The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before March 15, 2020. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2020 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 840-848

Problem 840. *Proposed by the editor*.

Consider the sequence $a_0 = 1,\ a_1 = 1,\ a_n = 2a_{n-1} + a_{n-2}$ which is 1,1,3,7,17,41,99,…

a) Prove that no term in the sequence ends in 5. [If you get this, send it.]

b) Prove that if p is prime, then $a_p \equiv 1 \pmod{p}$.

Problem 841. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain*.

Let *a,b,c* be the roots of the equation $x^3 - x^2 - 2x - 3 = 0$. Find the value of $a^5 + b^5 + c^5$.

Problem 842. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain*.

Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $4^{-(x+y)} \le \dfrac{f(x)f(y)}{(x^4+1)(y^4+1)} \le \dfrac{f(x+y)}{(x+y)^4+1}$

for all x,y $\in \mathbb{R}$.

Problem 843. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that in ΔABC, you have

$$\sqrt{(2^{h_a}+2^{h_b}+2^{h_c})(2^{m_a}+2^{m_b}+2^{m_c})}<2^a+3^b+4^c.$$

Problem 844. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that if 0<*a*<*b*<*c*<1, then

$$2\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a\ln a & b\ln b & c\ln c \end{vmatrix} > \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ (a-1)\ln(a^2+1) & (b-1)\ln(b^2+1) & (c-1)\ln(c^2+1) \end{vmatrix}$$

Problem 845. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

If $a,b,c \in [0,1)$, then

$$8\int_0^a\left(\int_0^b\left(\int_0^c \frac{\sin^{-1}x\cdot\sin^{-1}y\cdot\sin^{-1}z}{(1+\sin^{-1}x)(1+\sin^{-1})(1+\sin^{-1}z)}\,dz\right)dy\right)dx \le a^2b^2c^2.$$

.

Problem 846. *Proposed by Pedro H.O. Pantoja, Natal/RN, Brazil.*

Evaluate

$$\int_0^{\pi/4}\cos^2(x)\cdot\ln(1+\cos(4x))dx.$$

Problem 847. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let $(x\ ),(y_n)$ be positive sequences of real numbers such that $\lim_{n\to\infty}\frac{x_n}{n}=x$ and $\lim_{n\to\infty}(y_{n+1}-y_n)=y$. Evaluate $\lim_{n\to\infty}\left(\frac{y_{n+1}}{y_n}\right)^{x_n}$.

Problem 848. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

If $m \in (1,\infty)$, n an integer greater than 1, a_k positive reals and $\sum_{k=1}^{n} a_k = a,$ then

$$\sum_{k=1}^{n}\left(\frac{a_k}{a_{k+1}}+\frac{1}{a_k}\right)^{1/m} \geq \left(\frac{2}{\sqrt{a}}\right)^{1/m} n^{1+\frac{1}{2m}}$$ where $a_{n+1}=a_1$.