The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before May 31, 2026. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2025 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 937-944.

Problem 937. *Proposed by José Luis Díaz-Barrero, Barcelona Mathematical Circle, Barcelona, Spain.*

Three roots of the equation $x^4 - px^3 + qx^2 - rx + s = 0$ are tan A, tan B, tan C where A, B, C are the angles of a triangle ABC. Determine the fourth root as a function of only *p*, *q*, *r* and *s*.

Problem 938. . *Proposed by José Luis Díaz-Barrero, Barcelona Mathematical Circle, Barcelona, Spain.*

If *a*,*b*,*c* are positive reals no larger than one, prove that

$$\frac{2a - \sqrt[3]{abc}}{1+a} + \frac{2b - \sqrt[3]{abc}}{1+b} + \frac{2b - \sqrt[3]{abc}}{1+b} \geq \frac{3\sqrt[3]{abc}}{1+\sqrt[3]{abc}}$$

Problem 939. *Proposed by Toyesh Prakash Sharma and Etisha Sharma, Agra College, Agra, India.*

Find the highest power of 5 which is contained in 777!

Problem 940. *Proposed by John Zerger, Catawba College, Salisbury, NC.*

Show that if *p* and *q* are two consecutive odd prime numbers then *p* + *q* is the product of at least three prime numbers (not necessarily distinct),

Problem 941. *Proposed by Guillermo Garcia (student) and Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

Evaluate the sum $\int artanx\left(e^x+\frac{1}{e^x}\right)dx+\int\frac{1}{1+x^2}\left(e^x-\frac{1}{e^x}\right)dx.$

Problem 942. *Proposed D.M. Bătineţu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

If $((a_n)$ with n≥1 is a positive real sequence such that $\lim_{n\to\infty}\frac{a_{n+1}}{na_n\sqrt[n]{n!}}=a$ which is a positive real, then compute $\lim_{n\to\infty}\frac{1}{n}\left(\sqrt[n+1]{a_{n+1}}-\sqrt[n]{a_n}\right)$.

Problem 943. *Proposed D.M. Bătineţu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

If $((a_n)$ with n≥1 is a positive real sequence such that $\lim_{n\to\infty}\frac{a_{n+1}}{a_n\sqrt[n]{n!}}=\pi$, then compute $\lim_{n\to\infty}\left(\sqrt[n+1]{a_{n+1}}-\sqrt[n]{a_n}\right)$.

Problem 944. *Proposed by the editor*.

Find a positive integer *x* which is divisible by a fourth power and *x*+1 is divisible by a cube, and x+2 is divisible by a square.

SOLUTIONS TO PROBLEMS 920-927