The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before December 31, 2023. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2023 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 911-919.

Problem 911. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Solve for real numbers:

$$\begin{cases} 2\sin x + 1 = 2\sin y + 2\sin z \\ (\sin x + \sin y - \sin z)^2 + (\sin x - \sin y + \sin z)^2 + 1 = \sin x + \sin y + \sin z \end{cases}$$

Problem 912. *Proposed by Mihaly Bencze, Braşov, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Solve in real numbers the following equation:

$$x^2 - 5x - 2\sqrt{x-2} + 7 + \log_2 \frac{x^2 - 5x + 8}{\sqrt{x-2}} + \log_3 \frac{x^2 - 5x + 8}{2\sqrt{x-2}} = 0.$$

Problem 913. *Proposed by D.M. Bătineţu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Find $\lim_{n\to\infty}\left(\left(\frac{\pi^2}{6} - \sum_{k=1}^{n}\frac{1}{k^2}\right) * e^{x_n}\right)$ where $x_n = \sum_{k=1}^{n}\frac{1}{k}$.

Problem 914. *Proposed by D.M. Bătineţu-Giurgiu,"Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Compute $\lim_{n\to\infty} n\sqrt[n]{(2n-1)!!F_n}\sin\frac{1}{n^2}$ where F_n is the nth Fibonacci number.

Problem 915. *Proposed by Toyesh Prakash Sharma (student), Agra College, Agra, India.*

Evaluate the following integral

$$\int \ln(1+x)\cdot(e^x+\frac{1}{e^x})dx+\int\frac{1}{x+1}\cdot(e^x-\frac{1}{e^x})dx.$$

Problem 916. *Proposed by Raluca Maria Caraion and Forică Anastase, "Alexandru Odobescu" High School, Lehliu-Gară, Călăraşi, Romania.*

Find: $\Omega=\lim_{p\to\infty}\frac{1}{p^a}\cdot\sum_{m=1}^{p}\sum_{n=1}^{m}\sum_{k=1}^{n}\frac{k^2}{2k^2-2nk+n^2}$

Problem 917. *Proposed by Marian Ursărescu, "Roman Voda" College, Roman, Neamt, Romania, and Forică Anastase, "Alexandru Odobescu" High School, Lehliu-Gară, Călăraşi, Romania.*

Let $(a_n)_{n\geq1},(b_n)_{n\geq1}$ be two sequences of real numbers defined by

$$a_n=\int_1^n\left[\frac{n^2}{x}\right]dx;\ b_1>1,\ b_{n+1}=1+\log(b_n)$$

where [*] denotes the greatest integer function. Find L = $\lim_{n\to\infty}\frac{a_n\cdot\log\sqrt[n]{b_n}}{\log n^n}$

Problem 918. *Proposed by Seán Stewart, King Abdullah University of Science and Technology, Saudi Arabia.*

If k > 0 evaluate

$$\int_0^1\frac{\log(1+x^k+x^{2k})}{x}dx.$$

Problem 919. *Proposed by the editor.*

Find the error in the following proof:

Want to find $\lim\limits_{n\to\infty}\frac{4^n}{3^n}$. This is an $\frac{\infty}{\infty}$ form so we can apply L'Hopital's Rule.

Let L = $\lim\limits_{n\to\infty}\frac{4^n}{3^n}$. Then L = $\lim\limits_{n\to\infty}\frac{4^n}{3^n} = \lim\limits_{n\to\infty}\frac{4^n\cdot\ln 4}{3^n\cdot\ln 3}$ by L'Hopital's Rule

$$= \lim_{n\to\infty}\frac{4^n}{3^n}\cdot\lim_{n\to\infty}\frac{\ln 4}{\ln 3} = \mathrm{L}\cdot\frac{\ln 4}{\ln 3}$$

Subtracting L from both sides gives, $0 = L\cdot(\frac{\ln 4}{\ln 3}-1)$ but $\frac{\ln 4}{\ln 3}-1$ is not 0. Therefore, L = 0.