by Raymond Neth

Life of Descartes, by Imogene McCormick

Life of Einstein, by Maynard Cowan

Non-Euclidean Geometry, by James Leatherman

Calculating Machines, by James Idol

Development of Trigonometry, by Paul Swedburg

Contributions to Mathematics of People Who Are Little Known, by James Idol

Theory of Probability, by Ed Norvell

Mathematics in the Imagination, by Kathleen Ricketts

Nebraska Alpha, State Teachers College, Wayne

Review of *Mathematics for the Million,* by Mrs. Elizabeth Wool-ridge

Mathematical Preparation for Science, by Dick Campbell

New Jersey Alpha, Upsala College, East Orange

An Introduction to Nomography, by Marjorie Cohen

Various Methods of Proving the Theorem of Pythagoras, by Frances Rischmiller

Nomographic Chart for Quadratic Equations, by Robert Wallace

The Genesis of the Body of Mathematics as We Have It Today, by Professor M. A. Nordgaard

New Jersey Beta, State Teachers College, Montclair

Einstein's Theory of Relativity, by Rocco Feravolo and Shirley Taylor

Artillery Mathematics, by Robert Lundquist

Ohio Alpha, Bowling Green State University

History and Aims of the National Organization and the Local Group, by Dr. F. C. Ogg

A Phase of the Relativity Theory, by Dr. D. W. Bowman

Mathematics in Business and Economics, by Professor L. F. Manhart

Mathematics in Industry, by Dr. D. M. Krabill

Mathematics in Finance, by Professor H. Mathias

Ohio Beta, College of Wooster, Wooster

Topology, by Professor M. P. Fobes

Interesting Higher Plane Curves, by the initiates

Relativity, by Professor R. J. Stephenson

Ohio Gamma, Baldwin-Wallace College, Berea

Discussion on Pi, by Nelle Williams, Marian Harriger, and Ellie Longwell

Little Known Mathematicians, by Ralph Dietrich and Frank Recker

History of Greek Mathematics, by John Detlef

Japanese Mathematics, by Edward Johnson

Oklahoma Alpha, Northeastern State College, Tahlequah

Astronomy, by Professor L. P. Woods

Solution of Higher Equations, by Jim Barringer

Teaching Secondary Equations, by Professor Vella Frazee

Discussion and Solution of Determinants, by Charles Brown and Thomas Summers

Texas Delta, Texas Christian University, Fort Worth

Early Contributions of the Hindus to Mathematics, by French White

The Brink of Infinity, by Wayne McNutt

A Certain Integration by Method of Multiple Angles, by Robert McCarty

English Interpretation vs. Mathematical Equations, by Bruce D. Fallis

Wisconsin Alpha, Mount Mary College, Milwaukee

Symposium on Non-Euclidean Geometry, by Pat Farrell, Mary Ann Frodel, Eleanor Grogan, Eileen Ford, June Rose McDonald, Mary Lou Marquardt, Mildred Oestreich, and Marie Tronchuk