The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before November 1, 2020. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2020 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 849-858

Problem 849. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that in an acute ΔABC the following relationship holds:

$$\frac{1}{\sin A}+\frac{1}{\sin B}+\frac{1}{\sin C}+\frac{1}{\cos A}+\frac{1}{\cos B}+\frac{1}{\cos C}>6\sqrt{2}$$

Problem 850. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

If $x, y, z > 0$ and $x+y+z=2\pi$, prove

$$\frac{\cos^4 x}{y+z}+\frac{\cos^4 y}{z+x}+\frac{\cos^4(x+y)}{x+y}\geq\frac{9}{64\pi}$$

Problem 851. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Let $a < b$ and $f : [a,b] \to (0,\infty)$ be continuous. Prove

$$3(b-a)\int_a^b f^2(x)dx+(b-a)^2\geq 2(b-a)\int_a^b f(x)dx+2\left(\int_a^b f(x)dx\right)^2$$

Problem 852. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

If $a, b, c > 0$ and $a + b + c = 3$, prove

$$\sum_{cyc} \frac{a^3c(b+1)+b^3c(a+1)}{a^2b(b+1)+b^2a(a+1)} \geq 3$$

Problem 853. *Marcel Chirita, Bucharest, Romania.*

Let $x \in \mathbb{Z}$. If $x^5+5x^3+15x^2 > 21x$, prove $x^5+5x^3+15x^2-21x \geq 30$

Problem 854. *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

If f: $\mathbb{R} \to \mathbb{R}$ with $f(0) = 2019$ and $3f(x) = f(x+y)+2f(x-y)+y$ for any x, y $\in \mathbb{R}$, then compute $\int_e^\pi f(x)dx$

Problem 855. *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let $\{a_n\},\{b_n\},\{c_n\}$ be positive real sequences such that $\lim_{n\to\infty} \frac{a_{n+1}}{na_n} = a > 0$, $\lim_{n\to\infty} \frac{b_{n+1}}{nb_n} = b > 0$, $\lim_{n\to\infty} \frac{c_{n+1}}{nc_n} = c > 0$. Compute $\lim_{n\to\infty} \frac{1}{\sqrt[n]{a_n^3}} \sum_{k=1}^n (b_k c_k)^{1/k}$

Problem 856. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Each 1x1 square of a 7x211 rectangle is painted either black or white. Prove that it is possible to choose four rows and four columns of the rectangle so that the sixteen 1x1 squares in which they intersect are painted with the same color.

Problem 857. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let a, b, c, d be four positive real numbers. Find the maximum value of

$$\frac{\sqrt[4]{a}+\sqrt[4]{b}+\sqrt[4]{c}+\sqrt[4]{d}}{\sqrt[4]{a+b+c+d}}$$

Problem 858. *Proposed by Pedro H.O. Pantoja, University of Campina Grande, Brazil.*

Let $M = \frac{8}{7}\sin^2\frac{\pi}{7} + \frac{\sqrt{7}}{7}\cot\frac{\pi}{7}$. Is M irrational?