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Kappa Mu Epsilon National Officers

Ron Wasserstein *President*
American Statistical Association
732 N Washington Street
Alexandria, VA 22314-1943
ron@amstat.org

Rhonda McKee *President-Elect*
Department of Mathematics
University of Central Missouri
Warrensburg, MO 64093-5045
mckee@ucmo.edu

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Department of Mathematics and Computer Science
Texas Woman's University
Denton, TX 76204
mhamner@twu.edu

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Pittsburg State University
Pittsburg, KS 66762-7502
cwoodbur@pittstate.edu

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Saint Francis University
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pskoner@francis.edu

Kevin Reed *Webmaster*
Department of Science and Technology
Evangel University
1111 N. Glenstone Avenue
Springfield, MO 65802

KME National Website:
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Trigonometric Functions in the Biangular Plane: Part 2

Kathryn Wagner, *student*

MI Alpha

Albion College
Albion, MI 49224

Note: Part 1 of this article appears in the Fall 2011 issue of *The Pentagon*.

2. The Biangular Coordinate System

Function Family: $\phi = \sin(\theta)$

When we transform $\phi = \sin(\theta)$ into rectangular coordinates, the product resembles a sideways parabola. This relationship has a period of 2π. When both θ and ϕ are increasing together the curve rises faster which forms the steep slope of the tip of the parabola. As ϕ reaches a value of 1 and begins to decrease back towards 0 (since ϕ is bounded between -1 and 1) the slope decreases. When $\theta = \pi$ we get an indeterminate which can result in an asymptote shown in the center of some of the graphs (it varies depending on the period chosen). In the figure below, we allowed *Mathematica* to plot from 0 to 2π, which bypassed the asymptotes due to the technicalities of the program.

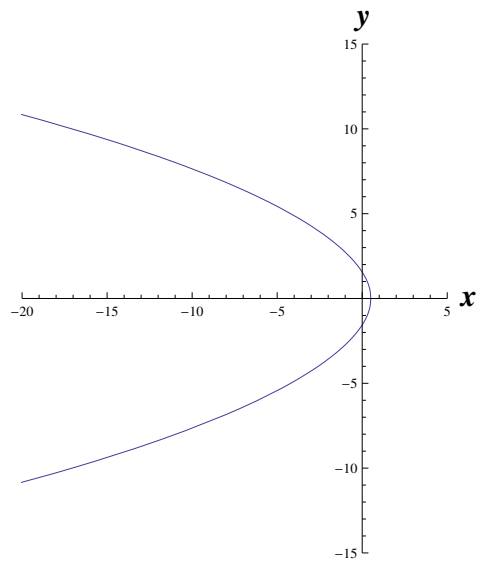

Figure 16

Adding a Constant outside of the Argument: $\phi = \sin(\theta) + k$

We are familiar with the concept that adding a constant k in the rectangular plane results in a vertical shift, but we already determined for $\phi = \cos(\theta) + k$ that a vertical shift is not displayed when we add a constant of 1.

We actually see an unusual shape for $\phi = \sin(\theta) + 1$. This graph possesses no loops, but does have a semi-circle-like structure in the center. When we change k to 2 we see a similar graph that appears to have been rotated around the pole (0,1). By Theorem 1 for $\phi = \sin(\theta) + k$, when $k = \pi$, our graph rotates back to the same position when $k = 0$. Thus, $\phi = \sin(\theta) + k$ in rectangular coordinates is a biperiodic relationship. The graphs for $k = 1$ and $k = 2$ do not resemble our original sideways parabola for $k = 0$, but we know that eventually the graph must return to this shape when $k = \pi$.

How exactly does the graph change? We start with our sideways parabola. The bottom half of the parabola becomes a large loop when $k = 0.1$ (the A graph in figure 18). This loop shrinks towards the point (0,0) as k approaches 1. We can clearly see that when $k = 0.3$ (B graph in figure 18), the loop shrinks. It shrinks even more when $k = 0.5$ (C graph in figure 18).

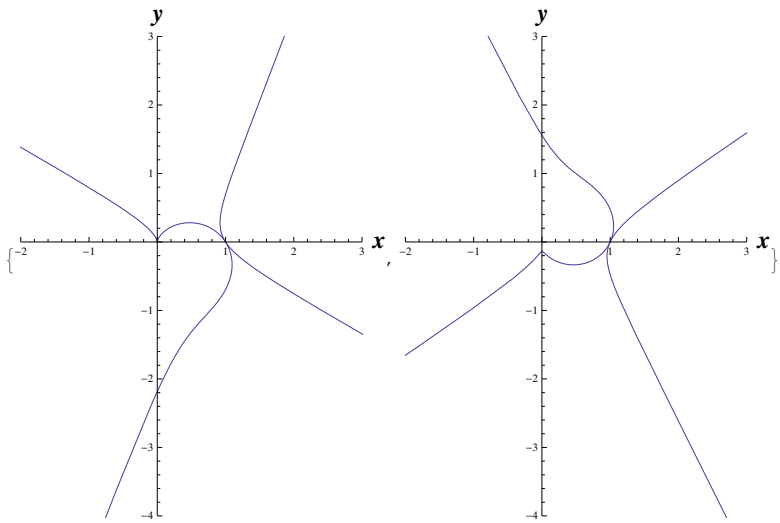

Figure 17

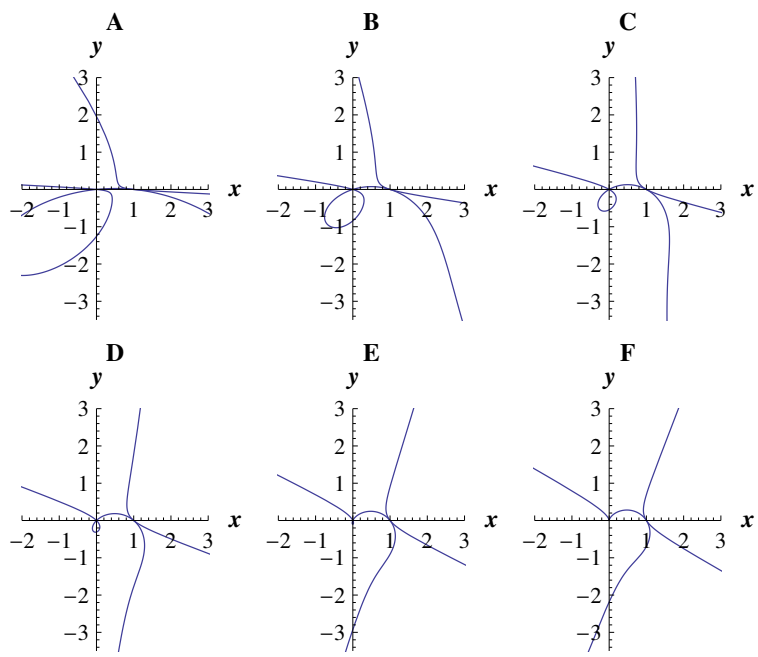

Figure 18

The loop continues to shrink until $k = 1$, when the loop disappears and the graph touches the point (0,0). The graph now possesses no loops but has an interesting shape (see F graph in figure 18).

When k grows greater than one, the graph retains a similar shape as when $k = 1$, but it rotates clockwise. In figure 19, the values of $k = 1.1, 1.3, 1.5, 1.7, 1.9,$ and 2 correspond to the A, B, C, D, E, and F graphs respectively. We can see that the higher the value of k, the farther the graph has rotated clockwise. We can also see that as it rotates, one of our sweeping arms begins to form a sort of tip which appears to be reaching towards the point (0,0). Once this tip of the arm touches the origin, loops begin to reappear. Unlike the first loop we saw, this loop grows in size, and when $k = \pi$, the graph returns to the original sideways parabolic form.

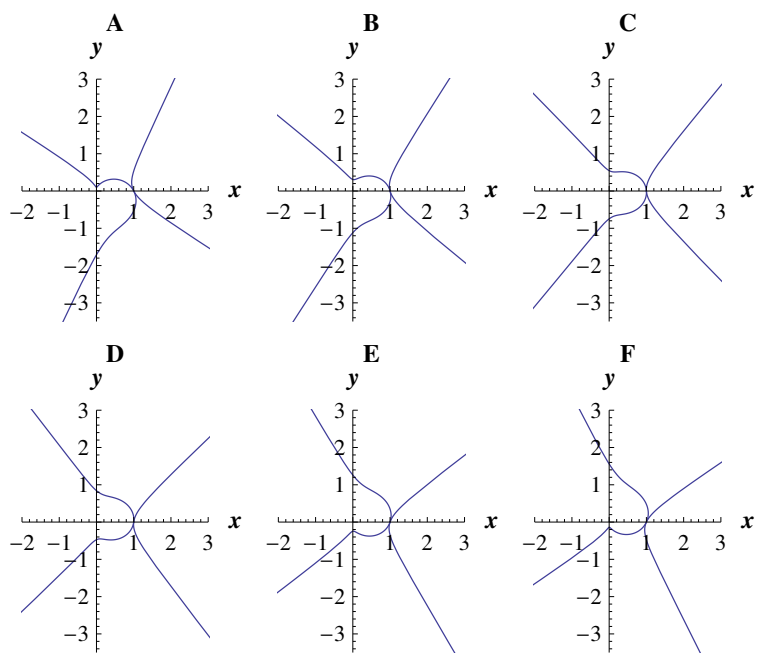

Figure 19

Our previous progression of loops shrunk in size, but this time we see a growth pattern. Note that the values of $k = 2.1, 2.3, 2.5, 2.7, 2.9,$ and 3 correspond to the A, B, C, D, E, and F graphs in figure 20 respectively. It almost appears to be the reverse of what took place when k progressed from 0 to 1. This makes sense, however, since when $k = \pi$, the graph returns to the sideways parabola that we saw when $k = 0$.

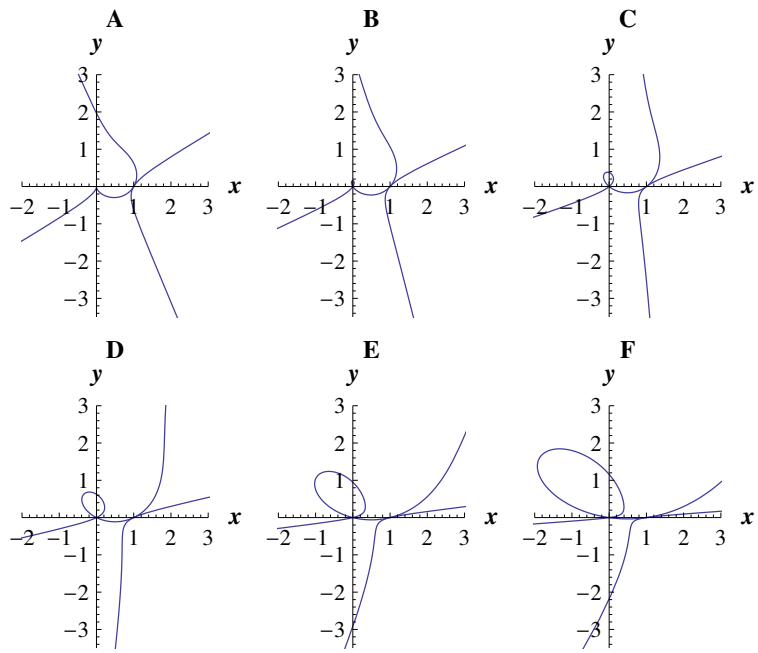

Figure 20

A subtraction of k simply reverses the rotation. Instead of rotating clockwise as we saw with positive values of k, subtracting values of k rotates the graph counterclockwise. Thus $\phi = \sin(\theta) + k$ is the reflection over the x-axis of $\phi = \sin(\theta) - k$. Therefore, we again see that for the sine family, reflection for biangular relationships is a subtraction of constants outside of the argument, not a multiplication by a negative constant.

Adding a Constant within the argument: $\phi = \sin(\theta + k)$

For rectangular relationships, we know that adding a constant k within the brackets of a sine or cosine function causes the graph to shift k units horizontally (either left or right depending on k's sign). It is also common knowledge that if we shift the sine graph by $\frac{\pi}{2}$, it becomes the original cosine function. While we have not seen a horizontal shift in the biangular coordinates, the property that $\sin(x + \frac{\pi}{2}) = \cos(x)$ still holds. For the relationship $\phi = \sin(\theta + \frac{\pi}{2} + k)$, we see a familiar set of graphs.

The graphs in figure 21 are the same that we saw previously when we examined $\phi = \cos(\theta + k)$. In actuality, for the graphs $\phi = \cos(\theta + k)$ and $\phi = \sin(\theta + k)$, if we allow k to fluctuate from 0 to π, they deliver the same graphs, but in a different order. This again can be shown analytically.

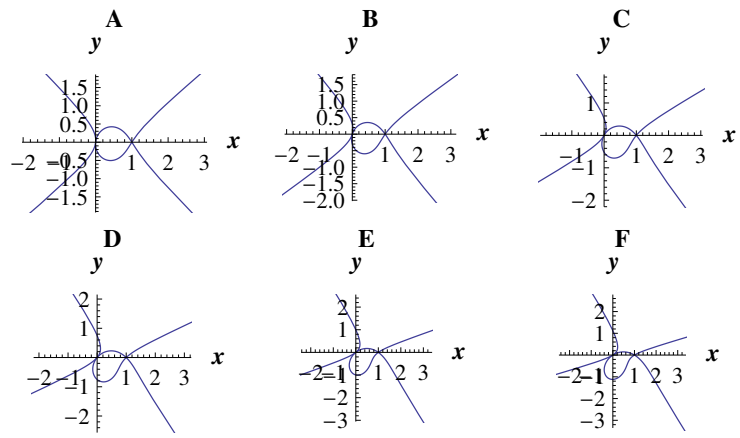

Figure 21

Theorem 4 *The equation* $\phi = \cos(\theta + k)$ *delivers the same graph as* $\phi = \sin\left(\theta + \frac{\pi}{2} + k\right)$.

Proof.

For $\phi = \sin\left(\theta + \dfrac{\pi}{2} + k\right)$ the abscissa is

$$\frac{\cos(\theta)\sin\left(\sin\left(\theta + \frac{\pi}{2} + k\right)\right)}{\sin\left(\theta + \sin\left(\theta + \frac{\pi}{2} + k\right)\right)}$$

$$= \frac{\cos(\theta)\sin\left(\sin(\theta + k)\cos\left(\frac{\pi}{2}\right) + \cos(\theta + k)\sin\left(\frac{\pi}{2}\right)\right)}{\sin\left(\theta + \sin(\theta + k)\cos\left(\frac{\pi}{2}\right) + \cos(\theta + k)\sin\left(\frac{\pi}{2}\right)\right)}.$$

But $\cos\left(\dfrac{\pi}{2}\right) = 0$ and $\sin\left(\dfrac{\pi}{2}\right) = 1$. Thus, the right hand side simplifies to

$$\frac{\cos(\theta)\sin(\cos(\theta + k))}{\sin(\theta + \cos(\theta + k))},$$

which is what we wished to show. The ordinate follows similarly. ■

The sideways parabola that we know as $\phi = \sin(\theta)$ appears in $\phi = \cos(\theta + k)$ when $k = \frac{\pi}{2}$. When $k = \frac{\pi}{2}$, $\phi = \sin(\theta + k)$ becomes the bow-tie shape $\phi = \cos(\theta)$.

This is intuitive given that ϕ takes on the values of these two functions. We can see that although the biangular coordinate system does not display the traditional horizontal shift that we are used to seeing, it produces a rotation-like movement as k fluctuates. For transformed biangular rela-

tionships, this rotational movement corresponds to the horizontal shift of the rectangular plane.

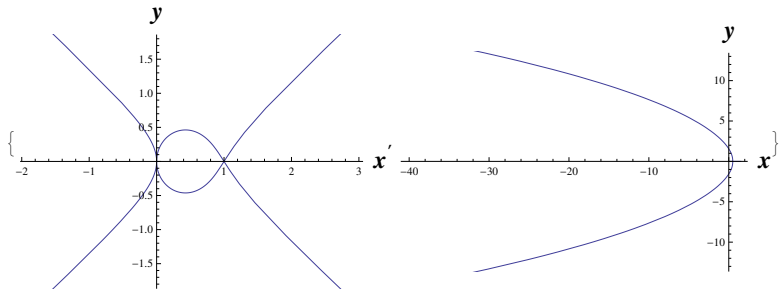

Figure 22

Multiplying by a constant within the argument: $\phi = \sin(k\theta)$

When we let $k = 2$, we advance from a sideways parabola to a lima bean like structure (see figure 23 left). When $k = 3$, however, we notice that we have a shape that I will denote as fairy-like (see figure 23 right). Notice that the "fairy's dress" is split into two pieces.

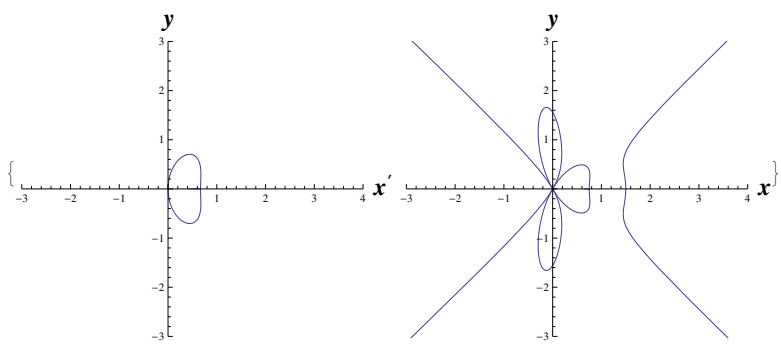

Figure 23

The top rests in the 2nd and 3rd quadrants while the bottom lies in the 1st and 4th quadrants. We have seen this type of situation occur often with biangular coordinates. As the intersection of our two rays directed by ϕ and θ diverts off to infinity on one side of the graph, the rotations of the two angles continue and eventually complete resulting in their intersection returning on another side of the graph.

These graphs are obviously different, but their constants only have a difference of 1. Both also have little resemblance to the original sideways parabola.

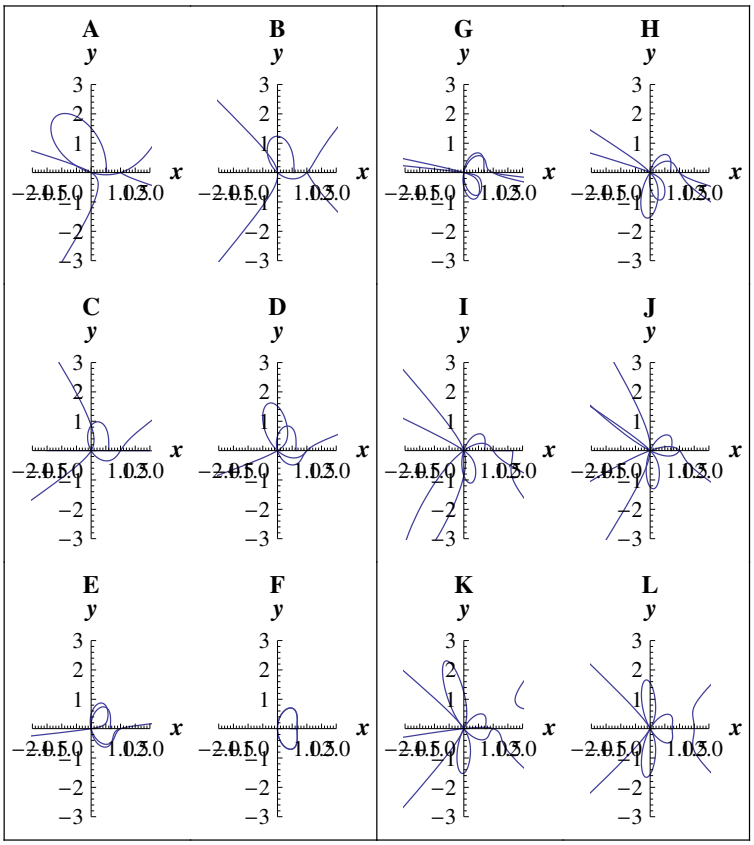

Figure 24

As we have seen, this condition results from a rotation-like movement that occurs when we change the characteristics. As k progresses from 1 to 2, the parabola shape forms loops in the 2nd quadrant that gradually decrease in size and shift position downward until we are left with the lima bean-like shape (see figure 24 left). Allowing k to go from 2 to 3 produces a graph with more loops. As k grows even larger than 3, more loops form. These loops rotate in a propeller-like motion, spinning clockwise, around the pole (0,0), adding more loops as k grows. As the blades rotate around, they are stretched outside forming what resembles a lopsided propeller with smaller blades on the right and large blades (which stretch out to infinity towards the left and return to the graph on the right) on the left (see figure 25).

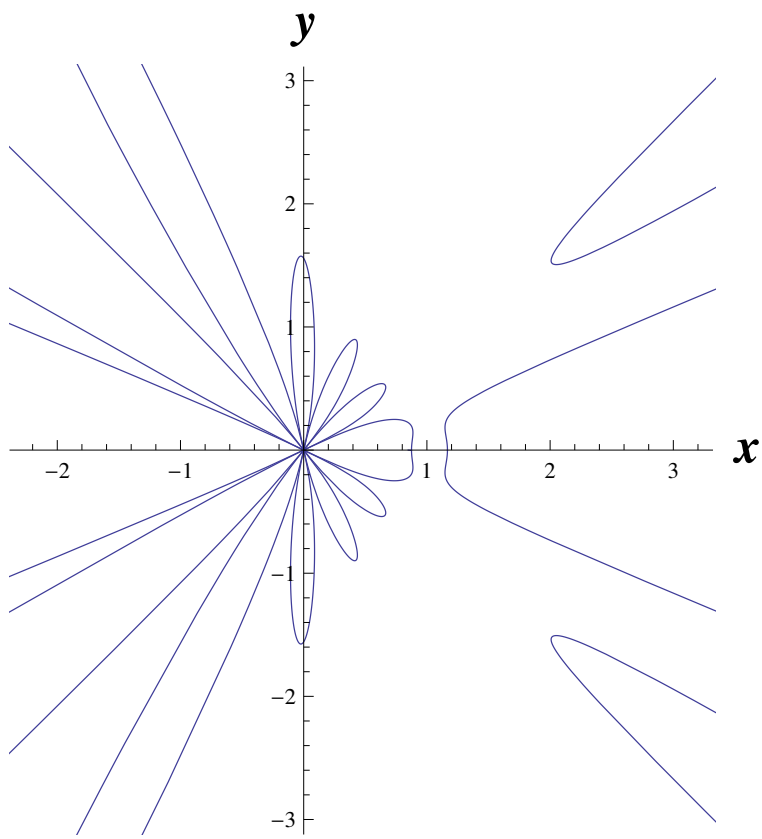

Figure 25

Like we saw with $\phi = \cos(k\theta)$, we see a difference between even and odd values of k. Those with an even constant have fewer "blades" than the odds surrounding them. A constant of even parity, results in a period of π and results in the differing number of blades between parity types (Theorem 2).

Multiplying by a constant outside of the argument: $\phi = k\sin(\theta)$

Again, we see that the larger the constant k, the less informative a graph is. For relatively small values of k though, we can observe some very peculiar shapes.

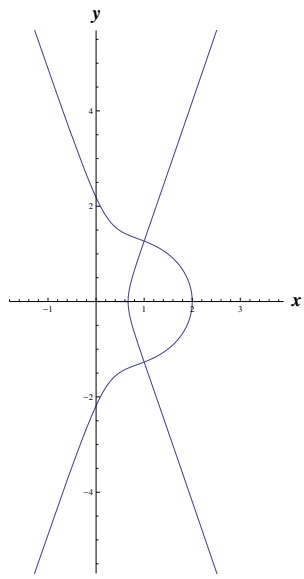

Figure 26

The v-shaped part of the graph in the 1st and 4th quadrants of figure 26 is actually the result of our original sideways parabola bending backwards. If we let k increase indefinitely, the semi-circle will eventually close in on itself at the point (0,0). After this occurs, two loops will grow out of this point, one in the upward direction and one in the downward direction. These loops will grow so large that their tips will appear on the opposite side of the graph, grow continuously towards the point (0,0) and form two new loops. This continues for as long as we let k increase. Thus it results in a graph in which it is difficult to observe a relationship because we have so many loops present in the graph. This is similar to what we saw when we multiplied $\cos(\theta)$ by a constant. ϕ grows dramatically faster than θ which causes numerous arms and loops to appear on the screen.

Raising the relationship to a power:$\phi = \sin(\theta)^k$

In figure 27, we see the graph of $\phi = \sin(\theta)^2$. It contains one elliptically shaped loop. This relationship has a period of π. Examining the values of ϕ, we notice that ϕ always has a positive value and is bounded above by 1 and bounded below by 0. When $\theta = \frac{\pi}{2}$, ϕ is at its maximum value of 1. This is the point where our graph crosses the vertical axis at approximately 1.6.

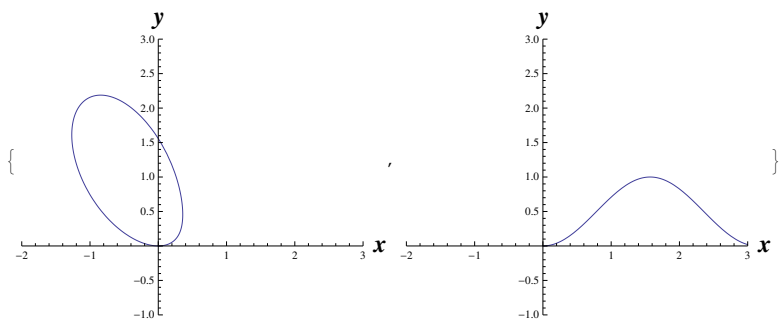

Figure 27

After reaching its maximum, ϕ begins its descent back towards zero, but θ continues towards π. This is how our circular shape results because ϕ is so restricted in that it only increases and then decreases exactly once. Thus, it does not alternate as often as we have seen in previous graphs.

When $\phi = \sin(\theta)^3$, however, we see two elliptical shapes appear in the graph (see figure 28, left). Because our value of k is odd in this case, ϕ is not restricted to positive numbers. Thus, when θ progresses into the 3rd and 4th quadrants, ϕ is decreasing towards -1 instead of 0. This results in the second loop on the lower half of the graph, forming a lopsided figure eight. Indeed, numerical evidence suggests that this pattern continues for powers greater than 3. Odd values allow ϕ to become negative, while even powers do not. Notice how the loops for $k = 3$ are narrower and not as tall as when $k = 2$. The larger the power, the thinner and shorter the loops become (see figure 28).

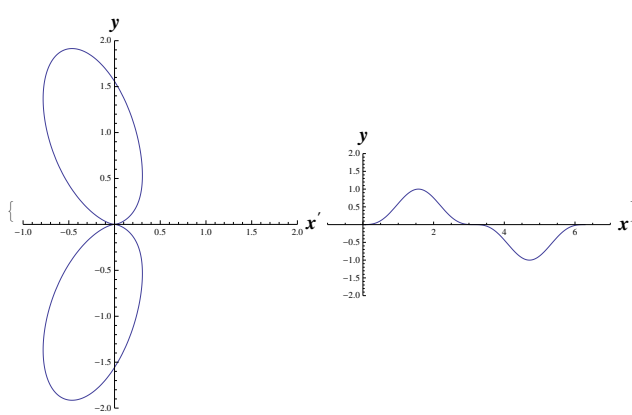

Figure 28

Function Family: $\phi = \sin(\theta)\cos(\theta)$

When we plot the relationship $y = \sin(x)\cos(x)$ in the rectangular plane, we see that y is bounded between -0.5 and 0.5 and has a period of π. It does, however, have a similar shape to a sine function so it would be reasonable to conclude that when we convert this to the biangular plane, it results in a graph resembling $\phi = \sin(\theta)$.

This, however, is not the case. $\phi = \sin(\theta)$ resulted in a sideways parabola, but $\phi = \sin(\theta)\cos(\theta)$ results in an ellipse. This makes sense, as we know from trigonometric identities that $\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$. It is a logical conclusion then that $\phi = \sin(\theta)\cos(\theta) = \frac{1}{2}\sin(2\theta)$. We have already been exposed to what happens to $\phi = \sin(\theta)$ when we multiply by constants inside and outside the argument. This is simply an example of what occurs with a combination of these effects.

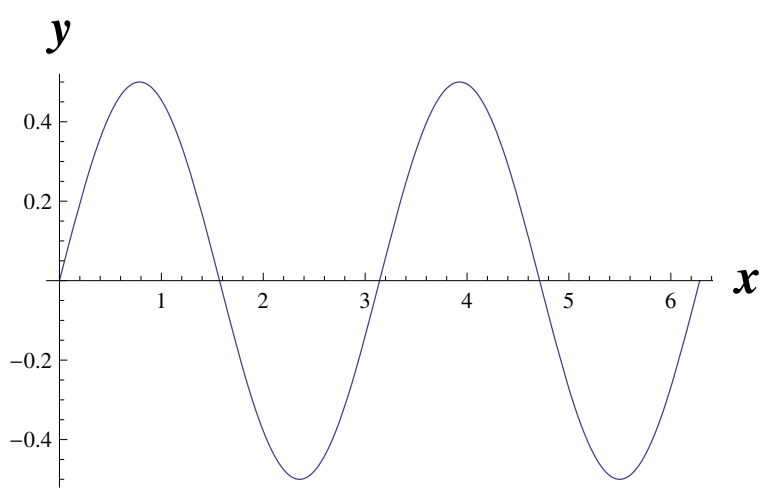

Figure 29

The value of ϕ begins at zero, increases to around 0.5, decreases back to 0, continues decreasing to approximately -0.5, and then increases back to zero. This process is then repeated. Thus ϕ has a period of π. Note that this is consistent with Theorem 2. θ is consistently increasing but ϕ alternates between increasing and decreasing. It is this alternation that results in the ellipse-like graph above. Initially ϕ increases with θ, creating a curve that rises above the x axis as ϕ rises, when ϕ begins to decrease, however, (at approx $\theta = .75$) θ continues to increase which results in the curve shifting back downwards towards the x axis. A similar process repeats as ϕ becomes a negative angle and then returns to 0. This creates the lower part

of the ellipse. When $\theta = 0$, so does ϕ, so at this point, technically any of the points on the polar axis are valid. We exclude this characteristic on the graph in figure 30 because it is irrelevant to our understanding of this relationship.

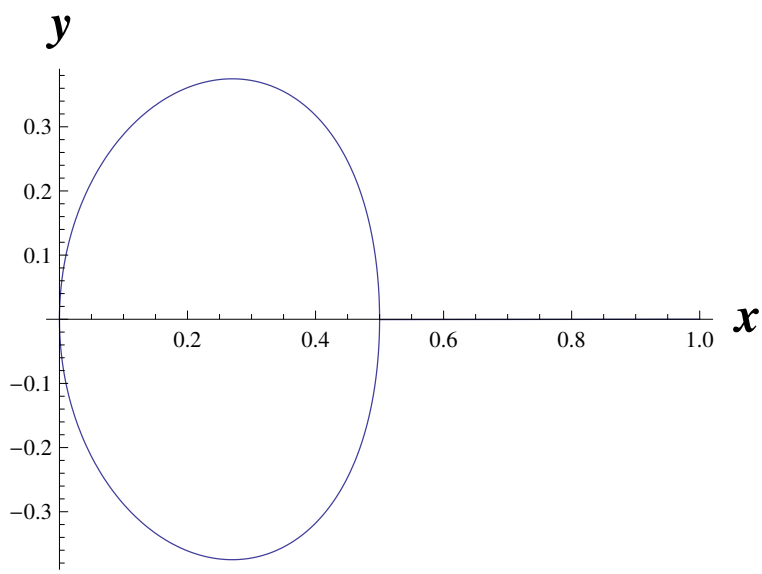

Figure 30

Multiplying this relationship by a constant of k, we see an increase in the size of the ellipse-like shape. When this occurs, however, the point that the graph crosses the polar axis is pulled back towards the origin while the top and bottom parts of the graph are stretched off to infinity.

This forms a boomerang-like shape (see figure 31, A-C). The tips of the boomerang eventually stretch so far out to infinity that they appear on the other side of the vertical axis, cross through the origin and repeat the growth again. After observing many values for k, computational evidence suggests that the higher the constant, the more times this process occurs, and the more lines that appear on the graph, making it harder to observe a relationship (see figure 31, D-F).

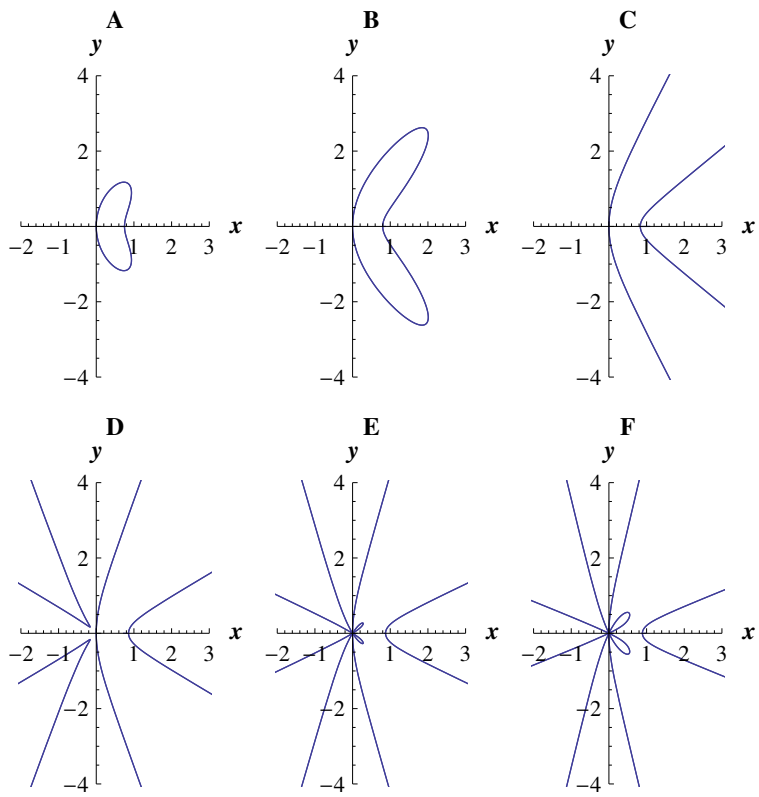

Figure 31

Some Interesting Graphs

Consider the two functions $\frac{\cos(\theta)}{\theta}$ and $\frac{\sin(\theta)}{\theta}$. Neither of these two functions have a period.

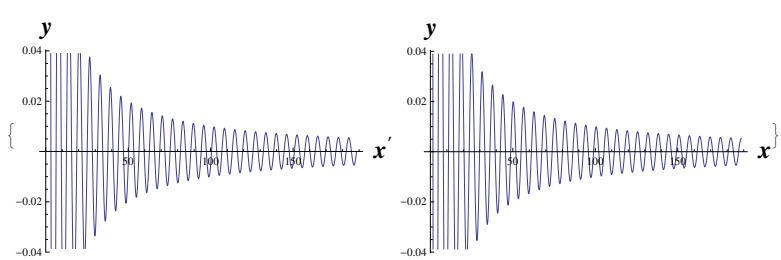

Figure 32

Despite having no period, these graphs possess a somewhat repeated pattern. If we plot this relationship between ϕ and θ we obtain the figure below.

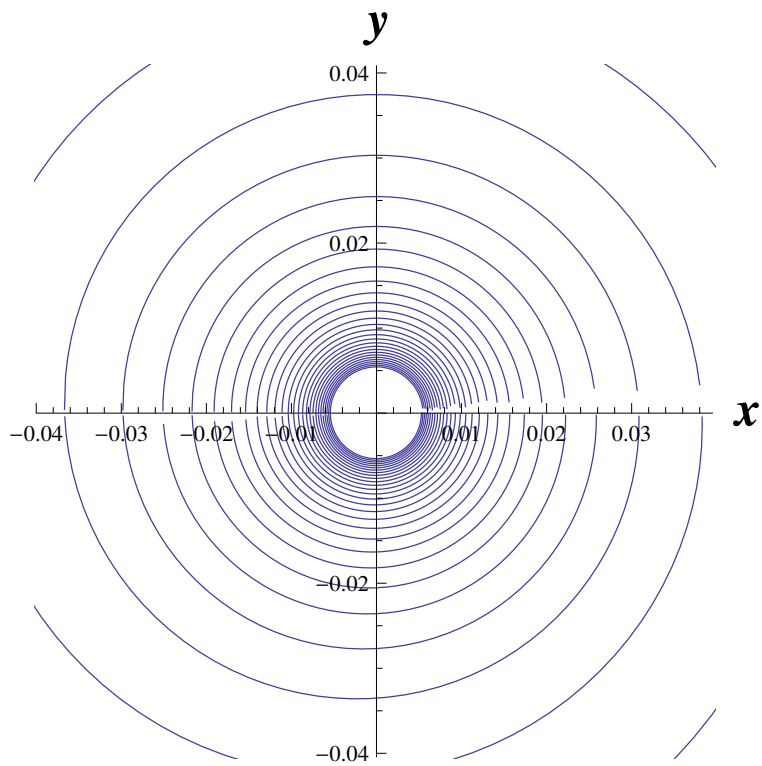

Figure 33

The graph resembles a spiral which would proceed farther if not for the bounding $0 \leq \theta \leq 60\pi$. We can see why this occurs by looking at the graph for ϕ displayed above (see figure 32, left). We see a repeated pattern of increasing to a local maximum and then decreasing to a local minimum. The peaks of this function gradually gain smaller amplitude as θ increases. This results in the spiral becoming smaller, generating a tunnel-like illusion in the graph.

If we change our plot range on this graph, we can gain a wider view.

The shape in the figure above resembles a shape that is commonly displayed in polar coordinates. It is called an Archimedean spiral and has the equation $r(\theta) = a + b\theta$ $(a, b \in \mathbb{R})$ in polar coordinates.

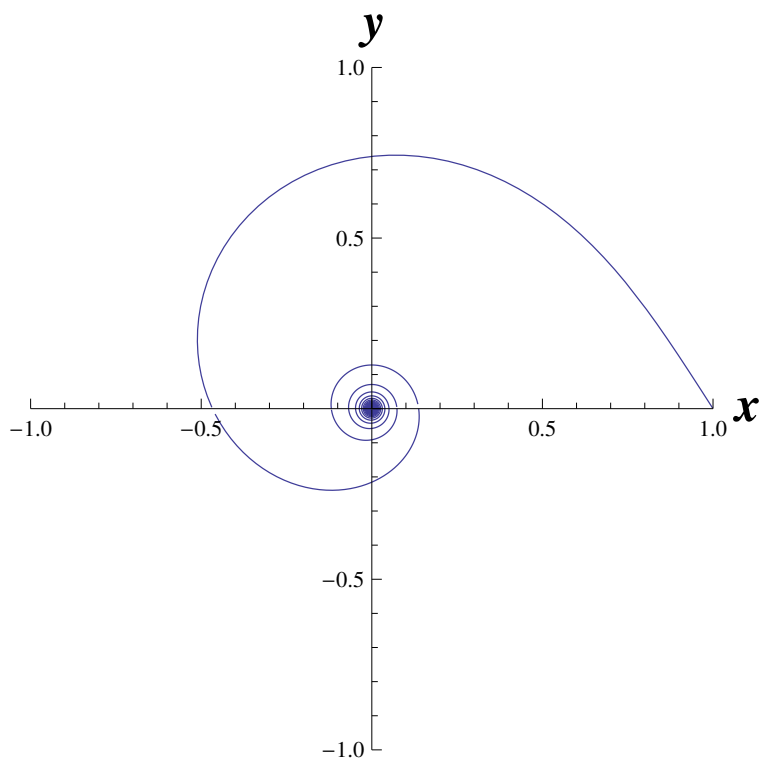

Figure 34

When we plot $\phi = \frac{\cos(\theta)}{\theta}$, we get something entirely different.

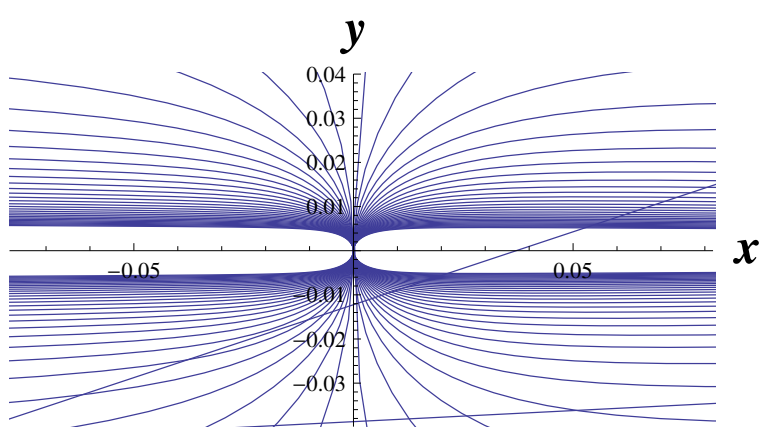

Figure 35

The tiniest difference results in a dramatic change. This graph still gives the illusion of a structure in the distance. Once again, if we widen our viewing screen for a better view we see that this new graph is not very intuitive. Its most revealing aspect appears right around the origin and we can see a dense area of graph in that location in figure 36. This does, however, explain the presence of the few asymptotes in figure 35. It results from the jumping of quadrants by the overall graph as opposed to what we observe around the origin.

Conclusion

Coordinate systems are a foundation of mathematics and can be employed for various uses because they are carefully designed to work properly. Before we use them, however, we must first understand how they work and the benefits of using a particular system.

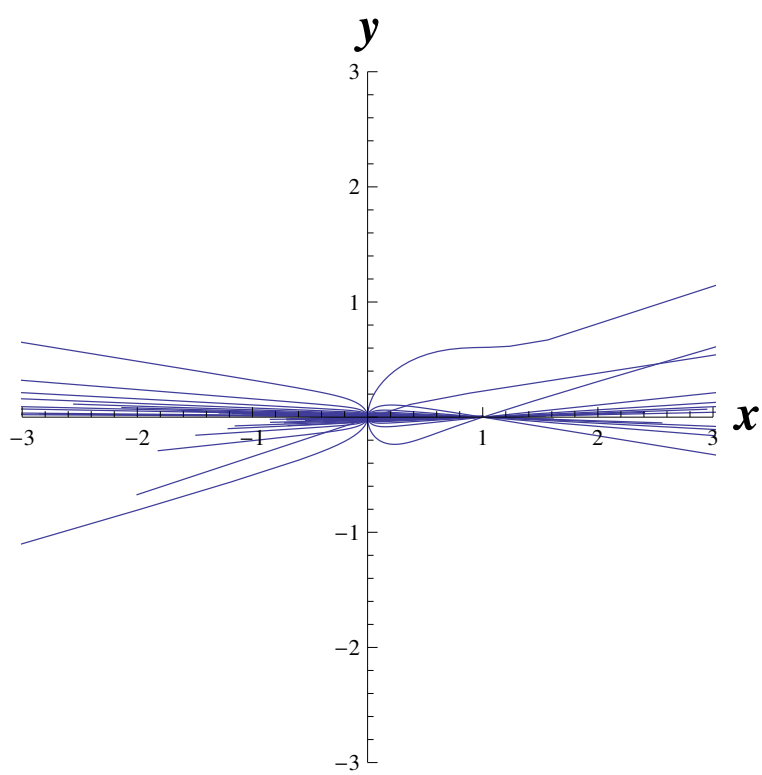

Figure 36

The introductory steps have been taken to understand the capabilities of the biangular coordinate system, and we find that we are in semi-familiar territory. Continued work on the topic will be required in order to fully understand the possibilities the biangular coordinate system can provide for mathematics and for other academic fields.

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[2] G.B.M. Zerr, Biangular coordinates, Amer. Math. Monthly, **17** (1910) 34-38.

Approximations of π

Anthony Cammarano and Gabriela Rodrigues, *students*

NY Omicron

St. Joseph's College
Patchogue, New York, 11772

1. Introduction

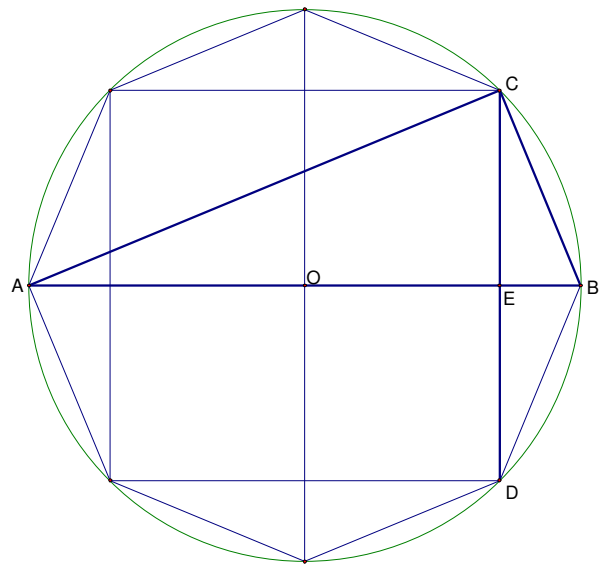
In this article, we examine several methods that mathematicians have used to approximate π. The race to calculate π to as many digits as possible has inspired many methods of calculation.

2. Archimedes' Approximation of π

Archimedes' idea was to compute the ratio of the perimeter to the diameter in a regular polygon with 2^n sides and a radius of 1. As n increases, the polygon more closely resembles a circle, and the ratio more closely approximates π.

Inscribed polygons

To illustrate Archimedes' method of approximating π, we use a diagram of an octagon and a square inscribed in a circle. Our goal here is to find the length of one side of the octagon. From that information, we can easily determine the approximation of π by finding the perimeter and dividing the perimeter by the diameter.



Since AB is a diameter, $\angle ACB$ is a right angle. Thus, the Pythagorean Theorem applied to triangle ACB implies that

$$(BC)^2 = (AB)^2 - (AC)^2 .$$

Recalling our assumption that the radius is 1, we have $AB = 2$, so that

$$(BC)^2 = 4 - (AC)^2 . \tag{1}$$

We note next that triangles ACB and AEC are similar. Thus,

$$\frac{AC}{AB} = \frac{AE}{AC},$$

that is,

$$\begin{aligned} (AC)^2 &= 2\,(AE) \\ &= 2\,(AB - EB) \\ &= 2\,(2 - EB) . \end{aligned}$$

Substituting in (1) gives

$$(BC)^2 = 2\,(EB) . \tag{2}$$

The Pythagorean Theorem applied to triangle BCE gives

$$\begin{aligned} (EB)^2 &= (BC)^2 - (EC)^2 \\ &= (BC)^2 - \left(\frac{1}{2}CD\right)^2 . \end{aligned}$$

Substituting in (2) gives

$$(BC)^2 = 2\sqrt{(BC)^2 - \left(\frac{1}{2}CD\right)^2}.$$

Squaring and rearranging gives

$$(BC)^4 - 4\,(BC)^2 + (CD)^2 = 0. \tag{3}$$

For positive integers k, we define s_k to be the length of the side of a regular k-gon inscribed in a circle of radius 1. Thus, in the diagram, $CD = s_4 = s_{2^2}$ and $BC = s_8 = s_{2^3}$. In general, if B, C, and D are consecutive vertices of a regular 2^n-gon inscribed in a circle of radius 1, then B and D are consecutive vertices of a regular 2^{n-1}-gon inscribed in the same circle. Thus,

$$BC = s_{2^n} \text{ and } CD = s_{2^{n-1}},$$

and equation (3) becomes

$$(s_{2^n})^4 - 4\,(s_{2^n})^2 + (s_{2^{n-1}})^2 = 0.$$

We apply the quadratic formula, regarding s_{2^n} as the variable, obtaining

$$(s_{2^n})^2 = \frac{4 - \sqrt{16 - 4\,(s_{2^{n-1}})^2}}{2},$$

where we have used the negative root, since otherwise the side length would be greater than the diameter of the circle. Simplifying and taking the square root gives the recursive formula

$$s_{2^n} = \sqrt{2 - \sqrt{4 - (s_{2^{n-1}})^2}}, n \geq 3.$$

Noting that the side lengths of the inscribed square have length $\sqrt{2}$, we obtain the following values.

n	Recursive formula for s_{2^n}	Value of s_{2^n}
3	$s_{2^3} = \sqrt{2 - \sqrt{4 - (s_{2^2})^2}}$	$s_{2^3} = \sqrt{2 - \sqrt{4 - \left(\sqrt{2}\right)^2}}$ $s_{2^3} = \sqrt{2 - \sqrt{2}}$ $s_{2^3} = 0.7653668647$
4	$s_{2^4} = \sqrt{2 - \sqrt{4 - (s_{2^3})^2}}$	$s_{2^4} = \sqrt{2 - \sqrt{4 - \left(\sqrt{2 - \sqrt{2}}\right)^2}}$ $s_{2^4} = \sqrt{2 - \sqrt{2 + \sqrt{2}}}$ $s_{2^4} = 0.390180644$

More generally, this process leads to

$$s_{2^n} = \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}}}}},$$

with a total of $n - 1$ 2's. For instance, the side length of a regular polygon with 2^{10} sides inscribed in a circle of radius 1 is

$$\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}}}}}} \approx 0.00613591.$$

Finally, as an approximation to π, we have

$$\begin{aligned} \pi &\approx \left(\frac{2^n}{2}\right) s_{2^n} = \left(\frac{2^n}{2}\right) \sqrt{2 - \sqrt{4 - \left(s_{2^{n-1}}\right)^2}} \\ &= \left(\frac{2^n}{2}\right) \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}}}}}, \end{aligned}$$

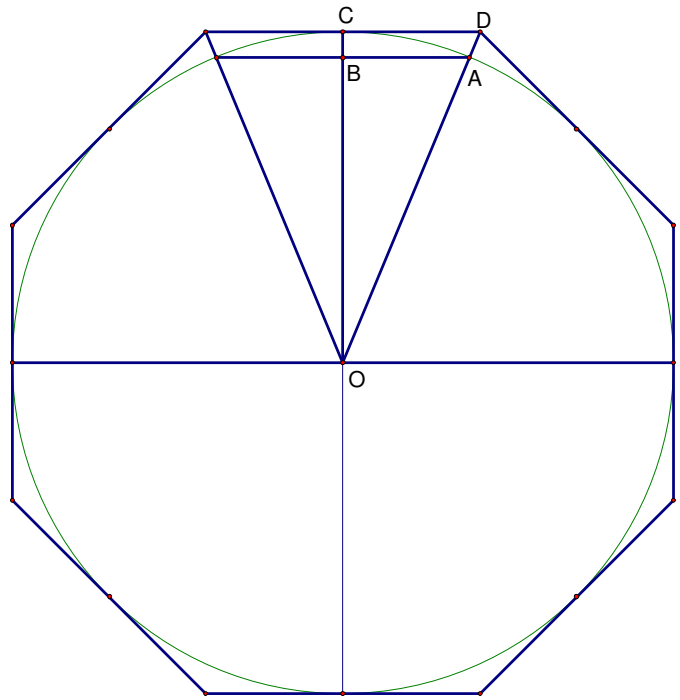
again with $n - 1$ 2's within the radicals. This gives the following values. Since we are using inscribed polygons, the approximations are much smaller than π and increase as n increases.

Number of Sides	$\mathbf{P}_n(\pi)$
4	2.828427125
8	3.061467459
16	3.121445152
32	3.136548491
64	3.140331157
$\vdots$	$\vdots$
1024	3.141587725

Circumscribed polygons

The next step in Archimedes' approximation of π is to determine a formula for the ratio of perimeter to diameter for regular polygons circumscribed about a circle of radius 1. As n increases, the polygon more closely resembles a circle, and the ratio more closely approximates π.

This time, we look for a relationship between the side lengths of an inscribed polygon and the circumscribed polygon of the same degree or number of sides. We can then use our previously-derived formulas. We use a regular octagon to illustrate the calculations.



We note that OA and OC are radii of the circle, and triangles AOB and DOC are similar right triangles, so that

$$\begin{aligned} \frac{CD}{OC} &= \frac{AB}{OB} = \frac{AB}{\sqrt{(OA)^2 - (AB)^2}} \\ CD &= \frac{AB}{\sqrt{1-(AB)^2}}. \end{aligned}$$

With s_n as before and c_n now representing the length of a side of a circumscribed 2^n-gon, we have

$$c_{2^n} = \frac{s_{2^n}}{\sqrt{1 - \frac{1}{4}\left(s_{2^n}\right)^2}}$$

and

$$\pi \approx \left(\frac{2^n}{2}\right) c_n = \left(\frac{2^n}{2}\right) \frac{s_{2^n}}{\sqrt{1 - \frac{1}{4}\left(s_{2^n}\right)^2}}.$$

The following tables gives values of the estimates obtained using both inscribed and circumscribed regular polygons.

n	Number of Sides	$\mathbf{P}_n$ Underestimate	$\mathbf{T}_n$ Overestimate
2	4	2.828427124746190	4
3	8	3.061467458920718	3.313708498984760
4	16	3.121445152258052	3.182597878074528
5	32	3.136548490545939	3.151724907429256
6	64	3.140331156954753	3.144118385245904
	$\vdots$	$\vdots$	$\vdots$
10	1024	3.141587725277160	3.141602510256809

3. MacLaurin Expansion for arctanx

Basic version

Since $\arctan 1 = \frac{\pi}{4}$, we can approximate π by computing the MacLaurin expansion of $\arctan x$, substituting 1 for x, evaluating a finite number of terms of the sum, and multiplying the result by 4. In general, the MacLaurin series for an infinitely-differentiable function f is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n.$$

Accordingly, we calculate successive derivatives of $f(x) = \arctan x$ and evaluate them at $x = 0$, obtaining

n	$f^{(n)}(x)$	$f^{(n)}(0)$
1	$\frac{1}{1+x^2}$	1
2	$-\frac{2x}{(1+x^2)^2}$	0
3	$\frac{8x^2}{(1+x^2)^3} - \frac{2}{(1+x^2)^2}$	-2
4	$-\frac{48x^3}{(x^2+1)^4} + \frac{24x}{(x^2+1)^3}$	0
5	$\frac{384x^4}{(x^2+1)^5} - \frac{288x^2}{(x^2+1)^4} + \frac{24}{(x^2+1)^3}$	24

This gives

$$\begin{aligned} f(x) &\sim \frac{0}{0!}x^0 + \frac{1}{1!}x^1 + \frac{0}{2!}x^2 - \frac{2}{3!}x^3 + \frac{0}{4!}x^4 + \frac{24}{5!}x^5 + \cdots \\ &= \frac{x^1}{1} - \frac{x^3}{3} + \frac{x^5}{5} - \cdots, \end{aligned}$$

and more generally,

$$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} \text{ for } -1 \le x \le 1.$$

In particular,

$$\pi = 4 \cdot \arctan 1 = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots\right).$$

Thus, with N terms, the approximation to π is

$$a(N) = 4 \sum_{n=0}^{N-1} \frac{(-1)^n}{2n+1}.$$

We will use this equation to determine how many terms in the series would be necessary to acquire a good approximation of π.

N	Approximation of π
100	3.131592
500	3.139592
1000	3.140592
5000	3.141392
20,000	3.141542

Although the MacLaurin Expansion did provide a valid approximation of π, the convergence was extremely slow – taking approximately 20,000 terms in the series to obtain a value correct to four decimal places.

An improvement

A variation on this approach is to use the formula

$$\frac{\pi}{4} = 4\arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right). \tag{4}$$

To prove that equation (4) is valid, set

$$\alpha = \arctan\left(\frac{1}{5}\right) \text{ and } \beta = \arctan\left(\frac{1}{239}\right).$$

Then equation (4) is equivalent to

$$\frac{\pi}{4} - 4\alpha = -\beta. \tag{5}$$

But

$$\begin{aligned}\tan\left(\frac{\pi}{4}-4\alpha\right) &= \frac{\tan\frac{\pi}{4}-\tan(4\alpha)}{1+\tan\frac{\pi}{4}\cdot\tan(4\alpha)}\\ &= \frac{1-\tan(4\alpha)}{1+\tan(4\alpha)}.\end{aligned}$$

Since $\tan\alpha=\frac{1}{5}$, we have

$$\begin{aligned}\tan(2\alpha) &= \frac{2\tan\alpha}{1-\tan^2 a}\\ &= \frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2}\\ &= \frac{10}{24}\\ &= \frac{5}{12}\end{aligned}$$

and so

$$\begin{aligned}\tan(4\alpha) &= \frac{2\tan(2\alpha)}{1-\tan^2(2\alpha)}\\ &=\\ &= \frac{2\left(\frac{5}{12}\right)}{1-\left(\frac{5}{12}\right)^2}\\ &= \frac{120}{119}.\end{aligned}$$

Thus,

$$\begin{aligned}\tan\left(\frac{\pi}{4}-4\alpha\right) &= \frac{1-\frac{120}{119}}{1+\frac{120}{119}}\\ &= -\frac{1}{239},\end{aligned}$$

proving (5) and thus (4).

Using the MacLaurin expansion for $\arctan$, equation (4) gives

$$\begin{aligned}\frac{\pi}{4} &= 4\left[\frac{1}{5}-\frac{1}{3}\left(\frac{1}{5}\right)^3+\frac{1}{5}\left(\frac{1}{5}\right)^5-\cdots\right]\\ &\quad-\left[\frac{1}{239}-\frac{1}{3}\left(\frac{1}{239}\right)^3+\frac{1}{5}\left(\frac{1}{239}\right)^5-\cdots\right].\end{aligned}$$

That is, with N terms, the approximation to π is

$$\begin{aligned} b(N) &= 4\left[4\sum_{n=0}^{N-1}\frac{(-1)^n}{2n+1}\cdot\left(\frac{1}{5}\right)^{2n+1} - \sum_{n=0}^{N-1}\frac{(-1)^n}{2n+1}\cdot\left(\frac{1}{239}\right)^{2n+1}\right] \\ b(N) &= 4\sum_{n=0}^{N-1}\frac{(-1)^n}{2n+1}\left[4\left(\frac{1}{5}\right)^{2n+1} - \left(\frac{1}{239}\right)^{2n+1}\right]. \end{aligned}$$

N	Approximation of π
4	3.141591772
6	3.141592653

Thus, with just 6 terms,we have a value correct to 9 decimal places.

4. Integration of Even and Odd Powers of Sine

The last approximation of Pi we examine uses the Wallis Product.

Recursive formula

Applying integration by parts to the integral

$$\int \sin^n x\,dx = \int \sin^{n-1} x \sin x\,dx$$

with $u = \sin^{n-1} x$ and $dv = \sin x\,dx$, we obtain

$$\begin{aligned} \int \sin^n x\,dx &= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\cos^2 x\,dx \\ &= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\left(1-\sin^2 x\right)\,dx \\ &= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx \\ &\quad -(n-1)\int \sin^n x\,dx. \end{aligned}$$

Rearranging gives

$$n\int \sin^n x\,dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx,$$

so that

$$\int \sin^n x\,dx = \frac{-\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx}{n}.$$

Evaluating on the interval $\left[0, \frac{\pi}{2}\right]$, we obtain

$$\int_0^{\pi/2} \sin^n \, dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x \, dx. \tag{6}$$

Even powers

For $n = 2m$, equation (6) gives

$$\int_0^{\pi/2} \sin^{2m} \, dx = \frac{2m-1}{2m} \int_0^{\pi/2} \sin^{2m-2} x \, dx.$$

Iterating gives

$$\begin{aligned}
\int_0^{\pi/2} \sin^{2m} \, dx &= \frac{2m-1}{2m} \cdot \frac{2m-3}{2m-2} \int_0^{\pi/2} \sin^{2m-4} x \, dx \\
&= \cdots \\
&= \frac{2m-1}{2m} \cdot \frac{2m-3}{2m-2} \cdot \frac{2m-5}{2m-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \int_0^{\pi/2} 1 \, dx \\
&= \frac{2m-1}{2m} \cdot \frac{2m-3}{2m-2} \cdot \frac{2m-5}{2m-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} \\
&= \left(\prod_{k=1}^{m} \frac{2k-1}{2k} \right) \cdot \frac{\pi}{2}.
\end{aligned}$$

Odd powers

For $n = 2m+1$, equation (6) gives successively

$$\begin{aligned}
\int_0^{\pi/2} \sin^{2m+1} x \, dx &= \frac{2m}{2m+1} \int_0^{\pi/2} \sin^{2m-1} x \, dx \\
&= \frac{2m}{2m+1} \cdot \frac{2m-2}{2m-1} \int_0^{\pi/2} \sin^{2m-3} x \, dx \\
&= \cdots \\
&= \frac{2m}{2m+1} \cdot \frac{2m-2}{2m-1} \cdots \frac{4}{5} \cdot \frac{2}{3} \int_0^{\pi/2} \sin x \, dx \\
&= \frac{2m}{2m+1} \cdot \frac{2m-2}{2m-1} \cdots \frac{4}{5} \cdot \frac{2}{3} \\
&= \prod_{k=1}^{m} \frac{2k}{2k+1}.
\end{aligned}$$

π as a product

For positive integers n, wet

$$I(n) = \int_0^{\pi/2} \sin^n x \, dx.$$

From the above, we have

$$I(2m) = \left(\prod_{k=1}^{m} \frac{2k-1}{2k} \right) \cdot \frac{\pi}{2}$$

and

$$I(2m+1) = \prod_{k=1}^{m} \frac{2k}{2k+1},$$

so that

$$\frac{I(2m)}{I(2m+1)} = \prod_{k=1}^{m} \frac{(2k+1)(2k-1)}{(2k)^2} \cdot \frac{\pi}{2}.$$

Hence,

$$\pi = 2 \cdot \frac{I(2m)}{I(2m+1)} \cdot \prod_{k=1}^{m} \frac{(2k)^2}{(2k+1)(2k-1)} \text{ for all positive integers } m.$$

We now estimate $\frac{I(2m)}{I(2m+1)}$. For all $x \in \left[0, \frac{\pi}{2}\right]$, we have

$$0 \leq \sin^{2m+1} x \leq \sin^{2m} x \leq \sin^{2m-1} x.$$

Integrating gives

$$0 < I(2m+1) \leq I(2m) \leq I(2m-1),$$

and dividing by $I(2m+1)$ gives

$$1 \leq \frac{I(2m)}{I(2m+1)} \leq \frac{I(2m-1)}{I(2m+1)} = \frac{2m+1}{2m}.$$

As $m \to \infty$, $\frac{2m+1}{2m} \to 1$, so by the Squeeze Theorem,

$$\lim_{m \to \infty} \frac{I(2m)}{I(2m+1)} = 1.$$

Hence,

$$\begin{aligned} \pi &= 2 \cdot \lim_{m \to \infty} \left[\frac{I(2m)}{I(2m+1)} \cdot \prod_{k=1}^{m} \frac{(2k)^2}{(2k+1)(2k-1)} \right] \\ &= 2 \prod_{k=1}^{\infty} \frac{(2k)^2}{(2k+1)(2k-1)}. \end{aligned}$$

From this product, we obtain the following values.

Number of Factors	Approximation of π
5	3.0022
20	3.1035
80	3.1319
640	3.1404

We note that the convergence is very slow.

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Twin Primes

Pedro Henrique Oliveira Pantoja, *student*

University of Lisbon, Portugal

1. Introduction

In this short essay we will explore some of the results of a special class of primes, twin primes. There is a well known open problem, the conjecture of the infinity of twin primes. First, we prove some elementary results and classics such as Clement's theorem. Therefore, some non-trivial, as Brun's theorem.

Since the time of Euclid is known of the existence of infinity of primes. Much later Dirichlet studied primes in arithmetic progressions. Recently in 2004, T. Tao and B. Green proved a result of arbitrarily long arithmetic progressions of primes, see [1]. The problem here is a little different. Consider the following trivial case:

It is easy to argue that the only difference is one whose cousins are (2, 3). However if we consider that the difference is 2, are there one or two such primes infinite? This question is far from being answered, and that is the famous twin prime conjecture.

The twin primes less than 300 are (3,5), (5,7), (11,13), (17,19), (29,31), (41, 43), (59, 61), (71, 73), (101, 103), (107, 109), (137, 139), (149, 151), (179, 181), (191, 193), (197, 199), (227, 229), (239, 241), (269, 271), (281, 283).

Interestingly, except for (3,5), all these pairs of primes can be written as $(6k-1, 6k+1)$ for some $k \in \mathbb{Z}$. For example, $5 = 6 \cdot 1 - 1$ and $7 = 6 \cdot 1 + 1$. This is easy to see, since every integer can be written as $6k + r$, where $k \in \mathbb{Z}$ and $r \in \{0, 1, 2, 3, 4, 5\}$[1]. Of these, the ones that can be prime numbers greater than 3 are those with r equal to 1 or 5. The others are multiples of 2 or 3. If $p = 6k+1$, however, then $p+2 = 6k+3$, which is a multiple of 3. Thus, for a pair $(p, p+2)$ of twin primes with $p > 3$, we must have $p = 6k + 5$, or equivalently, $p = 6k - 1$. By

[1] Of course {0,1,2,3,4,5} are different possible residues mod 6.

Dirichlet's theorem[2], there are infinitely many primes of the form $6k+1$ and $6k-1$ with $k \in \mathbb{Z}$. And every pair of twin primes except $(3,5)$ can be written as $(6k-1, 6k+1)$ with $k \in \mathbb{Z}$. We may be tempted to conclude that there infinitely many twin primes.

Unfortunately (or fortunately) this argument is flawed. Note that this problem is not so easy in fact it is an extremely difficult problem. Although there are a large amount of twin primes discovered, it does not guarantee its infinity. Dubner in 1993 showed that $459 \cdot 2^{8529} \pm 1$ are twin primes. There are other classes of prime numbers called prime triplets, for example, (3, 5, 7). In the last part of the text we will discuss generalizations made about twin prime conjecture and the twin primes.

2. Clement's Theorem and Catalan Numbers

The following result is a weak version of Clement's theorem.

Proposition 1 *If $(p, p+2)$ is a twin prime pair, then*

$$2^{p+2} \equiv 3p + 8 \pmod{p^2 + 2p}.$$

Proof. If p is an odd prime, then by Fermat's little theorem we have

$$2^{p-1} \equiv 1 \pmod{p} \Longrightarrow 2^{p+2} \equiv 8 \equiv 3p + 8 \pmod{p}.$$

Similarly, if $p+2$ is prime, then

$$\begin{aligned} 2^{p+1} \equiv 1 \pmod{p+2} \quad &\Longrightarrow \quad 2^{p+2} \equiv 2 \equiv 2 + 3(p+2) \\ &= \quad 3p + 8 \pmod{p+2}. \end{aligned}$$

Since $\gcd(p, p+2) = 1$, the result follows. ■

It is worth mentioning that the converse is not valid; a counterexample is the pair (561, 563). (Is it a coincidence that 561 is a Carmichael number?)

Let us consider now the Catalan numbers, defined by

$$\mathcal{C}_n = \frac{1}{n+1}\binom{2n}{n}.$$

There are many interesting properties of Catalan numbers, see [2]. The following lemma is immediate.

[2] (Dirichlet's theorem): Given an integer $n > 1$ and an integer a such that $\gcd(a, n) = 1$, there are infinitely many primes p such that $p \equiv a \pmod{n}$.

Lemma 1 *If a prime number p is odd, then*

$$(-1)^{\frac{p-1}{2}} \cdot \mathcal{C}_{(p-1)/2} \equiv 2 \pmod{p}.$$

Proof. We have $p - i \equiv -i \pmod{p}$ for any integer i; hence $(p-1)! = 1 \cdot 2 \cdot 3 \cdots$. Thus

$$\begin{aligned} & \left(\frac{p-1}{2}\right)\left(p - \left(\frac{p-1}{2}\right)\right)\left(p - \left(\frac{p-1}{2} - 1\right)\right) \cdots (p-1) \\ \equiv \quad & (-1)^{\frac{p-1}{2}} \left[\left(\frac{p-1}{2}\right)!\right]^2, \end{aligned}$$

so that

$$\begin{aligned} \mathcal{C}_{(p-1)/2} &\equiv (p+1)\,\mathcal{C}_{\frac{p-1}{2}} = 2\binom{p-1}{(p-1)/2} \\ &= 2 \cdot \frac{(p-1)!}{\left[\left(\frac{p-1}{2}\right)!\right]^2} \equiv 2\,(-1)^{\frac{p-1}{2}} \pmod{p}. \end{aligned}$$

The result follows. ■

Proposition 2 *If $(p, p+2)$ is a twin prime pair, then*

$$8\,(-1)^{\frac{p-1}{2}} \cdot \mathcal{C}_{(p-1)/2} \equiv 7p + 16 \pmod{p^2 + 2p}.$$

Proof. By the previous lemma, $8\,(-1)^{\frac{p-1}{2}} \cdot \mathcal{C}_{(p-1)/2} \equiv 8 \cdot 2 \equiv 7p + 16 \pmod{p}$. Similarly, as

$$\mathcal{C}_{(p-1)/2} = \frac{p+3}{4p} \cdot \mathcal{C}_{(p+1)/2},$$

it follows that

$$\begin{aligned} 8\,(-1)^{\frac{p-1}{2}} \cdot \mathcal{C}_{(p-1)/2} &= -8\,(-1)^{\frac{p-1}{2}} \cdot \frac{p+3}{4p} \cdot \mathcal{C}_{(p+1)/2} \\ &\equiv -4\left(\frac{p+3}{p}\right) \\ &\equiv 2 \equiv 2 + 7\,(p+2) = 7p + 16 \pmod{p+2} \end{aligned}$$

since $-4\,(p+3) \equiv 2p \pmod{p+2}$. ■

The following result was proved by P. A. Clement in 1949.

Theorem 1 *The pair $(p, p+2)$ is a twin prime pair if and only if*

$$4\left[(p-1)!+1\right]+p \equiv 0 \pmod{p^2+2p}.$$

Proof.

($\Longrightarrow$) Suppose $(p, p+2)$ is a twin prime pair. By Wilson's Theorem, $(p-1)!+1 \equiv 0 \pmod{p}$ and $(p+1)!+1 \equiv 0 \pmod{p+2}$. Thus $2(p-1)!+1 = k(p+2)$ for some $k \in \mathbb{Z}$, implying that $-1 \equiv 2k \pmod{p}$. In other words, $2k+1 \equiv 0 \pmod{p}$; hence, $4(p-1)!+2 = 2k(p+2)$. Thus

$$4\left[(p-1)!+1\right]+p = (2k+1)(p+2) \equiv 0 \pmod{p^2+2p}.$$

($\Longleftarrow$) Conversely, if $4\left[(p-1)!+1\right]+p \equiv 0 \pmod{p^2+2p}$, then $4\left[(p-1)!+1\right]+p \equiv 0 \pmod{p}$, so that $4\left[(p-1)!+1\right] \equiv 0 \pmod{p}$. If p divides 4, with p prime, then $p=2$, and $p+2=4$, which is not prime. Therefore $(p-1)!+1 \equiv 0 \pmod{p}$, so that by Wilson's theorem p is prime. Similarly, if $4\left[(p-1)!+1\right]+p \equiv 0 \pmod{p+2}$, we have

$$\begin{aligned} 4(p-1)!+p+4 &\equiv 2(-1)(-2)(p-1)!+2 \equiv 2(p+1)!+2 \\ &= 2\left[(p+1)!+1\right] \equiv 0 \pmod{p+2}. \end{aligned}$$

Since $p+2$ and 2 are relatively prime, $(p+1)!+1 \equiv 0 \pmod{p+2}$. By Wilson's Theorem, $p+2$ is prime. ■

The following can be proved following the same ideas:

Corollary 1 *The pair $(p, p+2)$ is a twin prime pair if and only if*

$$2\left[\left(\frac{p-1}{2}\right)!\right]^2+(-1)^{\frac{p-1}{2}}(5p+2) \equiv 0 \pmod{p^2+2p}.$$

The following result gives a characterization of twin primes.

Theorem 2 *The numbers p and $p+2$ are twin primes if and only if*

1. $\sigma(p)+\sigma(p+2) = 2(p+2)$;
2. $\varphi(p)+\varphi(p+2) = 2p$.

Here, the Euler function $\varphi(n)$ is defined as the number of positive integers, relatively prime to n and less than or equal to n, and $\sigma(n)$ is the sum of the divisors of n.

Proof.

1. ($\Longrightarrow$) If $(p, p+2)$ is a twin prime pair, then $\sigma(p) = p+1$ and $\sigma(p+2) = p+3$, so $\sigma(p) + \sigma(p+2) = 2(p+2)$.

   ($\Longleftarrow$) It is clear that $\sigma(n) \geq n+1$ for each integer n, so if p were not prime, then $p = ab$, with $1 < a, b < p$. By assumption, $\sigma(p) + \sigma(p+2) = 2(p+2)$. We conclude that $\sigma(p) \geq 1 + a + b + ab > 1 + 1 + 1 + p = p + 3$, implying that $\sigma(p) + \sigma(p+2) = 2(p+2) > \sigma(p+2) + p + 3$, i.e., $\sigma(p+2) < p+1$ which cannot happen since $\sigma(p+2) + 3 \geq p$. Thus, p is prime. Likewise, if $p+2$ were not prime, we would have a contradiction.

2. ($\Longrightarrow$) If $(p, p+2)$ is a twin prime pair, then $\varphi(p) = p-1$ and $\varphi(p+2) = p+1$; hence, $\varphi(p) + \varphi(p+2) = 2p$.

   ($\Longleftarrow$) It is clear that $\varphi(n) \leq n-1$ for each integer n. Suppose $\varphi(p) + \varphi(p+2) = 2p$. If p were not prime, say $p = cd$, $a < c$, $d < p$, then $\varphi(p) \leq (p-1) + 2$, so $\varphi(p) + \varphi(p+2) = 2p \leq \varphi(p+2) + p - 3$, implying that $\varphi(p+2) \geq p+3$, which is absurd. Therefore p is prime. The case $p+2$ not prime is analogous.

The proof is complete. ■

M. Ruiz gave the following characterization for twin primes.

Proposition 3 *The pair $(p, p+2)$ is a twin prime pair if and only if*

$$\sum_{i=1}^{p} i^a \left(\left\lfloor \frac{p+2}{i} \right\rfloor + \left\lfloor \frac{p}{i} \right\rfloor \right) = 2 + n^a + \sum_{i=1}^{p} i^a \left(\left\lfloor \frac{p+1}{i} \right\rfloor + \left\lfloor \frac{p-1}{i} \right\rfloor \right)$$

for $a \geq 0$, where $\lfloor x \rfloor$ denotes the integer part of x.

3. Brun's Theorem

The following result is completely unexpected, and is one of the most profound about twins primes. It states that the sum of the reciprocals of the pairs of twin primes converges.

Theorem 3 *(Brun) The series*

$$\sum_{p, p+2 \in \mathbb{P}} \left(\frac{1}{p} + \frac{1}{p+2} \right) = \left(\frac{1}{3} + \frac{1}{5} \right) + \left(\frac{1}{5} + \frac{1}{7} \right) + \left(\frac{1}{11} + \frac{1}{13} \right) + \cdots,$$

where $\mathbb{P}$ is the set of prime numbers, is convergent. The sum, denoted B_2, is called Brun's constant. It's value is approximately 1.99.

However, we know that the sum of the reciprocals of the primes diverges. Let's prove it by using the following inequality, well known in analytic number theory.

$$p_n \leq n\left(\log n + \log\log n\right) \text{ for } n \geq 6,$$

where p_n is the nth primes.

Theorem 4 *(Euler) The series*

$$\sum_{p\in\mathbb{P}} \frac{1}{p}$$

is divergent.

Proof. We have

$$\sum_{p\in\mathbb{P}} \frac{1}{p} \geq \sum_{n=6}^{\infty} \frac{1}{p_n} > \sum_{n=6}^{\infty} \frac{1}{n\log n + n\log\log n} > \frac{1}{2}\sum_{n=6}^{\infty} \frac{1}{n\log n} = \infty$$

since, by the integral test, the last series diverges. ■

Let $\pi_2(x)$ be the number of twin prime pairs with $p \leq x$. Brun proved that there exists an integer x_0 such that for $x \geq x_0$,

$$\pi_2(x) < \frac{100x}{(\log x)^2}.$$

It can be shown that

$$\pi_2(x) \leq c\prod_{p>2}\left(1 - \frac{1}{(p-1)^2}\right) \cdot \frac{x}{(\log x)^2} \cdot \left(1 + O\left(\frac{\log\log x}{x}\right)\right)$$

or

$$\pi_2(x) \leq c\Pi_2 \cdot \frac{x}{(\log x)^2} \cdot \left(1 + O\left(\frac{\log\log x}{x}\right)\right),^{3}$$

where Π_2 is known as the twin prime constant and c is another constant. Hardy and Littlewood conjectured that $c = 2$ and

$$\pi_2(x) \sim 2\Pi_2 \int_2^x \frac{dx}{(\log x)^2}.^{4}$$

The twin prime conjecture is the equivalent of $\liminf d_n = 2$, where $d_n = p_{n+1} - p_n$ is the difference between consecutive primes. Little is known of the behavior of this function. It is conjectured that

$$L = \liminf \frac{d_n}{\log p_n} = 0,$$

[3] Let f and g be functions, with g positive. If here is a positive constant c such that $|f(x)| \leq c \cdot g(x)$, we write $f(x) = O(g(x))$.

[4] The statement $f(x) \sim g(x)$ is equivalent to $\lim\limits_{n\to\infty} \frac{f(x)}{g(x)} = 1$.

but there is no proof. P. Erdos showed that $L < 1$. This result has been improved. The following result is simple.

Lemma 2

$$\limsup d_n = \infty.$$

Proof. Since all of the terms of the sequence

$$n! + 2, n! + 3, ..., n! + n$$

are composite, the difference between the largest prime number less than $n! + 2$ and the smallest prime number greater than $n! + n$ is at least n. Thus, $d_n > n$, and the result follows. ■

Westzynthius in 1931, proved that

$$\limsup \frac{d_n}{\log p_n} = \infty.$$

Finally, we give a proof of the theorem of Brun. We will use the following:

$$\pi_2(x) \ll \frac{x(\log\log x)^2}{(\log x)^2}.$$

For more details see the article by M. Faester [5].

Proof. Consider the sequence $\{a_n\}$ defined by

$$a_n = \begin{cases} 1 & \text{if} \quad n, n+2 \text{ are twin primes} \\ 0 & \text{if} \quad \text{otherwise} \end{cases}.$$

We have

$$\sum_{p,p+2\in\mathbb{P}} \left(\frac{1}{p} + \frac{1}{p+2}\right) \ll \sum_{n\geq 2} \frac{a_n}{n} \ll \lim_{x\to\infty} \frac{(\log\log x)^2}{(\log x)^2} + \int_2^\infty \frac{(\log\log t)^2}{t(\log t)^2}\, dt.$$

Making the substitution $u = \log t$, in the last integral, we have

$$\sum_{p,p+2\in\mathbb{P}} \left(\frac{1}{p} + \frac{1}{p+2}\right) \ll \lim_{x\to\infty} \frac{(\log\log x)^2}{(\log x)^2} + \int_{\ln 2}^\infty \left(\frac{\log u}{u}\right)^2 du.$$

A simple calculation shows that

$$\int \left(\frac{\log u}{u}\right)^2 du = -\frac{(\log u)^2 + 2\log u + 2}{u} + C.$$

Therefore the series in question converges. ■

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before February 1, 2013. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2013 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)-622-3051).

NEW PROBLEMS 699-709

Problem 699. *Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA.*

1. Find all positive integers x such that $2^x + 2^{11} + 2^8$ is a perfect square.
2. Find all positive integers x such that $4^x + 4^{11} + 4^8$ is a perfect square.

Problem 700. *Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA.*

The prime 19, when rotated 180 degrees yields the prime number 61. It is also true that the pairs 199 and 661, as well as 1999 and 6661, are prime numbers. The next occurrence of such pime pairs is $1999\cdots9$ and $6666\cdots61$, where the number of digits is 28. Prove that there are infinitely many occurrences where both $1999\cdots9$ and $6666\cdots61$ are simultaneously composite.

Problem 701. *Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA.*

Let φ be the Euler φ-function (so $\varphi(m)$ is the number of values less than m which are relatively prime to m). The congruence $a^m \equiv a \bmod \varphi(m)$ is true for all a relatively prime to $\varphi(m)$ when $m = 3$, 4, 5, and 6. Show that this congruence fails for infinitely many m.

Problem 702. *Proposed by Pedro H.O. Pantoja (student), University of Natal, Brazil.*

Let $\pi(n)$ be the number of primes less than or equal to n. Prove that the sum

$$\sum_{n=2}^{\infty} \frac{1}{n(\pi(n))^{1+\varepsilon}}$$

converges for all $\varepsilon > 0$.

Problem 703. *Proposed by Pedro H.O. Pantoja (student), University of Natal, Brazil.*

Let x, y, and z be positive real numbers. Prove that

$$\begin{aligned} & \left(x^3 + y^3 + z^3 + 3xyz\right)^{2(x+y+z)} \\ \geq\ & [2xy(x+y+z)]^{x+y} \cdot [2yz(x+y+z)]^{y+z} \cdot [2xz(x+y+z)]^{x+z}. \end{aligned}$$

Problem 704. *Proposed by D.M. Batinetu-Giurgiu, Matei Basarab National College, Bucharest, Romania and Neculai Stanciu, George Emil Palade Secondary School, Buzau, Romania.*

Let $\{a_n\}$ and $\{b_n\}$ be sequences of positive real numbers with

$$\lim_{n\to\infty} \frac{a_{n+1}}{n^2 a_n} = a \text{ and } \lim_{n\to\infty} \frac{b_{n+1}}{n^3 b_n} = b.$$

Find

$$\lim_{n\to\infty} \left(\sqrt[n+1]{\frac{b_{n+1}}{a_{n+1}}} - \sqrt[n]{\frac{b_n}{a_n}} \right).$$

Problem 705. *Proposed by Anastasios Kotronis, Athens, Greece.*

Let

$$a_n = \left(\sum_{k=0}^{n} \frac{n^k \left(n^k + 1\right)}{n^{2k} + 1} \right)^{\frac{1}{n(n+1)}}.$$

Find the following limits.

1. $\lim\limits_{n\to\infty} a_n$
2. $\lim\limits_{n\to\infty} \frac{n^2(a_n-1)}{\ln n}$
3. $\lim\limits_{n\to\infty} \frac{n^3(a_n-1)}{\ln n}$

Problem 706. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let x be a positive real number. Prove that

$$\frac{x + \{x\}}{[x]^2 + 2\{x\}^2} + \frac{x + [x]}{\{x\}^2 + 2[x]^2} < \frac{4}{x},$$

where $[x]$ and $\{x\}$ are the integer and fractional parts of x, respectively.

Problem 707. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let n be an odd positive integer and p a prime number of the form $3n + 2$. Prove that if

$$\frac{a}{b} = \sum_{i=1}^{2n+1} \frac{(-1)^{i+1}}{i},$$

then p divides a.

Problem 708. *Proposed by Mary Kay Schippers and Hongbiao Zeng, Fort Hays State University, Hays, KS.*

Let $\{P_n\}$ be a sequence defined as follows:

$$\begin{aligned} P_1 &= \frac{1}{4}, P_2 = \frac{1}{8} \\ 4P_n &= 2P_{n-1} + P_{n-2}, n \geq 3. \end{aligned}$$

Calculate the following values.

1. $\displaystyle\sum_{n=1}^{\infty} P_n$
2. $\displaystyle\sum_{n=1}^{\infty} nP_n$

Problem 709. *Proposed by Michael Woltermann, Washington and Jefferson College, Washington, PA.*

Prove the following identities:

1. $\cos a\theta \cos(a-1)\theta + \tan a\theta \sin a\theta \cos(a-1)\theta = \cos\theta + \sin\theta \tan a\theta$
2. $(2\cos a\theta)\left(\cos(a-1)\theta - \frac{\sin(a-1)\theta}{\tan 2a\theta}\right) = \cos\theta + \frac{\sin\theta}{\tan a\theta}$,

and give geometric interpretations of them when $a > 1$ and $0 < \theta < \frac{\pi}{2a}$.

SOLUTIONS TO PROBLEMS 679-688

Problem 679. *Proposed by Hongbiao Zeng, Fort Hays State University, Hays, KS.*

Suppose that $f(x)$ is continuous and bounded on $(0,\infty)$ and the sequence $\{f(n)\}_{n=1}^{\infty}$ doesn't converge. Show that for any positive constant M, there exists an $x_0 > M$ such that $f(x_0+1) > f(x_0)$.

Solution *by Anastasios Kotronis, Athens, Greece.*

Suppose that there exists a positive constant M such that for every $x > M$, $f(x) \geq f(x+1)$. Let $n_0 = \lceil M \rceil + 1$. Applying this condition to $x = n_0, n_0+1$,..., we get

$$f(n_0) \geq f(n_0+1) \geq f(n_0+2) \geq \cdots,$$

implying that the sequence $\{f(n)\}$ is eventually decreasing. Since it is bounded, it is convergent. This is a contradiction. Hence there is no such M. [Note: The condition of continuity is unnecessary.]

Also solved by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; and the proposer.

Problem 680. *Proposed by Hongbiao Zeng, Fort Hays State University, Hays, KS.*

Let

$$f_n(x) = \sum_{i=1}^{n}\sum_{j=1}^{n}\cos^i x \sin^j x - \sum_{i=1}^{n}\cos^i x - \sum_{j=1}^{n}\sin^j x + 1.$$

Show the following two things:

1. The function $f_n(x)$ has exactly two zeros in the interval $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ for $n = 2, 3, \ldots$.
2. If we denote the smaller zero and larger zero as a_n and b_n, respectively, then
$$\lim_{n\to\infty} a_n = \frac{\pi}{6} \text{ and } \lim_{n\to\infty} b_n = \frac{\pi}{3}.$$

Solution *by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO.*

We note that f_n factors as

$$f_n(x) = \left(-1 + \sum_{i=1}^{n} \cos^i x\right)\left(-1 + \sum_{i=1}^{n} \sin^i x\right),$$

so finding the zeros of f_n is equivalent to finding the zeros of $g_n(x) = -1 + \sum_{i=1}^{n} \cos^i x$ and $h_n(x) = -1 + \sum_{i=1}^{n} \sin^i x$. Since $\cos x$ and $\sin x$ are monotonic on the interval in question, so are g_n and h_n. So each can have at most one root in the interval. Now

$$g_n\left(\frac{\pi}{6}\right) = -1 + \sum_{i=1}^{n} \left(\frac{\sqrt{3}}{2}\right)^i \geq -1 + \frac{\sqrt{3}}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 > 0,$$

and

$$g_n\left(\frac{\pi}{3}\right) = -1 + \sum_{i=1}^{n} \left(\frac{1}{2}\right)^i = -1 + \left(1 - \frac{1}{2^n}\right) < 0.$$

By the Intermediate Value Theorem, g_n has at least one root in the interval $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$. Hence g_n has exactly one root in the interval $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$. Denote it γ_n. Since $h_n(x) = g_n\left(\frac{\pi}{2} - x\right)$ and $\frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$, h_n has exactly one root in the interval. Denote it δ_n. Thus, f_n has exactly two roots in the interval. Let $\alpha_n = \arccos\left(\frac{1}{2} + \frac{1}{2^n}\right)$. We have

$$\begin{aligned}
g(\alpha_n) &= -1 + \sum_{i=1}^{n} \left(\frac{1}{2} + \frac{1}{2^n}\right)^i \\
&> -1 + \sum_{i=1}^{n} \left[\left(\frac{1}{2}\right)^i + i\left(\frac{1}{2}\right)^{i-1} \cdot \frac{1}{2^n}\right] \\
&> -1 + \sum_{i=1}^{n} \left[\left(\frac{1}{2}\right)^i + \left(\frac{1}{2}\right)^{i-1} \cdot \frac{1}{2^n}\right] \\
&= -1 + \left(1 - \frac{1}{2^n}\right) + \left(2 - \frac{1}{2^{n-1}}\right) \cdot \frac{1}{2^n} \\
&= \frac{1}{2^n} - \frac{1}{2^{2n-1}} \\
&> 0 \text{ (since } n \geq 2).
\end{aligned}$$

By the Intermediate Value Theorem and the fact that γ_n is the unique root of g_n in the interval, we have $\alpha_n < \gamma_n < \frac{\pi}{3}$. Since $\lim_{n\to\infty} \alpha_n = \frac{\pi}{3}$, the

Squeeze Theorem implies that $\lim_{n\to\infty} \gamma_n = \frac{\pi}{3}$. Since $\delta_n = \frac{\pi}{2} - \gamma_n$, we have $\lim_{n\to\infty} \delta_n = \frac{\pi}{6}$. Since a_n is to represent the smaller, and b_n the larger, root of f_n, we have $\lim_{n\to\infty} a_n = \frac{\pi}{6}$ and $\lim_{n\to\infty} b_n = \frac{\pi}{3}$.

Also solved by the propser.

Problem 681. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let a, b, c be the lengths of the sides of an acute triangle ABC. Prove that

$$\sum_{\text{cyclic}} \left(\cos^a B \cos^b A\right)^{1/(a+b)} < 2.$$

Solution *by Neculai Stanciu, George Emil Palade Secondary School, Buzau, Romania, and Titu Zvonaru, Comanesti, Romania.*

Because $\cos A, \cos B, \cos C > 0$, we can apply the AM-GM inequality to deduce that

$$\cos^a B \cos^b A \le \left(\frac{a\cos B + b\cos A}{a+b}\right)^{a+b}.$$

Since $a\cos B + b\cos A = c$, we obtain

$$\left(\cos^a B \cos^b A\right)^{1/(a+b)} \le \frac{a\cos B + b\cos A}{a+b} = \frac{c}{a+b}.$$

It is thus sufficient to prove that

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} < 2. \tag{7}$$

By symmetry, we can assume that $a \le b \le c$. We then have

$$\begin{aligned} \frac{a}{b+c} &\le \frac{a}{a+b} \\ \frac{b}{c+a} &\le \frac{b}{a+b} \\ \frac{c}{a+b} &< 1. \end{aligned}$$

Adding these inequalities gives (1).

Also solved by Codreanunu Ioan Viorel, Satalung, Maramures; and the proposer.

Problem 682. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let a, b, c be three positive numbers such that $a^2+b^2+c^2=1$. Prove that

$$\left[\frac{1}{a^3(b+c)^5}+\frac{1}{b^3(c+a)^5}+\frac{1}{c^3(a+b)^5}\right]^{1/5}\geq\frac{3}{2}.$$

Solution *by the proposer*.

To prove the inequality, we consider the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(t)=t^5$. This function is convex on the interval $[0,\infty)$, as can be easily checked. We will use Jensen's inequality, namely,

$$\sum_{k=1}^{3}q_kf(x_k)\geq\sum_{k=1}^{3}q_k\cdot f\left(\frac{\sum_{k=1}^{3}q_kx_k}{\sum_{k=1}^{3}q_k}\right),$$

which gives

$$\sum_{k=1}^{3}q_kf(x_k)\geq f\left(\sum_{k=1}^{3}q_kx_k\right)$$

when $\sum_{k=1}^{3}q_k=1$. Choose

$$q_1=\frac{a^2}{a^2+b^2+c^2},\ q_2=\frac{b^2}{a^2+b^2+c^2},\ q_3=\frac{c^2}{a^2+b^2+c^2}$$

and

$$x_1=\frac{1}{a(b+c)},x_2=\frac{1}{b(c+a)},x_3=\frac{1}{c(a+b)}.$$

Then Jensen gives

$$\frac{1}{a^2+b^2+c^2}\left[a^2\left(\frac{1}{a(b+c)}\right)^5+b^2\left(\frac{1}{b(c+a)}\right)^5+c^2\left(\frac{1}{c(a+b)}\right)^5\right]$$
$$\geq\frac{\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)^5}{(a^2+b^2+c^2)^5}.$$

With the condition $a^2+b^2+c^2=1$, this is equivalent to

$$\frac{1}{a^3(b+c)^5}+\frac{1}{b^3(c+a)^5}+\frac{1}{c^3(a+b)^5}\geq\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)^5.$$

By Nesbitt's inequality, we have

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\geq\frac{3}{2},$$

and the conclusion follows.

Also solved by Pedro H.O. Pantoja (student), University of Natal, Brazil; Neculai Stanciu, George Emil Palade Secondary School, Buzau, Romania, and Titu Zvonaru, Comanesti, Romania; and Codreanunu Ioan Viorel, Satalung, Maramures.

Problem 683. *Proposed by Pedro H.O. Pantoja (student), University of Natal, Brazil.*

Let $F_n=2^{2^n}+1$, the nth Fermat number. Prove that

$$\sum_{n=1}^{\infty}\frac{1}{\pi\left(F_1\right)+\pi\left(F_2\right)+\cdots+\pi\left(F_n\right)}\leq 2,$$

where $\pi\left(x\right)$ denotes the number of primes less than or equal to x.

Solution *by the proposer*.

Lemma: Let p_n be the nth prime. Then $p_{n+1}\leq 1+\prod_{i=1}^{n}p_i$.

Proof: $1+\prod_{i=1}^{n}p$ is relatively prime to p_1, p_2, ..., p_n. Either it is prime or has a prime factor greater than p_n.
Now we prove by induction that $p_k<2^{2^k}$. We have $2=p_1<2^{2^1}=4$ and $3=p_2<2^{2^2}=16$. Assume that $p_k<2^{2^k}$ for $2<k<m$. By the Lemma,

$$p_{n+1}\leq 1+\prod_{i=1}^{n}p_i<1+\prod_{i=1}^{n}2^{2^i}=1+2^{2^1+2^2+\cdots 2^n}=1+2^{2^{n+1}-2}<2^{2^{n+1}}.$$

Hence

$$p_k<2^{2^k}<2^{2^k}+1=F_k.$$

Thus $\pi\left(F_k\right)\geq k$, implying that

$$\sum_{k=1}^{n}\pi\left(F_k\right)\geq\sum_{k=1}^{n}k=\frac{n\left(n+1\right)}{2}.$$

Finally, $\sum_{n=1}^{\infty}\frac{1}{\pi(F_1)+\cdots+\pi(F_n)}\leq\sum_{n=1}^{\infty}\frac{2}{n(n+1)}=2.$

Problem 684. *Proposed by Ovidiu Furdui, Campia Turzii, 405100, Cluj, Romania.*

Calculate $\int_0^1 x \ln\left(\sqrt{1+x}-\sqrt{1-x}\right)\, dx$.

Solution *by Seeka Yang and Yaneth Gonzalez (students), California State University - Fresno, Fresno, CA.*

We integrate by parts with $u = \ln\left(\sqrt{1+x}-\sqrt{1-x}\right)$ and $dv = xdx$. Then $du = \frac{1+\sqrt{1-x^2}}{2x\sqrt{1-x^2}}\, dx$ and $v = \frac{x^2}{2}$. We get

$$\begin{aligned}
& \int_0^1 x \ln\left(\sqrt{1+x}-\sqrt{1-x}\right)\, dx \\
= \; & \frac{x^2}{2} \ln\left(\sqrt{1+x}-\sqrt{1-x}\right)\Big|_0^1 \\
& - \int_0^1 \frac{x}{4} \cdot \frac{1+\sqrt{1-x^2}}{\sqrt{1-x^2}}\, dx \\
= \; & \frac{1}{2}\ln\sqrt{2} - \int_0^1 \frac{x}{4} \cdot \frac{1}{\sqrt{1-x^2}}\, dx - \int_0^1 \frac{x}{4}\, dx.
\end{aligned}$$

The first integral can be evaluated with the substitution $u = 1-x^2$, giving

$$\begin{aligned}
& \int_0^1 x \ln\left(\sqrt{1+x}-\sqrt{1-x}\right)\, dx \\
= \; & \frac{1}{2}\ln\sqrt{2} + \frac{1}{4}\sqrt{1-x^2}\Big|_0^1 - \frac{1}{8} \\
= \; & \frac{1}{2}\ln\sqrt{2} - \frac{1}{4} - \frac{1}{8} \\
= \; & \frac{1}{4}\ln 2 - \frac{3}{8}.
\end{aligned}$$

Also solved by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Fotini Kotroni, Athens, Greece; and the proposer.

[Editor's note: Mathematica correctly evaluates the integral.]

Problem 685. *Proposed by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, MO.*

Assume that $f(x)$ is continuous and the integral

$$I = \int_c^d \frac{f(a-x)}{f(a-x)+f(x-b)}\,dx$$

exists. Evaluate I.

Solution *by the proposers.*

Let $u = x - \alpha$, where $\alpha = \frac{c+d}{2}$, and let $\beta = \frac{d-c}{2}$. then

$$I = \int_{-\beta}^{\beta} \frac{f(a-u-\alpha)}{f(a-u-\alpha)+f(u+\alpha-b)}\,du.$$

But it is easy to check that $a - \alpha = \alpha - b$. It follows that

$$I = \int_{-\beta}^{\beta} \frac{f(a-u-\alpha)}{f(a-u-\alpha)+f(u+a-\alpha)}\,du. \tag{8}$$

Letting $t = -u$, we get

$$I = \int_{-\beta}^{\beta} \frac{f(a-\alpha+t)}{f(a-\alpha+t)+f(a-\alpha-t)}\,dt. \tag{9}$$

Adding (2) and (3), we get

$$2I = \int_{-\beta}^{\beta} \frac{f(a-\alpha-u)+f(u+a-\alpha)}{f(a-\alpha-u)+f(u+a-\alpha)} = 2\beta,$$

and so

$$I = \beta = \frac{d-c}{2}.$$

Also solved by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO.

Problem 686. *Proposed by Panagiote Ligouras, Leonardo da Vinci High School, Noci, Italy.*

The lengths of the sides of the hexagon $ABCDEF$ satisfy $3AB = BC$, $3CD = DE$, $3EF = FA$. Prove that

$$\frac{AF}{CF} + \frac{CB}{EB} + \frac{ED}{AD} \geq \frac{9}{4}.$$

Solution *by Scott Brown, Auburn University, Montgomery, AL.*

We apply a proof similar to Example 2.3.6 in the book Inequalities: A Mathematical Olympiad Approach by Radmila Manfrino (2009). Let $a = AC$, $b = CE$, $c = EA$. Using Ptolemy's inequality for the quadrilateral $ACEF$, we have

$$EA \cdot FC \leq FA \cdot CE + AC \cdot EF. \tag{10}$$

Substituting the condition $3EF = FA$ into (4) gives

$$cFC \leq FA \cdot b + \frac{1}{3} FA \cdot a,$$

from which it follows that

$$\frac{FA}{FC} \geq \frac{c}{b + (1/3)\, a}.$$

In a similar fashion for quadrilaterals $AECB$ and $EDCA$, we obtain the inequalities

$$\frac{BC}{EB} \geq \frac{a}{c + (1/3)\, b}$$

and

$$\frac{DE}{AD} \geq \frac{b}{a + (1/3)\, c}.$$

Combining these yields

$$\frac{AF}{CF} + \frac{CB}{EB} + \frac{ED}{AD} \geq \frac{a}{(1/3)\, b + c} + \frac{b}{(1/3)\, c + a} + \frac{c}{(1/3)\, a + b}.$$

We must show that right side is at least 9/4. Equivalently,

$$3\left(\frac{a}{b+3c} + \frac{b}{c+3a} + \frac{c}{a+3b}\right) \geq \frac{9}{4}.$$

Multiplying the first term by a/a, the second by b/b, and the third by c/c gives

$$3\left(\frac{a^2}{ab+3ac} + \frac{b^2}{bc+3ab} + \frac{c^2}{ac+3bc}\right) \geq \frac{9}{4}.$$

Applying the Cauchy-Schwarz inequality in Engel form from p. 35 of

Manfrino's book gives

$$3\left(\frac{a^2}{ab+3ac}+\frac{b^2}{bc+3ab}+\frac{c^2}{ac+3bc}\right)\geq\frac{3\left(a+b+c\right)^2}{4\left(ab+bc+ac\right)}.$$

Thus we need to prove that the right side is at least 9/4. This can be done by applying the following inequality from p. 37 of Manfrino:

$$\frac{\left(a+b+c\right)^2}{ab+bc+ac}\geq 3.$$

Also solved by the proposer.

Problem 687. *Proposed by the editor.*

On a calculus test, one student wrote that the derivative of the product of three functions $f\left(x\right)$, $g\left(x\right)$, $h\left(x\right)$ was equal to $f'\left(x\right)g'\left(x\right)h\left(x\right)+f'\left(x\right)g\left(x\right)h'\left(x\right)+f\left(x\right)g'\left(x\right)h'\left(x\right)$. While this is not the correct formula, it does work sometimes. Do the following two things:

1. Prove that if the functions are all linear functions and this formula holds, either the functions are all constant or one is the zero function.
2. Find an infinite collection of sets of three non-constant functions
$$\left\{f\left(x\right),g\left(x\right),h\left(x\right)\right\}$$
where this formula gives the correct derivative of the product of three functions.

Solution *by Joel Smith, California State University - Fresno, Fresno, CA.*

1. First select 3 general linear functions for f, g, and h:
$$\begin{aligned} f\left(x\right) &= m_1x+b_1\\ g\left(x\right) &= m_2x+b_2\\ h\left(x\right) &= m_3x+b_3 \end{aligned}$$
To find out when the student's formula works, we need only set the student's formula equal to the correct derivative and simplify. We get
$$\begin{aligned} & m_1m_2\left(m_3+b_3\right)+\left(m_1x+b_1\right)m_2m_3+m_1\left(m_2x+b_2\right)m_3\\ = \; & m_1\left(m_2x+b_2\right)\left(m_3x+b_3\right)+m_2\left(m_1x+b_1\right)\left(m_3x+b_3\right)\\ & +m_3\left(m_1x+b_1\right)\left(m_2x+b_2\right), \end{aligned}$$

which gives

$$\begin{aligned} & 3m_1m_2m_3x + (m_2m_3b_1 + m_1m_3b_2 + m_1m_2b_3) \\ = \; & 3m_1m_2m_3x^2 + (2m_1m_2b_3 + 2m_2m_3b_1 + 2m_1m_3b_2)\,x \\ & + (m_1b_2b_3 + m_2b_1b_3 + m_3b_1b_2)\,. \end{aligned}$$

Equating coefficients of powers of x gives

$$0 = 3m_1m_2m_3 \tag{11}$$

$$3m_1m_2m_3 = 2m_1m_2b_3 \quad +2m_2m_3b_1 + 2m_1m_3b_2 \tag{12}$$

$$m_2m_3b_1 + m_1m_3b_2 + m_1m_2b_3 = m_1b_2b_3 + m_2b_1b_3 + m_3b_1b_2 \tag{13}$$

Equation (5) implies that m_1 or m_2 or m_3 is 0. If all three are 0, then the three functions are constants. Consider cases where one or two slopes are zero.

- If only one slope is zero, we may assume $m_1 = 0$ and m_2 and m_3 are nonzero. Setting $m_1 = 0$ in (6) gives $0 = 2m_2m_3b_1$. Since $m_2m_3 \neq 0$, we must have $b_1 = 0$. This means $f(x)$ is the zero function.

- If exactly two slopes are zero, we may assume $m_1 = m_2 = 0$ and m_3 is nonzero. Setting $m_1 = m_2 = 0$ in (7) gives $0 = m_3b_1b_2$, so that $b_1 = 0$ or $b_2 = 0$. This means that either $f(x)$ or $g(x)$ is the zero function.

2. For an infinite set where not all three are constant functions and the student's formula works, consider

$$f_n(x) = e^x,\, g_n(x) = n,\, h_n(x) = ke^{(1/2)x^2}.$$

[The Missouri State University Problem Solving Group suggested

$$f(x) = e^{ax},\, g(x) = e^{bx},\, h(x) = e^{cx},$$

with $ab + bc + ca = a + b + c$. When $a + b \neq 1$, this determines c.]

Also solved by Missouri State University Problem Solving Group, Missouri State University, Springfield, MO; and the proposer.

Problem 688. *Proposed by the editor.*

Find a 6-digit prime integer x where all of its digits are prime and every pair of consecutive digits is a prime. Find a 12-digit prime integer y with the same property.

Solution *by North Carolina Wesleyan College Mathematics Senior Seminar Class (consisting of Matt Dougherty, Samantha House, Jessica Jackson, Melita Lewis, Denver Nixon, Deanna Petersen, Cecilia Thorpe, John Williamson (students), Bill Yankosky (professor)).*

There are actually two 6-digit primes which satisfy the conditions. They are $x = 237373$ and $x = 537373$. The only 12-digit prime which works is $y = 237373737373$. Since every digit must be prime, the only digits the integers can contain are 2, 3, 5, and 7. Every pair of consecutive digits must also be prime, and the only two-digit primes made up entirely of the digits 2, 3, 5, and 7 are 23, 37, 53, and 73. Both the 6-digit prime and 12-digit prime must be constructed using a sequence of these 2-digit primes concatenated in some way.

The only place a 2 can appear is as the first digit. Otherwise the 2 would necessarily be the second digit of a consecutive pair and hence that consecutive pair would not be prime since it would be divisible by 2. Similarly, the only place a 5 can appear is as the first digit.

We cannot have 77 appear as a string in the integers since 77 is not prime. Nor can we have 33. This means that any 3 must be followed by a 7 and vice-versa.

With all of these restrictions, the only possible values for the 6-digit integer are 237373, 373737, 537373, and 737373. Both 237373 and 537373 are, in fact, prime, as confirmed by the Maple command isprime. Both 373737 and 737373 are divisible by 3.

With the restrictions, the only possible values for the 12-digit integer are 237373737373, 373737373737, 537373737373, and 737373737373. Of these, only 237373737373 is prime. Both 373737373737 and 737373737373 are divisible by 3, while 537373737373 is divisible by 11.

Also solved by Christy Rickett, Chip Collins, Ryan Whaley, Ranthony Clark, and Lindsey Conrotto (students), Eastern Kentucky University, Richmond, KY; Carl Libis, Middle Tennessee State University, Murfreesboro, TN; Robyn McDonald and Elizabeth Brown (students), California State University - Fresno, Fresno, CA; the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO; Scott Brown, Auburn University, Montgomery, AL; and the proposer.

Pentagon Editor Search

The current editor of *The Pentagon* would like to step down within approximately the next year, so Kappa Mu Epsilon seeks a new editor to begin summer of 2013. Interested individuals should contact the KME National President, Ron Wasserstein. His contact information is listed on page 2 of this issue. Below is a description of the job.

Pentagon Editor Job Description

KME's official journal, *The Pentagon*, is published twice annually. It is the editor's job to prepare *The Pentagon* for publication. It can be quite gratifying to be involved in the process of bringing the work of multiple authors together into a completed publication.

Each issue of *The Pentagon* consists of articles (typically 2-4), the Problem Corner, KME News, a list of active chapters, and a list of national officers. Certain issues contain additional features, such as reports of the national conventions. The Problem Corner Editor collects and assembles the material (problems and solutions) for the Problem Corner. The KME Historian collects and assembles the material for KME News.

The Pentagon Editor attends the national convention to encourage student paper presenters to submit their articles for publication. The Editor responds to inquiries about publishing in the Pentagon. The Editor maintains a list of reviewers (obtained by sending out to past reviewers, KME sponsors and Corresponding Secretaries, and others a request to serve as a reviewer). The Editor forwards submitted articles to appropriate reviewers, and guided by their reviews, determines which articles to publish. The Editor communicates with the Problem Corner Editor and the KME Historian to obtain their components of each issue. The Editor selects which articles will appear in a particular issue. The Editor typesets all of the components of the issue, ultimately preparing a single .pdf file for the issue, which is then forwarded to the Business Manager of *The Pentagon*.

Kappa Mu Epsilon
National Convention
April 11-13, 2013
Hosted by Kansas Delta
Washburn University
Topeka, Kansas

The 2013 convention will mark the beginning of the 40th biennium of Kappa Mu Epsilon. Start making your chapter's plans now to be a part of the "40 Chapters for 40 Biennia" celebration! Registration forms and hotel information will be emailed this fall to the corresponding secretary of each chapter. Students start preparing your presentations! Check out the call for abstracts on the next page.

2013 KME National Convention
Call for Presentation Abstracts

The 2013 National Convention of Kappa Mu Epsilon will be held at Kansas Beta Chapter, Washburn University, Topeka, Kansas, April 11-13, 2013.

A significant feature of our national convention is presentations by student members of Kappa Mu Epsilon. The mathematical topic selected by each student should be of interest to the author and of such scope that it can be given adequate treatment in a timed oral presentation. Senior projects and seminar presentations have been a popular way for faculty to get students to investigate suitable topics. Student talks to be presented at the convention will be chosen prior to the convention on the basis of the materials submitted. At the convention, the Awards Committee will judge the selected talks on both content and presentation, and the top papers will receive an award.

Who may submit an abstract?

Any undergraduate member of Kappa Mu Epsilon may submit an abstract for consideration as a talk at the national convention. Presentations may be coauthored. Graduate students may submit an abstract for a talk, which will be considered if there is time on the schedule after undergraduates have been selected. Graduate students will not be considered for prizes.

Presentation topics

Presentations at the convention should discuss material understandable by undergraduate students. Abstracts that satisfy this criterion and which can be presented with reasonable completeness within the time allotted are preferred.

Presentations may be original research by the student(s) or exposition of interesting but not widely known results. Presenters should always cite authors if presenting exposition of known results.

Presentation time limits

Presentations at the convention will be 15 to 20 minutes in length.

How to submit an abstract

Students who wish to make a presentation at the national convention should:

1. Prepare an abstract of up to 500 words explaining the nature of the presentation and indicating the results.
2. Complete the abstract cover page (see below).
3. Send the abstract and cover page by email to the President Elect, address below.
4. Have the student's project advisor send an email to the President Elect, certifying that the student is doing the work specified in the abstract and the advisor's belief that the student will have a fully prepared presentation by the time of the convention.

Please send the abstract, cover page and advisor's note by electronic mail not later than March 1, 2013 to Dr. Rhonda McKee at mckee@ucmo.edu.

Selection of papers for presentation

From the abstracts submitted by undergraduate students, approximately twenty papers will be invited for presentations to be judged for awards at the convention. Graduate students and undergraduate students whose papers are not selected for judging may be offered the opportunity to present their papers at a parallel session of talks during the convention. The President Elect will notify all authors of the status of their submissions soon after they are received.

Judging criteria

Judging criteria include

- Choice and originality of topic
- Literature sources and references
- Depth, significance, and correctness of content
- Clarity and organization of materials
- Adherence to the time constraints
- Effective use of graphs and/or visual aids
- Overall effect

Prizes

Authors of the best presentations by undergraduates, as decided by the Awards Committees, will each receive a cash prize.

Publication

A presenter who has not prepared a formal written paper by the time of the convention is encouraged to do so soon after the convention, so that the paper can be submitted for possible publication in The Pentagon. Unless published elsewhere, papers prepared from the prize-winning presentations will be published in The Pentagon after any necessary revisions have been completed. All other papers will be considered for publication. The editor of The Pentagon will contact each author during or soon after the convention to review his or her presentation and discuss requirements for publication.

To have a paper considered for publication, prepare it as a Microsoft Word document or .tex file, and include it as an attachment to an email to the editor at curtis-c@mssu.edu. The electronic copy of the paper will be sent to a referee who will prepare an anonymous report. If the referee recommends publication and space is available, the paper will be published in one of the next several issues.

KME National Convention
Abstract Cover Sheet

Abstracts with cover sheet must be sent to Rhonda McKee, national president elect, by email (mckee@ucmo.edu) by March 1, 2013.

Presenter's name:

Presenter's email address:

Title of Presentation:

Presenter's KME chapter:
(state followed by Greek letter, e.g., Missouri Beta)

Presenter's college or university:

Project Advisor:

Project Advisor's email address:

☐ I have asked my project advisor to send an email to Rhonda McKee certifying that I am doing the work specified in the abstract and the advisor's belief that I will have a fully prepared presentation by the time of the convention.

A computer, data projector and screen will be provided. If you require any other equipment, please indicate the needed equipment. We will do our best to accommodate the need, but cannot guarantee it.

Abstract (or attach as separate sheet):

Kappa Mu Epsilon News

Edited by Peter Skoner, Historian

Updated information as of April 2012

Send news of chapter activities and other noteworthy KME events to

Peter Skoner, KME Historian
Saint Francis University
117 Evergreen Drive, 313 Scotus Hall
Loretto, PA 15940
or to
pskoner@francis.edu

Installation Report

Pennsylvania Tau Chapter
DeSales University

The installation of the Pennsylvania Tau Chapter of Kappa Mu Epsilon was held in the University Center on the campus of DeSales University in Center Valley, Pennsylvania on Sunday, April 29, 2012, at 3:30 PM.

The meeting was conducted by Dr. Bro. Daniel P. Wisniewski, and the installing officer was National President Ron Wasserstein. Two charter faculty members, Professor Wisniewski and Professor Annmarie Houck, were initiated into the chapter. Fifteen charter student members were also initiated: Austin M. Benner, Sarah A. Capano, Carrie A. Caswell, Jennifer L. Duncan, Kelsey R. Foster, Joseph A. Marlin, Lauren N. Metz, Tripty Modi, Daniel J. Papson, Michael P. Russo, Colleen M. Shelley, Mary E. Simone, Andrew J. Tevington, Robert A. Zanneo, and Rebecca A. Zysk.

Numerous family members and other faculty were in attendance, making the total attendance about 50 people.

Dr. Wisniewski welcomed the initiates and guests, who enjoyed finger food and soft drinks while Dr. Wasserstein presented a talk entitled "What Probability and Forrest Gump Teach Us About the Pennsylvania

Lottery." Then Dr. Wasserstein installed the chapter and its charter chapter officers, including President Sarah Capano (in absentia), Vice President Robert Zanneo, Secretary Kelsey Foster, Treasurer Michael Russo, and Corresponding Secretary/Faculty Advisor Dr. Wisniewski. Vice President Zanneo accepted the charter on behalf of the new chapter.

Installation of the Pennsylvania Tau Chapter at DeSales University on April 29, 2012

Chapter News

AL Alpha – Athens State University

Chapter President – Lauren Atkins; 10 Current Members

Other Fall 2011 Officers: Kayla Usery, Vice President; Dane Glover, Secretary; and Patricia Edge Glaze, Corresponding Sec. and Faculty Sponsor

AL Gamma – University of Montevallo

Chapter President – John David Herron; 19 New Members

New Initiates - Joel Barnett, Stephanie Browdy, Megan Carver, Ben Corbitt, Callie Ellis, Jamie Eloff, Curtis Foshee, James Gilbert, Cayman Honea, Alicia Johnson, Emily Johnson, Christina Leonard, Whittley Rasberry, Leslie Schmidt, Charles Smith, Chelsea Smith, Justin Smith, Pavel Teterin, and Chad Williams.

AL Eta – The University of West Alabama

Corresponding Secretary – Hazel Truelove; 11 New Members

New Initiates - David N. Brannan, Lonia Dancy, Clintarus Dear, Shenita Evans, Tommy Gibson Jr., Allen Martin, Deanna Maura, Courtney Mims, Ericka Reed, John Vaughn, and Jonathon Woodruff.

AL Theta – Jacksonville State University

Corresponding Secretary and Faculty Sponsor – Dr. David Dempsey; 24 New Members

New Initiates - Kyle Mason Bradshaw, Christopher Burdette, Nicholas Joseph Charles, Brittney G. Cole, Karen Davidson, Jennifer Gurley, Alexandra N. Hall, Jennifer A. Holthof, Joi Jacobs, Mary Kathryn Killion, Brittney Lauren Kingery, Tiffany Nicole McIlwain, Matthew Lee Miller, Laura Francis Morris, Meagan Ingram Morrow, Gabriel Phillips, LaShandra Rivers, Zachary Mark Searels, Emily Alyss Shelton, Katie LeAnna Smith, Alecia Brooke Stovall, Ashley Elizabeth Trotter, Ralph Rogers Wheeler III, and Ashley Danielle Wiggins.

AR Beta – Henderson State University

Corresponding Secretary – Fred Worth; 18 New Members

New Initiates - Amy E. Benzi, Jones Carlson, Debra Coventry Ph.D., Stefanie Davis, Carolyn S. Eoff Ph.D., Kayla Hood, Duane Jackson Ph.D., Constance Jones, Leonard Lawson, Michael Lloyd Ph.D., Holly Morado M.A., Devonta Morrison, Kayla Morrison, Kristina Stephens, Jeannie Toole, James Edward Turner III, Trae Warner, and Meredith Wright M.A.

CA Gamma – California Polytechnic State University

Corresponding Secretary – Jonathan Shapiro; 18 New Members

New Initiates - Alex Bozarth, Stephen Calabrese, Ashley Chandler, Katherine Chiccone, Jenna Colavincenzzo, Jacob Ferrarelli, Alyssa Hamlin, Michele Jenkins, Trevor Jones, Erin Kelly, Casey Kelleher, Anna Kopcrak, Logan Lossing, Elizabeth Owens, Cierra Rawlings, Matthew Rodrigues, Kaitlyn Sutter, Adrian Tamayo, and Maro Tsiifte.

CA Delta – Cal Poly Pomona

Corresponding Secretary – Patricia Hale; 16 New Members

New Initiates - Louis Boherquez, Mayra Cervantes, Andrew Chang, Adrian Escalera, Leah Fritter, Elizabeth Hardman, Christin Jow, Lindsay Kohorn, John Miller, Rafael Morales, Benjamin Roth, Christina Sargent, Alberto Soto, Laura Wakefield, Michelle Yang, and Xiao Zhang.

CO Gamma – Fort Lewis College

Corresponding Secretary – Erich McAlister; 4 New Members

New Initiates - Paul Havris, James Hurley, Matthew Klema, and Ty Tsinnijinnie.

FL Beta – Florida Southern College

Chapter President – Lindsay Snyder; 15 Current Members

Other Fall 2011 Officers: Melissa Adams, Vice President; Kelly Madden, Treasurer; and Shawn Hedman, Corresponding Secretary and Faculty Sponsor

FL Gamma – Southeastern University

Corresponding Secretary – Dr. Berhane Ghaim; 4 New Members

New Initiates - Brittany Burns, Joy Elizabeth Ferro, Victoria Kiss, and Janae Muckle.

GA Beta – Georgia College & State University

Corresponding Secretary – Rodica Cazacu; 11 New Members

New Initiates - James Baugh, Chelsea Davis, Colleen Foy, Sally Gilbreth, Aubrey Kemp, Jacqueline Merriman, Savannah Moore, Catherine Stein, Patricia Swift, Kelly Walter, and Kathy Westbrook.

GA Epsilon – Wesleyan College

Corresponding Secretary – Dr. Joe Iskra; 8 New Members

New Initiates - Abisola Akosile Han Aung, Alayne Brown, Aakriti Kharel, Katherine McIntosh, Rebecca Navarre, Nihit Pokhrel, and Humaira Taz.

HI Alpha – Hawaii Pacific University

Chapter Pres. – Janet Calkins; 9 Current Members and 8 New Members

Other Fall 2011 Officers: Isaac Kim, Vice President; Laura Mitchell, Secretary; Randolph Uclaray, Treasurer; and Tara Davis and Chez Nielson, Corresponding Secretaries

New Initiates - Nathaniel Befus, Stephanie Bovia, Janet Calkins, Devin Kibler, Laura M. Mitchell, Lucas Mohler, Colleen Anne Seibel, and Randolf Arellano Uclaray.

IA Alpha – University of Northern Iowa

Chapter President – Adam Feller; 35 Current Members; 6 New Members

Other Fall 2011 Officers: Melinda McDowell, Vice President; Hannah Andrews, Secretary; Adam Vaught, Treasurer; and Mark D. Ecker, Corresponding Secretary and Faculty Sponsor

Our first fall KME meeting was held on September 26, 2011 at Professor Mark Ecker's house where student member Hannah Andrews presented her paper entitled "Weather Changes." Student member Adam Feller

presented his paper entitled "Baseball Salary Regression" at our second meeting on November 7, 2011 at Professor Syed Kirmani's home. Student member Lisa Stoecken addressed the fall initiation banquet with "A Statistical Analysis of the Factors Affecting University Endowments." Our fall banquet was held at the Pepper's Grill and Sports Pub in Cedar Falls on December 5, 2011 where six new members were initiated.

New Initiates - Chanda Engle, Renee Greiman, Andrew Skinner, Shaina Steger, Nathan Temeyer, and Lucas Thomas.

IA Gamma – Morningside College

Corresponding Secretary – Eric Canning; 11 New Members

New Initiates - Jessie Marie Byrnes, Lyra Grace Christianson, Eric D. Gahlon, Claire Janelle Gibbons, John R. Mascarello, Breanna Jane Mathes, Cameron Michael Meter, Jedidiah J. McCoy, Preston T. Nibaur, Laura Beth Stokes, and Kendra Beth Timmerman.

IA Delta – Wartburg College

Corresponding Secretary – Brian Birgen; 20 New Members

New Initiates - Nicole Boesenberg, Marcela Correa, Oliver de Quadros, Courtney Egts, Ashley Freese, Alyssa Hanson, Allison Huedepohl, Loni Kringle, Matthew Kristensen, Adam Kucera, Joshua Lehman, Jennifer Lynes, Paul Masterson, Kristen Nielsen, Derek Norton, Nevena Ostojic, Megan Puls, Samuel Read, Alexander Schaefer, and Ellen Schwarz.

IL Zeta – Dominican University

Chapter President – Daniel Dziarkowski; 29 Current Members; 12 New Members

Other Fall 2011 Officers: Lisa Gullo, Vice President; Eva Mehta, Secretary; and Aliza Steurer, Corresponding Secretary and Faculty Sponsor

Our chapter of Kappa Mu Epsilon and Dominican University Math Club operate as one student organization. This past fall, we had bi-weekly meetings and participated in many other fun activities. We held a special lectures series about how math is used everyday and organized an undergraduate research information session. During the holiday season, we made and sold origami ornaments.

New Initiates - Courtney Adams, Drew Adduci, Lisa Gullo, Mark Hodges Ph.D., Magdalena Kolek, Ivonne Machuca, Joanna Sasara, Sara Seweryn, Willa Skeehan, Demirhan Tunc M.S., Michael Wesolowski, and Matthew Zitkus.

IN Beta – Butler University

Corresponding Secretary – Dr. Amos Carpenter; 5 New Members

New Initiates - Erica R. Gilliland, Stacey K. Havlin, Clare F. Hubbard, Anthony J. Miller, and Paige A. Sheraw.

KS Delta – Washburn University

Corresponding Secretary – Mike Mosier; 14 New Members

New Initiates - Carly Jania Boswell, Samantha Corber, Diana Crain, William Gahnstrom, Alex R. Hawkins, Thomas M. Keyes, Hee Soek Nam, Brian Steven Oxendine, Raj Birju Prakash Patel, Catherine E. Peppers, Stacy Rottinghaus, Matthew Rush, Rachel H. Schomacker, and Jeff Tanking.

MA Beta – Stonehill College

Chapter President – Cortney Logan; 11 Current Members; 12 New Members

Other Fall 2011 Officers: Laura Bercume, Vice President; Kathleen Zarnitz, Secretary; Timothy Woodcock, Corresponding Secretary; and Ralph Bravaco, Faculty Sponsor

In September, our chapter hosted an outdoor pizza social, on a beautiful evening, welcoming all mathematics majors to attend. We now look forward to our second annual initiation ceremony, later in the spring.

New Initiates - Corey Adams, Olivia Almeida, Daniel Bouchard, Patrick Clark, Joshua Cunningham, Lauren Hinchey, William Pellegrino, Elizabeth Perkins, Kelsey Roberts, Danielle Sutherby, Laura Sweeney, and Heiko Todt.

MD Alpha – Notre Dame of Maryland University

Corresponding Secretary – Margaret Sullivan; 22 New Members

New Initiates - Jincy Abraham, Fathima Asharaff, Kelcymarie Bye, Tasnim Choudhury, Melinda Daugherty, Caitlin Engel, Lindsey Frommeyer, Meghan Hartnett, Manpreet Kaur, Ashley Khan, Theresa Maseda, Sarita Mistry, Kathryn Murphy, Molly Murray, Samantha Pinder, Takia Raneri, India Scott, Kirsten Smulovitz, Nicholette Stachowiak, Maleeha Syed, Maria Tibbs, and Julia VanBuskirk.

MD Delta – Frostburg State University

Chapter President – Kevin Loftus; 20 Current Members; 12 New Members

Other Fall 2011 Officers: Justin Good, Vice President; Jesse Otto, Secretary; Marcus Carter, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet, Faculty Sponsor

During our first meeting in September we were treated to a lecture on the mathematics of juggling given by our chapter president Kevin Loftus. In addition to being a talented mathematics student he is an expect juggler. He was able to provide some impressive demonstrations! For our October meeting we watched a video on turning a sphere inside out and during our last meeting in December we watched a video on the geometry of soap bubbles. Other activities for the semester involved helping out with the Majors' Fair and with the Math Department's annual meet and greet.

New Initiates - Devota Aabel, Raymond Azenadaga, Joshua Green, Joshua McDonald, DeVonte' McGee, Steven Moon, Jacob Reed, Andrew Siemann, Anna Struhar, Nicholas Torgerson, Deborah K. Wiles, and Justin C. Zimmermann.

MD Epsilon – Stevenson University

Chapter Pres. – Rebecca Hollins; 38 Current Members; 14 New Members
Other Fall 2011 Officers: Kristina Pugh, Vice President; Charles Schuster, Secretary; Rachel Buchanan, Treasurer; and Dr. Christopher E. Barat, Corresponding Secretary and Faculty Sponsor

On September 22, 2011, the Chapter initiated 14 new members (13 students, 1 faculty) in an initiation ceremony in the Ratcliffe Board Room on Stevenson's Greenspring campus. The guest speaker, Dr. Betty Mayfield of Hood College, gave a presentation on "Women and Mathematics in the Time of Euler." The main fund-raising initiative during the Fall, the Worldwide Book Drive in November and December, collected over 600 new and used textbooks and raised a total of over $100.

New Initiates - Anthony Carle, Jenna Carr, Stephen Crowley, Kristen DeBaugh, Megan Good, Miranda Gregory, Megan Klein, Dr. Edwin Lo, Paige O'Connor, Kristina Pugh, Charles Schuster, Nickolas Shaw, Andrew Wegerski, and Rebecca Wong.

MI Alpha – Albion College

Corresponding Secretary – Mark Bollman; 4 New Members

New Initiates – Jacqueline Chung, Sarah Erdman, Alexandra Sovansky, and Laura VerHulst.

MI Delta – Hillsdale College

Chapter Pres. – Gladys Anyenya; 47 Current Members; 22 New Members
Other Fall 2011 Officers: David Montgomery, Vice President; Aubrey Annis, Secretary; Casey Gresenz, Treasurer; and Dr. David Murphy, Corresponding Secretary and Faculty Sponsor

During the Fall 2011 semester, we initiated 16 new members and watched the movie "A Beautiful Mind."

New Initiates - David W. Beatrice, Kevin A. Bishop, Sydney Therese Bruno, Stephen Davis, Branden John Harris, Andrea Hay, Josiah Martin Kollmeyer, Heather Van Law, Monica Leif, Linda Lizalek, Abigail M. Loxton, Joshua Mirth, Megan Moss, Brett R Pasche, Alisha Pehlert, Viktor Rozsa, Cody Kelly Sommer, Samuel Joseph Stoneburner, Olivia Tilly, Paulina Volosov, John William James Walsh, and Kelsie Jean White.

MI Epsilon – Kettering University

Chapter President – Jessi Harden (A Section); 186 Current Members; 16 New Members
Other Fall 2011 Officers: Ryan McGuire, President elect; Bryan Coburn (A Section) and Starla Walters (B Section), Vice Presidents; Derek Hazard and Michael Steinert (A Section) and Shahnoor Amin (B Section), Secretaries; Kasey Simons (A Section), Treasurer; Boyan N. Dimitrov, Corresponding Secretary; and Ruben Hayrapetyan (Section A) and Ada Cheng (Section B), Faculty Sponsors

Kettering University usually enjoys an active KME Society life. In the Summer 2011, Dr. Hayrapetyan offered the traditional Pizza/Movie twice in the second and ninth weeks. The movie choices were "The story of One," and "Decoding Nazi Secrets." The story of the number one is the story of Western civilization and recounts the amazing tale behind the world's simplest number. Using computer graphics, "One" is brought to life, in all his various guises. One's story reveals how celebrated civilizations in history were achieved, where our modern numbers came from and how the invention of zero changed the world forever, and saved us from having to use Roman numerals today. The second describes true stories for the work of mathematicians during the World War II, which contributed to development of machine computing power and busted the applications of mathematics. The attention of students was taken by the discoveries of Turing, by the first UK computers created then, by the fate of those mathematicians working in the group. The movie choice of Dr. Ada Cheng for the ninth week was "Julia Robinson and Hilbert's Tenth Problem." And the next week was the initiation ceremony when new eligible Kettering students joined MI Epsilon. The Mathematics Department at Kettering University hosted the 11th High School Mathematics Olympiad on December 3, 2011; it is a competition designed to identify and encourage students with interests and abilities in mathematics with a goal to develop the Olympiad into one of the most prestigious mathematical competitions in the region. The examination consists of six challenging problems, has a time limit of four hours, and is designed for students in grades 9 through 12. The problems range from "mind-benders" that require little mathematical skills to problems that require the knowledge of geometry, trigonometry and beginning calculus. The 2011 Kettering Math Olympiad Winners are:

First Place: David Lu from Bloomfield Hills who attends Detroit Country Day; Second Place: Mayank Patke from Okemos who attends Okemos High School; Third Place: Matthew Bauerle from Fenton who is homeschooled; Fourth - Seven Place: (In alphabetical order) Matthew Bagaziniski from Livonia; Kegan Thorrez from Cement City who attends Hillsdale Academy; Alan Xu from Ypsianti who attends Detroit Country Day, and Aaron Zeng from Novi who attends Detroit Country Day.

MO Alpha – Missouri State University

Chapter President – Christina Tharp; 31 Current Members and 8 New Members

Other Fall 2011 Officers: Brett Foster, Vice President; Sarah Kramer, Secretary; Lee Hicks, Treasurer; and Jorge Rebaza, Corresponding Secretary and Faculty Sponsor

The following events were held during the Fall: September 19. 2011–KME Annual Picnic (about 60 people attended); September 21, 2011–KME Seminar, Speaker Erin Buchanan (Psychology, MSU), Modeling Language in the Brain; October 19, 2011–KME Seminar, Speaker Les Reid (Mathematics, MSU), Magic Squares and Cubes; November 16, 2011–KME Seminar, Speaker Justin Millan (ANPAC, Springfield MO), Careers in Actuarial Math; December 7, 2011–KME Social with secret gift exchange, soda, snacks, games, etc.

New Initiates - Cody Crossley, Elena Janicik, Braden McCann, Marissa Mullen, Ashley Osborn, Alex Schwent, Ali Soliman, and Christina Usher.

MO Beta – University of Central Missouri

Corresponding Secretary – Rhonda McKee; 15 New Members

New Initiates - Stephen Abbott, Hannah Barnard, Carley A. Byus, Alexander Card, Philippe Chikuru, Christine L. Crawford, Mary Gossell, Logan Keeler, Danielle Orcutt, Ghana S. Paudel, LeighAnn Sherfey, Matthew Slack, Renee Stone, Samantha Tyler, and Alycia C. Wolfe.

MO Epsilon – Central Methodist University

Chapter President – Megan Davidson; 9 New Members

Other Fall 2011 Officers: Ashton Zimmerman, Vice President; Amber Strubberg, Secretary; Jacob Heppner, Treasurer; and Linda O. Lembke, Corresponding Secretary and Faculty Sponsor

The chapter will hold an initiation in late March or early April.

New Initiates - Kristen Bailey, Kaeleigh Brown, Harlan Fletcher, Novy Foland II, Kayla Leeser, Kate Otten, Boone Priddy, Taylor Reinkemeyer, and Melanie Wilmsmeyer.

MO Zeta – Missouri University of Science and Technology

Corresponding Secretary – Dr. Vy K. Le; 33 New Members

New Initiates - Vincent Allen, Stephen Banks, Kelsey Bass, Lindsay Brandt, Kyle Butts, Abbey Campbell, Caleb Collier, Kameron Davis, Daniel Ferdman, Kevin Gill, Katie Gray, Bernadette Heneghan, Joe Hoing, Jeremy Joh, Parker Kessler, Alex Korff, Ryan Luerssen, Patrick McCarver, Josh Manescalco, Jenny Nussbaum, Ashley Overschmidt, Travis Peeler, Joseph Pepple, David Pohlman, Patrick Schlotter, Dustin Specker, Kaylea Smith, Matthew Stone, Michael Tang, Sam Thebeau, Lynsey Toivonen, Jacob Tuia, and David Voong.

MO Theta – Evangel University

Chapter President – Katie Strand; 7 New Members

Other Fall 2011 Officers: Elizabeth Baumeister, Vice President; and Don Tosh, Corresponding Secretary and Faculty Sponsor

Meetings were held monthly. The October meeting was an ice cream social at the home of Don Tosh. The spring regional meeting for KME's North Central region will be hosted by Missouri Theta on Evangel's campus.

New Initiates - Laura E. Balch, Joshua Forsman, Ryan C. Geppert, Emily Johnson, Kevin Mackey, Hope Moorhead, and Jared Strader.

MO Iota – Missouri Southern State University
Corresponding Secretary – Charles Curtis
Other Fall 2011 Officers: Rich Laird and Grant Lathrom, Faculty Sponsors
MO Kappa – Drury University
Corresponding Secretary – Bob Robertson; 8 New Members
New Initiates - Lindsey Andrews, Ryan Eddleman, Samuel Gardner, Lauren Jones, Caitlin Krebs, Lindsay Lehmen, Scott Robinson, and Amanda Watson.
MO Lambda – Missouri Western State University
Corresponding Secretary – Dr. Steve Klassen; 13 New Members
New Initiates – Cody Beyers, David Carlisle, Andie Cassity, Melanie Edlin, Jonathan Guilkey, Hannah Huff, Bridget Janssen, Brad Isom, Joel Luzmoor, Alaina Rickard, Dylan Steelman, Bailee Testorff, and Sam Wold.
MO Mu – Harris-Stowe State University
Corresponding Secretary – Ann Podleski; 9 New Members
New Initiates - Ja'mar Anderson, Aleksandra Ceric, Jason Chapel, LaKeisha Jackson Ducote, Atteria Keely-Scott, Maya Morris, Brittney Robinson, Latifu Thompson, and Tihra Devres (posthumously).
MO Nu – Columbia College
Chapter Pres. – Tomas Horvath; 10 Current Members; 5 New Members
Other Fall 2011 Officers: Kyle Christian, Vice President; Carolyn Summers, Secretary; Ran Kim, Treasurer; and Dr. Kenny Felts, Corresponding Secretary and Faculty Sponsor
New Initiates - Rachel Garrett, Carroline Kirtley, Michael Little, Brandy Poag-Dorado, and Colleen Whalen.
MS Alpha – Mississippi University for Women
Chapter President – Chelsea Pugh; 7 Current Members; 2 New Members
Other Fall 2011 Officers: Meagan Vaughan, Vice President; Menuka Ban, Secretary; Joshua Hanes, Treasurer, Corresponding Secretary and Faculty Sponsor
New Initiates - Leigh Ellen Barefield and Tshering Lama Sherpa.
MS Gamma – University of Southern Mississippi
Corresponding Secretary – Jeremy Lyle; 6 New Members
New Initiates - Nicole Cotten, Abbie Desselle, Melissa Dyess, Chasmine Flax, Amber Sumner, and Kevin Tran.
MS Epsilon – Delta State University
Corresponding Secretary – Paula Morris; 3 New Members
New Initiates - Trent Calvin, Ian Stuart Campbell, and Catelin Steele Peay.
NC Epsilon – North Carolina Wesleyan College
Corresponding Secretary – Bill Yankosky; 5 New Members
New Initiates - Jeremy Davis, Steven Franklin, Fred Lemongo, Temple Annette Mills, and Zach Seitter.

NC Zeta – Catawba College

Chapter President – Mary M. McKee; 15 Current Members
Other Fall 2011 Officers: Spencer K. Ashley, Vice President; Bridgett Henderson, Secretary; Dustin Craft, Treasurer; and Douglas K. Brown, Corresponding Secretary and Faculty Sponsor

The students of the NC Zeta Chapter provided weekly Sunday evening Math Help Sessions.

NE Alpha – Wayne State College

Corresponding Secretary – Jennifer Langdon; 6 New Members

New Initiates - Krista Biernbaum, Baili Klein, Cassandra Kuiken, Hannah Lee, Jesse Smidt, and Katherine Svec.

NE Beta – University of Nebraska Kearney

Chapter Pres. – April Christman; 15 Current Members; 5 New Members
Other Fall 2011 Officers: Brent Wheaton, Vice President; Ali Titus, Secretary; and Brittany Spiehs, Treasurer; and Dr. Katherine Kime, Corresponding Secretary and Faculty Sponsor.

KME members held a Math Fun Day at a local middle school. We began planning for bringing a speaker to campus. New t-shirts were designed, voted upon and ordered.

New Initiates - Tyler Adelung, Roy Machamire, Grant Person, Li Tan, Sen Wang, and James Weese.

NE Delta – Nebraska Wesleyan University

Chapter Pres. – Joseph Menousek; 11 Current Members; 4 New Members
Other Fall 2011 Officers: Linda Arthur, Vice President; Laura Booton, Secretary; and Rebecca Swanson, Corresponding Secretary and Faculty Sponsor

In Fall 2011, we organized one large event each month. In September, we began the semester with a social event, Game Night. This was a chance to meet new students interested in mathematics as well as inform them of the rest of the semester's activities. In October, we held a "Summer Opportunities" panel. Students who had previously participated in summer programs or REUs discussed their experiences, and faculty and staff from the university discussed other programs and internships. In November, Derrick Stolee from the University of Nebraska gave a talk on his research. In December, we co-organized a holiday party with the Physics and Computer Science groups on campus. In the spring semester, we will kick off the year with another game night. In February, we are holding a career panel on careers in mathematics. Alumni from the university will return to speak about their careers. To celebrate Pi Day, we are holding a "Pi Mile Fun Run" on our campus. The event is open to the university and the greater Lincoln community. In April, we will have a student speaker at

the beginning of the month, and KME initiation at the end of the month.
New Initiates – Paul Dorenbach, Katie Fagot, Jayme Prenosil, and Alex Whigham.

NH Alpha – Keene State College

Corresponding Secretary – Vincent Ferlini; 12 New Members

New Initiates - Matthew Adams, Brittany Boscarino, Kristopher Bucyk, Filip Duz, Caitlin Fecteau, Alyssa Gilhooly, Julia Marrone, Sylvie McCarthy, Rachael Morgan, Alexandrea Murby, Katherine Nunes, and Kyle Virgin.

NJ Delta – Centenary College of New Jersey

Corresponding Secretary and Faculty Sponsor – Kathy Turrisi; 7 Current Members

Other Fall 2011 Officer: Linda Ritchie, Treasurer

We had one student, Johanne, this semester present at the 26th Moravian College Student Mathematics Conference. She collaborated with another student from Stevens Institute of Technology by chats and by practicing the presentation on Skype. They did a fantastic job and even taught us all something new (Mystery 24). See link below on Mystery 24 (the proof was fun to work out too) http://www.murderousmaths.co.uk/games/primcal.htm

NY Iota – Wagner College

Corresponding Secretaries – Zohreh Shahvar and Marisa Scarpa; 14 New Members

New Initiates - Bareah Alam, Stephanie Asusta, Ervila Behri, Jordan Frankel, Alaina Gennaoui, William Hedges, Jasmine Jusufi, Andrew Ledet, Vincent Lombardo, Joseph Mininni, Adam Nicolais, Amanda Spira, Bujar Tagani, and Ryan Van Spronsen.

NY Lambda – C.W. Post Campus of Long Island University

Chapter President – Daniel Barone; 12 New Members

Other Fall 2011 Officers: Ashley Vaughan, Vice President; Elyse Capozza, Secretary; Jennifer Hanly, Treasurer; Dr. Andrew M. Rockett, Corresponding Secretary; and Dr. James B. Peters, Corresponding Secretary and Faculty Sponsor

New Initiates - Michael Cohen, Thomas Fallon, Brittany Greene, Christine Koenigsmann, Jamianne Kruse, Matthew Lucas, Amanda Mazza, Gladney Nose, Mei Qiao, Christopher Salvato, Paul Samber, and Paige Wehren.

NY Mu – St. Thomas Aquinas College

Corresponding Secretary – Marie Postner; 4 New Members

New Initiates - James Michael Carey, Devin C. Meikle, Jose I. Rodriguez-Rojas, and Joseph M. Schragenheim.

NY Nu – Hartwick College

Chapter President – Ashley Hunt; 15 Current Members; 15 New Members

Other Fall 2011 Officers: Nicole Besancon, Vice President; Julie Kessler, Secretary; Rebecca Lounsbury, Treasurer; and Ron Brzenk, Corresponding Secretary.

The chapter sponsored two events: 1) An Open House for Alumni as part of Homecoming weekend, and 2) A talk by an alumnus who is an actuary and VP of Mass Mutual Life Insurance about becoming an actuary.

New Initiates - Mercy Alila, Jessica Bentley, Alyssa Failey, Matthew Feeman, Desiree Fuller, Steven Grzeskowiak, Brain Heller, Leanne Keeley, Jordan Liz, Rhianna Morgan, Nathan Nichols, James Orlando, Aaron Parisi, Joseph Seney, and Jaime Toboada.

NY Omicron – St. Joseph's College

Chapter President – Margaret Kumpas; 25 Current Members; 17 New Members

Other Fall 2011 Officers: Ricardo Campos, Vice President; Salvatore Alfredson, Secretary; Alexander De Ridder, Treasurer; Elana Reiser, Corresponding Secretary; and Dr. Donna Marie Pirich, Faculty Sponsor

Our chapter members volunteered to tutor high school students in nearby districts through our math clinic, which is held every Saturday.

New Initiates - Rachelle L. Amendola, Kimberly L. Avelin, Stephen A. Bates, Cassandra Benedict, Daniel G. Ferguson, Janéce S. Guerra, Mercedees E. Jordan, Jaclyn R. Lazio, Corine A. Maglione, Stephanie Mann, Kasey C. Melzer, Sarah E. Redding, Emma K. Schrader, Nicolas J. Simonetti, Douglas M. Smith, Christopher P. Vandenberge II, and Geeta Vir.

NY Pi – Mount Saint Mary College

Corresponding Secretary – Lee Fothergill; 14 New Members

New Initiates - Michael J. Amanti, Karen M. Borst, Brittany Botta, Allison Joy Cowan, Michael Pietro Del Rosso, Christopher J. DiRusso, Courtney A. Elmendorf, Nicole Havrilla, Jessica Alice Krawec, Tyler Laufersweiler, Bryan Polack, Alycia L. Scarpelli, Ellen Slocum, and Katharine Tischer.

NY Rho – Molloy College

Corresponding Secretary – Manyiu Tse; 21 New Members

New Initiates - Michael Barbosa, Cassandra Beggen, Kerry Butler, Claudia Connor, Michael Favuzzi, Erin Fay, Emily Fox, Alexis Garcia, Nicole Garcia, Kaitlin Hoberg, Lauren Kenedy, Natalie Khouryawad, Kayla Kirkwood, Stacey Mueller, Kerry Murphy, Dana Pickett, Adria Puma, Jennifer Schmoll, Allison Sloper, Kirsten Telfer, and Michelle Troici.

OH Gamma – Baldwin-Wallace College

Corresponding Secretary; 13 New Members

New Initiates - Devin G. Basile, Yuriy Y. Boyko, Gary B. Cefalo, Andrew R. Dibacco, Michael J. Gercevich, Rachel A. Kelly, Kayleigh M. Kushner, Michael W. Lamoreux, Meghan E. Mikolay, Bradley M. Minrovic, Rebecca E. Mittler, Keith D. Pech, and William R. Roth.

OH Epsilon – Marietta College

Chapter President – Lauren Litts; 25 Current Members

Other Fall 2011 Officer: John Tynan, Corresponding Secretary and Faculty Sponsor

OH Zeta – Muskingum University

Corresponding Secretary – Richard Daquila; 7 New Members

New Initiates - Megan Duke, Meng Li, Eli Morris, Lindsay Mullen, Ben Pasley, Ashley Reynolds, and Aaron Wagner.

OK Alpha – Northeastern State University

Chapter President – Zac Kindle; 53 Current Members, 18 New Members
Other Fall 2011 Officers: Tanisha Payne, Vice President; Gregory Palma, Secretary; Erik Friend, Treasurer; and Dr. Joan E. Bell, Corresponding Secretary and Faculty Sponsor

Our fall initiation brought 12 new members into our chapter. At our September meeting, Dr. Giovanni Petris, Department of Mathematical Sciences, University of Arkansas, spoke on "Applied Bayesian Statistics." Members worked on problems in The Pentagon and submitted one solution to the editor. Members also submitted solutions to problems in the College Math Journal. We spent one evening in September calling NSU alumni and asking for their support. In November, Gregory Palma, KME member, presented his summer research on "Non-Recursively Constructible Recursive Families of Graphs." The speaker at our Christmas party was Dr. Cynthia Woodburn, Pittsburg State University. The topic of her presentation was: "2012 and Mayan Math." At the conclusion of the party, Dr. Woodburn played holiday selections on the hand bells.

New Initiates - Ryan A. Berkley, Eric D. Butson, Jerry J. Capps, Mariam J. Corbett, Haley R. Crane, Lauren A. Davey, William H. Dodson, Anna E. Faina, Jacqueline R. Falk, Joshua L. Gregory, Chad T. Hollifield, Falicia R. Mansfield, Scott R. Martin, Curtis M. Roberts, Megan E. Rowan, Miranda A. Sawyer, James Q. Sherrell, and Shanna R. Vice.

OK Gamma – Southwestern Oklahoma State University

Corresponding Secretary – Thomas McNamara; 11 New Members

New Initiates - John Bui, Stephanie Chidester, Jordan Cotter, Lloyd Delua, Caitlin Dismore, Tuyen Doan, Tayler Eason, Luke Kraft, Ray Mosqueda, Matthew Stangl, and Yimfor Yimfor.

OK Epsilon – Oklahoma Christian University

Corresponding Secretary – Ray Hamlet; 12 New Members

New Initiates - Shandi Berridge, Colby Boone, Christina Deffenbaugh, Ryan Farthing, Naomi Fitzsimmons, Sara Gower, Jennifer Loe, Barthelemy Niyibizi, Amanda Roehrkasse, Jenny Tavares, Talon Unruh, and Tyler Wilson.

PA Alpha – Westminster College

Corresponding Secretary – Natacha Fontes-Merz; 13 New Members

New Initiates - William Armentrout, Anthony Caratelli, Todd Changoway, Branislav Cikel, Beth Ekimoff, Christina Erceg, Lisa Kaylor, Andrew Kieffer, Timothy Matyas, Jaclyn Miller, Hannah Raihall, Sydney Spain, and Chelsea Wesp.

PA Beta – La Salle University

Chapter President – Stephen Kernytsky; 9 Current Members; 19 New Members

Other Fall 2011 Officers: Ryan Cunningham, Treasurer; and Stephen Andrilli, Corresponding Secretary and Faculty Sponsor

During Fall 2011, our KME Chapter conducted an informal mathematics problem-solving competition (with several student teams participating) in November, held a Poker Night in October, and watched a math-related video in December. We also sent a delegation of students to the Careers in Mathematics conference at West Chester University on Saturday, October 15, 2011. For fund-raising purposes, a pretzel sale was held in October to aid the treasury.

New Initiates - D. Joseph Barron, Daniel Bowers, Kellen Burke, Salvatore Calvo, Rose-Mary Carberry, Anthony Carbone, John Celley, David Comberiate, Alex Confer, Meghan Dondero, Georgia Hansen, Joseph Jung, Dominick Macaluso, John Miller, Anna Nguyen, Steven Rose, Amanda Russo, Olivia Shoemaker, and Marina Solomos.

PA Gamma – Waynesburg University

Corresponding Secretary – James R. Bush; 9 New Members

New Initiates - Christina Beohmer, Andrew Harmon, Ashley Hull, Garett Johnston, Daniel Piett, Abbey Ritenour, Kaitlyn Smith, Lindsay Triola, and Kristen Wilkinson.

PA Eta – Grove City College

Corresponding Secretary – Dale L. McIntyre; 22 New Members

New Initiates - Kayla J. Burkett, Hannah E. Chapman, Mary P. Collins, Joseph J. Diani, Peter W. Foster, Esther J. Froberg, Rebecca E. Genet, Claire L. Grabowski, Seth R. Gregory, Timothy S. Leavitt, Robin E. Mabe, Christa E. Moore, Steven W. Muniak, Angela M. Palumbo, Jennifer L. Piscsalko, Emma M. Polaski, Paul M. Sangrey, Mark A. Schrecengost, Daniel R. Scofield, Samuel J. Shesman, Elaine M. Sotherden, and Elisabeth E. Stewart.

PA Theta – Susquehanna University

Corresponding Secretary – Kenneth Brakke; 12 New Members

New Initiates - Joshua Allen, Michael Brown, Nicole Caviris, Blake Chamberlain, Rebecca Grenell, Alexander Kahle, Jonathan Kennedy, Daniel Martin, Jeffrey Mazurek, Katelyn Reese, Aaron Schuettler, and Gregory Swierzewski.

PA Kappa – Holy Family University

Corresponding Secretary – Sister Marcella Wallowicz; 7 New Members

New Initiates - Ahmed Benjaani, Kaitlin Bonner, Carly Cofer, Michelle Green, Anthony Ivanoski, William Kane, and Alexander Sowa.

PA Lambda – Bloomsburg University

Chapter President – Carrie Mensch; 24 Current Members and 15 New Members

Other Fall 2011 Officers: Jenna Mordan, Vice President; Marissa McDowell, Secretary; Meagan Robinson, Treasurer; Elizabeth Mauch and Dr. Eric B. Kahn, Corresponding Secretaries; and Dr. William Calhoun, Faculty Sponsor

New Initiates - Edward Arnold, Ralph Beishline, Ashley Boehmer, Mary E. Breznitsky, Patrick Brinich, Jacob Jacavage, John Kelly, Marissa McDowell, Megan Miller, Travis Moyer, Frances Nicoletti, Sarah Reed, Meagan Robinson, Dan Stiglitz, and Ashley Williams.

PA Mu – Saint Francis University

Chapter President – Laura Stibich; Current Members 53; 17 New Members

Other Fall 2011 Officers: Katie Dacanay, Vice President; Marissa Basile, Secretary; Matthew Skoner, Treasurer; Peter Skoner, Corresponding Secretary; and Katherine Remillard, Faculty Sponsor

On October 5, several KME members participated in the Commissioning Service in the University Chapel for students who perform community service. At the 18th Annual Science Day held November 22, KME members served as session moderators for faculty making presentations, and moderators, judges, scorekeepers, and timers for the Science Bowl; a total of 418 high school students from 23 area high schools attended.

New Initiates - Abenezer Alemu, Jeffrey Chastel, Ann Colligan, Christopher DeLaney, Michael DeLyser, Collier Devlin, Phuong Minh N. Do, Julia Havko, Rob Hodgson, Bemnet Kebede, Elise Löfgren, Michelle Maher, Jacob McCloskey, Jessie Minor, James Shiring, Sean Veights, and Selam Woldemeskel.

PA Nu – Ursinus College

Corresponding Secretary – Jeff Neslen; 8 New Members

New Initiates - Seth Aaronson, Emma Cave, James Galgon, Kristin Hanratty, Tyler Lovelace, Gregory Martell, Samantha Mercer, and Alison Nolan.

PA Omicron – University of Pittsburgh at Johnstown

Corresponding Secretary – Dr. Nina Girard; 31 New Members

New Initiates - Jacqueline Baughman, Chad Beck, Adam Beilchick, Noam Berns, Elizabeth Blank, Kiersten Burkett, Benjamin Cairns, Vincent Cassone, Kathernie Dalzell, Lena Delucia, Brian Dvorsky, Eric Fromherz, Christopher Kakias, Kurt Kretchman, Rachel Lasko, Shiyi Li, Jeffrey Matevish, Patrick Moon-Rhoades, Ryan Nibert, Aaron O'Leary, Amber Reichard, Corey Rook, Wayde Simington, Steven Sulosky, Derik Swope, Cassandra Thomas, Katie Treese, Amanda Williams, Jade Zatek, Jennifer Zelinksy, and Alyssa Ziats.

PA Rho – Thiel College

Corresponding Secretary – Max Shellenbarger; 10 New Members

New Initiates - Paul Auchter, Alyssa Chine, Joslyne Beth Cook, Anthony Corso, Nicholas Cox, Kayce Estelle Grimm, Roger Dale Irby, Joseph John Janicki, Ryan T. Murphy, and Kirk James Russell.

PA Sigma – Lycoming College

Chapter President – Allyson Blizman; 20 Current Members and 5 New Members

Other Fall 2011 Officers: Cortney Schoemberger, Vice President; David Brown, Secretary; Christopher Reed, Treasurer; Santu de Silva, Corresponding Secretary; and Eileen Peluso, Faculty Sponsor.

The chapter has been largely inactive this year (including the Fall of 2011). We were so inactive that we forgot to report on the initiation of Spring 2011, and did not elect officers for 2011-2012 until February of 2012. Dr Michael Smith, a new faculty member hired for the Fall, was inducted to KME at another institution in 2008, which brings the number of faculty in the chapter to 9. Dr Philip Sprunger, who was appointed Dean and Provost in the Summer of 2011 is a member of the chapter as well. Five new members were inducted in January, bringing the roll up to 71, including inactive members.

PA Tau – DeSales University

Corresponding Secretary – Br. Daniel P. Wisniewski, O.S.F.S.; 17 New Members

New Initiates - Austin M. Benner, Sarah A. Capano, Carrie A. Caswell, Jennifer L. Duncan, Kelsey R. Foster, Annmarie Houck, Joseph A. Marlin, Lauren N. Metz, Tripty Modi, Daniel J. Papson, Michael P. Russo, Colleen M. Shelley, Mary E. Simone, Andrew J. Tevington, Robert A. Zanneo, Rebecca A. Zysk, and Daniel P. Wisniewski, O.S.F.S.

SC Epsilon – Francis Marion University

Corresponding Secretary – Damon Scott; 5 New Members

New Initiates - Drake Cole Brookins, Holly Michell Coleman, Kyle Gilley, Tia S. Myers, and Brian Thomas Owens.

TN Alpha – Tennessee Technological University

Corresponding Secretary – Andrew Hetzel; 24 New Members

New Initiates - Charles Batson, Dalton Blythe, Michael Boyd, John Boys, Zachary Brown, Cameron Chaparro, Clifford Clark, Matthew Collins, Timothy Eisenbraun, Joshua Escue, Zachary Farris, Stephen Graves, Heather Holman, Ashley Lawson, Hoang Le, Emily McDonald, Laura Patton, Matthew Schaller, Michele Smith, Madison Thomas, Christopher Tidwell, Tyler Travis, John Welchance, and Kyle Willoughby.

TN Beta – East Tennessee State University

Chapter President – Jessie Deering; 779 Current Members and 9 New Members

Other Fall 2011 Officers: Tony Rodriguez, Vice President and Corresponding Secretary; Jessica Lunsford, Secretary; Terrance McDermott, Treasurer; and Dr. Bob Gardner, Faculty Sponsor

New Initiates - Jack Hartsell, Chelsea Herald, Chelsea Holtzsclaw, Erin McMullen, Tabitha McCoy, Olivia Miller, Eric Seguin, Clayton Walvoort, and Andrew Young.

TN Gamma – Union University

Chapter Pres. – Emilie Huffman; 24 Current Members; 6 New Members
Other Fall 2011 Officers: Kimberly Lukens, Vice President; Rachel Carbonell, Secretary and Treasurer; Seth Kincaid, Historian/Webmaster; Michelle Nielsen, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor

New Initiates - Alexandria Archer, David Clark, Patrick Joseph, Caroline McConnell, Grace Morriss, and Corey Wilson.

TN Delta – Carson-Newman College

Chapter President – Hannah Whitaker; 19 Current Members
Other Fall 2011 Officers: Corbin Hedges, Vice President; and Kenneth Massey, Treasurer, Corresponding Secretary and Faculty Sponsor

We held our annual fall picnic at Cherokee Dam. Unfortunately, it was a busy Friday night for many folks, so we didn't have enough students to hold the traditional post-cookout lacrosse game. In December, we held a Christmas party at the Weaver Mansion. After dinner, everyone was required to make of fool of him/herself in a game of Quelf.

TN Epsilon – Bethel University

Corresponding Secretary – Mr. Russell Holder; 6 Current Members
Other Fall 2011 Officer: Mr. David Lankford, Faculty Sponsor

TX Alpha – Texas Tech University

Corresponding Secretary – Magdalena Toda; 5 New Members

New Initiates - Jonathan Adams, Eugenio Aulisa, Carmen Gogu, Judith Jones, and Morgan Rogers.

TX Gamma – Texas Woman's University

Corresponding Secretary – Dr. Mark Hamner; 14 New Members

New Initiates - Katherine Andrews, Maria Ayala, Rachel Berry, Valerie Clark, Staci Downe, Brigitte Hogan, Linh Huynh, Tammy Limoges, Kendra Murphy, Sarah Neal, Jessica Peralta, Kalli Porter, Jacquline Price, and Carol Schofield.

TX Iota – McMurry University

Corresponding Secretary – Dr. Kelly McCoun; 12 New Members

New Initiates - Lauren Alexander, Gerri Boggs, Kimberly Cline, Rance Cook, Drew Dutcher, Amanda Gillett, Jonathan Langford, Darrick Matthews, Arnett McClure, Danni McMahan, Shalisa Sites, and Danielle St. Jean.

TX Mu – Schreiner University

Chapter Pres. – Colin Lawson; 11 Current Members; 11 New Members
Other Fall 2011 Officers: James Heikkenen, Vice President; Will Keaton, Secretary; Amanda Ludwig, Treasurer; Dr. Stefan T. Mecay, Corresponding Secretary and Faculty Sponsor

New Initiates - Saira Avila Justin Butler, Elizabeth Fawcett, Raevin Butler, Teresa Gaitan, Tereso Hernandez, Daniel Hood, Kevin Reder, Seth Reed, Marcus Vargus, and Colin Williams.

WI Alpha – Mount Mary College

Corresponding Secretary – Roxanne Back; 5 New Members

New Initiates - Paulette Belling, Allison Dolnik, Linnea Esberg, Amy Ramirez, and Nerissa Seward.

WI Gamma – University of Wisconsin-Eau Claire

Corresponding Secretaries – Simei Tong (Fall 2011) and Carolyn Otto (Spring 2012); 50 New Members

New Initiates - Roxanne Accola, Ivan Alias, Sarah Baker, Robert M. Belau, Sophia Bolle, Roman D. Borisov, Meghan Christenson, Cassandra Dale, Cory Davis, Kayla S. Deutscher, Laura Elder, Robert Erickson, Michael Farrell, Daniel F. Ferrise Jr., Hannah J. Fredrickson, Sam Hendrickson, Ryan Horstman, Abagail Huebner, Nicholas Jaeger, Jessica Joniaux, Zachary Kelliher, Nolan Kriener, Jenniffer Kulesa, Hannah Leary, Sara Leffingwell, Michael Loper, Phillip McDaniel, Klayt W. Morfoot, Mackenzie Nelson, Thomas Nevins, Frederick Paffel, Bethanna Petersen, Dean Poppe, Stephanie Prahl, Daniel Putman, Jami Riley, Gregory James Samens, Dan Schilcher, Philip James Schumacher, Emily Simpson, Blake R. Smith, Nicholas Sortedahl, Sudharishini Subramani, Fallon Swope, Li Boon Tay, Kayla VandenLangenberg, Lan Pham Vu, Nancy Vue, Tzyy Ren Wong, and Kok Chee Yew.

WV Alpha – Bethany College

Corresponding Secretary – Adam C. Fletcher; 5 New Members

New Initiates - Melinda S. Bierhals, Seth A. Cawoski, Brittany L. Marsh, Katherine E. Robert, and Joseph P. Ward.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 March 1935
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, Jonesboro	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Domincan University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985

NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February 2001
GA Delta	Berry College, Mount Berry	21 April 2001
TX Mu	Schreiner University, Kerrville	28 April 2001
NJ Gamma	Monmouth University	21 April 2002
CA Epsilon	California Baptist University, Riverside	21 April 2003
PA Rho	Thiel College, Greenville	13 February 2004
VA Delta	Marymount University, Arlington	26 March 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 February 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 March 2005
SC Epsilon	Francis Marion University, Florence	18 March 2005
PA Sigma	Lycoming College, Williamsport	1 April 2005
MO Nu	Columbia College, Columbia	29 April 2005
MD Epsilon	Stevenson University, Stevenson	3 December 2005
NJ Delta	Centenary College, Hackettstown	1 December 2006
NY Pi	Mount Saint Mary College, Newburgh	20 March 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 April 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 October 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 March 2008

CA Zeta	Simpson University, Redding	4 April 2009
NY Rho	Molloy College, Rockville Center	21 April, 2009
NC Zeta	Catawba College, Salisbury	17 September, 2009
RI Alpha	Roger Williams University, Bristol	13 November, 2009
NJ Epsilon	New Jersey City University, Jersey City	22 February, 2010
NC Eta	Johnson C. Smith University, Charlotte	18 March, 2010
AL Theta	Jacksonville State University, Jacksonville	29 March, 2010
GA Epsilon	Wesleyan College, Macon	30 March, 2010
FL Gamma	Southeastern University, Lakeland	31 March, 2010
MA Beta	Stonehill College, Easton	8 April, 2011
AR Beta	Henderson State University, Arkadelphia	10 October, 2011
PA Tau	DeSales University, Center Valley	29 April, 2012