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A Semantical Approach to the Study of Mathematics

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A semantical[1] study of mathematics is of little interest to many pure mathematicians, and of less interest to applied mathematicians. Most mathematicians find that they get along quite well without considering problems such as symbol reference, the nature of true propositions, the status of postulates, or even the consistency of the mathematical system in which they are working. For this they can hardly be criticized, since these problems have little bearing on the particular, specialized fields in which they are working.

A study of the foundations of mathematics, however, cannot be made without some investigation of at least a theory of signs, the status of "truth" in mathematics, and the assumption of intuitive logic. In this paper such an investigation will be outlined, and we shall also consider certain other problems; some of these problems are essential to a (mathematical) study of the foundations of mathematics, whereas some are considered only because their omission would leave one with the feeling that he does not understand the subject he is studying.

Certain words will be assumed to be primitive, no definitions being given for them. This does not imply that they are assumed to be "ultimate" or "undefinable," but only that since we cannot point at or see such a thing as meaning, and can use only a finite number of words, we must start with some words which are assumed to be primitive. The words assumed to be primitive may very well be "simple," that is, not capable of being defined by words easier to

[1] The word "semantics" will be used in the same sense that Morris uses the word "semiosis." See C. W. Morris, *Foundations of the Theory of Signs*, International Encyclopedia of Unified Science, Volume 1, Number 2.

comprehend, but whether they are simple or not, no attempt will be made to define them. An attempt will be made, however, to indicate how these words will be used and to give some (necessarily circular) explanations of their meanings.

The words assumed to be primitive are "referent," "meaning," "same," and "design." The words "same" and "design" will be used in the conventional sense, and no attempt will be made to explain them.

The *referent* of a symbol is the object for which the symbol stands. The referent of a symbol may, of course, be another symbol, as, for example, the referent of "symbol" is 'symbol'.[2] Symbol reference will not be analyzed further than this.

The word "meaning" will be used in three senses sufficiently distinct that there is no possibility of ambiguity. They are as follows:

1. A symbol may *mean* its referent, as, for example, "dog" means the animal lying by the fire.
2. A symbol may *mean* its definiens, as, for example, "man" means "a featherless biped."
3. A symbol may *mean* one of its synonyms, as, for example, "blau" means "blue."

At no point will we consider "real meaning." While the necessity of undefined terms and of a certain intuitive understanding of these terms is admitted, the intuition of most non-philosophers about the words "real meaning" is notable for its absence. Further, existing attempts at definition of these words seem to make them no more comprehensible, except possibly to the definers. At any rate, the usage of the words "real meaning" can always be avoided in studies of mathematics, and in line with a general policy of keeping undefined terms to a minimum, we shall do so.

This completes the explanations and examples of usage of most of the words which are assumed in this paper to be primitive. The words listed are not all the words that will be used without being defined, nor are they all the words of

[2] For a detailed study of such cases as this, see W. V. Quine, *Mathematical Logic*, pp. 23-26.

the English language that would be assumed to be primitive in a more detailed study. However, the conventional usage of the other words which will be used is satisfactory, and the choice of primitives is to a large extent arbitrary.

The words "mathematician" and "mathematical" will be used in the broad conventional sense. Anyone whose profession is commonly called "mathematics" will be termed a "mathematician," and anything related to what is commonly called "mathematics" will be termed "mathematical."

The positions of modern mathematicians on the status of symbols in mathematics can roughly be divided into three classifications as follows:

1. The symbols of mathematics are *meaningless marks*, completely devoid of interpretation.
2. The symbols of mathematics are entities *referred to* by marks or sounds.
3. The symbols of mathematics are typographical designs, without consideration of their meaning.

The first position is not usually elaborated further than the above statement. It is open to criticism mainly because of its incompleteness. Several important questions are left unanswered, and without a radical alteration of the position are apparently unanswerable.

For example, if symbols are meaningless, how can it be possible to apply mathematics to physical situations? That is, how does a "meaningless mark" suddenly begin to stand for the velocity of a projectile, or the number of bank directors who are not shareholders? Are symbols assumed, rather, to have indeterminate meanings, assignable at will? Or is "mathematics" a shorthand for "pure mathematics," so that such a question need never arise?

What are we to say of such symbols as x, $\times$, and X? Are they the "same" symbol? Surely not; they are not in the same position, and the second is typographically quite distinct from the other two. Are they "occurrences" of the same symbol? If so, what is the significance of "occurrence of a mark"? Pencil marks, at least, can hardly be assumed to move about the paper, "occurring" first in one place, then in another.

The first position is seen to be untenable without extreme modification. The second position, however, solves the above problems readily, although the solutions are perhaps surprising.

Those who assume that symbols are "entities" referred to by marks (usually called "intuitive") apparently postulate the "existence" of a "mathematical reality" such as that described by G. H. Hardy.[3] Marks or sounds are to be termed "intuitive symbols," and are assumed to represent the "real" symbols of the "mathematical reality." In a similar fashion, such a string of marks as "p implies q" is not a proposition,[4] but instead *represents* a proposition. The question, "What *is* the proposition represented by 'p implies q'?" is not supposed to be asked, and will not be answered.

Now this usage of the word "symbol" easily answers the questions arising from the usage of symbols as meaningless marks. The application of mathematics consists merely in the assigning of referents to the *symbols* (not to the *intuitive symbols*, which have fixed referents). Since we do not know what a symbol is, we are in no position to question the possibility of such an assignment of referents, or, if it is assumed to be possible, to ask just how it is done.

Problems concerning the occurrence of symbols are as easily solved. Since it is the same symbol represented by the marks x, $\times$, and X, it makes little difference just where or in what form these marks occur.

As far as consistent usage goes, this is all very well. The usage of the word "symbol" is apparently non-contradictory, if nothing else. Yet one is impressed at once by its extreme artificiality. Many mathematicians will object strenuously to assuming the "existence" of "entities" in a "mathematical reality." It seems to be more nearly Platonism and Pythagoreanism than mathematics. There may, of course,

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[3] G. H. Hardy, *A Mathematician's Apology*, pp. 63-64.

[4] "Proposition" will be defined later. Roughly, a proposition is a string of marks which, when interpreted, forms a declarative sentence.

be mathematicians who are mystics, and they can not be criticized for pursuing their own interests. But in line with the modern tendency toward formalism,[5] something less mystical seems desirable.

The third position is not only less mystical; it is completely formalistic. To elaborate it, we shall need to clarify a usage of the word "mathematics."

We shall assume *mathematics* to be the deductive study of the structure of sign-arrangements. This is certainly an over-condensation of the formalistic thesis, but it will be sufficient for our needs, and the formalistic thesis will be stated more explicitly later.

The word "symbol" will be used as synonymous with "symbol design" or "typographical design." It is assumed that we understand the meaning of the word "design," and are able to recognize that certain marks have the *same* design, and that others do not. For example, we can see that x and *X* have the same design, but that x and y have different designs.

Of course, in any mathematical system it must be explicitly stated which marks are to be considered as having the same design. For certain purposes it may be desirable to consider z and *z* to have the same design, but to consider x and X to have different designs. Such a situation might arise in a group composed of both Germans and Americans, where the subject studied necessitated a distinction between, say, variables which represented integers, and variables which represented non-integral rational numbers.

The essential point, however, is that we can always recognize that such marks as y and y have the same design; that is, small differences in typography or handwriting do not alter the (tacitly) assumed sameness of design.

Using the concept of "symbol" described above, it is assumed that any proposition of mathematics is a predication of symbolic design. For example, if we postulate p implies

[5] For a detailed description of the formalistic position see Max Black, *The Nature of Mathematics*, Supplement A, pp. 147-167.

p, it is assumed that we are also postulating *p* implies *p*, P implies P, and all formulas[6] of the same design.[7]

In order to maintain that mathematics is the deductive study of the structure of sign-arrangements, we denote the application of mathematics and all deductive studies in which the symbols possess referents by the word "meta-mathematics."[8] There is then no contradiction in our usage of the words "symbol" and "mathematics," and studies which necessitate strong intuitive postulates are clearly classified as being partially intuitive, and not at the same level of rigor as mathematics.

When we speak of occurrence of a symbol we now mean occurrence of a design. It is probable that this involves no contradiction, for designs can and do recur, and the fact that these designs are conveyed by marks which are not in the same position and are not typographically identical is of little interest.

This position can be criticized mainly for its use of words which are not defined but whose meanings must be understood (at least in context); for example, consider the words "design" and "same." However, "design" and "same" are familiar to most people, and seem to be easily understood even though they are undefined. Furthermore, any thesis of symbolism, if it is to answer pertinent questions such as have been formulated here, appears to need the use of these words or their synonyms.

Mathematicians who are not formalists will probably prefer the second position, or some variation of that position; mathematicians who are completely uninterested in the foundations of mathematics may prefer the first position because of its simplicity. Formalists will prefer the third position; this latter definition of "symbol" will be used throughout the rest of this paper.

[6] A *formula* is a string of symbols. This word is assumed to be primitive. See A. Church, "A Set of Postulates for the Foundation of Logic," Annals of Mathematics, Vol. 33, 2d Ser., (1932).

[7] See Rudolph Carnap, *The Logical Syntax of Language*, pp. 15-16.

[8] *Meta-mathematics* is usually defined to be the science whose subject is the structure of mathematical formulas. For the purposes of this paper it is desirable to define "meta-mathematics" so as to include both this concept and the above.

It is customary to divide the symbols of mathematics into two classes: "variables" and "constants." The variables are usually assumed to be the symbols whose referents "vary," the constants to be the symbols whose referents do not "vary." This is saying nothing, of course, but the statements are typical of the usual definitions of "variable" and "constant," which are obscure and contradictory. Quite often the variables do not vary, or are termed "arbitrary constants," and have the outward characteristics of constants.

This confusion can be avoided. In order to see how to avoid it, let us consider the preliminary steps in the development of a mathematical system.

We begin with an infinite list of undefined symbols. We shall, *entirely arbitrarily*, call some of the symbols *constants*, and the rest of the symbols *variables*. We then construct postulates[9] about these symbols. In order that we may make use of the postulates, it is also essential at the start that we state certain rules of deduction, usually *modus ponens* and a rule for quantification.[10]

A typical postulate might be:

$$x \text{ implies } (y \text{ implies } x)$$

The letters involved in the statement of this postulate may be called *meta-variables*, and are used as follows: a meta-variable stands for *any one* of the variables; that is, whenever we see a formula which contains one or more meta-variables, it is to be understood that this formula is a shorthand for an infinity of formulas, each of these formulas to be obtained from the one containing meta-variables by replacing the meta-variables throughout by variables.

The meta-variables are not, strictly speaking, a part of the mathematical system. They are a device for talking *about* the system; they enable us to make an infinity of

[9] A *postulate* of mathematics is an initial unproved formula on which a system is based, as distinguished from the conventional usage, which makes "postulate" synonymous with "assumption."

[10] See R. Carnap, *Logical Syntax*, p. 94.

statements in one. The necessity for them is obvious; without meta-variables (or some similar device)[11] we could not deduce from a = a that b = b, unless we added new rules, which would only cause more complicated proofs.

Now what exactly is the difference between the variables and the constants from a *strictly mathematical* standpoint? The answer is that the difference is purely nominal! Of course, the constants have fixed (intuitive) referents, while the referents of the variables are arbitrary, but this is a meta-mathematical consideration. In any formula, or in the proof of any formula, the variables and the constants behave exactly the same. Difference in roles of symbols is to be found only between meta-variables and the primitive symbols. There is no reason, other than convenience or convention, for classifying the symbols into variables and constants, or into any other classes. Such classification is made only for meta-mathematical reasons.

Even with the aid of meta-variables, in order that the formulas of mathematics may be short enough to be manipulated easily, it is necessary that *definitions* be employed. The word "definition" will be used as an abbreviation for "a statement that a certain string of symbols will be used as an abbreviation for another string of symbols." This is a definition of "definition," and it can be proved to be so, thus giving a constructive proof that dialectic "proofs" of the impossibility of defining "definition" are invalid. For the sentence just given is itself "a statement that a certain string of symbols (viz., "definition") will be used as an abbreviation for another string of symbols (viz., "a statement that a certain string of symbols will be used as an abbreviation for another string of symbols")," and is therefore a definition, by definition.

Definitions in mathematics are usually written in a form similar to the following:

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[11] For a similar device, see Quine, *Mathematical Logic*, pp. 33-37.

Formula 1 for *Formula 2*[12]

This indicates that Formula 1 will be used as an abbreviation for Formula 2; that is, wherever Formula 1 appears, it is to be understood that it is merely a shorter way of writing Formula 2.[13] The definitions are thus not a part of mathematics; they are a meta-mathematical notation to simplify the formulas, and may be omitted whenever desired. Russell says: "It is to be observed that a definition is, strictly speaking, no part of the subject in which it occurs. For a definition is concerned wholly with the symbols, not with what they symbolize. Moreover, it is not true or false, being the expression of a volition, not of a proposition. . . . Theoretically, it is unnecessary ever to give a definition; we might always use the *definiens*[14] instead and thus wholly dispense with the definiendum. . . . The definitions are no part of our subject, but are, strictly speaking, mere typographical conveniences. Practically, of course, if we introduced no definitions, our formulae would very soon become so lengthy as to be unmanageable; but theoretically, all definitions are superfluous."[15]

The definitions are to a large extent arbitrary, in the sense that there are no restrictions on the choice of definiens and definiendum, with respect to the meta-mathematical interpretation of the symbols involved. There is, however, one syntactical prohibition; in the definiens no free variable[16] may occur which does not already occur in the definiendum. If this condition is not made, it is possible for definitions to

[12] Quine, *Mathematical Logic*, pp. 47-48.

[13] Rosser states this idea as follows: "'Def' between two sets of symbols will mean that wherever the set on the left appears, it is to be understood that it is really the set on the right which is there." J. B. Rosser, "On the consistency of Quine's *New Foundations for Mathematical Logic*, The Journal of Symbolic Logic, Volume 4, Number 1.

[14] The *definiendum* is that which is defined; the *definiens* that for which the definiendum is an abbreviation. See Carnap, *Logical Syntax*, p. 28.

[15] Whitehead and Russell, *Principia Mathematica*, Volume 1, p. 11.

[16] Roughly, a *free variable* is one that does not lie within the scope of any quantifier of that variable. A *bound variable* is any variable that is not free. For an accurate definition, see Quine, *Mathematical Logic*, pp. 76-80.

be framed by means of which a contradiction may be inferred.[17]

The choice of definitions is not, however, usually at random, or influenced only by typographical convenience. There are *usually* (not always) meta-mathematical considerations which cause particular choices of definitions. Of this Russell says: "In spite of the fact that definitions are theoretically superfluous, it is nevertheless true that they often convey more important information than is contained in the propositions in which they are used. This arises from two causes. First, a definition usually implies that the *definiens* is worthy of careful consideration. Hence the collection of definitions embodies our choice of subjects and our judgment as to what is most important. Secondly, when what is defined is (as often occurs) something already familiar, such as cardinal or ordinal numbers, the definition contains an analysis of a common idea, and may therefore express a notable advance."[18]

The one other type of definition which occurs frequently in mathematics,[19] the *recursive definition*, can always be written and treated in the manner described above, so that it need not be considered separately.[20]

One other kind of definition occurs in meta-mathematics; this may be called "ostensive definition" or "definition by enumeration." The definition of "variable" is such a definition; it consists simply in naming certain displayed symbols, it being understood that only the symbols displayed are so named.

The postulates of most mathematical systems are propositions.[21] As yet we have not given a definition of "postulate," and this has been intentional. Without careful usage of the words "postulate," "demonstration," and "conse-

[17] See Carnap, *Logical Syntax*, pp. 24-25. Carnap also states one other restriction, which does not seem to be necessary.

[18] Whitehead and Russell, *Principia Mathematica*, Vol. 1, pp. 11-12.

[19] Quine calls definitions of the type just described "formal definitions." See Quine, *Mathematical Logic*, p. 47.

[20] See Carnap, *Logical Syntax*, pp. 88-89.

[21] For an example of a mathematical system in which "propositions" are not defined, see A. Church, *The Calculi of Lambda Conversion*, pp. 68-71.

quence" it is easy to cause considerable ambiguity and difficulty in definition. In order to avoid this confusion we proceed as follows.

We define "postulate" ostensively, and only in context; that is, we state our definition of postulate thus:

Definition: The following is a *postulate*:

1. x implies (y implies x)

A postulate is thus seen to appear initially in the system only as a part of the definiens of a definition. There is thus no necessity for definitions of "assumption" and of "assertion."

We then make the following definitions:

Definition: Let S be a set of propositions, and let $q_1, q_2, \ldots q_n$ be a sequence of propositions. We say that this sequence is a *demonstration, in n steps ,that* q_n *is a consequence of S* if for each i from 1 to n inclusive either:

a) q_i is in the set S.
b) There are numbers, j and k, both less than i, such that q_j is the proposition q_k implies q_i.
c) There is a variable v and a number j, less than i, such that q_i is the proposition $(v)\,q_j$.

Definition: If a demonstration that q_n is a consequence of S exists, we say that q_n is a *consequence* of S.

Definition: If q_j is the proposition q_k implies q_i, then we say that q_i *is derived from* q_j *and* q_k *by use of Rule I.*

Definition: If q_i is the proposition $(v)q_j$, then we say that q_i *is derived from* q_j *by use of Rule Q with the variable* v.

Definition: If $p_1, p_2, \ldots, p_n, q$ are propositions we shall use the symbolism "$p_1, p_2, \ldots, p_n$ yield q" to denote "q is a consequence of $p_1, p_2, \ldots, p_n$, and the set of postulates." If n is zero, "yield q" denotes that q is a consequence of the postulates alone.

Definition: If "yield q," then q will be called a *theorem.* Any demonstration that yields q will be called a *proof* of q.[22]

Consideration of the above definitions indicates the procedure to be followed in proving theorems. But although

[22] From J. B. Rosser, *A Symbolic Logic* (not published), pp. 12-13, with slight alterations.

any one system is based on definitions which are entirely verbal, it is clear that a certain amount of what may be called "intuitive logic" is necessary to prove any theorems at all. About this Church says: "It is clear, however, that formulas composed of symbols to which no meaning is attached cannot define a procedure of proof or justify an inference from one formula to another. If our postulates were expressed wholly by means of the symbols of formal logic without use of any words or symbols having a meaning, there would be no theorems except the postulates themselves. We are therefore obliged to use in some at least of our postulates other symbols than the undefined terms of the formal system, and to presuppose a knowledge of the meaning of these symbols, as well as to assume an understanding of a certain body of principles which these symbols are used to express, and which belong to what we shall call intuitive logic. It seems desirable to make these presuppositions as few and as simple as we can, but there is no possibility of doing without them."[23]

The postulates, being only the definiens of a definition, are entirely arbitrary, with the sole restriction that they be *consistent*; that is, it is agreed that there must be no proofs of both p and not-p. It is usually considered desirable that a system be *complete*, and that the postulates be *independent*,[24] but neither of these conditions is considered mandatory. Other than the demand for consistency, the choice of postulates depends only on the interests of the inventor of the system. Professor Bell has said: "In any science or branch of mathematics we *must* start from a certain set of *undefined things*. . . . We then proceed to *make postulates* (pure assumptions) about our "indefinables." . . . We then try to see what our postulates *imply*. This leads us

[23] A. Church, "A set of Postulates for the Foundation of Logic," Annals of Mathematics, Vol. 33, 2d Ser., (1932).

[24] A system is *complete* if every proposition which contains no free variables is either provable or refutable; a set of postulates is *independent* if none of the postulates can be derived from the other postulates. K. Godel has proved (see "Uber formal unentscheidbare Setze der Principia Mathematica und verwandter Systeme I," Monatshefte fur Mathematik und Physik, vol. 38 (1931)) that logical systems suitable for the development of arithmetic can not be both consistent and complete.

to the *theorems* of our theory. Notice that *we ourselves* do all this; *we* lay down the postulates, which may have been suggested by "experience" or by pure whim; *we make* the theorems (we do not *discover* them, unless we happen to be four-dimensional mystics) according to the rules of a logical game which *we ourselves* have made and which *we* have agreed to abide by for the time being. The whole business from start to finish is a purely human activity, like banking or government or getting into debt and out again.[25]

Almost always, however, the inventor of a mathematical system has in mind some interpretation of the system, and this, of course, influences the choice of postulates. The interpretation may be to a widely speculative theory of the universe or to an observable physical situation, but the postulates will certainly not be chosen at random. It is this aspect of mathematical systems that makes them so often applicable to other sciences; it is not mere happen-so. The systems are originally constructed with the interpretations (or analogous interpretations) in mind.

Whether or not a system has a predetermined application, it seems desirable to consider it only as a study of symbolic structure. If we require mathematics to be correlated with physical phenomena or with other theories, the development of new systems will have to wait for the end (if any) of innumerable squabbles over the "truth" of the postulates. Further, rigor will be lessened by the unintentional introduction into the proofs of physical concepts. Bell says: "The difference between mathematics and the science of the speculators is simply this: the mathematician does not assert that his hypotheses give a correct picture of nature or of anything else beyond the marks he has put on paper; the speculator does. . . . Mathematical structures do not crash very often; the builders merely lose interest in them and go on to something else."[26]

Now let us consider the status of the word "true" in the meta-mathematics of such a system as the one described

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[25] E. T. Bell, *The Search for Truth*, p. 242.
[26] Ibid. p. 83.

above. We find that there are at least three frequent usages of "true."

First: A proposition may be said to be *true* if it is provable. This definition is apparently advocated by Bell for using "true" in any context.[27] Mathematically it is of little importance, for we can always use the word "provable" and avoid making an unnecessary definition.

Second: A proposition may be said to be *true* if it has a certain structure. For example, the tautologies of the propositional calculus can be determined by the so-called "truth-table" method.[28] This definition is of particular importance for proving the completeness of certain mathematical systems.

Third: A proposition may be said to be *true* if it expresses an actual occurrence in "reality." This is vague, and possibly nonsense, but it is used in the proof of at least one meta-mathematical theorem, the *Lowenheim Theorem.* For example, "the moon is blue" is true if and only if the moon is blue.

Sufficient use has been made of the words "mathematics" and "meta-mathematics" that their meanings are probably clear. However, to complete the discussion the following definitions are stated:

A *mathematical system* is any study of symbolic structure which is carried out by means of an explicitly stated list of postulates and rules, and the minimum amount of intuitive logic necessary to use these postulates and rules. The "minimum amount" of intuitive logic necessary may change as further study shows the need for weaker or stronger intuitive logics.

Mathematics is the totality of all mathematical systems which *exist at present.*

Meta-mathematics is the totality of all existing studies whose subject is one or more mathematical systems, or particular aspects of one or more mathematical systems.

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[27] Ibid. The definition is not explicitly stated, but is implied by statements throughout the book.

[28] Quine, *Mathematical Logic*, pp. 50-61.

The word "meta-mathematics" is also applied to studies which make use of mathematical systems whose symbols have been assigned referents.

It will be noticed that the above definitions have not been formulated as we have said definitions should be formulated. The usage at this point is merely a matter of literary style, and does not constitute a change in theory.

Extensively, these definitions seem to be in agreement with most of the well known definitions; intensively, there are wide differences. By way of illustration the following definitions are quoted:

"Hilbert's formalism reduces mathematics to a *meaningless* game, played according to certain simple rules, with meaningless symbols or marks.... *After* the game has been played we can begin to talk about what, if anything, it "means." But when we do so we are *outside* of mathematics and are in meta-mathematics."[29]

"Mathematics is the study of what is true of hypothetical states of things. That is its essence and definition."[30]

"The formalist thesis: pure mathematics is the science of the formal structure of symbols."[31]

"Pure mathematics is the class of all propositions of the form 'p implies q' where p and q are propositions containing one or more variables, the same in the two propositions, and neither p nor q contains any constants except logical constants. And logical constants are all notions definable in terms of the following: implication, the relation of a term to a class of which it is a member, the notion of *such that*, the notion of relation, and such further notions as may be involved in the general notion of propositions of the above form. In addition to these mathematics *uses* a notion which is not a constituent of the propositions which it considers, namely the notion of truth."[32]

[29] E. T. Bell, *The Search for Truth*, p. 239.
[30] *Collected Papers of Charles Sanders Peirce*, Vol. 4, p. 193.
[31] Max Black, *The Nature of Mathematics*, p. 8.
[32] Bertrand Russell, *The Principles of Mathematics*, p. 3.

The differences in these definitions reflect differences in the interests of their authors. Some mathematicians are more interested in applications, in "truth," or in logic than in symbolic structure, and prefer to define "mathematics" so that it will pertain to their interests. It is entirely a matter of personal preference.

An Interesting Algorithm

MERLE MITCHELL

President of Texas Beta, 1941-1942

The use of synthetic division to find the values of a polynomial at various points often becomes cumbersome and inconvenient because of the constant recopying of figures which is necessary. This duplication of figures may be avoided if we employ an algorithm which is similar to synthetic division but which is more convenient, compact, and concise.*

In the discussion which follows, the reader should keep in mind the fact that no proof of the algorithm is intended. This article is only an exposition of an interesting and useful device for evaluating a polynomial for various values of the variable. An understanding of the process of synthetic division is necessary for reading this work. If the reader is not acquainted with the process, a discussion of synthetic division may be found in any good college algebra text.

Although an explanation of the algorithm is rather long, the actual computation and space needed to carry it out is very short. After a step by step explanation of the process through a concrete example, there follows a condensation of the algorithm into its final compact form.

Suppose the polynominal $f(x) = 2x^4 - 3x^3 + x^2 - 6x + 1$ is to be evaluated at various successive integral values. The procedure as applied to this function follows.

* This algorithm was suggested to the author in 1941 by Dr. Gerald B. Huff, professor of mathematics at Southern Methodist University.

Copy the coefficients of f(x), and by synthetic division evaluate f(x) at the first point chosen. If this point is x = –2, the division will be:

(1)

2	–3	1	–6	1	
	–4	14	–30	72	
2	–7	15	–36	73	–2

Hence, it is known that f(– 2) = 73. Now draw a line between the remainder 73 and the rest of the numbers on the line.

Next, copy the figures appearing in the last line of (1), noting the position of the last line drawn above. (Note the next diagram.) For evaluation at –1, proceed as in synthetic division until the line is reached. To get the value which is to be written on the right side of the line, multiply the last figure (that is, –60) by +1, which is the result of subtracting –2 from –1. Then add the value obtained to 73 as indicated. Now draw two lines, one between 13 and –60, and one between –60 and 24.

(2)

2	–7	15	–36	73	
	–2	9	–24	–60	
2	–9	24	–60	13	–1

The next study will be concerned with the last line of figures in (2). The point now chosen for the evaluation of f(x) is x = 0. (See next diagram.) Divide synthetically until the first line is reached. To proceed from this point, multiply the last number (that is, 24) by +2, the difference, 0 – (–2). Then add this result to –60 to obtain –12. Next multiply –12 by +1, and add to 13 so that the result is 1. Now draw three lines as indicated below.

(3)

2	–9	24	–60	13	
	0	0	48	–12	
2	–9	24	–12	1	0

Now work with the last line of (3). Let the next point considered be x = +1. Divide synthetically until the first line is reached; multiply the last figure (that is, –7) by +3, which is the result obtained from 1 – (–2), and add the result to 24 to get 3. Multiply 3 by +2 and add to –12 to get –6. Multiply –6 by +1 and add to 1, obtaining the value –5. Now draw four lines as indicated below.

(4)

2	–9	24	–12	1	
	2	–21	6	–6	
2	–7	3	–6	–5	+1

If the process is continued in a similar manner, it is clear that the value of the polynomial at x = + 2 can be obtained as follows:

(5)

2	–7	3	–6	–5	
	8	3	12	6	
2	1	6	6	1	+2

At x = + 3, the value of f(x) is calculated thus:

(6)

2	1	6	6	1	
	8	27	66	72	
2	9	33	72	73	+3

A similar process is used to evaluate the function for consecutive integral values. In the example above, in which f(x) is a polynomial of the fourth degree, it is to be noticed that, after step (4), it is not necessary to perform any multiplication by a number greater than 4. In fact, through the use of this algorithm to evaluate a polynomial of degree n at various points, it is not necessary to multiply by a number greater than n after the nth step is reached.

Below is a condensation of the six steps just explained:

	(4)	(3)	(2)	(1)	
2	−3	1	−6	1	
	−4	14	−30	72	
2	−7	15	−36	73	−2 ; f(−2) = 73
	−2	9	−24	−60	
2	−9	24	−60	13	−1 ; f(−1) = 13
	0	0	48	−12	
2	−9	24	−12	1	0 ; f(0) = 1
	2	−21	6	−6	
2	−7	3	−6	−5	+1 ; f(+1) = −5
	8	3	12	6	
2	1	6	6	1	+2 ; f(+2) = 1
	8	27	66	72	
2	9	33	72	73	+3 ; f(+3) = 73

Another example follows:

$$f(x) = 2x^3 - 4x^2 - 2$$

	(3)	(2)	(1)	
2	−4	0	−2	
	−6	30	−90	
2	−10	30	−92	−3
	−4	28	58	
2	−14	58	−34	−2
	−2	−32	26	
2	−16	26	−8	−1
	6	−20	6	
2	−10	6	−2	0
	6	−8	−2	
2	−4	−2	−4	+1
	6	4	2	
2	2	2	−2	+2
	6	16	18	
2	8	18	16	+3
	6	28	46	
2	14	46	62	+4

With the values of the polynomial at successive integral points outlined in this way, it is easy to visualize the graph of the function and to determine the location of real roots between integral values. In this second example, it is clear there is a root between 2 and 3 since the value of the function changes from negative to positive between these points. By a continuation of this method, now using intervals of one-tenth instead of one, the root may be located between successive tenths.

2	−4	0	−2	
	4	0	0	
2	0	0	−2.000	2.0
	4.2	8.82	.882	
2	4.2	8.82	−1.118	2.1
	4.4	1.72	1.054	
2	8.6	10.54	−.064	2.2
	.6	1.84	1.238	
2	9.2	12.38	1.174	2.3

Hence there is a root between 2.2 and 2.3. The root could be located more exactly by continuing the process, now using intervals of one one-hundredth. An approximation to the root can be obtained by interpolation, as follows:

$$\frac{x}{.064}=\frac{.1}{1.238};\ x=.005$$

The root of the equation is, therefore, approximately 2.205.

The Mathametical Scrapbook

Cannon-balls may aid the truth,
But thought's a weapon stronger.
—Charles Mackay

For the mathematician, the present academic year has been a year of memories. Newton was born on Christmas Day, 1642, three centuries ago. The world was ready for a genius when Newton was born. That he was that genius has been attested to many times. The story of Newton may be told simply by means of the following quotations:

> There may have been minds as happily constituted as his for the cultivation of pure mathematical science; there may have been minds as happily constituted for the cultivation of science purely experimental; but in no other mind have the demonstrative faculty and the inductive faculty co-existed in such supreme excellence and perfect harmony.—Lord Macauley.

> This almost superhuman genius, whose powers and attainments at once make us proud of our common nature, and humble us with our disparity.—Thomas Brown.

> Newton and La Place need myriads of ages and thick-strewn celestial areas. One may say a gravitating solar system is already prophesied in the nature of Newton's mind.—Emerson.

> I don't know what I may seem to the world, but as to myself, I seem to have been only as a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay undiscovered before me.—Isaac Newton.

When we review his life, his idiosyncrasies, his periods of contrast, and his doubts and ambitions and desire for place, may we not take some pleasure in thinking of him as a man—a man like most other men save in one particular—he had genius—a greater touch of divinity than comes to the rest of us?—David Eugene Smith.

If I have seen further than Descartes it is because I stood on the shoulders of giants.—Isaac Newton.

The efforts of the great philosopher (Newton) were always superhuman; the questions which he did not solve were incapable of solution in his time.— Arago.

Taking mathematics from the beginning of the world to the time when Newton lived, what he had done was much the better half.—Leibnitz.

In the year that Newton was born, Galileo died. Also in 1642, Pascal invented the first calculating machine; this invention was the occasion for a great scientific celebration in London. Exactly one hundred years after the birth of Newton, the great astronomer Halley died. It was Halley who forecast the return of the comet named after him, and he personally subsidized the printing of Newton's famous *Principia.*

In 1443, five hundred years ago, Copernicus was born. It was Copernicus who asserted that the earth rotated on its axis, and he showed that the known motions of the planets could be explained by supposing that the planets revolved about the sun. Copernicus introduced a new era into mathematical astronomy, and scientists the world over are celebrating his birth date this year.

Some of the difficulties which Copernicus encountered are indicated by the following sections from the preface to his book, which he first submitted to Pope Paul III.

I can easily conceive, most Holy Father, that as soon as people learn that in this book I ascribe certain motions to the earth, they will cry out at once that I and my theory should be rejected. I am not so much in love with my conclusions as not to weigh what others will think about them. . . . Therefore, when I considered this carefully, the contempt which I had to fear because of the novelty and apparent

> absurdity of my viewpoint nearly induced me to abandon the work I had begun.
>
> If perchance there shall be idle talkers who, though they are ignorant of all mathematical science, nevertheless assume the right to pass judgment in these things, and if they should dare to criticize and attack this theory of mine because of some passage of Scripture which they have falsely distorted for their own purpose, I care not at all; I will even despise their judgment as foolish.

In this country, Thomas Jefferson was born April 13, 1743. All are familiar with Jefferson as one of the founders of American democracy. Few people realize that he was one of the fathers of science in this country. He was well-versed in engineering, astronomy, geology, agriculture, architecture, biology, and mathematics. It is said that he used the calculus with facility. Virtually every known science of his day felt the imprint of his genius. He insisted that mathematics should occupy a more prominent place in American education. Following the French trend, he urged that mathematics should occupy an important position in the curriculum of military academies. He advocated a more extensive use of the decimal scale. His work in astronomy was considerably in advance of that being taught in the universities of his day, and he proved to be a master in applying his mathematics to astronomical calculations. Every scientist in this country reveres the memory of Thomas Jefferson.

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ZENO'S PARADOXES IN MOTION

I. The Dichotomy.

The idea of motion is inconceivable for what moves must reach the middle of its course before it reaches the end. Hence the assumption of motion pre-supposes another motion, and that in turn another, and so ad infinitum.

II. The Achilles.

This asserts that the slower when running will never be overtaken by the quicker, for that which is pursuing must first reach the point from which that which is fleeing started so that the slower must necessarily always be some distance ahead.

III. The Arrow.

Since an arrow cannot move where it is not, and since also it cannot move where it is (in the space it exactly fills), it follows that it cannot move at all.

IV. The Stadium.

The fourth concerns two rows of bodies, each composed of an equal number of bodies of equal size, which pass one another on a race course as they proceed with equal velocity in opposite directions, one row starting from the end of the course and the other from the middle. This involves the conclusion that half a given time is equal to its double. Aristotle said that "The fallacy of the reasoning lies in the assumption that an equal magnitude occupies an equal time in passing with equal velocity a magnitude that is in motion and a magnitude that is at rest, an assumption that is false."

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Mathematics has a triple end. It should furnish an instrument for the study of nature. Furthermore it has a philosophic end, and, I venture to say, an end esthetic. It ought to incite the philosopher to search into the notions of number, space, and time; and, above all, adepts find in mathematics delights analogous to those that painting and music give. They admire the delicate harmony of number and of forms; they are amazed when a new discovery discloses for them an unlooked for perspective; and the joy they thus experience, has it not the esthetic character although the senses take no part in it? Only the privileged few are called to enjoy it fully, it is true; but is it not the same with all the noblest arts? Hence I do not hesitate to say that mathematics deserves to be cultivated for its own sake, and that the theories not admitting of application to physics deserve to be studied as well as others.—H. Poincare.

If logical training is to consist, not in repeating barbarous scholastic formulas or mechanically tacking together empty majors and minors, but in acquiring dexterity in the use of trustworthy methods of advancing from the known to the unknown, then mathematical investigation must ever remain one of its most indispensable instruments. Once inured to the habit of accurately imagining abstract relations, recognizing the true value of symbolic conceptions, and familiarized with a fixed standard of proof, the mind is equipped for the consideration of quite other objects than lines and angles. The twin treatises of Adam Smith on social science, wherein, by deducing all human phenomena first from the unchecked action of selfishness and then from the unchecked action of sympathy, he arrives at mutually-limiting conclusions of transcendent practical importance, furnish for all time a brilliant illustration of the value of mathematical methods and mathematical discipline.—John Fiske.

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From the very outset of his investigations the physicist has to rely constantly on the aid of the mathematician, for even in the simplest cases, the direct results of his measuring operations are entirely without meaning until they have been submitted to more or less of mathematical discussion. And when in this way some interpretation of the experimental results has been arrived at, and it has been proved that two or more physical quantities stand in a definite relation to each other, the mathematician is very often able to infer, from the existence of this relation, that the quantities in question also fulfill some other relation, that was previously unsuspected. Thus when Coulomb, combining the functions of experimentalist and mathematician, had discovered the law of the force exerted between two particles of electricity, it became a purely mathematical problem, not requiring any further experiment, to ascertain how electricity is distributed upon a charged conductor and this problem has been solved by mathematicians in several cases.—G. C. Foster.

It is plain that that part of geometry which bears upon strategy does concern us. For in pitching camps, or in occupying positions, or in closing or extending the lines of an army, and in all the other maneuvers of an army whether in battle or on the march, it will make a great difference to a general, whether he is a geometrician or not.—Plato.

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Newton had so remarkable a talent for mathematics that Euclid's Geometry seemed to him "a trifling book," and he wondered that any man should have taken the trouble to demonstrate propositions, the truth of which was so obvious to him at first glance. But, on attempting to read the more abstruse geometry of Descartes, without having mastered the elements of the science, he was baffled, and was glad to come back again to his Euclid.—James Parton.

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So long as a man imagines that he cannot do this or that, so long is he determined not to do it: and consequently so long it is impossible to him that he should do it.—Spinoza.

Directions in air navigation are measured in clockwise fashion from the north line as initial line. With this understanding, can you solve the following problem?

An aircraft carrier is cruising a course of 47° at a speed of 18 knots. A scouting plane takes off at a speed of 142 knots and flies a course of 192° for 36 minutes before the pilot spots something suspicious in the distance. The pilot of the plane then turns and flies a course of 270° for 20 minutes at a speed of 160 knots. After that he desires to return to the carrier. Assuming that he makes 160 knots on the home trip, what course should the pilot follow to exactly contact his carrier?

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The positions of three men form an equilateral triangle. They are all shooting at the same target which is 200 yards from the first man, 280 yards from the second man, and 310 yards from the third man. How far apart are the three men?

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What number divided by 2, 3, 4, 5, or 6 gives a remainder of 1, but when divided by 7, gives no remainder?

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What two numbers have the property that their product is equal to the difference of their squares, and the sum of their squares is equal to the difference of their cubes?

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What is the smallest integer having sixteen integral divisors?

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How much of the earth's surface would a man see if he were raised to the height of the radius above it?

=∇=

A is known to be three times as truthful as B. What is the chance of the truth of an assertion which B affirms and A denies?

=∇=

A person living in San Diego sent the following message:

tsgty elied einga laote fsthr.

Why was he arrested when the message was intercepted?

The cryptogram is of the simplest type, employing only transposition.

=∇=

The orbit of the earth is a circle; round the sphere to which this circle belongs, describe a dodecahedron; the sphere including this will give the orbit of Mars. Round Mars describe a tetrahedron; the circle including this will be the orbit of Jupiter. Describe a cube round Jupiter's orbit; the circle including this will be the orbit of Saturn. Now inscribe in the earth's orbit an icosahedron; the circle inscribed in it will be the orbit of Venus. Inscribe an octahedron in the orbit of Venus; the circle inscribed in it will be Mercury's orbit. This is the reason of the number of the planets.—Kepler.

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Within limits, the zoologist employs the formula that the time-rate of growth of a body or of an organ within a body varies directly as the magnitude of the body or the organ in question. The constant of variation is composed of two factors, a specific constant for the body and a factor G which measures the general conditions of growth as affected by age and environment.

Can you show, in agreement with Julian Huxley, that the weight x of an animal is related to the weight y of an organ within the animal through the formula

$$y = cx^n,$$

where n is equal to b, the specific constant for the organ, divided by a, the specific constant for the body.

Kappa Mu Epsilon News

Chapter 3. KANSAS ALPHA, Kansas State Teachers College, Pittsburg, Kansas.

Mrs. Faye Wallack, class of 1924, has returned to Kansas State Teachers College to work toward her Master's degree and to assist in the mathematics department this year. She was one of the last group initiated into Kansas Alpha.

Mrs. Mildred Bradshaw, the former Mildred Martin, who was a charter member of the Kansas Alpha Chapter of Kappa Mu Epsilon, is now teaching a few mathematics classes at her Alma Mater.

Harvey Lanier, president of the chapter, has been called to the service, and Billie Sherwood, the vice-president, is now acting president.

H. Franklin Lanier is teaching at Scott Field, Illinois. He was president of Kansas Alpha in 1940-41, while he was taking graduate work in mathematics and physics. He received his Master's degree in 1941, and took further graduate work at Ohio State University in 1941-42. In October 1942, he was married to Anna Rupert, who was secretary of Kansas Alpha. She is also teaching at Scott Field.

F. Earl Ludlum and Cecil Cooper became instructors in meteorology at Maxwell Field, Alabama, immediately after receiving their Master's degrees in the summer of 1942. Both had minored in physics, and had taken courses in the teaching of aeronautics.

Samuel J. Mark is a laboratory assistant in hydrodynamics and dynamic meteorology at the University of Chicago where he is in training to be a military meteorologist.

Marie Cowley is a senior clerk at Wright Field, Dayton, Ohio. She was selected for this position because of her excellent record in mathematics and physics, and her preparation as a teacher of aeronautics.

Prof. R. W. Hart is absent on leave from the Kansas State Teachers College. He is a lieutenant in the Navy and is teaching navigation at the Great Lakes Training School. Mr. James R. Foresman is teaching mathematics at the same place.

Chapter 6. MISSISSIPPI BETA, Mississippi State College, State College, Mississippi.

President D. E. Smith ---Mr. Reeves Power Cochran
Vice-President H. L. Reitz --Mr. Eugene Simon Rose
Secretary G. D. Birkhoff ---Mr. Archie Glenn McKee
Treasurer L. E. Dickson ---Mr. Charles W. Norman
Secretary Descartes ----------------Mr. C. R. Stark
Faculty Sponsor ---------Professor W. O. Spencer

In the recent graduating class, Kappa Mu Epsilon was represented by Tom Ledbetter and H. C. Leake, who graduated with honors, and by James W. Carr, who took highest honors. These men are now in military service.

All the members of Mississippi Beta are listed as reserves in some branch of the service; consequently Kappa Mu Epsilon will probably not have an active membership at Mississippi State College after the present semester. Arrangements have been made for keeping the organization intact for resumption of activities after the war. A standing committee has been appointed to contact Kappa Mu Epsilon men in the service.

Bi-monthly meetings have been held to instruct new members and to transact business. Programs have consisted of informal talks and a special discussion by Professor C. D. Smith on the "Origins of the Calculus." One initiation banquet was held with Professor Arthur Ollivier as principal speaker.

Chapter 7. NEBRASKA ALPHA, Nebraska State Teachers College, Wayne, Nebraska.

Ensign E. J. Huntemer is reported missing in action following the Battle of the Solomons on November 13, 1942.

Lieutenant Walter B. Thompson has been awarded the distinguished flying cross for his "extraordinary achievements" in Australia.

The Nebraska Alpha Chapter is represented in the Pacific War Zone by Lieutenant Walter B. Thompson and Ensigns James Doyle, Darel Bright, and David Garwood; in the North Africa region, by Lieutenants Franklin Victor and Gerald Johns.

The field of meteorology has attracted the following from our chapter: Captain John Jones who is stationed at Washington, D. C.; Lieutenant Lloyd Erxleben who is stationed at Memphis, Tennessee; Aviation Cadets Bob Dale, Eugene Everson, Don Strahan, and Russell Vlaanderen who will receive their commissions from Chicago this May; Aviation Cadets John Macklin and Russell Holdenreid also at Chicago; Aviation Cadet Arthur Gulliver at Grand Rapids, Michigan; and Private Clayton Christensen at Carleton College. Don Strahan and Bob Dale talked to the chapter in January telling of their work at the University of Chicago.

Lieutenant Van Bearinger is receiving training in radar at the Massachusetts Institute of Technology and at Harvard. Aviation Cadet Kenneth Peterson is receiving training in radio at Yale. Ensign Jack R. Davenport is instructing at Midshipmen's School at Northwestern University.

Connie Owen has been initiated into Pi Mu Epsilon at the University of Nebraska.

Jack F. Morgan will receive his Ph.D. at the University of Nebraska this summer, after which he will go to Easton, Pennsylvania, as a research chemist for the General Aniline and Film Corporation.

Active members of the Nebraska Alpha Chapter who were chosen to be listed in *Who's Who Among Students in American Universities and Colleges for 1942-43* were Marjorie Gnuse, Margie Morgan, Orin Currie, Dean Jensen, Craig Magwire, and Robert Westphal.

Lyle Seymour, Dean Jensen, Edwin Scott, and Elmer Hansen recently reported for service in the Air Corps.

Chapter 8. ILLINOIS ALPHA, Illinois State Normal University, Normal, Illinois.

Despite a reduction in the membership of Illinois Alpha to only twelve student and five faculty members, the group has carried on a full program. The annual homecoming breakfast was held on October 17, 1942, with twenty-nine persons present. Mr. Poppen, a member of the faculty, spoke about some of his experiences at George Peabody College, where he worked last year on his doctorate. Two other very successful social events were the Christmas party and the spring banquet. The decorations at the banquet planned by the social chairman, Amber Grauer, honored the forty former members of Illinois Alpha who are now in the service. Special recognition was given to the two members who have sacrificed their lives.

Especially interesting programs were given at the November, February, and March meetings. In November, the subject, "There's Secrecy in Numbers," led to a discussion of methods of coding and decoding. In February, Miss Atkin's class in the history of mathematics presented a program on methods of computation. In March, Dr. Mills gave an illustrated lecture on astronomy. A program of talks featuring mathematical recreations and a picnic will close the year's activities.

Chapter 9. KANSAS BETA, Kansas State Teachers College, Emporia, Kansas.

New initiates to Kansas Beta are Eloise Miller, Guy Grisell, Vernon Sheffield, Margaret Ellis, Wanda Rector, Frances Peterson, Gordon Pahrm, Philip Patrick, Jay Clothier, Raymond Sloan, Herbert Jackson, Laurel Fry, Bob Fry, Warren Jones, Irene Hageberg, and Richard Voots.

The following boys of this chapter are in the reserves: Gordon Pahrm, Warren Burns, and Richard Voots in the Army Air Corps; Jay Clothier in the Navy; and Philip Patrick in the Army. Merton Hoch and Virgil Kinnaman are both lieutenants in the Army Air Corps; Garrett Bartley is a Lieutenant in the Field Artillery. Donald Knop enlisted in the Army Air Corps, and Lester Meisenheimer enlisted in the Signal Corps.

The marriage of John Zimmerman and Lenore Stenzel, both members of Kansas Beta, took place recently. Mr. Zimmerman is now back at his Alma Mater teaching physics to the Army Air Cadets, and Mrs. Zimmerman is working in the general office.

Chapter 11. NEW MEXICO ALPHA, University of New Mexico, Albuquerque, New Mexico.

President Benjamin Peirce	Mr. Robert Hutchinson
Vice-President E. H. Moore	Mr. John Cunningham
Secretary Cajori	Miss Elena Davis
Treasurer Bocher	Mr. C. B. Barker
Secretary Descartes	Mr. C. A. Barnhart
Faculty Sponsor	Mr. C. A. Barnhart

Ruth Ford, president of New Mexico Alpha for the past year, graduated at mid-term and took a position with the T.V.A. Roberta Warren was elected to fill the vacancy in the office of president.

Programs for the last semester have been varied. The following topics have been discussed: "The History of the

Mathematical Association of America" by Dr. W. D. Cairns, president of the Association and now professor at New Mexico, and "The Extension and Period of a Conically Wound Spring" by Dr. O. B. Ader. An illustrated lecture was given on the subject, "Mathematics and Meteoritics" by Professor Lincoln LaPaz of Ohio State University, now research associate at the University. Student speakers have been Ruth Barnhart, who talked on "The Duplication of the Cube," and Roberta Warren who spoke on "Squaring the Circle."

The following new members have been initiated: Geary Allen, Marie Balling, Reka Lois Black, Elena Davis, Joe Harley, and Howard Muller.

Chapter 13. ALABAMA BETA, Alabama State Teachers College, Florence, Alabama.

The following students are to be initiated into Kappa Mu Epsilon on May 11, 1943: Joe Cade, Loreta Clark, Corinne Dickson, Elizabeth Gray, Rufus Hibbett, Richard McNeill, Thomas Mitchell, Russell Preuit, and Karl Tynee. Alabama Beta plans to present a second war bond to the Henry Jones Willingham Loan Fund in May.

Ensigns Cecil Harbin and Wilson Hite are studying at the Submarine Chaser Training Center, Miami, Florida. Private Lilbourne Hall, ex-president of Alabama Beta, is studying meteorology at the University of North Carolina. Hayden Hargett, class of 1935-'36, who entered the Air Corps in 1942, is now doing work in meteorology at the University of New York; previous to his entry into the service, he was an engineer for the E. I. Du Pont Company at Childersburg, Alabama. Mildred Burgess, who obtained her B.S. in 1942, is now employed in the Ground Signal Service at Fort Monmouth, New Jersey. She was formerly the secretary to Dean C. B. Collier and Dr. Eula P. Egan of State Teachers College, Florence, Alabama. Private Milton L. Baughn is in the Army Techical School at Sioux Falls, South Dakota.

Chapter 14. LOUISIANA ALPHA, Louisiana State University, University, Louisiana.

President Gauss ______________ Mr. Leland Morgan
Vice-President Poincare ________ Miss Nina Nichols
Secretary Fermat __________ Miss Gloria McCarthy
Treasurer Galois _______________ Miss Julia Weil
Historian Cajori _______________ Mr. William Wray
Secretary Descartes ________ Mr. Houston T. Karnes
Faculty Sponsor _____________ Miss Marelena White

Mr. Robert Anding recently received the Kappa Mu Epsilon Freshman Award which is awarded on the basis of an honors test in freshman mathematics. Mr. Sam Cunningham of Baton Rouge, Louisiana won the Kappa Mu Epsilon Senior Award which depends upon the quantity and the quality of work in mathematics. He is the first individual to have received both the freshman and the senior awards.

Chapter 18. MISSOURI BETA, Central Missouri State Teachers College, Warrensburg, Missouri.

President Laplace _____________ Mr. Robert Wilde
Vice-President Pascal _________ Miss Evelyn Cowan
Secretary Gauss _______________ Miss Velda Keeney
Treasurer Galois ____________ Mr. Arthur Anderson
Historian ____________________ Mr. Edwin Basham
Secretary Descartes _________ Miss Berne Heberling
Faculty Sponsor ______________ Mr. Fred W. Urban

On February 1, 1943, Missouri Beta initiated the following members: Evelyn Cowan, Pat Donahoe, Dean Grigsby, Henry Hironaka, Wade Mummo, Claude Norcross, and H. C. Stossberg. Several days after the initiation, all the new members with the exception of Miss Cowan were called into the service. A luncheon was held on February 16 in honor of the members who were leaving. All of the members of Missouri Beta except two are now in some branch of the service.

Chapter 19. SOUTH CAROLINA ALPHA, Coker College, Hartsville, South Carolina.

President Leibnitz	Miss Dorothy Boykin
Vice-President Pascal	Miss Betty McIntosh
Secretary Thales	Miss Katherine Still
Treasurer Gauss	Miss Elinor McIntosh
Secretary Descrates	Miss Caroline M. Reaves
Faculty Sponsor	Miss Caroline M. Reaves

Besides carrying on the regular business and program meetings for the year, South Carolina Alpha has worked as a group for two hours each month making surgical dressings for the Red Cross.

The following are new initiates: Lorraine Carlisle, Frankie Ellis, Miriam McCormic, Betty McIntosh, Elinor McIntosh, Rebecca Smith, Katherine Still, and Eloise Bellflower.

Chapter 21. TEXAS BETA, Southern Methodist University, Dallas, Texas.

President Galois	Miss Marian Weaver
Secretary Pascal	Miss Mary Moseley
Treasurer Fermat	Mr. Roland Porth
Secretary Descartes	Mr. Paul K. Rees
Faculty Sponsor	Mr. Gerald B. Huff

Jane Taylor, vice-president of Texas Beta, graduated at mid-term leaving her office vacant. Billy Parham, former treasurer, was awarded a fellowship in chemistry at the University of Illinois. Mary Moseley has been awarded a fellowship at the State University of Iowa for the year 1943-'44.

Student speakers for the programs this year have been: Milton Drandell, who spoke on "Geometric and Arithmetic Means and Trigonometric Solutions of Triangles," and Marian Weaver, who talked on "Convergence of Infinite Series."

Chapter 22. KANSAS GAMMA, Mount St. Scholastica College, Atchison, Kansas.

Presidet Leibnitz ....Miss Virginia Meyers
Vice-President Sylvester ......Miss Rosemary Solas
Secretary Newton ....Miss Rosa Garcia
Treasurer Euler ....Miss Margaret Mary Wolters
Secretary Descartes ...Sister Helen Sullivan, O.S.B.
Faculty Sponsor ....Sister Helen Sullivan, O.S.B.

The activities of Kansas Gamma for the past year included a panel discussion on "Mathematics and its Place in National Defense," a question and answer program on the cultural, disciplinary, practical, and statistical aspects of mathematics; the traditional Christmas party; a round table discussion on "Famous Women in Mathematics," and a report on the life and work of William Rowan Hamilton.

Bobbie Powers, class of 1942, and former president of Kansas Gamma, is an instructor in the Chicago School of Aircraft Instruments. Jane Schweizer, treasurer of the chapter last year, is employed at the Newark Prudential Insurance Company, Newark, New Jersey.

Margaret Molloy, Margaret Mary Kennedy, and Mary Margaret Downs, all of whom have held office in Kansas Gamma, have been elected to Kappa Gamma Phi for outstanding achievement.

Chapter 24. NEW JERSEY ALPHA, Upsala College, East Orange, New Jersey.

Zelda Meisel was elected to fill the vacancy in the office of recording secretary occasioned by the withdrawal of Marjorie Nicols from school last semester.

Edward Cohen, former vice-president of New Jersey Alpha, is teaching radio in the training station at Scott Field, Illinois. Anne Zmurkiewicz, instructor in mathematics at Upsala College, resigned at the end of the first semester of the past year in order to do research work in defense industries. Lieutenant Richard Rasmussen, class of 1940, one of the eleven charter members of New Jersey Alpha, was killed in action in the Solomons area; he was, as far as is known, the first casualty of the war from Upsala College.

Speakers and topics for the year were as follows: "Why Teach Mathematics" by Phyllis Gustafson; "Problems from Weierstrass" by Joseph Prieto; "Divine Proportion in Classic Art" by Lillian Meisel; "Hyperbolic Functions" by Elizabeth Evel; "Sources of Euclid" by Marjorie Wolfe; "Dynamic Symmetry" by William O'Brien of Central High School in Newark; "The History of Pi" by Zelda Meisel; "Statistical Research" by Hirsch Geeler.

Chapter 26. TENNESSEE ALPHA, Tennessee Polytechnic Institute, Cookeville, Tennessee.

The following new members were initiated into Tennessee Alpha on February 9, 1943: Ollie James Agee, W. J. Blevins, Pierce Brown, Don Ferguson, Harry Grisham, Frances Hollis, Cooper Loftis, Cordell Moore, Fred Paulk, and John Roy. After the ritual, Dr. R. O. Hutchinson and Dr. R. H. Moorman presented short talks concerning the place of Kappa Mu Epsilon on the campus. The following old members were in attendance at the initiation banquet and ceremony: Charles Cagle, William Fitzgerald, Howard Herndon, Dr. R. O. Hutchinson, Robert Johnson, Wilma Leonard, Dr. R. H. Moorman, Margaret Plumlee, Charles Tabor, and Mildred Murphy.

Chapter 29. ILLINOIS GAMMA, Chicago Teachers College, Chicago, Illinois.

President Archimedes .... Mr. Robert Healy
Vice-President Euler .... Mr. John T. Wiegand
Secretary Galileo .... Miss Arletta Mae Loomis
Treasurer Kepler .... Miss Rita Cooney

Secretary Descartes ____________Mr. J. J. Urbancek
Faculty Sponsor ______________Mr. J. J. Urbancek

The program of Illinois Beta for the year consisted of a series of talks. Dr. Ralph Mansfield spoke on developing a mathematieal formula in regard to the struggle for existence of the mosquito; Lieutenant F. C. Chunn lectured on "Celestial Navigation"; Dr. Lester R. Ford, chairman of the mathematics department of the Illinois Institute of Technology, talked on "Old and New Ways of Solving Equations"; Dr. William C. Krathwahl, Department of Tests and Measurements at the Illinois Institute of Technology, spoke on "How Mathematics can be used to obtain Vital Information for an Educational Testing Program."

The following new members have been initiated: Lt. Albert J. Belanger (USMCR), Boris Bruce (Quartermaster School USN), Private Rudolph Jezek (USA), William J. Coyne, Marcella Crossen, Dorothy DeVries, Mary Dillon, Dan Drennan, Dorothea Reidy, Leslie Sissman, Charles E. Stanley, Allan Wilson, Charles Craig Wilson, Helen Wright, Lois Hinkle, Rose Hornacek, Joseph Kubal, and Ray Lane.