The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before May 31, 2024. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2024 issue of *The Pentagon*.

Prefere nce will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 920-927.

Problem 920. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Let α be the golden ratio. Show that

$$\sum_{i=0}^{\infty} \frac{i}{\alpha^i} \left(\sum_{j=0}^{\infty} \left[\lim_{n\to\infty} \frac{F_n^2 + F_{n+2}^2 - F_{n+1}F_{n+3}}{F_{n-1}F_{n+2}}\right]^j\right)^{-1} = \alpha$$

Problem 921. *Proposed by Mihaly Bencze, Braşov, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Solve in real numbers the following equation:

$$\log_2(x^2 + 2^x) + (x^2 - 1) * 2^{x+1} + x^4 + x^2 + 2^x = 3 * 4^x + x + 1$$

Problem 922. *Proposed by Toyesh Prakash Sharma,*

Let F_n be the nth Fibonacci number defined by $F_1 = 1, F_2 = 1$ and for all n≥3,$F_n = F_{n-1} + F_{n-2}$. Prove that $\sum_{n=1}^{\infty} (\frac{1}{9})^{F_{n+2}}$ is an irrational number but not a transcendental number.

Problem 923. *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain.*

Let n≥1 be an integer. Compute

$$\lim_{n\to\infty} \frac{\binom{n+1}{2}}{2^{n*1}} \sum_{k=0}^{\infty} \frac{k+4}{(k+1)(k+2)(k+3)} \binom{n}{k}$$

Problem 924. *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain.*

Let *a,b,c* be three positive real numbers. Prove that

$$\frac{a}{4b+7\sqrt{ab}} + \frac{b}{4c+7\sqrt{bc}} + \frac{c}{4a+7\sqrt{ca}} \geq \frac{3}{11}$$

Problem 925. *Proposed Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Prove that in any triangle ABC with usual notations (R = circumradius, r = inradius, s=semiperimeter, m_a = median from vertex A) the following inequality is true:

$$2\sum m_a \leq 3\sqrt{\frac{R(s^2+r^2+Rr)}{2r}}$$

Problem 926. *Proposed by the editor*.
Prove that the sequence
$a_1 = 1, a_2 = 1, a_n = a_{n-1} + 2 * a_{n-2}$ for all $n > 2$
gives the number of integers between 2^n and 2^{n+1} which are divisible by 3.

Problem 927. *Proposed by the editor*.

Find the area below the two lines 8x+5y= 976 and 6x+5y = 792 that lies in the first quadrant.