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## WHO'S WHO IN KAPPA MU EPSILON

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Kappa Mu Epsilon, national : nnorary mathematics fraternity, was founded in 1931. The object 1 the fraternity is four-fold: to further the interests of mathemal in those schools which place their primary emphasis on the undergia 'aate program; to help the undergraduate realize the important roie that mathematics has played in the development of western civilization; to develop an appreciation oi the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievement in the study of mathem:tics in the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

## THE NATURE OF MATHEMATICAL REASONING

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In his book, The Degrees of Knowledge [6]*, Jacques Maritain outlines the three degrees of abstraction of the human mind. The second degree of abstraction is mathematics resolving itself into quantity as such. He does not place mathematics in the same line with physics and metaphysics. His reason for this is that mathematical abstraction is a thing by itself. The purpose of this paper is to study the reasoning employed in mathematical abstraction and to show that it merits being placed in the degrees of abstraction above the sciences dependent upon the scientific method.

Such a study can be of value in understanding the nature of mathematics and exactly what a mathematical system involves. It leads to a clearer understanding of the difference between truth and validity in a system of knowledge and also a knowledge of the fact that mathematical systems can be developed independent of present physical experience. Alfred Tarski says in his book, Introduction to Logic [11, p. 117], "For anyone who intends to study or advance some science it is undoubtedly important to be conscious of the method which is employed in the construction of that science; and . . . in the case of mathematics, the knowledge of that method is of particularly far-reaching importance for lacking such knowledge it is impossible to comprehend the nature of mathematics."

The method employed in the construction of mathematics is commonly known as the axiomatic method. This method starts with certain undefined terms. They must be undefined, for if they were not it would mean that theirdefinitions would involve terms defined by means of others and such a regress ultimately leads us around in a vicious circle. The method involves axioms or postulates which are unproved and which state various properties of the undefined terms. After the undefined terms and axioms

[^0]have been drawn up, other statements are accepted as valid only when they have been established to be so by using only those axioms, definitions, and the statements that have been proved previously. Such statements are known as theorems and the process by which their validity is established is the proof.

Upon the axioms which constitute the foundation of the system rest all the conclusions that are deduced from them. Therefore they are of the utmost importance and must of necessity possess certain qualities. The first is consistency. By this is meant that no axioms should be adopted that are contradictory or which, although not obviously contradictory in themselves, lead to contradictory conclusions. A certain degree of simplicity is important for it must be known at the beginning just what is agreed upon. The third property of the axioms is that they should be independent. This is necessary because if one could be derived from another, the first would be a theorem and not an axiom. This property helps keep the number of axioms fairly small. But of the above properties only the first is truly essential to the axioms of a mathematical system [2, pp. 537-538].

Although the axioms are unproved, they are not accepted as self-evident truths that must hold true in nature. If they do, it is, of course, better for practical reasons. But the system built upon axioms which do not conform to nature is not invalid [ $8, \mathrm{p} .172$ ]. Pure mathematicians are concerned only with developing a logical system upon the axioms they choose; whether or not this system can be used to interpret the physical world is not their concern. Many abstract theories of pure mathematics which were initially developed only for the sake of mathematics have proved of definite practical use. An example of this is the algebra of complex quantities that was first developed without reference to the physical world, but which proved to be an invaluable tool to the electrical engineers who were developing the theory of alternating currents. [15, p. 166]. The theory of conic sections is another example of this, for it was found that it described the motion of celestial bodies.

The importance of the axioms of a system can be seen by noting the changes that would result if one were altered in any way. It was by this method that nonEuclidean geometry was brought into existence. Up until the middle of the nineteenth century it was believed that Euclid's geometry gave an exact account of physical space. Some attested to the fact that it was the only description of the universe. At this time certain mathematicians were beginning to question Euclid's fundamental axiom concerning parallel lines. They had come to the conclusion that it was independent of the rest and could be replaced by a different axiom. This is exactly what Bolyai and Lobachewsky did. The result was a new geometry known as non-Euclidean geometry. Riemann rejected not only the parallel axiom but also the axiom that a straight line is infinite; thus he developed still another geometry. Of great importance was the work of these three men. NonEuclidean geometry enabled scientists to advance more satisfactory theories concerning the physical universe [2, pp. 519-530].

The process by which the theorems are proved is that of deduction. It develops all the implications of the axioms and what follows if the first principles are accepted. The argument follows from premise to conclusion, and from one conclusion as a premise to another, as in the following example:

$$
\begin{aligned}
& \text { But } z=y . \\
& z=x . \\
& \text { Therefore } z=y . \\
& \text { But } w=z . \\
& \text { Therefore } w=y .
\end{aligned}
$$

In proving a theorem by deduction the principle of formal implication taken from logic, or the "if-then" principle, is widely used. Many mathematical theorems are stated in this form, the "if" clause being the hypothesis and the "then" clause being the conclusion [11, p. 28]. For instance, the theorem, "If $x$ is a positive number, then $2 x$ is a positive number." Other examples are seen in the statement of the theorems of plane geometry such as "If
a triangle is isoceles, then the angles opposite the equal sides are equal." :

More readily than other systems of mathematics, the whole of geometry shows that it is logically deduced from and built upon the unproved postulates. Just one example will indicate the role of deduction in geometry. Take the proof of the theorem stated above:


Hypothesis: In $\triangle A B C, A C=B C$. Conclusion: $\angle A=\angle B$.

Proof:
Let $C D$ bisect $\angle C$.

Then $A C=B C$, $C D=C D$,
and $\angle 1=\angle 2$.
Hence,
$\triangle A C D=\triangle B C D$. and
$\angle A=\angle B$.

Given.
A part is identical with itself.
Construction.
Two sides and the included angle are equal.
Corresponding parts of congruent triangles are equal.

The justification for each statement is given at each step and the proof is based on the hypothesis and other theorems that have been proved previously.

Geometry is presented to the student in a compact deductive form, and it is most frequently given to illustrate the axiomatic method of mathematics. Arithmetic and algebra are not taught to the student in this way, and
thus the true character of the reasoning employed in these branches is not apparent until a careful study is made of the nature of mathematics. But they are both very logical systems whose conclusions are deduced from axioms and definitions. E. V. Huntington says that algebra is better suited than geometry to illustrate what is involved in mathematical reasoning because the concepts with which it deals are of a much simpler nature [15, p. 152]. Since the axiomatic method of geometry is better known on the undergraduate level, I have used examples from it to illustrate my point.

Geometry reached the position of a deductive science so long ago that it is hard to realize at the present time that it was ever in an inductive stage. But this fact is attested to by history. The geometry of the Egyptians consisted of rules for measurement which grew out of experience and observation. This was really not mathematics, because mathematics insists upon deductive proof for workable rules arrived at inductively before they are accepted as a valid part of its system. There is a great breach between the workable rules of the Egyptians and the geometry of the Greeks in the sixth century B.C. The first is what preceded mathematics, the latter is mathematics. The criterion for judging mathematics from non-mathematics is deduction [1, p. 4].

The above discussion does not deny that intuition, induction, and experiment play an important role in mathematical invention and discovery. Induction, the process of developing conclusions from the observations of a series of instances, helps to formulate hypotheses which can then be proved by deduction. Induction and experiment suggest to the mind many ideas that can be used in setting up a mathematical system. They also suggest ideas of things that have no material existence as they are conceived in the system. This is true of Euclid's circle and straight line which do not exist as he defined them, but which are abstracted from such figures as the moon and a taut string. By thus abstracting and simplifying the evidence of the senses, mathematics brings the world of science into focus.

This abstractness is the chief glory of mathematics and the reason for its great usefulness [1, p. 8].

Aristotle says that major premises are ultimately the result of the faculty of intuition. Our ordinary major premises can be traced back to ultimate principles that are the major premises of the particular science [ $8, \mathrm{p} .196$ ]. Thus intuition, a natural power of the mind, helps in establishing the foundation of a mathematical system.

Lastly this discussion is concerned with the use of logic. Mathematics is related to logic in that the methods of logic are used in its deductive proofs. This relation is a very close one. In every argument, concepts are taken from logic. It is not essential in elementary mathematics that logic be known for correct thinking, but certainly it will be helpful for critical cases in higher mathematics [11, p. 109]. Logic helps the mathematician to decide whether or not a given statement follows from the premise. It also helps by showing if inconsistencies exist in a set of supposedly true statements and offers grounds for rejecting them. In return mathematics has given much to logic itself in the way of new methods. It has helped to put the type of reasoning known as the probable inference on a sound basis [2, p. 599].

With all this in mind it is seen that mathematics really merits being placed above the physical sciences, which discover their truths by induction using the scientific method. This method is subject to many errors. With mathematics it is known that a conclusion is valid if it has been logically deduced from a set of postulates that are initially agreed upon. If the postulates are true, the conclusions will be true; if the postulates are false, the conclusions may be false but they will be valid. Mathematical abstraction is above the physical which considers objects still clothed in the sensible, but it is below the metaphysical which considers abstract objects entirely divorced from matter. For the mathematical considers only one property of the bodies-what remains when the sensible is removed, that is, quantity, number, and extension [6, pp. 45-46]. Thus the abstract, formal, deductive character of mathematics places
it in close affinity to metaphysics and certainly above the sciences of an inductive character [9, p. 120].

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# LITTLE-KNOWN CONTRIBUTORS TO MATHEMATICS 

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We are all acquainted with well-known mathematicians such as Newton, Archimedes, and Descartes, but not all of us know that there are a host of unsung contributors to mathematics. There are many famous figures who, although renowned for accomplishments in entirely different fields, have loved mathematics and have made worthwhile contributions to it.

One of these is a man who is a stranger to no one, but who is not often thought of as a mathematician: Thomas Jefferson. We think of Jefferson as president, statesman, or; as he wished inscribed on his tombstone, "-Author of the Declaration of Independence, of the Statute of Virginia for religious freedom, and father of the University of Virginia." We rarely think of him as a mathematician. Jefferson never taught a mathematics class, never wrote a book on mathematics, and never added any new numbers or concepts to mathematics, but mathematics is still more indebted to him than to many professional mathematicians.

Jefferson's love and respect for mathematics is obvious in many of his writings as well as his actions. He once wrote, "Nature intended me for the tranquil pursuits of science, by rendering them my supreme delight," and he included mathematics in his "pursuits." Years after his retirement from public office, he wrote to a friend: "Having to conduct my grandson through his course of mathematics, I have renewed that study with great avidity. It was ever my favorite one. We have no theories there, no uncertainties remain in the mind; all is demonstration and satisfaction."

Jefferson once wrote George Wythe: "I have reflected on your idea of wooden or ivory diagrams for the geometrical demonstrations. I think wood as good as ivory; and in this case it might add to the improvement of the
young gentlemen that they make the figures themselves." Thus his interest in geometry is also evident.

When the metric system was receiving the attention of scientists, Jefferson wrote to David Rittenhouse, a famed astronomer, asking for his help in working out a plan for establishing assystem of uniform weights and measures and coins in the United States. Jefferson's interest in placing the tables of measurement on a decimal basis first showed itself in the United States monetary system. Jefferson was also interested in mathematical astronomy.

Jefferson viewed mathematics from above and made possible the work of its devotees. His contributions to the field may be classed under four divisions:

1. He gave mathematics a more prominent place at the University of Virginia than it had at any other American college.
2. He was influential in carrying out Washington's plan of a military academy and was influential in giving mathematics a French trend, which led to a more progressive system than was then used in English schools.
3. He encouraged Americans to study the achievements of scholars in other lands.
4. He awakened a spirit of research among New World scholars.

Jefferson was one of the first trustees of the University of Virginia and it was because of his influence that men like Key of Cambridge were imported to build up the mathematical courses in this school. Jefferson was a practical man; he studied mathematics for its own sake, then turned it to practical use. Jefferson asserted that the main purpose of the sciences, including mathematics, is the happiness of man, and that science is more essential in a republican form of government than any other. Thus we see that Jefferson was a patron of mathematics in its truest sense.

In the cemetery at Guildford in Surrey there is a grave marked with two names: Charles Lutwidge Dodgson and Lewis Carroll. Everyone knows that Lewis Carroll was
the author of the incomparable Alice In Wonderland and Alice Through the Looking Glass, but not everyone knows that Charles L. Dodgson was an able mathematician.

Scores of old Oxonians remember Lewis Carroll-he died in 1898-as a lean, dark-haired lecturer at Christ Church College, who imparted his instruction in a singularly dry and perfunctory manner. Carroll was not a great mathematician, but he wrote several books on mathematics, some of which are still used, and although he did not propound any new theories, he contributed to mathematics by his contribution to literature. And mathematics contributed to him-his knowledge of it gives his books precision and charm.

Queen Victoria liked Alice in Wonderland so much that she wrote to Carroll and asked him for the rest of his works. With his humble duty, C. L. Dodgson presented Her Majesty with a number of volumes, all mathematical, and including a treatise on The Condensation of Determinants. Some of his other works were a Syllabus of Plane Algebraical Geometry, Formulae of Plane Trigonometry, and a Guide to the Mathematical Student. It is interesting to know that the king of classical nonsense could reign also in the realm of logic and reason.

There is another outstanding literary figure, who, had he been consulted, would have preferred to be remembered for his mathematical achievements rather than his literary ones. That figure is Omar Khayyam. Omar ibn Ibrahim alKhayyam, Gujat ed-din Abul Fath was born at Nishapur, Persia, in 1044 A.D. and died in 1123. The name Khayyam means tentmaker, but Omar probably never practiced the profession. He is known to the West as a poet and philosopher, to the East as an astronomer and mathematician. The fate which made him famous as a poet and philosopher rather than a scientist and mathematician robbed him of a fame that would have pleased him more.

Omar Khayyam's Rubaiyat is well known, but his Algebra is not so well known, despite its greatness. The Algebra first became known in 1742 when Gerard Newman, discussing the development of analytical calculus,
cited an Arabic manuscript on algebra by Omar Khayyam. It was Newman's contention that the manuscript might contain the algebraic solution of cubic equations. This was not the case, but the work did contain a discussion of geometric construction and solution of the cubic equation by means of the intersection of conic equations.

The material in Omar's Algebra falls under six topics:

1. Introduction, including preface and definition of fundamental notions of algebra.
2. Tables of equations proposed for solution, including simple (binomial) equations and compound equations (trinomial and tetranominal).
3. Numerical solutions and geometric constructions of first and second degree equations.
4. Constructions and demonstrations of cubic equations by means of intersection of conic sections.
5. Discussion of fractional equations.
6. Remarks on the work of Abul Jud.

Khayyam was appointed Royal Astronomer to the court and was one of the principal authors of the reformed calendar introduced by the sultan in 1079. In addition to his astronomical computations, Omar composed a treatise on the extraction of roots of higher order and made a systematic theory of cubic equations.

Khayyam's conception of algebra is apparent in his definition of it as "the science that aims at the determination of numeral and geometrical unknowns." He never lost sight of the object he set at the beginning: to supplement numerical solutions of equations by geometric construction. As for the number of roots of an equation, Omar, like his predecessors, considers only one single positive root and neglects the negative as well as the imaginary roots. When the equation does not yield to a positive root, he maintains that it is impossible. It must be remembered that mathematicians who came along years later than Omar made the same assumption.

It is in considering Omar's cubic equations that we find his outstanding contribution to mathematics. In this he was the mathematician of his time. Although he did
not make any addition to the theory of Apollonius, he did apply the principle of intersecting conic sections in solving algebraic problems. He not only knew the intrinsic properties of conic sections, he also made use of them. He gave a complete classification of forms of cubic equations and constructed the geometrical solution for each type. He applied algebra to geometry and established the foundation of the interreiation of the two.

Thus his principal accomplishment is in the development of the algebraic science. He gave a complete classification of equations through the third degree with respect to the number of terms, then after setting himself the task of verifying its algebraic solutions by geometric construction, and vice versa, he followed the problem systematically. In other words, he made an effort to unite algebra and geometry. It can be noted that today the more progressive high schools are working toward this "new" idea in their mathematics courses.

Thus Omar Khayyam's contributions to mathematics, especially in the field of cubic equations, rank him as the greatest and most original mathematician of his time, for all that the Occidental world knows him only as a philosophical Oriental who wrote, "A book of verse beneath the bough-." We can imagine that the man who wrote
"Into this Universe and Why not knowing, Nor whence, like water, willy-nilly flowing And out of it, as Wind along the Waste I know not Whither, willy-nilly blowing." must have gotten an infinite amount of joy in working with things that did have a "whence" and a "why."

There is yet another man of letters and philosophy who contributed to mathematics. That is Rabbi Ben Ezra, or Ibn Ezra. Rabbi Ben Ezra was a learned Jew who lived from 1092 till 1167 A.D. Like Khayyam, Rabbi Ben Ezra is known as a great philosopher, one of the four greatest, in fact, of his time, and one of the lights of the Jews in the Middle Ages. Ben Ezra is known best by most people, and certainly by all students of sophomore literature, as the
subject of a poem by Browning for which Ezra and his philosophy were the inspiration:
"Grow old along with me, The best is yet to be, The last of life, for which the first was made."
Born at Toledo, Spain, Rabbi Ben Ezra travelled widely , studied much and wrote poetry and treatises on mathematics, Hebrew grammar, and astronomy. He helped introduce learning into Europe. He is best known for his Bible commentaries, but he was also an excellent writer on mathematics. In his Arithmetic, he explains the Arabic System of Numeration, and is perhaps partially responsible for its spread.

Sir Percival Christopher Wren is known everywhere as a great architect. When the Fire of London occurred in 1666, Wren had his first chance to use his gifts as an architect and for forty years there was hardly a structure built around London that did not have a dash of Wren in it. He rebuilt St. Paul's Cathedral after the Fire. Some of his other constructions are the Customs House, the Royal Exchange; Marlborough House, Buckingham House, and the Hall of the College of Physicians. All these have been destroyed, but Wren's wor': still stands in the Tom Tower of Christ and Qucen's College Chapel, the library of Trinity College, and the Chapel of Pembroke at Cambridge.

It is hardly to be expected that a man who constructed all these buildings would have time for anything else, but Wren was very much interested in both physics and mathematics and made several contributions to both. Born in 1632, Wren, while still at Oxford, distinguished himself in geometry and applied mathematics. He was a friend of Newton and Newton speaks very highly in his Principia of Wren's work as a geometrician.

As an architect, Wren was naturally interested.in geometrical forms of construction. His interest in the arch and his need for understanding it led him to investigate the cycloid and to discover that the cycloidal arch is superior to any other. He determined the length of any arc of the curve and its center of gravity, which was later introduced into the construction of the pendulum clock.

At the invitation of the Royal Society, Wren, with: Wallis and Huygens in 1668 and 1669, investigated the impact of elastic and inelastic bodies. Wren was at one time president of the Royal Society, and he and Halley once questioned Newton's statement that the law of the inverse squares accounted for the elliptical orbits of the planets. As a result of their inquiry, Newton retraced his operations and discovered a mistake in his calculation.

There are two great artists who have been excellent mathematicians. One of these is Albrecht Durer, painter and wood carver of the fifteenth and sixteenth century, in whose engraving of Melancholia can be seen a magic square. Durer is well known for his beautiful paintings and woodcuts, but not so well known for his mathematical work.

Duer gave an approximate construction for a regular pentagon thus: Let $A B C$ be an equilateral triangle. With centers $A, B, C$, and radius $A B$, circles are drawn. Circles with centers $A$ and $C$ meet in $F . \quad G$ is the mid-point of the minor $\operatorname{arc} A B$ of circle of center $C . F G$ produced meets circle of center $B$ at $H$. The angle $A B H$ is approximately the angle of the pentagon.

Durer also showed that the side of a regular pentagon is nearly half the side of an equilateral triangle of equal area.

Last in our list of mathematical artists, but far from least, is Leonardo da Vinci. It would be safe to say, without knowing anything of da Vinci's mathematical achievements, that he was an excellent mathematician because da Vinci excelled in everything. He was one of those rare individuals who are richly endowed with nearly every conceivable talent and aptitude. Known as the painter of the Last Supper and La Giaconda, da Vinci was also sculptor, poet, inventor-he made a design for a flying machine -physicist, philosopher, and mathematician. When he studied arithmetic, he made such rapid progress that he often astounded his teacher by his work and his questions. He helped the mathematician Luci Pacioli in one of his works and is credited (somewhat doubtfully, however) with having invented the algebraic signs of plus and minus.

The best way of learning of da Vinci's accomplishments as a mathematician is to study his notebooks. Here are snatches from his notebook on mathematics:
"There is no certainty where one can neither apply any of the mathematical sciences nor any of those which are based upon the mathematical sciences."
"Arithmetic is a mental science and forms its calculations with true and perfect denominations; but it has not the power in its continuing quantities which irrational or surd roots have, for these divide the quantities without numerical denomination."
"Every odd number multiplied by odd numbers remains odd; every odd number multiplied by even numbers becomes even."
"The circle that touches the three angles of an equilateral triangle is triple the triangle that touches the three sides of the same triangle."
"The greatest pyramid that can be drawn from a cube will be one-third of the whole cube."

Leonardo also played with the ever-present game of squaring the circle. He says: "-Animals that draw chariots afford us a very simple demonstration of the squaring of a circle which is made by the wheels of these chariots by means of the track of the circumference which forms a straight line." Some other jottings in his notebook are:
"Of the elements: A point is that which has no center. A line is a length produced by the movement of a point, and its extremities are points."
"To obtain the cube of a sphere: When you have obtained the surface of a circle, divide the square into as many small equal squares as you please, make each square the base of a pyramid, of which the axis is half the diameter of the sphere of which you wish to obtain the cube, let them all be equal."

Some of the headings give us further insight into his mathematical accomplishments:

> "Actual Proof of a Square."
> "Definition of Four Groups of Parallels."
> "Methods of Measuring a Height."

All this in the fifteenth and sixteenth century by a man who accomplished just as much as this in five or six other fields!

This group of unsung mathematicians would not be complete without a mention of a woman who was an able mathematician. She is Alice Duer Miller, the author of the well-known White Cliffs of Dover. A reigning beauty of New York society, Alice Duer Miller was the daughter of an old and wealthy family, but she was also a scholar. When her father's firm collapsed in 1895, Alice shocked not only her family but also her friends by announcing her intention of working her way through Barnard College. Today no one is astonished by a girl's planning to go to college, but in 1895 it was no joking matter, especially in the case of a girl in Alice's situation. Mrs. Astor's remark on the subject, "What a pity, that lovely girl going to college," has been treasured in the family ever since.

Despite all protests, Alice went to college and her advance there in mathematics and astronomy was rapid. Her interest in higher mathematics remained with her throughout her life.

In 1899 one of the unsolved problems of mathematics related to the application of numbers, which are discrete, in the continuity of space. It was later found that the gap between the rational system of numbers and linear space could be filled by the introduction of irrational numbers of a one-to-one relation with space thus established. The results both in mathematics and physics were farreaching, and, though Alice's prize-winning thesis in her senior year, "Dedekind's Theory of the Irrational Number," did not contribute to the final solution, it did anticipate the research which finally resulted in that solution.

In fact, at this time Alice Duer Miller had but one thought in life, to pursue her studies in mathematics. There is no doubt what would have happened if she had. Her ability to apply her powers of imagination, and her capacity for hard work, would have brought her distinction in the field of applied mathematics, the field more truly in keeping with her nature. However, when she became engaged to Henry Wise Miller, she said good-bye to
mathematics. She never lost touch or interest with it, though. A Curtiss Scholar in Pure Science, she was also Phi Beta Kappa, holder of the Columbia University Medal, and Doctor of Letters.

Mathematics is even more inextricably woven into the life of Alice Duer Miller than most people's. Her husband says: "It was, I think, her rigorous early training in mathematics that gave Alice the skill in writing that characterized all her work. Mathematics may almost be defined as the science of relations. This principle was not an abstraction to Alice but fundamental in her character. She saw at once what was essential or significant. The relative value of one part to another, and the proper emphasis that follows, she felt instinctively, which accounts for the convincing rapidity with which her stories unfold. She early developed a pattern for herself by which she could best tell a story. She developed that pattern steadily, and much of her success is due to the flawless neatness with which her stories are presented.

Thus we come to know many unsung contributors to mathematics, and we see also that mathematics, with its broad and numerous possibilities has interested some of the greatest people of all times, even though they were in entirely different fields.

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# PROPERTIES OF THE NUMBER NINE 

## Jane Rourke <br> College of St. Francis

The number nine has often been called the most romantic of all numbers because of its striking and peculiar properties. This number truly possesses the most remarkable properties of any of the natural numbers. A great many of these properties have been known for centuries and have aroused much interest among both mathematicians and ordinary scholars.

Because of its relation to the numerical scale, a factor 9 in a number will cling to the expression and turn up in a variety of ways, now in one place and now in another, in a very surprising manner. It reminds one of a mountain streamlet which ripples beside the pathway, now and then buried and hidden from our sight, but sooner or later gurgling to the surface at the most unexpected moment. It leaves no wonder that the property has been regarded as magical and the number has been called the "magical number."

The first property of this number which we will note is that all through the column of "nines" in the multiplication table, the sum of the digits in each product is 9 or a multiple of 9 . Begin with $9 \times 2=18$; adding the digits of the product, 1 and 8 are 9. Following this through to $9 \times 11=99$, we see that the digits, 9 and 9 , added together are 18, and 8 and 1
 are 9. Going further, multiply 326 by 9 and we have 2934, the sum of whose digits is 18 , which in turn has a digit sum 9 . The principle involved is this: all these numbers being divisible by 9 , the sums of their digits must be 9 , or a multiple of 9 .
A. second curious property of the number nine is that if you take any row of figures and change their order as you
please, the numbers thus obtained, when divided by 9 , leave the same remainder. Thus, 64783, 76438, 43678, etc., when divided by 9 all give the same remainder, 1. The reason is that the sum of the digits, in any order, is the same; and the remainder from dividing a number by 9 is the same as the remainder from dividing the sum of its digits by 9 .

Another interesting principle is presented in the following puzzle which, if understood, is very simple. Take a number consisting of two figures, invert them, and take the difference between the resulting number and the first number; now tell me one figure of the remainder and I will name the other. The secret again is that the sum of the two digits of the remainder will always equal 9 . Thus take 85 ; invert the two digits to form 58; take the difference of the two numbers and we have 27 , of which the digit sum is 9 . Therefore, told one of the numbers I can readily name the other. The reason behind the principle is this: both numbers have the same remainder when divided by 9 ; hence, their difference is an exact multiple of 9 , and consequently the sum of the two digits will equal 9.

There is another puzzle arising from the principle of the divisibility by 9 . Take any number, divide it by 9 , and name the remainder; multiply the number taken by some number which I name, divide the product by 9 , and I will name the remainder. To tell the remainder, I multiply the first remainder by the number which I named as a multiplier, and divide this product by 9 . The remainder thus arising will be the same as the remainder which you obtained.

From the law of the divisibility by nine, several other properties, especially interesting to the young arithmetician, may be derived. First, take any number consisting of three consecutive digits and, by changing the order of the digits, make two other numbers; the sum of these three numbers will be divisible by 9. Second, subtract the sum of the digits from any number; the difference will be exactly divisible by 9 . Third, take two numbers in which the sum of the digits is the same; the difference of the two numbers will be divisible by 9 .

Such properties as these must have seemed exceedingly curious to the early arithmeticians, and fully entitle the number nine to be regarded as a magical number.

## HYPATIA'

## Walter J. Parker, Jr.

## Albion College

Hypatia was the first known woman mathematician and philosopher. She was famous for her beauty, purity, wisdom, and tragic death.

Being the daughter of Theon, a famous astronomer, mathematician, and philosopher at the University of Alexandria, she was trained by the most able instructors. She was born around 370 A.D., and by the time she was an adult it was said she knew more about astronomy than her father. At length she became so capable in all fields of learning that she succeeded Theon as Lecturer at the University.

Not only was she brilliant as a student, but the remarkable eloquence and clarity of her lectures, to say nothing of her beauty and purity of character, made her a very popular instructor with students from Europe, Asia, and Africa. She was respected by all in Alexandria, and many of the high officials consulted her about their various major problems. Orestes, the Roman Governor of Alexandria, was one of those who often sought Hypatia's advice. It was her intimacy with Orestes that resulted in her death.

On October 12, 412 A.D., Theophilus, the Bishop of Alexandria, died and six days later his nephew, Cyril, took his place. Cyril, swell headed with authority, placed many unnecessary restrictions on the Jews and unorthodox Christians; eventually, his self-made authority conflicted seriously with that of Orestes, the Governor. Socrates, then the Church Historian, relates what happened to Hypatia when the powers of the two authorities overlapped and Cyril vowed her destruction: "It was commonly reported among the Christians that it was by her influence he (Orestes) was prevented from being reconciled by Cyril. Some of them,

[^1]therefore, whose ringleader was a reader named Peter, hurried away by a fierce and bigoted zeal, entered into a conspiracy against her; and observing her as she returned home in a carriage, they dragged her from it, and carried her to a church called Caesarum, where they completely stripped her and then murdered her with (oyster) shells. After tearing her body to pieces, they took her mangled limbs to a place called Cinaron, and then burned them." This brutal murder happened in March, 415 A.D.

None of the writings of Hypatia exist today, but Suidas, an historian during her time, names these three: a commentary on the Arithmetica of Diophantus of Alexandria, a commentary on the Conics of Apollonius Pergassus, and a commentary on the Astronomical Canon of Ptolemy.

Hypatia's life was unusual not only because she was the first famous woman mathematician and philosopher, but because her life ended in such a tragic way. Her research papers may remain extinct, but the legend of her life will live forever.

## \%

It was only years after the body of Evariste Galois had turned to dust and ashes that his work began to be recognized and acclaimed for something like its true worth. More than a generation later, a French mathematician, C. Jordan, undertook the task of "discovering" Galois to the scientific world. Jordan found he had to devote a lifetime to the task-and still had to leave it unfinished, although he wrote a ponderous commentary of several hundred pages in an attempt to elucidate what Galois, at nineteen, had set down on a few sheets of foolscap!
-Gustav Davidson.

# GALILEO GALILEI 

## Wiliam L. Pillinger

## Albion College

The day, February 15th, 1564, was indeed worthy of note-a great star had fallen, Michelangelo was dead; a new star, yet invisible, had been born to the noble but impoverished Florentine family of Galilei. The father, Vincenzio, a competent mathematician and musician, had resorted to boot-making in order to support his family. His determination was that his son should study medicine, gain fortune, and restore the family name and wealth.

Thus one day Galileo found himself at the University of Pisa. Here, sickened by the smell of dead bodies, he soon became bored and dissatisfied. Once, while artfully dodging a medical lecture, he strayed by chance onto a lecture in geometry. So inspired was Galileo by this chance lesson that he would not rest until he had devoured all of Euclid's book and plied every professor with questions about his new idol's life. His father had carefully avoided even the mention of mathematics to Galileo, fearing just this result-the total alienation of his son from medicine. The mysteries of algebra and geometry quickly unfolded to Galileo. Such was his aptitude in this new field that a reluctant father was persuaded to allow Galileo to begin a study of mathematics and science. Unfortunately, lack of funds made it necessary for him to withdraw from Pisa before he had received his degree.

Vincenzio now endeavored to put Galileo to work in the bootshop, but to no avail-Galileo's mind was elsewhere. Borrowed books and inventions cluttering the bootshop had convinced his father that he was a wastrel, that is, until his hydrostatic balance won praise and favor from the Grand Duke of Tuscany. This remarkable invention was followed by a treatise on the center of gravity in solids. Galileo's fame spread and his reward was the chair of mathematics at the University of Pisa. While in that position he advanced the first principle of dynamics and proved that the path of a projectile was a parabola. His
contempt for the theories of Aristotle, a near blasphemy in those days, resulted in his unpopularity among his colleagues and, finally, his resignation.

In 1592, with the aid of interested and influential friends, he was appointed professor of mathematics at Padua. Here in this liberal school he enjoyed great popularity and his lectures were always overcrowded. His salary advanced until it surpassed any figure ever before given a professor of mathematics at Padua.

In the year 1610 Galileo abandoned this lucrative position at Padua and traveled to Rome. Quite early he must have adopted the Copernican theory of the solar system but did not avow his opinions for fear of ridicule. Now he was ready to begin a long battle with the Roman Catholic Church. In his Letters on the Solar Spots he openly defended Copernicus. This was met with strict censure from the Holy Office and Galileo was forbidden by Pope Paul V to hold, teach, or defend the condemned Copernican doctrine.

Seven years later a new pope was installed, a pope who in former days had been a very close friend and admirer of Galileo. After many audiences with Pope Urban and numerous visits to high church dignitaries, Galileo won high favor in ecclesiastical circles. He now ventured publication of his famous Saggitore. This treatise in dialogue, containing a carefully implied defense of the Copernican theory, was accepted and acclaimed by ecclesiastical and scientific authorities alike. Galileo's fame spread throughout the world, and likewise the doctrine of Copernicus. However, in his condemnation of Aristotle, Galileo had incurred the wrath of several powerful factions. These enemies slowly incited Pope Urban against Galileo and finally convinced him that Galileo had made him out a fool in the Saggitore. In 1633 Galileo was summoned before the dreaded "Holy Inquisition," condemned, and forced to renounce the whole Copernican theory.

This terrible ordeal greatly impaired Galileo's health and in 1637 he became completely blind. Galileo spent this last period of his life in seclusion with pupils sent to him by old friends. Notable among these young men were

Vivani and Toricellio. To these few he passed his latest ideas and theories of impact. The pendulum clock was Galileo's parting gift.

A young poet visited Galileo shortly before his death. Galileo's encouragement meant much to this young man, and you will find Galileo mentioned in Milton's Paradise Lost. Then came a slow fever and death on January 8th, 1642.

The great advances made by Galileo in science are attributed directly to his powerful applications of mathematical analysis to physical problems. His writings are vivid and suggestive; they inspired his successors to develop and formulate the laws of dynamics; foundations were given to men such as Newton. Undoubtedly Galileo's greatest contribution was the kindling of a great scientific "Renaissance" which freed science and mathematics forever from the yoke of Church and the Aristotelian grave.

## O

Conterminous with space and coeval with time is the kingdom of Mathematics; within this range her dominion is supreme; otherwise than according to her order nothing can exist; in contradiction to her laws nothing takes place. On her mysterious scroll is to be found written for those who can read it that which has been, that which is, and that which is to come.
-W. Spottiswoode.

## PYTHAGORAS

William F. Powlison

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Pythagoras, one of the most interesting figures in the history of mathematics, was born in Greece about 580 B.C. When he was still a youth he traveled to Egypt, and many of his greatest achievements were the result of the teachings he received there. In his later years, Pythagoras founded a secret brotherhood for the study of mathematics, the members of which called themselves Pythagoreans. If one wished to join this organization, he first had to spend a period of about three years as a "probationer," during which he spent most of his time listening to the lectures of Pythagoras. The followers of Pythagoras were highly devoted both to their master and to one another, and they seemed to possess a high moral creed.

Pythagoras' greatest contribution to science was the introduction of proof into mathematics since he was the first man to consider things in a really scientific manner. To most people, however, Pythagoras is best known for the theory which bears his name. There is some doubt as to whether or not he originated its idea, but Pythagoras did give the first real demonstration of its truth. Outside the realm of mathematics, Pythagoras seems to have been the first man to declare that the Earth was not the center of the universe but that it travels about the sun. Pythagoras made many other contributions to the sciences, chiefly in the field of mathematics, but these are too numerous to include here.
.Pythagoras' death was the result of a popular revolt about 500 B.C. during which all the Pythagorean schools were destroyed by fire. Nevertheless, his teachings have been continued, and today many of Pythagoras' ideas are basic scientific facts.

## TOPICS FOR CHAPTER PROGRAMS-IV

This is the fourth in a series of bibliographies on topics which are suitable for chapter programs. It is hoped that these bibliographies will encourage students to prepare many interesting papers, not only for presentation on chapter programs, but also for publication in the PENTAGON. Previous articles have presented bibliographies concerning the following nine subjects: (1) Women as Mathematicians; (2) The Cattle Problem of Archimedes; (3) Paper Folding; (4) Mathematical Prodigies; (5) Calculating Machines; (6) The Bee as a Mathematician; (7) Solutions of the Quadratic Equation; (8) Scales of Notation; and (9) The Planimeter. The editor will welcome contributions to this series of bibliographies.

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## Q

I like to look at mathematics almost more as an art than as a science; for the activity of the mathematician, constantly creating as he is, guided although not controlled by the external world of senses, bears a resemblance, not fanciful, I believe, but real, to the activities of the artist, of a painter, let us say. Rigorous deductive reasoning on the part of the mathematician may be likened here to the technical skill in drawing on the part of the painter. Just as one cannot become a painter without a certain amount of skill, so no one can become a mathematician without the power to reason accurately up to a certain point. Yet these qualities, fundamental though they are, do not make a painter or a mathematician worthy of the name, nor indeed are they the most important factors in the case. Other qualities of a far more subtle sort, chief among which in both cases is imagination, go to the making of a good artist or a good mathematician.
-Maxime Bocher.

## A NUMERICAL TEST**

1. The Musketeers.
2. ___th Century Limited.
3. 


4. $\qquad$ cheers.
5. A perfect $\qquad$
6. The House of $\qquad$ Gables.
7. The Spirit of $\qquad$
8. Pieces of
9. Aper cent American.
10. Friday the th.
11. $\qquad$ Nights in a Barroom.
12. Around the World in $\qquad$ Days.
13. The $\qquad$ Seas.
14. The $\qquad$ Muses.
15. The $\qquad$ Fates.
16. Fair, fat, and
17. $\qquad$ strikes and out.
18. A cat has $\qquad$ lives.
19. The Wonderful $\qquad$ Hoss Shay.
20. $\qquad$ days hath September.
21. $\qquad$ and $\qquad$ per cent pure.
22. ___ varieties.
23. $\qquad$ —.
24. The R's.
25. $\qquad$ winks.
26. As useless as a $\qquad$ th wheel.
27. $\qquad$ is company, $\qquad$ is a crowd.
28. The $\qquad$ Horsemen.
29. Sweet $\qquad$ and never been kissed.
30. Into the Valley of Death rode the $\qquad$
31. The first $\qquad$ years are the hardest.
32. $\qquad$ and $\qquad$ blackbirds baked in a pie.
33. Walk a flight and save \$ $\qquad$
34. The night has $\qquad$ eyes, the day but $\qquad$
35. As I was going to St. Ives, I met a man with wives.
36. Columbus sailed the ocean blue in $\qquad$

[^2]37. And the rain was upon the earth days and —_ nights.
38. $\qquad$ for defense, but not $\qquad$ cent for tribute.
39. The $\qquad$ and $\qquad$ Tales of the Arabian Nights.
40. $\qquad$ th Heaven.
41. $\qquad$ th Night.
42. The $\qquad$ Bears.
43. ___ shooter.
44. $\qquad$ wheel brakes.
45. A $\qquad$ th sense.
46. At the th hour.
47. $\qquad$ blind mice.
48. The Gay $\qquad$ 's.
49. $\qquad$ Lakes.
50. skiddoo.
51. -_ Little Peppers.
02. $\qquad$ Freedoms.
53. The $\qquad$
54. $\qquad$ Wise Men.
55. __ Years Before the Mast.
56. $\qquad$ th Day Adventists.
57. Cat o' $\qquad$ Tails.
58. __ corners of the globe.
59. $\qquad$ Keys to Baldpate.
60. The Tale of $\qquad$ Cities.
61. The earth was made in $\qquad$ days.
62. Commandments.
63. The $\qquad$ Colonies.
64. Wilson's $\qquad$ points.
65. Tea for $\qquad$
66. Under Flags.
67. The __ Disciples.
68. Louis the $\qquad$ th furniture.
69. Leagues under the Sea.
70. league boots.
71. Rain before $\longrightarrow$ stop before $\qquad$
72. The $\qquad$ Wonders of the World.
73. To kill birds with $\qquad$ stone.
74. Ali Baba and the $\qquad$ Thieves.
75. The __ Books of Euclid.
76. 'Twas the $\qquad$ th of April in ' $\qquad$
$\qquad$
78. The roaring 's.
79. ___ good men and true.
80. _ lay them straight.

Did you like this test? If so, then no doubt you have thought of similar questions which might be added to the list. Send these questions to the Editor for publication in a future number of the PENTAGON.

## (2)

Mathematics represents one of the great imaginative activities of mankind. It is the language of ideas. In ordinary language, words are symbols which stand for abstractions of varying degree. In mathematics, symbols are chosen which represent ideas abstractly also, but more simply because they carry in themselves and their combinations the logical connection of ideas. In difficult processes of thought no other tool is adequate.
-G. C. Evans.

## THE PROBLEM CORNER

## Edited by Judson W. Foust <br> Central Michigan College of Education

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solutions should not demand any tools beyond the calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before March 1, 1948. The best solutions submitted by students will be published in the Spring number of THE PENTAGON. Credit will be given for all correct solutions received. Address all communications to Dr. Judson W. Foust, Central Michigan College of Education, Miount Pleasant, Michigan.

## PROBLEMS PROPOSED

1. Selected from the second Stanford University Mathematics Examination, April 19, 1947.

To number the pages of a bulky volume the printer used 1890 digits. How many pages has the volume?
2. Selected from the second Stanford University Mathematics Examination, April 19, 1947.

Among grandfather's papers a bill was found: 72 turkeys \$-67.9-

The first and last digits of the number that obviously represented the total price of these fowls are replaced here by blanks, for they have faded and are now illegible. What are the two faded digits and what was the price of one turkey?

3. Proposed by the Editor.

Each side of triangle $A B C$ is trisected. Show that the area of the shaded part in the figure is exactly one-seventh of the whole triangle. (See H. Steinhaus, Mathematical Snapshots, p. 8.)

## 4. Proposed by the Editor.

Into a full conical wineglass of depth a and generating angle a there is carefully dropped a sphere of such size as to cause the greatest overflow. Show that the radius of the sphere is $a \sin a /(\sin a+\cos 2 \alpha)$. (See Granville, Smith, Longley, Calculus, p. 114.)
O

The object of mathematics is to prove that certain premises imply certain conclusions; and the fact that the conclusions may be as obvious as the premises never detracts from the necessity, and often not even from the interest of the proof.
G. H. Hardy.

## THE MATHEMATICAL SCRAPBOOK

In the name of the gods I beg you to think.
-Demosthenes.

$$
=\nabla=
$$

How can you drop an egg three feet without breaking the shell?

$$
=\nabla=
$$

The thirteenth day of the month is more likely to be Friday than any other day of the week.

$$
=\nabla=
$$

In a class there are twice as many girls as boys. Each girl makes a bow to every other girl, to every boy, and to the teacher. Each boy makes a bow to every boy, to every girl, and to the teacher. In all there were 900 bows made. How many boys are in the class?

$$
=\nabla=
$$

The following original demonstration of the Pythagorean Theorem was developed by President Garfield when
 he was in college. Let $A B C$ be a right triangle. On the hypotenuse $C B$ describe the half square $B C E$, and from $E$ draw $D E$ perpendicular to $A C$ produced. Now area $A B E D=A D \cdot(D E+A B) / 2$ $=1(A C+A B)^{2}$. But area $A B E D=A B C+D C E+E C B$ $=1 B C^{2}+A C \cdot A B$. Hence, $(A C+A B)^{2}=B C^{2}+2$ $A C \cdot A B$, so that $A C^{2}+A B^{2}$ $=B C^{\prime}$.
-School Science \& Math.

Decode the following division.
$A B) B C A D A(B E D$
CFE
$G E D$
GFG
FEA
FEA
$=\nabla=$
Byerly suggested the abbreviation cis $\theta$ for $\cos \theta+$ $\mathrm{i} \sin \theta$.

$$
=\nabla=
$$

The area of the plane quadrilateral $P_{1} P_{2} P_{3} P_{4}$ is equal to

$$
\begin{gathered}
1 / 2\left|\begin{array}{llll}
x_{1} & y_{1} & 1 & 1 \\
x_{2} & y_{2} & 0 & 1 \\
x_{3} & y_{3} & 1 & 1 \\
x_{4} & y_{4} & 0 & 1
\end{array}\right| \\
=\nabla=
\end{gathered}
$$

"Paradoxes and contradictions to the scientist are but the foundations upon which to plan and structure new truth."
-C. S. SLichter.

$$
=\nabla=
$$

The comma was in general use as a decimal point long before the period, and is still used for this purpose in France, Germany, Italy, Sweden, and other countries.

$$
\begin{aligned}
& =\nabla= \\
8712 & =4 \times 2178 \\
9801 & =9 \times 1089
\end{aligned}
$$

We know that

$$
\text { Let } \begin{aligned}
& \log 2=1-1 / 2+1 / 3-1 / 4+1 / 6-\cdots \\
& \text { Then } \begin{aligned}
\log 2+S & = \\
& (1+11 / 2+1 / 3+1 / 4+1 / 6+\cdots \\
& +2(1 / 2+1 / 4+1 / 4+1 / 6) \\
= & 1+1 / 2+1 / 3+1 / 4+\cdots) \\
& =\nabla=
\end{aligned} \\
&=\nabla .
\end{aligned}
$$

"Archimedes the wise, the famous maker of engines, was a Syracusen by race, and worked at geometry till old age, surviving five-and-seventy years; he reduced to his service many mechanical powers, and with his triple-pulley device, using only his left hand, he drew a vessel of fifty thousand medimni burden. Once, when Marcellus, the Roman general, was assaulting Syracuse by land and sea, first by his engines he drew up some merchant-vessels, lifted them up against the wall of Syracuse, and sent them in a heap again to the bottom, crews and all. When Marcellus had withdrawn his ships a little distance, the old man gave all the Syracusans power to lift stones large enough to load a waggon and, hurling them one after the other, to sink the ships. When Marcellus withdrew them a bowshot, the old man constructed a kind of hexagonal mirror, and at an interval proportionate to the size of the mirror he set similar small mirrors with four edges, moved by links and by a form of hinge, and made it the centre of the sun's beams-its noon-tide beam, whether in summer or in mid-winter. Afterwards, when the beams were reflected in the mirror, a fearful kindling of fire was raised in the ships, and at the distance of bow-shot he turned them into ashes. In this way did the old man prevail over Marcellus with his weapons. In his Doric dialect, and in its Syracusan variant, he declared: 'If I have somewhere to stand, I will move the whole earth with my charistion.' Whether, as Diodorus asserts, Syracuse was betrayed and the citizens went in a body to Marcellus, or, as Dion tells, it was plundered by the Romans, while the citizens were keeping a night festival to Artemis, he died in this fashion at the hands of one of the Romans. He was stooping down,
drawing some diagram in mechanics, when a Roman came up and began to drag him away to take him prisoner. But he, being wholly intent at the time on the diagram, and not perceiving who was tugging at him, said to the man: 'Stand away, fellow, from my diagram.' As the man continued pulling, he turned round and, realizing that he was a Roman, he cried, 'Somebody give me one of my engines.' But the Roman, scared, straightway slew him, a feeble old man but wonderful in his works."

Translated by Ivor Thomas. Reprinted by permission of the publishers from the Loeb Classical Library edition of Selections Illustrating the History of Greek Mathematics, Vol. II, pp. 19-23. Harvard University Press, 1941.

$$
=\nabla=
$$

A garrison of 500 men was victualled for 48 days; after 15 days it was reinforced, and then the provisions were exhausted in 11 days; required the number of men in the reinforcement.
-School Science \& Math.

$$
=\nabla=
$$

What is the greatest possible number of one-inch cubes that can be placed into an empty box, four inches wide, ten centimeters long, and one-third fathom deep?

$$
=\nabla=
$$

If $A, B, C$, and $D$ each speak the truth once in three times (independently), and $A$ affirms that $B$ denies that $C$ declares that $D$ is a liar, what is the probability that $D$ was speaking the truth? (Sir A.S. Eddington solves this problem in the Mathematical Gazette, Vol. 19, 1935, p. 256. He obtained $p=25 / 27$.)

$$
=\nabla=
$$

Leibnitz (1646-1716) is the earliest known writer on determinants.
"There's no expedient to which a man will not go to avoid the labor of thinking.".
-Thomas A. Edison.

$$
=\nabla=
$$

The centipede was happy till
One day the toad in fun
Said, "Pray, which leg comes after which?"
This raised his thoughts to such a pitch,
He lay distracted in the ditch,
Not knowing how to run.
-School Science \& Matif

$$
=\nabla=
$$

To prove that all triangles are isosceles: Let $A B C$ be any triangle. Bisect the angle $A$ by the line $A O$ and let this line meet the perpendicular bisector of $B C$ in $O$. Draw $O S$ perpendicular to $A C$
 and $O R$ perpendicular to $A B$. Draw OB and $O C$. Then $R O=$ $O S, O B=O C, \angle B R O$ $=\angle C S O$ being right angles, and therefore $R B=S C$. Likewise $A R=A S$. By addition, $A B=A C$.

$$
=\nabla=
$$

When Henry Briggs first met John Napier, he exclaimed: "My lord, I have undertaken this long journey purposely to see your person, and to know by what engine of wit or ingenuity you came first to think of this most excellent help in astronomy, viz., the logs."

$$
=\nabla=
$$

Show that the sum of an infinite geometric progression may be written in the form, $S=a_{1}^{2} /\left(a_{1}-a_{2}\right)$. This form is often more convenient for calculating $S$ than the usual form.

Pythagoras is said to have known that a triangle whose sides are $2 n+1,2 n^{2}+2 n$, and $2 n^{2}+2 n+1$ was a right triangle. This may be written $m=2 n+1,\left(m^{2}-1\right) / 2$, ( $m^{2}+1$ )/2. According to Proclus the following rule was obtained by Plato: If $2 n$ is one side, then $n^{2}-1$ and $n^{2}+1$ are the other two sides.

$$
\begin{gathered}
=\nabla= \\
\left(\sin 80^{\circ}\right)^{2}=\sin ^{2} 6400^{\circ} \\
=\nabla=
\end{gathered}
$$

E. A. Abbott, the author of that charming little book, Flatland, was formerly Master of the City of London School. He wrote over thirty theological works, and a dozen literary and pedagogical works, but Flatland was. his only venture in the field of mathematics.

$$
=\nabla=
$$

Show that if any even number of terms is taken in the series $1,3,5,7, \cdots$, the sum of the first half is one-third of the sum of the second half.

$$
=\nabla=
$$

Prove that the quadrilateral formed by the four bisectors of the angles of a parallelogram is a rectangle.

$$
=\nabla=
$$

It is annoying to dwell upon such trifles, but there is a time for trifling.
-Blaise Pascal.

## INSTALLATION OF NEW CHAPTERS

The PENTAGON takes pleasure in reporting the installation of five new chapters of Kappa Mu Epsilon.

## MISSOURI GAMMA

A mathematics honorary club, Mu Sigma Alpha, was organized in 1943 at William Jewell College, Liberty, Missouri. The club earned a place of distinction and respect among the honor societies on the campus. During 1946-47 the following talks were made by members and guests: "Contribution of Einstein toward the Atomic Bomb, and his Mass-Velocity Relationship," by Dr. W. A. Hilton; "The Invention and Development of the Concept of Zero," by Professor L. O. Jones; "Fallacies in Mathematics," by Nicholas Housley; "Short Cuts in Mathematics," by Professor C. O. Van Dyke; "Geometric Designs in Lighting," by LeRoy Heaton; "Telling Time by the Stars," by Lloyd Elrod; "Improvement of High School Mathematics Courses," by Paul Curau; "The Case for Mathematics," by Woody Rixey. When several members of the club became interested in affiliating with a national mathematics fraternity, a petition for membership was presented to Kappa Mu Epsilon.

The Missouri Gamma Chapter was duly installed at William Jewell College on May 7, 1947. The installation team from Missouri Beta at Warrensburg State College consisted of Dr. Claud Brown, installing officer, Mrs. Virginia Moore, Ronald Evans, Patricia Stewart, Berma Deane Rist, Norman Hoover, and Samuel Herndon. Other members of Missouri Beta at the ceremony were Gerald Hart, Robert Ellis, Kenneth Heying, Gerhardt Jaeger, Professor Fred W. Urban, and Dr. Reid Hemphill.

The charter members of Missouri Gamma are Walter Binns, Ellen Buckley, Harry Burress, Mary Ruth Carney, Maynard Cowan, Wiley Crawford, Paul Curau, Lloyd Elrod, Robert Evans, LeRoy Heaton, Wallace Hilton, Lee Jones, Verne Lafrenz, James Leatherman, Elsie Lewis, Imogene McCormick, Truett Neese, Raymond Neth, William Ridell, Jr., Herbert Ross, Howard Skeen, Anna Mae Stigers, Paul Swedberg, and Edwin Watson. The following officers were
installed: President, Mary Ruth Carney; Vice-President, Truett Neese; Secretary, Edwin Watson; Treasurer, LeRoy Heaton; Corresponding Secretary, Professor L. O. Jones. :

## TEXAS GAMMA

The Texas Gamma Chapter was installed on May 7, 1947, at the Texas State College for Women, Denton, Texas. The installing officers, all from Southern Methodist University, were Professor Edwin D. Mouzon, Jr., chairman of the mathematics department, Professor David W. Starr, and Gene Archer, Joseale Hulse, and Doris Wyatt, student officers of Texas Beta Chapter. The installation ceremony was followed by a banquet given jointly by the Texas Gamma Chapter and the E. V. White Mathematics Club which was celebrating the twenty-fifth anniversary of its founding. Members of the installation team were honorary guests for the dinner. Following the dinner, Dr. Mouzon spoke on "A new Definition of the Angle of Inclination" and Doris Wyatt gave a brief report of the sixth biennial convention of Kappa Mu Epsilon.

In order to be eligible for membership in Texas Gamma, a student must have completed one semester of calculus and be enrolled in the second semester, and she must have an over-all average of $B$ as well as a $B$ average in mathematics. The charter members are Professor Andrew Ashburn, Jane Beale, Charlotte Bell, Joy Harris Conley, Alice Edrington, Lila Ruth Gillen, Betty Ruth Johnston, Beverly Lamp, Professor Harlan Miller, Professor Josephine Mitchell, Bettye Reed, Robbie Jane Stanley, Ann Lauree Tillman, Alice Weeks, Imagene White, and Johnnie Williams. The following officers were installed: President, Johnnie Williams; Vice-President, Beverly Lamp; Secretary, Alice Weeks; Treasurer, Jane Beale; Corresponding Secretary and Faculty Sponsor, Professor Harlan Miller.

## WISCONSIN ALPHA

Mount Mary College of Milwaukee, Wisconsin, can boast of having the first chapter in Wisconsin of Kappa Mu Epsilon. The installation ceremony was performed on May 11, 1947, by a team from the Illinois Gamma Chapter lo-
cated at the Chicago Teachers College. The team consisted of President Cloda Augelli, Vice-President Joseph M. Duffy, Secretary Mary T. Graham, Treasurer Dolores Grien, and the Faculty Sponsor, Joseph J. Urbancek, who acted as the installing officer. The team was accompanied by Mrs. Urbancek, Kathryn Graham, and Helen Mae Grunder. The installation ceremony was preceded by a tour of the campus and a dinner in honor of the occasion. Members of the installation team wore doctoral robes which added dignity and solemnity to the impressive ceremony.

The charter members of Wisconsin Alpha are Sister Mary Felice, Sister Mary Therese, Sister Mary Thomasine, Sister Mary Xavier, Patricia Farrell, Eileen Ford, Elizabeth Forrestal, Mary Ann Frodel, Eleanor Grogan, Mary Alice Gourke, Norma Harding, Dorothy Karner, Mary Lou Marquardt, Jean Matela, June Rose McDonald, Dorothy Nilles, Mildred Oestreich, Bernadine Spitznogle, Marie Tronchuk, and Elizabeth Williams. The officers installed were Dorothy Nilles, President, Patricia Farrell, Secretary-Treasurer, and Sister Mary Felice, Faculty Sponsor.

## TEXAS DELTA

The Texas Delta Chapter was installed on May 13, 1947, at Texas Christian University, Fort Worth, Texas. The installation was conducted by a delegation from Texas Beta Chapter consisting of Dr. Edwin Mouzon, Doris Wyatt, Marion Mart, Joseale Hulse, and Dr. David Starr. Other memkers of Texas Beta who attended the installation were Everett Brown, Betty McKnight, and Paul Minton. The ceremony was held in Jarvis Hall with Dr. Mouzon acting as installing officer. An informal reception followed.

A grade point average of 2.5 in mathematics is a prerequisite to membership in Texas Delta. The charter members are Elizabeth Baker, Lillian Bales, Robert Blackwell, Ina Bramblett, Beatrice Bukowski, Janie Callahan, Ellaveen Childress, Charles Cook, Donald Cowan, Bruce Fallis, Louise Grady, Iva Helen Lee, Elva Lerret, Paul Lerret, Mary Ann Macke, Roert McCarty, Edith Morgan, W. Harvey Neely, Robbie Rutherford, Clarence Sale, Douglas Schwartz, Charles Sherer, and Ernest Wolff.

The chapter installed the following officers: President, Robert J. Blackwell; Vice-President, Louise Grady; Secretary, Elizabeth Baker; Treasurer, Robbie Rutherford; Corresponding Secretary, Ina Bramblett.

## OHIO GAMMA

June 6, 1947, marked the installation of the Ohio Gamma Chapter at Baldwin-Wallace College, Berea, Ohio. Twenty-six charter members were initiated at the ceremony which folowed dinner at a local restaurant. The installing officers were Dr. and Mrs. F. C. Ogg and Professor H. R. Mathias of Bowling Green University. The chapter president, Thaddeus Curtz, presented the new members to the installation team who responded with brief talks on the principles of the fraternity and the meaning of the insignia. The ceremony was concluded with the signing of the constitution by the new members.

The charter members of Ohio Gamma are Arthur Abloott, Professor Paul Annear, Richard Arbuckle, Jan Arnold, Herbert Baker, Professor Theodore Bogardus, Edward Cook, 'Thaddeus Curtz, Harry Davis, Ralph Deitrick, John Detlef, Richard Fitz, John Galvin, Theodore Goodson, Mona Gurney, Marian Harriger, Betty Hostetler, Joseph Lewandowski, .John Ranchoff, George Salabak, Rose Shalla, Professor John Sinnema, Herbert Smith, Wayne Smith, Eugene Socha, and Kenneth Vander Sluis. Edward Johnson, Eleanor Longwell, Elizabeth Plas, Professor Dean Robb, and Herbert Thoma who were unable to attend the installation ceremony were initiated on June 13.

The chapter installed the following officers: President, 'Thaddeus Curtz; Vice-President, Joseph Lewandowski; Secretary, John Detlef; Treasurer, Kenneth Vander Sluis; Corresponding Secretary, Professor Paul Annear.


I see more men to acknowledge the benefit of number, than I can espy willing to study, to attain the benefits of it. -Robert Recorde.

## CHAPTERS OF KAPPA MU EPSILON

ALABAMA ALPHA, Athens College, Athens.
ALABAMA BETA, Alabama State Teachers College, Florence. aLABAMA GAMMA, Alabama College, Montevallo.
ILLINOIS ALPHA, Illinois State Normal University, Normal.
ILLINOIS BETA, Illinois State Teachers College, Charleston.
ILLINOIS GAMMA, Chicago Teachers College, Chicago.
ILLINOIS DELTA, College of St. Francis, Joliet. IOWA ALPHA, Iowa State Teachers College, Cedar Falls. IOWA BETA, Drake University, Des Moines.
KANSAS ALPHA, Kansas State Teachers College, Pittsburg.
KANSAS BETA, Kansas State Teachers College, Emporia.
KANSAS GAMMA, Mount St. Scholastica College, Atchison. KANSAS DELTA, Washburn Municipal University, Topeka. MICHIGAN ALPHA, Albion College, Albion.
MICHIGAN BETA, Central Michigan College, Mount Pleasant. MICHIGAN GAMMA, Wayne University, Detroit, Michigan. MISSISSIPPI ALPHA, State College for Women, Columbus. MISSISSIPPI BETA, Mississippi State College, State College. MISSOURI ALPHA, Missouri State Teachers College, Springfield. MISSOURI BETA, Missouri State Teachers College, Warrensburg. MISSOURI GAMMA, William Jewell College, Liberty. NEBRASKA ALPHA, Nebraska State Teachers College, Wayne. NEW JERSEY ALPHA, Upsala College, East Orange. NEW JERSEY BETA, New Jersey State Teachers College, Montclair. NEW MEXICO ALPHA, University of New Mexico, Albuquerque. NEW YORK ALPHA, Hofstra College, Hempstead. OHIO ALPHA, Bowling Green State University, Bowling Green. OHIO BETA, College of Wooster, Wooster. OHIO GAMMA, Baldwin-Wallace College, Berea. OKLAHOMA ALPHA, Northeastern State College, Tahlequah. SOUTH CAROLINA ALPHA, Coker College, Hartsville. TENNESSEE ALPHA, Tennessee Polytechnic Institute, Cookeville. TEXAS ALPHA, Texas Technological College, Lubbock. TEXAS BETA, Southern Methodist University, Dallas. TEXAS GAMMA, Texas State College for Women, Denton. TEXAS DELTA, Texas Christian University, Fort Worth. WISCONSIN ALPHA; Mount Mary College, Milwaukee.


[^0]:    - Numbers in brackets refer to the literature cited at the end of this paper.

[^1]:    With this article, THE PENTAGON inaugurates a series of biographies of great men ant woumen of mathematics. Students are cordially invited to contribute to this series. The liree biographies presented in this number were prepared by new mem. hers of the Michigall Npha Chapter as part fo the requirements for initiation, October 28, 1947.

[^2]:    * Contributed by E. Marie Hove.

