

## THE PENTAGON

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### Consecutive Numbers in Lotteries Reconsidered

We wish to thank Andrew Reed of Evangel University for his work with sequences of non-consecutive lottery numbers in the Fall 2006 issue of The Pentagon. Our senior seminar found his results intriguing. In this paper we, like Reed, will also not improve the chances of winning the lottery, however we will present what we believe to be a clear way of counting sequences of non-consecutive positive integers.

Reed showed that

$$NC(n, k) = \frac{(n - k + 1)!}{k!(n - 2k + 1)!}.$$

We observed that

$$NC(n, k) = \frac{(n - k + 1)!}{k!(n - 2k + 1)!} = \binom{n - (k - 1)}{k}.$$

We realized a bijection between the sequences of  $k$  non-consecutive integers chosen from  $n$  integers and the combinations of  $k$  integers chosen from  $n - (k - 1)$  integers would prove Reed's result in an alternate manner.

**Theorem:** A bijection exists between the sequences of  $k$  non-consecutive integers chosen from  $n$  integers and the combinations of  $k$  integers chosen from  $n - (k - 1)$  integers.

**Proof.** We start with a sequence of  $k$  non-consecutive positive integers less than or equal to an integer  $n$ . Put the sequence in ascending order,

$$x_0, x_1, x_2, \dots, x_{k-1}.$$

Now map this sequence to

$$x_0, x_1 - 1, x_2 - 2, \dots, x_{k-1} - (k - 1).$$

This is an element of the combinations of  $k$  integers chosen from  $n - (k - 1)$  possible integers. We see this map is one-to-one and onto. ■

Now we use an example from the Nebraska lottery, which chooses 5 numbers from 55. Say a person chooses the non-consecutive balls

$$21, 55, 8, 17, 42.$$

First, we put them in ascending order

$$8, 17, 21, 42, 55.$$

Next map them to

$$8, 16, 19, 39, 51,$$

one of the 2,349,060 combinations of 5 of 51 elements.

Let us provide an example in the other direction. Choose any combination of 5 of 51 elements, say

13, 14, 5, 27, 26.

Put it in ascending order

5, 13, 14, 26, 27.

This would correspond to the nonconsecutive lottery sequence

5, 14, 16, 29, 31.

We invite the reader to try other examples.

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## ***Who Dun'it?: Investigations in the Mathematics of Forensic Science***

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Presented at the 2007 National Convention and awarded “top three” status by the Awards Committee.

### **1. Purpose**

Who Dun'it? This is the million-dollar question to investigators on homicide cases. A number of times the investigators find suspects and must take them to court to prove they are guilty. A crucial piece of evidence in the courtroom is reconciling the victim's time of death and the suspect's alibis. Determining the time of death can lead to putting a person behind bars or the person going free. This paper will discuss the investigation of the application of mathematics in forensic science with the specific focus of developing a robust model for the prediction of time of death. Historically, the underlying fundamental mathematics relies on the solution of differential equations and the application of Newton's law of cooling. The initial phase of the project involved establishing the sensitivity of Newton's law of cooling to various environmental factors. The second phase of the project focused on developing a more robust mathematical model, which could be applied to an organic system. This work was guided by the existing forensic science literature, as well as, standard mathematical modeling techniques. The project used MAPLE (software that is used for mathematical purposes: graphs, solving equations, etc.) to help with establishing the sensitivity of Newton's law of cooling to the different environmental factors, to explore a new model, and to investigate the problem from three perspectives: graphically, numerically, and analytically.

## 2. Assumptions

In a project dealing with bodies, environmental factors, and other issues, there are a couple of assumptions that had to be made to narrow the focus of the project. First, the victims prior to being murdered were relatively healthy. This implies the next assumption made, which is, that at the time of death; the victim's body temperature was approximately  $37^\circ$ , which is considered to be normal. Typically, all healthy humans have a temperature of  $37^\circ$ , with a  $\pm 0.6^\circ C$  range. Changing the normal body temperature does affect the time of death; it is not a major contributor until it gets out of the  $\pm 0.6^\circ C$  range. In this paper, we will keep it a constant and realize that the time of death may be altered a few minutes in the end, depending on the victim's normal body temperature.

The next assumption is crucial; the models that are discussed will be analyzed in the time frame of the first 48 hours after death or less. Though the data will include the temperature measurements up to 48 hours after the body was murdered, the specific part that will be looked at in the three-stage approach is the first 24 hours after the time of death. If a body is found and it is concluded that the body has been there over 24 hours, the methods in this paper will not be the best approach to solving for the time of death. After the first 24 hours, the standard practice is to consider other evidence, for example, the presence of bugs. This work will focus in on if the body is found sometime within the first 24 hours of being murdered.

The last assumption is that the ambient temperature, the room/outdoor air temperature in which the body is found, stays constant. This is not the most realistic, but considering that for the best fit for the data is that the body is found within less than 24 hours after being murdered, we will assume that the ambient temperature does not waiver enough to change the time of death.

## 3. Symbolization/Equations

Five main equations were used in this project to produce results. The first two equations are:

$$T_c - T_a = (T_0 - T_a) e^{-k\Delta t} \quad (1)$$

and

$$T_0 - T_a = (37 - T_a) e^{-k\Delta t}. \quad (2)$$



The symbolization is as follows:

- $T_c$  = 2nd body temperature
- $T_a$  = ambient temperature (room/outdoor temperature)
- $T_o$  = 1st body temperature
- $k$  = constant
- $\Delta t$  = time between 1st and 2nd body temperature measurements
- 37 = Normal body temperature (in °C)

These two equations are Newton's law of cooling. Throughout the paper they will be referenced back as equation 1 and equation 2. The two equations were used to see what environmental factors have the most impact on Newton's law of cooling. The first equation is to solve for the constant  $k$ , using the information that would be found on the scene with the victim. The second equation is where the time of death will actually be established. The value of  $\Delta t$  subtracted from the time of the first measurement of body temperature equals the time of death. Throughout the project the equations were used to change the ambient temperature, the second measurement of body temperature, the first measurement of body temperature, and the time between temperature measurements to see what factors had the greatest impact on Newton's law of cooling.

The next set of equations all work together. It is actually a set of six equations, but three of them helped obtain the constants used in the second set of three. The six equations were found in the journal article [5]. The three equations for the constants were omitted, the three equations I used are as follows:

$$T_r = T_m + \left(T_r^{(o)} - T_m\right) e^{-k_r \Delta t} \quad (3)$$

$$T_m = m \left[ T_s + \left(T_m^{(o)} - T_s\right) e^{-k_m \Delta t} \right] \quad (4)$$

$$+ (1 - m) \left[ T_r + \left(T_m^{(o)} - T_r\right) e^{-k_m \Delta t} \right] \quad (5)$$

$$T_s = s \left[ T_a + \left(T_s^{(o)} - T_a\right) e^{-k_s \Delta t} \right] \quad (6)$$

$$+ (1 - s) \left[ T_m + \left(T_s^{(o)} - T_m\right) e^{-k_s \Delta t} \right] \quad (7)$$

The symbolization is as follows:

- $T_a$  = ambient temperature
- $T_r$  = rectal temperature
- $T_s$  = skin temperature
- $T_m$  = muscle temperature
- $m, s, k_r, k_s, k_m$  = constants

■  $\Delta t$  = change in time between the initial measurement and the measurement taken

■  $T_r^{(o)}, T_s^{(o)}, T_m^{(o)}$  = the initial values of rectal, skin, and muscle temperatures

For purposes of this project the equations needed are equations 3, 4, and 5. The other three gave the constants for equations 3, 4, and 5, which throughout the project were kept the same as what were given in the journal article. The constants are given as:  $m = 0.85$ ,  $s = 0.95$ ,  $k_r = 0.169$ ,  $k_m = 0.761$ ,  $k_s = 0.0625$ . These constants were used for the first 48 hours after death, which is the part that we are concerned about. Using this model, the following numbers were used throughout the entire project unless otherwise stated:  $T_s^{(o)} = 26.5^\circ C$ ,  $T_r^{(o)} = 38^\circ C$ ,  $T_m^{(o)} = 32.5^\circ C$ , and  $T_a = -6^\circ C$ .

#### 4. Newton's Law of Cooling

To explore the robustness of Newton's law of cooling, equations 1 and 2 were used with MAPLE to investigate the effect of changing the ambient temperature, the second measurement of body temperature, the first measurement of body temperature, and the time between temperature measurements on the time of death. The original data that was used before altering the different variables is as follows:

- $T_c = 29.9^\circ C$  at time 6:30 a.m.
- $T_o = 31.1^\circ C$  at time 4:30 a.m.
- $T_a = 24.4^\circ C$

The times picked for  $T_c$  and  $T_o$  are arbitrary. By plugging this information into equation 1, MAPLE then solves for  $k$ , the constant. Then, using the second equation, MAPLE solves for  $\Delta t$ , which gives the time of death. For example, with the given data,  $k = .10261321$  and  $\Delta t = 6.251661628$ . The  $t$  is in terms of hours, so it really is six hours and about fifteen minutes. By taking 4:30 a.m. and subtracting the six hours and fifteen minutes we arrive at the approximation of time of death of 10:15 p.m. Everything else was based on these numbers. Using these numbers was a way to understand the equations and to have a set of numbers to work with.

Using MAPLE and the numbers stated above, tests were run to see what the affects of changing these variables would have on the time of death. The first variable that was tested was the first body temperature measurement. In the original example the first body temperature was  $31.0^\circ C$ . In the spreadsheet below, the first body temperature was altered, by starting at  $30.0^\circ C$  and going up to  $36.9^\circ$ . The reason that the range starts at  $30.0^\circ C$

is because going any lower, the temperature then goes below what the second body temperature measurement was, and this would be impossible. The temperature of a deceased person cannot rise under the stated circumstances. The range then ended at  $36.9^{\circ}\text{C}$  because any higher would be higher than what is assumed as the normal body temperature,  $37^{\circ}\text{C}$  and once again, the person's body temperature cannot rise after dying. The following is the spreadsheet of the altered first body temperature measurement data.

1st Body Temp. in $^{\circ}\text{C}$	$k$	$\Delta t$	Time of death
30.0	0.0101014	80.7180008	3 days and then it would be 7:47 PM
30.3	0.0344964	22.22187385	6:17 AM
30.6	0.0577564	12.46709687	4:02 PM
30.8	0.0799823	8.446904764	8:03 PM
31.1	0.1012621	6.251529998	10:15 PM
31.4	0.1216731	4.867314649	11:38 PM
31.7	0.1412835	3.914119829	12:35 AM
31.9	0.1601536	3.217287784	1:17 AM
32.2	0.1783375	2.685316194	1:49 AM
32.5	0.1958831	2.265643421	2:14 AM
32.8	0.2128339	1.925913612	2:34 AM
33.1	0.2292288	1.645124358	2:51 AM
33.3	0.2451032	1.409043962	3:05 AM
33.6	0.2604890	1.20768833	3:18 AM
33.9	0.2754155	1.033843637	3:28 AM
34.2	0.2899092	0.882169401	3:37 AM
34.4	0.3039947	0.748625382	3:45 AM
34.7	0.3176942	0.630100185	3:52 AM
35.0	0.3310283	0.524157382	3:59 AM
35.3	0.3440160	0.428862148	4:04 AM
35.6	0.3566749	0.34265835	4:09 AM
35.8	0.3690213	0.264280192	4:14 AM
36.1	0.3810700	0.192687613	4:18 AM
36.4	0.3928353	0.127017534	4:22 AM
36.7	0.4043300	0.066548248	4:26 AM
36.9	0.4155665	0.010671209	4:29 AM

As the reader can see, by changing the first body temperature and leaving everything else the same, the time of death has a wide range. We can conclude from this that the first body temperature does have a large affect on Newton's law of cooling if changed even  $.5^{\circ}$  C. This was the first indication of a problem with this approach.

Similar data was collected for altering the time between the two temperature measurements, the second body temperature measurement, and the ambient temperature. By changing the time between the two measurements, there was a significant affect on the time of death, though not as much as there was by changing the first body temperature. It was a significant enough change that would lead to the conclusion that the time between measurements has a great impact on the time of death. Changing the second body temperature measurement was very similar to the data collected by changing the first body temperature measurement. By changing the second body temperature, the times of death went up or down by hours, so that was recorded as a significant change. Once again data was run with altering the ambient temperature, which is the room or outdoor temperature that the body was found in, depending on the case. The original ambient temperature was  $34.4^{\circ}$ C and by changing it even by  $1^{\circ}$ C changed the time of death by over two hours.

After running the data and coming up with the above conclusions, we can see that Newton's law of cooling is not robust. It is very sensitive to the changes that were made to the different variables, which led to considering a better model to determine the time of death.

### **5. Three –Stage Approach**

The three-stage approach, from [5] takes into consideration the rectal, skin and muscle temperature measurements. After a death of a little girl, there was need to model the body cooling of her body to determine her time of death. She was found 4 days after she had gone missing in the snow and her body was in a frozen state, which meant she had been murdered awhile before they had found her, the time and day was unsure. To model the body cooling of the little girl, two dogs were taken about her size and similar insulation was put on them, like the girl would have had, and they were killed and put through the same environmental factors, including laying in snow. One dog was covered and one dog was uncovered. Three temperatures were collected from each dog, the rectal, skin, and muscle temperatures and after collecting the data, only the covered dog's data was taken to model. The covered dog was a closer resemblance of the girl. Taking the data produced a three-equation system. Equation 3

is the equation that models the rectal temperature data. It is very similar to Newton's law of cooling, after coming up with equation 3, the authors' realized that it was not good enough, they needed a more complex model, because having just one equation did not take in account the body having different temperatures. Next, a two-equation model was formed, this being equations 3 and 4, which now includes the skin temperatures. This made a good fit for the rectal temperature, but not with the muscle temperatures. Thus, a third equation, equation 5, was introduced involving the muscle temperatures. This resulted in a reasonable fit to the data from the dog and then the constants were formed from that.

Taking into consideration the three different temperatures produces a more robust model to predict the time of death. The more measurements and specifically, these three measurements help produce a more realistic model. The skin is the outer shell of the body, the muscle is the inner shell of the body, and the rectum is the core of the body. This model takes more work for investigators to use; they must have multiple measurements, which is more difficult to produce on the scene.

The article did not have a data set with it. It gave the equations, initial conditions, and the constants, graphs, but a data set of time and temperature was needed. Using the equations 3, 4, and 5 with the given constants and the given conditions in the symbolization/equation section, experimental data set was generated. The three equations and their constants were plugged into MAPLE. Then a "t" was picked, starting at .25 of an hour (15 minutes) and going up to 48 hours by increments of .25 of an hour. Each time was entered and then MAPLE solved the three-equation system for  $T_s$ ,  $T_r$ , and  $T_m$ .

The data set is assumed experimental data. As seen through other studies that have been done, their data sets are experimental. To test the environmental conditions, dummies are sometimes used. The dummies are there to try to resemble a person as close as possible. The article [1] investigated the effects on time of death when a homicide victim was found either on top of leaves or under leaves. To construct a data table for the article, a corpse-like dummy was used and put under certain conditions. Then measurements of temperature of the dummy were taken every thirty minutes. For any type of project dealing with a person, using real data with a real human would be preferred, but killing a person just to construct a data set is out of the question.

The following is the data table of the collected information:

Time	$T_s$ -°C	$T_r$ -°C	$T_m$ -°C	Time	$T_s$ -°C	$T_r$ -°C	$T_m$ -°C
0.25	26.05345	37.73865	31.68268	12.25	9.54637	13.67941	10.16832
0.5	25.55679	37.42193	30.86587	12.5	9.31094	13.30913	9.91234
0.75	25.09428	37.05471	30.0595	12.75	9.07898	12.94588	9.66041
1	24.63777	36.64263	29.27043	13	8.85045	12.58959	9.41249
1.25	24.18723	36.19156	28.50324	13.25	8.62529	12.24016	9.1685
1.5	23.74265	35.70723	27.76079	13.5	8.40346	11.89749	8.92838
1.75	23.30399	35.19513	27.04463	13.75	8.18492	11.56145	8.69209
2	22.87124	34.66025	26.35532	14	7.96961	11.23208	8.45955
2.25	22.44434	34.10709	25.69271	14.25	7.75749	10.90914	8.23071
2.5	22.02334	33.53962	25.05613	14.5	7.54853	10.59256	8.00553
2.75	21.60813	32.96133	24.44453	14.75	7.34266	10.28226	7.78393
3	21.19868	32.3752	23.85668	15	7.13985	9.97813	7.56587
3.25	20.79495	31.78379	23.29116	15.25	6.94006	9.68006	7.35129
3.5	20.39689	31.18927	22.74654	15.5	6.74324	9.38795	7.14014
3.75	20.00444	30.59348	22.22214	15.75	6.54936	9.10169	6.93237
4	19.61754	29.99793	21.71419	16	6.35836	8.82118	6.72792
4.25	19.23614	29.403	21.22369	16.25	6.17021	8.54632	6.52674
4.5	18.86017	28.81246	20.74859	16.5	5.98487	8.27699	6.32878
4.75	18.48957	28.22449	20.2877	16.75	5.8023	8.01309	6.13399
5	18.12426	27.64073	19.83995	17	5.62246	7.75452	5.94233
5.25	17.76419	27.06178	19.40434	17.25	5.4453	7.50118	5.75374
5.5	17.40928	26.48816	18.97999	17.5	5.2708	7.25296	5.56817
5.75	17.05947	25.92028	18.56609	17.75	5.09891	7.00976	5.38558
6	16.71469	25.35849	18.16193	18	4.9296	6.77148	5.02591
6.25	16.37487	24.80309	17.76687	18.25	4.76282	6.53802	5.02913
6.5	16.03995	24.25434	17.38035	18.5	4.59855	6.30928	4.85518
6.75	15.70986	23.71243	17.00187	18.75	4.43674	6.08516	4.68402
7	15.38454	23.17755	16.63097	19	4.27737	5.86558	4.51561
7.25	15.06392	22.64984	16.26728	19.25	4.12038	5.65043	4.3499
7.5	14.74794	22.12941	15.91044	19.5	3.96576	5.43961	4.18685
7.75	14.43654	21.61639	15.56015	19.75	3.81347	5.23304	4.02641
8	14.12964	21.11084	15.21614	20	3.66346	5.03064	3.86855
8.25	13.8272	20.61283	14.87818	20.25	3.51572	4.8323	3.71321
8.5	13.52915	20.12241	14.54604	20.5	3.3702	4.63793	3.56037
8.75	13.23543	19.63964	14.21954	20.75	3.22688	4.44747	3.41
9	12.94598	19.16452	13.89851	21	3.08571	4.26082	3.26198
9.25	12.66074	18.69708	13.58279	21.25	2.94668	4.07789	3.11637
9.5	12.37966	18.23732	13.27226	21.5	2.80975	3.89861	2.97308
9.75	12.10268	17.78523	12.96678	21.75	2.67488	3.7229	2.83209
10	11.82974	17.34081	12.66234	22	2.54205	3.55068	2.69335
10.25	11.56079	16.90402	12.37053	22.25	2.41123	3.38187	2.55683
10.5	11.29578	16.47484	12.07956	22.5	2.41123	3.38187	2.55683
10.75	11.03464	16.95322	11.79323	22.75	2.1555	3.0542	2.2903
11	10.77733	15.63913	11.51146	23	2.03053	2.89518	2.16023
11.25	10.5238	15.2325	11.23417	23.25	1.90745	2.73929	2.03222
11.5	10.27399	14.83328	10.96129	23.5	1.78623	2.58646	1.90627
11.75	10.02784	14.44141	10.69272	23.75	1.66685	2.43661	1.78232
12	9.785322	14.05681	10.42843	24	1.54928	2.28968	1.66034

Time	$T_s$ -°C	$T_r$ -°C	$T_m$ -°C	Time	$T_s$ -°C	$T_r$ -°C	$T_m$ -°C
24.25	1.4335	2.14561	1.54031	36.25	-2.46869	-2.36471	-2.45309
24.5	1.31947	2.00433	1.4222	36.5	-2.52316	-2.42335	-2.50819
24.75	1.20717	1.86579	1.30596	36.75	-2.5768	-2.48099	-2.56243
25	1.09658	1.72991	1.19158	37	-2.62961	-2.53764	-2.61581
25.25	0.98766	1.59665	1.07901	37.25	-2.68161	-2.59333	-2.66837
25.5	0.8804	1.46595	0.96824	37.5	-2.73281	-2.64808	-2.7201
25.75	0.77477	1.33778	0.85922	37.75	-2.78323	-2.7019	-2.77103
26	0.67075	1.212	0.75194	38	-2.83287	-2.75481	-2.82116
26.25	0.56831	1.08862	0.64636	38.25	-2.88175	-2.80683	-2.87051
26.5	0.46742	0.96761	0.54245	38.5	-2.92988	-2.85797	-2.91909
26.75	0.36808	0.84888	0.4402	38.75	-2.97727	-2.90825	-2.96691
27	0.27024	0.73239	0.33956	39	-3.02393	-2.95769	-3.01399
27.25	0.17389	0.61809	0.24052	39.25	-3.06987	-3.0063	-3.06034
27.5	0.07901	0.50593	0.14305	39.5	-3.11511	-3.0541	-3.10596
27.75	-0.01442	0.39587	0.04712	39.75	-3.15965	-3.1011	-3.15087
28	-0.10643	0.28786	-0.04729	40	-3.2035	-3.14732	-3.19508
28.25	-0.19704	0.18187	-0.1402	40.25	-3.24669	-3.19277	-3.2386
28.5	-0.28627	0.07784	-0.23165	40.5	-3.2892	-3.23746	-3.28144
28.75	-0.37413	-0.02427	-0.32165	40.75	-3.33107	-3.28142	-3.32362
29	-0.46066	-0.12449	-0.41023	41	-3.37228	-3.32464	-3.36514
29.25	-0.54586	-0.22287	-0.49741	41.25	-3.41287	-3.36715	-3.40601
29.5	-0.62977	-0.31945	-0.58322	41.5	-3.45283	-3.40896	-3.44625
29.75	-0.71239	-0.41426	-0.66767	41.75	-3.49218	-3.45008	-3.48586
30	-0.79375	-0.50734	-0.75079	42	-3.53092	-3.49053	-3.52486
30.25	-0.87387	-0.59873	-0.8326	42.25	-3.56906	-3.5303	-3.56324
30.5	-0.95277	-0.68846	-0.91312	42.5	-3.60662	-3.56943	-3.60104
30.75	-1.03046	-0.77657	-0.99237	42.75	-3.64359	-3.60792	-3.63824
31	-1.10696	-0.86309	-1.07038	43	-3.68	-3.64577	-3.67487
31.25	-1.18229	-0.94805	-1.14716	43.25	-3.71585	-3.68301	-3.71092
31.5	-1.25647	-1.03149	-1.22272	43.5	-3.75115	-3.71963	-3.74642
31.75	-1.32952	-1.11343	-1.2971	43.75	-3.7859	-3.75566	-3.78136
32	-1.40145	-1.19391	-1.37032	44	-3.82011	-3.79111	-3.81576
32.25	-1.47228	-1.27296	-1.44238	44.25	-3.8538	-3.82597	-3.84963
32.5	-1.54202	-1.35061	-1.514202	44.5	-3.88697	-3.86027	-3.88297
32.75	-1.6107	-1.42688	-1.58312	44.75	-3.91963	-3.8994	-3.91579
33	-1.67832	-1.50181	-1.65184	45	-3.95179	-3.92722	-3.9481
33.25	-1.74491	-1.57541	-1.71948	45.25	-3.98345	-3.95987	-3.97991
33.5	-1.81048	-1.64772	-1.78607	45.5	-4.01462	-3.99201	-4.01123
33.75	-1.87505	-1.71876	-1.8516	45.75	-4.04531	-4.02362	-4.04206
34	-1.93862	-1.78856	-1.91611	46	-4.07553	-4.05472	-4.07241
34.25	-2.00122	-1.85714	-1.97961	46.25	-4.10539	-4.08532	-4.10229
34.5	-2.06287	-1.92453	-2.04211	46.5	-4.13458	-4.11543	-4.13171
34.75	-2.12356	-1.99074	-2.10364	46.75	-4.16342	-4.14505	-4.16067
35	-2.18333	-2.05581	-2.1642	47	-4.19182	-4.17419	-4.18918
35.25	-2.24179	-2.11976	-2.22382	47.25	-4.21978	-4.20287	-4.21724
35.5	-2.30013	-2.1826	-2.2825	47.5	-4.24731	-4.23109	-4.24488
35.75	-2.35719	-2.24436	-2.34026	47.75	-4.27441	-4.25885	-4.27208
36	-2.41337	-2.30505	-2.39712	48	-4.3011	-4.28617	-4.29886

After getting the initial data, the variables needed to be changed to see the affects on the rectal, muscle, and skin temperatures. In equations 3, 4, and 5, the following variables will be looked at:  $T_a$ ,  $m$ , and  $s$ .

While leaving all other numbers as stated in the symbolization/equations section,  $T_a$  was changed. The original temperature was  $-6^\circ C$ . A range from  $-14^\circ C$  to  $40^\circ C$  was plugged into the equations to see what happened to the rectal, skin, and muscle temperatures. The values of  $T_m$  and  $T_r$  were the least affected;  $T_s$  was the most affected. The values of  $T_m$  and  $T_r$  were changed enough by the time the outside/indoor temperature reached  $40^\circ$  to know that is a very important factor in the three-stage approach.

The variables  $m$  and  $s$  reflect the input that the ambient temperature had on the muscle and skin;  $m$  to reflect the effects that the ambient temperature had on the muscle temperature measurement and  $s$  to reflect the effects that the ambient temperature had on the skin temperature measurement. First it is important to know that  $m$  and  $s$  must be between 0 and 1. Notice in equations 4 and 5 that if the value of  $m$  or  $s$  is either 0 or 1, then parts of the equation become zero and thus parts of the model are zero and the model does not account for all three measurements accurately. After plugging in different values between 0 and 1 for  $m$  and  $s$ , a small change, of about  $3^\circ C$  or less, in the rectal, skin, and muscle temperature can be seen. Though any change is significant, a change of less than  $3^\circ C$  is going to be assumed to not have a significant role in the three-model stage approach. By noting that there is a small change in the rectal, skin, and muscle temperatures when  $m$  and  $s$  are changed, is enough for right now. After getting a feel for the changes that occur when altering the three variables, we need to find a curve that fits the three-stage approach data. Finding a curve that fits the data will give the ability to make a prediction of the time of death with collected data from a crime scene.

## 6. Curve Fitting

From the three-stage approach, many data points were gathered. As stated, the first 24 hours is the part that we are most interested in. After 24 hours, these methods are just not the most reliable techniques to find the time of death. Therefore, the first 24 hours of data from the three-stage approach is the set of data that a curve will be fit to. A curve must be fit to the  $T_r$ ,  $T_s$ , and  $T_m$  data. To do this, interpolation was used. Interpolation "is the process of estimating the value of a function corresponding to a particular value of the independent variable  $x$  when the function is described by a set of tabular data" ([7], p. 365). There are several numerical techniques for interpolation; for example, some are done by using polyno-



mials, spline functions, Fourier series, and many others. To start to fit a curve to the data of  $T_r$ ,  $T_s$ , and  $T_m$ , the following three forms were used: linear spline, cubic spline, and polynomial interpolation.

For the sets of  $T_r$ ,  $T_s$ , and  $T_m$  data, the data was run through MAPLE using the three different interpolation forms (linear spline, cubic spline, and polynomial interpolation) and the best-fit method is shown below for each of the data sets. The two types that ended up being the best-fit were the polynomial interpolation and linear spline. Using these methods, the first five points ( $t = 1, 2, 3, 4, 5$ ) of the original data sets were used to construct the data.

From the data we can see that  $T_r$  and  $T_m$  data best fits using the linear spline method and  $T_s$  data best fits using the polynomial interpolation method. Linear spline, otherwise known as first-order spline, creates a straight line between data points. The linear spline does not form a smooth line. When wanting to find a  $y$  value, linear spline takes two known points around the point needing to be calculated, and uses the two known points to form an equation, piecewise linear equations, to find the unknown  $y$  value. A polynomial interpolation is a function that goes through all of the data points. There are numerous types of polynomial interpolation, some of which are Lagrange interpolation, Newton's interpolation, and Hermite interpolation. The one that we are concerned about is the Lagrange interpolation, though the results using the others would be similar, we will stick to discussing the Lagrange interpolation. The Lagrange interpolation, like the others, does provide an exact fit to the data set. The solving of the Lagrange interpolation needs a computer program, which is where MAPLE took over and solved for the data points needed to retrieve the table that is above.

As we can see, for the first 12 hours the original data and the interpolation method are a close match to each other and after 12 hours they begin to differ more. This is a concern and would be addressed, if further research is done. As of right now, the data will be taken with a known fact that it is not the best fit that could be done.

Time	Temp. Lin. Spline	Temperature $T_r$	Time	Temp. Poly. Interp.	Temperature $T_s$
6	25.2835	25.35849	6	16.7143	16.71469
6.5	24.1049	24.25434	6.5	16.0388	16.03995
7	22.9263	23.17755	7	15.382	15.38454
7.5	21.7477	22.12941	7.5	14.743	14.74794
8	20.5691	21.11084	8	14.1208	14.12964
8.5	19.3905	20.12241	8.5	13.5146	13.52915
9	18.2119	19.16452	9	12.9232	12.94598
9.5	17.0333	18.23732	9.5	12.3455	12.37966
10	15.8547	17.34081	10	11.7805	11.82974
10.5	14.6761	16.47484	10.5	11.2267	11.29578
11	13.4975	15.63913	11	10.683	10.77733
11.5	12.3189	14.83328	11.5	10.148	10.27399
12	11.1403	14.05681	12	9.6202	9.785322
12.5	9.9617	13.30913	12.5	9.0982	9.31094
13	8.7831	12.58959	13	8.5805	8.85045
13.5	7.6045	11.89749	13.5	8.0653	8.40346
14	6.4259	11.23208	14	7.5511	7.96961
14.5	5.2473	10.59256	14.5	7.0361	7.54853
15	4.0687	9.97813	15	6.5184	7.13985
15.5	2.8901	9.38795	15.5	5.9963	6.74324
16	1.7115	8.82118	16	5.4678	6.35836
16.5	0.5329	8.27699	16.5	4.9309	5.98487
17	-0.64567	7.75452	17	4.3835	5.62246
17.5	-1.8243	7.25296	17.5	3.8236	5.2708
18	-3.0029	6.77148	18	3.2489	4.9296
18.5	-4.1815	6.30928	18.5	2.6572	4.59855
19	-5.3601	5.86558	19	2.0462	4.27737
19.5	-6.5387	5.43961	19.5	1.4136	3.96576
20	-7.7173	5.03064	20	0.7569	3.66346
20.5	-8.8959	4.63793	20.5	0.0735	3.3702
21	-10.0745	4.26082	21	-0.639	3.08571
21.5	-11.2531	3.89861	21.5	-1.3833	2.80975
22	-12.4317	3.55068	22	-2.162	2.54205
22.5	-13.6103	3.38187	22.5	-2.9779	2.41123
23	-14.7889	2.89518	23	-3.834	2.03053
23.5	-15.9675	2.58646	23.5	-4.7329	1.78623
24	-17.1461	2.28968	24	-5.6776	1.54928

Time	Temp. Lin. Spline	Temperature $T_m$
6	17.9657	18.16193
6.5	17.0286	17.38035
7	16.0915	16.63097
7.5	15.1544	15.91044
8	14.2172	15.21614
8.5	13.2801	14.54604
9	12.343	13.89851
9.5	11.4059	13.27226
10	10.4686	12.66234
10.5	9.5316	12.07956
11	8.5945	11.51146
11.5	7.6574	10.96129
12	6.7203	10.42843
12.5	5.7832	9.91234
13	4.846	9.41249
13.5	3.9089	8.92838
14	2.9718	8.45955
14.5	2.0347	8.00553
15	1.0976	7.56587
15.5	0.1604	7.14014
16	-0.7767	6.72792
16.5	-1.7138	6.32878
17	-2.6509	5.94233
17.5	-3.5881	5.56817
18	-4.5252	5.02591
18.5	-5.4623	4.85518
19	-6.3995	4.51561
19.5	-7.3365	4.18685
20	-8.277	3.86855
20.5	-9.2108	3.56037
21	-10.1479	3.26198
21.5	-11.085	2.97308
22	-12.0221	2.69335
22.5	-12.9593	2.55683
23	-13.8964	2.16023
23.5	-14.833	1.90627
24	-15.7706	1.66034

The purpose of the interpolation is to be able to plug in data and to predict the time of death. Normally, crime scene investigators would have data collected and then with the interpolation they would be able to match up their data with the data above, assuming their conditions are similar to the above example. After their data is matched, they would run the interpolation function and come up with a time, if they got the time to equal six, this would mean the murder was committed six hours prior to the temperature measurements taken on the scene. For example, a murder case dealing with a girl that was found in snow and had been there for awhile could be applied to the data given. It means that she had approximately normal body temperatures for rectal, skin and muscle prior to her death and the investigators had the rectal, skin, and muscle measurements at the time they found her; say 3 o'clock in the afternoon with an ambient temperature of  $-5^{\circ}$ . Since her conditions match closely, the data from the interpolation methods would be pulled up on a computer screen for an investigator to look at. Her temperature measurements which were the following:  $T_r = 12.8^{\circ}$ ,  $T_s = 10.8^{\circ}C$ ,  $T_m = 7.5^{\circ}$ . These then would be matched in the data and the times would then be gathered. In this case, her time of death would be approximately 10.5 to 11.5 hours before the on-scene measurements were taken. This means that the girl was murdered between 3:30 and 4:30 in the morning. This is not precise, but will give an approximate time of death without having to run dummies to exactly match each murder case, which would be very time consuming and costly.

## 7. Conclusion

Completing the interpolation best-fit models gives a decent method to predict the time of death, with the one set of conditions that were listed. Realistically, not everyone dies with this one set of conditions, though it would be nice. Therefore, the ideal situation is to have numerous sets of data and to be able to eliminate some of the assumptions. Then, when an investigator gathers data on the scene, they can pick the set of data that matches up with their conditions and then generate the time of death.

A project is never ending; this is the basis of a better model for time of death. It shows one set of conditions and the interpolation model. Next to do would be to refine the interpolation and have only one method, not the two that it was narrowed down to, as shown above. There would be further investigation to try different interpolation methods to continue to find a better fit to the data. There is always room for improvement. Preferably, there would be many more data sets to make a database bank, to allow investigators to simply search for the data that best matches their case.

The more improvement that can be made to the way that time of death is calculated, the better. Accurately calculating the time of death will help put the guilty behind bars and allow those that are innocent to go free.

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## *Sudoku for Dummies: A Mathematician's Approach*

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### **1. Background and Recent Research**

Sudoku is a game that originated in Asia and has rapidly taken the United States by storm in the previous year. A traditional Sudoku consists of a nine by nine grid which is then split into nine three by three sub-grids. While its specific origination is sometimes questionable, the rules are always the same: each row, column, and sub-grid of nine squares must contain the numbers one through nine exactly once. The player is always given a grid with at least seventeen numbers filled in, and each valid Sudoku has only one unique solution that can be reached logically (the minimum is seventeen since no one has been able to demonstrate the uniqueness of a sudoku with only sixteen givens). An example sudoku is given below.

Since these number games have just recently caught fire, most of the research on them is fairly new. For example, in June 2005, Bertram Felgenhauer and Frazer Jarvis found the possible number of Sudoku games as 6,670,903,752,021,072,936,960, or  $6.671 \times 10^{21}$ . This is quite amazing since this is only the possible number of solutions and not the possible number of games, when one solution can render a vast number of games (as the creator can continually take more and more numbers off of the grid).

8		7				9		2
	1		4	6	2			
	2			8				
	7	2		5				9
	6			4			2	
3				2		5	1	
				1	8		4	
2		1				8		7

## 2. Different Variations of Sudoku

While 9x9 sudoku grids are the most popular and most-often printed, there are other variations of these games. Some newspapers publish sudoku grids that are larger than 9x9 (12x12, 15x15, and 16x16 are common), which are generalized into the category of superdokus. Shidakus are 4x4 mini games, and godokus are 5x5 grids in which the sub-grids look more like Tetris pieces than squares, since it is a prime sudoku. Another common smaller sudoku is the 6x6 roku doku, and these smaller games fall in to the category of subdokus. We will be spending the rest of our time focusing on the properties of a 4x4 shidoku.

## 3. How Many Possible Solutions for a Shidoku?

Before we can discuss what we can do with a shidoku, we must find all of the possible solutions. Given the same rules as for the original sudoku (except now applying to the numbers one through four), a simple enumeration is below.

The first sub-grid (we will use the upper left sub-grid for convenience) can easily be set at 4! possibilities since there are no other numbers in the grid yet. Following this, I originally assumed that thus the upper right

and bottom right sub-grid could each be set at 22 possibilities. Add the fact that this procedure would make the remaining sub-grid unique and one quickly arrives at 384 possible solutions. However, out of the sixteen possible solutions rendered when given a fixed upper left sub-grid (or any sub-grid for that matter), four solutions will always be impossible. This then lowers the number of possible solutions to 288. Dr. Don Tosh of Evangel University also created a program in APL which found the same number of solutions in October 2005.

Note: While doing this research in early September, I found 384 solutions posted on one of my reference pages, but when I noticed the error and went back to the page to examine the logic, the number had been changed to 288.

The following grids are the four impossible solutions when the upper left box is fixed in a 1, 2, 3, 4 fashion. It is interesting (although not unexpected) that they are all variations of one another.

1	2	4	3	1	2	3	4	1	2	4	3	1	2	3	4
3	4	1	2	3	4	2	1	3	4	1	2	3	4	2	1
2	3	X		4	1	X		4	1		X	2	3		X
4	1		X	2	3		X	2	3	X		4	1	X	

#### 4. What is the Minimum Number of Clues Necessary?

The best way to proceed with this question would be to try to determine the minimum number of clues and then prove that it is impossible to have less than this number. To do this, we will use a model that is similar to one by Sourendu Gupta, where we proceed with a blank grid just listing all of the possibilities for a given square.

1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4
1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4
1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4
1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4

Since we desire to place numbers in positions that will create the most impact, we will use caution and always use different numbers. Let us start by putting a '1' in the upper left corner.



1	3	2 4	2 3	2 4	2 3
3	2 4	3 4	1 3	2 4	1 3
3	2 4	1 3	2 4	1 3	2 4
3	2 4	1 3	2 4	1 3	2 4

Putting the '1' in this position has now affected the squares shaded in green (there are less possibilities for these boxes). The box in the lower right would now be the most logical box to explore since any number placed there will affect six already untouched boxes and still allow for symmetry.

1	3	2 4	2 3		4
3	2 4	3 4	1 3	2 4	1 3
3	2 4	1 3	2 4	1 3	2 4
3	4	1 3	1 4		2

With just these two clues we have affected all but two of the boxes of the grid, but cannot derive a unique solution. To try and accomplish a unique shidoku with just three solutions, we will put a '3' in the top box that has not yet been affected.

1	3	2 4	2 4		4
	2 4	2 4	3	1 4	
3	2 4	3 4	1 4	2 3	1 4
3	4	1 3	1 4		2

The shidoku with three clues does look promising, but still does not render a unique solution. We will continue and put a '4' in the only box that has not yet been affected.

1	3	2	2	4
2	4	2	3	1
3	2	4	1	1
3	1	1	4	2

Remark: As long as there are numbers in three of the  $2 \times 2$  boxes and all four columns or all four rows, there is a unique solution for four numbers.

To prove that a unique solution is impossible with three numbers, it is best to continue with cases.

It can be easily shown that any three clues that are the same (e.g. all '2's) will never render anything unique, as well as any three numbers that are in the same row, column, or sub-grid. The third case, consisting of three clues with only two different numbers, in any position, will not complete a unique grid either. This brings our focus to the fourth case in which the three numbers are distinct and spanning different rows, columns, and sub-grids. However, this proves incomplete as well, as only eight boxes can be determined.

The fifth case is the most interesting, since it involves having three distinct numbers, but with two in the same row, column, or sub-grid. By doing this, it is often possible to 'fix' one sub-grid, but only two unknown numbers can be found outside of this sub-grid.

In my opinion, the inability to create a unique game with just three clues stems from the fact that each clue only affects seven other positions on the game board. If the clues were able to affect more positions this may not be the case.

## 5. Different Game Levels

While all of these small shidoku seem trivial to mathematics students and mathematicians, they may actually have applications in mathematics education. One of the chief criticisms of Jean Piaget's work is that many people believe that his stages of development are inaccurately timed, and that children may be able to think about possibilities and abstract ideas long before his predetermined timeframe. These smaller puzzles may be helpful to elementary school and middle school math teachers, showing the teacher how well the student can assess possibilities and think abstractly, thus allowing the teacher to determine where to place the student academically or better tailor mathematics education to their level. This being said,

I have categorized possible games into three categories: easy, medium, and hard. After exploring many ways to assign shidokus to these groups, I have decided on basing it upon the number of boxes removed from the unique solutions. Easy games have four to six boxes removed, medium games have seven to nine boxes removed, and hard games have ten to twelve boxes removed. Following are examples of each level of game.

3	1		
	2	1	3
1			4
2		3	1

EASY

	1	3	
4			2
	4		3
	2	4	

MEDIUM

	1		
4			
		2	
3			

HARD

This method of classifying is useful, since games with more numbers removed have fewer squares that can be filled in solely by already given information. This means that before students could find some of the clues, they must realize that other squares are necessary first. Following are the same three puzzles, but with the squares highlighted that can be derived solely off of the original clues. Notice how the number of highlighted boxes decreases with each level of difficulty.

3	1		
	2	1	3
1			4
2		3	1

EASY

	1	3	
4			2
	4		3
	2	4	

MEDIUM

	1		
4			
		2	
3			

HARD

## 6. Concluding Remarks and Further Research

While we do know the number of solutions for both the  $4 \times 4$  shidoku and the  $9 \times 9$  sudoku, no one has discovered how many possible games can be generated out of these solutions. I estimate that for a shidoku, at least 250,000 to 500,000 games can be created out of the 288 solutions. If this estimate is correct, the number of games for a  $9 \times 9$  sudoku would be astronomical.

**References**

- [1] <http://theory.tifr.res.in/~sgupta/sudoku/shidoku.html> a website created by Sourendu Gupta, a member of the Department of Theoretical Physics in the Tata Institute of Fundamental Research. He discusses all types of sudokus on these pages.
- [2] <http://research.att.com/cgi-bin/access.cgi/as/njas/sequences/eisA.cgi?Anum=a107739> this provides an alternate proof to the 288 solutions.
- [3] <http://www.shef.ac.uk/~pmlafj/sudoku/sudoku.pdf> This is the article written by Frazer Jarvis and Bertram Felgenhauer with the enumeration of the  $9 \times 9$  sudoku grid.

## *Generalized Geometric Series, The Ratio Comparison Test, and Raabe's Test*

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The goal of this paper is to examine the convergence of a type of infinite series in which the summands are products of numbers in arithmetic progression, a type of infinite series we will call a *generalized geometric series*. These series are studied in order to state and to prove Raabe's Test, an application of a little-known test for infinite series known as the Ratio Comparison Test, which we will also state and prove. Statements and proofs of these little-known results are taken from [1]. This paper assumes the reader is familiar with the standard tests for convergence of infinite series, specifically, the Ratio Test, the Comparison Test, and the Integral Test.

Let's begin by recalling a standard result from the topic of infinite series:

**Theorem 1 (Geometric Series).** *Consider an infinite series of the form*

$$a_0 + a_0r + a_0r^2 + a_0r^3 + \cdots + a_0r^n + \cdots$$

*This series converges if and only if  $|r| < 1$ . If this series converges, its sum is*

$$\frac{a_0}{1-r}.$$

We would like to consider what I will call *generalized geometric series*:

**Definition 1.** A *generalized geometric series* is an infinite series of the form

$$\frac{a}{b} + \frac{a \cdot (a+d)}{b \cdot (b+e)} + \frac{a \cdot (a+d) \cdot (a+2d)}{b \cdot (b+e) \cdot (b+2e)} + \cdots + \frac{a \cdot (a+d) \cdots (a+nd)}{b \cdot (b+e) \cdots (b+ne)} + \cdots$$

Notice that if  $d = e = 0$ , we get an ordinary geometric series with  $a_0 = r = \frac{a}{b}$ . In particular, for  $d = e = 0$ , this series converges if and only if  $|a| < |b|$ .

At this point, let's recall (one version of) the Ratio Test:

**Theorem 2 (Ratio Test).** Suppose  $a_n > 0$  for all  $n$  and suppose  $L = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$  exists. Then  $\sum_n a_n$  converges if  $L < 1$  and diverges if  $L > 1$ . If  $L = 1$ , no conclusion can be reached about the behavior of  $\sum_n a_n$ .

If we apply the Ratio Test to a generalized geometric series, we have

$$\lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \rightarrow \infty} \frac{|a + (n+1)d|}{|b + (n+1)e|} = \frac{|d|}{|e|}.$$

By the Ratio Test, if  $|d| < |e|$ , the generalized geometric series converges absolutely and if  $|d| > |e|$ , the generalized geometric series diverges. Now we ask what happens if  $d = e > 0$ .

Let's begin by investigating a particular example:

**Problem:** Consider the infinite series

$$\frac{1}{4} + \frac{1 \cdot 3}{4 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{4 \cdot 6 \cdot 8} + \cdots + \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{4 \cdot 6 \cdot 8 \cdots (2n+2)} + \cdots$$

Does this infinite series converge or diverge?

We notice that this is a generalized geometric series with  $a = 1$ ,  $b = 4$ , and  $d = e = 2$ . Since  $d = e$ , we already know the Ratio Test fails.

We might tackle this problem by writing the products in the numerator and denominator in terms of factorials, according to the following lemma:

Let  $n$  be a natural number. Then

- $2 \cdot 4 \cdot 6 \cdots (2n) = 2^n n!$
- $1 \cdot 3 \cdot 5 \cdots (2n-1) = \frac{(2n)!}{2^n n!}.$

**Proof.** Let  $n$  be a natural number. We prove (i) by factoring 2 from each of the  $n$  factors and regrouping:

$$2 \cdot 4 \cdot 6 \cdots (2n) = (2 \cdot 1) \cdot (2 \cdot 2) \cdot (2 \cdot 3) \cdots (2 \cdot n) = 2^n (1 \cdot 2 \cdot 3 \cdots n) = 2^n n!.$$

To prove (ii), set

$$P = 1 \cdot 3 \cdot 5 \cdots (2n-1).$$

Multiplying this equation by equation (i), we have

$$\begin{aligned} 2^n n! P &= [1 \cdot 3 \cdot 5 \cdots (2n-1)] \cdot [2 \cdot 4 \cdot 6 \cdots (2n)] \\ &= 1 \cdot 2 \cdot 3 \cdots (2n-1) \cdot (2n) \\ &= (2n)! \end{aligned}$$

Solving for  $P$  yields the desired result. ■

Returning to the series in question, we can use the preceding lemma to write the series in closed form:

$$\sum_{n=1}^{\infty} \frac{(2n)!/(2^n n!)}{2^n (n+1)!} = \sum_{n=1}^{\infty} \frac{(2n)!}{4^n (n+1)! n!}.$$

This form has the virtue of expressing the infinite series as a sum of terms where the terms themselves are in closed form, i.e. not written as products. However, I don't think this closed form renders this series any more accessible!

In order to tackle this problem, we need a modification of the Ratio Test which is slightly more sensitive, a little-known test which I happened to run across in my random readings: The Ratio Comparison Test.

**Theorem 3 (Ratio Comparison Test).** *If  $a_n, b_n > 0$  for all  $n$ , and  $\sum b_n$  converges, and if for all sufficiently large  $n$ ,*

$$\frac{a_{n+1}}{a_n} \leq \frac{b_{n+1}}{b_n},$$

*then  $\sum a_n$  converges.*

**Proof.** Let  $\sum a_n$  and  $\sum b_n$  be series having positive terms and satisfying

$$\frac{a_{n+1}}{a_n} \leq \frac{b_{n+1}}{b_n}$$

for all sufficiently large  $n$ . If we rewrite this inequality as

$$\frac{a_{n+1}}{b_{n+1}} \leq \frac{a_n}{b_n},$$

we see that the sequence  $\{a_n/b_n\}$  is monotone decreasing. Since  $a_n, b_n \geq 0$ , this sequence is bounded below, and it follows that  $\{a_n/b_n\}$  converges by the completeness of the real numbers. In particular,  $\{a_n/b_n\}$  is bounded above by some positive number  $M$ , so  $a_n \leq Mb_n$  for all  $n$  sufficiently large. Since  $\sum b_n$  converges, so does  $\sum Mb_n$ , and by the Comparison Test,  $\sum a_n$  converges. ■

As I tell my own students, you only have two friends when it comes to infinite series:

- geometric series
- $p$ -series

We have already recalled our first friend, the geometric series. Our second friend, the  $p$ -series, derives its usefulness as a corollary of the Integral Test:

**Theorem 4 (Integral Test).** If  $f$  is positive and continuous on the interval  $1 \leq x < \infty$  and monotonic decreasing with  $\lim_{x \rightarrow \infty} f(x) = 0$ , then the series  $\sum_1^\infty f(n)$  and the improper integral  $\int_1^\infty f(x) dx$  are either both convergent or both divergent.

**Corollary 1 ( $p$ -series).** The series  $\sum_0^\infty 1/n^p$  converges when  $p > 1$  and diverges when  $p \leq 1$ .

We wish to use the Ratio Comparison Test using a  $p$ -series as the series known to be convergence. If we implement this approach utilizing the following lemma, we get Raabe's Test.

**Lemma 2.** If  $p > 1$  and  $0 < x < 1$ , then

$$1 - px \leq (1 - x)^p.$$

**Proof.** Set  $g(x) = px + (1 - x)^p$ . Then

$$g'(x) = p - p(1 - x)^{p-1} = p[1 - (1 - x)^{p-1}] \geq 0$$

for  $0 < x < 1$ , so  $g$  is increasing on this interval. Since  $g$  is continuous on  $[0, 1]$  and  $g(0) = 1$ , we have  $g(x) \geq 1$  for  $0 < x < 1$ . Substituting  $g(x)$  into this inequality, we have

$$1 - px \leq (1 - x)^p,$$

as desired. ■

**Theorem 5. (Raabe's Test)** Let  $\sum a_n$  be a series with positive terms and let  $p > 1$  and suppose  $\frac{a_{n+1}}{a_n} \leq 1 - \frac{p}{n}$  for all sufficiently large  $n$ . Then, the series  $\sum a_n$  converges.

**Proof.** Let  $\sum a_n$  be a series with positive terms and let  $p > 1$  and suppose  $\frac{a_{n+1}}{a_n} \leq 1 - \frac{p}{n}$  for all sufficiently large  $n$ . Setting  $x = 1/n$  in Lemma 2, we have

$$\frac{a_{n+1}}{a_n} \leq 1 - \frac{p}{n} \leq \left(1 - \frac{1}{n}\right)^p = \left(\frac{n-1}{n}\right)^p = \frac{b_{n+1}}{b_n},$$

where  $b_{n+1} = \frac{1}{n^p}$ . Since the series  $\sum b_n$  is a  $p$ -series with  $p > 1$ , the series  $\sum b_n$  converges. Applying the Ratio Comparison Test,  $\sum a_n$  also converges. ■

**Solution to our problem:** Consider the series

$$\frac{1}{4} + \frac{1 \cdot 3}{4 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{4 \cdot 6 \cdot 8} + \cdots + \frac{1 \cdot 3 \cdots (2n-1)}{4 \cdot 6 \cdots (2n+2)} + \cdots$$

If we apply Raabe's Theorem, we have

$$\frac{a_{n+1}}{a_n} = \frac{2n-1}{2n+2} = \frac{(2n+2)-3}{2n+2} = 1 - \frac{3/2}{n+1},$$



so  $p = \frac{3}{2}$  and the series

$$\frac{1}{4} + \frac{1 \cdot 3}{4 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{4 \cdot 6 \cdot 8} + \cdots + \frac{1 \cdot 3 \cdot \cdots \cdot (2n-1)}{4 \cdot 6 \cdot \cdots \cdot (2n+2)} + \cdots$$

converges.

Returning to an arbitrary generalized geometric series (with  $d = e > 0$ ), we apply Raabe's Test

$$\begin{aligned} \frac{a_{n+1}}{a_n} &= \frac{a + (n+1)d}{b + (n+1)d} \\ &= \frac{b + (n+1)d + (a-b)}{b + (n+1)d} \\ &= 1 - \frac{b-a}{b + (n+1)d} \\ &= 1 - \frac{(b-a)/d}{(b/d) + (n+1)}. \end{aligned}$$

Now,

$$\frac{b}{d} + n + 1 \leq n + K \text{ for some constant } K,$$

so

$$\begin{aligned} \frac{b}{d} + n + 1 &\leq n + K \\ \frac{1}{\frac{b}{d} + n + 1} &\geq \frac{1}{n + K} \\ \frac{(b-a)/d}{\frac{b}{d} + n + 1} &\geq \frac{(b-a)/d}{n + K} \\ 1 - \frac{(b-a)/d}{\frac{b}{d} + n + 1} &\leq 1 - \frac{(b-a)/d}{n + K}. \end{aligned}$$

Hence,

$$\begin{aligned} \frac{a_{n+1}}{a_n} &= 1 - \frac{(b-a)/d}{(b/d) + (n+1)} \\ &\leq 1 - \frac{(b-a)/d}{n + K}. \end{aligned}$$

This is sufficient to show that the generalized geometric series converges if  $(b-a)/d > 1$ , that is, if  $b > a + d$ . Notice that if  $d = e = 0$ , so that the generalized geometric series degenerates to an ordinary geometric series, we get the result we already know: The geometric series converges if  $b > a$ , that is, if the common ratio  $r$  is less than one.

We state our result as a

**Theorem.** Consider the generalized geometric series

$$\frac{a}{b} + \frac{a \cdot (a+d)}{b \cdot (b+e)} + \frac{a \cdot (a+d) \cdot (a+2d)}{b \cdot (b+e) \cdot (b+2e)} + \dots + \frac{a \cdot (a+d) \cdot \dots \cdot (a+nd)}{b \cdot (b+e) \cdot \dots \cdot (b+ne)} + \dots$$

converges absolutely if  $|d| < |e|$  and diverges if  $|d| > |e|$ . Further, if  $d = e > 0$ , this series converges if  $b > a + d$ .

### References

- [1] R. C. Buck, *Advanced Calculus*, Third Edition, McGraw-Hill, 1978.

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# *An Introduction to Genetic Algorithms*

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## **1. Purpose**

Genetic algorithms are a computer programming method used since 1975 to tackle problems that are impossible or prohibitively difficult to solve using conventional programming methods. The genetic algorithm models its behavior after evolution in nature, especially the processes of DNA crossover and mutation which are essential to sexual reproduction. Implementations of genetic algorithms have been used in many fields, including biology, mathematics, computational modeling, business, and every type of engineering.

This paper provides an introduction to genetic algorithms, with specific emphasis on an interdisciplinary, straightforward approach. The paper uses, as a simplified example, the Traveling Salesman problem. The purpose of the paper is to provide an overview of a specific genetic algorithm and the benefits of such an approach to problem-solving.

## **2. Anatomy of a Genetic Algorithm**

A genetic algorithm is a searching algorithm. It uses the concepts of natural selection and evolution to allow solutions to a particular problem to compete like a pack of lions on the African savannah. This leads to the most favorable solutions appearing more frequently over time. "Thus," Forrest [1, p. 872], explains, "genetic algorithms allow engineers to use a computer to evolve problem solutions over time, instead of designing them

by hand". Genetic algorithms are not simply a problem solving method; they can also be used as simple models for the more complex evolution found in the natural world.

Solutions are called individuals, which make up a population. Individuals are comprised of genes, which hold information regarding the solution. The information could be the number of cogs in a watch, the third variable in a function, or the location of a methyl group in an organic compound. The population is evolved over generations through selection of good individuals from the population, crossover between individuals, and random mutation to encourage diversity. Better individuals are said to have more fitness. The fitness of an individual is determined by the fitness function, which tries out each solution and scores them.

The process the population progresses through over the course of a trial can be best explained through pseudocode, which is a less-technical, programming language-agnostic version of the actual code. Here is what it looks like:

- Randomly create an initial population
- Until the program is finished (after a period of time or until there is a very good solution):
  - Determine each individual's fitness.
  - Select, based on fitness (the higher the better), two individuals and create one or two children, through crossover and place them in a new population.
  - Randomly mutate genes in a small percentage of the new population.
  - Replace the old population with the new population.
- The individual with the highest fitness in the final population is the solution to the problem.

### **3. Design Considerations and Data Representation**

Those few basic ideas are the building blocks of a genetic algorithm. However, there are many intricacies and difficulties involved in creating one which is successful. For example, based on the data in the genes, certain crossover and mutation functions will work, while others will not. The termination conditions will also need to be determined: will the program halt after a certain number of generations, or will it stop once the fitness

of the individuals stop increasing by a certain amount over time? These are but a few of the many variables programmers have to consider before writing their first lines of code.

The gene is the basic unit of any genetic algorithm, and is responsible for holding the information for the solutions the individuals contain. One of the most important decisions the programmer has to make involves the format of these genes. In the early genetic algorithms, all encoding was done in binary: sequences of ones and zeroes. A one could mean “on” or “true”, while a zero could mean “off” or “false”. Every gene might not mean something unique, but taken together, they would form a pattern or description of the solution. The fitness function would be designed to understand and process them.

Later, other methods of representation were developed, using more familiar data types in the common programming languages. These data types include integers, or whole numbers (e.g., 36); floating-point numbers, (e.g., 2.39074); and strings (e.g., “Miami”). Each has its own advantages and disadvantages. Integers can be manipulated easily and are easy to understand at face value, but require the use of arrays, which are like Excel spreadsheets. Arrays are more difficult to work with than a simple series of ones and zeroes, which means they require more code to work effectively. Floating-point numbers are perfect for many mathematical problems, and also lend themselves well to engineering applications, but they incur much of the same problems as integers. Strings are not used as frequently as any of the other data types, but there are problems which can only be solved using strings, so they are quite useful.

#### **4. Determining Fitness of Individuals**

Once the representation of the individual has been determined, the next hurdle is the fitness function. The programmer needs to decide what separates a “good” solution from a “bad” solution. Some problems are more complex than others, and since the individuals do not “know” what they are solving, they need to be coaxed and guided toward better fitness. For example, suppose there is an oil company which needs to find the ideal path to lay a pipeline through the Ural Mountains in Russia. There are millions of possible routes, so a programmer designs a genetic algorithm to find a way through. The company is looking for a route which does not require very much pipe, but which also does not require much blasting to clear away rock. The fitness function should reward individuals with short paths, while simultaneously rewarding individuals that use the curves and bends of the mountains to their advantage. Most of this process is trial and

error, tweaking the fitness function after several test runs, and gauging the results with a keen eye.

Another factor to keep in mind is the size of the population. If there are too few individuals, there will not be enough diversity in the “gene pool” and the answer they return may not be good enough. Yet, the more individuals there are, the longer the program will run. Also, for certain problems it appears there is a maximum effective number for population size beyond which the return on investment is poor. In other words, the extras sit around, without contributing their fair share. However, if the time to run is not an issue, the programmer will often set the population size to an arbitrarily large number and forget about it.

### **5. Operators: Crossover and Mutation**

Once the fitness can be calculated for the individuals, it is time to do something with that information. Operators such as crossover and mutation use the fitness to determine whether to perform their functions on each individual. Usually crossover acts first on the population. There are several different crossover functions, depending on the representation (binary, integer, etc.) of the genes, the problem being solved, and design considerations such as program size and speed. Generally, two individuals are picked from the population, then chopped in two parts and swapped, so the first part of individual A is attached to the second part of individual B, and vice versa. The two resulting children are then placed in the new population. Once the new population is full, the mutation function may pick random genes and replace them with a random value. This encourages changes in the population that may not occur otherwise, due to the inherent homogeneity of the population.

### **6. Real-world Application: TSP**

This paper has mentioned applications, implementations, and problems, but very vaguely. An ideal application is the Traveling Salesman Problem (TSP). The problem is actually very old, evolving out of several mathematical puzzles from the 1700’s. The TSP as we know it was created by the RAND Corporation as a combinatorial problem in 1948. The concept of the problem is quite simple: a traveling salesman must visit every city in an area exactly once and then return to his starting city. Because the cost to travel from any city to any other city is known, what is the route through the cities that yields the minimum cost?

Mathematically speaking, the search space, which is the list of all possible routes the salesman can take, is a set of permutations of  $n$  cities. The size of the search space is  $n!$ , or  $n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$ . To illustrate the daunting scale of the problem, while a TSP with 3 cities has a search space of  $3\cdot 2\cdot 1 = 6$  possible routes, a TSP with 11 cities has  $11\cdot 10\cdot 9\cdots 3\cdot 2\cdot 1 = 39,916,800$  possible routes! Due to the difficulty involved with calculating so many possible routes, the TSP is an NP-Hard problem, which means that it is almost impossible to solve in an exponentially long time. The TSP is also classified as an optimization problem, which Merino and Reyes [2, p. 21] define as a problem in which “one wishes to find an optimal solution from a space of solutions that minimizes (or maximizes) an objective function”.

How long would it take to solve? A fellow student and roommate, Ian Hixson, developed a brute-force program to solve the TSP. It took thirty minutes for the program to solve a 13-city TSP, and at the same speed, it would take seven times longer than the estimated lifespan of the universe to solve a 39-city TSP. This leads to the logical conclusion that a more elegant and faster alternative must be employed if one wishes to produce a solution to any TSP of average size. This is where genetic algorithms shine, because instead of calculating every possible route the salesman can take, they calculate random routes and let them compete and evolve until the best possible route stands victorious.

## 7. TSP Data Representation

Recalling the steps enumerated earlier, in order to design a genetic algorithm to solve the TSP, the programmer must first decide how to represent the data. The data can be represented in multiple ways, but an array of numbered cities, in the order in which they are visited by the traveling salesman, seems most intuitive:

1 4 2 6 3 0 5

In this route, the number of cities is 7, and the salesman started at city one, and canvassed each city (zero is a city like the others which is used because in computer science, lists are numbered starting with zero) until reaching city five, after which he returned to city one. The cost to travel can be stored in a special table with seven rows and seven columns, with the cost in each cell:

Table 1. Travel cost between cities

City	0	1	2	3	4	5	6
0	X	50	19	78	9	31	67
1	50	X	82	33	90	16	54
2	19	82	X	48	63	57	5
3	78	33	48	X	24	99	47
4	9	90	63	24	X	41	72
5	31	16	57	99	41	X	87
6	67	54	5	47	72	87	X

For example, to travel from city 2 to city 5, or vice versa, the cost would be 57. Note that the costs to travel from one city to the same city are marked 'X', because such travel costs nothing at all.

The TSP is an interesting case of genetic algorithm, in part because there are stipulations placed on whether an individual is legal or illegal. The de [3, p. 121] summarized the problem quite well:

"The bit string model is not very useful, as the cross-overs and mutations can easily produce a tour that is invalid – remember that every city must occur in the tour exactly once".

For example, an individual with city 5 in two separate places would be illegal, because the very definition of a TSP prohibits revisiting a city. Therefore, not only does that restrict the ways in which the data is represented, but in the ways operators act upon that data.

## 8. TSP Functions and Operators

With the representation of the data determined, the TSP can now be described in terms of a genetic algorithm. The population is composed of routes the salesman can take, each of which is stored within an individual. A gene in an individual denotes the city at which the salesman stops at that point in the route. In each generation, the population undergoes crossover and mutation, creating the children that will become the next generation, who hopefully have better fitness, as determined by the fitness function. The main parts of the program can now be generally determined. The initialization function is first, in which the initial population is randomly generated. Next in line is the fitness function, which will have to be carefully designed to optimize the individuals' fitness. Third is the crossover function, which is critical for advancing the population while encouraging fitness. The last function is the mutation function, which diversifies the population. A closer look at these functions will enable us to divine how



the algorithm functions and why it leads to accurate results.

The initialization function creates our initial population in a series of steps. First, it creates an  $n$  by  $m$  list, where  $n$  is the number of cities in the route, and  $m$  is the number of individuals in the population. This list holds the population. Next, the function selects the first individual and places a random number between 0 and  $n-1$  in the first gene. It then places another number in the next gene, making sure it does not place the same number as it placed in the first gene. It proceeds through the individual, checking the other genes each time, until each gene holds a unique number between 0 and  $n-1$ . It has now created a random route. The function moves onto the next individual, repeating the procedure until the entire population is full of randomly-generated individuals.

Next is the all-important fitness function, which will evaluate the fitness of the individuals in each generation, starting with the initial generation. Since each individual represents a route, they are essentially solutions to the TSP, which simply need to be summed. For example, from the table it can be determined that the cost to travel from city two to city three or vice versa is 48. The fitness function adds together all of the costs between the cities (genes) in the individual, to arrive at the total cost of travel. Since a lower cost is preferable, in the earliest GA code for this project the costs were simply inverted, turning a total cost of 10 into 0.1 and a total cost of 100 into 0.01. This was an easy way to assign lower costs a higher fitness, though it was later replaced with a more complex system.

The resultant fitnesses of these individuals are evaluated in the crossover function, giving individuals with higher fitness a greater chance to be selected for crossover. To perform crossover, the function performs a set of calculations and manipulations. First, it randomly generates a number in a range between the highest and lowest fitnesses in the population. Then, it compares the fitness of a randomly selected individual to that number. If the fitness of the individual is higher, it is selected for crossover and saved as Parent 1. It does the same two calculations again, and the next individual to fit the criteria is saved as Parent 2. These two parents undergo a process called Single-Point Order Crossover (OX). First, a random gene between 2 and  $n-1$  is selected. All genes in the Parent 1 up to that gene are copied into the child:

- Random gene =  $4, n = 7$
- Parent 1:     1 4 2 6 3 0 5
- Child:         1 4 2 6 - - -

Next, all of the selected cities in Parent 1 are removed from Parent 2:

- Parent 2 (before):     5 3 6 2 0 1 4
- Parent 2 (after):     5 3 - - 0 - -

Finally, the remaining cities in Parent 2 are copied in order into the child, yielding a valid route:

- Child:             1 4 2 6 5 3 0

The child is then copied into the new population. This process repeats until the new population is full, at which time the mutation operator takes control.

The mutation function mutates a certain percent of the individuals, which is determined by a variable in the program called the mutation rate. The function starts at the first individual, and generates a decimal number between 0 and 1. If the number is less than or equal to the mutation rate, the individual will undergo mutation. If it is not, then the mutation operator moves on. Mutation in the TSP is tricky, because the function cannot simply change a gene to any random value, because the resulting route may be illegal. Therefore, it chooses two genes at random and swaps their contents. In this way, it is assured that no cities have been skipped or visited more than once, because the only thing that has changed is the order in which the cities have been visited. Diversification has been achieved within the rules of the TSP.

These functions are repeated in the order Fitness, Crossover, and Mutation until the pre-determined generation limit is reached. The final generation should contain individuals with exceptional fitness, especially if the programmer has developed a good algorithm. In many cases, genetic algorithms have achieved human-competitive results even with a very high  $n$ . Usually, however, the genetic algorithms involved are much more sophisticated and complex than the one presented in this paper. Yet, deep within their complexity lies the simple functions described here. How human-competitive are the results of these algorithms? The algorithm written for this research found an optimized solution to a 39-city problem in a little over a minute, as opposed to the brute-force technique's multi-eon running time.

Yet, the TSP is quite a narrow, specialized problem. There is not much demand for traveling salesmen in today's world. Why does this problem continue to concern mathematicians, computer programmers and engineers? It is because the TSP can be applied to many diverse applications. For example, there are millions of tiny transistors in a modern computer processor. Each transistor must be connected, while minimizing the distance between all of them. The number of possible physical arrangements of these transistors is staggering, but not to a genetic algorithm. It is not too hard to imagine the distance a signal has to travel through the transistors as the total cost, and with a few modifications in the code, a program can be created that outputs the optimal arrangement of the processor, ready for fabrication and sale. There are a myriad of such applications for TSP genetic algorithms in use today.

## 9. Conclusion

Genetic algorithms have many benefits: they are easy to reuse due to the similarities in the general design and they are able to solve problems that would otherwise be prohibitively difficult to solve. Plus, the solutions they develop are not constrained by the ways in which the human mind processes and evaluates information: they can create totally new solutions to problems because they have an innate randomness of which the human mind is not capable.

## References

- [1] S. Forrest, Genetic algorithms: Principles of adaptation applied to computation, *Science*, **261** (1993), 872-878.
- [2] D. Merino and E. Reyes, Genetic Algorithms: A Tutorial, *Journal of Computing Sciences in Colleges*, **13**(4) (1998), 21-26.
- [3] S. Thede, An Introduction to Genetic Algorithms, *Journal of Computing Sciences in Colleges*, **20**(1) (2004), 115-123.

## The Problem Corner

Edited by Pat Costello

*The Problem Corner* invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before August 1, 2008. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall, 2008 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)-622-3051)

### NEW PROBLEMS 616-623

**Problem 616.** *Proposed by Melissa Erdmann, Nebraska Wesleyan University, Lincoln, NE.*

The birthday paradox is that in a room with 23 people, the probability that two or more of them will have the same birthday (month and day) is at least 50%. Find the number of people needed so that there is a 50% probability that at least three or more of them will have the same birthday. Find the formula that will represent the probability that at least  $k$  of  $n$  total people in a room share a birthday.

**Problem 617.** *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Find all triplets  $(x, y, z)$  of real numbers such that

$$\sqrt{3^x(4y+5z)} + \sqrt{4y(3^x+5z)} + \sqrt{5z(3^x+4y)} = \sqrt{2}(3^x+4y+5z).$$

**Problem 618.** *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let  $a, b, c$  be real numbers such that  $0 < a \leq b \leq c < \frac{\pi}{2}$ . Prove that

$$\frac{\sin a + \sin b + \sin c}{\cos a (\tan b + \tan c) + \cos b (\tan c + \tan a) + \cos c (\tan a + \tan b)} \leq \frac{1}{2}.$$

**Problem 619.** *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, IL.*

Several identical square napkins are placed on a table. They are placed in such a way that any two of them have a common area which is greater than half of the area of one of them. Is it always possible to pierce all the napkins with a needle going perpendicular to the plane of the table? If yes, prove it. If not, provide a counterexample.

**Problem 620.** *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, IL.*

Nick chooses 81 consecutive integers, rearranges them, and concatenates them to form one long, multi-digit number  $N$ . Michael chooses 80 consecutive integers, rearranges them, and concatenates them to obtain the number  $M$ . Is it possible that  $M = N$ ? If yes, provide an example. If not, prove it.

**Problem 621.** *Proposed by Lisa Hernandez, Jim Buchholz, Doug Martin (jointly), California Baptist University, Riverside, CA.*

A standard technique for showing that  $.999\dots = 1$  is to let  $x = .999\dots$  and then  $10x = 9.999\dots$  and so  $9x = 10x - x = 9$ , which gives  $x = 1$ . Can you derive an alternative proof that  $.999\dots = 1$ , perhaps using proof by contradiction?

**Problem 622.** *Proposed by the editor.*

Prove that there are infinitely many positive, palindromic integers containing just the digits 2, 7, and 9, which are divisible by 2, 7, and 9.

**Problem 623.** *Proposed by the editor.*

Let  $f(n)$  be the number of rationals between 0 and 1 (noninclusive) that have denominator less than or equal to  $n$ . Prove that  $f(n) > \frac{2}{3}n^{3/2}$  for all integers  $n \geq 2$ .

## SOLUTIONS 600, 602, 604-610

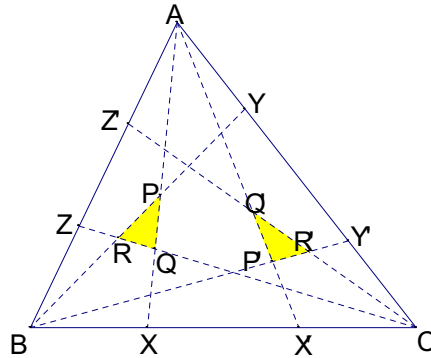
**Problem 600.** Proposed by Stanley Rabinowitz, MathPro Press, Chelmsford, MA.

In  $\triangle ABC$ , let  $X$ ,  $Y$ , and  $Z$  be points on sides  $BC$ ,  $CA$ , and  $AB$ , respectively. Let

$$x = \frac{BX}{XC}, \quad y = \frac{CY}{YA}, \quad \text{and} \quad z = \frac{AZ}{ZB}.$$

The lines  $AX$ ,  $BY$ ,  $CZ$  bound a central triangle  $PQR$ . Let  $X'$ ,  $Y'$ , and  $Z'$  be the reflections of  $X$ ,  $Y$ , and  $Z$ , respectively, about the midpoints of the sides of the triangle upon which they reside. These give rise to a central triangle  $P'Q'R'$ . Prove that the area of  $\triangle PQR$  is equal to the area of  $\triangle P'Q'R'$  if and only if either

$$x = y \text{ or } y = z \text{ or } z = x.$$



**Solution** by the proposer.

Let  $[ABC]$  denote the area of triangle  $ABC$ . By Routh's Theorem (<http://mathworld.wolfram.com/RouthsTheorem.html>),

$$\frac{[PQR]}{[ABC]} = \frac{(xyz - 1)^2}{(xy + x + 1)(yz + y + 1)(zx + z + 1)}.$$

Since  $\frac{BX'}{X'C} = \frac{1}{x}$ ,  $\frac{CY'}{Y'A} = \frac{1}{y}$ , and  $\frac{AZ'}{Z'B} = \frac{1}{z}$ , we have (after simplifying)

$$\frac{[P'Q'R']}{[ABC]} = \frac{(xyz - 1)^2}{(xy + y + 1)(yz + z + 1)(zx + x + 1)}.$$

Thus  $[P'Q'R'] = [PQR]$  if and only if

$$\begin{aligned} & (xy + y + 1)(yz + z + 1)(zx + x + 1) \\ &= (xy + x + 1)(yz + y + 1)(zx + z + 1). \end{aligned}$$

A little algebra shows that this condition is equivalent to

$$(x - y)(y - z)(z - x) = 0.$$

Thus we see that  $[P'Q'R'] = [PQR]$  if and only if  $x = y$  or  $y = z$  or  $x = z$ .

**Problem 602.** *Proposed by the editor.*

Consider the sequence of polynomials recursively defined by

$$\begin{aligned} p_1(x) &= (x - 2)^2 \\ p_2(x) &= [p_1(x) - 2]^2 \\ &\vdots \\ p_n(x) &= [p_{n-1}(x) - 2]^2 \\ &= x^m + a_{m-1}x^{m-1} + a_{m-2}x^{m-2} + \cdots + a_2x^2 + a_1x + 4, \end{aligned}$$

where  $m = 2^n$ . Find closed formulas for the coefficients  $a_{m-1}$ ,  $a_{m-2}$ ,  $a_2$ ,  $a_1$ .

**Solution** by Harrison Potter, student, Marietta College, Marietta, OH.

It is easier if we rewrite the general polynomial as

$$p_n(x) = x^m + A_n x^{m-1} + B_n x^{m-2} + \cdots + b_n x^2 + a_n x + 4,$$

where  $m = 2^n$ . Looking at the way in which each polynomial was found from its predecessor, we can deduce the following recursion relations for the specified coefficients. For  $n \geq 2$ , we have  $A_n = 2A_{n-1}$ , with  $A_1 = -4$ , and  $a_n = 4a_{n-1}$ , with  $a_1 = -4$ . Likewise, for  $n \geq 3$ , we have  $B_n = 2B_{n-1} + A_{n-1}^2$ , with  $B_2 = 20$ , and  $b_n = 4b_{n-1} + a_{n-1}^2$ , with  $b_2 = 20$ . We proceed by finding  $A_n$  and  $a_n$  first. We have

$$A_n = 2A_{n-1} = 2(2A_{n-2}) = 2(2(2A_{n-3})) = \cdots = 2^{n-1}A_1.$$

Since  $A_1 = -4$ , we see that

$$A_n = -4 \cdot 2^{n-1} = -2^{n+1}.$$

Likewise,

$$a_n = 4^{n-1}a_1.$$

Since  $a_1 = -4$ , we see that

$$a_n = -4 \cdot 4^{n-1} = -4^n.$$

Now we can find  $B_n$  and  $b_n$ . We have

$$\begin{aligned}
 B_n &= 2B_{n-1} + A_{n-1}^2 = 2(2B_{n-2} + A_{n-2}^2) + A_{n-1}^2 \\
 &= 2(2(2B_{n-3} + A_{n-3}^2) + A_{n-2}^2) + A_{n-1}^2 = \cdots \\
 &= A_{n-1}^2 + 2A_{n-2}^2 + 4A_{n-3}^2 + \cdots + 2^{n-3}A_2^2 + 2^{n-2}B_2 \\
 &= 20 \cdot 2^{n-2} + \sum_{k=2}^{n-1} A_k^2 \cdot 2^{n-1-k} \\
 &= 20 \cdot 2^{n-2} + \sum_{k=2}^{n-1} 16 \cdot 2^{2k-2+n-1-k} \\
 &= 20 \cdot 2^{n-2} + 16 \cdot 2^{n-1} \cdot \sum_{k=2}^{n-1} 2^{k-2} \\
 &= 20 \cdot 2^{n-2} + 16 \cdot 2^{n-1} \cdot \sum_{k=0}^{n-3} 2^k \\
 &= 20 \cdot 2^{n-2} + 16 \cdot 2^{n-1} \cdot (2^{n-2} - 1) \\
 &= 2^{2n+1} - 3 \cdot 2^n.
 \end{aligned}$$

Finally,

$$\begin{aligned}
 b_n &= 4b_{n-1} + a_{n-1}^2 \\
 &= 4(4b_{n-2} + a_{n-2}^2) + a_{n-1}^2 = \cdots \\
 &= a_{n-1}^2 + 4a_{n-2}^2 + 16a_{n-3}^2 + \cdots + 4^{n-3}a_2^2 + 4^{n-2}b_2 \\
 &= 20 \cdot 4^{n-2} + \sum_{k=2}^{n-1} a_k^2 \cdot 4^{n-1-k} \\
 &= 20 \cdot 4^{n-2} + 16 \cdot 4^{n-1} \cdot \sum_{k=0}^{n-3} 4^k \\
 &= 20 \cdot 4^{n-2} + 16 \cdot 4^{n-1} \cdot \frac{4^{n-2} - 1}{3} \\
 &= \frac{4^{2n-1} - 4^{n-1}}{3}.
 \end{aligned}$$

*Also solved by the Missouri State Problem Solving Group, Springfield, MO, and the proposer.*

A solution by Russ Euler, Northwest Missouri State University, Maryville, MO to Problem 603 was submitted just after the Spring 2007 issue was typed.



**Problem 604.** Proposed by Ovidiu Furdui, Western Michigan University, Kalamazoo, MI.

Find the sum  $\sum_{n=1}^{\infty} (-1)^{n-1} \left( 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n - \gamma \right)$ , where  $\gamma$  is the Euler-Mascheroni constant.

**Solution** by the proposer.

The sum is equal to  $-\frac{\gamma}{2} + \frac{\ln \pi}{2}$ . We will use Wallis' Product Formula:

$$\begin{aligned} & \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdots (2n)(2n)}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdots (2n-1)(2n+1)} \\ &= \left( \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{3 \cdot 5 \cdot 7 \cdots (2n-1)} \right)^2 \cdot \frac{1}{2n+1} \rightarrow \frac{\pi}{2}. \end{aligned}$$

Let

$$\begin{aligned} S_{2n} &= \sum_{k=1}^{2n} (-1)^{k-1} \left( 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k} - \ln k - \gamma \right) \\ &= -\frac{1}{2} - \frac{1}{4} - \cdots - \frac{1}{2n} + \ln \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{1 \cdot 3 \cdot 5 \cdots (2n-1)} \\ &= -\frac{1}{2} \left( 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln n \right) - \frac{\ln n}{2} + \ln \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{1 \cdot 3 \cdot 5 \cdots (2n-1)} \\ &= -\frac{1}{2} \left( 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln n \right) + \ln \left[ \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{1 \cdot 3 \cdot 5 \cdots (2n-1)} \cdot \frac{1}{\sqrt{n}} \right] \\ &= -\frac{\gamma}{2} + \frac{\ln \pi}{2} \text{ by Wallis.} \end{aligned}$$

On the other hand, since

$$S_{2n+1} = S_{2n} + (-1)^{2n} \left( 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2n+1} - \ln(2n+1) - \gamma \right),$$

we get that

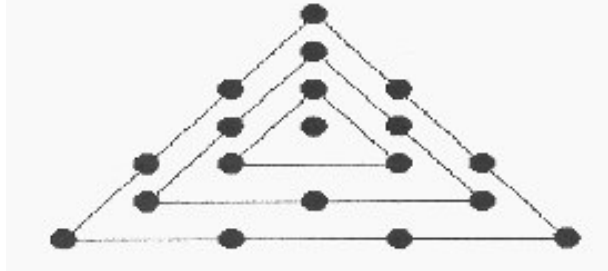
$$S_{2n+1} \rightarrow -\frac{\gamma}{2} + \frac{\ln \pi}{2},$$

which implies the desired result.

**Problem 605.** Proposed by Cathy George (student) and Russell Euler, Northwest Missouri state University, Maryville, MO.

Geometrically, *centered triangular numbers* consist of a central dot with three dots around it and then additional dots in the gaps between adjacent dots (see figure). The first four centered triangular numbers are 1, 4,

10, 19. Prove that a positive integer  $m \geq 5$  is a centered triangular number if and only if  $m$  is the sum of three consecutive triangular numbers.



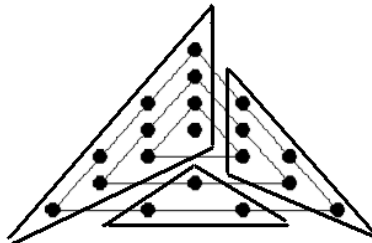
**Solution** by the proposers.

The  $n^{\text{th}}$  triangular number is given by  $t_n = \frac{n(n+1)}{2}$ , and the  $n^{\text{th}}$  centered triangular number is given by  $T_n = \frac{3n^2 - 3n + 2}{2}$  for  $n \geq 1$ . If  $m$  is a centered triangular number, then for some  $n$ ,

$$\begin{aligned} m &= T_n \\ &= \frac{(n^2 + n) + (n^2 - n) + (n^2 - 3n + 2)}{2} \\ &= \frac{n(n+1)}{2} + \frac{(n-1)n}{2} + \frac{(n-2)(n-1)}{2} \\ &= t_n + t_{n-1} + t_{n-2}. \end{aligned}$$

Also solved by Donald Poynter, Eastern Kentucky University, Richmond, KY and Diana Linville, Eastern Kentucky University, Richmond, KY.

*Editor's note:* You can see the three consecutive triangular numbers pictorially in the following figure.



**Problem 606.** *Proposed by Mathew Cropper and Bangteng Xu (jointly), Eastern Kentucky University, Richmond, KY.*

Show that for any nonnegative integer  $n$ ,

$$a_n = \frac{2}{7}(-1)^n + \frac{1}{2(4+\sqrt{2})} \left(\frac{2}{2+\sqrt{2}}\right)^{n+1} + \frac{1+2\sqrt{2}}{14\sqrt{2}} \left(\frac{2}{2-\sqrt{2}}\right)^{n+1}$$

is an integer, and  $a_n$  is even when  $n \geq 2$ .

**Solution** by the proposers.

It is true for  $n = 0$  because  $a_0 = 1$ . It is also true for  $n = 1$  since  $a_1 = 2$ . Assume that  $n > 1$  and that  $a_0, a_1, \dots, a_{n-1}$  are integers. Now we show that  $a_n$  is an integer. Let  $s_{n-1} = a_0 + a_1 + \dots + a_{n-1}$ . By assumption,  $s_{n-1}$  must be an integer, and

$$\begin{aligned} s_{n-1} &= \frac{2}{7} \sum_{k=0}^{n-1} (-1)^k + \frac{1}{2(4+\sqrt{2})} \sum_{k=1}^n \left(\frac{2}{2+\sqrt{2}}\right)^k \\ &\quad + \frac{1+2\sqrt{2}}{14\sqrt{2}} \sum_{k=1}^n \left(\frac{2}{2-\sqrt{2}}\right)^k \\ &= \frac{2}{7} \cdot \frac{1 - (-1)^n}{2} \\ &\quad + \frac{1}{2(4+\sqrt{2})} \left[ \frac{2/(2+\sqrt{2}) - (2/(2+\sqrt{2}))^{n+1}}{1 - 2/(2+\sqrt{2})} \right] \\ &\quad + \frac{1+2\sqrt{2}}{14\sqrt{2}} \left[ \frac{2/(2-\sqrt{2}) - (2/(2-\sqrt{2}))^{n+1}}{1 - 2/(2-\sqrt{2})} \right] \\ &= \frac{2}{7} \cdot \frac{1 - (-1)^n}{2} + \frac{\sqrt{2}}{2(4+\sqrt{2})} \left[ 1 - \left(\frac{2}{2+\sqrt{2}}\right)^n \right] \\ &\quad + \frac{1+2\sqrt{2}}{14\sqrt{2}} (-\sqrt{2}) \left[ 1 - \left(\frac{2}{2-\sqrt{2}}\right)^n \right]. \end{aligned}$$

Thus,

$$\begin{aligned}
& s_{n-1} + a_{n-2} \\
= & \frac{2}{7} \left[ \frac{1 - (-1)^n}{2} + (-1)^{n-2} \right] \\
& + \frac{1}{2(4 + \sqrt{2})} \left[ \sqrt{2} - \sqrt{2} \left( \frac{2}{2 + \sqrt{2}} \right)^n + \left( \frac{2}{2 + \sqrt{2}} \right)^{n-1} \right] \\
& - \frac{1 + 2\sqrt{2}}{14\sqrt{2}} \left[ \sqrt{2} - \sqrt{2} \left( \frac{2}{2 - \sqrt{2}} \right)^n - \left( \frac{2}{2 - \sqrt{2}} \right)^{n-1} \right] \\
= & \frac{2}{7} \cdot \frac{1 + (-1)^n}{2} + \frac{1}{2(4 + \sqrt{2})} \left[ \sqrt{2} + \frac{1}{2} \left( \frac{2}{2 + \sqrt{2}} \right)^{n+1} \right] \\
& + \frac{1 + 2\sqrt{2}}{14\sqrt{2}} \left[ -\sqrt{2} + \frac{1}{2} \left( \frac{2}{2 - \sqrt{2}} \right)^{n+1} \right] \\
= & \frac{2}{7} \cdot \frac{(-1)^n}{2} + \frac{1}{2} \cdot \frac{1}{2(4 + \sqrt{2})} \left( \frac{2}{2 + \sqrt{2}} \right)^{n+1} \\
& + \frac{1}{2} \cdot \frac{1 + 2\sqrt{2}}{14\sqrt{2}} \left( \frac{2}{2 - \sqrt{2}} \right)^{n+1} \\
= & \frac{1}{2} a_n.
\end{aligned}$$

Thus,

$$a_n = 2(s_{n-1} + a_{n-2})$$

is an integer, and even when  $n \geq 2$ .

[The formula for  $a_n$  actually comes from solving the recurrence  $a_n = 2 \left( \sum_{i=0}^{n-1} a_i + a_{n-2} \right)$ .]

**Problem 607.** *Proposed by Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, MO.*

A point  $P$  is moving on a quarter circle with center  $O$ , and  $P$  is bounded by two points  $A$  and  $B$ . Let  $PQ$  be the perpendicular from  $P$  to the radius  $OA$ . Choose a point  $M$  on the ray  $OP$  such that

$$\text{length of } OM = \text{length of } OQ + \text{length of } QP.$$

Find the locus of the points  $M$  as  $P$  moves on the quarter circle.

**Solution** by the proposers.

Let  $AN$  be the perpendicular from  $A$  to  $OP$ . The two triangles  $OPQ$  and  $OAN$  are congruent. It follows that  $AN = PQ$  and  $OQ = ON$ . Since  $ON + MN = OQ + QP$ , we have  $AN = MN$ . This implies that the angle  $OMA$  is  $45^\circ$ . Since the points  $A$  and  $O$  are fixed and the angle  $OMA$  is constant, the locus of  $M$  is the arc  $AMB$  of the circle circumscribed about the triangle  $OAM$ .

**Problem 608.** Proposed by Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, MO.

Let  $C$  be a circle with center  $O$  and radius  $R$ . Let  $ABCD$  be a parallelogram circumscribed about  $C$ . Express

$$\frac{1}{(AC)^2} + \frac{1}{(BD)^2}$$

in terms of  $R$ .

**Solution** by the proposers.

It can be easily shown that  $ABCD$  is actually a rhombus with side length  $a$ . We have, by the law of diagonals in a parallelogram,  $(AC)^2 + (BD)^2 = 4a^2$ . This implies that

$$\frac{1}{(AC)^2} + \frac{1}{(BD)^2} = \frac{4a^2}{(AC)^2(BD)^2}.$$

In the right triangle  $AOD$ , the height,  $OH$  say, satisfies the relation

$$OH \cdot AD = \frac{BD}{2} \cdot \frac{AC}{2}.$$

By substitution of  $OH = R$ , this gives  $4Ra = BD \cdot AC$ . Substituting this into the above equation, we get

$$\frac{1}{(AC)^2} + \frac{1}{(BD)^2} = \frac{4a^2}{16R^2a^2} = \frac{1}{4R^2},$$

which is the desired expression.

**Problem 609.** Proposed by Mathew Cropper, Eastern Kentucky University, Richmond, KY.

Let the  $n$  vertices of a given graph  $G$  be labeled  $v_1, v_2, \dots, v_n$ . Form a new graph  $M(G)$  from  $G$  in the following way:

1. Add to  $G$  an additional  $n + 1$  vertices,  $u_1, u_2, \dots, u_n, w$ .
2. Connect each vertex  $u_i$  by a new edge to every vertex  $v_j$  where there

is an edge from  $v_i$  to  $v_j$  in  $G$ .

3. Connect each vertex  $u_i$  by an edge to vertex  $w$ .

Starting with a graph which is just two vertices and one edge between the two vertices, find a formula for the number of edges in the iterated graph  $M^{(n)}(G) = M(M(M \cdots (M(G))))$ , where the graph formation is iterated  $n$  times.

**Solution** by the proposer.

The new graph constructed from  $G$  is called the Mycielski graph of  $G$ ,  $M(G)$ . The iterated graphs form examples of triangle-free graphs with large chromatic numbers. By the construction, if  $G$  has  $n$  vertices and  $m$  edges, then  $M(G)$  has  $2n + 1$  vertices and  $3m + n$  edges. Let  $V_n$  be the number of vertices in the iterated graph  $M^{(n)}(G)$  and  $E_n$  the number of edges in  $M^{(n)}(G)$ . Then the recursions satisfied by these sequences are

$$V_n = 2V_{n-1} + 1 \text{ and } E_n = 3E_{n-1} + V_{n-1}.$$

Solving the first recursion, we have  $V_n = 2^n V_0 + (2^n - 1)$ . Since  $V_0 = 2$  in the initial graph  $G$ , we get  $V_n = 2^n \cdot 2 + (2^n - 1) = 3 \cdot 2^n - 1$ . Working on the second recursion,

$$E_n = 3E_{n-1} + V_{n-1} = 3E_{n-1} + 3 \cdot 2^{n-1} - 1. (*)$$

Suppose the sequence forms the coefficients of a power series

$$E(x) = E_0 + E_1x + E_2x^2 + E_3x^3 + \cdots,$$

where  $E_0 = 1$  and  $E_n = 3E_{n-1} + 3 \cdot 2^{n-1} - 1$ . We have

$$\begin{aligned} 3xE(x) &= 3E_0x + 3E_1x^2 + 3E_2x^3 + 3E_3x^4 + \cdots \\ \frac{3x}{1-2x} &= 3x + 3 \cdot 2x^2 + 3 \cdot 2^2x^3 + 3 \cdot 2^3x^4 + \cdots \\ \frac{-1}{1-x} &= -1 - x - x^2 - x^3 - x^4 - \cdots \end{aligned}$$

Adding 2 to the sum of the three power series gives the series for  $E(x)$ . Hence

$$E(x) = 3xE(x) + \frac{3x}{1-2x} + \frac{-1}{1-x} + 2,$$

which implies that

$$E(x)(1-3x) = \frac{3x}{1-2x} + \frac{-1}{1-x} + 2.$$

Dividing by  $(1-3x)$  gives

$$E(x) = \frac{3x}{(1-2x)(1-3x)} + \frac{-1}{(1-x)(1-3x)} + \frac{2}{1-3x}.$$

Using the technique of partial fraction decomposition, we get

$$\begin{aligned} E(x) &= \frac{7/2}{1-3x} + \frac{-3}{1-2x} + \frac{1/2}{1-x} \\ &= \sum_{n=0}^{\infty} \left( \frac{7}{2} \cdot 3^n - 3 \cdot 2^n + \frac{1}{2} \right) x^n. \end{aligned}$$

Thus,

$$E_n = \frac{7}{2} \cdot 3^n - 3 \cdot 2^n + \frac{1}{2}.$$

**Problem 610.** *Proposed by the editor.*

Let  $f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + 4$ , where all coefficients  $a_i$  are positive reals. If  $f(x)$  has  $n$  real roots, prove that  $f(1) \geq 2^{n+1}$ .

**Solution** *by the proposer.*

Since  $a_i > 0$ , the real roots must all be negative, say  $-r_i$ . Then

$$f(x) = (x + r_1)(x + r_2) \cdots (x + r_n),$$

with

$$r_1 r_2 \cdots r_n = 4.$$

By the Arithmetic-Geometric Mean Inequality,

$$\frac{1 + r_i}{2} \geq \sqrt{1 \cdot r_i},$$

which implies that

$$1 + r_i \geq 2\sqrt{r_i}.$$

Thus

$$\begin{aligned} f(1) &= (1 + r_1)(1 + r_2) \cdots (1 + r_n) \\ &\geq 2\sqrt{r_1} \cdot 2\sqrt{r_2} \cdots 2\sqrt{r_n} \\ &= 2^n \sqrt{r_1 r_2 \cdots r_n} \\ &= 2^n \sqrt{4} = 2^{n+1}. \end{aligned}$$

## ***Kappa Mu Epsilon News***

Edited by Connie Schrock, Historian

**Updated information as of August 2007**

Send news of chapter activities and other noteworthy KME events to

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### Installation Report

OK Epsilon  
Oklahoma Christian University, Edmond, Oklahoma

The Oklahoma Epsilon Chapter of Kappa Mu Epsilon, Mathematics Honor Society Installation Ceremony was held in room 203 of the Prince Engineering Building on the campus of Oklahoma Christian University in Edmond, Oklahoma, on Friday, April 27, 2007 at 6 p.m.

KME National President Dr. Don Tosh was the installing officer. Dr. Robert Mitchell, Associate Dean of the College of Engineering, was the conductor. Dr. Tosh began with a brief history of KME and discussed its goals and purposes. During the ceremony he presented the chapter charter and the chapter crest, and pointed out the various symbols and their meanings.

Harrison Barbarick was introduced as the Vice President and instructed his fellow initiates on the purposes of KME. The initiates, after having heard these purposes, were asked to reaffirm their wish to join in the pursuit of these ideals, and to agree to participate actively in KME and abide by the society's regulations. All agreed, and, one-by-one, each initiate's name was read and each signed the Oklahoma Epsilon Chapter Roll. Each initiate was presented with a membership certificate and pin. All joined in applause in welcoming the new members.



The charter members of Oklahoma Epsilon are: Harrison Barbarick, Adam Barnes, Marianne Bentley Strande, Kenneth Bowling, Jennifer Bryan, Cortney Butler, Jason Cain, Johnnie Frye, Ray Hamlett, Jacob Higginbotham, Travis Hughes, Craig Johnson, Sean Lacey, Don Leftwich, William Martin, Samantha Marshall, Monica McDalton, Michael Mceuen, Leighton Minor, Amber Newell, Stephanie Rummel, Joel Scarsella, Tara Triplett, Arlette Umuhoza, and Megan Willson.

Next, the officers of Oklahoma Epsilon were installed. Each officer was charged with the responsibilities of his or her new office, and each chose to accept those responsibilities. Sean Lacey was installed as President. Harrison Barbarick was installed as Vice President. Marianne Bentley was installed as Secretary. Amber Newell was installed as Treasurer. Dr. Ray Hamlett was installed as the Corresponding Secretary, and Dr. Ray Hamlett and Dr. Craig Johnson were installed as Faculty Co-Sponsors.

Following the ceremony everyone joined in for a time of fellowship and refreshments.

## Chapter News

### **AL Alpha – Athens State University**

*Chapter President– Jenna O’Neal*

*Other spring 2007 officers: Curt Merchant, Vice-President; Susan Webb, Secretary; Dottie Gasbarro, Corresponding Secretary.*

Alabama Alpha Chapter of Kappa Mu Epsilon held its annual initiation of new members on April 4, 2007 in the Ballroom of the Sandridge Student Center on the historic Athens State campus in Athens, Alabama. Mariel Gray, President; Allison Stanford, Vice-President; Nick Retherford, Secretary; and Meaghan Mitchell, Treasurer conducted the initiation ceremonies. Dottie Gasbarro, advisor and Mathematics professor, assisted in the ceremony. Dr. Ron Fritze, Dean of the College of Arts and Sciences, welcomed members, initiates, faculty, students, and other guests to the gathering, and Dr. Neal Fentress, Mathematics Department chair, spoke about the importance of KME and its legacy on the Athens State campus. About 50 guests attended. A reception was held immediately after the ceremony for initiates, members, and guests and many browsed through the chapter scrapbook and history.

New officers were selected for the 2007-2008 academic year and inducted one week later in a small ceremony for mathematics students and faculty. Jenna O’Neal was inducted as President, Curt Merchant as Vice-President, and Susan Webb as Secretary. A Treasurer will be named at a later date.

No activities are planned for the summer semester, but we look forward to our service work at the 2007 Old Time Fiddler's Convention the first weekend in October. KME works with the Math and Computer Science Club to sell food and drink to raise money for scholarships and travel stipends for Mathematics students at the convention which attracts thousands of competitors and visitors from all over the United States.

New Initiates – Christy Lela Burleson, Carol M. Dearman, Kenneth Ray Downs, Jr., Suzanne Drake Giles, Cheryl Kelley, Curtis Merchant, Jenna N. O'Neal, Peggy Nuss, Michelle Russell, Tonya Taber, Susan Webb.

**AL Eta – University of West Alabama**

*Spring 2007 officer: Hazel Truelove, Corresponding Secretary.*

New Initiates – Cody Brown, Theresa A. Clark, Daniel Crowe, Deivid Delgado, Devin Fields, Derrick Hester, DeVeeta Hines, Mr. Mose Hunt, Dr. Frank Rogers.

**AL Gamma – Montevallo University**

*Spring 2007 officer: John Herron, Corresponding Secretary.*

New Initiates – Lean Jennings, Stefanie Morlan, Claire K. McCall, Bethany Gellings, Bandy Howard, Mellissa Cooke, Alyse Young, Krista Rondenberry, Justin Cannady, Kimberly Auer.

**AL Zeta – Birmingham Southern College**

*Chapter President – Kelly Bragan, Current Members, 4 New Members*

*Other spring 2007 officers: Jill Stupiansky, Vice-President; David Ray, Secretary; David Ray, Treasurer; Mary Jane Turner, Corresponding Secretary.*

Senior Project Research Presentation by Kelly Bragan and Jill Stupiansky. Topic; Rollercoaster's

New Initiates – Bryant Kendall Allen, Kelsey Harris Jordan, Manuel DeJesus Martinez, Jason Abbott Sabio.

**CA Delta – Cal Poly Pomona**

*Spring 2007 officer: Patricia Hale, Corresponding Secretary.*

New Initiates – Kendra Parsons, Tony Nguyen, Eddy Arbouet, Veselka Danova, Lisa Morales, Kristen Mendel, Long J. Yu, Edward Reyes, Christopher Phillips, Matthew Krupa, Maria Marisol Perez, Ben Miller, Michal Green.

**CO Delta – Mesa State College**

*Chapter President – Joshua Garland, 198 Current Members, 9 New Members*

*Other spring 2007 officers: Steven Hartman, Vice-President; Christopher Aquinto, Secretary; Heather Boette, Treasurer; Erik Packard, Corresponding Secretary.*

We met in May and initiated 9 new members.

**CT Beta – Eastern Connecticut State University**

*Chapter President – Current Members, 15 New Members*

*Other spring 2007 officers: Mizan R. Khan, Treasurer; Christian L. Yankov, Corresponding Secretary.*

New Initiates – Taylor L. Ackert, Melissa L. Cassidy, Chad H. Estabrooks, Cassie M. Filteau, Carrie E. Fleischer, Michael R. Jacques, Rebecca L. Jordan, William R. Kopplin, Daniel J. Landeck, Matthew A. Loehr, Stephanie L. Morris, Albert M. Navetta, Erin L. Stoecker, John H. Taylor, Cory R. Thompson.

**FL Beta – Florida Southern College**

*Chapter President – Melissa Nolet, 338 Current Members, 6 New Members*

*Other spring 2007 officers: Jackie Boreborg, Vice-President; Allen Wuertz, Corresponding Secretary.*

New Initiates – Patrick M. Cummings, Alan M. George, Shawn C. Hedman, Travis C. Hobgood, Alyssa R. Huebner, David A. Rose.

**GA Beta – Georgia College and State University**

*Spring 2007 officer: Dr. Jason Huffman, Corresponding Secretary.*

New Initiates – Matthew W. Baxter, Ryan Brown, Kayla M. Cook, Jeffrey M. Creasy, Ryan R. Davis, Michael B. Ferra, Desale H. Habtzghi, Michael P. James, Riaz R. Khan, Nathaniel M. Langley, Patrick R. McDowell, Ryan D. Mickey, Lisa A. Murphy, Nathaniel R. Overall, Andrew M. Smith, Jesse C. Smith, Walt T. Wilson.

**IA Alpha – University of Northern Iowa**

*Chapter President – Erin Conrad.*

*Other spring 2007 officers: Adam Schneberger, Vice-President; Bill Freese, Secretary; Brenda Funke, Treasurer; Mark D. Ecker, Corresponding Secretary.*

Our first spring KME meeting was held on February 17, 2007 at Professor Syed Kirmani's residence. Our second meeting was held on March 8, 2007 at Professor Mark Ecker's residence where student member Joe Decker presented his paper on "Major League Baseball Park Factor Analysis". Student member Andy Quint addressed the spring initiation banquet with "What Makes a Winning Baseball Team". Our banquet was held at the Brown Bottle restaurant in Cedar Falls on April 26, 2007 where eleven new members were initiated.

New Initiates – Laura Bader, Maria Garcia, Amanda Goepferich, Jenah Harris, Megan Klein, Beth Kolsrud, Kellen Miller, Krista Schares, Alyssa Soenksen, Rebecca Thayer, Catherine Wojtecki.

**IA Delta – Wartburg College**

*Chapter President – Tim Schwickerath, 31 Current Members, 17 New Members*

*Other spring 2007 officers: Jill Wiebke, Vice-President; Sarah Danner, Secretary; Man-Ling Fan, Treasurer; Dr. Brian Birgen, Corresponding Secretary.*

**IL Eta – Western Illinois University**

*Spring 2007 officer: Boris Petracovici, Corresponding Secretary.*

New Initiates – Kimberly Cramer, Daniel Gustafson, Jeremiah Simmons, Antje Catherine Thauer, Carla Webb.

**IL Theta – Benedictine University**

This semester we had 6 students give talks at state conferences, and celebrated PI day (on another date, as 3/14 is during spring break). The speaker on campus spoke about the 10 mathematics equations that changed biology.

**IL Zeta – Dominican University**

*Chapter President – Francesca Mazei, 30 Current Members, 8 New Members*

*Other spring 2007 officers: Melisa Fleming, Vice-President; Maksymilian Dervlo, Secretary; Cassie Hileman, Treasurer; Marion Weedermann, Corresponding Secretary.*

In January, the club held its annual “Donut-Day” to introduce the club and its activities to students enrolled in various calculus courses. Six members of the club volunteered one hour per week for the entire semester in the Math Academic Resource Center on campus. The club sponsored and held several preparation sessions for the annual calculus competition organized by the Association of Colleges in the Chicago Area. Several members participated in the competition. The club designed and ordered new T-shirts for its members. Induction was held on March 25th.

**IN Alpha – Manchester College**

*Spring 2007 officer: Stanley Beery, Corresponding Secretary.*

New Initiates – Richelle Reed, Rachelle Steggerda.

**IN Delta – University of Evansville**

*Spring 2007 officer: Dr. Adam Salminen, Corresponding Secretary.*

New Initiates – Bethany Brimberry, Kevin Brittain, Cody Buckley, Caleb Butterfield, Sara Michelle Hagen, Desthy Kardell Matouba, Joel Oliver Melby, Craig Miller, Micah Noelle Nave, Childiebere Chukwujekwu Okoye, Daniel L. Price, Jacqueline A. Rice, Billy Rickey, Casey Schu, Amanda Senechal, Brian Michael Wallace, Adam Salminen, J. Christopher Tweddle.

**IN Gamma – Anderson University**

*Chapter President – Bradley Mitchell, 8 Current Members, 1 New Member*

*Other spring 2007 officers: Melissa Bailey, Vice-President; Stanley L. Stephens, Corresponding Secretary.*

We had no planned activities this past year. Several of the students did participate in a tutoring program for local public school students in our area. Two of our senior members have accepted offers to begin graduate study with graduate assistantships next year, one at Indiana University and one at Boston University.

New Initiate – Jimmy R. Lanphear.

#### **KS Alpha – Pittsburg State University**

*Chapter President – Dusty Peterson, 32 Current Members, 7 New Members*

*Other spring 2007 officers: Luke Henke and Erin Wells, Vice-President; Heather Haselrick, Secretary; Heidi Eck, Treasurer; Dr. Tim Flood, Corresponding Secretary.*

Monthly meetings, with pizza and drinks provided, were held February 15 (panel Discussion on Experiences in Student Teaching), March 8 (initiation and a presentation by student Luke Henke on “Evariste Galois and His Mathematical Contributions”), and April 19 (presentation by Jim Rowland, FCAS, MAAA, an actuary with All-State Insurance).

This spring the chapter also held a “kiss-the-pig” contest. A jar for each faculty member was placed in the department commons area and students and faculty were encouraged to contribute to the jar of their “favorite” instructor. The “winner” collected over \$300. This event brought in \$800 for the chapter.

We also had a delegation of 11 students and 2 faculty members attend the Thirty-Sixth Biennial Convention, April 12-14, 2007 hosted by MO Beta in Springfield, MO.

New Initiates – Benjamin Buttler, Linda Crum, Karalyn Lenox, Christine Wilson, Heidi Eck, Nicolaus Prelogar, Gustavo Pezzi.

#### **KS Beta – Emporia State University**

New Initiates – Cassie Norton, Melissa Swager, Cory Preister, Whitney Turley, Amanda Anders, Andrew Groene, Heath Yates, Jonathan Haney.

#### **KS Delta – Washburn University**

*Chapter President – Kristin Ranum, 35 Current Members, 16 New Members*

*Other spring 2007 officers: Tamela Bolen, Vice-President; Fai Ng, Secretary; Fai Ng, Treasurer; Kevin Charlwood, Corresponding Secretary.*

The Kansas Delta chapter of KME met for three luncheon meetings with the Washburn Math Club during the semester. Our student (Kristin Ranum) preparing to present her research work at the KME national convention at Missouri State University gave a practice talk at one of those meetings. The chapter’s annual KME initiation banquet was held on February 26, 2007 with 16 new initiates. Faculty members Kevin

Charlwood, Donna LaLonde, Al Riveland and Ron Wasserstein took students Tamela Bolen, Emily Huelskamp, Alexandria Jeannin, Clint Kendrick, Brandy Mann, Richard Nelson II, and Kristin Ranum to the National KME Convention at MO Alpha in Springfield in April of 2007. Kristin presented at the meeting, winning a “top three” placement prize in the judging of both written and oral presentations of her paper. Professor Al Riveland was recognized at the KME banquet at the convention with the George R. Mach Service Award. Al served in numerous capacities for KME over the past 32 years, including 8 years as national Treasurer from 1995 - 2003.

New Initiates – Andrea Broadbent, Gina Burdick, Sarah R. Butler, Riley Harrington, Kristen Hearrell, Quintin Kahmeyer, Samuel Loganbill, Misty Long, Halee Markos, Ivan Monroy, Jake Peterson, Aaron J. Podschun, Amanda Reese, Alissa Ross, Allyson Slupianek, Jackson Waechter.

**KS Epsilon – Fort Hays State University**

Spring 2007 officer: Jeffrey Sadler, Corresponding Secretary.

New Initiates – Donald Clewett, Jeremy Dreiling, Thomas Hornung, Andrew Leiker, Catelyn Manly, Craig Stramel, Derek Vonada, Chelsey Weber.

**KS Gamma – Benedictine College**

*Chapter President – Chris G'Sell, 4 Current Members, 9 New Members  
Other spring 2007 officers: Erica Goedken, Vice-President; Josie Villa, Secretary; Dr. Linda Herndon, Corresponding Secretary.*

Dr. Eric West, faculty sponsor, and three students attended the national convention at Missouri State University in April. The Kansas Gamma chapter was excited to initiate 9 new members this spring and since we had no graduates this year, we look forward to having a large, active chapter during the 2007-08 school year.

New Initiates – Vicki Bentley, Mark Johl, Linda Myers, James, Jocelyn Stureck, Jordan Hill, Erik Klinchman, Natalie Russo, Elaina Schaefer.

**KY Beta – University of the Cumberland**

*Chapter President- Sarah Strunk, 29 Current Members, 14 New Members  
Other spring 2007 officers: Lane Royer, Vice-President; John Steely, Secretary; Charle Delph, Treasurer; Dr. Jonathan Ramey, Corresponding Secretary.*

On February 23, 2007, the Kentucky Beta chapter held an initiation and a joint banquet with Sigma Pi Sigma, physics honor society, at the Cumberland Inn. Kappa Mu Epsilon inducted thirteen new student members at the banquet, presided over by outgoing president, Sarah Strunk. As an additional feature, senior awards were given by the department at the banquet. One additional member was inducted later in the semester.

On March 28, the chapter helped to host Ms. Pam Armentrout from University of Tennessee graduate school. Jointly with the Mathematics and Physics Club, the Kentucky Beta Chapter hosted Dr. Carroll Wells from David Lipscomb University on April 12. He spoke about "Euler – After 300 Years We Still Can't." On April 13, members also assisted in hosting a regional high school math contest, held annually at the University of Cumberland. On April 26, the entire department, including the Math and Physics Club, Sigma Pi Sigma (Physics Honors Society), and the Kentucky Beta Chapter, held a pizza party to celebrate the end of the semester.

**LA Delta – University of Louisiana at Monroe**

*Chapter President – Anna Beaubouef, 17 Current Members, 9 New Members*

*Other spring 2007 officers: Jennifer Ellerbe, Vice-President; Ruby Riles, Secretary; Amanda Bartlett and Sunny Brown, Treasurer; Dr. David Hare, Corresponding Secretary.*

The Louisiana Delta Chapter of KME met four times during the school year. We had three pizza socials, one Christmas party and we inducted 9 students and 3 faculty members into KME in April. The socials were particularly enjoyable as there are a variety of majors represented in our chapter and so this was the primary opportunity for those who did not have classes with other members to interact and get to know one another socially.

Among the activities that KME members participated in through the ULM Office of Student Life and Leadership was one in which each organization painted a banner to be used at a home ULM football game. The KME banner had both the ULM and KME emblems. Finally, several members took the initiative to have KME t-shirts made. This created minimal profit as their primary purpose was simply to provide, at cost, the ULM KME t-shirts to those who wanted them.

New Initiates – Andrew Arrant, Amy Dawson, Allison Fisher, Beth Hannahs, Jessica Hunter, Chris McElduff, Jacob Myers, Lindsey Rogers, Crystal Woodard, Maritte Maroun, Christine Cumming-Strunk, Brent Strunk.

**MD Alpha – College of Notre Dame of Maryland**

*Chapter President – Kimberly Wall, 13 Current Members, 9 New Members*

*Other spring 2007 officers: Neeraj Sharma, Vice-President; Nicole Kotulak, Secretary; Vera Ulanowicz, Treasurer; Dr. Margaret Sullivan, Corresponding Secretary.*

Our KME chapter continued to offer twice weekly tutoring sessions. Our Spring Induction included a presentation by Dr. Richard Fahey on mathematical ideas underlying gravity (or the lack thereof).

**MD Delta – Frostburg State University**

*Chapter President – Timothy Smith, 30 Current Members, 8 New Members. Other spring 2007 officers: Kyle Conroy, Vice-President; Nicole Garber, Secretary; Bradley Yoder, Treasurer; Dr. Mark Hughes, Corresponding Secretary.*

The Maryland Delta chapter held two fundraisers during the spring semester. Our traditional Pi Day bake sale rose over \$70 and a similar amount was raised by our candy Easter egg sale. Eight new members were inducted in March and Dr. Mark Hughes gave a lecture on partitions during the induction ceremony. Later that month, Dr. Hughes gave a lecture on algebraic solutions of polynomials during our March meeting. Our April meeting saw the election of new officers: Shay Mallory as president, Matthew Bucchino as vice president, Courtney Kamauf as treasurer and Nicole Garber continuing as secretary. Special mention goes to outgoing president Tim Smith for his energy and hard work during the past year. As usual, our May picnic featured cool and drizzly weather, but the food was great and the weather cleared up enough for a game of kickball. A good time was had by all.

**MD Epsilon – Villa Julie College**

*Chapter President – Richard Haney, 24 Current Members, 20 New Members*

*Other spring 2007 officers: Steven Mrozinski, Vice-President; Courtney Naff, Secretary; Emily Clemens, Treasurer; Dr. Christopher E. Barat, Corresponding Secretary.*

On October 14, 2006, the MD Epsilon Chapter initiated 20 new members, including five members of the full- and part-time faculty. The guest speaker was Dr. James Lightner, professor emeritus at McDaniel College. During the academic year, the Chapter sponsored a fund-raising raffle for a laser jet printer and, as part of Mathematics Awareness Month, invited several VJC graduates with Mathematics degrees to speak to the membership on their career goals and experiences. The third chapter initiation is currently scheduled for September 2007.

**MI Alpha – Albion College**

*Spring 2007 officer: Mark Bollman, Corresponding Secretary.*

New Initiates – Allyson Bush, Neil Kocan, Kristen Krum, Jaclyn Sweeney, Jeremy Troisi, Justin Willbrandt, Cayley Pendergrass, Norma Taber.

**MI Epsilon – Kettering University**

*Spring 2007 officer: Boyan Dimitrov, Corresponding Secretary.*

New Initiates – Munaf K. Assaf, Ife Christelle Bancole, Joey Michael Campbell, Brittany Ann Cunningham, Paul Thomas Duehlmeier, Joshua David Eaton, Denver Bennet Gallardy, Gadrin Christopher Higgs, David Michael Keyser, Randell Krug, Paul Steven



Kruse, Kennet August Mangus, Rafal Lucjan Metkowski, Simon Nicholas Murphy, Herman Joseph Orgeron, Jarrad A. Pouncil, William Christophe Quinn, Casey Octavia Stevenson, Nicholas Paul Timkovich, Christopher Mich Zavicar.

**MO Alpha – Missouri State University**

*Chapter President– Uriah Williams, 36 Current Members, 14 New Members*

*Other spring 2007 officers: Megan Reineke, Vice-President; Annie Johnson, Secretary; Thomas Buck, Treasurer; John Kubicek, Corresponding Secretary.*

For the Spring 2007 semester the Missouri Alpha Chapter held two regular meeting with presentations made by faculty members Dr. Paula Kemp and Dr Kishor Shah. In April we hosted the KME National convention which was attended by approximately 100 faculty and students from 14 chapters.

New Initiates – Kate Hercules, Caleb Floyd, Bobbi Sue Gregory, Nicole Jones, Aaron Yeager, Tom Mangan, Michael McDonald, Julia Roark, Eric Stephens, Lee R. Stevens, Kimberly White, Chris Wunderlich, Jeffrey Doak, Elizabeth Higgins.

**MO Beta – University of Central Missouri**

*Spring 2007 officer: Dr. Rhonda McKee, Corresponding Secretary.*

New Initiates – Thomas Gossell, Andrew Kuttenkuler, Melissa LeFevre, Allison B. Monroe, John Westbrook.

**MO Kappa – Drury University**

New Initiates – James Adrian, Katie Ambrose, Johathan Brownsworth, Amanda Johnson, Sarah Radke, Ben Taylor.

**MO Iota – Missouri Southern State University**

*Chapter President – Ben Cartmill, 15 Current Members, 0 New Members*

*Other spring 2007 officer: Rikki McCullough, Vice-President; David Smith, Secretary; David Smith, Treasurer; Chip Curtis, Corresponding Secretary.*

In addition to holding monthly meetings, representatives attended the national convention in Springfield, MO, at which one member, John Carr, presented a paper chosen for “top 3” status.

New Initiates – James Baldwin, Michael B. Bertelsen, John R. Carr, Jonathan S. Evans, Thanh K. Nauven, Josiah A. Roelfsema, Katherine Hiebert-Brumley, Heather Dickson, Matthew R. Gentry, Kaylee N. Parker, David M. Smith.

**MO Nu – Columbia College**

*Chapter President – Mandy Jorgenson, 30 Current Members, 6 New Members*

*Other spring 2007 officers: Laurie Weaver, Vice-President; Christina Schoonever, Secretary; Laurie Weaver, Treasurer; Dr. Ann Bledsoe, Corresponding Secretary.*

KME members upgraded wallet size tip tables and posted them on the KME bulletin board; organized a celebration of the Pi day in collaboration with the Math Center; and organized and conducted the spring 2007 KME initiation ceremony, held on April 13.

New Initiates – Sasia Barnett, Sara Ann Demma, Catryna Palmer, Adam Spudich, Lindsey D. Voqt, Ashley Wesche.

**MO Theta – Evangel University**

*Chapter President – Lindsay Hull, 6 Current Members, 3 New Members.*

*Other spring 2007 officers: Monica Pedersen, Vice-President; Don Tosh, Corresponding Secretary.*

Meetings were held every month. In January we initiated 3 members and elected new officers. Six members were able to attend the national convention. The final meeting of the semester was held at the home of Dr. Tosh at which time we honored graduating seniors.

New Initiates – W. Paul Griffin, Jeremy W. Grisbee, Micah L. Ruth.

**MO Zeta – University of Missouri – Rolla**

*Spring 2007 officer: Roger Hering, Corresponding Secretary.*

New Initiates – Frank Morrissey, Steve Puzach, Ryan Miller, Susie Phillips, Lauren Bengston, Margaret Gardiner, Jacob Herzog, Trevor Gladback, Jessica Owens, Austin Shaw, Christopher A. Hill, Christopher Baker, Marsha Kiphart.

**MS Alpha – Mississippi University for Women**

*Chapter President – Johnatan Dillon, 16 Current Members, 4 New Members*

*Other spring 2007 officers: May Hawkins, Vice-President; David Wages, Secretary; Vasile (Johnny) Bratan, Treasurer; Dr. Shaochen Yang, Corresponding Secretary.*

Activities for the semester included a 1/25/2007 meeting, 2/8/2007 initiation, and 4/24/2007 officer election for Fall 2007.

New Initiates – Kelsey Adams, Dana Derrick, Michelle Hitt, Kaycee McMullan.

**NE Beta – University of Nebraska at Kearney**

*Spring 2007 officer: Dr. Katherine Kime, Corresponding Secretary.*

New Initiates – Cory Hatt, Courtney Johnson, Drew Rische, Sandra Torres.

**NE Delta – Nebraska Wesleyan University**

*Spring 2007 officer: Erdmann, Corresponding Secretary.*

New Initiates – Rebecca Brown, Ashley Dorwart, Brian Grummert, Jessica Haight, Andrew Synhorst.

**NH Alpha – Keene State College**

*Spring 2007 officer: Vincent J. Ferlini, Corresponding Secretary.*

New Initiates – Jenna Burns, William Clarke, Torey Cutting, Abby Dutch, Ryan Farnsworth, Holly Lamport, Bethanie San Martino, Kevin Seymour, Hamilton Williams, Alexandra Wilson, Caitlin Coulombe, Erin Connell.

**NY Iota – Wagner College**

*Spring 2007 officer: Dr. Zohreh Shahvar, Corresponding Secretary.*

New Initiates – Brian Halling, Kurt Larser, Alla Bronskaya, Kayla Zeller, Steven Herman, Jessica Bajkowski, Stephanie McNamara, Kristina Eells, Melissa Cambria, Nadwa Ibrahim, Daniele Mancuso, Dana DiLullo.

**NY Kappa – Pace University**

New Initiates – Jason Yarmish, Annmarie Smith, Dhvani Patel, James Lentz, Nira Hermann, Christopher Hemsley, Matthew Chapnick.

**NY Lambda – C.W. Post Campus of Long Island University**

*Spring 2007 officer: Andrew Rockett, Corresponding Secretary.*

New Initiates – Ozlem N. Altop, Salvatore Cardillo, Zhen Chen, Michael D. Coleman, Joseph Cornwell, Christina A. Del Giudice, Keith W. Gaylord, Rosemarie Goldman, Daniela F. Iacovissi, Tara J. Koebel, Richard F. Mazanek, Kathleen P. Moore, Isha S. Morgenstern, Daniel J. Resch, Thomas M. Rouge, Celia M. Santorello.

**NY Mu – St. Thomas Aquinas College**

New Initiates – Ashley Bass, Anthony Bucci, Lou Gabrilli, Jr., Matthew Marino, Martina Morenberg, Dana Samko, Kelly Sullivan, Alyssa Butterfield, Jessica Gosda, Scott Miller, Venessa Porrata, Jennifer Spadaccini.

**NY Nu – Hartwick College**

*Chapter President – Joseph Fayton, 25 Current Members, 13 New Members*

*Other spring 2007 officers: Caitlin Gilman, Vice-President; Anees Gharzita, Secretary; Dustin Jones, Treasurer; Ron Brzenk, Corresponding Secretary.*

New Initiates – Ian Avenia-Tapper, Amelia Berchtold, Meghan Buckner, Andrew Haverly, Dustin Jones.

**NY Omicron – St. Joseph’s College**

*Chapter President – Christine Vaccaro, 22 Current Members, 8 New Members*

*Other spring 2007 officers: Jaclyn Pirrotta, Vice-President; Adrienne Eterno, Secretary; Alicia Gervasi, Treasurer; Elana Epstein, Corresponding Secretary.*

During the semester, the chapter met once a month, organized a math colloquium on “space and mathematics”, and inducted 8 new members.

New Initiates – Kai Lian Chen, Mirljinda Krivca, Courtney Leja, Svetlana Medvedeva, Francisco Rangel, Dan Ristea, Jason Stancati, Karen Zhao.

**NY Pi – Mount Saint Mary College**

*Chapter President – Kelly Reid, 0 Current Members, 14 New Members*

*Other spring 2007 officers: Amy LaPointe, Vice-President; Brianne Beebe, Secretary; Erica Lauffer, Treasurer; Dr. Lee Fothergill, Corresponding Secretary.*

Our chapter was installed on March 29, 2007 in the Villa Library of Mount Saint Mary College in Newburgh, New York. Fourteen charter members were initiated. Dr. Andrew M. Rockett was the installing officer. The new members received membership certificates and KME pins. They received their honor cords at a college wide honor society celebration on April 28, 2007.

Members of the honor society provided free math tutoring at the end of the semester for any students at Mount Saint Mary College. They look forward to planning more events when they return in the fall.

New Initiates – Walter O. Krawec, Alison L. Marrine, Christopher M. Schacca, Erica Lauffer, Erin Dever, Brianne Beebe, Kelly A. Reid, Felicia Leung, Jamie J. Woodcock, Kathryn M. Goodrich, Amy LaPointe, Diana R. Thibault, Michael Redmond.

#### **OH Alpha – Bowling Green State University**

*Spring 2007 officer: Dr. David E. Meel, Corresponding Secretary.*

New Initiates – Deborah L. Alesch, Stephanie C. Brazie, Rachel G. Childers, Anna K. Glett, Nicholas M. Inbody, Lynn E. Lottman, Emily L. Nicol, Eric T. Radabaugh, Scott W. Sisco, John T. Strobel, Rebecca A. Sustersic, Zachary D. Taylor, Amanda M. Waldeck, Tonya M. Wharton, Tessa M. Yahr.

#### **OH Epsilon – Marietta College**

*Chapter President – Phil DeOrsey, 20 Current Members, 12 New Members  
Other spring 2007 officers: Kelsie McCartney, Vice-President; Dr. John C. Tynan, Corresponding Secretary.*

New Initiates – Paul Bodager, Megan Brothers, Elmo Cecchetti, Joshua Downey, Matthew Dutko, Elisabeth Kager, Daniel Lopata, Ashleigh Pottmeyer, Laura-Elizabeth Smith, Robert Vazquez, Keith Zeigler, Elizabeth Jeffers.

#### **OH Gamma– Baldwin-Wallace College**

*Chapter President – Stacey Batcha, 28 Current Members, 20 New Members*

*Other spring 2007 officers: Jennifer Cawrse, Vice-President; Kali Dye, Secretary; Lindsay Moomaw, Treasurer; Dr. David Calvis, Corresponding Secretary.*

New Initiates – Zachary Balash, Kristen Balogh, Katherine Barath, Madeline Black, Rachel Bouw, Rachel Bryda, Randy Deighton, Kali Dye, Dean Glass, Bryan Haslett, Benjamin Horton, Katherine Komar, Nicole Lashock, Amanda Mansell, Nathan Miller, Ilirjan Pipa, Brenna Posa, Amy Powers, Matthew Rodriguez, Jessica Schutrum, Gregory Skupski, Maria Sorgi, Maria Stopak, Kelly Stoyanoff.

#### **OH Zeta – Muskingum College**

*Spring 2007 officer: Dr. Richard Daquila, Corresponding Secretary.*

New Initiates – Jessica Bosway, Dirk Chisnell, Randi Dance, Rahil Devgan, Dr. Victor Keiser, Kelley Kubik, Renee Metzger, Stephen Michel, Michael Neff, Sankalp Sharma.

**OK Alpha – Northeastern State University**

*Chapter President – Lindsey Box, 62 Current Members, 14 New Members  
Other spring 2007 officers: Bobbie Back, Vice-President; Seana Smith,  
Secretary; Jeff Smith, Treasurer; Dr. Joan E. Bell, Corresponding  
Secretary.*

Our spring activities began with a presentation by Dr. Yungchen, the Mathematics Department Chairman at Missouri State University. The spring initiation of fourteen new members was held at a local Chinese restaurant. Our chapter again designed a math T-shirt and sold over 75 shirts! Many of the KME meetings this semester involved preparations for the annual Oklahoma Arkansas section meeting, which Northeastern State University hosted in late March. The meeting was a big success, with about 200 faculty and students attending. Dr. Joan E. Bell, the KME OK Alpha sponsor for the past 22 years, was named the 2007 OK-AR Mathematical Association of American Distinguished Teacher of the Year at the 69th annual meeting of this organization.

One of the founders of KME is Professor L. P. Woods. His son, also L. P. Woods, emailed this message to Dr. Bell, to share with those attending the 36th Biennial Convention of KME, held at Missouri State University in Springfield, Missouri. "I hope you have a rewarding meeting. I enjoyed the national KME convention in Springfield over 50 years ago, and still remember the thrill of seeing so many high achievers in one room at one time. Please give my best to all my brothers and sisters and hold the pentagon high! Sincerely yours, L. P. Woods, Oklahoma Alpha, 1952"

The last activity of the semester was the annual ice cream social. Here members presented both Mr. Maurice Turney, KME OK Alpha member since 1945, and his wife, with this year's OK Alpha math shirt.

New Initiates – Amanda A. Barker, Wendy J. Fehrenbach, Holly C. Floyd, Rachel L. Keck, Caleb D. Knowlton, Phyllis A. Magady, Ramona A. Medlin, Crystal Merritt, Junel R. Miller, Steven Ostrander, Sarah M. Owens, Renee P. Rabovsky, Jerry A. Smith, Amber R. Warren.

**OK Delta – Oral Roberts University**

*Spring 2007 officer: Vincent Dimiceli, Corresponding Secretary.*

New Initiates – Jonathan Isaiah Barrett, Bonnie Jean Bavido, William Butron, Sean Estes, Stefanie Gurley, Nathaniel Hunt, David Kobilnyk, Nathan Christopher Marth, Timothy Ryan Onarecker, Nathaniel Roman, Jason Vermette.

**OK Epsilon – Oklahoma Christian University**

*Spring 2007 officer: Ray Hamlett, Corresponding Secretary.*

New Initiates – Harrison Barbarick, Adam Barnes, Marianne Bentley Strande, Kenneth Bowling, Jennifer Bryan, Cortney Butler, Jason Cain, Johnnie Frye, Ray Hamlett, Jacob Higginbotham, Travis Hughes, Craig Johnson, Sean Lacey, Don Leftwich, William Martin,

Samantha Marshall, Monica McDalton, Michael Mceuen, Leighton Minor, Amber Newell, Stephanie Rummel, Joel Scarsella, Tara Triplett, Arlette Umuhoza, Megan Willson.

**OK Gamma – Southwest Oklahoma State University**

*Spring 2007 officer: Bill Sticka, Corresponding Secretary.*

New Initiates – Pankaj Mishra, Sarah Morris, Lindsay Opper, Gaurav Poudyal.

**PA Alpha – Westminster College**

*Chapter President – Nicole Panza, 17 Current Members, 8 New Members  
Other spring 2007 officers: Larissa Fortna, Vice-President; Dana Larson, Secretary; Allison McNary, Treasurer; Natacha Fontes-Merz, Corresponding Secretary.*

Some activities for this semester included pizza and donuts for students during finals week and participation in Westminster's "Geek Week" (an academic competition between the chemistry, biology, math, psychology, and physics departments).

New Initiates – Callie Croushore, Larissa Fortna, Elizabeth Kribel, Amy Leicht, Anna Sparacino, Lindsey Stankiewicz, Randy Sylvester, Brandon Phillian.

**PA Beta – LaSalle University**

*Chapter President – Brian Story, 11 Current Members, 3 New Members  
Other spring 2007 officers: Joseph Cosella, Vice-President; Danielle Forsythe, Secretary; Jeremiah Noll, Treasurer; Dr. Anne E. Edlin, Corresponding Secretary.*

This year's activities by our KME students included a "Bowling for Primes" competition in the Fall (at a bowling alley in West Philadelphia), attendance (by KME Moderator Dr. Anne Edlin and several KME students) at the MAA MathFest in August, 2006, a monthly symposium at La Salle on topics of interest (featured speakers were department faculty/students from mathematics or computer science), a Problem-of-the-Week competition for students, a celebration of Pi Day (3/14) on campus, assistance at "Open House" Days (recruitment of new students) at La Salle, and, finally, a KME induction ceremony for new members in April, 2007 at which Dr. Stephen Andrilli gave an invited lecture on "The Vanishing Square" (a Fibonacci number-related puzzle). (Note: Dr. Stephen Andrilli will temporarily take over as KME Moderator for Dr. Anne Edlin while she is on maternity leave during fall of 2007.)

New Initiates – Stephen J. Muscelli, Joseph A. Cosella III, Catherine Gulkis.

**PA Epsilon – Kutztown University**

*Spring 2007 officer: Randy S. Schaeffer, Corresponding Secretary.*

New Initiates – Angela Davis, Scott Kallal, LeeAnn Watts, Jeromy Guistwite, Amanda Latz, Jayme Mahle, Amber Snyder, Brian Walsh.

**PA Gamma – Waynesburg College**

*Spring 2007 officer: James R. Bush, Corresponding Secretary.*

New Initiates – Ashley Bernola, Christopher Canning, Brad Davis, Todd Day, Emily Dooley, Valerie Higinbotham, Danielle King, Brian Lucarelli, Tara Marcischak, Erik Murphy, Katie Slaughter, Megan Steiminger.

**PA Kappa – Holy Family University**

*Chapter President – Patrick Heasley, 10 Current Members, 1 New Member  
Other spring 2007 officers: Shawn Kane, Vice-President; Tiffany Young,  
Secretary; Tiffany Young, Treasurer; Sister Marcella Louise Wallowicz,  
Corresponding Secretary.*

On March 14, 2007 the Chapter hosted a “PI Day” Celebration. The day’s events included a pie eating competition, the “digits of pi” recitation contest and the annual high school mathematics competition.

The winner of the pie eating contest “downed” a blueberry pie in little less than 10 minutes. The winner of the pi recitation contest correctly recited the first 41 digits of pi. Both events were covered by the local newspaper. Participants in both events were students at Holy Family University. The winners received a certificate and a \$10 gift certificate to the cafeteria or book store (winner’s choice).

The high school math competition drew 35 students from 5 local area high schools. There were two levels of competition: Division I (9th and 10th graders) and Division II (11th and 12th graders). Each school was permitted to enter a maximum of 5 students in each division. Plaques were awarded to the top student and top school in each division. Refreshments were served after the competition – soda and pizza pie (what else!)

The annual induction ceremony was held on Friday, March 23. The lone inductee was Samuel Ganta, a math minor. The keynote address was given by Dr. Agnieszka Lekka, a visiting professor from the Catholic University of Lublin, Poland.

On April 16th, the chapter members, in conjunction with the History of Math class, celebrated the 300th Anniversary of Euler’s birth. Activities included “pin the bridge on Konigsberg,” Fast Euler Facts and finding Euler circuits in graphs. Of course, there was a birthday cake and other refreshments to mark the occasion.

New Initiate – Samuel Ganta.

**PA Lambda – Bloomsburg University of Pennsylvania**

New Initiates – Christopher Bennett, Katlyn Boiwka, Katie Burns, Bethany Cummings, Melissa Josuweit, Shun Chi Vencent Lam, Katie Lechner, Katie Lyle, Matthew M. Mills, Kasey Motter, Adrienne Rebarchak, Adam Saunders, James Till Matta, Tara Werley, Caitlin Zeuggin.

**PA Nu – Ursinus College**

*Spring 2007 officer: Jeffrey Neslen, Corresponding Secretary.*

New Initiates – Nicole Rascavage, Jenna Lesky, Erin Bender, Chelsey Morien, Anastasia Salaneck, Jamie Doperak, Carla Giampa, Laura Ng, Jason Knapp, Alexander Clayton, Stephen Krol, Kelly Hosier, Jennie Moore.

**PA Pi – Slippery Rock University**

*Chapter President – Emily Hendrickson, 15 Current Members, 6 New Members*

*Other spring 2007 officers: Michelle Komo, Vice-President; Tyler Druschel, Secretary; Dr. Elise Grabner, Corresponding Secretary.*

New Initiates – Tory Corbett, Tyler Druschel, Emily Elder, Michelle George, Eric Hardin, Diane Kszastowski, Scott Rager, Duayne Rieger, Robert Rifenburg, Kimberly Romanelli, Kevin Sobczak, Christopher Zorich.

**PA Rho – Thiel College**

*Chapter President – Angela Crone, 21 Current Members, 13 New Members*

*Other spring 2007 officers: Kaitlyn Scherer, Vice-President; Kari Zeleznick, Secretary; Cassandra Beck, Treasurer; Max Shellenbarger, Corresponding Secretary.*

New Initiates – Cassandra M. Beck, Angela L. Crone, Adam Evans, Michael Dworakowski, Yoshitaka Homma, Anna Nakashima, Michael Ryan, Nicholas A. Scaife, Kaitlyn M. Scherer, Punit Upadhyaya, Anthony White, Kari Zeleznick, Professor Max Shellenbarger.

**PA Sigma– Lycoming College**

*Spring 2007 officer: Dr Santu de Silva, Corresponding Secretary.*

New Initiates – Seth Burns, Stephanie L. Hartman, Adam Hughes, Justin Hughes, Dr. Philip Sprunger.

**PA Theta – Susquehanna University**

*Spring 2007 officer: Kenneth A. Brakke, Corresponding Secretary.*

New Initiates – Laurie Blaszk, Anurodh Joshi, Ashley Moormann, Eric Peterson, Claire Polcrack, Deanna Snyder, Alexander Ulsh, Andrew White.

**PA Xi – Cedar Crest College**

*Spring 2007 officer: Marie Wilde, Corresponding Secretary.*

New Initiates – Erika Asselin, Katelyn Burgett, Patricia Egner, Stephanie Frace, Jennifer Resling, Gwendolyn Rogers, Tova Samuels, Rachel Scholla, Adina Smith, Moira Stanley, Jennifer Stone, Athena Strobl, Bridget Szala, Allison Taylor, Ashley Tillette, Zoe Tuck, Heather Wert, Renee Zimmerman.

**SC Gamma – Winthrop University**

*Spring 2007 officer: Dawn M. Strickland, Corresponding Secretary.*

New Initiates – Jozef Richard Brewer, Lauren Elizabeth Cairco, Kristen Elizabeth Huete.



**TN Alpha – Tennessee Technological University**

*Spring 2007 officer: Andrew J. Hetzel, Corresponding Secretary.*

New Initiates – Matthew Clayton Bedford, Donald Lance Brady, Staci Leigh-Anne Daniel, Wesley Stuart Ingram, Jacky King, Jr., Justin Tyler Medley, Brett Alan Steigerwaldt, James Allen Sunkes, III, Kimberly Michelle Tidman.

**TN Beta – East Tennessee State University**

*Chapter President – Glenn Quarles, 13 Current Members, 7 New Members  
Other spring 2007 officers: Hamilton Scott, Vice-President; Jennifer Woodell, Secretary; Courtney Sanders, Treasurer; Lyndell Kerley, Corresponding Secretary.*

Tennessee Beta chapter held its annual initiation April 12 in the Tennessee Room of D. P. Culp University Center. Officers of the chapter Glenn Quarles-President, Hamilton Scott-Vice President, Jennifer Woodell-Secretary, and Courtney Sanders-Treasurer initiated the following: Elizabeth Adkins, Tommy Byrd, Shawna Cox, Ryan Deskins, Elizabeth Harris, Joshua Odom, and Johnny Odom. In addition to the initiation, two KME members were recognized. David Simpson won the Mathematics Faculty Award and Brett Kindle won the Mathematics Award. The evening ended with a talk by Dr. Sloan Despeaux of Western Carolina University. The title of her talk was “The first Attacks on a ‘Man-Eating Problem’: The Four Color Problem in Nineteenth-Century Britain”

New Initiates – Elizabeth Adkins, Tommy Byrd, Shawna Cox, Walter Deskins, Elizabeth Harris, Joshua Odom, Johnny Odom.

**TN Delta – Carson-Newman College**

*Chapter President – Alex Cate, 17 Current Members, 5 New Members  
Other spring 2007 officers; Brittany Hall, Vice-President; Brian A. McLaughlin, Secretary; B. A. Starnes, Corresponding Secretary.*

New Initiates – Jacob L. Allen, Whitney P. Jacobsen, Brittany Rae Morrisett, Emily S. Brann, Benjamin C. Love.

**TN Epsilon – Bethel College**

*Spring 2007 officer: Russell Holder, Corresponding Secretary.*

New Initiates – Justin C. Dubruiel, Chastity Fortenberry.

**TN Gamma – Union University**

*Chapter President – Kendal Hershberger. 15 Current Members, 4 New Members*

*Other spring 2007 officers: Joshua Shrewsberry, Vice-President; Matthew Dawson, Secretary; Matthew Dawson, Treasurer; David Moses, Webmaster; Bryan Dawson, Corresponding Secretary.*

The Tennessee Gamma chapter held its annual initiation banquet at the Old Country Store on March 27. Four students were initiated. Our chapter was represented at the biennial national convention by one faculty

member and one student, Bryan Dawson and Matthew Dawson, both of whom served on the Awards Committee. Our primary social event was a trip to Lambert's in southeast Missouri, a restaurant that is famous for its "threwed rolls." The end-of-year picnic was held May 3; graduating seniors were recognized.

New Initiates – Joshua Brooks, Heather Nicole Higdon, Blake J. Waggoner, William Thompson White.

#### **TX Alpha – Texas Tech University**

New Initiates – Andrew C. Adams, Ashley Lefler, Jennifer Dlynn McCracken, Trey Porter, Derek Allison, Andy Hannan, Matthew Monk, Kyle Lovorn, Clinton Sheffer, Laura Doi, Aaron Phillips, Sawyer Sams, Ryan Conklin, Todd Buckingham, Martyn Robertson, Ellen Durant, Cody Hukill, Lee Porter, Daniel P Dukich, Hyo Yun, Alex Boothby, Kristen Godeke, Hector Guardado, Robyn Veraa, Kyle Romero, Scott Blissit, Jared Casey, Alan Kuhn, Nick Jencopale, Tewobista G. Metaferia, Beau J. Chambers, Kathryn E. Carter, Jacob Smalts, Tyler Thompson, Ryan Humphrey, Eric Greenberg.

#### **TX Gamma – Texas Women's University**

*Spring 2007 officer: Dr. Mark Hammer, Corresponding Secretary.*

New Initiates – Katie Schniebs, Rae Lynn McFallin, Michelle Haubrich, Zevnep Seven Marisol Guzman, Melissa Bird, Jessica Kosine, Tera Smith, Alexandra Noriega, Andy Hall, Cindy Tempel.

#### **TX Kappa – University of Mary Hardin-Baylor**

*Chapter President – Peggy Cain, 10 Current Members, 4 New Members  
Other spring 2007 officers: Mark Leech, Vice-President; Kristin Chamberlain, Secretary; Dr. Peter H. Chen, Corresponding Secretary.*

The Texas Kappa chapter of Kappa Mu Epsilon initiated four students into its membership at its meeting on April 23, 2007.

New Initiates – Andrew Brenek, Dephne Kahlig, Brittany Mumme, Joel Munoz.

#### **TX Mu – Schreiner University**

*Chapter President – Matthew Casey, 13 Current Members, 5 New Members*

*Other spring 2007 officers: Sean Collins, Vice-President; Christian Saaam, Secretary; Meagan Goodson, Treasurer; William Sliva, Corresponding Secretary.*

New Initiates – Guadalupe Garza, Ashley E. Moore, Lynn M. Stow, Timothy J. Tucker, Amy L. Vickers.

#### **VA Alpha – Virginia State University**

*Spring 2007 officer: V. Sagar Bakhshi, Corresponding Secretary.*

New Initiates – Goerge W. Carstens, Aysa S. Monds.

**VA Gamma – Liberty University**

*Spring 2007 officer: Glyn Wooldridge, Corresponding Secretary.*

New Initiates – Khue Anh Bui, Jacob Christian Burke, Megan Beth Claar, Casey Rae Ellison, Braden J. Eriksen, Jarrett Nelson Eriksen, Kristina Elizabeth Finch, Jessica Michelle Harley, Rachel Savannah Hassenpflug, Lydia Im, James Samuel Jackson, Christopher Harold-Grove Johnson, Matthew David Lombardo, Matthew Henry Luthman, Rebecca Lynne Mix, Emily Diane Munton, Rei Joyce Gregorio Taman, Zhongxiao (David) Wang, Tom Zara.

**WI Gamma – University of Wisconsin-Eau Claire**

*Spring 2007 officer: Dr. Simei Tong, Corresponding Secretary.*

New Initiates – Christopher Joseph Declene, Sara Ann Grassel, James Patrick Hahn, Emily Sue Kopp, Kevin J. Kropp, Cassandra Marie Lawler, Julie A. Meis, Jessica Jeanne Porath, Alexandra Marie Redpath, Emily F. Vance-Curzan, Frank Lee III Emmert, Julianne Marie Fuhrmann, Rochelle Lee Halama, Benjamin Andrew Lebeau.

**WV Alpha – Bethany College**

*Spring 2007 officer: Dr. Mary Ellen Komorowski, Corresponding Secretary.*

New Initiates – Courtney Leigh McKean, Joseph Michael McLane, Adam Michael Tomaino, Briana R. Boyd, Michelle L. Fowler, Megan K. Pisterius, William S. Grant, Kenneth B. Kearns, William Alan Seelhorst, Jr., Anthony Michael Talotta, James Thomas Buchanan, Gregory S. Mowrer, Alycia A. Forney, Gerald Lee Shultze, Suzanne M. Wonderle.

## ***Report of the 36th Biennial Convention***

Kappa Mu Epsilon  
April 12-14, 2007  
Missouri State University  
Springfield, MO

### **Thursday, April 12, 2007**

Registration and a mixer took place at the Robert W. Plaster Student Union from 6:00-10:00 p.m. Mixer activities included bowling, pool and lots of snacks. The National Council and regional directors met at 8:00 p.m.

### **Friday, April 13, 2007**

#### **First General Session and Business Meeting**

The first general session began at 8:30 on Friday morning. Participants were welcomed to Missouri State University (MSU) by Michael Nietzel, president of MSU, and were also greeted by Uriah Williams, current president of the Missouri Alpha chapter, and April Williams, past president of the Missouri Alpha chapter.

The first business meeting began with roll call by National Secretary Rhonda McKee. Fifty-eight students and 24 faculty members (total 82) from 14 chapters were present.

A summary of a new chapter petition from Hawaii Pacific University was distributed by National President Don Tosh. A vote to approve the petition was to take place at the final business meeting on Saturday.

Al Riveland, from the nominating committee, introduced the nominees for the offices to be filled. Rhonda McKee has agreed to serve another term as National Secretary, and, as allowed by the KME Constitution, will be the nominating committee's only nominee for that position. The nominees for National Treasurer were Mark Hamner, of Texas Gamma, and Cynthia Woodburn, of Kansas Alpha. Information about each of these candidates was distributed. The vote on the nominees was to take place at the final business meeting on Saturday.

#### **First Paper Session**

The first paper session was presided over by National President Elect Ron Wasserstein. In honor of the fact that the convention was being held in Springfield, Missouri, Dr. Wasserstein showed internet maps of other cities named Springfield and had the audience guess the state in which the

city was located. If no one guessed correctly, he zoomed out until someone was able to recognize the state. Those who guessed correctly were given prizes.

Three papers were presented during the Friday morning session.

- “Maximizing Functionals: A Generalization of Problem B5 from the 67th Putnam Competition”, by Nathan Bloomfield, MO Kappa
- “Evariste Galois and His Mathematical Contributions”, by Luke Henke, KS Alpha
- “An Inquiry into LC-Loops”, by Jacob Magnusson, KS Beta

After lunch, the video “The Great  $\pi$  v.  $e$  Debate: Which is the Better Number?” was shown.

#### Second Paper Session

The second paper session began with another of Dr. Wasserstein’s “Springfield” quizzes and concluded with two more student papers.

- “PascGalois Triangles: Visualizing Abstract Algebra Concepts Using Pascal’s Triangle and Group Theory”, by Casey Kuhn, KS Alpha
- “Mathematically Modeling Drug Dosages of Quinine”, by John Carr, MO Iota

After a refreshment break, students met in one room, while faculty met in another. Both groups were charged with discussing issues related to KME.

#### Banquet

On Friday evening, a banquet was held in Crusader Hall on the nearby campus of Evangel University. After dinner, the George R. Mach Distinguished Service Award was presented to Al Riveland of the Kansas Delta chapter. Dr. Riveland served as a faculty advisor and/or corresponding secretary of the Kansas Delta Chapter for over 30 years. He served as an advisor for dozens of Washburn students presenting papers at national and regional meetings. Twice under Dr. Riveland’s leadership, Washburn University hosted the national convention of KME, and twice also hosted regional conventions. Further, Dr. Riveland provided leadership at the national level, serving as national treasurer for eight years.

Following the presentation of the Mach Award, Dr. Larry Campbell, of Missouri State University gave the keynote address, titled “Celebrating Mathematics.”

**Saturday, April 14, 2007**

Saturday morning brought an unusually late spring snow storm to Springfield, but the convention proceeded.

**Third Paper Session**

The third paper session began with more “Springfield” quizzes and concluded with three more student papers.

- “The Mathematics of Music”, by John Estes, OK Delta
- “Who Dun It: Investigations in the Mathematics of Forensics Science”, by Kristin Ranum, KS Delta
- “Cellular Automata”, by Christy Baker, KS Alpha

**Second General Session and Business Meeting**

Don Tosh, National President, presided over the second general session and business meeting. The national officers each made an oral report and distributed copies of these reports to each chapter in attendance. President Tosh presented a plaque to John Kubicek in honor of his service as National Treasurer of KME for the past four years.

The Auditing and Resolutions Committees gave reports and then elections were held. Rhonda McKee was elected to another four-year term as National Secretary and Cynthia Woodburn was elected to a four-year term as National Treasurer. They were formally installed to their offices by President Tosh.

At this time all paper presenters were called to the front and congratulated. The Awards Committee had completed their task of choosing the top three papers. The presenters of the top three papers (in no particular order) were Jacob Magnusson, John Carr and Kristin Ranum. Each received a check in the amount of \$100. All other paper presenters received \$50 checks.

After National Treasurer John Kubicek distributed travel allowance checks, the meeting adjourned.

**List of Committee Members**

- Auditing Committee: Cynthia Woodburn, KS Alpha; JoAnn Fellin, KS Gamma; Vince Dimicelli, OK Delta; Dusty Peterson, KS Alpha; Danny Noonan, KS Gamma; Jason Vermette, OK Delta

- Awards Committee: Bryan Dawson, TN Gamma; Lanie Schaefer, KS Gamma; Kimberly Moss, MO Alpha; Matthew Dawson, TN Gamma; Steve Shattuck, MO Beta; Bill Sticka, OK Gamma
- Nominating Committee: Al Riveland, KS Delta; Robert Bailey, NY Eta
- Resolutions Committee: Kevin Charlwood, KS Delta; Lurena Erickson, MO Theta; Brian Hollenbeck, KS Beta; Heather Julian, KS Beta
- Paper Selection Committee: Roxanne Back, WI Alpha; Elana Epstein, NY Omicron; Eric West, KS Gamma; Kiersten Purves, WI Alpha; Vicki Bentley, KS Gamma

Summary submitted by Rhonda McKee, National Secretary

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#### Report of the National President

I was installed as president at the national convention in Kerrville, TX, in April, 2005. In the last two years I have attempted to represent Kappa Mu Epsilon, and work toward maintaining and improving the level of service we provide to our constituents.

The main responsibility of the president is to be the contact person for schools wishing to form new chapters. Requests for information about forming new chapters come at various times, and the president has to be ready to provide information and advice in response to these requests. I am hoping to develop our website to help in this regard. In fact, during this convention, we are voting on the petition of a school, Hawaii Pacific University, which made the initial contact, obtained all the forms, and filled them out completely from what was available on our website. I am hoping to make that even easier and more common in the future.

The four chapters that have been installed during the past biennium are:

- Missouri Nu at Columbia College in St. Louis on April 29, 2005
- Maryland Epsilon at Villa Julie College in Stevenson on December 3, 2005
- New Jersey Delta at Centenary College in Hackettstown on December 1, 2006
- New York Pi at Mount Saint Mary College in Newburgh on March 29, 2007

Next week I will be traveling to Edmond, Oklahoma to install Oklahoma Epsilon at Oklahoma Christian University on Friday, April 20, 2007.

In February, 2006, I attended the Association of Collegiate Honor Societies meeting in Portland, OR. KME has been a member of ACHS for many years and the president represents KME at this on alternate years. At the meeting I was able to network with other honor society executives and also meet with companies that offer goods and services to honor societies. That lead to renegotiating the pin pricing and design and we will begin sending out the new pins beginning with this month's initiation reports. We will also be able to sell pins to chapters who want to stock up at a reduced price. I was also able to get a very competitive quote on honor cords that I would be happy to share with chapters that are interested in offering cords to graduates.

Corresponding Secretaries are the heart of KME, and without their faithful service to their students and to KME the society would not be able to function. We are always looking for ways to encourage Corresponding Secretaries and show them our appreciation for what they do. Corresponding Secretaries at this convention will be given laser pens identifying them as Corresponding Secretaries. These pens will be included in certificate orders processed within the next year. It is a small gesture, but we want you to know we appreciate your service to Kappa Mu Epsilon.

I also took over some of the webmaster responsibilities and tried to make the web site more current than it had been. I was partially successful and managed to update information regarding this convention and also some chapter information. I have added several useful forms to the site, such as the constitution and initiation ceremony. We will also be posting the current and past Problem Corner items on the web site. The web site needs some major work and I am happy to announce that Kevin Reed, a KME member and former president of Missouri Theta, has agreed to accept the responsibilities of webmaster. He has web expertise and will be making several changes to the site in the next few months. You may want to check out the site from time to time. Any suggestions you have can be sent to [webmaster@kappamuepsilon.org](mailto:webmaster@kappamuepsilon.org).

I am constantly amazed by the quality of the people who contribute to the success of this organization. It is a pleasure to work with individuals who are so selfless and considerate. I believe that Kappa Mu Epsilon has some of the best servant-leaders that any organization could have and I am grateful for their contributions to KME. I am looking forward to working with them for the last two years of my term as president. One of the people whose services we will dearly miss is that of the outgoing treasurer, John Kubicek. John has served very effectively in his post and decided to exit



in grand style, by hosting this convention. Many thanks to John and the members of Missouri Alpha who helped him prepare for this convention.

Don Tosh  
National President

#### Report of the President Elect

The 36th Biennial Convention at Springfield, MO, April 12-14, 2007, was a success, as measured by the feedback from participants and the quality of the papers presented. My thanks in advance to President Tosh and the other officers for all the advice and guidance they provided, and special thanks to John Kubicek and Missouri Alpha at Missouri State University for being outstanding hosts. While the quality of papers was good, I am concerned about the small number of submissions received, and about the rather low turnout at the convention. The National Council will no doubt address these issues at its Fall 2007 meeting. We will be seeking hosts for regional conventions in the Spring of 2008, and a host for the 37th Biennial Convention during Spring, 2009.

Ron Wasserstein  
President-Elect

#### Report of the National Secretary

Kappa Mu Epsilon, National Mathematics Honor Society initiated 2,705 new members in 122 chapters during the 36th Biennium that ended March 9, 2007. That brings the total membership of KME to 72,893. Eighteen active chapters did not report any initiates during the 36th biennium.

As National Secretary, I receive all initiation reports from chapters, make a record of those reports, up-date mailing list information for corresponding secretaries and forward copies of the reports to other officers. In the fall of each year, I send out supplies to each chapter. The supplies include information brochures, membership cards, a hard copy of the initiation report form and one or two copies of the brochure "A Matter of Honor." I also take minutes of all business meetings of the organization and all meetings of the national council. When a college or university petitions for a new chapter of KME, I send out a summary of the petition to each chapter and receive the chapter ballots.

Rhonda McKee  
National Secretary

## Report of the National Treasurer

36th Biennium (March 21, 2005 – March 20, 2007)

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows end-of-biennium

## BIENNIUM ASSET REPORT

Total Assets (March 20, 2005)		\$61,094.50
Current Assets		
Great Southern Bank		
Checking	59,910.82	
Savings	8,739.57	
Total Current Assets		\$68,650.39

## BIENNIUM CASH FLOW REPORT

Receipts		
Initiation fees received	54,100.00	
Installation fees received	300.00	
Interest income	2,738.14	
Overpayment in	20.00	
Gifts & misc. income	220.00	
Total Biennium Receipts		\$57,378.14
Expenditures		
Association of College Honor Soc	1,889.83	
Administrative expenses	3,075.73	
National Convention expenses	11,267.28	
Regional Convention expenses	1,601.75	
Council meetings travel	1,528.13	
Certificates, jewelry, & shipping	16,943.58	
Installation expenses	1.00	
Overpayment returns	00.00	
Pentagon expenses	13,385.95	
Miscellaneous expenses	129.00	
Total biennium Expenses		\$49,822.25
Biennium Cash Flow		+ \$7,555.89

The 36th biennium Cash Flow was + \$ 7,555.89. We continue to realize the savings from having the pins and membership certificates processed through Evangel University. We have maintained our goal of maintaining assets of at least \$40,000. The financial condition of Kappa Mu Epsilon is sound.

I want to thank the National Officers for their continued efforts to keep Kappa Mu Epsilon expenses at a relative low level. Their watchfulness over expenses has been a tremendous help to my work as National Treasurer. I wish Kappa Mu Epsilon continued success.

John Kubicek  
National Treasurer.

#### Report of the Pentagon Editor

*The Pentagon*, introduced in 1941, is the official publication of Kappa Mu Epsilon. Publication of student papers continues to be the central theme of *The Pentagon*. Continuing a tradition, papers given "Top 4 Status" by the Awards Committee at the KME National Convention are guaranteed an opportunity to be published. Many other student papers presented at the KME National and regional conventions are included as well. The key to having one's paper published in *The Pentagon* is submission of all of the necessary materials. For additional information about having an article published in *The Pentagon*, please see the sheet entitled "Information for Authors".

All new initiates receive a two-year subscription to *The Pentagon* and can continue their subscriptions for \$5.00 per year. Having a current address is obviously vital in insuring that you receive your copy. Please check the address on your most recent issue and e-mail corrections to the Business Manager if necessary at toshd@evangel.edu. Don Tosh has undertaken the laborious task of updating the mailing list, and discussion is underway about the possibility of establishing an on-line database, which would, among other benefits, make it easier to keep this mailing list current.

Manuscripts received by *The Pentagon* other than those presented at our conventions are refereed by faculty volunteers. Over the past several months, dozens of referees have volunteered. The service of these individuals is invaluable.

Another regular feature of *The Pentagon* is the Problem Corner, to which you are encouraged to submit problems and solutions. After a transition period of approximately two years, during which remaining problems and solutions were dealt with jointly by outgoing longtime Prob-

lem Corner Editor Ken Wilke and incoming Problem Corner Editor Pat Costello, the Problem Corner is now fully under the direction of Pat Costello. It is already apparent that Pat's enthusiasm for and attention to the Problem Corner will ensure its continuation as a quality component of The Pentagon. Thank you, Pat.

I'd like to extend many thanks to Connie Shrock, National Historian. I can practically set my clock by the arrival of her e-mails containing the Chapter News. The care she brings to the job is obvious in the quality of the materials she sends. Thank you as well to the regional directors for contributing reports of the regional conventions and forwarding student papers from the conventions, and to the National Committee for its assistance with the many details crucial to the success of *The Pentagon*.

Finally, a special thank you to Rich Laird and Don Tosh, outgoing and incoming Business Managers, respectively. They are the ones who get the Pentagon where it is supposed to go.

Charles Curtis  
Pentagon Editor

#### Report of the Pentagon Business Manager

One item that has taken considerable effort is the transition in The Pentagon business manager position. The new editor, Charles Curtis, is doing an excellent job, and we are thankful for all the time and effort he puts into the publication. However, the business manager transition has been a challenge. The replacement manager became overwhelmed with the responsibilities of the position. Negotiating reduced postal rates with the post office was time consuming and frustrating. Entering the 1200 annual new names and addresses into the database became smothering and didn't get done. I took over the responsibilities in November of 2006 with secretarial support was able to get most of the new names entered and arrangements made with the post office in Springfield. However, there was some damage to the database and we are still trying to correct all the errors in the mailing list of about 4000 names and addresses. The Fall, 2006, issue was mailed out in March and we expect that the Spring, 2007, issue will go out on time. If you were supposed to have received the fall issue and didn't, please contact me and I will check to make sure your mailing information is correct in our database. Getting most of the errors out of the mailing list is going to be a time consuming and ongoing process.

Don Tosh  
Pentagon Business Manager

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## *Kappa Mu Epsilon National Officers*

Don Tosh	Department of Science and Technology Evangel University 1111 N. Glenstone Avenue Springfield, MO 65802 toshd@evangel.edu	<i>President</i>
Ron Wasserstein	262 Morgan Hall Washburn University 1700 SW College Avenue Topeka, KS 66621 ron.wasserstein@washburn.edu	<i>President-Elect</i>
Rhonda McKee	Department of Mathematics University of Central Missouri Warrensburg, MO 64093-5045 mckee@ucmo.edu	<i>Secretary</i>
Cynthia Woodburn	Department of Mathematics Pittsburg State University Pittsburg, KS 66762-7502 cwoodbur@pittstate.edu	<i>Treasurer</i>
Connie Schrock	Department of Mathematics Emporia State University Emporia, KS 66801-5087 schrockc@emporia.edu	<i>Historian</i>
Kevin Reed	Department of Science and Technology Evangel University 1111 N. Glenstone Avenue Springfield, MO 65802	<i>Webmaster</i>

KME National Website:  
<http://www.kappamuepsilon.org/>

## *Active Chapters of Kappa Mu Epsilon*

*Listed by date of installation*

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 March 1935
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Alpha	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960

MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986

TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February 2001
GA Delta	Berry College, Mount Berry	21 April 2001
TX Mu	Schreiner University, Kerrville	28 April 2001
NJ Gamma	Monmouth University	21 April 2002
CA Epsilon	California Baptist University, Riverside	21 April 2003
PA Rho	Thiel College, Greenville	13 February 2004
VA Delta	Marymount University, Arlington	26 March 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 February 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 March 2005
SC Epsilon	Francis Marion University, Florence	18 March 2005
PA Sigma	Lycoming College, Williamsport	1 April 2005
MO Nu	Columbia College, Columbia	29 April 2005
MD Epsilon	Villa Julie College, Stevenson	3 December 2005
NJ Delta	Centenary College, Hackettstown	1 December 2006
NY Pi	Mount Saint Mary College, Newburgh	20 March 2007