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OPPORTUNITIES FOR WOMEN TRAINED IN MATHEMATICS

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Sooner or later, in a generation of doers, the college teacher finds herself confronted with the question, "What can I do with mathematics?" She realizes that no mere shrug of the shoulder or a general statement as to its later applicability will satisfy the serious-minded youth who feels that his college should prepare him for life. An attempt to answer this question for herself, so as to be able to give adequate response to the many interested inquirers, was the remote stimulus for this paper. The immediate reason is the desire to share with the fellow mathematicians the findings of a study which was carried on rather extensively at Mount St. Scholastica College. The writer is fully aware that in treating this subject, "Opportunities for Women Trained in Mathematics," she is writing about a minority group and for a limited reading public, yet it is a subject that must be considered because of the unusual importance which mathematics enjoys in the female ranks at the present time.

An outstanding issue of the fashionable magazine *Mademoiselle* carried an article which stated, "According to the National Roster of Scientific and Specialized Personnel there are only 916 women in this country who are mathematicians of full professional standing. There are fewer than 5,000 women mathematics majors now in our colleges."[1] A bit of reflection shows us that this is not surprising. Liberal education, which is America's proud boast, aims to afford each individual the opportunity to develop fully and to produce the utmost according to his nature. Hence, in the case of woman, liberal education means the unhampered

[1] July, 1943. p. 82.

opportunity for the free evolution of her natural powers unto perfect womanhood.

Woman's greatness and power rests in the acquisition and development of such qualities as "insight, tact, tenderness, sympathy, sensibility, practicality, and understanding."[2] The role she is meant to play in life, and one which was designed by her Creator, is that of the universalist—she must needs be an authority in a limited degree on everything in her household. The great social thinkers of our times protest that national stability and security are unthinkable unless the nucleus of the nation—the home—is sound. Hence, specialization, except in the case of the minority who will be career women, is fatal to woman. This is not tantamount to saying that her destiny in life is *inferior* to that of man—rather it is *different*. The nation needs wise women and learned men. "Woman acquires (wisdom) and maturity of mind through the concrete handling of life, through the exercise of practical and rapid intuitions applied to the different problems that present themselves to her for solution, in the exercise of her characteristic role"[3]—of universalist. Deep, prolonged and specialized study, such as is required for professional mathematics, does not equip her for this task; hence, women mathematical specialists will ever be few in number. If we can trust the Hollywood version of the life of Madame Curie, most of us are agreed that while her scientific achievements are most admirable we should not like to have been a member of her neglected household.

The present war, fought on a scientific basis wherein each of the warring parties attempts to conquer the aggressive host by the discovery and use of deadlier scientific weapons, has given an unprecedented importance to mathematics. With the departure of men to the front lines we have witnessed the simultaneous action on the part of women to replace men in the vital industries. They are to be commended for this step; they are being true to their

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2 Edward Leen, *What is Education.* London and Dublin, Burns Oates and Washbourne Ltd., 1943.

3 *Ibid.* p. 233.

calling—that of the universalist. As in times past, just so now, woman adapts and adjusts herself to answer any need. Just how many of these same women will be actually needed in peace time is difficult to say. It is an admitted fact that women trained in mathematics today have opportunities hitherto unknown. This increase in opportunities was born of necessity. There is no doubt that industry needs them badly.

The preceding general introduction shows that while there are more opportunities for mathematically trained women now than heretofore, still it is not a desideratum for all women. The remainder of this paper will be given over to a rather detailed discussion of these opportunities. It does not represent in any sense an exhaustive study nor even a statistical experiment. It does show trends and possibilities. Extraordinary war-time demands have made mathematics courses popular. Hence, college girls frequently ask themselves, "What are my chances for utilizing this training when my college days are over?" The effort to answer these questions in our own local group gave rise to the investigation herein described.

Shortly after the present term got under way a committee of interested and mathematically-bent collegians assembled a form letter. It carried a request for information concerning "post-war positions open to women with college mathematical training, their specific requirements, opportunities for advancement, and salaries." Printed material relative to these positions was also solicited. Approximately fifty letters were sent to the leading industrial organizations and professional groups. In determining the class of firms to which letters were to be sent preference was given to the old, well-established, peace-time industries. Without going into detail on this matter it will suffice to mention: Southern Pacific Railroads; Pennsylvania-Central Airlines; United Steel Corporation; Westinghouse Electric Company; General Motors Corporation; Actuarial Departments of some of the leading Universities (notably Iowa); Commercial National Bank; U. S. Office of Education; U. S. Department of Agriculture; U. S. Department of Commerce; U. S. Weather Bureau; and others of like importance.

Besides these form letters, direct appeal by personal letter was made to former mathematics graduates of Mount St. Scholastica College who are employed by similar firms. The results in both cases have been quite significant and open to our youth hitherto unthought-of fields.

For purposes of clarification we shall report our findings from the form letters in group results according to the rather general classification suggested by Professor T. H. Hildebrandt of the University of Michigan. It is his conviction that woman's greatest opportunities are to be had in one or the other of the following:

(a) teaching of mathematics in secondary schools;
(b) actuarial work in connection with insurance companies;
(c) statistical work with industrial concerns or with governmental agencies;
(d) skilled work requiring knowledge of mathematics and physics, in connection with big industries.

No one doubts the direct application of mathematical knowledge in the field of teaching and we shall not devote any space here to this matter. Concerning actuarial work, which is defined as the knowledge and skill requisite for making the computations used in the running of a life-insurance company, a very encouraging picture is presented to us. We find that some of the specific mathematical computations involve the calculation of premium rates, policy reserves, distribution of surplus, and allied problems. We quote directly from the *Actuarial Science Bulletin* published by the University of Iowa: "Although the actuary has a wide range of duties his primary duty is to see that the business is conducted on a basis that will insure permanent solvency of the company." The University of Iowa, the University of Michigan, and Hunter College give complete actuarial courses. Some insurance companies offer classes to supplement the training of those in their employment who seem fitted for the work. Professor Lloyd Knowler of the

University of Iowa offers the following statement from the writings of an insurance company representative: "Those qualifications which are important for success in an actuarial career include an intellectual ability of a high order, an *unusual capacity* in mathematics with a greater interest in the application of mathematics to practical problems than in advanced theoretical mathematics, the strength and enthusiasm to study long and arduously for several years following completion of a college career and the personality and other qualifications that go to make a person, at least potentially, a good business executive."

A former student of Mount St. Scholastica College who is employed in the Rates and Values Section of the Actuarial Department of the Prudential Life Insurance Company at Newark, N. J., has this to say concerning her work: "We figure premiums, annuities, and extended-insurance values. Then there are claims——either death claims or matured endowments. Then, too, there are several settlement options that people can select if they do not choose a cash settlement. If they take their money out in installments, interest rates are involved. Usually we have tables to consult, but occasionally things turn up which demand good knowledge of algebra and calculus. One definitely needs higher college mathematics to pass the difficult tests preparatory to becoming an assistant actuary."

The third general field open to those trained in mathematics deals with statistical work. Statisticians are employed in government agencies, business concerns, or endowed institutions devoted to scientific research. The government functions through many organizations concerned with economic and social statistics. Agricultural colleges and experimental stations, besides their statistical studies on crop yields and animal products, also carry big production experiments. The Weather Bureau and the Civil Service Commission encounter manifold statistical problems in their work of weather forecasting and in devising means for measuring personal ability. Business concerns employ statisticians for three main objects. Usually it is the job of these persons to forecast demand and prices, to evaluate the factors governing sales, and sometimes merely to record and tabulate facts. In lesser numbers we find privately employed statisticians in industrial and technical research groups. After commenting on the demand for women with mathematics training in laboratories and subprofessional jobs, Professor Harold Hotelling of Columbia University's Statistical Department writes: "I receive an even greater number of requests for women having a thorough training in mathematical statistics. While this is partly because of the selective fact of my teaching mathematical statistics, it does seem that the possibilities here are very great. I am now continually receiving requests for such people. . . . Two weeks ago I received a request from a man working on some mental testing problems who wants an assistant, at a salary of $3,200 per year. The principal qualification needed for this assistant was an uncommonly good knowledge of mathematics and mathematical statistics. There is also apparently a great field for women in the planning of agricultural experimentation by means of the principles of mathematical statistics and in the new field of industry quality control by sampling inspection." Mr. Clarence Brewer, Personnel Officer in the Bureau of Agricultural Economics, has this to say regarding his department: "It is concerned with the collection and analysis of statistical data relating to the production of agricultural products. From this basic data periodic estimates or forecasts are made of prospective agricultural production. The basic requirements for this type of work call for persons trained in statistics and statistical techniques and analysis with an agricultural background either by experience or training." Accompanying his letter was a civil-service announcement for *Mathematician.* Although this offer was made primarily for the duration it is valuable in that it shows the wide range of applicability of mathematical training. First-hand information, limited because engaged in defense work, regarding the importance of mathematical statistics comes from a "Mountie" employed with the Division of War Research,

U. S. Navy Laboratories at San Diego, California. She writes that in her work much use is made of correlation coefficients and standard deviations. Kenneth A. Meade, Director of Technical Employment with General Motors Corporation in Detroit, states in a recent letter that girls with mathematical training are being employed "in work connected with accounting statistics and engineering calculations."

Still a fourth field of opportunity for the trained woman mathematician is in industry which employs skilled workers who, besides a knowledge of mathematics, have also good training in the exact sciences. Again we quote from the recent letter of Professor Harold Hotelling of Columbia University: "I have many reasons for believing that after the war there will be a very great expansion of positions open to women with suitable training in mathematics if their training includes also the foundations of one or another of the subjects with which mathematics can be applied. . . . I receive frequent requests (from heads of laboratories and technical shops) indicating that they would like to have girls who have majored in mathematics and physics in college who can do mechanical drawing and computing and contribute occasional ideas to the developmental and production problems. It does seem to me, on the basis of numerous conversations with such men that there is a real opportunity here for a girl with brains, ambition, and mathematical training. The heights to which such a girl might rise seem to be limited definitely, only, by the amount of time she will put into this kind of work and into educational preparation for it. The one complaint (the writer feels that it is a complaint only from the viewpoint of the employer) I hear about women in such positions is that they frequently get married before reaching the stage of exploiting their maximum scientific possibilities. Apart from this factor there seems a great readiness to accept women into these technical fields." The preceding is but a proof that, for the generality, woman's true place is in the home, as co-founder of a family, and not in industry.

That science training is most beneficial for job seekers is also the view taken by Mr. William T. Clark, specialist in the U. S. Office of Education. He makes casual mention of the applications of pure mathematics in the research departments of large corporations and in such fields as astronomy and ballistics calculations, and then adds, "It is my personal opinion, however, that vastly greater opportunities are open to the mathematics major who has leavened his mathematical proficiency with a sound knowledge of some physical science such as physics, biometrics, chemistry, or engineering. I feel that this is true because the employment opportunities for strictly specialized mathematicians in industry are far more limited than those which await the mathematically proficient scientist."

Mr. H. C. Madsen of Westinghouse Electric has this to offer: "We use very few straight mathematicians. The best opportunities for women with a college education in science are in our engineering and research laboratories. Here we need those with training in chemistry, physics, and metallurgy with emphasis on laboratory technique."

It is also the opinion of other industrial concerns that mathematical training to be really effective must be combined with training in the related sciences. The Eastman Kodak Company employs women graduates in mathematics, physics, and chemistry. The applicants receive $35.00 per week and after a trial period are assigned to work that is most suited to them. The Atlantic Refining Company seems to prefer the major in physics or chemistry with a minor in mathematics.

Before bringing this discussion of opportunities to a conclusion we shall make brief mention of several other occupational possibilities not included in the four-fold classification as treated in this paper. Mr. Edward S. Sullivan, General Sales Manager for Pennsylvania Central Airlines, writes that "women with this particular type of training should be capable of holding positions in our treasury and accounting departments or in our maintenance department where knowledge of mathematics, engineering, and accounting could be put to best advantage." Mr. P. J. Kendall of the Southern Pacific Company reports that their accounting department normally employs 1,500 or more people of whom a large number are women. It is his opinion that in the post-war period many openings will be available and that "college mathematical training would be an asset for these positions, in fact would be advantageous in almost any clerical positions we have to offer."

Miss Mary Ann Flaherty, Mount graduate of '42, who is employed as Senior Assistant in the Engineering Department of the Illinois Bell Telephone Company, at Chicago, quotes a significant statement made by the executive when she was interviewed for the job. After inquiring of her how much mathematics she had studied in college, he gave, as his motive for inquiry, the fact that such training provides "an index into the mental agility we can expect of you." Apparently this executive upholds the doctrine of transfer of training and believes that there is a high correlation between proficiency in mathematics and general intelligence.

It also seems worthy of mention that the *Kansas City Star* attached great publicity and devoted paper space to the fact that in the past month a woman was elected to the office of Vice-President in one of Kansas City's largest banks. According to the story it is the first time such an honor has been accorded a Kansas City woman.

The title of this paper requires that we treat primarily of opportunities for women mathematicians. Yet we feel that mention of a current article of general interest is in place. Professor James H. Zant of the department of mathematics at Oklahoma A. & M. College is concerned with a similar problem which he discussed in the *Bulletin of the Kansas Association of Teachers of Mathematics*[4] under the title "Jobs Available for Mathematics Students." In addition to those jobs which have been mentioned earlier in this paper he lists marine and air navigators which of course would be suitable only for men.

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4 February, 1945.

In conclusion we wish to state that the research underlying this paper has been most enlightening as was evidenced when these same results were presented in the form of a symposium to the entire student body. We feel it is a definite step in answering the question, "What can one do with mathematics?," and that those few women who will choose mathematics as basic to a career need have no reasonable fear of unemployment.

"A few of the most beautiful discoveries of Archimedes (287-212 B.C.) relate to the area and volume of a sphere. While his discoveries in mechanics and mechanical devices seemed to have contributed most forcibly towards the local reputation of Archimedes, he himself seems to have delighted mostly in those relating to pure mathematics, as may be seen from the fact that he desired to have engraved on his tombstone the figure of a sphere inscribed in a cylinder, together with an inscription giving the ratio which the surface and the volume of the cylinder bear to the corresponding elements of the sphere."

—G. A. MILLER.

THE HISTORY OF THE NUMBER PI

DAVID A. LAWSON, '39
University of New Mexico

The easiest and probably the earliest area computed by man was the area of the square. So it was only natural that attempts to find the area of a circle, which presented a far more difficult problem, gave rise to the idea of considering a square of equal area. This idea developed into one of the classical problems of geometry, "the squaring of the circle." It is found that much of the early history of the number represented by π is connected with this problem.

The Egyptians had a method for finding the area of a circle by comparing it with a square. The rule as presented in the *Rhind Papyrus* assumed that the area of a circle is equivalent to that of a square whose side is eight-ninths of the diameter of the circle. This amounts to stating that the area of a circle of radius R is $(256/81)R^2$. Upon comparison with the true area, πR^2, it may be seen that the ancient Egyptian rule is equivalent to having $\pi = 256/81 = 3.16049+$. This is not a very close approximation to the true value of π, yet it is closer than other approximations obtained before the time of the Greek mathematicians.

While the value of π was obtained by the Egyptians as a result of their attempts to "square the circle," the Babylonians, on the other hand, were interested in the rectification of the circumference; that is, in finding directly the relationship between the radius and the circumference. The Babylonians reached the conclusion that the circumference of a circle is equal to a line which is "a little more than six times the radius."

The Hebrews considered the circumference of a circle as equal to three times the diameter. This may be seen in at least two places in the Old Testament, 2 Chronicles 4:2, and 1 Kings 7:23. The first of these two verses is as follows: "Also he made a molten sea of ten cubits from brim to brim, round in compass, and five cubits the height thereof; and a line of thirty cubits did compass it about."

The problem of squaring the circle was a problem which the Greeks took up with zest the moment they realized its difficulty. Although many Greek mathematicians and non-mathematicians became interested, the contributions of several men stand out. First, Antiphon of Athens must be mentioned. "Antiphon inscribed within a circle some one of the regular inscribed polygons which can be inscribed. On each side of the inscribed polygon as a base he described an isosceles triangle with its vertex on the arc of the segment of the circle subtended by the side. This gave him a regular inscribed polygon with double the number of sides. Repeating the construction with the new polygon, he had an inscribed polygon with four times as many sides as the original polygon. Continuing the process, Antiphon thought that in this way the area of the circle would be used up, and he would some time have a polygon inscribed in the circle the sides of which would, owing to their smallness, coincide with the circumference of the circle" [4, p. 222]*. Antiphon assumed that he could make a square equal in area to any regular polygon, an impossible assumption. Otherwise, his method is still an approximation.

Antiphon started the "ball rolling" in the right direction and soon afterward Bryson of Heraclea gave it another push. His method was similar to that of Antiphon but with the addition of a circumscribed polygon. Bryson was the first to introduce into mathematics "the concept of upper and lower limits in approximations" [8, p. 125], comparing a circle with its regular inscribed and circumscribed polygons. By using a modification of Bryson's method, Archimedes was later able to calculate his approximation to π.

It is interesting to note that Euclid, in his Elements, made no effort to find the area of a circle or to calculate the ratio of the circumference to the radius.

Following Euclid there lived "the greatest mathemati-

*Numbers in brackets refer to the literature cited at the end of this paper.

cian of antiquity" [6, p. 166], Archimedes. In using the method originated by Bryson, Archimedes made one important change; he considered the perimeters of the polygons and the radius of the circle rather than the area. This method for finding the limits between which π must lie was practically the only one used for about two thousand years preceding the invention of the differential calculus. Archimedes found the value of π to lie in the range

$$22/7 > \pi > 223/71.$$

Later he made even a better approximation, his figures giving

$$195882/62351 > \pi > 211872/67441,$$

or

$$3.1416016 > \pi > 3.1415904.$$

The arithmetic mean between these two limits gives the close approximation 3.141596.

Archimedes' calculations were most remarkable considering the "unwearied perseverance" [8, p. 127] he must have employed to get such results by using the crude system of Greek notation. Anyone who is familiar with the Greek system of numbers will agree that even the calculation of π to 707 decimal places is much less a wonder than Archimedes' results, correct to only four decimal places.

One other result obtained in the Grecian school might be mentioned. The astronomer, Ptolemy, who lived in Alexandria about 150 A. D., expressed π as the sexagesimal fraction $3 + 8/60 + 30/3600$, or $3.14166\cdots$.

The Romans added nothing to the work of the Greeks. Instead, they seemed to have lost much of the exactness which Archimedes had contributed. Even though the Romans seemed to realize that 3 1/7 was closer to the true value of π than 3 1/8, they frequently employed the latter fraction because it was "more convenient" [1, p. 351].

A Roman treatise on surveying contains the following instructions for squaring the circle: "Divide the circumference of a circle into four parts and make one part the side of a square; this square will be equal in area to the circle" [8, p. 128]. Although this is actually an impossible construction, if the construction were possible, π is found to be equal to 4. This is more inexact than any other known computation for the number π.

For the thousand years following the decline of the Greeks, the center of mathematical activity shifted eastward. The Hindus, in particular, were very active during this period. Their mathematicians carried the method of Archimedes far enough to get an answer closer to the true value than either Archimedes or Ptolemy. In spite of the fact that Aryabhatta (about 500 A. D.) calculated π correctly to at least four decimal places, the great Hindu mathematician Brahmagupta gave the value $\sqrt{10}$ which equals 3.16228. Unfortunately, it was this latter value for π which spread to Europe and was used quite extensively during the middle ages.

The Chinese mathematician Tsu Ch'ungchih (fifth century A. D.) should not be overlooked. Probably by using the method of Archimedes, he found that the true value of π lies between 3.1415926 and 3.1415927.

The Arabians must be remembered in view of the fact that they handed down the results of the Greek and Hindu mathematicians to the awakening countries of Europe. In this way many of these results were probably preserved. The Arab scholar, Muhammed Ibn Musa Alchwarizmi, who brought the principles of our present system of numerical notation from India and introduced it to the Mohammedan world, brought together the various Greek and Hindu approximations for the number π.

Going back to Europe, it is found that little was done in mathematics during the Dark Ages. The value of π was calculated on more than one occasion, but all these results were less accurate than those of the Greeks and Hindus. For example, Michael Psellus was a scholar who lived in the latter part of the eleventh century. Although his contemporaries called him "first of philosophers," what survives of his mathematical work is very inaccurate. "In a book purporting to be by Psellus on the four mathematical sciences, arithmetic, music, geometry, and astronomy, the author's favorite method to find the area of a cirlce is given. The area was taken as the geometric mean between the inscribed and circumscribed squares; this gives a value to π equal to the square root of 8, or 2.8284271" [5, p. 545]. "The greatest mathematical genius of the middle ages" [9, p. 395], Leonardo of Pisa (thirteenth century), was able to get a little closer in his calculations; he gave π equal to the value 3.1418.

During the fifteenth century the sciences began to revive. Greater interest was shown in mathematics, and especially, at first, in the quadrature of the circle. This interest was, to a large extent, aroused by Cardinal Nicolas de Cusa who claimed to have discovered a method for squaring the circle. None doubted that the cardinal had solved this famous problem until his construction was proved false by Regiomontanus.

For the next couple of hundred years the circle-squarers as well as the calculators were very active. But, during this period, the reputable mathematicians began to realize that the ancient problem of quadrature was an impossibility. They wasted little time upon it except to show that the results of the various circle-squarers were incorrect. Of course, these demonstrations had little effect on the circle-squarers. "In the future as in the past, there will be people who know nothing of this demonstration and will not care to know anything, and who believe that they cannot help succeeding in a matter in which others have failed, and that just they have been appointed by Providence to solve the famous puzzle" [8, p. 116].

A few years after the revival of interest in mathematics, or about 1500 A. D., mathematicians began to extend the value of π to more places of decimals. Adrian Metius published his value of π correct to six decimal places, and Vieta, in 1579, calculated the value correct to nine decimal places. In 1593, Adrian Romanus determined π to 15 places, but in order to do so he had to calculate the perimeter of an inscribed regular polygon of 2^{30} sides, where

$$2^{39} = 1,073,741,824.$$

All these results were outdone by Ludolf Van Ceulen who carried Archimedes' method to a calculation of π correct to 35 decimal places. He was so proud of his work that he requested in his will that his results be engraved upon his tombstone. Herman Schubert tells us that in honor of Ludolf π is called today in Germany the Ludolfian number. The history of Archimedes' method of calculating π was closed in 1630 when Grienberger, the last to employ the method, announced his result correct to 39 places of decimals.

A new period in the solution of our problem began in the second half of the seventeenth century with the development of the calculus. New analysis came to the aid of the investigators, and the method of Archimedes became obsolete. The new methods attempted to express π analytically by developing it as an infinite product or series. The first important new result was produced by John Wallis (1616-1703) who proved the two relationships,

$$\pi/2 = 2/1 \cdot 2/3 \cdot 4/3 \cdot 4/5 \cdot 6/5 \cdot 6/7 \cdot 8/7 \cdot 8/9 \cdots,$$

and

$$\frac{4}{\pi} = 1 + \cfrac{1}{2 + \cfrac{9}{2 + \cfrac{25}{2 + \cfrac{49}{2 + \cfrac{81}{2 + \cdots}}}}}.$$

The continued fraction form had previously been expressed without proof by Lord Brouncker (1620-84).

The first infinite series developed for the study of the circle was the series,

$$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + \cdots.$$

Although others knew it previously, this series was published by Leibnitz and bears his name. The Leibnitz series converges so slowly as to be inconvenient in practice. It is the series obtained from the expansion of arctangent x,

$$\arctan x = x - x^3/3 + x^5/5 - x^7/7 + \cdots,$$

when x is set equal to 1.

If x is taken equal to $\sqrt{1/3}$, the arctangent series becomes

$$\pi/6 = \sqrt{1/3} \cdot (1 - 1/3 \cdot 3 + 1/3^2 \cdot 5 - 1/3^3 \cdot 7 + 1/3^4 \cdot 9 - 1/3^5 \cdot 11 + \ldots),$$

a series which converges much more rapidly. This general series was discovered by James Gregory independently of Leibnitz. The series is frequently called Gregory's series.

By using various infinite series, the following men extended the value of π to more and more decimal places during the next two hundred years [1, pp. 356-7]:

Abraham Sharp, in 1699, to 71 correct decimal places;
Machin, about 1706, to 100 correct decimal places;
De Lagney, in 1719, to 122 correct decimal places;
Vega, in 1789, to 126 correct decimal places;
Vega, in 1794, to 136 correct decimal places;
Rutherford, in 1841, to 152 correct decimal places;
Dase, in 1844, to 200 correct decimal places;
Clausen, in 1847, to 248 correct decimal places;
Rutherford, in 1853, to 440 correct decimal places;
William Shanks, in 1873, to 707 decimal places.

What about the circle-squarers while all this was going on? Of course they were as busy as ever. But, at the same time, various mathematicians were trying to prove that the quadrature of the circle is an impossibility. The first step was made in 1761 by the French mathematician Lambert who proved that π is not a rational number. In 1794, Legendre showed that π cannot be the root of a quadratic equation with rational coefficients. "This definitely disposed of the question of squaring the circle, without, of course, dampening in the least the ardor of the circle squarers" [3, p. 117].

The intimate connection between the number e and π had been well known for some years; so when, in 1873, Hermite proved that e was transcendental, the efforts were redoubled to prove π was also a transcendental number. Nine years later, Professor Lindeman of Freiburg, Germany, was successful in proving this fact.

We are so accustomed to the use of the symbol π to express the ratio of the circumference of a circle to the diameter that we are in danger of overlooking the fact that the use of the symbol π is quite recent. It was apparently used in this connection by William Jones in 1706. But it was Euler, "the most prolific mathematical writer who ever lived" [6, p. 168], who made this symbol popular by using it consistently after 1737.

The number π has properties of which many of us are unaware. This is especially true in the field of probability. An interesting experiment was conducted by Professor Wolff of Zurich some years ago. "The floor of a room was divided up into equal squares, so as to resemble a huge chessboard, and a needle exactly equal to the side of these squares was cast haphazardly upon the floor. If we calculate, now, the probabilities of the needle so falling as to lie wholly within one of the squares, that is, so that it does not cross any of the parallel lines forming the squares, the result of the calculation for this probability will be found to be exactly equal to $\pi - 3$. Consequently, a sufficient number of casts of the needle according to the law of large numbers must give the value of π approximately. As a matter of fact, Professor Wolff, after 10,000 trials, obtained the value of π correct to 3 decimal places" [8, p. 140].

There have been other methods of this type employed to calculate π. For example, if two numbers are written down at random, it has been found that the probability that they will be prime to each other is $6/\pi^2$. "Thus, in one case where each of 50 students wrote down 5 pairs of numbers at random, 154 of the pairs were found to consist of numbers prime to each other. This gives $6/\pi^2 = 154/250$, from which we get $\pi = 3.12$" [1, p. 359].

Let us consider the question of benefits which might be derived from calculating the value of π to a large number of decimal places. Such calculations show the power of modern methods compared with some of the older ones. But, for practical use, the general opinion seems to be that there is no need to have the value of π to more than 10 or 15 decimal places. Measurements are seldom correct to as many as 10 decimal places, and if π is used to many more places, the result would have fictitious accuracy.

In 1899, Hermann Schubert gave an example "to show that the calculation of π to 100 or 500 decimal places is wholly useless. Imagine a circle to be described with Berlin as centre, and the circumference to pass through Hamburg; then let the circumference of the circle be computed by multiplying its diameter by the value of π to 15 decimal places, and then conceive it to be actually measured. The deviation from the true length in so large a circle as this even could not be as great as the 18 millionth part of a millimetre" [8, p. 398]. Some years ago the late Professor Newcomb remarked. "Ten decimals are sufficient to give the circumference of the earth to the fraction of an inch, and thirty decimals would give the circumference of the whole visible universe to a quantity imperceptible with the most powerful microscope" [9, p. 398].

Curiously, attempts have been made to "fix" the value of π by law. Typical of these attempts was the bill presented to the legislature of Indiana in 1897. The bill was suggested by a local circle-squarer who said that "the present rule in computing the circle's area is entirely wrong." The bill was introduced as "A bill for an act introducing a new mathematical truth and offered as a contribution to education to be used only by the state of Indiana free of cost by paying any royalties whatever on the same, provided it is accepted and adopted by the official action of the legislature of 1897." The bill was considered by the Committee on Education which recommended that it "do pass." The bill passed the house but was lost in the state senate.

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"A natural science is a science only in so far as it is mathematical."

—KANT.

FINITE DIFFERENCES

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This paper will deal with first and second differences in so far as they are of use in the discovery of empirical formulas.

First, a word about finite differences in general. Let u_x be a rational integral function of x and let x be given values $1, 2, 3, \cdots, n$ such that a series is formed: $u_1, u_2, u_3, \cdots, u_n$. Then subtract each term in this series from the term which immediately follows it, and a new series is formed: $u_2 - u_1, u_3 - u_2, u_4 - u_3, \cdots, u_n - u_{n-1}$. The series thus found is called the series of the first order of differences. By subtracting each term of this series from the term that immediately follows it, the series of the second order of differences is found. In like manner, the series of the third, fourth, $\cdots$, n^{th} orders of differences may be found. First differences are denoted by prefixing Δ to the expression of the function of x; i. e., Δu_x. Second differences are denoted by $\Delta^2 u_x$, third differences by $\Delta^3 u_x$, and so on.

If we suppose $u_x = x^3$, the successive values of u_x with their successive differences of the first, second, and third orders may be represented in the following manner:

x	u_x	Δu_x	$\Delta^2 u_x$	$\Delta^3 u_x$
1	1	7	12	6
2	8	19	18	6
3	27	37	24	6
4	64	61	30	
5	125	91		
6	216			

It will be noted that the third differences are constant. Generally, if u_x be a rational and integral function of x of the nth degree, its nth differences will be constant. [1, p. 51][1]. Inspection of similar tabulations for $u_x = x^2$ and $u_x = x^4$ will bear this out.

1 Numbers in brackets refer to the literature cited at the end of this paper.

Next, a word about empirical formulas. Suppose that n pairs of values of the variables x and y are known as in tables I and II [2, pp. 248, 252] and it is desired to express y in

TABLE I

x	5	10	15	20	25	30	35	40
y	31.0	28.5	26.1	23.5	20.8	18.3	15.8	13.4

TABLE II

x	0	5	10	15	20	25	30
y	15.1	20.0	24.4	28.4	32.0	35.2	38.1

the form $y = f(x)$. The problems stated are clearly insolvable, for while substitution of the values of x and y in an equation involving n arbitrary constants will yield n simultaneous equations which may be solved for the unknown constants, it cannot be assumed that any pair of values other than those given would satisfy the resulting equation. However, the given values of x and y usually represent measurements of some kind and are themselves not exact. It then remains to find a simple formula *nearly* satisfied by the measured values; i. e., one in which the calculated values of the dependent variable differ from the measured or observed values by less than the errors due to unavoidable imperfections in measurements. Such a formula is an empirical formula.

In finding an empirical formula, the work falls into three parts: (1) choosing the type of equation; (2) finding the best values of the arbitrary constants; (3) calculating the deviations between the observed values of the dependent variable and those given by the formula. This paper will be concerned with the first two parts, even though finite differences enter only into the first part.

The choice of the type of equation is sometimes dictated by theoretical considerations. If there is no theoretical reason for preferring a particular type, the choice is a matter of intelligent guesswork and experiment. Ordinarily, the first thing to do would be to plot the data and draw a graph. Its shape will often indicate the proper equation. However, by the use of first or second differences, the determination of the type of equation may possibly be made without plotting the data.

The case of the linear equation, $y = b + mx$, will first be considered. In general, if y is a linear function of x and therefore changes at a constant rate, it is clear that for equal values of Δx (the change in x), the corresponding values of Δy (the change in y) must be equal, and conversely. To illustrate, let $b = 2$ and $m = 3$ in the linear equation $y = b + mx$. Now give to x values such that Δx is a constant; in this illustration, $\Delta x = 2$. Then the following table of differences results:

x	Δx	$y = 2 + 3x$	Δy
1	2	5	6
3	2	11	6
5	2	17	6
7	2	23	6
9	2	29	6
.	.	.	.
.	.	.	.
.	.	.	.

The fact that the first differences of a linear function are constant affords a method for deciding upon the availability of a linear formula without plotting the data. For, if the values of the independent variable are given at equal intervals and the first differences for the dependent variable are approximately constant, a linear relation may be assumed.

Considering the data in Table I again, it is seen that the values of the independent variable are given at equal intervals. It is found that the first differences of the dependent variable are approximately constant:

x	y	Δy
5	31.0	–2.5
10	28.5	–2.4
15	26.1	–2.6
20	23.5	–2.7
25	20.8	–2.5
30	18.3	–2.5
35	15.8	–2.4
40	13.4	

Therefore, it is reasonable to assume a linear relation of the form $y = b + mx$.

We have now to determine the proper values of the arbitrary constants b and m (part 2 of the procedure). This may be done by the method of averages:

$$\begin{array}{rl} 31.0 = & b + 5m \\ 28.5 = & b + 10m \\ 26.1 = & b + 15m \\ 23.5 = & b + 20m \\ \hline 109.1 = & 4b + 50m \end{array} \qquad \begin{array}{rl} 20.8 = & b + 25m \\ 18.3 = & b + 30m \\ 15.8 = & b + 35m \\ 13.4 = & b + 40m \\ \hline 68.3 = & 4b + 130m \end{array}$$

Solving the resulting equations simultaneously, it is found that $m = -0.51$ and $b = 33.65$. Therefore, the linear formula approximately representing the dependent variable y as a function of the independent variable x is

$$y = 33.65 - 0.51x.$$

The case of the parabola $y = a + bx + cx^2$ will now be considered. It may be shown that the following theorem applies to this case: If $y = a + bx + cx^2$, and Δx is a constant, then Δy is a linear function of x. Proof: Let $\Delta x = h$. Then for any value of x, $y = a + bx + cx^2$ and

$$\begin{aligned} y + \Delta y &= a + b(x + h) + c(x + h)^2 \\ &= (a + bx + cx^2) + bh + 2chx + ch^2. \end{aligned}$$

Hence, subtracting, $\Delta y = bh + 2chx + ch^2$, which is a linear function of x. Conversely, if Δy is a linear function of x and Δx is a constant, then $y = a + bx + cx^2$. Hence, if it can be found from pairs of values of the variables x and y that Δy is linear and Δx is a constant, it will be known that the formula $y = a + bx + cx^2$ fits the data. From the previous case considered, it is known that a linear equation may be assumed when the first differences for the dependent variable are constant. Therefore, if the first differences of the set of values of Δy (i. e., the second differences of y, or $\Delta^2 y$) are constant, Δy may be assumed to be linear. The corollary of the above theorem then follows: If $y = a + bx + cx^2$ and Δx is constant, then $\Delta^2 y$ is constant, and conversely.

From these results we conclude that if the values of the independent variable are given at equal intervals and the second differences for the dependent variable are approximately equal, the proper formula is of the type

$$y = a + bx + cx^2.$$

Now, considering the data in table II again, it is seen that the values of the independent variable are given at equal intervals. It is found that the second differences are approximately constant:

x	y	Δy	$\Delta^2 y$
0	15.1	4.9	–0.5
5	20.0	4.4	–0.4
10	24.4	4.0	–0.4
15	28.4	3.6	–0.4
20	32.0	3.2	–0.3
25	35.2	2.9	
30	38.1		

Therefore it is reasonable to assume a relation of the form $y = a + bx + cx^2$.

We have now to determine the proper values of the arbitrary constants a, b, and c. This may be done by the method of averages:

$$\begin{array}{rl} 15.1 = & a \\ 20.0 = & a + 5b + 25c \\ 24.4 = & a + 10b + 100c \\ \hline 59.5 = & 3a + 15b + 125c \end{array} \qquad \begin{array}{rl} 28.4 = & a + 15b + 225c \\ 32.0 = & a + 20b + 400c \\ \hline 60.4 = & 2a + 35b + 625c \end{array}$$

$$\begin{array}{rl} 35.2 = & a + 25b + 625c \\ 38.1 = & a + 30b + 900c \\ \hline 73.3 = & 2a + 55b + 1525c \end{array}$$

Solving the resulting equations simultaneously, it is found that $a = 15.2$, $b = 1.00$, and $c = -0.0079$. Therefore, the formula of the type $y = a + bx + cx^2$ approximately representing the dependent variable y as a function of the independent variable x is $y = 15.2 + 1.00x - 0.0079x^2$.

One practical example will now be considered. The following table gives the time of flight (t seconds) of a certain projectile for various ranges (R thousand yards). Express t as a function of R. [2, p. 254].

R	t	Δt	$\Delta^2 t$
1	2.1	2.6	0.5
2	4.7	3.1	0.3
3	7.8	3.4	0.5
4	11.2	3.9	0.4
5	15.1	4.3	
6	19.4		

Since values of the independent variable R are given at equal intervals, we immediately take first differences of the dependent variable t. Since these are not constant, we assume that the formula we are seeking is not linear. Taking second differences, we note that they are approximately constant. We therefore assume the formula to be of the type $t = a + bR + cR^2$. We now determine the proper values of the arbitrary constants a, b, and c by the method of averages.

$$\begin{array}{l} 2.1 = a + b + c \\ 4.7 = a + 2b + 4c \\ \hline 6.8 = 2a + 3b + 5c \end{array} \qquad \begin{array}{l} 7.8 = a + 3b + 9c \\ 11.2 = a + 4b + 16c \\ \hline 19.0 = 2a + 7b + 25c \end{array}$$

$$\begin{array}{l} 15.1 = a + 5b + 25c \\ 19.4 = a + 6b + 36c \\ \hline 34.5 = 2a + 11b + 61c \end{array}$$

Solving these equations simultaneously, it is found that $a = -0.14$, $b = 2.02$, and $c = 0.206$. Therefore, t expressed as a function of R is $t = -0.14 + 2.02R + 0.206R^2$.

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BRIEF HISTORY OF GENERAL SOLUTIONS OF ALGEBRAIC EQUATIONS

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This paper results from the desire of the writer to see in logical, summarized form the steps and the modes of thought used by mathematicians to generalize the solution of algebraic equations. Two problems are involved in the solution of algebraic equations. In pure mathematics, the most important consideration is the setting up of all functions of the literal coefficients which reduce the equation to an identity, these expressions being obtained by a finite number of rational operations and root-extractions. In applied mathematics, more stress is laid on the process of computing roots from the numerical coefficients. It is the purpose of this paper to trace the progress which has been made in the treatment of the first of these problems.

The formation of linear equations involving numerical coefficients dates back to about 1700 B. C. Word problems using for the unknown quantity the term "heap" are found on the Ahmes papyrus, an Egyptian work of this period. More frequent evidence of linear equations is found in subsequent centuries, but in every instance the ancients sought merely the numerical values of the unknowns in a particular system of equations. Although Aryabhata, a Hindu mathematician of 600 A. D., solved a literal linear equation in one unknown, it was not until the 17th century when negative and complex roots were accepted as solutions that the solving of linear equations was generalized. All linear equations in one unknown are reducible to the form, $ax = b$, in which a and b can be any constants except a cannot be zero. To solve this equation, all that is required is to divide each member by a, thereby yielding the general solution, $x = b/a$.

The quadratic equation opened two great avenues of thought; namely, the question of the number of roots in an equation and the consideration of irrational and complex roots. Diophantus accepted only one positive rational root of a quadratic equation, ignoring the negative and complex roots. Bhaskara, a Hindu mathematician of the 12th century, observed that some quadratic equations have two roots but, since he did not use complex numbers, he could find no roots for certain other quadratics. Factorization is one of the simplest methods of solving a quadratic equation, but it is a specialized (not general) method in that it is practicable only when f(x) is factorable into rational factors. Modern textbooks give the completion of the square as a distinct method of solution of quadratic equations; in the case of the general quadratic $ax^2 + bx + c = 0$, it leads to the standard quadratic formula, $x = (-b \pm \sqrt{b^2 - 4ac})/2a$.

The solution of the cubic equation has a richer historical background than either the linear or the quadratic equation. A cubic equation which is "reducible" can be factored into a linear and a quadratic factor or into three linear factors. Hence, specific reducible cubics were solved long before the discovery of the general solution whose authorship was claimed by Tartaglia. In the later 17th century, Wang Hs'iao-t'ung gave several problems dealing with cubics which he solved by a process similar to that of extracting cube roots. The Arabian poet, Omar Khayyam, about the year 1100 divided cubics into two classes and these he subdivided into species. Then by the method of intersecting conics he was able to formulate general plans for the solution of each species. All attempts to solve the general cubic failed until the 16th century when several men were successful. Scipio del Ferro found a method of solving equations of the type $x^3 + bx + c = 0$. Cardan published this solution in his *Ars Magna* in 1545, and it is therefore known as Cardan's solution. The general cubic $x^3 + bx^2 + cx + d = 0$ is transformed into the standard form $x^3 + qx + r = 0$. By means of a suitable substitution, this reduced cubic is changed to a sextic in y containing the terms to the sixth power, the third power, and the constant. This equation can be solved as a quadratic for y^3. The cube root is extracted from the two values of y^3 thus obtained. It can be shown by proper combination of these six roots of the sextic that the desired roots of the given cubic are

$$x_1 = y_1 + y_2, \quad x_2 = y_1\omega + y_2\omega^2,$$

and

$$x_3 = y_1\omega^2 + y_2\omega,$$

where ω and ω^2 are two of the cube roots of unity.

Since the time of Cardan a large number of general solutions of the cubic have been advanced. In 1591, Vieta developed an elegant and much briefer algebraic solution. In very recent times graphic methods have been presented which even take care of the complex roots.* However, unless the roots are rational, graphic solutions can never be more than fair approximations. Undoubtedly, the most important solution since the time of Cardan was Vieta's trigonometric solution which is used in the irreducible case to eliminate the difficulty of extracting the cube roots of complex numbers. Vieta is responsible for the pronounced uniformity and generality which was introduced into the solving of equations at this time.

Once the cubic was solved for the general case, it was not such a difficult step to the solution of the quartic since the operations which enter into its solution are similar to those required for the cubic. Ferrari, a pupil of Cardan, reduced the general quartic to the form

$$(x^2 + \tfrac{1}{2}bx)^2 = (\tfrac{1}{4}b^2 - c)x^2 - dx - e.$$

Adding $(x^2 + \frac{1}{2}bx)y + \frac{1}{4}y^2$ to each member he obtained a new expression in y,

$$(x^2 + \tfrac{1}{2}bx + \tfrac{1}{2}y)^2 =$$
$$(\tfrac{1}{4}b^2 - c + y)x^2 + (\tfrac{1}{2}by - d)x + \tfrac{1}{4}y^2 - e.$$

The condition that the right-hand member be a perfect square is that its discriminate equals zero. By setting the

*Yanosik, "Graphic Solution of Cubic," *National Mathematics Magazine*, Vol. X, pp. 139-140 and Vol. XVII, pp. 147-150.

discriminant equal to zero, a resolvent cubic in y is obtained. Solving the cubic for y and substituting, it remains only to extract the square root of both members of the bi-quadratic and solve the resulting quadratic for x. This is the most general solution of the quartic; but, about 1770, Euler published another general solution slightly different in method from Ferrari's.

For several hundred years after Ferrari solved the quartic, mathematicians tried intensively but in vain to find a general method for solving the quintic by a finite number of rational operations and root-extractions. Lagrange found in solving 2nd, 3rd, and 4th degree equations that, in every case, the solution was reducible to an equation of lower degree whose roots are linear functions of the roots of the given equation and the roots of unity. But this seemingly general method when applied to the quintic produced a sextic. Tschirnhausen, in 1683, generalized the removal of the second term from cubics and quartics by making certain rational substitutions. The Swedish mathematician, Bring, succeeded in reducing the general quintic to the form $x^5 + ax + b = 0$ by a Tschirnhausen transformation. These discoveries, although they neither solved the quintic nor proved it incapable of solution, are very important in the theory of equations because they generalize the results mathematicians had obtained earlier by means of ingenious devices which often had no apparent logical basis.

Attempts to solve the general algebraic equation of higher degree than four by means of the previously mentioned elementary algebraic operations ended in a rather vague proof by Ruffini that such solutions do not exist. The first rigorous proof of this theorem was published in 1824 by Abel, a Norwegian mathematician.

The general solution of equations of degree greater than four was effected by using elliptic modular functions, the discussion of which is beyond the scope of this paper. More recently, roots of equations of the nth degree were given in terms of the coefficients by means of Fuchsian functions.

The most general solutions of equations usually involve complicated calculations which necessarily limit their use. In specific applied problems, a specialized or approximate method is usually employed. The importance of the general solution lies in the contribution it makes to algebraic theory. Mathematics as a branch of knowledge enlarges and extends the field of thought in so far as it can make and prove such generalizations.

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"Mathematics is the glory of the human mind."

—LEIBNITZ.

BOOKS FOR CHAPTER PROGRAMS

The arrangement of interesting and varied programs is frequently a perplexing problem to chapter officers. A bibliography of books on "popular" mathematics should aid in solving this problem. Accordingly, we have compiled a list of books which contain material suitable for chapter programs. This list is not intended to be complete. Prices are given for the convenience of chapters who may wish to expand their libraries. Although these prices are not guaranteed, they should be useful in comparing relative costs. Some of the older books are now out of print and are available only in used condition.

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THE MATHEMATICAL SCRAPBOOK

To business that we love we rise betime,
And go to't with delight.

—SHAKESPEARE.

=∇=

In a game of "Heads or Tails," a gambler bets half of the money in his possession on each toss of the coin. If he wins on exactly half of a series of tosses, does he gain or lose?

=∇=

$$\begin{array}{r}
1 + 2 = 3 \\
4 + 5 + 6 = 7 + 8 \\
9 + 10 + 11 + 12 = 13 + 14 + 15 \\
16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24, \text{ etc.}
\end{array}$$

Note that the middle term is the product of the number of terms on each side.

—SCRIPTA.

=∇=

A and B are playing a game at a round table using an unlimited number of flat discs of equal radii. Each player, in turn, places a disc flat against the top of the table. After a disc is once placed on the table, its position can not be altered. The winner is the player who places a disc in the last available space on the table. In order to win, should A move first or second, and what should be his method of play?

=∇=

Choose any number except 0 or 1.

a) subtract from 1;
b) find the reciprocal;
c) subtract from 1;
d) find the reciprocal;
e) subtract from 1;
f) find the reciprocal.

The answer is the original number.

—SCH. SCI. & MATH.

$$i^i = 0.207879576351.$$

=∇=

"One per cent inspiration, 99 per cent perspiration."

—EDISON.

=∇=

A grocer attempts to weigh out identical amounts of sugar to two customers, but his scales are false. The first time he puts the weight in one pan and the sugar in the other, the second time he reverses the procedure. Does he gain or lose?

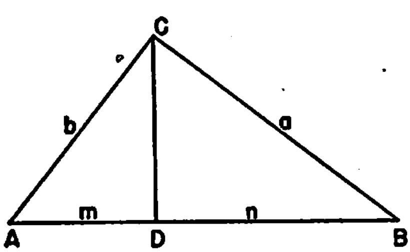
—KRAITCHICK, MATHEMATICAL RECREATIONS.

=∇=

"It is the perennial youthfulness of mathematics itself which marks it off with a disconcerting immortality from the sciences."

—E. T. BELL.

=∇=



Over 90 proofs of the Pythagorean theorem have been published. The following is Legendre's proof. In the figure, triangles ABC, ACD, and BCD are similar. Hence, $n/a = a/c$ and $m/b = b/c$, so that $a^2 = nc$ and $b^2 = mc$. Therefore, $a^2 + b^2 = nc + mc = c(n + m) = c^2$.

=∇=

A box with one small opening is resting on some scales. A bird weighing 5 ounces flies into the box and continues to fly round and round inside the box. Is the weight of the box increased or decreased?

=∇=

$$365 = 10^2 + 11^2 + 12^2 = 13^2 + 14^2.$$

An alert copyreader on the *Illinois State Journal* couldn't believe it—the reporter's story of the theft of 2025 pigs.

"That's a lot of pigs," he growled, and called the farmer to check the copy.

"Was it 2025 pigs that were stolen?"

The victimized farmer replied, "Yeth."

"Thanks," said the copyreader, and corrected the copy to read two sows and 25 pigs.

—NEWSPAPER ITEM (INS.).

=∇=

Can you form four equilateral triangles with six matches?

=∇=

Wisely and slow; they stumble that run fast.

—SHAKESPEARE.

=∇=

The identity $a^2 = (a + b)(a - b) + b^2$ is useful in squaring numbers mentally. Thus,

$$78^2 = (78 + 2)(78 - 2) + 2^2 = (80)(76) + 4 = 6084.$$

Similarly,

$$83^2 = (83 - 3)(83 + 3) + 3^2 = (80)(86) + 9 = 6889.$$

=∇=

3.1415926536

And I wish I could recollect
My number, known and select.

=∇=

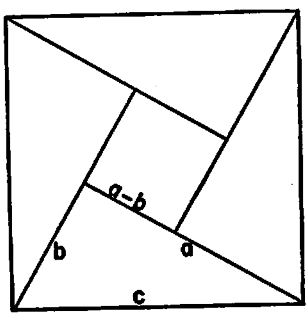
"The proof . . . by *reductio ad absurdum,* which Euclid loved so much, is one of the mathematician's finest weapons. It is a far better gambit than any chess gambit: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers *the game.*"

—G. H. HARDY.

Can you decode the following problem in division by assigning digits to each letter?

$$\begin{array}{r@{}l} & FCE \\ AB\,) & \overline{CADB} \\ & \underline{FE} \\ & DD \\ & \underline{CG} \\ & HB \\ & \underline{HB} \end{array}$$

=∇=



It is said that Bhaskara's proof of the Pythagorean theorem consisted of a figure (called the "bride's chair") and the single word, "behold." Note that

$$c^2 = 2ab + (a-b)^2 = a^2 + b^2.$$

=∇=

Mrs. A, Mrs. B, and Mrs. C and their three daughters each bought cloth and lace. Each of the six bought as many yards as she paid cents per yard and each daughter paid 63 cents less than her mother. Jane bought 23 yards less than Mrs. A; Elizabeth bought 11 yards less than Mrs. B; the other daughter was named Ann. Whose daughter was each girl? —MATHEMATICS TEACHER.

=∇=

One freshman's "proof" of the Remainder Theorem:

$$\begin{array}{r@{}l} & f \\ x-a\,) & \overline{f(x)} \\ & \underline{f(x)\ -f(a)} \\ & f(a) = R \end{array}$$

=∇=

A monkey hangs on one end of a rope which passes through a pulley. On the other end of the rope hangs a weight equal to that of the monkey. The monkey decides to climb the rope. What happens?

Form the statement I = VII using seven matches. Can you change this to a true statement by displacing just one match?

=∇=

Carl Friederick Gauss (1777-1855) when three years old taught himself some arithmetical processes, and astonished his father by correcting him in his calculations of certain payments for overtime.

—BALL, MATHEMATICAL RECREATIONS.

=∇=

Shanks used the formulas

$$\pi/4 = 4 \arctan (1/5) - \arctan (1/239)$$

and

$$\arctan x = x - 1/3\, x^3 + 1/5\, x^5 + \cdots,$$

to compute π to 707 decimal places. Only 35 terms suffice for 50 place accuracy.

=∇=

Write down any three consecutive numbers, the largest of which is a multiple of 3. Add. Then add the digits in the sum. Continue adding the digits in each sum until a single digit is reached. It will always be 6.

$$\begin{array}{r} 7279 \\ 7280 \\ \underline{7281} \\ 21840 \\ 15 \\ 6 \end{array}$$

=∇=

Note three arithmetic progressions in the following products:

$$\begin{array}{l} 1 \times 91 = 091 \\ 2 \times 91 = 182 \\ 3 \times 91 = 273 \\ 4 \times 91 = 364 \\ 5 \times 91 = 455 \\ 6 \times 91 = 546 \\ 7 \times 91 = 637 \\ 8 \times 91 = 728 \\ 9 \times 91 = 819 \end{array}$$

=∇=

Solve: $1/(x+1) - 2/(x+2) + 1/(x+3) = 0.$

"One merit of mathematics few will deny; it says more and in fewer words than any other science. The formula, $e^{\pi i} = -1$, expresses a world of thought, of truth, of poetry, and of religious spirit for 'God eternally geometrizes.'"

—DAVID EUGENE SMITH.

$=\nabla=$

Three gentlemen, A, B, and C, engage in a conversation; from it we are to decide whether each is a Noble or a Hunter. A Noble always tells the truth, whereas a Hunter always lies. A begins the conversation and says either "I am a Noble" or "I am a Hunter," but we don't know which. B says to A, "You said you were a Hunter." B says to C, "You are a Noble." C says to A, "You are a Noble."

$=\nabla=$

"No mathematician should ever allow himself to forget that mathematics, more than any other art or science is a young man's game." —G. H. HARDY.

$=\nabla=$

Let $x =$ your money, $y =$ Uncle Sam's money, and $x + y = z$. Then, multiplying by $x - y$, $x^2 - y^2 = zx - zy$, or $x^2 - zy = y^2 - zy$. Then $x^2 - zy + \frac{1}{4}z^2 = y^2 - zy + \frac{1}{4}z^2$ or $(x - \frac{1}{2}z)^2 = (y - \frac{1}{2}z)^2$. Thus, $x - \frac{1}{2}z = y - \frac{1}{2}z$, so that $x = y$ and you have as much money as Uncle Sam!

$=\nabla=$

Many interesting and important situations arise in arithmetic which depend upon a process known as "casting out nines." By the "excess of nines" in a given number is meant the remainder that is obtained when that number is divided by nine. The following theorem is easily established: The excess of the nines in a number is equal to the excess of nines in the sum of its digits. The latter excess can be found rather quickly by mental arithmetic. The process may be speeded up in various ways; in particular, it should be noted that all the nines can be discarded while the digits are being added just as well as to wait and discard them after the sum has been obtained.

A second theorem is useful in conjunction with the first theorem: If the order of the digits of a number are altered in any way to form a new number, then the difference between the old number and the new number is divisible by nine.

The two principles just stated are the basis for the "bookkeeper's check." In double-entry bookkeeping, for each debit entry there is always an equal credit entry, and vice versa. Hence, at all times the books must "balance"; i. e., the sum of the debits must equal the sum of the credits. Now perhaps the most frequent error made in arithmetic is that of transposing the digits of a number. In bookkeeping this error occurs quite frequently; a debit is entered correctly as $237.86, for example, whereas the credit might be entered as $238.67. Such an error is commonly called a "slide." In the event that his books fail to balance, the bookkeeper obtains the difference between the debits and credits and proceeds to cast out nines. If the excess of nines is equal to zero, he is reasonably sure that his mistake was a transposition of digits and begins the laborious task of checking each debit against the corresponding credit. If the excess of nines is not zero, he looks for his mistake in some other direction.

The two principles are also the basis of several tricks with numbers. To illustrate, take any number and form a new number by reversing its digits. Subtract the smaller number from the larger number, and multiply the difference by any number you choose. Now scratch out any non-zero digit in your product. If you read me the result, I'll tell you the digit scratched out. For I only need to cast out nines from the result you read to me and subtract the excess from 9.

=∇=

The development of arithmetic was primarily due to the practical needs of everyday life. However, mathematicians very early began to observe properties of individual numbers and attached a peculiar mysticism to these properties. This mysticism reached its peak in the philosophy of the Pythagorean school.

Pythagoras (about 550 B. C.) discovered the numerical intervals of the musical scale. In particular, he noticed that three vibrating strings whose lengths were in the ratio 6:4:3 and which were under the same tension would sound a given note, its fifth, and its octave. Pythagoras was struck by the fact that this physical law was explained by the use of integers. He inferred that integers played a similar role in other physical laws, and concluded that nature had imposed mathematical laws on the course of human events. Developing this philosophy, Pythagoras and his disciples believed that number was everything and that everything was number. Accordingly, they assigned mystic properties to numbers and sought to explain their good or evil fortune by means of these properties. Like the Hollywood numerologists of today, the Pythagoreans regarded numbers as possessing powers for good and evil. Numbers were the cause of events and any natural law was produced by the proper combination of numerical operations. When two warriors met in a battle, for example, it was firmly believed that the victor would be the one whose name had the more letters in it. This was the mystic reason given for Achilles' triumph over Hector.

Some of the mystic meanings given to numbers by the Greeks are of interest. The number one itself was not regarded as a number, but rather as the source of all numbers. The even numbers, being divisible by two, were regarded as soluble, and hence weak; they were accordingly considered as possessing a feminine attribute. The odd numbers, on the other hand, were regarded as dissoluble, hence strong or masculine. The number five was considered as representing marriage since $5 = 2 + 3$ denotes the union of the first feminine and masculine numbers.

The number eight was considered as representing death, since the sum of the digits in successive multiples of eight decrease successively by one:

$$
\begin{aligned}
&1 \times 8 = 8 \\
&2 \times 8 = 16 \text{ and } 1 + 6 = 7 \\
&3 \times 8 = 24 \text{ and } 2 + 4 = 6 \\
&4 \times 8 = 32 \text{ and } 3 + 2 = 5 \\
&5 \times 8 = 40 \text{ and } 4 + 0 = 4 \\
&6 \times 8 = 48 \text{ and } 4 + 8 = 12,\ 1 + 2 = 3 \\
&7 \times 8 = 56 \text{ and } 5 + 6 = 11,\ 1 + 1 = 2 \\
&8 \times 8 = 64 \text{ and } 6 + 4 = 10,\ 1 + 0 = 1.
\end{aligned}
$$

Similarly, the number nine was considered as representing immortality, since the sum of the digits in successive multiples of nine remain constant:

$$
\begin{aligned}
&1 \times 9 = 9 \\
&2 \times 9 = 18 \text{ and } 1 + 8 = 9 \\
&3 \times 9 = 27 \text{ and } 2 + 7 = 9 \\
&4 \times 9 = 36 \text{ and } 3 + 6 = 9 \\
&5 \times 9 = 45 \text{ and } 4 + 5 = 9 \\
&6 \times 9 = 54 \text{ and } 5 + 4 = 9 \\
&7 \times 9 = 63 \text{ and } 6 + 3 = 9 \\
&8 \times 9 = 72 \text{ and } 7 + 2 = 9 \\
&9 \times 9 = 81 \text{ and } 8 + 1 = 9.
\end{aligned}
$$

=∇=

Any number consisting of an even number of digits which reads the same from either end is divisible by 11. Likewise, any number formed by writing down an odd number of digits and then repeating them in the same order is divisible by 11.

=∇=

There are 10 black socks and 10 white socks in a dresser drawer. If you reach into the drawer in the dark, what is the minimum number of socks you must take out before you are sure of having a pair of socks that match?

=∇=

If thy wits run the wild-goose chase, I have done.

—SHAKESPEARE.

FROM PRESIDENT PYTHAGORAS

E. R. SLEIGHT

The National Council of Kappa Mu Epsilon met at Iowa State Teachers College, Cedar Falls, Iowa, on May 25th and 26th, 1944, with all members of the Council present. Professor H. Van Engen, past treasurer and head of the department of mathematics at State Teachers College, together with the other members of his department arranged for the meeting and entertained the Council in a splendid manner. It was very fortunate that Miss E. Marie Hove, our capable and efficient secretary, was teaching in this institution at this time.

Many changes in the national constitution as well as methods of procedure were discussed at this meeting. All chapters were advised of these proposals, and all have been adopted. The Council wishes to express its appreciation for your implied confidence and your coöperation. We confidently expect that these changes will result in more satisfactory and efficient working ideals for the fraternity.

We realize that these are strenuous times, and any apparent lack of interest in suggestions by members of the Council quite likely is caused by lack of time. Also with many of us our great interests are elsewhere. Nevertheless, we had hoped for greater participation in the fraternity-song contest, as well as more contributions from chapters of papers representing some degree of undergraduate research. Kappa Mu Epsilon should encourage a high degree of excellence in the papers presented, many of them being worthy of consideration for a place in the PENTAGON. Some chapters require that a copy of all contributions to club programs be placed on file. This tends to increase the interest of the individual and improves the program. This is merely a suggestion, but might be worth introducing into your program if you already have not done so. In regard to the song contest, the Council plans to continue its offer until we have secured something that will add dignity and interest to our meetings and initiation ceremonies.

Just recently arrangements were made with the L. G. Balfour Company to furnish all fraternity pins and keys. By this change two objectives have been attained—immediate service and a reduction in price. The Council is now considering the matter of a fraternity crest. We expect to have something to report later on this subject.

We shall be unable to have our national convention this spring, but the Council plans to hold a meeting in May or early June. We shall be very glad for suggestions from any individual chapter, and assure you that all such will receive our consideration. It is our desire to serve the best interests of Kappa Mu Epsilon, and to that purpose, we pledge you our best efforts.

FROM SECRETARY DIOPHANTUS

E. MARIE HOVE

In spite of war's demands and the restrictions due to war-time conditions, Kappa Mu Epsilon goes forward. Professor Sleight, your president, has given you the highlights of the past year's achievements.

During the past year, 457 new members have been initiated into Kappa Mu Epsilon, bringing our total membership to 4,445 in the thirty chapters. Because of decreased enrollments some chapters have initiated no new members, while other chapters at schools with large navy units have been able to initiate large groups.

We realize the difficulty of carrying on without our usual biennial convention, but your fine coöperation in handling the business by correspondence is fully appreciated. What measures of success we have depend upon each and every chapter of the fraternity. With your coöperation we can steadily go forward. Every chapter and every member has an equal responsibility for the prosperity and reputation of Kappa Mu Epsilon.

REPORT OF TREASURER NEWTON

LOYAL F. OLLMANN

The last previous report was submitted to the chapters on November 1, 1944. The following is a summary of receipts and expenditures from November 1, 1944, to February 1, 1945.

Cash balance, November 1, 1944	$ 514.32
Receipts	159.05
Expenditures	37.35
Cash balance, February 1, 1944	$ 636.02
U. S. bonds on hand	1,775.00
Total balance, February 1, 1945	$2,411.02

KAPPA MU EPSILON NEWS

SISTER HELEN SULLIVAN, O. S. B.

Chapter 1. OKLAHOMA ALPHA, Northeastern State College, Tahlequah, Oklahoma.

President	Miss Thelma Jean Crane
Vice-President	Miss Lucile Maddux
Secretary-Treasurer	Miss Barbara Butler
Secretary Descartes	Miss Mary K. Stewart
Faculty Sponsor	Mr. W. S. Bishop

The general theme for this year in Oklahoma Alpha was *Lives of Great Men of Mathematics.* Mr. Elton J. Green, chapter vice-president in '43-'44, is with the Navy V-12 Unit at Tulane University. War demands have curtailed the membership considerably but regular semi-monthly meetings are held.

Chapter 2. IOWA ALPHA, Iowa State Teachers College, Cedar Falls, Iowa.

Due to lack of student personnel, Iowa Alpha has been forced to inactivity for the present year. The mathematics faculty of Iowa State Teachers College acted as hosts to the National Council of Kappa Mu Epsilon which met at Cedar Falls on May 25 and 26, 1944. The college facilities were at the disposal of the council members who had a very enjoyable time along with the regular business sessions.

Chapter 3. KANSAS ALPHA, Kansas State Teachers College, Pittsburg, Kansas.

President	Mrs. Dorothy Breiner-Bradshaw
Vice-President	Miss Dorothy Jo Bernhardt
Secretary	Miss Nancy Lee Horton
Treasurer	Miss Carolyn Ann Berry
Secretary Descartes	Professor W. H. Hill
Faculty Sponsor	Professor J. A. G. Shirk

Professor W. H. Hill retired from active duty in the department of mathematics at Kansas State Teachers College on November 1, 1944. He had been associated with the department since 1921. Since 1925 he had complete charge of the teacher-training program in secondary school mathematics and has a total of two hundred eighty-three trained teachers to his credit. Professor Hill holds membership in the National Council of Teachers of Mathematics, the Kansas Association of Teachers of Mathematics, and is a charter member of the Mathematical Association of America.

Word was received by the chapter president, Mrs. Dorothy Breiner-Bradshaw, that her husband, Cpl. Charles H. Bradshaw, was killled in action in France on December 11, 1944. Mr. and Mrs. Bradshaw were married June 9, 1944.

The following members of Kansas Alpha are engaged in teaching: Miss Helen Kriegsmann, chapter president '43-'44, at Senior High School, Altamont, Kansas; Miss Mary Lou Ralston, secretary '43-'44, at High School in Gorham, Kansas; Miss Virginia Prussing, treasurer '43-'44, at High School in Chetopa, Kansas; Miss Madge Sneller at Russell, Kansas; Mrs. Faye Wallack at Senior High School in Arkansas City, Kansas.

Mrs. Mildred Bradshaw, vice-president '43-'44, and her two young sons have joined Captain Walter Bradshaw at Little Rock, Arkansas. Captain Bradshaw has seen two and one-half years service in the Aleutians.

Besides the local chapter meetings Kansas Alpha also sponsors the college mathematics club which holds interesting meetings periodically.

Chapter 4. MISSOURI ALPHA, Southwest Missouri State Teachers College, Springfield, Missouri.

President	Miss Estaleta Dole
Vice-President	Miss Wilma Reynolds
Secretary	Miss Dotty Carter
Treasurer	Mr. William Compton
Secretary Descartes	Mr. Carl V. Fronabarger
Faculty Sponsor	Mr. L. E. Pummill

Missouri Alpha has thirteen active members and is carrying on as well as possible with this limited membership. Of last year's members the teaching profession claims Miss Wilma Tolbert, Miss Rosalie Covert, and Miss Dorothy Harrell. Mr. David Ellis is doing graduate work at the University of Missouri. Mr. William Hoff and Mr. Byron Maine are with the U. S. Navy.

Chapter 7. NEBRASKA ALPHA, Nebraska State Teachers College, Wayne, Nebraska.

President	Miss Jeanne Richards
Vice-President	Mr. Paul Petersen
Secretary	Miss Ann Mathena
Treasurer	Miss Barbara Hook
Historian	Mr. Joseph Scollard
Reporter	Mr. James Hanson
Secretary Descartes	Mr. C. H. Lindahl
Faculty Sponsor	Miss Jessie Boyce

Nebraska Alpha's membership has been curtailed due to war-time conditions but with nine active members is managing to keep the chapter alive and functioning.

Miss Barbara Hook is teaching in the High School at Coleridge, Nebraska. Miss Joyce Adams is teaching at Winside, Nebraska. The V-12 Navy Unit at Ames, Iowa, includes Mr. Harold Foecke. Miss Mary Clare Jordan is doing graduate work at the University of Iowa.

The following members of Nebraska Alpha contributed articles which were published in the *Nebraska Bulletin of the National Council of Teachers of Mathematics:* Mr. Paul Petersen, "Trisection"; Miss Ann Mathena, "Transcendental Pi."

Chapter 8. ILLINOIS ALPHA, Illinois State Normal University, Normal, Illinois.

President	Miss Edith Robinson
Vice-President	Miss Bertha Harper
Secretary	Miss Alice McCarkle
Treasurer	Miss Maxine Sponsler
Historian	Miss Mary Donnell
Social Chairman	Miss Janice Posey
Secretary Descartes	Dr. C. N. Mills
Faculty Sponsor	Miss Elinor B. Flagg

Special features of regular meetings included the following: Mathematics in the Navy Hydrographic Office; the Mathematical Interests of Thomas Jefferson; Practical Applications of Mathematics in Architecture. Because of decreased membership this year, Illinois Alpha found it necessary to abandon the usual custom of assembling and circulating the annual News-Letter to all former members. The Homecoming breakfast, the Christmas party, and the spring banquet were marked by the usual fine attendance.

Illinois Alpha is proud of her membership which totals two hundred forty-four members of whom fifty-eight are in the service of the United States. The chapter's representation in the officer's groups is as follows: one lieutenant colonel, four captains, seven lieutenants (j. g.), thirteen lieutenants (1st and 2nd), fourteen ensigns, one staff sergeant, and two WAVES.

Chapter 10. ALABAMA ALPHA, Athens College, Athens, Alabama.

President	Mr. Verner Anderson
Vice-President	Miss Florence Tilman
Secretary	Mr. Randall Freeman
Treasurer	Miss Lona Ruth Auxford
Secretary Descartes	Dr. Mary E. Renich
Faculty Sponsor	Dr. Mary E. Renich

Although Alabama Alpha must operate with but four active members, the fraternal spirit flourishes and meetings are held whenever possible. The teaching profession includes the following '43-'44 members: Richie Christopher, Oleta Garrison, Hazel Harrison, and Thelma Long. Miss Opal Whitten has a position with Tennessee Valley Association.

Chapter 11. NEW MEXICO ALPHA, University of New Mexico, Albuquerque.

President	Mr. Robert Hansen
Vice-President	Miss Virginia Schmitt
Secretary	Miss Katherine Trible
Treasurer	Mr. Anchard Zeller
Secretary Descartes	Miss Merle Mitchell
Faculty Sponsor	Miss Merle Mitchell

Dr. C. V. Newsom, former head of the mathematics department at the University of New Mexico and national president of Kappa Mu Epsilon '39-'41, is now head of the mathematics department at Oberlin College, Oberlin, Ohio. Dr. Frank Gentry, also of the mathematics staff, is engaged in research work at Massachusetts Institute of Technology. New Mexico Alpha is honored in having Miss E. Marie Hove on the mathematics faculty this semester at the University of New Mexico. Miss Hove is national secretary of Kappa Mu Epsilon.

Miss Elena Davis, former president of New Mexico Alpha, is an ensign in the WAVES and is stationed in Washington, D. C. Roy Frame and Rollin Schneider are students in the University of Mexico's engineering school. Reka Lois Black is attending Yale Divinity School. Marx Brook, Margaret Johnson, and W. C. Scrivner are engaged in war research. Miss Carol Williams is employed in the business office of the University of New Mexico. Past-president Esther Barnhart is teaching high school.

Chapter 12. ILLINOIS BETA, Eastern Illinois State Teachers College, Charleston, Illinois.

Due to the fact that Illinois Beta has but two student members there is no opportunity for the chapter to function.

At the close of the '43-'44 term a testimonial dinner was given for Dr. E. H. Taylor at the U. S. Grant Hotel in Mattoon. Out of deference to his spirit of modesty, no great display was made. The chapter members presented him with a fountain pen in recognition of his fine services and a symbol of his productiveness to come. Dr. Taylor continues to live in Charleston and comes to his office on the campus to work on the revision of his "Arithmetic for Teacher-Training Classes." Professor Taylor was head of the mathematics department at Eastern Illinois State Teachers College for forty-five years. Dr. Taylor also served as National Vice-President of Kappa Mu Epsilon '39-'41.

Chapter 13. ALABAMA BETA, Alabama State Teachers College, Florence, Alabama.

President	Miss Lilbourne Hall
Vice-President	Mr. Thomas Mitchell
Secretary-Treasurer	Miss Elizabeth Gray
Secretary Descartes	Miss Orpha Ann Culmer
Faculty Sponsor	Miss Orpha Ann Culmer

The monthly meetings of Alabama Beta are held in the homes of the members residing in Florence. Each year the chapter purchases a war bond and all activities are directed toward this goal. Miss Bernice Posey, the only girl graduate of '43-'44, is a Junior Hydraulic Engineer for the T. V. A. at Murray, Kentucky. Miss Lilbourne Hall, chapter president, and Miss Elizabeth Gray, chapter secretary-treasurer, were among those elected to membership in the '44-'45 Who's Who in American Universities and Colleges.

Chapter 15. ALABAMA GAMMA, Alabama College, Montevallo, Alabama.

President	Miss Carolyn Irwin
Vice-President	Miss Betty Perryman
Secretary	Miss Juanita Jernigan
Treasurer	Miss Annie Dillard
Secretary Descartes	Miss Rosa Lea Jackson
Faculty Sponsor	Miss Rosa Lea Jackson

The chapter held monthly meetings on various mathematical topics.

Miss Ann Cooper, chapter president for '43-'44 and now Mrs. Robert Kelley, is with her husband at Roswell, New Mexico. Miss Virginia Jernigan is teaching in Capital Heights Junior High School, Montgomery, Alabama. Miss Virginia Windham, chapter treasurer for '43-'44, is a chemist for Tennessee Valley Association, Wilson Dam. Miss Peggy Kirk, '43, is studying medicine at Tulane University.

Chapter 16. OHIO ALPHA, Bowling Green State University, Bowling Green, Ohio.

President	Miss Winifred Cole
Vice-President	Miss Lois Perrin
Secretary	Mr. Clayton McDole
Treasurer	Mr. W. F. Cornell
Secretary Descartes	Mr. Harry R. Mathias
Faculty Sponsor	Mr. F. C. Ogg

Due to the fact that Ohio Alpha has only four student members it has not been possible to hold regular meetings.

Chapter 17. MICHIGAN ALPHA, Albion College, Albion, Michigan.

President	Miss Virginia Tripp
Vice-President	Mr. Fusajiro Aburano
Secretary-Treasurer	Mr. Marion L. Bunte
Secretary Descartes	Mr. E. R. Sleight
Faculty Sponsor	Mr. E. R. Sleight

Michigan Alpha held monthly meetings on mathematical subjects and allied topics. Several joint meetings with the Physics and Chemistry clubs were held. The three groups also united in various social functions on the campus.

Betty Hossfeld Brigham, chapter secretary-treasurer for '43-'44 and a very outstanding student, was married to Ensign Jesse Brigham in January. They are living in Boston where Ensign Brigham is pursuing studies at Harvard

University. Mr. William Voglesong is a technical assistant with Tennessee Eastman Corporation at Oak Ridge, Tennessee. Pvt. Kenneth R. Ferguson, chapter president for '43-'44, will be remembered for his excellent record at Albion. He is now in a special Engineer's Detachment at Oak Ridge, Tennessee. Russell Barrow is serving the United States with a marine unit in the Pacific area. Earl Dinger is doing graduate work at the University of Michigan. Edward Brender is with the Navy V-12 unit on the campus of the University of Wisconsin at Madison. Glenn Frye is with the navy V-12 unit at the University of Michigan. George Kawano was taken into the Army Intelligence Division because of his proficiency in the Chinese and Japanese languages.

Chapter 19. SOUTH CAROLINA ALPHA, Coker College, Hartsville, South Carolina.

President	Miss Betty McIntosh
Vice-President	Miss Elinor McIntosh
Secretary	Miss Rebecca Smith
Treasurer	Miss Kathryn Love
Secretary Descartes	Miss Caroline Reaves
Faculty Sponsor	Miss Caroline Reaves

South Carolina Alpha is functioning very effectively with nine active members who meet regularly twice each month. The chapter sponsored chapel talks to the entire student body on "Mathematics and the Challenge it Faces Today."

The teaching profession claims three of last year's members: Miss Dorothy Boykin at Wilmington, North Carolina; Miss Addie McIntosh at Kershan, South Carolina; and Miss Betty Singleton at Conway, South Carolina. Miss Anne Ludham is employed in a bank in Conway, South Carolina. Miss Eleanor Vanse won the Freshman Award for '43-'44 in recognition of her superior work. Miss Betty McIntosh won the award for contributing most to the chapter.

Chapter 20. TEXAS ALPHA, Texas Technological College, Lubbock, Texas.

President ---------------------- Miss Betty Grace Pugh
Vice-President ------------------ Miss Beverly Price
Secretary ------------------------ Miss Jane Burton
Treasurer ------------------ Miss Betty Jane Morris
Secretary Descartes ---------------- Miss Lida B. May
Faculty Sponsor ---------------- Dr. Donald L. Webb

Texas Alpha has twenty-four active members and holds regular monthly meetings at which students and professors give papers on interesting topics. Special mention is made of the following: "The Three Theories of Color Vision," by Beverly Price; "The Life of Pythagoras," by Edward Turrentine.

Chapter 22. KANSAS GAMMA, Mount St. Scholastica College, Atchison, Kansas.

President --------------------- Miss Mary Lou Maloney
Vice-President --------- Miss Mary Catherine Growney
Secretary ---------------------- Miss Katherine Zeller
Treasurer -------------------------- Miss Ann Hughes
Secretary Descartes ------- Sister Helen Sullivan, O.S.B.
Faculty Sponsor ---------- Sister Helen Sullivan, O.S.B.

The topic for research and intensive study during the past year has been "Opportunities for Women Trained in Mathematics." A symposium based on the results of this study was presented at an all-school assembly January 24, 1945. Those participating in the symposium were the following: Patricia Warwick, Ann Hughes, Mary Catherine Growney, Mary Davis, Virginia Harrison, Katherine Zeller, and Jane Hajovsky.

Kansas Gamma published a departmental newspaper *The Exponent,* which was distributed to interested collegians as well as to former chapter members. All the members and pledges are continuing to fill in the Defense Stamp Albums which were unanimously voted on last year in the attempt to promote the war effort. The high-light of the year was the lecture on "Mathematics and Philosophy," given by Reverend Malachy Sullivan, O.S.B., professor of philosophy at St. Benedicts College, Atchison, Kansas.

Miss Virginia Meyers, chapter president for '43-'44 and valedictorian of her class, entered the Benedictine novitiate in June 1944 and is now known as Sister Mary Joyce. Sarah Alice Woodhouse, first president of Kansas Gamma and also the valedictorian of her class, was married February 5, 1945, to Mr. Myles Joseph Sheehy in La Jolla, California. Miss Marjorie Dorney was married to Mr. Dale Howey at Monte Vista, Colorado.

Chapter 24. NEW JERSEY ALPHA, Upsala College, East Orange, New Jersey.

President	Miss Betty Rudebock
Vice-President	Mr. Joseph Prieto
Secretary	Miss Elizabeth Ebel
Treasurer	Mrs. Mary Louise McKim
Secretary Descartes	Mr. M. A. Nordgaard
Faculty Sponsor	Mr. M. A. Nordgaard

A report on last year's members reveals the following: Mr. Ingald Opsal, an ensign in the navy, is on duty in the Pacific. Miss Dorothy Shaw is employed with the Bell Telephone Company in Hoboken, New Jersey. Miss Zilda Meisel is teaching in Caldwell, New Jersey. Miss Lillian Meisel is teaching in Newark, New Jersey. Miss Elizabeth Ebel is in the Actuarial Department of the Metropolitan Life Insurance Company in New York City.

Interesting papers presented at chapter meetings include the following: "Pre-modern Algebra," by Dr. M. A. Nordgaard; "An Approximation Method of Finding Roots," by Mrs. Mary L. McKim; "Comparative Study of Copernican and Ptolemaic Systems," by Miss Betty Rudebock.

Unusual chapter features were: Mid-year social meeting at which Mr. James Price, Wright Aeronautical Corporation, Patterson, New Jersey, spoke on "Mathematical Curves and Formulas Involved in Gears." The entire chapter attended Christmas presentation of "The Star of Bethlehem" at the Hayden Planetarium in New York City. A discussion meeting by the members took place in the Rotunda immediately after the presentation.

Chapter 25. OHIO BETA, The College of Wooster, Wooster, Ohio.

Due to a very marked decrease in the enrollment in mathematics classes, Ohio Beta's membership is too small to permit regular meetings. Former members are all serving in the armed forces. The mathematics faculty is employed in teaching the navy unit on the campus.

Chapter 26. TENNESSEE ALPHA, Tennessee Polytechnic Institute, Cookeville, Tennessee.

President	Sarah Francis Hollis
Vice-President	Wallace S. Prescott
Secretary-Treasurer	Ann Gwaltney
Secretary Descartes	R. O. Hutchinson
Faculty Sponsor	R. O. Hutchinson

No regular meetings were held because of the small number of active members. The chapter gave a program before the Mathematics Club on Magic Squares.

Dr. R. H. Moorman of the mathematics faculty is on leave of absence and is acting head of the department of mathematics in the College of Charleston, Charleston, S. C. Mr. Ray Kinslow, also of the faculty, is engaged in research work with the Tennessee Engineers at Oak Ridge, Tenn.

Mildred Murphy is employed by the First National Bank, Cookeville, Tenn., and is maintaining active membership in the chapter.

Chapter 27. NEW YORK ALPHA, Hofstra College, Hempstead, New York.

President	Miss Joan Schuppel
Vice-President	Mr. E. R. Stabler
Secretary	Mr. Robert Ackerson
Treasurer	Miss Ruth Mayer
Historian	Miss Wanda Scala
Secretary Descartes	Mr. E. R. Stabler
Faculty Sponsor	Mr. Albert Capuro

New York Alpha has fifteen active members who meet regularly each month. Interesting lectures given at the various meetings were the following: "Mathematics of Finance," by L. F. Ollmann; "Photography," by Mr. Rodgers; "Motion of Projectiles," by W. L. Ayres of Purdue University; "A Problem with Large Numbers," by E. R. Stabler.

Miss Edith Hufman, chapter president '43-'44, is engaged in designing special slide rules for the navy. Miss Anna Taussig and Miss Dorothy McFarland are employed by the Sperry Gyroscope Company in Garden City, New York. Two pairs of charter members of New York Alpha were married in 1944; Ellen Fletcher and Robert Beyer in February; Phyllis Reynolds and Paul Camp in September.

Chapter 28. MICHIGAN BETA, Central Michigan College, Mount Pleasant, Michigan.

Secretary Descartes ________________ L. Cecil Gray
Faculty Sponsors ____________ Mr. C. C. Richtmeyer, Mr. J. W. Foust

Thirty-one new members, the largest class of initiates in its history, were elected to the chapter in January. The meetings have featured reports on "Mathematicians and Their Contributions to the Science of Mathematics." A report on "Aeronautical Navigation" by Lester Serier showed the applications of mathematics to plotting a course and the need for accuracy in computation.

The chapter is working on a pamphlet for secondary schools concerning the courses in mathematics that high-school students should study in preparation for various vocations such as engineering, law, medicine, pharmacy, nursing, laboratory technicians, and flying.