THE PENTAGON

A Mathematics Magazine for Students

Volume 82 Number 2 Spring 2023

Contents

Typeset in WinEdt. Printed in the United States of America.

The Pentagon (ISSN 0031-4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Papers written by undergraduate mathematics students for undergraduate mathematics students are solicited. Papers written by graduate students or faculty will be considered on a space-available basis. Submissions should be made by means of an attachment to an e-mail sent to the editor. Either a TeX file or Word document is acceptable. An additional copy of the article as a pdf file is desirable. Standard notational conventions should be respected. Graphs, tables, or other materials taken from copyrighted works MUST be accompanied by an appropriate release form the copyright holder permitting their further reproduction. Student authors should include the names and addresses of their faculty advisors. Contributors to The Problem Corner or Kappa Mu Epsilon News are invited to correspond directly with the appropriate Associate Editor.

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Minimal Solutions of Linear Diophantine Equations

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Abstract

Linear Diophantine equations have been studied since the second century, introduced by Diophantus of Alexandria [2]. A linear Diophantine equation in two variables is one of the form $ax+by=c$ where a,b,c, as well as x and y are integers with $c \geq 0$. In [1], R. J. Levit investigates solutions which are minimal in the sense that $|x|+|y|$ is as small as possible. In this paper, we determine the values of c which do have a minimal solution with respect to x, with respect to y, or both x and y. We further provide a modular arithmetic approach to finding these solutions which often requires fewer steps than the Euclidean Algorithm.

Preliminaries and Definitions

Although there are lots of applications to Diophantine equations, we give a simple example of where integer solutions are required. Suppose you have a water faucet, a 3 gallon bucket and a 5 gallon bucket. You need to have precisely 4 gallons of water. This is exactly the conundrum faced in the movie *Die Hard with a Vengeance*. Algebraically, this is represented as $3x+5y=4$. As you are only able to fill a bucket from the faucet, dump the contents of one bucket into the other, or simply empty a bucket onto the ground, we must have integer solutions for x and y. In this case, we can fairly easily come up with a solution by guessing "small" values of x and y.

Definition. *A* linear Diophantine equation *in two variables has the form*

$$(*) \qquad ax+by=c$$

where a,b,c, as well as x and y are integers with $c \geq 0$.

In general, solving a linear Diophantine equation in two variables given the integer coefficients a and b raises the two questions: Does there exist a solution? If there does exist a solution, how can we find one? The answers to both of these are typically discovered in an Elementary Number Theory course. The answers are given in the following theorem.

Theorem (A)**.** *The linear Diophantine equation $ax+by=c$ has a solution if and only if the $d=\gcd(a,b)$ divides c. If x_0,y_0 is any particular solution of this equation, then all other solutions are given by $x=x_0+\left(\frac{b}{d}\right)t$ and $y=y_0-\left(\frac{a}{d}\right)t$ where t is an arbitrary integer.*

Proof. See [3]. ■

Obviously, thanks to this theorem there are an infinite number of solutions, if any exist. So, we need to find only one solution to our equation. The method usually given in introductory courses is to apply the Euclidean Algorithm. The following theorem, often referred to as Bezout's theorem, guarantees this is possible.

Theorem (B)**.** *If a and b are integers such that the $\gcd(a,b)$ is defined, then there exist integers x and y such that $ax+by=\gcd(a,b)$.*

Proof. See [4]. ■

Definition. *If a and b are integers such that the $\gcd(a,b)=1$, we say that a and b are* relatively prime.

Here is how the Euclidean Algorithm is applied.

Example
Determine the solutions to the Diophantine equation $3x+5y=4$.
We first note that since 3 and 5 are relatively prime the above theorems guarantee that we can solve this one. The Euclidean Algorithm gives us the x and y in the previous theorem:

$$\begin{aligned} 5&=3(1)+2\\ 3&=2(1)+1\\ 2&=1(2)+0. \end{aligned}$$

This gives us the obvious $\gcd(3,5)=1$. To write the $\gcd(3,5)=1$ as a linear combination of 3 and 5, we simply solve these equations backwards.

$$\begin{aligned} 1&=3-2(1)\\ &=3-(5-3(1))1\\ &=-5+3(2). \end{aligned}$$

Therefore, we have $3(2)+5(-1)=1$. Since we wanted to solve $3x+5y=4$ we multiply the equation by 4 to obtain

$$3(8)+5(-4)=4.$$

As noted in the above theorem, there are infinitely many solutions to this equation. In this case, we have $x = 8 + 5t,\ y = -4 - 3t$ for any integer t. Clearly, there are "smaller" solutions; for example, $(x,y) = (-2,2)$.

In this paper, we investigate the requirements to find "smaller" solutions to (*). Furthermore, we give a method that is different from the Euclidean Algorithm method to finding these solutions. First, we define what we mean by "smaller."

Definition. *We call a solution* $(x,\ y)$ *a* **minimal solution with respect to x** *if* $(x,\ y)$ *is a solution to* $(*)$ *with* $|x| \leq \frac{b}{2}$.
Similarly, we call a solution $(x,\ y)$ **a minimal solution with respect to y** *if* $(x,\ y)$ *is a solution to* $(*)$ *with* $|y| \leq \frac{a}{2}$.
We say that $(*)$ *has a* **definitely least solution** *if* $(x,\ y)$ *is a minimal solution with respect to both x and y.*

Note: Since both x and y are integers, if b is odd we actually have that $|x| \leq \frac{b-1}{2}$ in the minimal with respect to x case. Similarly, if a is odd we have that $|y| \leq \frac{a-1}{2}$ in the minimal with respect to y case. Of course, if both a and b are odd then a definitely least solution would have both $|x| \leq \frac{b-1}{2}$ *and* $|y| \leq \frac{a-1}{2}$.

Before proceeding we first list a couple of elementary results from number theory. The proofs can be found in any elementary text on Number Theory. See, for example, [3], or [4].

Definition. *Let m and n be integers. We say that m* **divides** *n if there exists an integer k such that* $mk = n$ *and denote this by* $m|n$.

Lemma (**Euclid's Lemma**). *If a and b be integers are integers with* $\gcd(a,b) = 1$ *and* $a|bc$ *then* $a|c$.

Definition (**Congruence Modulo** n). *Let* a,b, *and n be integers with* $n > 0$. *Two integers a and b are* **said to be congruent modulo** n, *notated as* $a \equiv b \mod n$, *if n divides* $a - b$, *that is,* $n|(a-b)$.

Lemma. *Let n be a positive integer. Congruence modulo n is an equivalence relation on the set of integers,* $\mathbb{Z}$. *Thus, congruence modulo n partitions the set of integers.*

The set of integers modulo n is often denoted by $\mathbb{Z}_n$ and is the set $\mathbb{Z}_n = \{0,\ 1,\ 2,\ \ldots,\ n-1\}$. By definition of congruence modulo n and using that congruence $\mod n$ is an equivalence relation, we can also represent $\mathbb{Z}_n$ as

$$\left\{1 - \frac{n}{2}, \ldots, -1,\ 0,\ 1, \ldots, \frac{n}{2}\right\} \text{ if } n \text{ is even,}$$

and

$$\left\{\frac{1-n}{2}, \ldots, -1, 0, 1, \ldots, \frac{n-1}{2}\right\} \text{if } n \text{ is odd.}$$

It is this latter notation that we will use throughout this paper.

The Modular Approach

We now record our first result in the following lemma.

Lemma 1. *Let a, m, and n be integers where a and n are relatively prime and $a \geq 1$, $m \geq 1, n \geq 0$. Then $ax \equiv m \mod n$ has a unique integer solution $x \mod n$. Furthermore, we can choose x so that $|x| \leq \frac{n}{2}$.*

Proof. Without loss of generality, we always reduce a, x, and m modulo n. To first show that $ax \equiv m \mod n$ has a solution, we look at the equivalent equation $ax + ny = m$. From Theorem 2, and the assumption that $\gcd(a,n) = 1$, there exists integers x_0, y_0 such that $ax_0 + ny_0 = 1$. Multiplying by m gives us, $a(x_0 m) + n(y_0 m) = m$, and thus $= x_0 m$ is a solution to the equation $ax \equiv m \mod n$.
Now to show uniqueness of the solution modulo n. Suppose, by way of contradiction, that both (x_1, y_1) and (x_2, y_2) are solutions. Then $ax_1 + ny_1 = ax_2 + ny_2 = m$. Taking the equation modulo n, we have $ax_1 \equiv m \mod n$ and $ax_2 \equiv m \mod n$. Subtracting yields $a(x_1 - x_2) \equiv 0 \mod n$, so $n | a(x_1 - x_2)$. Since $\gcd(a,n) = 1$, Euclid's Lemma gives us that $n | (x_1 - x_2)$. Therefore, $x_1 \equiv x_2 \mod n$. Since $ax \equiv m \mod n$ has a unique solution in $\mathbb{Z}_n$, as noted above we have that

$$x \in \left\{1 - \frac{n}{2}, \ldots, -1,\ 0,\ 1, \ldots, \frac{n}{2}\right\}$$

if n is even and

$$x \in \left\{\frac{1-n}{2}, \ldots, -1, 0, 1, \ldots, \frac{n-1}{2}\right\}$$

if n is odd. In either case, we have the desired $|x| \leq \frac{n}{2}$. This completes the proof of lemma 1. ■

We now make the assumption that the coefficients a and b are relatively prime so that we are guaranteed any solutions. This restriction is addressed at the end of this paper.

Lemma 2. *Consider the equation $ax + by = c$ where a, b, c, as well as x and y are integers with $gcd(a,b) = 1, c \geq 0$. This equation always has a unique minimal solution with respect to x, and always has a unique minimal solution with respect to y. Furthermore, if there is a definitely least solution then it is identical to the minimal solution with respect to either x or y.*

Proof. By Theorem A, we know there are an infinite number of solutions to the equation. Reducing the equation modulo b we have $ax \equiv c \mod b$. By Lemma 1, this has a unique solution with $|x| \leq \frac{b}{2}$. This is the definition of a minimal solution with respect to x.
Similarly, reducing the equation modulo a, we have $by \equiv c \mod a$. Again, Lemma 1 provides the unique integer solution with $|y| \leq \frac{a}{2}$. This is the definition of a minimal solution with respect to y. Since the minimal solutions with respect to both x and y are unique, if there is a definitely least solution it is identical to both of the minimal solutions. This completes our proof. ■

These lemmas give us the modular approach algorithm to find the minimal solutions.

Modular Approach Solve the equation $ax+by=c$ where a,b,c, as well as x and y are integers with $\gcd(a,b)=1, c\geq 0$. To find the minimal solution with respect to x, solve the equation modulo b. That is:

- solve $ax \equiv c \mod b$ for x in the applicable set for $\mathbb{Z}_n$. This forces the guaranteed minimum with respect to x; that is, $|x| \leq \frac{b}{2}$.
- Note: It may be possible to reduce a and/or c modulo b before solving.
- Once you have your minimal x, substitute this into the original equation and solve for y.

If the y value you found has the property $|y| \leq \frac{a}{2}$, then you have found the definitely least solution. If the y value does not have this property, then the equation does not have a definitely least solution. You can find the minimal solution with respect to y by solving the original equation modulo a. That is, solve $by \equiv c \mod a$. Again, be sure to choose the y in the appropriate set for $\mathbb{Z}_n$ so you have the minimal solution with respect to y. To find the corresponding x coordinate, just substitute this y value into the original equation and solve for x.

Here are a couple of examples:

Example
Solve the equation $3x+5y=4$ for integers x and y. We have seen the Euclidean algorithm approach which gave the solution $(8,-4)$. Here is the modular approach.

1. Find the minimal solution with respect to x. The equation modulo 5 is $3x \equiv 4 \mod 5$.
   - This has solution $x=-2$ (note that we chose $x=-2$ to force the minimal solution; $|x|=|-2|\leq 5/2$).
   - The y coordinate is then $y=\frac{1}{5}(4-3(-2))=2$. Thus we have the solution $(-2,2)$.
   - Note that since the y-coordinate $2>3/2$, this equation does not have a definitely least solution.
2. Find the minimal solution with respect to y. The equation modulo 3 is $2y \equiv 1 \mod 3$.
   - Note we reduced both 5 and 4 modulo 3.

- This has solution $y = -1$ which is a minimum with respect to y since $|-1| \leq 3/2$.
- Substituting into the original equation and solving for x gives $x = 3$. Thus, we have the solution $(3, -1)$.

We note that either of these two solutions will solve the water-jug problem posed earlier; the positive values indicate fill and negatives indicate to dump that bucket.

Example
Solve the Diophantine equation $34x + 55y = 26$.

To find the minimal solution with respect to x, first reduce the equation modulo 34. That is, $21y \equiv 26 \mod 34$. This is equivalent to $13y \equiv 8 \mod 34$. Solving, we find that $y = -2$. We substitute this into the original equation to find that $x = 4$ giving $(4, -2)$ as a solution. Notice that since $|y| = |-2| = 2 \leq \frac{34}{2}$ and $|x| = |4| = 4 \leq \frac{55}{2}$, we have that $(4, -2)$ is a minimal solution with respect to both x and y, therefore the definitely least solution.
For comparison, the Euclidean approach and back-substituting for this equation gives $(-546, 338)$ as the solution.

We now turn our attention to the work of determining the values of c for which there is a definitely least solution.

Key Results

We have two cases as we are still assuming that a and b are relatively prime. Namely, either a and b are both odd integers, or one is even and the other is odd. We begin with a lemma.

Lemma 3. *Let a and b both be odd positive relatively prime integers and c a nonnegative integer with $0 \leq c \leq ab$. Then either $(x,\ y)$ is a definitely least solution of $ax + by = c$ or $(-x,\ a - y)$ is a definitely least solution of $ax + by = ab - c$, but not both.*

Proof. Note that if $(x,\ y)$ is a solution to $ax + by = c$ then $(-x,\ a - y)$ is a solution of $ax + by = ab - c$. Just check that

$$(-x) + b\,(a - y) = -ax + ab - by = ab - (ax + by) = ab - c.$$

Assume that $(x,\ y)$ is a definitely least solution of $ax + by = c$. We will show that $(-x, a - y)$ is not a definitely least solution of $ax + by = ab - c$. Since a and b are both odd, we have that $|x| < \frac{b}{2}$ and $|y| < \frac{a}{2}$. Obviously, $|-x| = |x| < \frac{b}{2}$ and $|a - y| \geq a - |y| > a - \frac{a}{2} = \frac{a}{2}$. Thus, $(-x,\ a - y)$ is a minimal solution with respect to x but not a definitely least solution to $ax + by = ab - c$.
From what we just proved, the contrapositive; namely, if $(-x,\ a - y)$ is a definitely least solution of $ax + by = ab - c$ then $(x,\ y)$ is not a definitely least solution of $ax + by = c$, completes the proof of our lemma. ■

Theorem 1. *Let a and b both be odd positive relatively prime integers and c a nonnegative integer. Then $ax+by=c$ has:*

1. *A definitely least solution for every $0 \le c < \frac{1}{2}(a+b)$.*
2. *A definitely least solution for exactly half the c values with*

$$\frac{1}{2}(a+b) \le c \le ab - \frac{1}{2}(a+b).$$

   Furthermore, for integers k and r in the interval $\left[\frac{1}{2}(a+b), ab-\frac{1}{2}(a+b)\right]$ with $k+r=ab$, either $c=k$ or $c=r$ has a definitely least solution, but not both.

3. *Does not have a definitely least solution for every $c > ab - \frac{1}{2}(a+b)$.*

Proof. Note first that both a and b are odd. Therefore, if (x,y) is a definitely least solution we have $|x| \le \frac{b-1}{2}$ and $|y| \le \frac{a-1}{2}$. Furthermore, if (x,y) is a definitely least solution we have

$$c = ax+by \le a|x|+b|y| \le \frac{a(b-1)}{2} + \frac{b(a-1)}{2} = ab - \frac{1}{2}(a+b).$$

Therefore, if $c > ab - \frac{1}{2}(a+b)$ then there is no definitely least solution. This proves (3) of the theorem.

For the remainder of the proof, we can assume $0 \le c \le ab - \frac{1}{2}(a+b)$.

Now we prove part (1) of the theorem. Since (3) holds, we have that there is no definitely least solution (x,y) for c values with $ab - \frac{1}{2}(a+b) < c \le ab$. Then lemma 3 gives that $(-x, a-y)$ is a definitely least solution of $ax+by = ab-c$ where $ab - \frac{1}{2}(a+b) < ab-c \le ab$. Subtracting ab and then multiplying the inequality by -1 gives us $0 \le c < \frac{1}{2}(a+b)$. This proves (1) of the theorem.

Now consider the c values in $\frac{1}{2}(a+b) \le c \le ab - \frac{1}{2}(a+b)$. Let k and r be integers in the interval $\left[\frac{1}{2}(a+b),\ ab-\frac{1}{2}(a+b)\right]$ such that $k+r=ab$. We will show that either k results in a definitely least solution or r does, but not both. Note first that for $r = ab - \frac{1}{2}(a+b)$, the values $x = \frac{b-1}{2}$ and $y = \frac{a-1}{2}$ give $ax+by=r$ a definitely least solution. Therefore, lemma 3 shows that $ax+by = ab-r = ab - \left(ab - \frac{1}{2}(a+b)\right) = \frac{1}{2}(a+b) = k$ does not have a definitely least solution. Similarly, for any other values of k and r the above claim proves that either $ax+by=r$ or $ax+by = ab-r = k$ has a definitely least solution. Of course, since the length of the interval is odd, for every k there will be an r and vice versa. This proves part (2) of the theorem and hence Theorem 2 is proved. ■

Since a and b both cannot be even, as they are relatively prime, we now consider our last case. We begin with a lemma.

Lemma 4. *Let a and b be positive integers where a is an even integer and b is an odd integer. Further assume that a and b are relatively prime and that c is a nonnegative integer with $0 \leq c \leq ab+b-c$. Either (x,y) is a definitely least solution of $ax+by=c$ or $(-x,a-y+1)$ is a definitely least solution of $ax+by=ab+b-c$, but not both.*

Proof. Note that if (x,y) is a solution of $ax+by=c$, then $(-x,a-y+1)$ is a solution of $ax+by=ab+b-c$: just check that

$$\begin{aligned} a(-x)+b(a-y+1) &= -ax+ab-by+b \\ &= ab+b-(ax+by) = ab+b-c. \end{aligned}$$

Assume that (x,y) is a definitely least solution of $ax+by=c$. We will show that $(-x,a-y+1)$ is not a definitely least solution of $ax+by=ab+b-c$. Since a is even and b is odd, we have that $|x|<\frac{b}{2}$ and $|y|\leq\frac{a}{2}$. Note that $|-x|=|x|<\frac{b}{2}$ and

$$|a-y+1| \geq 1+|a-y| > 1+a-|y| \geq 1+a-\frac{a}{2} = 1+\frac{a}{2} > \frac{a}{2}.$$

Thus, $(-x,1+a-y)$ is a minimal solution with respect to x, but not a definitely least solution of $ax+by=ab+b-c$. From what we just proved, the contrapositive; namely, if $(-x,1+a-y)$ is a definitely least solution of $ax+by=ab+b-c$ then (x,y) is not a definitely least solution of $ax+by=c$, completes the proof of lemma 4. ■

Theorem 2. *Let a and b be positive integers where a is an even integer and b is an odd integer. Further assume that a and b are relatively prime and that c is a nonnegative integer. Then $ax+by=c$ has:*

1. *A definitely least solution for every $0 \leq c < \frac{1}{2}a+b$.*
2. *A definitely least solution for exactly half the c values with*

$$\frac{1}{2}a+b \leq c \leq ab-\frac{1}{2}a.$$

   Furthermore, for integers k and r in the interval $\left[\frac{1}{2}a+b,\ ab-\frac{1}{2}a\right]$ with $k+r=ab+b$, either $c=k$ or $c=r$ has a definitely least solution, but not both.

3. *Does not have a definitely least solution for every $c > ab-\frac{1}{2}a$.*

Proof. Note first that if (x,y) is a definitely least solution so that $|x|\leq\frac{b}{2}$ and $|y|\leq\frac{a}{2}$, then $c \leq ab-\frac{1}{2}(a+b)$. Since b is odd, we obtain

$$c = ax+by \leq a|x|+b|y| \leq \frac{a(b-1)}{2}+\frac{ab}{2} = ab-\frac{1}{2}a.$$

Therefore, if $c > ab - \frac{1}{2}a$ then there is not a definitely least solution. This proves (3) of the theorem.

For the remainder of the proof, we can assume that $0 \leq c \leq ab - \frac{1}{2}a$.

Now we prove part (1). Since (3) holds, we have that there is no definitely least solution (x,y) for $ax+by=c$ with c values $ab-\frac{1}{2}a < c \leq ab+b$. Then lemma 4 gives that $(-x, a-y+1)$ is a definitely least solution of $ax+by = ab+b-c$ for c values with $ab-\frac{1}{2}a < ab+b-c \leq ab+b$. Subtracting $ab-b$ and then multiplying the equality by -1 gives us $0 \leq c < \frac{1}{2}a+b$. This proves (1).

Finally, consider the c values in $\frac{1}{2}a+b \leq c \leq ab-\frac{1}{2}a$. Let k and r be integers in the interval $[\frac{1}{2}a+b, ab-\frac{1}{2}a]$ such that $k+r = ab+b$. We will show that either k results in a definitely least solution or r does but not both. Note first that for $r = ab-\frac{1}{2}a$ the values $x = \frac{b-1}{2}$ and $y = \frac{a}{2}$ give that $ax+by = r$ a definitely least solution. Indeed, $ax+by = \frac{a(b-1)}{2} + \frac{ab}{2} = ab - \frac{1}{2}a = r$. Therefore, lemma 4 shows that $ax+by = ab+b-r = ab+b-\left(ab-\frac{1}{2}a\right) = \frac{1}{2}a+b = k$ does not have a definitely least solution. Similarly, for any other values of k and r the above claim proves that either $ax+by = r$ or $ax+by = ab+b-r = k$ has a definitely least solution. This proves part (2) of the theorem.
Hence, Theorem 2 is proved. ■

Example
Consider the Diophantine equation $9x+89y = c$.

> Since both 9 and 89 are odd integers, Theorem 1 applies. We have $\frac{1}{2}(a+b) = 49$ and $ab - \frac{1}{2}(a+b) = 752$. Part (2) of Theorem 1 gives that for any k and r in $[49, 749]$ with $k+r = ab = 801$ either $9x+89y = k$ or $9x+89y = r$ has a definitely least solution, but not both. For example, let $k = 52$ so that $r = 749$. Let's solve $9x+89y = 749$ first. Since 9 is smaller that 89, we'll find the minimum solution with respect to y. Reducing the equation modulo 9, we have $8y \equiv 2 \mod 9$ which is equivalent to $-y \equiv 2 \mod 9$ which gives $y = -2$. Notice this has the required $2 = |y| \leq \frac{a-1}{2} = 4$. Substituting this into our equation and solving for x yields $x = 103$. Since $|x| = 103 > \frac{b-1}{2} = 44$, we do not have a definitely least solution to the equation $9x+89y = 749$. Therefore, we know the equation $9x+89y = 52$ does have a definitely least solution. Again, we'll find the minimum solution with respect to y since y is smaller than 89 (and so easier to solve). Reducing the equation modulo 9, we have $-y \equiv -2 \mod 9$ which is equivalent to $y \equiv 2 \mod 9$. Therefore, $y = 2$. Substituting this into our equation and solving for x yields $x = -14$. Of course, we now have the definitely least solution $(2, -14)$ of $9x+89y = 52$.

The following corollaries give a couple of special cases.

Corollary 1. *Let a and b both be odd positive relatively prime integers and con-*

sider the equations

$$ax+by=\frac{1}{2}(a+b) \tag{1}$$

and,

$$ax+by=ab-\frac{1}{2}(a+b). \tag{2}$$

The minimal solutions to (1) are $\left(\frac{1-b}{2},\frac{1+a}{2}\right)$ *and* $\left(\frac{1+b}{2},\frac{1-a}{2}\right)$. *The definitely least solution of (2) is* $(\frac{b-1}{2},\frac{a-1}{2})$.

Proof. We showed in our proof of part 2 of Theorem 1 that equation (1) has no definitely least solution. So we first check that both $\left(\frac{1-b}{2},\frac{1+a}{2}\right)$ and $\left(\frac{1+b}{2},\frac{1-a}{2}\right)$ satisfy the equation in (1). Note that

$$ax+by=a\left(\frac{1-b}{2}\right)+b\left(\frac{1+a}{2}\right)=\frac{1}{2}(a-ab+b+ab)=\frac{1}{2}(a+b)$$

and that

$$ax+by=a\left(\frac{1+b}{2}\right)+b\left(\frac{1-a}{2}\right)=\frac{1}{2}(a+ab+b-ab)=\frac{1}{2}(a+b),$$

so we have two solutions.
Now, since $1-b\leq 0$, we have $|x|=\left|\frac{1-b}{2}\right|=\frac{b-1}{2}\leq\frac{b}{2}$ and so $(\frac{1-b}{2},\frac{1+a}{2})$ is the minimal solution with respect to x. Similarly, since $1-a\leq 0$, we have $|y|=\left|\frac{1-a}{2}\right|=\frac{a-1}{2}\leq\frac{a}{2}$ and so $(\frac{1+b}{2},\frac{1-a}{2})$ is the minimal solution with respect to y. Interestingly, this gives (x,y) such that $ax+by=|x|+|y|$. Similarly, we also showed in part 2 of Theorem 1 that equation (2) has a definitely least solution. Since

$$ax+by=a\left(\frac{b-1}{2}\right)+b\left(\frac{a-1}{2}\right)=\frac{ab}{2}-\frac{a}{2}+\frac{ab}{2}-\frac{b}{2}=ab-\frac{1}{2}(a+b),$$

we have a solution. Showing minimality with respect to both x and y, $|x|=\left|\frac{b-1}{2}\right|=\frac{b-1}{2}\leq\frac{b}{2}$ and $|y|=\left|\frac{a-1}{2}\right|=\frac{a-1}{2}\leq\frac{a}{2}$, and so we have the definitely least solution. This completes the proof of Corollary 1. ■

Corollary 2. *Let a and b be positive relatively prime integers where a is an even integer and b is an odd integer and consider the equations*

$$ax+by=\frac{1}{2}a+b \tag{3}$$

and,

$$ax+by=ab-\frac{1}{2}a \tag{4}$$

The minimal solutions to (3) are $(\frac{1-b}{2},1+\frac{a}{2})$ and $(\frac{1+b}{2},1-\frac{a}{2})$. The definitely least solution of (4) is $(\frac{b-1}{2},\frac{a}{2})$.

Proof. We showed in our proof of part 2 of Theorem 2 that equation (3) has no definitely least solution. So we first check that both $\left(\frac{1-b}{2},1+\frac{a}{2}\right)$ and $(\frac{1+b}{2},1-\frac{a}{2})$ satisfy the equation (3). Note that

$$ax+by=a\left(\frac{1-b}{2}\right)+b\left(1+\frac{a}{2}\right)=\frac{1}{2}a-\frac{ab}{2}+b+\frac{ab}{2}=\frac{1}{2}a+b$$

and that

$$ax+by=a\left(\frac{1+b}{2}\right)+b\left(1-\frac{a}{2}\right)=\frac{1}{2}a+\frac{ab}{2}+b-\frac{ab}{2}=\frac{1}{2}a+b,$$

so we have two solutions.
Since $1-b\leq 0$ we have $|x|=\left|\frac{1-b}{2}\right|=\frac{b-1}{2}\leq\frac{b}{2}$ and so $(\frac{1-b}{2},1+\frac{a}{2})$ is the minimal solution with respect to x. Similarly, since $1-\frac{a}{2}\leq 0$, we have $|y|=\left|1-\frac{a}{2}\right|=\frac{a}{2}-1\leq\frac{a}{2}$ and so $(\frac{1+b}{2},1-\frac{a}{2})$ is the minimal solution with respect to y. We also showed in part 2 of Theorem 2 that (4) has a definitely least solution. So we first check that $(\frac{b-1}{2},\frac{a}{2})$ satisfies the equation. Since

$$ax+by=a\left(\frac{b-1}{2}\right)+b\left(\frac{a}{2}\right)=\frac{ab}{2}-\frac{1}{2}a+\frac{ab}{2}=ab-\frac{1}{2}a,$$

we have a solution. Since $|x|=\left|\frac{b-1}{2}\right|=\frac{b-1}{2}\leq\frac{b}{2}$ and $|y|=\left|\frac{a}{2}\right|=\frac{a}{2}\leq\frac{a}{2}$, we have the definitely least solution.
This completes the proof of Corollary 2. ■

Thus far, we have assumed that the $\gcd(a,b)=1$, however, we will now remove this condition. Namely, to solve $ax+by=c$ with $\gcd(a,b)\neq 1$. As stated in our first theorem, we must have that $\gcd(a,b)|c$ for there to be any solutions. With this necessary assumption, we first compute

$$\frac{a}{d}x+\frac{b}{d}y=\frac{c}{d}\text{ where }d=\gcd(a,b). \tag{5}$$

For simplicity, we will set $a'=\frac{a}{d},b'=\frac{b}{d}$, and $c'=\frac{c}{d}$. Since we are dividing a and b by $\gcd(a,b)=d,a'$ and b' are now relatively prime, so we can proceed as before to find the minimal solutions with respect to both x and y for the equation

$a'x+b'y=c'$.
The minimal solutions with respect to both x and y, or the definitely least solution of $a'x+b'y=c'$ will also be just that for the equation $ax+by=c$ with $\gcd(a,b)\neq 1$. After all, any solution (x,y) to $ax+by=c$ is also a solution to $k[ax+by=c]$ where k is a nonzero integer.

We will illustrate this in the following example.

Example
Consider the Diophantine equation $68x+110y=52$.

> The $\gcd(68,110)=2$ so we divide the equation by 2 and obtain the equation $34x+55y=26$. Thanks to part 1 of Theorem 2 we know that this equation has a definitely least solution. We found this solution, namely $(4,-2)$, in an earlier example. This, of course, is also the definitely least solution to $68x+110y=52$.

For further investigation, we can attempt to generalize to a Linear Diophantine equation in more than two variables.

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A 4×4 Magic Square Whose Entries Are Consecutive Integers

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Abstract

The main goal of this paper is to construct and explore the hidden and surprising properties of a 4×4 magic square whose entries are consecutive integers. As an application, we answer a puzzle considered by Les Reid, which yields the solution as a special case. Furthermore, we deduce a sprinkling of enigmatic 4×4 magic squares, recorded by Ramanujan in his second notebook.

Introduction

A ***magic square*** is a square array of numbers with the property that the sum of the numbers in each row, column and diagonal is the same, known as the "magic constant." A ***Gnomon Magic Square*** is a 4×4 magic square in which the sums of the entries in each 2x2 corner is equal to the magic constant. A ***broken diagonal*** is traced out by starting from any cell and moving parallel to a principal diagonal $(n-1)$-times, one cell at a time, cyclically; that is as though each edge of the square were joined to the opposite edge. A magic square is ***pandiagonal*** if the sum of elements in each broken diagonal is the magic constant.

While magic squares have been known and studied for many centuries, it is astonishing that we still do not know today,the hidden and surprising properties of certain types of magic squares. In an effort to make progress on these surprising properties, here we construct and explore the hidden and surprising properties of a 4×4 magic square whose entries are consecutive integers.

4×4 Magic Square Whose Entries Are Consecutive Integers

Consider the following array:

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(1)

The reader can verify that the rows, columns and diagonals (left and right) add to the same sum, the magic constant $M = 4x + 30$.

In [4], Prof. Les Reid poses the puzzle of finding a 4×4 magic square whose entries are consecutive integers such that the sum is 2014. Letting x=496, the above 4×4 magic square reduces to the 4×4 magic square below, which answers the puzzle posed by Dr. Reid.

503	506	509	496
508	497	502	507
498	511	504	501
505	500	499	510

Letting $x = 1$, (1) produces the 4×4 magic square below, which is a magic square with magic constant 34 [1, chap. I, p. 6], [3, chap. I, p. 10].

8	11	14	1
13	2	7	12
3	16	9	6
10	5	4	15

Letting $x = 9$, (1) produces the 4×4 magic square below, which is a magic square with magic constant 66 [1, chap. I, p. 6], [3, chap. I, p. 10].

16	19	22	9
21	10	15	20
11	24	17	14
18	13	12	23

Fascinating Properties of the Magic Square (1)

What is so different about this 4×4 magic square of consecutive integers? This 4×4 magic square of consecutive integers has the following interesting properties, which makes the square even more "magical."

(1) The sums of the entries in the four sets of like colored cells are again equal to the magic constant $4x + 30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(2) The sums of the entries in the red and yellow sets are yet again equal to the magic constant $4x+30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(3) The sums of the entries in the four sets of like colored cells are again equal to the magic constant $4x+30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(4) The sums of the entries in the red and yellow sets are again equal to the magic constant $4x+30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(5) The sums of the entries in the red and yellow sets are again equal to the magic constant $4x+30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(6) The sums of the entries in the four sets of like colored cells are again equal to the magic constant $4x+30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(7) The sum of the corner entries in the red 3×3 is again equal to the magic constant $4x+30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(8) The sum of the corner entries in the red 3×3 is again equal to the magic constant $4x+30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(9) The sum of the corner entries in the red 3×3 is again equal to the magic constant $4x+30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(10) The sum of the corner entries in the red 3×3 is again equal to the magic constant $4x+30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

(11) The sums of the entries in the four sets of like colored cells are again equal to the magic constant $4x+30$.

$x+7$	$x+10$	$x+13$	x
$x+12$	$x+1$	$x+6$	$x+11$
$x+2$	$x+15$	$x+8$	$x+5$
$x+9$	$x+4$	$x+3$	$x+14$

Intriguingly, from properties (1), (3), (4) and (5) we conclude that the sum of the entries in each of the 2×2 subsquares of the original 4×4 magic square is equal to the magic constant $4x+30$.

Property (3) indicates that the original 4×4 magic square is also a Gnomom magic square [2].

From properties (2) and (6) we see that this 4×4 magic square of consecutive integers is a pandiagonal magic square [5].

Finally, from properties (7), (8), (9), and (10), we conclude that the sums of the corners in each 3×3 subsquare of this 4×4 magic square is equal to the magic constant.

Acknowledgment

I am very grateful to the two anonymous referees for their remarks and suggestions, especially those concerning red and yellow sets.

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before July 31, 2024. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2024 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 920 - 927

Problem 920. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*
Let α be the golden ratio. Show that

$$\sum_{i=0}^{\infty} \frac{i}{\alpha^i} \left(\sum_{j=0}^{\infty} \left[\lim_{n\to\infty} \frac{F_n^2 + F_{n+2}^2 - F_{n+1}F_{n+3}}{F_{n-1}F_{n+2}} \right]^j \right)^{-1} = \alpha.$$

Problem 921. *Proposed by Mihaly Bencze, Braşov, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*
Solve in real numbers the following equation:

$$\log_2(x^2 + 2^x) + (x^2 - 1) * 2^{x+1} + x^4 + x^2 + 2^x = 3 * 4^x + x + 1.$$

Problem 922. *Proposed by Toyesh Prakash Sharma (student), Agra College, Agra, India.*
Let F_n be the n^{th} Fibonacci number defined by $F_1 = 1, F_2 = 1$ and, for all $n \geq 3, F_n = F_{n-1} + F_{n-2}$. Prove that $\sum_{n=1}^{\infty} \left(\frac{1}{9}\right)^{F_{n+2}}$ is an irrational number but not a transcendental number.

Problem 923. *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain.*
Let $n \geq 1$ be an integer. Compute

$$\lim_{n\to\infty} \frac{\binom{n+1}{2}}{2^{n-1}} \sum_{k=0}^{\infty} \frac{k+4}{(k+1)(k+2)(k+3)} \binom{n}{k}.$$

Problem 924. *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain.*
Let a,b,c be three positive real numbers. Prove that

$$\frac{a}{4b+7\sqrt{ab}} + \frac{b}{4c+7\sqrt{bc}} + \frac{c}{4a+7\sqrt{ca}} \geq \frac{3}{11}.$$

Problem 925. *Proposed by Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*
Prove that in any triangle ABC with usual notations (R = circumradius, r = inradius, s = semiperimeter, ma = median from vertex A) the following inequality is true:

$$2\sum m_a \leq 3\sqrt{\frac{R\left(s^2+r^2+Rr\right)}{2r}}.$$

Problem 926. *Proposed by the editor.*
Prove that the sequence $a_1 = 1, a_2 = 1, a_n = a_{n-1} + 2 * a_{n-2}$ for all $n > 2$ gives the number of integers between 2^n and 2^{n+1} which are divisible by 3.

Problem 927. *Proposed by the editor.*
Find the area below the two lines $8x+5y = 976$ and $6x+5y = 792$ that lies in the first quadrant.

SOLUTIONS TO PROBLEMS 901 - 910

Problem 901. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*
Suppose for some integer $k \geq 2$ that $a_1 < a_2 < \ldots < a_k$ are positive integers, and that A is their least common multiple. Prove that

$$a_1a_2 + a_2a_3 + \ldots + a_{k-1}a_k + a_ka_1 \leq A^2.$$

Solution *by the proposer.*

Each of $a_i (1 \leq i \leq k)$ is a divisor or A, therefore $\frac{A}{a_i}$ is a positive integer for each i. Also, the inequalities on a_i imply that

$$\frac{A}{a_1} > \frac{A}{a_2} > \ldots > \frac{A}{a_k}.$$

Therefore, $\frac{A}{a_k} \geq 1, \frac{A}{a_{k-1}} \geq 2$ and in general, $\frac{A}{a_i} \geq k+1-i$. Accordingly, for all i, we have

$$a_i \leq \frac{A}{k+1-i},$$

and

$$\begin{aligned}
&a_1a_2 + a_2a_3 + \ldots + a_{k-1}a_k + a_ka_1 \\
&\leq A/k * A/(k-1) + A/(k-2) * A/(k-3) + \cdots + A/2 * A/1 + A/1 * A/k \\
&\leq A^2(1/(k(k-1)) + 1/((k-1)(k-2)) + n + 1/(2*1) + 1/(1*k)).
\end{aligned}$$

Noticing that $\frac{1}{a(a-1)} = \frac{1}{a-1} - \frac{1}{a}$, we have that

$$\begin{aligned}
&A^2\left(\frac{1}{k(k-1)} + \frac{1}{(k-1)(k-2)} + \ldots + \frac{1}{2*1} + \frac{1}{1*k}\right) \\
&= A^2\left(\frac{1}{k-1} - \frac{1}{k} + \frac{1}{k-2} - \frac{1}{k-1} + \ldots + \frac{1}{1} - \frac{1}{2} + \frac{1}{k}\right) \\
&= A^2.
\end{aligned}$$

All terms in parentheses cancel except for $\frac{1}{1}$. Equality holds when $k = 2, a_1 = 1$ and $a_2 = 2$.

Also solved by Ioan Viorel Codreanu, Satulung, Maramures, Romania; Ivan Hadinata (student), Gadjah Mada University, Yogyakarta, Indonesia; and Kee- Wai Lau, Hong Kong, China.

Problem 902. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Without a computer, find $\Omega = \int_0^{\pi/30} \frac{\sin 5x \ sin 7x \ sin 8x}{\cos 2x \cos 3x \cos 5x \cos 10x} \ dx$.

Solution *by the proposer.*

Lemma. *If a,b,c are reals and $\cos a \cos b \cos c \cos(a+b+c) \neq 0$ then*

$$\tan(a+b+c) - (\tan a + \tan b + \tan c) = \frac{\sin(a+b)\sin(b+c)\sin(c+a)}{\cos a \cos b \cos c \cos(a+b+c)}.$$

Proof.

$$\begin{aligned}
&\tan(a+b+c) - (\tan a + \tan b + \tan c) \\
&\qquad = \tan(a+b+c) - \tan a - (\tan b + \tan c) \\
&\qquad = \frac{\sin(b+c)}{\cos a \cos(a+b+c)} - \frac{\sin(b+c)}{\cos b \cos c} \\
&\qquad = \frac{\sin(b+c)}{\cos a \cos b \cos c \cos(a+b+c)} (\cos b \cos c - \cos a \cos(a+b+c)).
\end{aligned}$$

It remains to prove that

$$\cos b \cos c - \cos a \cos(a+b+c) = \sin(a+b)\sin(c+a).$$

We have:

$$\begin{aligned}
&\cos b \cos c - \cos(a+b+c) = \cos b \cos c \\
&\qquad - \cos a[\cos a \cos(b+c) - \sin a \sin(b+c)] \\
&= \cos b \cos c - \cos a[\cos a(\cos b \cos c - \sin b \sin c) \\
&\qquad - \sin a(\sin b \cos c + \cos b \sin c)] \\
&= \cos b \cos c - \cos^2 a(\cos b \cos c - \sin b \sin c) \\
&\qquad + \cos a \sin a(\sin b \cos c + \cos b \sin c) \\
&= \cos b \cos c - \cos^2 a \cos b \cos c + \cos^2 a \sin b \sin c + \cos a \sin a \sin c \cos c \\
&\qquad + \cos a \sin a \cos b \sin c - \sin^2 a \cos b \cos c + \sin^2 a \cos b \cos c \\
&= \cos b \cos c - (\cos^2 a + \sin^2 a) \cos b \cos c + \\
&\qquad (\sin a \cos b + \cos a \sin b)(\sin c \cos a + \cos c \sin a) \\
&= \sin(a+b)\sin(c+a)
\end{aligned}$$

■

Now

$$\begin{aligned}
\Omega &= \int_0^{\pi/30} \frac{\sin 5x \sin 7x \sin 8x}{\cos 2x \cos 3x \cos 5x \cos 10x}\, dx \\
&= \int_0^{\pi/30} \frac{\sin(2x+3x)\sin(3x+5x)\sin(5x+2x)}{\cos 2x \cos 3x \cos 5x \cos(2x+3x+5x)}\, dx \\
&= \int_0^{\pi/30} (\tan(2x+3x+5x) - (\tan 2x + \tan 3x + \tan 5x))\, dx \quad \text{by the lemma} \\
&= \int_0^{\pi/30} \tan(10x - \tan 2x - \tan 3x - \tan 5x)\, dx \\
&= \left(\frac{-1}{10} \ln|\cos 10x| + \frac{1}{2} \ln|\cos 2x| + \frac{1}{3} \ln|\cos 3x| + \frac{1}{5} \ln|\cos 5x| \right) \Bigg|_0^{\pi/30} \\
&= \frac{-1}{10} \ln \frac{1}{2} + \frac{1}{2} \ln\left(\cos \frac{\pi}{15}\right) + \frac{1}{3} \ln\cos\left(\frac{\pi}{10}\right) + \frac{1}{5} \ln\left(\frac{\sqrt{3}}{2}\right).
\end{aligned}$$

Also solved by Kee-Wai Lau, Hong Kong, China; and Albert Stadler, Herrliberg, Switzerland.

Problem 903. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*
Solve for complex numbers:

$$\begin{vmatrix} x^2 & x^2+3x & x^2+6x+9 \\ x^2+2x+1 & x^2+5x+4 & x^2+8x+16 \\ x^4+2x^2+1 & x^4+3x^2+2 & x^4+4x^2+4 \end{vmatrix} = 0.$$

Solution *by Brian Beasley, Simpsonville, SC.*

Let $a = x, b = x+3, c = x+1, d = x+4, e = x^2+1$, and $f = x^2+2$. Then the given equation has the form

$$\begin{vmatrix} a^2 & ab & b^2 \\ c^2 & cd & d^2 \\ e^2 & ef & f^2 \end{vmatrix}.$$

Thus we obtain

$$\begin{aligned}
a^2cdf^2 + abd^2e^2 + b^2c^2ef - b^2cde^2 - a^2d^2ef - abc^2f^2 \\
= (ad-bc)(cf-de)(af-be) = 0,
\end{aligned}$$

which simplifies to

$$-3\left(-3x^2+x-2\right)\left(-3x^2+x-3\right) = 0.$$

Hence the four complex solutions are

$$x = \frac{1 \pm i\sqrt{23}}{6} \quad \text{and} \quad x = \frac{1 \pm i\sqrt{35}}{6}$$

Also solved by Kee-Wai Lau, Hong Kong, China; Missouri State Problem Solving Group, Springfield, MO; Ioannis Sfikas, National and Kapodistrian University of Athen, Greece; Etisha Sharma, Agra College, Agra, India; Ioannis Sfikas, Athen, Greece; Albert Stadler, Herrliberg, Switzerland; and the proposer.

Problem 904. *Proposed by Albert Natian, Los Angeles Valley College, Valley Glen, CA.*
Find the n^{th} term of the sequence $(a_n)_{n\geq 0}$ defined recursively as follows:

$$a_0 = 0,\ \ a_1 = 1,\ \ a_2 = 0$$
$$\forall\, n \geq 3:\ a_n = \sum_{k=1}^{n-1} (n-k)(n-k-4)a_k.$$

Solution *by the proposer.*

It follows that

$$a_{n-1} = \textstyle\sum_{k=1}^{n-2} (n-1-k)\,(n-1-k-4)\,a_k. \tag{1}$$

We have

$$\begin{aligned}
a_n &= \textstyle\sum_{k=1}^{n-1} (n-k)(n-k-4)a_k \\
&= -3a_{n-1} + \textstyle\sum_{k=1}^{n-2} (n-k)(n-k-4)a_k \\
&= -3a_{n-1} + \textstyle\sum_{k=1}^{n-2} (n-1-k+1)\,(n-1-k-4+1)\,a_k \\
&= -3a_{n-1} + \textstyle\sum_{k=1}^{n-2} (n-1-k)\,(n-1-k-4)\,a_k + \sum_{k=1}^{n-2} (2n-2k-5)a_k \\
&= -3a_{n-1} + a_{n-1} + \textstyle\sum_{k=1}^{n-2} (2n-2k-5)\,a_k \quad \text{by (1)} \\
&= a_{n-1} + \textstyle\sum_{k=1}^{n-1} (2n-2k-5)\,a_k.
\end{aligned}$$

From this we get

$$a_n - \ a_{n-1} = \textstyle\sum_{k=1}^{n-1} (2n-2k-5)a_k$$

so

$$a_{n-1} - a_{n-2} = \textstyle\sum_{k=1}^{n-2} (2[n-1]-2k-5)\,a_k. \tag{2}$$

We have

$$\begin{aligned}
a_n &= a_{n-1} + \textstyle\sum_{k=1}^{n-1} (2n-2k-5)\,a_k \\
&= -2a_{n-1} + \textstyle\sum_{k=1}^{n-2} (2n-2k-5)\,a_k \\
&= -2a_{n-1} + \textstyle\sum_{k=1}^{n-2} (2[n-1]-2k-5+2)\,a_k \\
&= -2a_{n-1} + \textstyle\sum_{k=1}^{n-2} (2[n-1]-2k-5)\,a_k + 2\sum_{k=1}^{n-2} a_k \\
&= -2a_{n-1} + a_{n-1} - a_{n-2} + 2\textstyle\sum_{k=1}^{n-2} a_k \quad \text{by (2)} \\
&= -a_{n-1} - a_{n-2} + 2\textstyle\sum_{k=1}^{n-2} a_k.
\end{aligned}$$

Then $a_n = (-1)^{n-1} + F_{n-1}$ where F_n is the n^{th} Fibonacci number.

Also solved by Kee-Wai Lau, Hong Kong, China; and Albert Stadler, Herrliberg, Switzerland.

Problem 905. *Proposed by Vasile Mircea Popa, Lucian Blaga University, Sibiu, Romania.*
Calculate the following integral without a computer:

$$\int_1^\infty \frac{x\sqrt{x}\ln x}{(x+1)(x^2+1)}\,dx.$$

Solution *by the proposer.*

Let $A = \int_1^\infty \frac{x\sqrt{x}\ln x}{(x+1)(x^2+1)}\,dx$. In the integral make the variable change $x = \frac{1}{y}$. We obtain

$$A = -\int_0^1 \frac{y^{-1/2}\ln y}{(y+1)(y^2+1)}\,dy.$$

Let us denote

$$B = \int_0^1 \frac{y^{-\frac{1}{2}}\ln y}{(y+1)\,(y^2+1)}\,dy.$$

We have, successively,

$$\begin{aligned} B &= \int_0^1 \frac{(1-y)y^{-\frac{1}{2}}\ln y}{(1-y^4)}\,dy \\ &= \int_0^1 \frac{y^{-1/2}\ln y}{1-y^4}dy - \int_0^1 \frac{y^{1/2}\ln y}{1-y^4}dy \\ &= \int_0^1 \sum_{n=0}^{\infty} y^{4n-1/2}\ln y\,dy - \int_0^1 \sum_{n=0}^{\infty} y^{4n+1/2}dy) \\ &= \sum_{n=0}^{\infty}(\int_0^1 y^{4n-1/2}\ln y\,dy - \int_0^1 y^{4n+1/2}\ln y\,dy). \end{aligned}$$

We will use the following relationship:

$$\int_0^1 x^a \ln x\,dx = -\frac{1}{(a+1)^2}$$

We obtain

$$B = \sum_{n=0}^{\infty}\left[\frac{1}{(4n+3/2)^2} - \frac{1}{(4n+1/2)^2}\right] = \sum_{n=0}^{\infty}\left[\frac{1/16}{(n+3/8)^2} - \frac{1/16}{(n+1/8)^2}\right].$$

We now use the following relationship:

$$\psi_1(x) = \sum_{n=0}^{\infty}\frac{1}{(x+n)^2}$$

where $\psi_1(x)$ is the trigamma function. We obtain the value of B:

$$B = \frac{1}{16}\left[\psi_1\left(\frac{3}{8}\right) - \psi_1\left(\frac{1}{8}\right)\right]$$

. Hence

$$A = \frac{1}{16}\left[\psi_1\left(\frac{1}{8}\right) - \psi_1\left(\frac{3}{8}\right)\right].$$

Also solved by Kee-Wai Lau, Hong Kong, China; Kamaal Mirza, University of Punjab, Pakistan; Kaushik Mahanta, National Institute of Technology, Silchar, Assam, India;. Ankush Parcha, Indira Gandhi National Open University, New Delhi, India; and Ioannis Sfikas, Athen, Greece; Albert Stadler, Herrliberg, Switzerland.

Problem 906. *Proposed by Mihaly Bencze, Braşov, Romania and Neculai Stanciu, "George Emil Palade" School, Buzǎ, Romania.*
If $\lambda \geq 1$ and *ABC* is a triangle, prove that $\sum \left(\tan\frac{A}{4}\right)^{\lambda} \geq 3\left(2-\sqrt{3}\right)^{\lambda}$.

Solution *by Kee-Wai Lau, Hong Kong, China.*

For $0 < x < \frac{\pi}{2}$, let $f(x) = (\tan x)^{\lambda}$, so that

$$f''(x) = \lambda(\tan x)^{\lambda-2}(\sec x)^2\left(2(\tan x)^2 + (\lambda - 1)(\sec x)^2\right) > 0.$$

By Jensen's Inequality, we have

$$\sum\left(\tan\frac{A}{4}\right)^{\lambda} \geq 3\left(\tan\frac{A+B+C}{12}\right)^{\lambda} = 3(\tan\frac{\pi}{12})^{\lambda} = 3\left(2-\sqrt{3}\right)^{\lambda}.$$

Also solved by Ioan Viorel Codreanu, Satulung, Maramures, Romania; Ivan Hadinata (student), Gadjah Mada University, Yogyakarta, Indonesia; Toyesh Prakash Sharma, (student), Agra College, Agra, India; Ioannis Sfikas, Athen, Greece; Albert Stadler, Herrliberg, Switzerland; and the proposers.

Problem 907. *Proposed by Toyesh Prakash Sharma (student), Agra College, Agra, India.*
If $a,b,c > 0$, then show that

$$\left(\frac{a}{b+c}\right)^{\frac{a}{b+c}} + \left(\frac{b}{c+a}\right)^{\frac{b}{c+a}} + \left(\frac{c}{a+b}\right)^{\frac{c}{a+b}} \geq 3^{\frac{2}{3}}.$$

Solution *by Henry Ricardo, Westchester Area Math Circle, Purchase, NY.* It is

easy to determine that $f(x) = x^x$ is convex for $x > 0$ and increasing for $x > \frac{1}{e} \approx$

0.37. Now let $S = \frac{a}{(b+c)} + \frac{b}{(c+a)} + \frac{c}{(a+b)}$. Nesbitt's inequality gives us $S \geq \frac{3}{2}$. Applying Jensen's inequality yields

$$\begin{aligned}\left(\frac{a}{b+c}\right)^{\frac{a}{b+c}} + \left(\frac{b}{c+a}\right)^{\frac{b}{c+a}} + \left(\frac{c}{a+b}\right)^{\frac{c}{a+b}} \\ \geq 3\left(\frac{S}{3}\right)^{\frac{S}{3}} \geq 3\left(\frac{\frac{3}{2}}{3}\right)^{\frac{\left(\frac{3}{2}\right)}{3}} \\ \geq 3\left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{3\sqrt{2}}{2} > 3^{\frac{2}{3}}.\end{aligned}$$

Also solved by Ivan Hadinata (student), Gadjah Mada University, Yogyakarta, Indonesia; Kee-Wai Lau, Hong Kong, China; Ioannis Sfikas, Athen, Greece; Albert Stadler, Herrliberg, Switzerland; and the proposer.

Problem 908. *Proposed by Raluca Maria Caraion and Forică Anastase, "Alexandru Odobescu" High School, Lehliu-Gară, Călăraşi, Romania.*
If $a,b,c > 0$, then show that

$$\prod \frac{(1+ab)(1+ac)}{1+a\sqrt{bc}} \geq \left(1+\sqrt[3]{a^2b^2c^2}\right)^3.$$

Solution *by Henry Ricardo, Westchester Area Math Circle, New York.*

The AGM inequality gives us

$$\begin{aligned}\prod_{cyc} \frac{(1+ab)(1+ac)}{1+a\sqrt{bc}} &= \prod_{cyc} \frac{1+a(b+c)+a^2bc}{1+a\sqrt{bc}} \\ &\geq \prod_{cyc} \frac{1+2a\sqrt{bc}+a^2bc}{1+a\sqrt{bc}} \geq \prod_{cyc} \frac{\left(1+a\sqrt{bc}\right)^2}{1+a\sqrt{bc}} \\ &= \prod_{cyc} (1+a\sqrt{bc}) \geq (1+\sqrt{a^2b^2c^2})^3,\end{aligned}$$

where the last inequality is a known consequence of the AGM inequality:

$$\sqrt[3]{(a_1+b_1)(a_2+b_2)(a_3+b_3)} \geq \sqrt[3]{a_1a_2a_3} + \sqrt[3]{b_1b_2b_3}.$$

Equality holds in the original problem if and only if $a = b = c$.

Also solved by Ioan Viorel Codreanu, Satulung, Maramures, Romania; Ivan Hadinata (student), Gadjah Mada University, Yogyakarta, Indonesia; Kee-Wai Lau, Hong Kong, China; Ioannis Sfikas, Athen, Greece; Albert Stadler, Herrliberg, Switzerland; and the proposers.

Problem 909. *Proposed by Seán Stewart, King Abdullah University of Science and Technology, Saudi Arabia.*
If $m > 1$, without a computer evaluate

$$\int_0^{\pi/2} \frac{\cot\left(\frac{x}{2}\right)\sec x \log(\cos x)}{\sqrt[m]{\sec x - 1}}\,dx.$$

Solution *by Ankush Parcha, Indira Gandhi National Open University, New Delhi, India.*

$$\begin{aligned}
&\int_0^{\pi/2} \frac{\cot\left(\frac{x}{2}\right)\sec x \ln(\cos x)}{\sqrt[m]{\sec x - 1}}\,dx \\
&= \frac{1}{2}\int_0^{\pi/2} \frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt[m]{\cos x}}{\cos x\left(1-\cos^2\left(\frac{x}{2}\right)\right)\sqrt[m]{1-\cos x}}\ln(\cos x)\,dx \\
&= \frac{1}{2}\int_0^{\pi/2} \frac{\sin x\sqrt[m]{\cos x}}{\cos x\left(\frac{2-\cos x-1}{2}\right)\sqrt[m]{1-\cos x}}\ln(\cos x)\,dx
\end{aligned}$$

$$\begin{aligned}
&= \int_0^1 \frac{\sqrt[m]{x}}{x(1-x)\sqrt[m]{1-x}}\ln(x)dx \\
&= \int_0^1 x^{\frac{1}{m}-1}(1-x)^{-\frac{1}{m}-1}\ln(x)\,dx \\
&= \beta\left(\frac{1}{m},-\frac{1}{m}\right)\left[\psi^{(0)}\left(-\frac{1}{m}\right)-\lim_{x\to 0}\psi^{(0)}(x)\right] \\
&= \Gamma\left(\frac{1}{m}\right)\Gamma\left(-\frac{1}{m}\right)\left[\lim_{x\to 0}\frac{\psi^{(0)}\left(-\frac{1}{m}\right)}{\Gamma(x)}-\lim_{x\to 0}\frac{\psi^{(0)}(x)}{\Gamma(x)}\right] \\
&= \Gamma\left(\frac{1}{m}\right)\Gamma\left(-\frac{1}{m}\right)\left[\lim_{x\to 0}\frac{x\psi^{(0)}(1+x)}{\Gamma(1+x)}-\lim_{x\to 0}\frac{1}{\Gamma(1+x)}\right] \\
&= \Gamma\left(\frac{1}{m}\right)\Gamma\left(-\frac{1}{m}\right).
\end{aligned}$$

Also solved by Kee-Wai Lau, Hong Kong, China; Albert Stadler, Herrliberg, Switzerland; and the proposer.

Problem 910. *Proposed by the editor.*
Prove that the sum of the last two digits of 2^n is never 17.

Solution 1 *by Hyunbin Yoo, South Korea.*

Define $a_n = 2^n$. Suppose there exists some positive integer k so that the sum of the last two digits of a_k is 17. That means that a_k ends in 98 or 89. Since a_k is even, we are left with only 98. We can express a_k as $100m+98$ where m is a positive integer. Then $a_k - 1 = 50m + 49$. If m is odd, $50m + 49 = 50(2m' + 1) + 49 = 100m' + 99$ where m' is a non-negative integer. If m is even, $50m + 49 = 50(2m') + 49 = 100m'z = 49$ where m' is a positive integer. Thus $a_k - 1$ ends in 99 or 49 which contradicts that it is even. Therefore the sum cannot be 17.

Solution 2 *by Albert Stadler, Herrliberg, Switzerland.*

The first 22 terms of the sequence $2^n(\mod 100)$ for $n \geq 1$ are 2, 4, 8, 16, 32, 64, 28, 56, 12, 24, 48, 96, 92, 84, 68, 36, 72, 44, 88, 76, 52, 4 which shows that the sequence is periodic with period 20. The sum of the last two digits assumes all the values between 2 and 16, but never 17.

Also solved by Brian Beasley, Simpsonville, SC; Ioan Viorel Codreanu, Satulung, Maramures, Romania; Ivan Hadinata (student), Gadjah Mada University, Yogyakarta, Indonesia; Kee-Wai Lau, Hong Kong, China; Albert Stadler, Herrliberg, Switzerland; John Zerger, Catawba College, Salisbury, NC; and the proposer.

Thanks to all who worked diligently on these challenging problems.
Congratulations to Kee-Wai Lau on successfully solving **all** of them!

Citation for Dr. Cynthia J. Huffman

The George R. Mach Distinguished Service Award Recipient

April 15, 2023

The George R. Mach Distinguished Service Award is presented each biennium to an individual who has made major contributions to the Society. Nominations are solicited from the chapters and the National Council determines the recipient. The chapter with which the recipient is affiliated receives a monetary award of $500.

The George Mach Distinguished Service Award is KME's most important award, and this year the recipient is Cynthia J. Huffman. Cynthia has been a member of Kansas Alpha for 40 years. She was Kansas Alpha's Faculty Advisor for 13 years and Corresponding Secretary for 3 years. She was the North Central Regional Director from 2001 to 2009, the National Treasurer from 2007 to 2015, and the National Historian from 2017 to 2021. Two items of special interest are that she was the creator of the KME Facebook page, and she served as the editor of the 90th Anniversary Edition of KME's History. She has served with distinction in all of her many roles. KME is a better organization because of her.

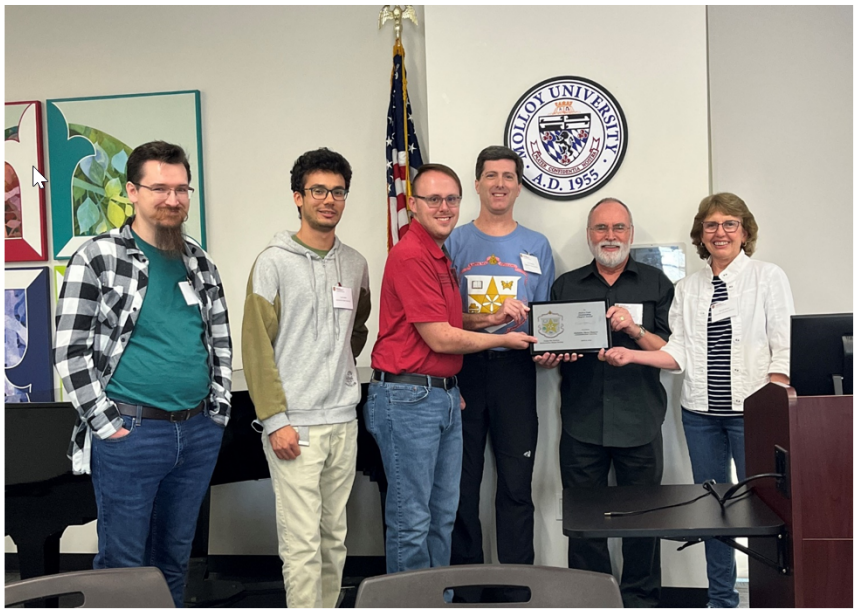
Dr. Cynthia J. Huffman

Alabama Theta

The Rhonda McKee and Don Tosh Outstanding Chapter Award

April 15, 2023

Established in 2021, the Rhonda McKee and Don Tosh Outstanding Chapter Award is granted biennially to recognize an active chapter which exemplifies the purposes of KME. The chapter receiving the award this biennium is Alabama Theta, at Jacksonville State University in Jacksonville, Alabama. In the description of this award there are several criteria listed. These criteria include regular initiations, filing regular reports, attending regional and national conventions, having student presenters, holding regular chapter meetings, and hosting conventions. The hope is that qualifying chapters will strive to attain many of these goals. Alabama Theta met every one of these goals, and has been active in KME since their installation in 2010. This chapter indeed exemplifies the purposes of KME. The award includes a plaque and a check, and Rhonda McKee and Don Tosh had the pleasure to make this presentation to the Corresponding Secretary of Alabama Theta, David Dempsey, and student members of the chapter at the national convention.



Pictured left to right: Marcus Shell, Lucas Saone, Dakota Heathcock, David Dempsey, Don Tosh, and Rhonda McKee

Report of the 46th Biennial National Convention

Kappa Mu Epsilon
April 13-15, 2023
Molloy University, New York Rho
Rockville Center, New York

The Forty-sixth Biennial Convention of Kappa Mu Epsilon was held April 13-15, 2023 and was hosted by the New York Rho chapter at Molloy University in Rockville Centre, NY. It should be noted here that the National Council voted previously to update the numbering of the biennia to reflect the actual number of years that KME has been in existence. Thus, the numbering of the biennia skipped from the 43rd in 2021 to the 46th in 2023. The numbering discrepancy resulted from having no conventions during the period from 1941-1947 due to conditions resulting from World War II.

Thursday, April 14, 2023

A reception and mixer took place from 7:00-9:00 p.m. in Room 339 of the Hagan building. Chapters could pick up their registration folders and there was plenty of food and socializing. At 8:00 p.m. the National Council, regional directors, and Pentagon editor met in Hagan 014. Some regional directors were unable to attend in person, but were able to join by a Zoom conference call.

Friday, April 13, 2023

Friday's activities began with breakfast in Hagan 339, followed by the First General Session.

First General Session
The first General Session began at 8:30 Friday morning. Don Tosh, KME National President, welcomed all participants. Rhonda McKee, acting KME National Secretary, called the roll. At the time of the roll call, 17 faculty members and 29 students were present, representing 13 chapters.

After the roll call, President Tosh introduced the national officers. He expressed the deep sympathy of the National Council at the passing of National Secretary, Steven Shattuck. Steve had been involved in KME for many years, from the time he was a student through his faculty years. He will be greatly missed by the National Council and all of KME, and in particular by his home chapter, MO Beta.

President Tosh, then introduced the candidates for National Secretary and National Treasurer. These candidates were nominated by the Nominating Committee. Dr. David Dempsey from AL Theta chapter was nominated for the secretary

position and Dr. Rajarshi (Raj) Dey from the KS Beta chapter was nominated for the treasurer position. Brief biographies of the candidates were distributed to the delegates. President Tosh called for nominations from the floor. Seeing none, nominations ceased and the two candidates were elected by acclamation.

Paper Session #1
President Elect Scott Thuong presided over the paper sessions. The first session of student presentations began at 9:00. Presentations during this session were:

- Peter Russell, Missouri Theta. Title: *Circular Projections of Ellipsoids*
- Catherine McClure, New York Rho. Title: *Perfect Numbers in Quadratic Fields*
- Julia DiCianni, New York Rho. Title: *COVID. Math Anxiety. Student Performance*

Paper Session #2
At 10:10, after a brief break, the second student paper session began.

- Ian Coleman-Hull, Kansas Beta. Title: *Explaining Economic Connectedness in Kansas*
- Abigail Branson, Tennessee Gamma. Title: *Lost in the Divergence Zone*
- Celine Lukito, New York Rho. Title: *An Introduction to Topological Data Analysis Through the Lens of Persistent Homology*
- John Mancuso, Rhode Island Beta. Title: *Song Recommendation using K-nearest neighbors Algorithm*

Group Photo and Lunch
After the second paper session, a group photo was taken outside the building. Lunch was served in the same room as the presentations, and enough time was allowed for participants to socialize or go for a walk on the beautiful campus.

Keynote Address
Following lunch, Jamie Stone of Johnson and Wales University gave a keynote address titled, *Pop Culture in the Mathematics Classroom*. The speaker shared examples of many projects he has assigned in mathematics and statistics courses to engage students by using pop culture. Projects included such things as predicting Oscar winners, Family Feud, and Disney bracketology. Students and faculty alike found the talk to be entertaining and resourceful.

Paper Session #3
Three more students presented papers during the third session.

- Sanskar Neupane, Kansas Delta. Title: *Exploring Ways to Model Stock Prices*
- Perry Wu, New York Rho. Title: *A Study on the (Mis)use of Mathematical Definition*

- James Gillin, Kansas Delta. Title: *Eulerian Numbers and Relations to Number Triangles and Infinite Series*

Faculty-Led Workshop
Following a short break, Dr. John Snow of Texas Kappa presented a workshop titled, *Programming in the Liberal Arts Classroom*. Faculty and students both participated in the workshop which was held in a computer lab. Participants were introduced to a programming package which they used to create works of art. The project was a fun learning experience.

Friday Evening
Friday evening was intentionally left open so that participants could explore New York. Some students took public transportation to Times Square, others went to Brooklyn to try out a restaurant, and others drove to the beach.

Saturday, April 15, 2023

The meetings on Saturday were held in the Larini room in the Public Square building. The day began with breakfast at 8:30 a.m., followed by student presentations and a faculty workshop.

Paper Session #3
The last session of student papers began at 9:00 a.m.

- Jack Lin and Victoria Risner, Missouri Theta. Title: *Triangles with Interlacing Rows*
- Austin Crabtree, Kansas Beta. Title: *What is a Sandwich? A Fuzzy Approach*
- Dakota Heathcock, Alabama Theta. Title: *To Know or Not to Know: A Bayesian Argument in Favor of Decision Theory*

Faculty-Led Workshop
Rajarshi Dey, of Kansas Beta chapter, led this second workshop, titled, *An Introduction to R*. This workshop was also held in a computer lab, so participants could do some "hands-on" programming in R. R is an open-source programming language that is mostly used for exploratory and inferential data analysis and creating user-friendly plots.

Committee Meetings
While the workshop was taking place, the Awards Committee and the Resolutions Committee met. The Awards Committee ranked the student presentations and chose the winners (see below). The Resolutions Committee wrote up resolutions to be adopted by the convention. Their resolutions can be found in their

committee report attached.

Concluding Business Meeting
The final meeting of the convention began at 11:20 a.m.

Officer and Committee Reports
President Don Tosh presided over concluding session. The following National Officers gave brief reports. Hard copies of their reports are attached to this convention report.

For the Continuation of New Business, the following national officers made reports:

- John W. Snow, Webmaster
- Doug Brown, Editor, *The Pentagon*
- Mark Hughes, Historian
- David Dempsey, Treasurer
- Rhonda McKee, Acting Secretary
- Scott Thuong, President-Elect
- Don Tosh, President

Manyiu Tse gave a report of the auditing committee. His committee joined him at the front of the room. His committee consisted of Manyiu Tse, New York Rho, chair; Rajarshi Dey, Kansas Beta; Abigail Branson, Tennessee Gamma; and James Gillin, Kansas Delta.

The Resolutions Committee gave their report. The committee consisted of Dianne Twigger, Missouri Theta, chair; Dakota Heathcock, Alabama Theta; Victoria Risner, Missouri Theta; and Lucas Saone, Alabama Theta.

Presentation Awards
The Awards Committee came forward to give their report, which consisted of announcing the top three papers. The committee members were Scott Thuong, Kansas Alpha, chair; Rhonda McKee, Missouri Beta; Deborah Upton, New York Rho; Stephanie Marcella, Missouri Beta; and Dylan Miller, Missouri Beta. The award-winning papers were:

- Third place, tie:
  - Celine Lukito, New York Rho. Title: *An Introduction to Topological Data Analysis Through the Lens of Persistent Homology*
  - Sanskar Neupane, Kansas Delta. Title: *Exploring Ways to Model Stock Prices*
- Second place: Austin Crabtree, Kansas Beta. Title: *What is a Sandwich? A Fuzzy Approach*

- First place: James Gillin, Kansas Delta. Title: *Eulerian Numbers and Relations to Number Triangles and Infinite Series*

The winning presenters were given cash awards and lots of photographs were taken.

Mach Award
President Tosh announced that the recipient of the George R. Mach Distinguished Service Award is Cynthia Huffman. Cynthia has been a member of Kansas Alpha for 40 years. She was Kansas Alpha's Faculty Advisor for 13 years and Corresponding Secretary for 3 years. She was the North Central Regional Director from 2001 to 2009, the National Treasurer from 2007 to 2015, and the National Historian from 2017 to 2021. Two items of special interest are that she was the creator of the KME Facebook page, and she served as the editor of the 90th Anniversary Edition of KME's History. She has served with distinction in all of her many roles. KME is a better organization because of her.

McKee-Tosh Award
Established in 2021, the Rhonda McKee and Don Tosh Outstanding Chapter Award is granted biennially to recognize an active chapter which exemplifies the purposes of KME. The chapter receiving the award this biennium is Alabama Theta, at Jacksonville State University in Jacksonville, Alabama. The criteria for this award include regular initiations, filing regular reports, attending regional and national conventions, having student presenters, holding regular chapter meetings, and hosting conventions. The hope is that qualifying chapters will strive to attain many of these goals. Alabama Theta met every one of these goals, and has been active in KME since their installation in 2010. This chapter indeed exemplifies the purposes of KME. The award includes a plaque and a check, and Rhonda McKee and Don Tosh had the pleasure to make this presentation to the Corresponding Secretary of Alabama Theta, David Dempsey, and other members of the chapter.

Installation of New Officers
President Tosh installed David Dempsey as the new National Secretary and Rajarshi Dey as the new National Treasurer. The new officers' terms began at their installation.

President Tosh also thanked David Dempsey for his many years as KME National Treasurer and presented him with a plaque in honor of his service. A plaque in honor of Steve Shattuck's year's of service to KME as National Secretary was presented to Rhonda McKee, who will present it to Steve's family.

Call for Hosts
The convention ended with a call for hosts for the 2025 convention.

Rhonda McKee
Acting National Secretary

Report of the National President

I was installed as president at the virtual 43rd National Convention hosted by Missouri Beta in April, 2021. In the last two years I have attempted to represent Kappa Mu Epsilon, and work toward maintaining the level of service we provide to our constituents.

One of the main responsibilities of the president is to be the contact person for schools wishing to form new chapters. Schools will typically make initial contact with KME through our website, and we have chapters which made the initial contact, obtained all the forms, and filled them out completely from what was available on our website. But at some point they need to have person-to-person contact with a representative of KME in order to complete the process.

The three chapters that have been installed during the past biennium are:

- Arkansas Gamma at Harding University in Searcy (virtual, April 27, 2021)
- Georgia Theta at College of Coastal Georgia in Brunswick (virtual, Oct. 22, 2021)
- California Theta at William Jessup University in Rocklin (in-person, Oct. 17, 2022)

There were 3 regional conventions held in 2022 in the following regions:

- South Central (virtual, March 4-5) hosted by Texas Kappa at the University of Mary Hardin-Baylor in Belton
- North Central (in-person, April 1-2) hosted by Kansas Alpha at Pittsburg State University in Pittsburg
- South Eastern (in-person, April 22-23) hosted by Alabama Beta at the University of Northern Alabama in Florence

In spite of the impact that Covid has had on American society, I am pleased that KME has been able to continue functioning. Initiations and installations have both dropped, but we are seeing chapter activity rebound and we are hoping for a full recovery. As you saw in the treasurer's report, we have not been impacted financially.

I am constantly amazed by the quality of the people who contribute to the success of this organization. It is a pleasure to work with individuals who are so selfless and considerate. I believe that Kappa Mu Epsilon has some of the best servant-leaders that any organization could hope for and I am grateful for their contributions to KME. One such person is our secretary, Steve Shattuck, who tragically passed away on April 10, 2023. He worked faithfully in his duties until cancer made that impossible and Rhonda McKee graciously stepped in to help. We are going to miss Steve very much.

Don Tosh
National President

Report of the National President-Elect

This is my second year as president-elect, and first year organizing the national convention. Despite my experience organizing the 2022 regional North-Central convention, I found out that organizing the national convention is quite a different beast. I wish to thank Manyiu Tse and Debora Upton for their hard work as local organizers. Without them, the convention would not be possible. I am quite pleased with the nearly 45 attendees from 14 chapters, 13 student presentations, and 2 faculty workshops. This was a stronger turnout than expected. I also want to thank the national council members for being supportive of me as I transition to my new role as president-elect.

Scott Thuong
President-Elect

Report of the National Secretary

Kappa Mu Epsilon, National Mathematics Honor Society initiated 1,593 new members in 134 chapters during the 46th Biennium that ended March 31, 2023. That brings the total membership of KME to 91,317. Thirty active chapters did not report any initiates during the 46th biennium.

The National Secretary, receives all initiation reports from chapters, makes a record of those reports, up-dates mailing list information for corresponding secretaries and forwards copies of the reports to other officers. At the beginning of each new biennium, the secretary prepares a new KME brochure. During an academic year, the secretary sends out supplies to each chapter. The supplies include information brochures, membership cards and one or two copies of the brochure "A Matter of Honor." When a college or university petitions for a new chapter of KME, the secretary sends out a summary of the petition, prepared by the president, to each chapter and receives the chapter ballots.

I was pleased to share the National Secretary job with Steve Shattuck for the past year or so, as Steve battled cancer. And I am very sad to report that Steve lost that battle just a few days before this convention began. Steve was a student of mine in both his undergraduate and graduate programs and then became a colleague and close friend. Steve was initiated into the Missouri Beta chapter of KME as an undergraduate and later became a faculty sponsor of that chapter. He attended every national and regional convention for more than 20 years, served on many committees, and was elected to the National Secretary position in 2019. Steve was a trusted friend, exceptional teacher, and great colleague. He will be sorely missed by his KME friends.

Rhonda McKee
Acting National Secretary

Report of the National Historian

I have enjoyed serving as Kappa Mu Epsilon's National Historian during the past two years. There are a number of people that I would like to thank. Cynthia Huffman, my immediate predecessor as National Historian provided a lot of help and guidance to me when I first took on this role in April, 2021. It has been a pleasure to work with Doug Brown, the editor of The Pentagon. Thanks also goes to Pete Skoner, another former National Historian, who encouraged me to become more involved in KME. Of course, great thanks goes to the corresponding secretaries and faculty sponsors of the various chapters throughout the country.

The primary duty of the National Historian is the preparation of a Chapter News Report for publication in The Pentagon every spring and fall. This is done after soliciting and collecting chapter news reports from corresponding secretaries. During this past biennium from March 2021 to March 2023, 56 chapters responded at least once to the request for chapter news with 35 chapters responding more than once. Special thanks goes to the 14 chapters that responded to every request: AL Theta, CT Beta, IA Alpha, IL Zeta, KS Beta, KS Delta, MD Delta, MO Theta, NC Zeta, PA Pi, PA Rho, RI Beta, TX Lambda, and WV Alpha. I've very much enjoyed my interaction with corresponding secretaries. It has been fun to see some of the creative activities going on in some of our chapters, for instance, celebrating Pi Day with a pie recipe booklet from PA Mu and a math luncheon with a mathematical version of Bingo from MD Alpha. The Chapter News Report is also where installation reports appear. During this past biennium, KME has welcomed three new chapters: AR Gamma installed on April 27, 2021, GA Theta installed on October 22, 2021, and CA Theta installed on October 17, 2022. Another event worthy of special notice was the 50th anniversary of MO Theta which was celebrated in October, 2021.

Another task of the National Historian is to maintain custody of the KME archive which is stored in several boxes. Among the items archived are printed copies of The Pentagon from its first issue of Fall 1941 through its final edition available in print from Spring 2013. When I looked through this collection, I realized that one issue was missing (Fall 1947). Now, our math department at Frostburg State University has a small number of old issues of The Pentagon. Amazingly, we had a copy of the missing issue and so the archive's collection is now complete! Happily, all issues of The Pentagon are available in pdf format at the organization's website.

This April 18 marks the 92nd anniversary of the founding of Kappa Mu Epsilon and so we are here gathered for KME's 46th Biennial Convention. It was 1931 when OK Alpha became KME's founding chapter at Northeastern State University in Tahlequah, Oklahoma. Tahlequah was also the location of KME's first national convention which was held in April, 1933. From that first national meeting

through our current one, 45 national conventions have been held. (Two meetings were not held during the WWII years and an extra one was held in 2014.)

With that, I offer my best wishes to all until our next gathering.

Mark Hughes
National Historian

Report of the National Treasurer

46th* Biennium (April 7, 2021 – March 31, 2023)
(*Note that Biennium numbering was changed to reflect the actual number of biennia since the founding of KME, rather than the number of conventions held.)

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows biennium assets of **$118,773.50**. The Cash Flow Report shows that we have an **asset gain of $5,915.76** this biennium.

BIENNIUM ASSET REPORT

Total Assets at beginning of 46th Biennium (April 7, 2021)		**$112,857.74**
Current Assets (Wells Fargo Bank)		
Checking	$ 36,733.22	
Savings	$ 40,276.29	
Time Account xxx7120	$ 10,125.00	
Time Account xxx7138	$ 10,220.82	
Time Account xxx7146	$ 10,247.22	
Time Account xxx7153	$ 10,247.22	
Total	**$ 117,849.77**	
Total Current Assets (as of March 31, 2023)		**$ 118,793.50**
- uncleared check #1744		- $ 20.00
Total Current Register Assets (as of March 31, 2023)		**$ 118,773.50**
- activity since end of 46th Biennium (March 31, 2023)		- $ 943.73
Total Assets (Register) at end of 46th Biennium(4/11/23)		**$ 117,829.77**

BIENNIUM CASH FLOW REPORT

Receipts		
Initiation fees received	$ 31,080.00	
Installation fees received	$ 365.00	
Interest income	$ 136.12	
Gifts & misc. income	$ 2441.42	
Total Biennium Inflow		**$ 34,022.54**

Expenditures		
Association of College Honor Societies	$ 858.00	
Administrative expenses	$ 6,582.41	
National Convention expenses	$ 6,648.40	
Regional Convention expenses	$ 2,600.00	
Initiation expenses: Certificates/jewelry	$ 10,088.59	
Installation expenses	$ 1,315.39	
Miscellaneous	$ 13.99	
Total Biennium Outflow		($ 28,106.78)
Biennium Cash Flow		**$ 5,915.76**

The cash flow last biennium (2017–19) was **$5,897.18**. Both receipts and expenditures were slightly up this biennium, coming out of the COVID-19 pandemic. Note that: (1) Initiation fees were steady $(+ < 1\%)$; (2) National Convention (2021) expenditures were down $10,677.45 $(–62\%)$ since there were no travel expenses for officers or chapters—costs were mainly for virtual hosting; (3) Regional Conventions resumed in 2022; (4) Initiation expenses were up $8002.23 $(+384\%)$ because the jewelry (pin) supply was replenished in bulk this biennium (2000 pins @ $8085.40). The National Council is likely to continue to pass on part of the increase in net revenue to the students and chapters with extra travel funding to future National and Regional Conventions. We have easily continued to meet our National Council goal of maintaining assets of at least $40,000. **The financial condition of Kappa Mu Epsilon is sound.** We have additional assurance by maintaining a fidelity bond insuring Kappa Mu Epsilon, Inc., against any losses resulting from dishonesty of the 5 main officers (however unlikely). I want to thank my colleagues on the National Council for their untiring dedication, as well as the corresponding secretaries who maintain such a vital role in Kappa Mu Epsilon. I am grateful for the opportunity to serve with such outstanding individuals in encouraging and recognizing students for accomplishments in mathematics.

David Dempsey
KME National Treasurer

KME BIENNIUM CASH FLOW HISTORY

Years	Number of Initiates in Biennium	End of Biennium Assets ($)	Biennium Cash Flow ($)	Biennium End Date
81-83	2377	25,378.36		
83-85	2542	37,184.70	11,806.34	
85-87	2679	47,378.50	10,193.80	
87-89	2570	53,008.08	5,629.58	
89-91	2315	61,661.79	8,653.71	
91-93	2574	55,780.57	-5,881.22	
93-95	2554	45,455.94	-10,324.63	
95-97	2042	34,594.12	-10,861.82	
97-99	2261	44,365.53	9,771.41	
99-01	2089	37,924.96	-6,440.57	
01-03	2400	48,982.24	11,057.28	
03-05	2598	61,094.50	12,112.26	3/20/05
05-07	2705	68,650.29	7,555.79	3/20/07
07-09	2444	67,002.51	-1647.78	2/27/09 (3/10/09 for Quicken)
09-11	2894	70,268.42	3,190.91	4/14/11
11-13	2452	88,900.25	18,533.69	3/15/13
13–15	2573	96,048.36	7,047.15	3/15/15
15–17	2601	105,180.96	9,132.60	3/15/17
17–19	2232	106,960.56	1,779.60	3/15/19
19–21	1464	112,857.74	5,897.18	4/6/21
21–23	1593	118,773.50	5,915.76	3/31/23

Report of *The Pentagon* Editor

Introduced in 1941, *The Pentagon* is the official publication of Kappa Mu Epsilon. Publication of student papers continues to be the focus of *The Pentagon*. Following tradition, papers given "top" status and other recognition by the Awards Committee at the KME National Convention are guaranteed an opportunity to be published. *The Pentagon* is now completely electronic and available for free online via the KME website:

`www.kappamuepsilon.org`

I have been the Editor of *The Pentagon* since November 2016. The typical duties involve corresponding with authors of potential articles for submission and facilitating referee feedback and author corrections for upcoming issues. Since the last national convention, four issues of *The Pentagon* (Fall 2020, Spring 2021, Fall 2021 and Spring 2022) have been published and made available on the website. The *Problem Corner* continues to be made available on the KME website ahead of publication of the full issue.

As of April 2023, there are a sufficient number of articles approved by the referees to produce the Fall 2022 issue (which should be completed by the end of June), and one approved article available for the Spring 2023 issue. A regular publication schedule requires a steady stream of articles from which to select, so assistance from the chapters in encouraging students to submit papers would be greatly appreciated. Students presenting at this conference and any of the regional conferences are especially urged to submit their work to *The Pentagon*.

The publication of *The Pentagon* would not be possible without the dedication of the referees, whose thoughtful reviews have been invaluable in helping authors fine-tune their submissions.

Stephen Andrilli	LaSalle University
Peter Chen	The University of Mary Hardin–Baylor
Chip Curtis	Missouri Southern State Univ.
Tara Davis	Hawaii Pacific University
Vincent Ferlini	Keene State College
Adam Fletcher	Bethany College
Tim Flood	Pittsburg State Univeristy
Mark Hughes	Frostburg Stae University
Thomas Kent	Marywood University
Tom McNamara	Southwestern OK State Univ.
J. Lyn Miller	Slippery Rock University
Lloyd Moya	Henderson State University
Ann Podleski	Harris-Stowe State Univ.
Sara Quinn	Dominican University
Adam Salminen	University of Evansville
Manyiu Tse	Molloy College
John Zerger	Catawba College

Finally, I am very grateful to the Associate Editors: Pat Costello, who organizes the *Problem Corner* for each issue, and KME Historian Mark Hughes, who collects and prepares the KME News Items, as well as KME Webmaster John Snow. Their patience and attention to detail are very much appreciated.

Doug Brown
Editor, *The Pentagon*

Report of the Webmaster

Webmaster activities this biennium have been mostly routine maintenance – posting news and issues of the *Pentagon* and the *Problem Corner* as they arrive. Additional activity included:

- We posted information and videos for last year's regional conventions, and we posted videos for the last national convention.
- We cleaned up after two php-reinfection attacks.
- We enlisted Sucuri to provide a firewall, scanning, and a secure connection for site visitors.
- In January of 2023, Media Temple migrated all of its accounts (including ours) to GoDaddy. This changed our billing to a monthly cycle, and it changed how we interact with the account and files related to the site. However, these changes should be invisible to visitors.

John Snow
Webmaster

Report of the Audit Committee

Audit Committee Members:

Manyiu Tse, NY Rho, faculty, Chair
Rajarshi Dey, Kansas Beta, faculty
James Gillin, Kansas Delta, student
Abigail Branson, Tennessee Gamma, student

Audit Process

1. Treasurer David Dempsey provided electronic copies of the biennium financial summary data to the committee chair to facilitate verification of asset account totals prior to the convention. The Audit Committee Chair was then given guest user credentials to Wells Fargo Bank. The account balances for the Kappa Mu Epsilon Business Checking account, Platinum Business Savings accounts, and the four Time accounts were verified to correspond to the associated totals found on Treasurer Dempsey's biennium reports and current records. This verification was conducted on April 12, 2023 for the balances as submitted by Treasurer Dempsey on April 6, 2023.
2. Treasurer Dempsey provided the committee with detailed documentation for receipt and payment transactions, monthly bank account statements and reconciliation documentation, expense reports, receipts, income information, as well as his own reports and summary for the full biennium. These documents were shared electronically with committee members approximately one week prior to the convention.
3. On April 12, the Audit Committee Chair sent an email to President Don Tosh and interim secretary Rhonda McKee to determine their impressions of the accuracy and completeness of the recording of the financial transactions throughout the biennium.

Findings

1. The bank information provided by Treasurer Dempsey (as of April 6, 2023) was verified by the committee chair on April 11, 2023 via logging into http://www.wellsfargo.com.
2. The committee spot-checked the Secretary and corresponding Treasurer reports and found no inconsistencies. The chair corresponded with the secretary via email on 4/12/23. She stated that the financial accuracy and completeness of the recording of KME financial transactions to be exemplary.
3. The committee spot checked the expense payment reports and receipts provided and found no inconsistencies. The President expressed total satisfaction with the integrity of the process.

4. The committee spot-checked the expense payment reports and receipts provided and found no concerns. However, we noted some inconsistencies that were followed up with the Treasurer:

    - 1715 has a negative $320 and is not on the Biennium Expense Summary (found in INCOME)
    - 1722 was lost in the mail and replaced with 1723, which is not on the Biennium Expense Summary (found in INCOME)
    - #9353 for CT Beta initiation was missing in the Biennium Expense Summary (found in INCOME)
    - 1744 for initiation overage does not appear in the Biennium Expense Summary (found in INCOME)

5. The chair corresponded with the president via email on 4/12/23. The President expressed that he has complete faith in the accuracy and completeness of the recordings of the KME financial transactions this biennium.
6. The committee spot-checked monthly financial institution statements, quarterly interest statements, and CD interest reports, comparing them to the Treasurer's quarterly reports. The committee did not find any inconsistencies.

Recommendations

1. Financial Information is electronically shared by the Treasurer to the audit committee prior to the national convention. This provides the opportunity for easier access to the verification process in a careful and timely manner. This process should be continued for future audit committees.
2. The organization files an electronic tax notecard annually even though no taxes are required. The committee recommends that this practice continue.
3. The internal checks built into the regular financial processing between the Treasurer, President, and Secretary provide an important safeguard to the integrity of the office of the Treasurer and help avoid the necessity of an expensive external audit. These ongoing internal audit processes should be continued and updated by the National Council as needed.

Commendations

1. The committee commends Treasurer David Dempsey for his exemplary maintenance, management, and organization of the financial records, and is thankful for his exemplary services as treasurer.
2. We further commend Treasurer Dempsey for his valuable input and transparency throughout the audit process. His detailed written guidelines and timely responses to questions were extremely helpful for the Audit Committee.
3. The committee commends the national President, Secretary, and Treasurer for the manner in which they communicate and cooperate to maintain the internal checks that preserve the integrity of the office of Treasurer.
4. The committee appreciates the sample reports provided by the Treasurer to ensure consistency of the audit process.

Manyiu Tse
Chair, Audit Committee

Report of the Resolutions Committee

The Resolutions Committee consisted of: Dianne Twigger, Faculty, Missouri Theta
Dakota Heathcock, Student, Alabama Theta
Victoria Risner, Student, Missouri Theta
Lucas Saone, Student, Alabama Theta
Lisa Reed, Student, Tennessee Gamma

This Committee hereby proposes the following resolutions.

Whereas the success of any undertaking relies heavily upon the dedication and ability of its leaders, be it resolved that this Forty- Sixth National Convention express its gratitude

1. to Don Tosh (national president), Scott Thuong (president elect), Steve Shattuck (national secretary), David Dempsey (national treasurer), Mark Hughes (national historian), and John Snow (national webmaster);
2. to Rhonda McKee for filling in as acting National Secretary for the Convention;
3. to Doug Brown for his service as editor of *The Pentagon*;
4. to Donna Marie Pirich, Pete Skoner, Jamye Curry, Katherine Kime, and John Diamantopoulos for their service as regional directors; and
5. to the students and faculty who served on the Auditing, Awards, Nominating, and Resolutions committees, which is so essential for the success of the meeting.

Whereas the primary purpose of Kappa Mu Epsilon is to encourage participation in mathematics and the development of an appreciation for its beauty, be it further resolved

1. That students Abigail Branson, Ian Coleman-Hull, Austin Crabtree, Julia DiCianni, James Gillin, Dakota Heathcock, Jack Lin, Celine Lukito, John Mancuso, Catherine McClure, Sanskar Neupane, Victoria Risner, Peter Russell, and Perry Wu, who prepared, submitted, and presented their work be given special commendation by this Forty-Sixth Biennial Convention for their enthusiasm and dedication.
2. That this Convention express its thanks
   (a) to workshop host John Snow for his interactive presentation on Programming in the Liberal Arts Classroom, special thanks to Dr. Scott Thuong for showing us what NOT to do with java script;
   (b) to workshop host John Snow for his interactive presentation on Programming in the Liberal Arts Classroom, special thanks to Dr. Scott Thuong for showing us what NOT to do with java script;

(c) to Jaimie Stone for his keynote address on Pop Culture in the Mathematics classroom while we are eagerly anticipating his role as Ken in the new Barbie Movie.

Finally, whereas Molloy University and the surrounding community of the Village of Rockville Centre has provided this Convention with gracious hospitality and enthusiasm, be it resolved:

1. That this Forty-Sixth Biennial Convention express its heartfelt appreciation to the New York Rho chapter for the careful arrangements they have planned and carried out so successfully;
2. We express our appreciation to the American Mathematical Society and to the American Statistical Association, through whose grants support the operations of this Convention;
3. That this Convention recognize and thank Dr. Maniyu Tse and Dr. Deborah Upton, together with all the other members of New York Rho, who devoted their time and talents to ensure the success of this meeting.

These resolutions respectfully submitted,
Dianne Twigger, Chair

Kappa Mu Epsilon News

Edited by Mark Hughes, Historian
Updated information as of June 2023

News of chapter activities and other noteworthy KME events should be sent to

Mark Hughes, KME Historian
Frostburg State University
Department of Mathematics
Frostburg, MD 21532
or to
mhughes@frostburg.edu

Chapter News

AL Gamma – University of Montevallo
Chapter President – Cody McLain; 724 Total Members; 7 New Members
Other Spring 2023 Officers: Cole Swain, Vice President; Samantha Cost, Secretary; Dr. George Lytle, Corresponding Secretary and Faculty Sponsor.
New Initiates – Samantha Cost, Carter Giles, Cody Mclain, Timothy Medewase, Claudia Morales, Cole Swain, and John Tidwell.

AL Eta – The University of West Alabama
Chapter President – Carleigh Cameron; 159 Total Members; 3 New Members
Other Spring 2023 Officer: Ms. Kim Giles, Corresponding Secretary and Faculty Sponsor.
New Initiates – Ashley Clark, Zoe Horton, and Mary Grace Sheffield.

AL Theta – Jacksonville State University
Chapter President – Dakota Heathcock; 339 Total Members; 15 New Members
Other Spring 2023 Officers: Nicholas Covalsen, Vice President; Adam Parton, Secretary; Suneet Sharma, Treasurer; Dr. David Dempsey, Corresponding Secretary; and Dr. Jason Cleveland, Faculty Sponsor.
The Alabama Theta chapter met at least monthly during Spring 2023 for game nights. We held an in-person initiation ceremony on March 10, 2023, inducting 15 new members. In addition, four of us enjoyed a 1900-mile road trip to the KME National Convention in New York in April. We were so honored to win the McKee-Tosh Outstanding Chapter Award, especially since both of the namesakes were there to make the award! We look forward to starting fresh in the fall with the election of new officers.

New Initiates – Ashlynn Elizabeth Abernathy, Elijah Beverly, Esat Ergin, Mason H. Faircloth, Emma-Leigh Grizzard, Kody Brock Gross, Carmen E. Johnson, Kalee Nicole Johnson, Jailon Lawrence, Kristopher McAnally, William Logan Nelms, Alexandria Nicholson, Elise McKenna Roberts, Lucas Saone, and Micah Alexander Smith.

AR Beta – Henderson State University
Chapter President – Alex Hunter; 74 Total Members; 6 New Members
Other Spring 2023 Officers: Kristen Harper, Vice President; Trenton Moore, Secretary; Catherine Leach, Corresponding Secretary and Faculty Sponsor.

CA Epsilon – California Baptist University
Corresponding Secretary and Faculty Sponsor – Dr. James Buchholz; 315 Total Members; 15 New Members
Other Fall 2017 Officers: ETC
New Initiates – Timothy Tamashiro, Krystal Karman, Crystal Garduno, Jamie Breslin, Melissa Araujo, Jessica Dreschler, Sarah Boyd, David Howerzyl, Anna Chiu, Victoria Bucy, Flint Freshwater, Audrey Pina, Alexander Akers, Tam Schroeder, Trinity Rosales, and Kenan Karabas.

CA Theta – William Jessup University
Chapter President – Elizabeth Salvato; 9 Total Members
Other Spring 2023 Officers: Tsz (Stephen) Chau, Vice President; Mikayla Erickson, Secretary; Samuel Smith, Treasurer; Bradley Wagner, Corresponding Secretary; and Michelle Clark, Faculty Sponsor.
This past semester our chapter, California Theta, saw its first graduating members. Our president, Elizabeth Salvato, our treasurer, Samuel Smith, and member Michael Beishline all graduated with degrees in mathematics. The entire chapter wishes them well as they move on to various enterprises. In anticipation of several of the officers of our chapter graduating we spent one meeting identifying new potential officers. New officers will be installed at the first meeting this fall. Our chapter also hosted several events this semester. We gathered for a group hike at Hidden Falls Regional Park in Placer County, California, where we hiked three miles out and back to a waterfall. Several members continued on to complete a full loop of the park totaling 9 miles! Additionally, we hosted a booth at Jessup's International Night Market. At our booth students could win candy by solving math puzzles in the game Krypto.

CO Delta – Colorado Mesa University
Chapter President – Matthew Montoni-Tiller; 334 Total Members; 6 New Members
Other Spring 2023 Officers: Ian Cropp, Vice President; Sierra Stevens, Secretary; Drake Cullen, Treasurer; Erik Packard, Corresponding Secretary; and Phil Gustafson, Faculty Sponsor.

CT Beta – Eastern Connecticut State University
Corresponding Secretary and Faculty Sponsor – Dr. Mehdi Khorami; 553 Total Members; 6 New Members
New Initiates – Juan Baltazar Tapia, Maya Brody, Kevin Flaherty, Liam Hemingway, Kennedy Jakan, and Sister Alexandria Smith.

CT Gamma – Central Connecticut State University
Corresponding Secretary – Gurbakhshash Singh ; 78 Total Members
Other Spring 2023 Officer: Nelson Castaneda, Faculty Sponsor.

GA Zeta – Georgia Gwinnett College
Chapter President – Hope Doherty; 69 Total Members; 3 New Members
Other Spring 2023 Officers: Gabriel Amat, Vice President; William Watts, Secretary; Matt Elenteny, Treasurer; Dr. Jamye Curry Savage, Corresponding Secretary and Faculty Sponsor; and Dr. Livy Uko, Faculty Sponsor.
Our chapter had 2 members to graduate: Hope Doherty (Pure Math) and William Watts (Applied – IT Data Analysis). We also had 3 members join our chapter this semester and we will hold a formal initiation ceremony in the fall to recognize these students and their achievement.

GA Theta – College of Coastal Georgia
Chapter President – Casey Griffin; 29 Total Members; 9 New Members
Other Spring 2023 Officers: Andrea Olvera, Vice President; Justin Von Gartzen, Secretary; Zach Atkinson, Treasurer; Aaron Yeager, Corresponding Secretary and Faculty Sponsor.
Over the Spring Semester the Georgia Theta Chapter of Kappa Mu Epsilon inducted nine new members and we had three meetings.

IA Alpha – University of Northern Iowa
Chapter President – Grace Croat; 1119 Total Members; 2 New Members
Other Spring 2023 Officers: Quinn Robinson, Vice President; Jackson Walter, Secretary; Rachel Wohlgemuth, Treasurer; and Dr. Mark D. Ecker, Corresponding Secretary and Faculty Sponsor.
Nine current student members of KME and four faculty met on Monday, May 1, 2023 in Wright Hall for our Spring KME Banquet. Sage Rodriguez presented her senior seminar project entitled “Analysis of Gun Death Rates by State” and two new student members were initiated at the banquet.

IA Gamma – Morningside University
Chapter President – Taylor Pierce; 445 Total Members; 3 New Members
Other Spring 2023 Officers: Taylor Pierce, Vice President; Isaiah Hinners, Secretary and Treasurer; and Dr. Eric Canning, Corresponding Secretary and Faculty Sponsor.
There were three new initiates in Spring 2023. Our KME math club met 8 dif-

ferent evenings during the Spring Semester. At these meetings, 4 times there was a guest speaker, we watched the movie *The Man Who Knew Infinity*, had a craft night, and a couple of game and pizza nights.
New Initiates – Fred Lageschulte, Austin Ramold, and Kelsey L. Schieffer.

IA Delta – Wartburg College
Chapter President – Gavin Foust-Wollenberg; 778 Total Members; 5 New Members
Other Spring 2023 Officers: Aden Stroup, Vice President; Beth Johll, Secretary; Paul Zelle, Treasurer; Brian Birgen, Corresponding Secretary; and Mariah Birgen, Faculty Sponsor.
Our initiation ceremony was held on April 15, 2023. Emily Dorway Upton, a 1997 alum, spoke.

IL Zeta – Dominican University
Corresponding Secretary – Mihaela Blanariu; 461 Total Members; 3 New Members
We have inducted three new members in Spring 2023.
New Initiates – Isabela Flores, Steve Pardinas, and Crystal Sotelo.

IN Beta – Butler University
Chapter President – Kelly Ryan; 452 Total Members; 8 New Members
Other Spring 2023 Officers: Nicole Dickson, Vice President; Aaron Marshall, Secretary; and Rasitha Jayasekare, Corresponding Secretary and Faculty Sponsor.
Our KME member Nicole Dickson worked with one of our faculty members, Dr. Amber Russell, on "Surjectivity of Natural Maps between Rings of Invariants" and gave a presentation at the Pi Mu Epsilon Contributed Session on Research by Undergraduates at JMM 2023. We initiated eight new KME members during the Department of Mathematical Sciences awards ceremony in April.
New Initiates – Chloe Enk, Chloe Helmreich, Teresa Dold, Grace Nedza, Rachel Grote, Sara Grace Moore, Eleanor Waiss, and Jena Lane.

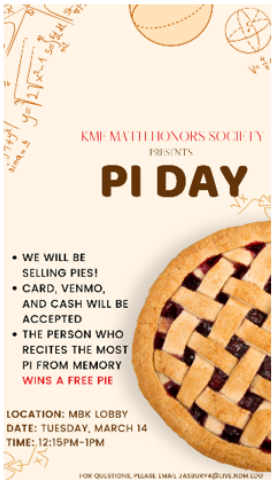
Indiana Beta initiation: (left to right) Rachel Grote, Jenna Lane, Sara Moore, Chloe Enk, Grace Nedza, Chloe Helmreich, and Teresa Dold. (Not pictured: Eleanor Waiss)

KS Beta – Emporia State University
Chapter President – Joe Rose; 1544 Total Members; 2 New Members
Other Spring 2023 Officers: Julia Whitaker, Vice President; Sky Decker, Secretary; Joey Feuerborn, Treasurer; Tom Mahoney, Corresponding Secretary; and Brian Hollenbeck, Faculty Sponsor.

KS Gamma – Benedictine College
Chapter President – William Westhoff; 509 Total Members; 11 New Members
Other Spring 2023 Officers: Kieran Amsberry, Vice President; and Dr. Josh Cole, Corresponding Secretary and Faculty Sponsor.

KS Delta – Washburn University
Chapter President – Ajar Basnet; 837 Total Members; 3 New Members
Other Spring 2023 Officers: Sanskar Neupane, Vice President; Graci Postma, Secretary; Sara Johnson, Treasurer; and Sarah Cook, Corresponding Secretary and Faculty Sponsor.
Two students and one faculty were initiated into the Kansas Delta Chapter of Kappa Mu Epsilon on March 9 through a virtual ceremony. In April, two faculty members (Beth McNamee and Gaspar Porta) and five students (Ajar Basnet, James Gillin, Sara Johnson, Sanskar Neupane, Graci Postma) attended the 46th Biennial National Convention at Molloy University in Rockville Centre, NY. Students Sanskar Neupane and James Gillin gave presentations on their research projects. Sanskar won 3rd place for his presentation on "Exploring Ways to Model Stock Prices" and James won 1st place for his project "Eulerian Numbers and Relations to Number Triangles."
New Initiates – Ellynor Marie George, James Gillin, and Jillian Kimzey.

MD Alpha – Notre Dame of Maryland University
Chapter President – Julie Asbury; 413 Total Members; 6 New Members
Other Spring 2023 Officers: Nana Fremah Nyarko, Vice President; Aissata Timbine, Secretary; Konstantina Vavaroutsos, Treasurer; and Charles Buehrle, Corresponding Secretary and Faculty Sponsor.
MD Alpha items are below.



MD Delta – Frostburg State University
Chapter President – Adam Sullivan; 546 Total Members; 3 New Members
Other Spring 2023 Officers: Kaitlyn Henderson-Adams, Vice President; Faith James Sergent, Secretary; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet, Faculty Sponsor.
Maryland Delta Chapter had its first meeting in February where we made plans for the semester. We were pleased to welcome three new members at our March 5 initiation ceremony. At this ceremony, faculty sponsor Dr. Frank Barnet gave an interesting presentation entitled "The Mathematics of the Diffraction of Light." On March 14, we had our annual Pi-Day Bake Sale which was a great success. After Spring Break, we had our March meeting where we enjoyed pizza, math videos and puzzles. Two student members, Kaitlyn Custer and Dawson Hormuth, attended the KME 46th Biennial National Convention at Molloy University along with faculty sponsor Dr. Mark Hughes. We were accompanied by Dr. Brendon LaBuz, a former president of Maryland Delta Chapter and currently a mathematics professor at St. Francis University in Pennsylvania. We had a great time at the convention and were able to visit the National Museum of Mathematics in Manhattan on our way back home. We had our final meeting of the semester towards the end of April where we elected some new officers: Kaitlyn Custer as president, Richard Day as vice president, and Dawson Hormuth as treasurer. Faith James Sergent will continue as secretary. We offer congratulations to graduating members Adam Sullivan and Kaitlyn Henderson-Adams.

New Initiates – Kaitlyn Custer, Richard Day, and Dawson Hormuth.

MI Beta – Central Michigan University
Chapter President – Maleia Thompson; 1762 Total Members; 1 New Member
Other Spring 2023 Officers: Julia Savage, Vice President; Travis Miller, Secretary; Elijah Hayes, Treasurer; and Dr. Dmitry Zakharov, Corresponding Secretary and Faculty Sponsor.
In the spring semester, the Michigan Beta Chapter held 6 general meetings, a fundraiser for Special Olympics, and a volunteer tutoring event. The chapter hosted a talk given by one of our CMU professors, Dr. Douglas Lapp, on the use of technology in undergraduate level linear algebra classes. Another talk was given by Evan Henning, a graduate student at Grand Valley State University, on edge covers and Fibonacci numbers. Other meetings included a math bingo night, a Pi Day event, and a scavenger hunt.

MO Beta – University of Central Missouri
Chapter President – Luke George; 1555 Total Members, 2 New Members
Other Spring 2023 Officers: Stephanie Marcella, Vice President; Sabrina Wolfman, Secretary and Treasurer; Dylan Miller, Historian; Blaise Heider, Corresponding Secretary and Faculty Sponsor; and Paul Plummer, Faculty Sponsor.

MO Epsilon – Central Methodist University
Chapter President – Jayklin Smith; 468 Total Members, 7 New Members
Other Spring 2023 Officers: Ashley Flowers, Vice President; Tessa Maidens, Secretary; Dillan Kessing, Treasurer; and Pam Gordy, Corresponding Secretary and Faculty Sponsor.
New Initiates – Jennifer Durbin, Lydia Elder, Ashlee Flowers, Joshua Henderson, Dillan Kessing, Dillan Lembke, and Tessa Maidens.

MO Theta – Evangel University
Chapter President – Jack Lin; 305 Total Members; 6 New Members
Other Spring 2023 Officers: Ericsson McDermott, Vice President; and Dianne Twigger, Corresponding Secretary and Faculty Sponsor.
Missouri Theta initiated 6 new members this spring, bringing our total number of members to 305. We also elected new leadership, with Jack Lin as president and Ericsson McDermott as vice president. Along with monthly meetings, three students and three faculty attended the biennial convention at Molloy University. All students attending presented at the conference.

MO Iota – Missouri Southern State University
Chapter President – Ashley Stokes; 442 Total Members; 3 New Members
Other Spring 2023 Officer: Dr. Amila Appuhamy, Treasurer, Corresponding Secretary and Faculty Sponsor.
On April 18, 2023, we successfully conducted the initiation ceremony for Kappa

Mu Epsilon, following the established tradition. The event was accompanied by a dinner to celebrate the occasion. The ceremony welcomed three new members into our esteemed organization. The total number of participants in the event was ten, including the current student president Ashley Stokes, Dr. Jacob Boswell, Dr. Chip Curtis, and myself. The newly initiated members are as follows:

1. Westin Allen
2. Ethan Coggeshell
3. Lauren Emanuel

We congratulated Weston, Ethan, and Lauren on their successful initiation into Kappa Mu Epsilon. It was a great honor to have them join our community of dedicated and passionate individuals. We extended our gratitude to everyone who participated in the ceremony and made this event memorable. Their presence and support added to the significance of the occasion.

NC Zeta – Catawba College
Chapter President – Leon Heiermann; 96 Total Members; 4 New Members
Other Spring 2023 Officers: Morgan Childress, Vice President; Hunter Sjobom, Secretary; and Dr. Katherine Baker, Corresponding Secretary and Faculty Sponsor.

NE Beta – University of Nebraska Kearney
Corresponding Secretary and Faculty Sponsor – Dr. Katherine Kime; 939 Total Members; 2 New Members
Brooke Carlson, KME member, visited classes to speak to students about membership in KME and give out membership applications. We had an initiation with two initiates in May, in which Brooke played the role of President and Dr. Kime played the roles of the other needed officers. Brooke graduated with degrees in Math and Physics and will be entering graduate school at the University of Nebraska Lincoln, Dept. of Mechanical Engineering. She was selected as an Outstanding Senior (one of four) by the UNK Alumni Association. Dr. Kaye Sorensen, Senior Lecturer, who became a KME member as an undergraduate at this institution (then Kearney State College) retired at the end of the semester. Her teaching and service have been much appreciated.

OH Gamma – Baldwin Wallace University
Chapter President – Moore Bright; 1053 Total Members; 12 New Members
Other Spring 2023 Officers: Izzy Andrews, Vice President; Julia Gersey, Secretary; and David Calvis, Corresponding Secretary and Faculty Sponsor.
Our annual initiation ceremony was held on April 23, 2023.
New Initiates – Noah Bartos, Ashley Blum, Rebecca DiScipio, Malini Gaddamanugu, Alek Johnson, Lukas McCain, YaEl Noir, Kathryn Raubolt, Caely Ressler, Esther Scott, Morgan Stoltz, and Justin Verhosek.

PA Kappa – Holy Family University
Corresponding Secretary and Faculty Sponsor – Dr. William Worden; 168 Total Members; 5 New Members

PA Mu – Saint Francis University
Chapter President – Nathan Wolfe; 519 Total Members; 9 New Members
Other Spring 2023 Officers: Regina Edgington, Vice President; Isabelle Cunningham, Secretary; Alexandra Ochs, Treasurer; and Dr. Brendon LaBuz, Corresponding Secretary and Faculty Sponsor.
The Pennsylvania Mu Chapter of Kappa Mu Epsilon National Mathematics Honor Society held its initiation ceremony on Monday April 3rd at 5:00 pm in the John N. Wozniak Atrium in the Science Center. After dinner Dr. Brendon LaBuz, faculty sponsor, gave a short talk on examples of exotic topological spaces. Dr. Rachel Wagner was initiated along with students Andrew Christoff, Aurembiaix Pifarre Planes, Caleb Stivanelli, Daniel Card, Josh Koval, McKenzie Watt, Meagan Wheeler, and Nathan Zini.
Faculty sponsor Brendon LaBuz attended the forty-sixth Biennial Convention of Kappa Mu Epsilon on April 13-15 at Molloy University.

PA Pi – Slippery Rock University
Chapter President – Spencer Kahley; 145 Total Members
Other Spring 2023 Officers: Boris Brimkov, Corresponding Secretary; and Amanda Goodrick, Faculty Sponsor.
We did not have any activities in Spring 2023, but we started talking to students to recruit them as new members for next semester.

PA Rho – Thiel College
Chapter President – Hunter Gray; 148 Total Members; 3 New Members
Other Spring 2023 Officers: Devyn Bossard, Vice President; Cassy Brown, Secretary; Brandon Forrest, Treasurer; Dr. Russell Richins, Corresponding Secretary; and Dr. Jie Wu, Faculty Sponsor.
We had several activities during spring 2023, including a Challenge 24 competition benefitting the local food bank, a Pi Day activity where mathematics professors took pies in the face for charity, and our initiation ceremony.

PA Sigma – Lycoming College
Chapter President – Haley Seebold; 169 Total Members; 13 New Members
Other Spring 2023 Officers: Kaitlyn Haefner, Vice President; Allison Kelly, Secretary; Zoe Stauffer, Treasurer; Dr. Andrew Brandon, Corresponding Secretary and Faculty Sponsor.

RI Beta – Bryant University
Corresponding Secretary – Prof. John Quinn; 217 Total Members; 17 New Members

Other Spring 2023 Officer: Prof. Gao Niu, Faculty Sponsor.
We held our KME initiation ceremony on April 27, 2023 during which we welcomed 17 new student members into our Rhode Island Beta chapter. Our student, John Mancuso, presented at the KME 46^{th} Biennial Convention. His research was entitled "Song Recommendation using K-nearest Neighbors Algorithm."

TN Gamma – Union University
Chapter President – Rylee Iorio; 534 Total Members; 10 New Members
Other Spring 2023 Officers: Taylor Overcast, Vice President; Sam Burket, Secretary and Treasurer; Joy Lewis, Webmaster and Historian; Bryan Dawson, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor.
Our annual initiation banquet was held at Brooksie's Barn. One student and one alumna attended the national conference at Molloy University. The student, Abby Branson, presented her work at the conference.
New Initiates – Nathaniel Barnard, Jonathan Brewer, Josiah Hays, Laura Jin, Tabitha Keylon, Erik Lewis, Meilyn Massie, Georgia Morgan, Michael Tankersley, and Allison Winkler.

TX Lambda – Trinity University
Corresponding Secretary and Faculty Sponsor – Dr. Hoa Nguyen; 326 Total Members; 11 New Members

WI Alpha – Mount Mary University
Chapter President – Mary Parlier; 310 Total Members; 1 New Member
Other Spring 2023 Officers: Marissa Heraly, Vice President; Megan Schmitz, Secretary and Treasurer; Sherrie Serros, Corresponding Secretary; and Jeremy Edison, Faculty Sponsor.
This semester we held our initiation ceremony on May1, 2023. Genesis Encarnacion was initiated and current student member and vice-president Marissa Heraly presented about her study of data related to Wisconsin's health professional shortage related to COVID-19 cases and deaths.
WV Alpha – Bethany College
Chapter President – Cullen J. Wise; 198 Total Members
Other Spring 2023 Officers: Lauren E. Starr, Vice President; Patrick M. Gleason, Secretary; Ian A. Nelson, Treasurer; and Dr. Adam C. Fletcher, Corresponding Secretary and Faculty Sponsor.
West Virginia Alpha had a rather quiet spring semester. The chapter and our local Mathematics and Computer Science Club attended a handful of conferences virtually and hosted small chess and gaming tournaments on campus. They successfully executed the return of the annual Math/Science Day Competition for local high school students to an in-person event this year after a three-year hiatus.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
TN Gamma	Union University, Jackson	24 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973

NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011
PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014
CA Eta	Fresno Pacific University, Fresno	24 Mar 2015
OH Theta	Capital University, Bexley	24 Apr 2015
GA Zeta	Georgia Gwinnett College, Lawrenceville	28 Apr 2015
MO Xi	William Woods University, Fulton	17 Feb 2016
IL Kappa	Aurora University, Aurora	3 May 2016
GA Eta	Atlanta Metropolitan University, Atlanta	1 Jan 2017
CT Gamma	Central Connecticut University, New Britan	24 Mar 2017
KS Eta	Sterling College, Sterling	30 Nov 2017
NY Sigma	College of Mount Saint Vincent, The Bronx	4 Apr 2018
PA Upsilon	Seton Hill University, Greensburg	5 May 2018

KY Gamma	Bellarmine University, Louisville	23 Apr 2019
MO Omicron	Rockhurst University, Kansas City	13 Nov 2020
AK Gamma	Harding University, Searcy	27 Apr 2021
GA Theta	College of Coastal Georgia, Brunswick	22 Oct 2021
CA Theta	William Jessup University, Rocklin	17 Oct 2022