THE PENTAGON

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Going Up?

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Elevator technology has been evolving for about 4,600 years. In 2600 BC, Egyptians used hoists to move 200,000 pound blocks that were used to build the pyramids, some of which stand over 500 feet tall. Around 80 AD, crude elevators were used in the coliseum in Rome to lift gladiators and wild animals up to the arena. The first stream powered elevator was made in the early 1800s. The first hydraulic elevator was made in 1878. In 1889 Otis Elevators introduced electric elevators. Today elevators are everywhere. Otis Elevators' web page states that "elevators move the equivalent of the world's population every three days."

The intent of this discussion is to develop a simple model of a single elevator's travel time within a more complex bank of elevators servicing a medium range, high rise building.

The first step of our process is to develop an acceptable velocity curve $v(t)$ for an elevator traveling from some starting point to some later stopping point. An appropriate velocity graph for our elevator might look like that in Figure 1.
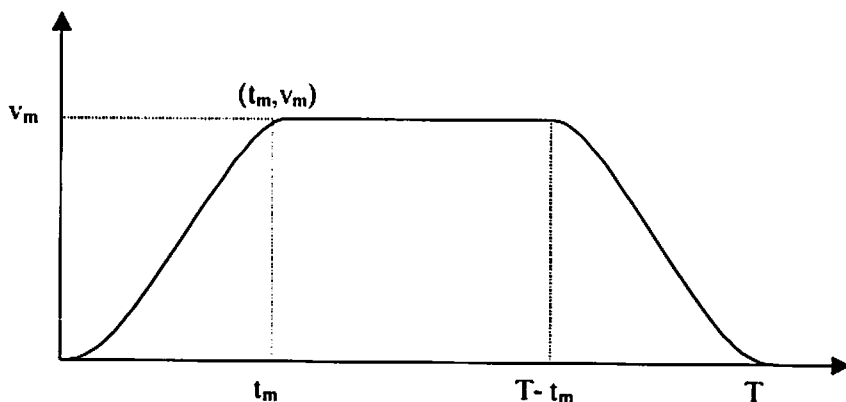
Figure 1

v_m =elevator maximum velocity.
t_m =time required for elevator to reach maximum velocity.
T =total time from start to stop.

Note that, starting from $t = 0$, the elevator's velocity starts out smoothly, gradually increases, smooths out to a constant where $t = t_m$, remains constant until $T - t_m$ and begins its deceleration in the reverse motion of its acceleration and stops at time T.

To develop a suitable equation for the velocity function $v(t)$, $0 \le t \ge T$ we make several assumptions.

1. $v(0) = v(T) = 0$ The elevator starts from rest and ends at rest seconds later.)

2. As t goes from 0 to t_m the velocity of the elevator can be effectively approximated by a cubic equation, $v(t) = bt^3 + ct^2 + dt + e$.

3. $v'(0) = v'(t_m) = 0$ (To insure a smooth start and a smooth transition to maximum velocity.)

4. As t goes from t_m to $T - t_m$ the velocity of the elevator remains at the constant maximum velocity v_m, which equals $v(t_m)$.

5. As t goes from $T - t_m$ to T the elevator slows down with a motion which "reverses" the motion described in (2) above.

Using the cubic velocity equation in (2) and $v(0) = v'(0) = 0$ we quickly get $d = e = 0$. Using $v(t_m) = v_m$ and $v'(t_m) = 0$ we get $b = \frac{-2v_m}{t_m^3}$ and $\frac{3v_m}{t_m^2}$.

Accordingly, the cubic equation for $v(t)$ is

$$v(t) = \frac{-2v_m}{t_m^3} t^3 + \frac{3v_m}{t_m^2} t^2 \text{ for } 0 \le t \le t_m.$$

To determine the "slowing down" velocity function, we need only reflect the "speeding up" velocity function across the vertical axis to get $v(-t)$ and then translate the result T units to the right to get $v(T - t)$. The resulting velocity function is:

$$v(t) = \frac{-2v_m}{t_m^3} (T - t)^3 + \frac{3v_m}{t_m^2} (T - t)^2 \text{ for } T - t_m \le t \le T.$$

The first step in our process is complete. The equation of the velocity for $0 \le t \le t_m$ is:

$$v(t) = \begin{cases} \frac{-2v_m}{t_m{}^3}t^3 + \frac{3v_m}{t_m{}^2}t^2 & 0 \le t \le t_m \\ v_m & t_m \le t \le (T - t_m) \\ \frac{-2v_m}{t_m{}^3}(T - t)^3 + \frac{3v_m}{t_m{}^2}(T - t)^2 & (T - t_m) \le t \le T \end{cases}$$

The next step is developing a position function, $s(t)$, which equals the distance traveled by the elevator at time t seconds, while traveling from one floor to another. To find $s(t)$ we need only anti-differentiate $v(t)$ while requiring that $s(t)$ be continuous on the whole interval from $0 \le t \le T$ and thus at the particular values t_m and $T - t_m$. Although this process is not difficult, it is very messy and the details are excluded.

The position function simplifies to:

$$s(t) = \begin{cases} \frac{-v_m}{2t_m{}^3}t_4 + \frac{v_m}{t_m{}^2}t_3 & 0 \le t \le t_m \\ v_m t - \frac{v_m t_m}{2} & t_m \le t \le (T - t_m) \\ \frac{v_m}{2t_m{}^3}(T - t_m)^4 - \frac{v_m}{t_m{}^2}(T - t_m)^3 \\ \quad + (T - t_m)v_m & (T - t_m) \le t \le T \end{cases}$$

Now that the velocity and position functions have been developed for the elevator, the running time, $r(D)$, can now be developed. $r(D)$ will represent the running time, in seconds, for an elevator to move D feet with the elevator starting and ending at rest.

The development of $r(D)$ comes from looking at $s(t)$ and determining the relationship between the distances traveled by the elevator and the amount of time it takes to cover those distances. The first step comes from finding the distance the elevator travels during its acceleration and deceleration times. We first look at the distance traveled while the elevator is accelerating. To do this we evaluate $s(t_m)$, i.e. plug t_m into the equation $\frac{-v_m}{2t_m{}^3}t^4 + \frac{v_m}{t_m{}^2}t^3$. The answer simplifies to $\frac{v_m t_m}{2}$. Since the acceleration motion is equal to the deceleration, the elevator also travels $\frac{v_m t_m}{2}$ feet while decelerating. The total distance traveled while the elevator speeds up and slows down is $v_m t_m$. The remaining distance traveled by the elevator is $D - v_m t_m$. The time it will take the elevator to travel this remaining distance is $\frac{D - v_m t_m}{v_m}$, it is divided by v_m because at this point the elevator is traveling at its maximum constant velocity, v_m. Now to find the running time, $r(D)$, we add the time it takes to accelerate (t_m) to the time it travels at constant velocity $\left(\frac{D - v_m t_m}{v_m}\right)$ to the time it takes to decelerate (t_m). $r(D) = \frac{D - v_m t_m}{v_m} + 2t_m$ which simplifies to $r(D) = \frac{D}{v_m} + t_m$. Now this

equation will hold as long as the total distance that is traveled by the elevator (D) is at least the distance traveled while the elevator is speeding up and slowing down $(v_m t_m)$.

If D is less than $v_m t_m$, then there is no period where the elevator is traveling at its constant maximum velocity. This means basically that the elevator will have a period of acceleration followed immediately by a period of deceleration. This change from acceleration to deceleration will occur at some time T_0 where $0 \leq T_0 \leq t_m$ and, in our model, could cause a slight "jolt" since we are losing our gradual change of acceleration in the cubic equation. T_0 is multiplied by 2 to account for both the speeding up and slowing down of the elevator. Therefore

$$r(D) = \begin{cases} \frac{D}{v_m} + t_m & for \quad D \geq v_m t_m \\ 2T_0 & for \quad D < v_m t_m \end{cases}$$

We have now completed the development of our running time formula $r(D)$. Note that $r(D)$ will be used to determine the time it takes for an elevator to travel D feet, starting and ending at rest. This formula will be used repeatedly in the remaining analysis.

We return to the general elevator problem.

Recall that we are considering only a single elevator within a larger bank of elevators. Accordingly, our elevator will only service a specific set of N consecutive floors of the building. We will denote the floors beyond the main floor #1, #2, #3, etc. If we let $\#L$ be the lowest floor served by our elevator then the N floors serviced are $\#L, \#L+1, \#L+2, ...\#L+N-1$. Since not <u>all</u> of those N floors will be served on a given run of the elevator, we must try to determine the typical or average total time for the elevator to complete a passenger run. More particularly, we will determine the time required to:

1. Pick up the passengers at the main floor.

2. Move the elevator to the lowest floor being served where at least 1 passenger will get off.

3. Continue moving upwards, unloading the passengers as needed.

4. Return to the main floor after the last passenger has left the elevator.

The methods we use are motivated by Bruce Powell's work in an article "Mathematical Modeling of Elevator Systems." Powell's article was based on work done for the Research and Development Center of the Westinghouse Corporation. Westinghouse is involved in the elevator industry. In

some cases we will parrot Powell's approach. In other cases we modify or extend his results. For example, in the development of our elevator velocity function, we used a cubic equation. Powell instead used a linear equation. Our velocity curve was much more difficult to develop, is likely more accurate, and is certainly more mathematically interesting than Powell's. Of course the small differences resulting from the two approaches would be insignificant in the context of the solution to the whole modeling problem with which we are dealing.

The general formula we will develop for the total time required for the elevator to make a round trip run, loading and unloading passengers, will be developed in 5 segments which we will symbolize $T_1, T_2, T_3, T_4,$ and T_5.

T_1 will represent the time, in seconds, to load each passenger at the main entrance and unload them at their destination floors. We let P denote the number of passengers that are loaded on a given run; l (seconds) the loading time for each passenger; u (seconds) the unloading time for each passenger. Since each passenger requires $l + u$ seconds to load and unload and there are P passengers, we have:

$$T_1 = P \cdot (l + u) .$$

T_2 represents the running time for the elevator to reach the first floor say $\#L + b (b \in \{0, 1, ... N - 1\})$ being served on a given run. Suppose that the distance between any two consecutive floors in the building is f feet. Then to reach floor $\#L + b$ the elevator must travel $(L + b) \cdot f$ feet. We apply our running time formula to obtain:

$$T_2 = r [(L + b) \cdot f]$$

T_3 is the time it takes to open and close the doors when unloading passengers at each stop. We let d (seconds) denote the time it takes to open and close the doors at the main entrance and on an unloading floor. We will mirror Powell's approach, developing an estimate of the number of stops needed to unload the P passengers on a given run. Of course the number of stops will change from run to run.

We list a logical sequence of probabilities:
Prob(a passenger randomly chooses Floor $\#L + i$ out of N floors serviced) $= \frac{1}{N}$.

Prob(a passenger does not chose Floor $\#L + i$) $= 1 - \frac{1}{N}$.

Prob(none of the P passengers choose Floor $\#L + i$) $= \left(1 - \frac{1}{N}\right)^P$.

Prob(at least one passenger chooses Floor $\#L + i$) $= 1 - \left(1 - \frac{1}{N}\right)^P$.

If we introduce the symbol S_r to represent the number of stops on a given run then:

$$S_r = \sum_{k=1}^{N}\left[1 - \left(1 - \frac{1}{N}\right)^P\right]$$

Since the probability of stopping at a given floor is the same for all N floors, we are summing a series of constants, so:

$$S_r = N\left[1 - \left(1 - \frac{1}{N}\right)^P\right]$$

Since S_r represents the expected number of stops on a given run, then (including the main floor) the elevator doors open and close approximately $S_r + 1$ times on a given run. If we assume that each stop requires d seconds for opening and closing the doors, then:

$$T_3 = d \cdot (S_r + 1)$$

T_4 represents the running time from the highest floor served, down to the main floor. Just as the number of stops varies from run to run, so also does the highest floor served. We again mirror Powell's approach and estimate the expected number of floors (N_r) reached on a given run beyond Floor $\#L$, as the P passengers are unloaded. Note $0 \le N_r < N$. The probability that a single passenger gets off the elevator at or before the kth floor serviced is $\frac{k}{N}$. The probability that all P passengers get off at or before the Kth floor serviced is $\left(\frac{k}{N}\right)^P$. Keeping this in mind, realize that N_r "is equivalent to the largest value in a sample of size P from a discrete uniform distribution with replacement," so:

Prob $(N_r = k)$ = Prob(largest value = k)

= Prob(largest value $\le k$) − Prob(largest value $\le (k-1)$

So, $N_r = \sum_{k=1}^{N} k\left[\left(\frac{k}{N}\right)^P - \left(\frac{k-1}{N}\right)^P\right]$ when we sum over the N floors.

Recall that the first floor being serviced by this elevator on any run is Floor $\#L$. There are an estimated $N_r - 1$ more floors beyond Floor $\#L$ to the top floor being serviced on this run (although the elevator stops at only S_r of these floors). Recall the distance between floors is f feet. Here the elevator travels $[L + (N_r - 1)] \cdot f$ feet as the elevator descends to the main floor from its highest point. Accordingly, the running time formula, $r(D)$, gives the result:

$$T_4 = r\left[(L + (N_r - 1)) \cdot f\right]$$

We have one step left in our process. We need to determine the run time T_5 of the elevator as it moves from stop to stop as it unloads passengers. Recall there are an expected S_r stops over an expected range of N_r

floors. We let Floor $\#L_r$ be the lowest expected floor at which passengers are unloaded. The general discussion gets rather involved so we illustrate with a simplified example. Consider starting floor, Floor $\#L = \#11$, $S_r=5$ and $N_r=15$. On this run our elevator goes from Floor #11 to Floor #25 making 5 stops. One of those stops is at Floor #11 and one is at Floor #25 and the other 3 are somewhere in the remaining 13 floors. Using the combination formula C(13,3) we find that there are 286 different possible "stop-configurations."

Two of these stop-configurations are sketched in Figure 2.

	#25	
	#24	
	#23	
	#22	
	#21	
	#20	
	#19	
	#18	
	#17	
	#16	
	#15	
	#14	
	#13	
	#12	
	#11	
C1		C2
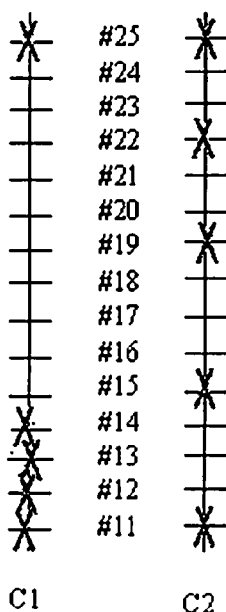
Figure 2

Configuration C1 has the elevator stopping at Floors #11, #12, #13, #14 and #25. Configuration C2 at #11, #15,#19, #22, and #25.

It can be argued that configuration C1 yields the fastest run time of all 286 possible configurations and that C2 yields the slowest possible run time. A reasonable approximation for T_5 (expected total run time) would be to average these two extremes.

C1 is a configuration in which the elevator travels the maximum total distance at <u>maximum</u> velocity. Although other configurations may involve the same total run time as C1, none of the other 285 run times can be faster than that of C1.Using our run time formula $r(D)$ with f feet floor distance we get:

$$C1 \text{ Run Time} = 3r(f) + r(11f)$$

In the general case the fastest possible run-time will occur with a configuration with $S_r - 2$ stops immediately after Floor #L_r followed by an uninterrupted run to the top floor, #$L_r + (N_r - 1)$.

Fastest run time $= (S_r - 2) \cdot r(f) + r[(N_r - S_r + 1) \cdot f]$

Configuration C2 in Figure 2 is a configuration in which the elevator travels the <u>minimum</u> total distance at maximum velocity. Although other configurations may involve the same total run time as C2, none of the other 285 configurations can be slower than C2. The key to insuring the slowest possible run time is to position the stops at floors, which will equalize, as much as possible, the number of floors between stops.

$$C2 \text{ run time} = 2r(4f) + 2r(3f)$$

In our generalization we will use an integer approximation for our expected values N_r and S_r. We let $\overline{N_r} = $ the first integer greater than or equal to N_r, and $\overline{S_r} = $ the first integer greater than or equal to S_r.

Generalizing this formula is rather complicated. There are $N_r - 1$ floors each separated by f feet. We wish to equalize our elevator's stop-to-stop runs as much as possible. We divide $\overline{N_r} - 1$ by $\overline{S_r} - 1$ and get a quotient of q and a remainder of z where $0 \leq z \leq \overline{S_r} - 1$ (using the division algorithm). The quotient q is the number of floors covered between the last $\overline{S_r} - 1$ elevator stops, but there are z floors remaining which must be "distributed uniformly." We do so by adding 1 to q, z times.

Shortest run time $= z \cdot r[(q + 1) \cdot f] + (\overline{S_r} - 1 - z) \cdot r[q \cdot f]$

We determine by averaging the two extremes:
$$T_5 = \frac{(S_r - 2) \cdot r(f) + r[(N_r - S_r + 1) \cdot f] + z \cdot r[(q+1) \cdot f] + (\overline{S_r} - 1 - z) \cdot r[q \cdot f]}{2}$$
where $\overline{N_r} - 1 = q \cdot (\overline{S_r} - 1) + z$ and $0 \leq z \leq \overline{S_r} - 1$.

It is interesting to note that Powell simply used what we called the "Fastest Run Time" in calculating <u>his</u> run time when unloading passengers. He ignored the fact that there are many different stop-configurations, which may produce different run times, all somewhat slower than his run time. In his defense, it can be argued that if an elevator could reach maximum speed in half of the floor-to-floor distance f then all stop-configurations would yield exactly the same total run times - but this is a rather unrealistic assumption. It is also true that the difference between the fastest and slowest stop-configuration run times is likely insignificant when taken in the context of all the other simplifying assumptions used in the real world modeling problem.

Summing up the segments of our formula gives the following:

$$\text{Total Run Time} = T_1 + T_2 + T_3 + T_4 + T_5$$
$$= P \cdot (l + u) + r\left[(L + b) \cdot f\right]$$
$$+ d \cdot (S_r + 1) + r\left[L + (N_r - 1)) \cdot f\right]$$
$$+ \frac{(S_r - 2) \cdot r(f) + r[(N_r - S_r + 1) \cdot f] + z \cdot r[(q+1) \cdot f] + \left(S_r - 1 - z\right) \cdot r[q \cdot f]}{2}$$

where $\overline{N_r} - 1 = q \cdot \left(\overline{S_r} - 1\right) + z; 0 \leq z \leq \overline{S_r} - 1;$

$r(D) = $ running time for the elevator to travel D feet;

$P = $ number of passengers loaded on a given run;

$l = $ loading time for each passenger;

$u = $ unloading time for each passenger;

$f = $ distance between any two consecutive floors;

$S_r = $ estimated number of stops on a given run;

$\overline{S_r} = $ the first integer greater than or equal to S_r;

$N_r = $ estimated number of floors reached beyond Floor #L;

$\overline{S_r} = $ the first integer greater than or equal to N_r.

This concludes our discussion on the development of a model of a single elevator's travel time while servicing a specified number of floors of a medium range high rise building. One can now extend the results of this model to examine the much more complex modeling problem involving a sequence, or bank, of elevators, which service all the floors of the building.

Perhaps a new high rise building is being designed. In the extended model one can investigate how many elevators would be needed to handle a peak service time (rush hour). What speeds and capacities would be needed?

Perhaps a high rise building is in place with elevators already installed. In the extended model one could determine an elevator scheduling assignment which would most effectively service the entire building.

The model we have completed is just a small first step. An interested reader can learn much more about these difficult questions by referring to (1) and (2) in the References.

References

1. Giordano, Frank R., Weir, Maurice D., Elevator Problem, *A First Course in Mathematical Models*, Brooks/Cole Publishing Company, 1985

2. Powell, Bruce A., Mathematical Modeling of Elevator Systems, *Case Studies in Mathematical Modeling*, Pittman Publishing Inc., 1981.

An Investigation of Elliptic Curves to Find Solutions to Special Cubic Equations in Three Variables

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Fermat's Last Theorem states that the equation $a^n + b^n = c^n$ where a, b, c ϵ has no solutions for $n \geq 3$. This result has been proven in general, but it has been known for quite some time in the $n = 3$ case that there are no solutions. What if we introduce a constant integer coefficient, call it f, for the c^3 term (i.e., $a^3 + b^3 = f * c^3$)? Does this have any solutions? That was the motivation for this investigation and it has lead to quite a few places that I never would have dreamed of going along with some unanswered topics for future consideration.

Part I: A "Simple" Example

Let's start off with a simple example:

$$x^3 + y^3 = 7 \tag{1}$$

where $x, y \notin Q$. We can see that $(x, y) = (2, -1)$ is a solution to this equation, which we shall use in a bit. We now take a look at the graph of this equation:
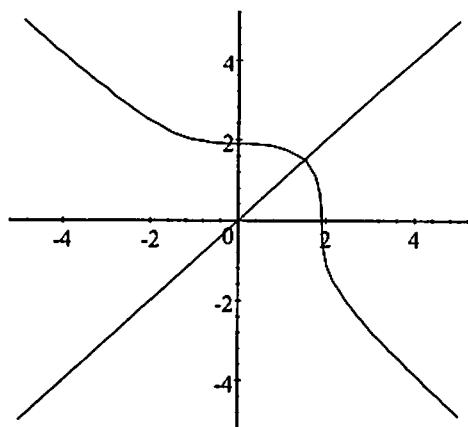
From this, we see two things. First, $(2, -1)$ is on the graph of this curve and that it is symmetric about the line $y = x$.

In general, when we draw a tangent line through a point on a cubic curve, it will cross the curve in at most three places. If we found the tangent line to our cubic equation (1), it would cross (probably) in only two points as we need to remember that this point will be a double root of our equation. The reason for this comes from calculus as if we imagine a line parallel to the tangent line that cuts the curve in three places. As we move this line closer and closer to the point of tangency, the points will merge and we will have a double root, just like in the case of $y = x^2$ at $x = 0$.

Now, we shall find the tangent line at $(2, -1)$ by doing some differentiation.

We have:

$$x^3 + y^3 = 7$$

Which, when differentiated implicitly yields

$$3x^2 + 3y^2 \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x^2}{y^2} \tag{2}$$

The equation of the tangent line at this point is of the form $y = mx + b$, therefore,

$$y = \frac{dy}{dx} x + b$$

$$\Rightarrow -1 = \frac{-(2)^2}{1^2} (2) + b$$

$$\Rightarrow b = 7$$

$$\Rightarrow y = -4x + 7 \tag{3}$$

Let's look at the graph with the tangent line drawn in now:
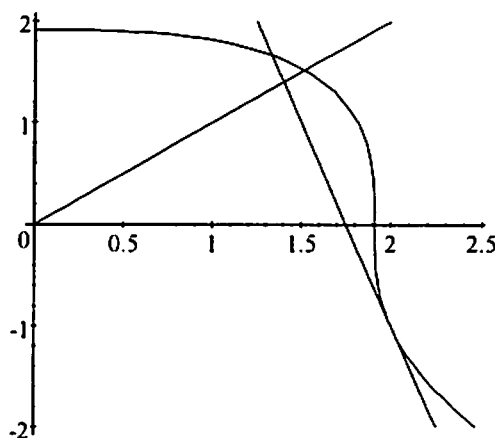
We now want to find where the tangent line and the curve intersects. This is done by substituting (3) into (1) and solving the equation for x. This would be hard, but we already know what two of the roots are as at the point of intersection of the tangent line with the curve yields a double root. Doing this, we now have:

$$x^3 + (-4x + 7)^3 = 7$$

$$\Rightarrow x^3 + \left(-64x^3 + 336x^2 - 588x + 343\right) = 7$$

$$\Rightarrow -63x^3 + 336x^2 - 588x + 336 = 0 \qquad (4)$$

I claim that $x = 2$ is a double root of this equation. To verify this, we perform synthetic division upon (4):

As a check, by performing a Simplify[ ] on (4), *Mathematica* gives us

$$(-2 + x)^2 (-4 + 3x) = 0 \qquad (5)$$

Which tells us that we do have double roots at $x = 2$ and that the third root is $x = \frac{4}{3}$. I am going to introduce the notation x to represent this third root. We now need to find the y coordinate that corresponds to this x coordinate, call it y. Since we have a nice equation for y (i.e., (3)), we shall use it to determine y. We now have:

$$y = -4\left(\frac{4}{3}\right) - 7$$

$$\Rightarrow y = \frac{5}{3}$$

Therefore, $(x, y) = \left(\frac{4}{3}, \frac{5}{3}\right)$, but this point is going to be part of an abelian group with the operation $\oplus$ and in order to preserve associativity, we must reflect these coordinates in $y = x$. This gives the (x, y) coordinate of this new point, $p \oplus p$, of $\left(\frac{5}{3}, \frac{4}{3}\right)$ where p was our first coordinate $(2, -1)$. Using this new operation, we can generate as many solutions as we want following the same process as before. As a check, let's verify that $2p$ (i.e., $p \oplus p$) is a solution to (1) before proceeding.

$$\left(\frac{5}{3}\right)^3 + \left(\frac{4}{3}\right)^3 = \frac{125}{27} + \frac{64}{27} = 7$$

Now, let's find $3p$ (i.e., $((p \oplus p) \oplus p)$). Since we have two points, $\left(\frac{5}{3}, \frac{4}{3}\right)$ and $(2, 1)$, the slope is:

$\frac{-\frac{7}{3}}{\frac{1}{3}} = -7$

$\Rightarrow y = -7x + b$

$\Rightarrow b = y + 7x$

$\Rightarrow b = \frac{4}{3} + 7 * \frac{5}{3} = 13$

$\Rightarrow y = -7x + 13$

Now, find the intersection on $x^3 + y^3 = 7$:

$\Rightarrow x^3 + (-7x + 13)^3 = 7$

Using $Mathematica$, we get

$$2190 - 3549x + 1911x^2 - 342x^3 = 0$$

Now, we make the cubic term's coefficient 1 and we have:

$$x^3 - \frac{1911}{342}x^2 + \frac{3549}{342}x - \frac{2190}{342} = 0$$

which becomes $(-2 + x)(-5 + 3x)(-73 + 38x) = 0$ as we used two of the roots to generate this equation.

$\Rightarrow x = \frac{73}{38}$

$\Rightarrow y = \frac{17}{38}$

Now, switching the coordinates, we get $((p \oplus p) \oplus p)$, which is $\left(-\frac{17}{38}, \frac{73}{38}\right)$. Let's check and see if this really is a solution to our equation:

$$\left(-\frac{17}{38}\right)^3 + \left(\frac{73}{38}\right)^3 = \frac{4913}{54872} + \frac{389017}{54872} = 7$$

Therefore, $3p$ is our new addition is $\left(-\frac{17}{38}, \frac{73}{38}\right)$. This process can be

repeated to continually find new solutions to $x^3 + y^3 = 7$. Since all of these solutions are rational and if we clear the denominators of x and y, we are actually finding solutions to the equation $a^3 + b^3 = 7c^3$ where $a, b, c \epsilon Z$!

Part II: $x^3 + y^3 = L$ for Generic Points

Let's now develop the equation for two generic points p and q on the curve with the $\oplus$ operator. We shall denote p as the point (x_1, y_1) and q as (x_2, y_2) where $p, q \epsilon Q$. The equation for the tangent line through p and q will be:

$$y = mx + b \text{ where } m = \frac{y_2 - y_1}{x_2 - x_1}$$

At p, b will be:

$$y_1 = \frac{y_2 - y_1}{x_2 - x_1} x_1 + b$$

$$b = y_1 - \frac{y_2 - y_1}{x_2 - x_1} x_1$$

Therefore, the general equation is:

$$y = \frac{y_2 - y_1}{x_2 - x_1} x + y_1 - \frac{y_2 - y_1}{x_2 - x_1} x_1$$

Plugging this into our original equation (1) yields

$$x^3 + \left(\frac{y_2 - y_1}{x_2 - x_1} x + y_1 - \frac{y_2 - y_1}{x_2 - x_1} x_1 \right)^3 = L$$

This becomes (with the help of *Mathematica*):

$$-L + x^3 + y_1^3 - \frac{x^3 y_1^3}{(-x_1+x_2)^3} + \frac{3x^2 x_1 y_1^3}{(-x_1+x_2)^3} - \frac{3xx_1^2 y_1^3}{(-x_1+x_2)^3} + \frac{x_1^3 y_1^3}{(-x_1+x_2)3} +$$

$$\frac{3x^2 y_1^3}{(-x_1+x_2)^2} - \frac{6xx_1 y_1^3}{(-x_1+x_2)^2} + \frac{3x_1^2 y_1^3}{(-x_1+x_2)^2} - \frac{3xy_1^3}{-x_1+x_2} + \frac{3x_1 y_1^3}{-x_1+x_2} +$$

$$\frac{3x^3 y_1^2 y_2}{(-x_1+x_2)^3} - \frac{9x^2 x_1 y_1^2 y_2}{(-x_1+x_2)^3} + \frac{9xx_1^2 y_1^2 y_2}{(-x_1+x_2)^3} - \frac{3x_1^3 y_1^2 y_2}{(-x_1+x_2)^3} - \frac{6x^2 y_1^2 y_2}{(-x_1+x_2)^2} +$$

$$\frac{12xx_1 y_1^2 y_2}{(-x_1+x_2)^2} - \frac{6x_1^2 y_1^2 y_2}{(-x_1+x_2)^2} + \frac{3xy_1^2 y_2}{-x_1+x_2} - \frac{3x_1 y_1^2 y_2}{-x_1+x_2} - \frac{3x^3 y_1 y_2^2}{(-x_1+x_2)^3} +$$

$$\frac{9x^2 x_1 y_1 y_2^2}{(-x_1+x_2)^3} - \frac{9xx_1^2 y_1 y_2^2}{(-x_1+x_2)^3} + \frac{3x_1^3 y_1 y_2^2}{(-x_1+x_2)^3} + \frac{3x^2 y_1 y_2^2}{(-x_1+x_2)^2} - \frac{6xx_1 y_1 y_2^2}{(-x_1+x_2)^2} +$$

$$\frac{3x_1^2 y_1 y_2^2}{(-x_1+x_2)^2} + \frac{x^3 y_2^3}{(-x_1+x_2)^3} - \frac{3x^2 x_1 y_2^3}{(-x_1+x_2)^3} + \frac{3xx_1^2 y_2^3}{(-x_1+x_2)^3} - \frac{x_1^3 y_2^3}{(-x_1+x_2)^3}$$

$\Longrightarrow \frac{1}{(x_1-x_2)^3}(-Lx_1^3 + x^3x_1^3 + 3Lx_1^2x_2 - 3x^3x_1^2x_2 - 3Lx_1x_2^2+$
$3x^3x_1x_2^2 + Lx_2^3 - x^3x_2^3 + x^3y_1^3 - 3x^2x_2y_1^3 + 3xx_2^2y_1^3 - x_2^3y_1^3-$
$3x^3y_1^2y_2 + 3x^2x_1y_1^2y_2 + 6x^2x_2y_1^2y_2 - 6xx_1x_2y_1^2y_2 - 3xx_2^2y_1^2y_2+$
$3x_1x_2^2y_1^2y_2 + 3x^3y_1y_2^2 - 6x^2x_1y_1y_2^2 + 3xx_1^2y_1y_2^2 - 3x^2x_2y_1y_2^2+$
$6xx_1x_2y_1y_2^2 - 3x_1^2x_2y_1y_2^2 - x^3y_2^3 + 3x^2x_1y_2^3 - 3xx_1^2y_2^3 + x_1^3y_2^3)$

Since $y_1^3 = L - x_1^3$ and $y_2^3 = L - x_2^3$, these are replaced and we get:

$Lx_1^3 - x^3x_1^3 - x^3(L - x_1^3) - 3Lx_1^2x_2 + 3x^3x_1^2x_2 + 3x^2(L - x_1^3)x_2+$
$3Lx_1x_2^2 - 3x^3x_1x_2^2 - 3x(L - x_1^3)x_2^2 - Lx_2^3 + x^3x_2^3+$
$(L - x_1^3)x_2^3 + x^3(L - x_2^3) - 3x^2x_1(L - x_2^3) + 3xx_1^2(L - x_2^3)-$
$x_1^3(L - x_2^3) + 3x^3y_1^2y_2 - 3x^2x_1y_1^2y_2 - 6x^2x_2y_1^2y_2 + 6xx_1x_2y_1^2y_2+$
$3xx_2^2y_1^2y_2 - 3x_1x_2^2y_1^2y_2 - 3x^3y_1y_2^2 + 6x^2x_1y_1y_2^2 - 3xx_1^2y_1y_2^2+$
$3x^2x_2y_1y_2^2 - 6xx_1x_2y_1y_2^2 + 3x_1^2x_2y_1y_2^2$

Finally, performing a Factor[%]on this yields:

$(-x + x_1)(x - x_2)Lx_1 - Lx_2 - xx_1^2x_2+$
$xx_1x_2^2 - xy_1^2y_2 + x_2y_1^2y_2 + xy_1y_2^2 - x_1y_1y_2^2$

As we can see, x_1 and x_2 are roots of this equation and our x will be where

$(Lx_1 - Lx_2 - xx_1^2x_2 + xx_1x_2^2 - xy_1^2y_2 + x_2y_1^2y_2 + xy_1y_2^2 - x_1y_1y_2^2)$

is equal to zero. Once again, we use *Mathematica* and find that:

$$x = \frac{-Lx_1 + Lx_2 - x_2y_1^2y_2 + x_1y_1y_2^2}{-x_1^2x_2 + x_1x_2^2 - y_1^2y_2 + y_1y_2^2}$$

This means that y will be:

$$y = mx + b$$

$$\Longrightarrow y = \frac{y_2 - y_1}{x_2 - x_1}\left(\frac{-Lx_1 + Lx_2 - x_2y_1^2y_2 + x_1y_1y_2^2}{-x_1^2x_2 + x_1x_2^2 - y_1^2y_2 + y_1y_2^2}\right) + y_1 - \frac{y_2 - y_1}{x_2 - x_1}x_1$$

Finally, to find the (x, y) coordinates of $p \oplus q$ we switch x and y and find that:

$$p \oplus q = \frac{y_2 - y_1}{x_2 - x_1}\left(\frac{-Lx_1 + Lx_2 - x_2y_1^2y_2 + x_1y_1y_2^2}{-x_1^2x_2 + x_1x_2^2 - y_1^2y_2 + y_1y_2^2}\right) + y_1$$
$$- \frac{y_2 - y_1}{x_2 - x_1}x_1, \frac{-Lx_1 + Lx_2 - x_2y_1^2y_2 + x_1y_1y_2^2}{-x_1^2x_2 + x_1x_2^2 - y_1^2y_2 + y_1y_2^2}$$

As a check, let's plug in $p = (2, -1)$, $q = \left(\frac{5}{3}, \frac{4}{3}\right)$, and $L = 7$ as we did

in Part I. By entering these values into *Mathematica*, we get

$$\frac{y_2-y_1}{x_2-x_1}\left(\frac{-Lx_1+Lx_2-x_2y_1^2y_2+x_1y_1y_2^2}{-x_1^2x_2+x_1x_2^2-y_1^2y_2+y_1y_2^2}\right)+y_1-\frac{y_2-y_1}{x_2-x_1}x_1=\frac{17}{38}$$

and

$$\frac{-Lx_1+Lx_2-x_2y_1^2y_2+x_1y_1y_2^2}{-x_1^2x_2+x_1x_2^2-y_1^2y_2+y_1y_2^2}=\frac{73}{38}$$

which is precisely what we found in Part I for these two particular points.

Part III : Topics for Future Investigation

1. Fermat's Last Theorem states that $a^n+b^n=c^n$ for $n\geq 3$ where $x,y,z\in Z$ has no solutions. However, we have shown that there are integer solutions of the form $a^3+b^3=7*c^3$, but this is not the case for $a^3+b^3=3*c^3$. Does $a^3+b^3=3*7*c^3$ have any solutions?

2. Show the associativity of the $\oplus$ operation, i.e., $(p\oplus q)\oplus r=p\oplus(q\oplus r)$.

Before you enter on the study of law a sufficient ground work must be laid $\cdots$.Mathematics and natural philosophy are so useful in the most familiar occurrences of life and are so engaging and delightful as would induce everyone to wish an acquaintance with them. Besides this, the faculties of the mind, like the members of a body, are strengthened and improved by exercise. Mathematical reasoning and deductions are, therefore, a fine preparation for investigating the abstruse speculations of the law.

- Thomas Jefferson

Matrix Multiplication Using Strassen's Algorithm

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A common operation done in many linear algebra applications is matrix multiplication. In this paper, I will discuss several of these methods and analyze each. Besides reviewing matrix multiplication as it is defined, I will also illustrate two divide-and-conquer algorithms. One of these, which actually has a name, is called Strassen's Algorithm. Although it was first published in 1969, it has only more recently been showing up in numerous algorithms books and journals.

In order to analyze these methods, I will present a technique for determining the total number of individual multiplications for each algorithm. The total number of additions (and subtractions) can also be calculated in this way.

Things to keep in mind:

A common notation: $lg x = log2 x$
Property of logarithms: $a^{\log b} = b^{\log a}$
   Note: $\log a^{\log b} = \log b \log a = \log a \, log b = \log b^{\log a}$

Defined Matrix Multiplication

Let $A = (a_{ij})$, $B = (b_{ij})$ be $n \; x \; n$ matrices. The product of A ann B, let's call it C, is defined as follows:

$$C = (c_{ij}) \text{ where } c_{ij} \sum_{k-1}^{n} = a_{ik} * b_{kj}$$

The 2 x 2 case: =

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} * \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

A concrete 2 x 2 example:

$$\begin{bmatrix} 7 & 3 \\ 6 & 8 \end{bmatrix} * \begin{bmatrix} 1 & 4 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} 7*1+3*6 & 7*4+3*5 \\ 6*1+8*6 & 6*4+8*5 \end{bmatrix} = \begin{bmatrix} 25 & 43 \\ 54 & 64 \end{bmatrix}$$

Now that matrix multiplication has been properly defined, I will proceed to analyze the total number of operations performed for a given n.

First, let $T_M(n)$ represent the total number of multiplications when multiplying two $n x n$ matrices. Also, let $T_A(n)$ be the total number of additions done during the matrix multiplication.

For the definition of matrix multiplication, $T_M(n)$ can be determined intuitively. Each entry in the product has n multiplications and the product consists of n^2 entries. Hence, the $T_M(n)$ is equal to $n * n^2 = n^3$.

$T_A(n)$ can be determined in a similar fashion. Again, there are n^2 entries. This time, however, there are only $(n - 1)$ additions per entry. $T_A(n)$ is, therefore, equal to $n^2 * (n - 1) = n^3 - n^2$.

The total number of operations done with the multiplication as it is defined is valid for any positive (or non-negative, if you desire) integer n.

Divide and Conquer Matrix multiplication can also be represented by a recursive divide-and-conquer algorithm (Cormen, Leiserson, Rivest 739).

First take an $n x n$ matrix and divide it into four submatrices. Each submatrix is of size $\frac{n}{2} x \frac{n}{2}$. These can then be multiplied separately and then combined to form something strikingly similar to the definition stated previously. At each step, n must be an even number, so this specific algorithm works only for $n = 2^k$ for $k \in Z^+$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} e & g \\ f & h \end{bmatrix} = \begin{bmatrix} [a I e] + [b I f] & [a I g] + [b I h] \\ [c I e] + [d I f] & [c I g] + [d I h] \end{bmatrix}$$

We can now analyze this particular algorithm.

In order to analyze this problem, we must also keep in mind that when multiplying two $1 x 1$ matrices, the number of multiplications needed is 1. That is, $T_m(1) = 1$.

Due to the recursive definition, $T_m(n)$ must be figured differently than before. We start with the recursive definition:

$T_m(n) = 8 * T_M(n/2)$ 8 submatrix multiplications

Let $n = 2k$ (*Also note that* $k = \log n$)

$$\begin{aligned} T_m(2^k) &= 8 * T_M(2^k/2) \\ &= 8 * T_M(2^{k-1}) \end{aligned}$$

The next step is to create a recurrence equation from our recursive definition. A recurrence equation is an equation where the value of a function is determined by the values of the function at a smaller k. To do this, we

replace entries of the form $T_m\left(2^k\right)$ by t_k.

$$t_k \qquad\qquad = 8 * t_{k-1}$$
$$t_k - 8 * t_{k-1} \; = 0$$

We now need to transform our recurrence equation into a characteristic equation, so its roots can be found. The basic concept is to replace the term with the smallest subscript with r^0 and work your way up until you have substituted the term with the largest subscript with r^d where $d = $ (*largest subscript-smallest subscript*). The actual definition of Neapolitan and Naimipour is given in Appendix A.

$$0 \; = r^1 - 8 * r^0$$
$$= (r - 8)$$

Solving for r gives us the root of $r = 8$. Neapolitan and Naimipour continue and prove that these roots solve the recurrence equation. The theorem is also in Appendix A. The general solution is then given by

$$t_k = c_1 * 8^k$$

Reversing the substitution of $T_M(2^k)$ by t_k and n by 2^k and recalling that $k = \log n$ gives

$$T_M(2^k) \; = c_1 * 8^k$$
$$T_M(2^k) \; = c_1 * 8^{\log n}$$
$$T_M(n) \; = c_1 * 8^{\log n}$$

We now use our rule of logarithms stated previously.

$$T_M(N) \; = c_1 * n^{\log 8}$$
$$= c_1 * n^3$$

Our initial value equation of $T_M(1) = 1$ is used to solve for the constant c_1.

$$T_M(1) \; = 1$$
$$T_M(1) \; = c_1 * 1^3$$
$$1 \qquad = c_1 * 1^3$$
$$= c_1 * 1$$
$$= c_1$$

The particular solution is given by the equation $T_M(n) = n^3$. Note this is the same solution as the defined matrix multiplication algorithm.

$T_A(n)$ is solved in a similar.

$$\begin{aligned} T_A(1) &= 0 \\ T_A(2) &= 4 \\ T_A(n) &= 8 * T_A(n/2) + 4 * (n/2)^2 \\ &= 8 * T_A(n/2) + n^2 \end{aligned}$$

Let $n = 2k$

$$\begin{aligned} T_A(2^k) &= 8 * T_A(2^k/2) + (2^k)^2 \\ &= 8 * T_A(2^k/2) + (2^2)^k \\ &= 8 * T_A(2^k/2) + 4^k \\ &= 8 * T_A(2^{k-1}) + 4^k \end{aligned}$$

Let $T_A(2^k) = t_k$.

$$\begin{aligned} t_k &= 8 * t_{k-1} + 4^k \\ t_k - 8 * t_{k-1} &= 4^k \end{aligned}$$

Now would be the time to transform our recurrence equation into a characteristic equation. Unfortunately, doing it now would not produce a homogeneous characteristic equation, which is necessary for finding the roots. Fortunately, however, we are able to manipulate our recurrence equation into a homogeneous equation.

The first step is to "roll back" our equation by replacing each k by $(k - 1)$.

$$4^{k-1} = t_{k-1} - 8 * t_{k-2}$$

We also want to divide our original recurrence equation by 4 giving:

$$\begin{aligned} \frac{4^k}{4} &= \frac{t_k - 8 * t_{k-1}}{4} \\ 4^{k-1} &= \tfrac{1}{4} t_k - 2 * t_{k-1} \end{aligned}$$

We now have two expressions both equal to 4^{k-1}, hence we can set them equal to each other.

$$\begin{aligned} \tfrac{1}{4} t_k - 2 * t_{k-1} &= t_{k-1} - 8 * t_{k-2} \\ t_k - 8 * t_{k-1} &= 4 * t_{k-1} - 32 * t_{k-2} \\ t_k - 12 * t_{k-1} + 32 * t_{k-2} &= 0 \end{aligned}$$

Now we may successfully do our transformation into the characteristic equation.

$$\begin{aligned} 0 &= r^2 - 12 * r^1 + 32 * r^0 \\ &= r^2 - 12 * r + 32 \\ &= (r - 4) * (r - 8) \end{aligned}$$

Solving for r gives us the roots of $r = 4$ and $r = 8$. The general

solution is then given by

$$t_k = c_1 * 4^k + c_2 * 8^k$$

Reversing the previous substitutions gives

$$
\begin{aligned}
T_a\left(2^k\right) &= c_1 * 4^k + c_2 * 8^k \\
T_a\left(2^k\right) &= c_1 * 4^{\log n} + c_2 * 8^{\log n} \\
T_a(n) &= c_1 * 4^{\log n} + c_2 * 8^{\log n}
\end{aligned}
$$

We again do our switch with the logarithms.

$$
\begin{aligned}
T_A(n) &= c_1 * n^{\log 4} + c_2 * n^{\log 8} \\
&= c_1 * n^2 + c_2 * n^3
\end{aligned}
$$

Initial values are used to solve for the constants. Since we have two constants to solve for, we must have two initial value equations.

$$
\begin{aligned}
T_A(1) &= 0 \\
T_A(2) &= 4 \\
T_A(1) &= c_1 * 1^2 + c_2 * 1^3 \\
T_A(2) &= c_1 * 2^2 + c_2 * 2^3 \\
0 &= c_1 * 1^2 + c_2 * 1^3 \\
&= c_1 + c_2 \\
4 &= c_1 * 2^2 + c_2 * 2^3 \\
&= 4 * c_1 + 8 * c_2
\end{aligned}
$$

The solution to this system of equations is $c_1 = -1$ and $c_2 = 1$. This leaves us with a particular solution of

$$T_A(2^k) = n^3 - n^2$$

The solution for divide-and-conquer's T_A is again identical to our first T_A. In this instance, the divide and conquer algorithm offers us no benefit. When used in a different way, however, this method of problem-solving gives us better results.

Strassen's Algorithm

Strassen's Algorithm is related to the previous algorithm of divide and conquer. Instead of doing straightforward multiplication and addition, the submatrices are manipulated in a special way. Again, we will assume that n is equal to a power of 2.

In 1969, Strassen published his algorithm that is described below (Cormen, Leiserson, Rivest 740-3).

$$
\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} e & g \\ f & h \end{bmatrix} =
$$

$$\begin{bmatrix} P5 + P4 - P2 + P6 & P1 + P2 \\ P3 + P4 & P5 + P1 - P3 - P7 \end{bmatrix}$$

where $P1, P2, ..., P7$ are $\frac{n}{2} x \frac{n}{2}$ matrices defined as:

$P1 = a * (g - h)$
$P2 = (a + b) * h$
$P3 = (c + d) * e$
$P4 = d * (f - e)$
$P5 = (a + d) * (e + h)$
$P6 = (b - d) * (f + h)$
$P7 = (a - c) * (e + g)$

A better way of explaining this algorithm is to work alongside an example. We will use our previously solved example. In this simple case, each submatrix is actually a $1x1$ *matrix*. Hence, multiplications and additions are between two integers.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} e & g \\ f & h \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 6 & 8 \end{bmatrix} * \begin{bmatrix} 1 & 4 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} 25 & 43 \\ 54 & 64 \end{bmatrix}$$

$$\begin{aligned}
P1 &= a * (g - h) & &= 7 * (4 - 5) & &= -7 \\
P2 &= (a + b) * h & &= (7 + 3) * 5 & &= 50 \\
P3 &= (c + d) * e & &= (6 + 8) * 1 & &= 14 \\
P4 &= d * (f - e) & &= 8 * (6 - 1) & &= 40 \\
P5 &= (a + d) * (e + h) & &= (7 + 8) * (1 + 5) & &= 90 \\
P6 &= (b - d) * (f + h) & &= (3 - 8) * (6 + 5) & &= -55 \\
P7 &= (a - c) * (e + g) & &= (7 - 6) * (1 + 4) & &= 5
\end{aligned}$$

$$\begin{bmatrix} P5 + P4 - P2 + P6 & P1 + P2 \\ P3 + P4 & P5 + P1 - P3 - P7 \end{bmatrix}$$
$$= \begin{bmatrix} 90 + 40 - 50 + (-55) & (-7) + 50 \\ 14 + 40 & 90 + (-7) - 14 - 5 \end{bmatrix}$$
$$= \begin{bmatrix} 25 & 43 \\ 54 & 64 \end{bmatrix}$$

A proof of Strassen's algorithm can be accomplished with a few steps.

$$\begin{bmatrix} P5 + P4 - P2 + P6 & P1 + P2 \\ P3 + P4 & P5 + P1 - P3 - P7 \end{bmatrix}$$
$$= \begin{bmatrix} \begin{aligned} &(ae + ah + de + dh) + \\ &(df - de) - (ah + bh) \\ &+ (bf + bh - df - dh) \end{aligned} & (ag - ah) + (ah + bh) \\ (ce + de) + (df - de) & \begin{aligned} &(ae + ah + de + dh) + \\ &(ag - ah) - (ce + de) \\ &- (ae + ag - ce - cg) \end{aligned} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{array}{l} (ae + ah + de + dh) + \\ (df - de) + (ah + bh) \\ + (bf + bh - df - dh) \end{array} & (ag - ah) + (ah + bh) \\[2em] (ce + de) + (df - de) & \begin{array}{l} (ae + ah + de + dh) \\ + (ag - ah) - (ce + de) \\ - (ae + ag - ce - cg) \end{array} \end{bmatrix}$$

$$= \begin{bmatrix} ae + bf & ag + bh \\ ce + df & cg + dh \end{bmatrix}$$

An analysis of Strassen's algorithm may now be conducted. We shall proceed similarly to the last analysis.

$$\begin{aligned} T_M(1) &= 1 \\ T_M(n) &= 7 T_M(\tfrac{n}{2}) \end{aligned}$$

Let $n = 2^k$

$$\begin{aligned} T_M(2^k) &= 7 * T_M(\tfrac{2^k}{2}) \\ &= 7 * T_M(2^{k-1}) \end{aligned}$$

Let $T_M(2^k) = t_k$.

$$\begin{aligned} t_k &= 7 t_{k-1} \\ t_k - 7 t_{k-1} &= 0 \end{aligned}$$

Transform recurrence into the characteristic equation.

$$\begin{aligned} 0 &= r^1 - 7 r^0 \\ &= (r - 7) \end{aligned}$$

Solving for r gives us $r = 7$.

The general solution is given by

$$t_k = c_1 * 7^k$$

Reversing the substitution for $T_M(2^k)$ from above gives

$$\begin{aligned} T_M(2^k) &= c_1 * 7^k \\ T_M(2^k) &= c_1 * 7^{\log n} \\ T_M(n) &= c_1 * 7^{\log n} \end{aligned}$$

The logarithm switch is done.

$$T_M(n) = c_1 * n^{\log 7}$$

Initial values are used to solve for c_1

$$
\begin{aligned}
T_M(1) &= 1 \\
T_M(1) &= c_1 * 1^{\log 7} \\
1 &= c_1 * 1^{\log 7} \\
&= c_1 * 1 \\
&= c_1
\end{aligned}
$$

Our solution becomes

$$
\begin{aligned}
T_M(n) &= n^{\log 7} \\
&= n^{2.81}
\end{aligned}
$$

The number of additions can be calculated in a similar fashion

$$
\begin{aligned}
T_A(1) &= 0 \\
T_A(2) &= 18 \\
T_A(n) &= 7T_A(\tfrac{n}{2}) + 18(\tfrac{n}{2})^2 \\
&= 7T_A(\tfrac{n}{2}) + \tfrac{9}{2}n^2
\end{aligned}
$$

Let $n = 2^k$

$$
\begin{aligned}
T_A(2^k) &= 7 * T_A(\tfrac{2^k}{2}) + \tfrac{9}{2}(2^k)^2 \\
&= 7 * T_A(\tfrac{2^k}{2}) + \tfrac{9}{2}(2^2)^k \\
&= 7 * T_A(\tfrac{2^k}{2}) + \tfrac{9}{2} * 4^k \\
&= 7 * T_A(2^{k-1}) + \tfrac{9}{2} * 4^k
\end{aligned}
$$

Let $T_A(2^k) = t_k$.

$$
\begin{aligned}
t_k &= 7 * t_{k-1} + \tfrac{9}{2} * 4^k \\
t_k - 7 * t_{k-1} &= \tfrac{9}{2} * 4^k
\end{aligned}
$$

"Roll back" and divide our equation to produce 2 equal expressions.

$$
\tfrac{9}{2} * 4^{k-1} = t_{k-1} - 7 * t_{k-2}
$$

and

$$
\begin{aligned}
\left(\tfrac{9}{2} * 4^k\right)/4 &= \tfrac{t_k - 7 * t_{k-1}}{4} \\
\tfrac{9}{2} * 4^{k-1} &= \tfrac{1}{4} * t_k - \tfrac{7}{4} * t_{k-1}
\end{aligned}
$$

Setting them equal to each other, we obtain

$$
\begin{aligned}
\tfrac{1}{4}t_k - \tfrac{7}{4} * t_{k-1} &= t_{k-1} - 7 * t_{k-2} \\
t_k - 7 * t_{k-1} &= 4 * t_{k-1} - 28 * t_{k-2} \\
t_k - 11 * t_{k-1} + 28 * t_{k-2} &= 0
\end{aligned}
$$

Transform the recurrence relation into the characteristic equation.

$$
\begin{aligned}
0 &= r^2 - 11 * r^1 + 28 * r^0 \\
&= r^2 - 11 * r + 28 \\
&= (r - 4) * (r - 7)
\end{aligned}
$$

We now have $r = 4$ and $r = 8$.

The general solution is

$$
t_k = c_1 * 4^k + c_2 * 7^k
$$

Reversing the previous substitutions gives

$$
\begin{aligned}
T_A(2^k) &= c_1 * 4^k + c_2 * 7^k \\
T_A(2^k) &= c_1 * 4^{\log n} + c_2 * 7^{\log n} \\
T_A(n) &= c_1 * 4^{\log n} + c_2 * 7^{\log n}
\end{aligned}
$$

Do the logarithm switch.

$$
\begin{aligned}
T_A(n) &= c_1 * n^{\log 4} + c_2 * n^{\log 7} \\
&= c_1 * n^2 + c_2 * n^{\log 7}
\end{aligned}
$$

Use initial values to determine the constants c_1 and c_2.

$$
\begin{aligned}
T_A(1) &= 0 \\
T_A(2) &= 18 \\
T_A(1) &= c_1 * 1^2 + c_2 * 1^{\log 7} \\
T_A(2) &= c_1 * 2^2 + c_2 * 2^{\log 7} \\
0 &= c_1 * 1^2 + c_2 * 1^{\log 7} \\
18 &= c_1 * 2^2 + c_2 * 2^{\log 7}
\end{aligned}
$$

Solving this small system of equations yields $c_1 = -6$ and $c_2 = 6$. Thus, the particular solution is

$$
\begin{aligned}
T_A(n) &= 6 * n^{\log 7} - 6 * n^2 \\
&\approx 6 * n^{2.81} - 6 * n^2
\end{aligned}
$$

Results

The results of the analyses are shown in the table below.

Size	Standard Multiplication		Strassen's Algorithm	
	Multiplications	Additions	Multiplications	Additions
$1x1$	1	0	1	0
$2x2$	8	4	7	18
$4x4$	64	48	49	198
$16x16$	4096	3841	2401	12870
$64x64$	262144	258048	117649	681318
nxn	n^3	$n^3 - n^2$	$n^{\log 7}$	$6 * n^{\log 7} - 6 * n^2$

In the simple $2x2$ case, the consequence for eliminating one multiplication step is the tabulation of 14 extra addition steps. Due to the relatively large constant of 6 associated with the number of additions, this algorithm performs very poorly with a small n. As n grows, however, the usefulness of the algorithm increases proportionally.

Various attempts have been made to implement Strassen's Algorithm. Included in this group is my own attempt. Unfortunately, my program actually ran slower as n increased. One of the tricks to implement this algorithm is to know when not to use the algorithm and use the straightforward approach.

A group of computer scientists associated with the Center for Computing Sciences out of Maryland have successfully implemented this algorithm (Huss-Lederman). Many analyzations were done to incorporate the best possible algorithm hybrid. The algorithm used uses a variant of Strassen's Algorithm known as Winograd's variant. The Winograd variant is slightly more efficient than the original algorithm. Instead of using 18 additions in the 2 x 2 case, only 15 are used. Computing the number of additions required gives $T_A(n) = 5 * n^{lg7} - 5 * n^2$.

Further Comments

Regressing to the beginning of the paper, we made the assumption that n is a power of 2. This was necessary for our recursive algorithm to be properly analyzed. In actuality, however, n does not need to be of this form.

Several ways of fixing this problem exist. The methods I will explain both are types of padding. The first is the simplest and easiest. The original nxn matrix is increased in size up to the next power of two. Each new

space is filled with a zero. After calculations are complete, these same entries are still equal to zero and may be eliminated.

The second and more elegant padding solution is to pad the matrix with zeros only when necessary, that is n is odd. The advantage to this solution is the less amount of temporary storage used to save the original and working matrices.

References

1. Cormen, Thomas H., Charles E. Leiserson, and Ronald L. Rivest. Introduction to Algorithms. Cambridge: MIT P, 1990.

2. Neapolitan, Richard E., and Kumarss Naimipour. Foundations of Algorithms. Lexington: Heath and Company, 1996.

3. Huss-Lederman, Steven, et al. "Implementation of Strassen's Algorithm for Matrix Multiplication." Oct. 1996. The Supercomputing Conferences Organization. http://www.supercomp.org/sc96/proceedings/sc96proc/jacobson/index.htm

Appendix A:

Definitions and Theorems for solving recurrence equations

Definition:
The recurrence of the form

$$a_0 t_n + a_1 t_{n-1} + ... + a_k t_{n-k} = 0$$

where k and the a_i terms are constants, is called a homogenous linear recurrence equation with constant coefficients.

Definition:
The characteristic equation for the homogeneous linear recurrence equation with constant coefficients

$$a_0 t_n + a_1 t_{n-1} + ... + a_k t_{n-k} = 0 \quad .$$

is defined as

$$a_0 r^k + a_1 r^{k-1} + ... + a_k r^0 = 0$$

Theorem B.1 in Foundations of Algorithms by Neapolitan and Naimipour states the following:
Let the homogeneous linear recurrence equation with constant coefficients

$$a_0 t_n + a_1 t_{n-1} + ... + a_k t_{n-k} = 0$$

be given. If its characteristic equation

$$a_0 r^k + a_1 r^{k-1} + ... + a_k r^0 = 0$$

has k distinct solutions $r_1, r_2, ..., r_k$, then the only solutions to the recurrence are

$$t_n = c_1 r_1^n + c_2 r_2^n + ... + c_k r_k^n$$

where the c_i terms are arbitrary constants.
The preceding was taken from "Appendix B: Solving Recurrence Equations: With Applications to Analysis of Recursive Algorithms" from Foundations of Algorithms.

KME Website

The national KME website can be found at
            http://www.cst.cmich.edu/org/kme_nat/
This is a new URL, so please update your bookmarks and links from chapter pages! Below is a partial list of items that are available on the site:

- How to start a KME Chapter

- Information on KME conventions

- The cumulative subject index of *The Pentagon*

You can get a web page template from the Kentucky Alpha chapter. Its URL is
            eagle.eku.edu/faculty/pjcostello/kme/
When you design a chapter homepage, please remember to make it clear that your page is for your chapter, and not for the national organization. Also, please include a link to the national homepage and submit your local chapter webpage's URL to the national webmaster. By doing so, other chapters can explore activities of your chapter and borrow some great ideas from you!

On the Density of Birthday Sets

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Terminology, Standard and Otherwise

Throughout this paper, I will use some terminology and notation that some of you may not be used to. I have collected the more standard ones in this section, roughly in the order they appear in the paper. (The terminology that I made up myself I'll introduce as I go along.)

Two integers a and b are said to be congruent with respect to the modulus n, or just congruent mod n, if and only if $b - a = kn$, for some integer k. This is also symbolized as $a \equiv b \pmod{n}$. The least nonnegative number k such that $b \equiv k \pmod{n}$ is called the residue of b mod n. I use somewhat outdated notation $\#A$ to denote the cardinality (or number of elements) of the set A. If we let $\#A(n)$ denote the number of elements of A less than or equal to n, then the natural (or asymptotic) density of the set A is equal to $\lim_{n \to \infty} (\#A(n))/n$. Two numbers are said to be relatively prime if they have no factors in common other than 1. And, throughout this paper, whenever I say "number", I will usually mean "integer".

Get On With It

Anyway, I happened to be speaking with a professor recently when he remarked to me that a friend of his was celebrating his 71st birthday, and wondered if I knew of any "interesting" properties of the number 71. After a little bit of thought, I replied that since it was between 70 and 72, it was one away from a multiple of 2, of 3, of 4, and so on up to 10. I promptly christened collections of all such numbers, "birthday sets". For example, 71 is an element of the birthday set of order 10, which I will denote by S_{10}.

It is easy to see that for any given n, the S_n is infinite; consider numbers of the form $k(n!) \pm 1$, for any integer k. The question we shall ask is, for a given n, what is the natural density of these numbers? We shall come up with a specific answer for the given problem ($n = 10$) and on the way, generalize where we can. Our approach will be this: since our set S_n will by definition be a subset of S_{n-1}, we can start with $n = 2$ and trim the set.

If a number m is one away from a multiple of 2, then it is fairly clear that m is odd. Thus, we have reduced the set of natural numbers to the set S_2, the set of odd natural numbers, which obviously has density 1/2.

If a number m is one away from a multiple of 3, then it is congruent to 1 or 2 mod 3. If we look at the odd numbers mod 3, we see that they form the pattern $1, 0, 2, 1, 0, 2, \ldots$. Since all the three possible moduli occur equally often, the we know that the ones we want (1 and 2) occur 2/3 of the time; thus we need to multiply the density of our previous set by 2/3. (I shall call this the "multiplication factor" for $n = 3$). Then when $n = 3$, the natural density of our set S_3 is $1/2 \times 2/3 = 1/3$.

If a number m is one away from a multiple of 4, then it is congruent to 1 or 3 mod 4. But the natural numbers of the form $4k + 1$ and $4k + 3$ are precisely the odd numbers, so our entire set S_3 is contained in S_4. Thus the multiplication factor for $n = 4$ is 1.

To establish our first generalization, we will need the following technical result:

Lemma. If a and b are relatively prime, then the set of integers congruent to a given number c mod a will contain all the possible residues mod b.

Proof. The set of numbers congruent to c mod a is just $c + ka$, where k ranges freely over the integers. Consider the sequence $a, 2a, 3a, \ldots, ba, (b + 1)a$. Clearly $a \equiv a \pmod{b}$, and $(b + 1)a = ba + a \equiv a \pmod{b}$. Since congruence is an equivalence relation, we then know that $2a \equiv (b + 2)a \pmod{b}$, $3a \equiv (b + 3)a \pmod{b}$, and so on through repeated addition. Thus the residues mod b of the first sequence are cyclic, repeating after at most b terms. Assume there exists a $k, 1 < k < b$, such that $a \equiv ka \pmod{b}$. But since a is relatively prime to $b, 1 \equiv k \pmod{b}$. (For a proof of this fact, see Corollary 3.3.1 in [1].) But then either $k = 1$ or $k > b$, neither of which is consistent with $1 < k < b$. Then the b numbers $a, 2a, 3a, \ldots, ba$ all have different residues mod b. We can add c to each of these numbers to get the sequence $c + a$, $c + 2a, , c + ba$, which must still contain b distinct residues. Since there are only b residues mod b, all of them must be represented, and since the sequence is cyclic, all of them occur equally often.

This leads to:

Generalization 1. If p is an odd prime, then the multiplication factor for $n = p$ is $2/p$.

Proof. The set S_{p-1} is dependent on congruences mod smaller numbers. By the lemma, these are in some sense independent of residues mod p. Therefore, all residues mod p occur equally often.

A number can be congruent to $0, 1, \ldots, p-2, p-1 \bmod p$, a total of p possibilities. One and
$p-1$ are the numbers we are interested in, and the set S_{p-1} is equally distributed throughout these congruences. Thus the number of members of S_p is $2/p$ times the number of members of S_{p-1}.

Thus we have that the multiplication factor for $n = 5$ is $2/5$, and therefore the natural density of S_5 is $1/3 \times 2/5 = 2/15$. Let us now consider $n = 6$. But we know already that any element m of S_5 is odd, and m is one away from a multiple of 3. But this multiple of 3, being one away from an odd number, is even. Thus it is a multiple of 6. Thus the multiplication factor for $n = 6$ is 1. This reasoning is clearly extensible to the following:

Generalization 2. If k is odd, then the multiplication factor of $n = 2k$ is 1.

The multiplication factor for $n = 7$ is $2/7$, so the density of S_7 is $2/15 \times 2/7 = 4/105$. Let us now consider $n = 8$. If m is an element of S_7, m is odd; hence m must be congruent to $1, 3, 5,$ or $7 \bmod 8$ (and these will occur equally often). We are only interested in the cases when m is congruent to 1 or 7; thus the multiplication factor for $n = 8$ is $1/2$ and the density of S_8 is $4/105 \times 1/2 = 2/105$. For $n = 9$, the fact that any m in S_8 one away from a multiple of 3 means that m is congruent to $1, 2, 4, 5, 7,$ or $8 \bmod 9$. We are interested in the cases when m is congruent to 1 or 8; thus the multiplication factor for $n = 9$ is $1/3$, and the density of S_9 is $2/105 \times 1/3 = 2/315$.

These cases lead us to:

Generalization 3. If p is prime, $k > 1$, and not both p and k are 2, then the multiplication factor for $n = p^k$ is $1/p$.

Proof. For a number m to be one away from a multiple of p^{k-1}, m must be congruent to $1, p^{k-1} - 1,$
$p^{k-1} + 1, 2p^{k-1} - 1, , (p-1)p^{k-1} - 1, (p-1)p^{k-1} + 1,$ or $p^k - 1$; in other words, all numbers of the form $lp^{k-1} \pm 1$, where l ranges from 0 to $p - 1$ (remember that $p^k - 1$ is congruent to $-1 \bmod p^k$). Since we are interested only in 1 and -1, 2 out of $2p$ numbers, the multiplication factor for $n = p^k$ is $1/p$.

Since $10 = 2 \times 5$ and 5 is odd, the multiplication factor for $n = 10$ is 1; therefore, the density of our original set, the set of all numbers one away from a multiple of 2, a multiple of 3, and so on up to 10, is $2/315$. However, the generalizations we have so far derived are not complete; for example, none of them will help us find the multiplication factor for $n = 12$.

So, let's look at some simple examples. What if n is the product of two odd primes? For example, $15 = 3 \times 5$. If m is one away from a multiple of 3, it is congruent to 1, 2, 4, 5, 7, 8, 10, 11, 13, or 14 mod 15. Similarly, if m is one away from a multiple of 5, m is congruent to 1, 4, 6, 9, 11, or 14 mod 15. Then since m must be congruent to the same number as itself, m must be congruent to 1, 4,11, or 14. Then the multiplication factor for $n = 15$ is $1/2$. What about $35 = 5 \times 7$? Then, using 5, m is congruent to 1, 4, 6, 9,11, 14, 16, 19, 21, 24, 26, 29, 31, 34 mod 35; and using 7, m is congruent to 1, 6, 8, 13, 15, 20, 22, 27, 29, 34 mod 35. They agree at 1, 6, 29, and 34. This is not a coincidence, for we can show the following:

Generalization 4. If $n = pq$, where p and q are odd primes, then the multiplication factor for n is $1/2$.

Proof. We are essentially looking for solutions to $ap + 1 = bq - 1$ and $ap - 1 = bq + 1$ where both a and b are restricted to keep all the numbers involved less than n. So essentially we are trying to solve $ap - bq = \pm 2$. But p and q are relatively prime, so there is an entire family of solutions $a = x_0 + kq$ and $b = x_1 - kp$; needless to say, there can only be one solution (a, b) where both $a < q$ and $b < p$. (A discussion of this topic can be found, among other places, in chapter 2 of [2].)

What if n is the product of (at least) three odd primes, say p, q, and r? Well, we can use the above technique with pq and r, since they are relatively prime, to come up with four possible moduli that a member of S_{n-1} could take. But you will find that, except for 1 and -1, neither of these will handle, for example, qr. But you shouldn't just take my word for it; here's a proof.

Generalization 5. The multiplication factor for $n = pqr$ is 1.

Proof. Suppose we have found a solution (a, b) to the equation $apq - 1 = br + 1$, where both $a < r$ and $b < pq$. We know that $l = apq - 1 = br + 1$ is congruent to 1 mod r; let us assume that is congruent to 1 mod qr. Thus $l = cqr + 1$. This tells us that $l \equiv 1 \pmod{q}$. But by the first equation, we have that $l \equiv -1 \pmod{q}$. For this to be true, q must equal 2; but we assumed that q was an odd prime. Thus, the natural numbers corresponding to the solution (a, b) will not be found in the set $S_{n-1}(m)$. Therefore the multiplication factor for $n = pqr$ equals 1.

This argument holds as well when $p = q$, as well as when other factors are thrown in. Thus if the prime factorization of n contains three odd primes, at least two of which are distinct, then the multiplication factor of n is 1.

We are now left with the cases $2^a p$ and $2^a pq$, where a is at least two. When $a = 2$, a proof almost identical to the one given for Generalization 4 tells us that the multiplication factor for $n = 4p$ is $1/2$. However, the case for $4pq$ follows the proof for Generalization 5, using 4 in place of r. And, the case for $8p$ also follows the proof for Generalization 5, starting with 8 in place of pq and p in place of r, and then considering $4r$. (Notice that once we have a Generalization 5-type result for a certain power of 2, it holds for all higher powers of 2.)

Every integer falls into one of these classes; therefore we have the multiplication factor for all n. Then for any specific n, we can find the density of the set S_n by taking the product of all these multiplication factors for all natural numbers less than or equal to n. Notice also that this sequence is strictly nonincreasing (since each set is a subset of the preceding one). Plus, the sequence of multiplication factors has a subsequence converging to zero (e.g., whenever n is a prime). Thus, the density can be made as small as one likes, i.e.,

$$\lim_{n \to \infty} \left(\lim_{m \to \infty} \frac{S_n(m)}{m} \right) = 0;$$

but the density of S_n for any particular (finite) n is positive.

References

1. Joseph B. Dence and Thomas P. Dence, Elements of the Theory of Numbers, Harcourt Academic Press, San Diego, 1999.

2. William J. LeVeque, Elementary Theory of Numbers, Dover, New York, 1990.

The sum of the tenth powers of the first thousand natural numbers is

91,409,924,241,424,243,424,241,924,242,500.

James Bernoulli mentions that it took him rather less than seven and a half minutes to obtain this result.

Wavelets

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Researchers began looking for new ways to analyze functions after they found that Fourier Analysis was relatively sensitive to noise. Fourier Analysis concentrates on frequency, which makes it sensitive to noise. Analysis that concentrated on scale would be much less sensitive to noise. This type of analysis would require the construction of a function that varied in scale. This is where wavelets came into the picture. A wavelet is a mathematical function that can be used to analyze functions at different scales. Wavelet Analysis is less sensitive to noise because it measures average fluctuations at different scales.

Before a discussion of what some wavelets look like or how they work, it is important to recall a small bit about Fourier Analysis. First, sines and cosines are the basis functions for Fourier Analysis. A function that is 2π-periodic can be represented by a trigonometrical series of the form:

$$f(x) = a_n \cos(nx) + b_n \sin(nx)$$

The basis functions are orthogonal. Instead of looking at orthogonality as having a dot product of two vectors equaling zero, it will be looked at as the integral of the product of two functions equaling zero. For example,

$$\int_0^{2\pi} \sin x \cdot \cos = 0$$

sin(x)cos(x)

.6

0.5

f(x)   0

-.6   -0.5

0   1   2   3   4   5   6

0   x   2π
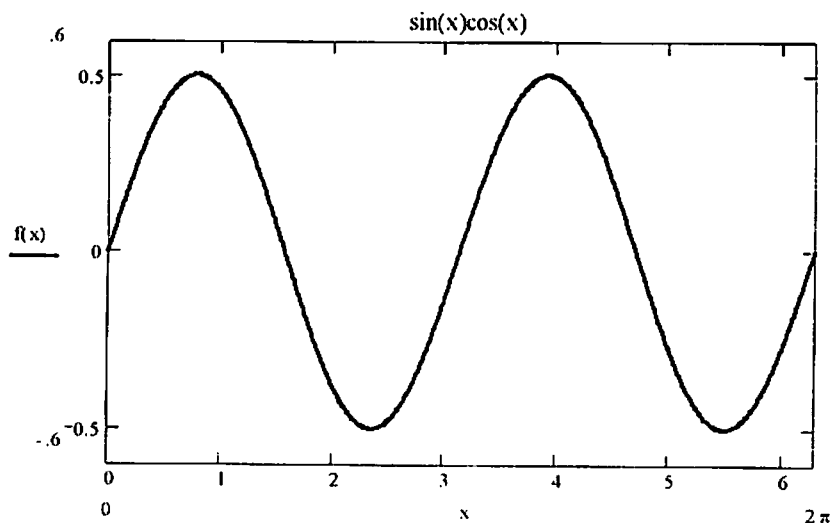
Wavelets work in a similar way. A function can be represented as a linear combination of a set of wavelets.

The history of wavelets is vast. The first mention of wavelets came in the appendix to the thesis of A. Haar in 1909. Paul Levy used wavelets to investigate Brownian Motion in the 1930s because wavelets helped him study complicated details in the Brownian motion. Between 1960 and 1980, two mathematicians, Guido Weiss and Ronald R. Coifman, used wavelets to reconstruct all of the elements of a function space from its atoms, the simplest elements of a function space. In 1980, Grossman and Morlet, a physicist and an engineer, defined wavelets in the context of Quantum Physics. David Marr developed an algorithm for numerical image processing using wavelets during the 1980s. Stephane Mallat used wavelets for digital signal processing and Multiresolutional Analysis (MRA) in 1985. Y. Meyer used Mallat's work to construct the first set of non-trivial wavelets. The derivatives of Shannon's wavelets exist and are continuous, but their support is all of R. In 1987, Ingrid Daubechies also used Mallat's work to construct a set of wavelet basis functions that have become the cornerstone of wavelet applications today. Daubechies Wavelets compromise between the property of compact support of Haar Wavelets and the smoothness property of Shannon's wavelets. Daubechies Wavelets have become the foundation of wavelet applications today.

An important property of wavelets is that they integrate to zero. The first set of wavelets that will be investigated are the Haar Wavelets. Haar Wavelets have compact support, which means they vanish outside of some desired interval, span the Hilbert Space, L^2, and are discontinuous, as are

its derivatives. Haar's basic structure is usually called his Mother Wavelet. The rest of the set of his wavelets is generated by shifting and scaling the Mother Wavelet. These generated functions can be called "the children." The Haar Mother Wavelet is a step function that takes a value of 1 on $[0, \frac{1}{2})$ and a value of -1 on $[\frac{1}{2}, 1)$. The following formula can be used to generate the rest of the set of wavelets:

$$\psi_{jk} = c \cdot \psi_{00} \left(2^j x - k\right)$$

where c is some constant, j ranges from 1 to $a - 1$, and $2a$ is the size of our data set. The subscript j shows how many levels a set of wavelets will have, and the k gives a clue as to how many sublevels each level will have.

To relate this to the ideas discussed regarding Fourier Analysis, these wavelets are the basis functions for Wavelet Analysis. It is important to see what happens if the product of two of these basis functions is integrated. Look at

$$\int \psi_{jk} \cdot \psi_{j'k'}$$

When $j \neq j'$, say $j < j'$, then the nonzero values of $\psi_{j'k'}$ are contained in the set where ψ_{jk} is constant. This reveals that $\int \psi_{jk} \cdot \psi_{j'k'} = 0$.

When $j = j'$ and is not satisfied simultaneously, $\int \psi_{jk} \cdot \psi_{j'k'} = 0$.

When $j = j'$ and $k \neq k'$, at least one of the factors of the product is zero. This also reveals that $\int \psi_{jk} \cdot \psi_{j'k'} = 0$

Therefore, the set of wavelets, $\{\psi_{jk}\}$, form an orthogonal basis in the Hilbert Space.

The set of wavelets, $\{\psi_{jk}\}$, actually forms an orthonormal basis in the Hilbert Space. To show this, the definition of $norm^2$ in the Hilbert Space is needed:

$$1 = c^2 \int \psi^2 \left(2^j x - k\right) dx \quad \text{where } c \text{ is some constant}$$
$$1 = c^2 \cdot 2^{-j} \int \psi^2 (t) \, dt \quad \text{by a change of variables}$$
$$1 = c^2 \cdot 2^{-j} \quad \text{because the integral of } \psi^2 = 1$$

Hence $c = 2^{j/2}$.

This gives the result that the basis is orthonormal by a factor of $2^{j/2}$. The following formula that will generate the rest of the set of wavelets from The Mother Wavelet:

$$\psi_{jk} = 2^{j/2} \psi \left(2^j x - k\right)$$

$$\psi_{10}(x) = \sqrt{2} \cdot \psi(2x) \qquad \text{makes the wavelet move twice as fast}$$
$$\text{and gives it an amplitude of } \sqrt{2}$$

$$\psi_{01}(x) = \psi(x - 1) \qquad \text{shifts the wavelet to the right one unit}$$

One might wonder what some wavelets look like by now. Plots of Haar's wavelets will follow. Mathcad was used to produce these graphs and for computational purposes following.

First, a small set of data will be used that allows the performance of some simple matrix computations. Then, a much larger set of data will be investigated that consists of hourly tide heights in feet from Bridgeport Harbor for approximately a year. Computer software will be used to perform the actual Wavelet Transform on this larger data set.

The following will show wavelets in action. A scaling function that lives where the wavelets live is needed to make the wavelet transformation work. For the Haar Wavelets, it is simple; denote it by ϕ_{00}, and it takes a value of 1 on $[0, 1)$. Then, take some data vector, $y = (1, 0, 1, 2, 1, 0, -2, 3)$. It is of length 8 or 2^3. This shows that there are $3 - 1 = 2$ different levels of wavelets: $\psi_{1m} and \psi_{2n}$, where m ranges from 0 to $2^1 - 1$ or 0 to1, and n ranges from 0 to $2^2 - 1$ or 0 to 3. The complete set of functions used to transform the data is $\left\{ \phi_{00}, \psi_{00}, \psi_{10}, \psi_{11}, \psi_{20}, \psi_{21}, \psi_{22}, \psi_{23} \right\}$.

Scaling Function

Haar Mother Wavelet

First Level of Wavelets

Second Level of Wavelets
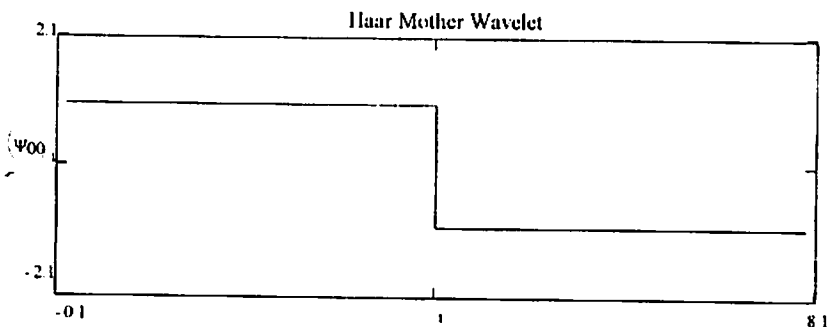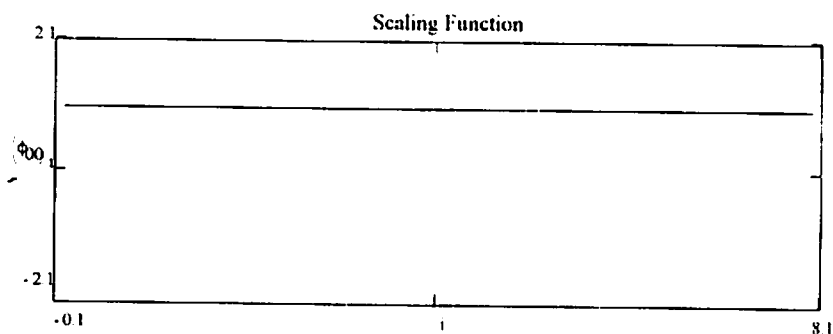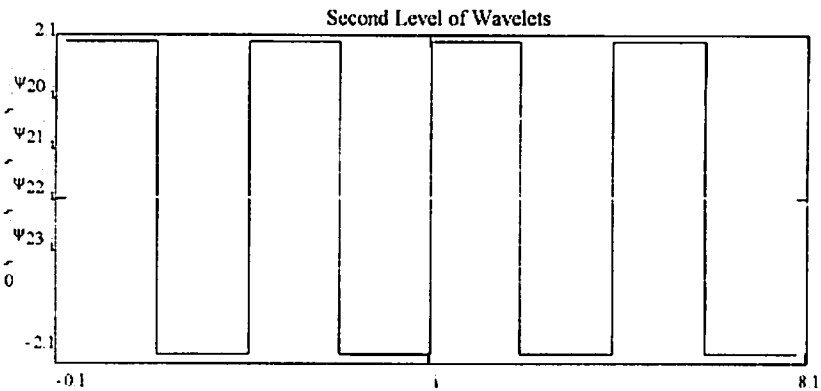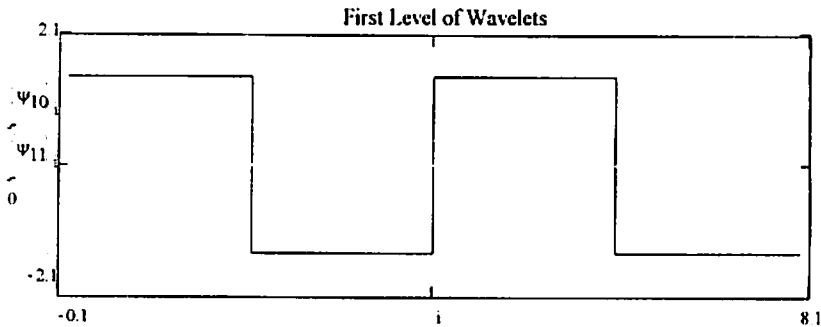
$$y := \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \\ 1 \\ 0 \\ -3 \\ 2 \end{bmatrix} \qquad W := \begin{bmatrix} 1 & 1 & \sqrt{2} & 0 & 2 & 0 & 0 & 0 \\ 1 & 1 & \sqrt{2} & 0 & -2 & 0 & 0 & 0 \\ 1 & 1 & -\sqrt{2} & 0 & 0 & 2 & 0 & 0 \\ 1 & 1 & -\sqrt{2} & 0 & 0 & -2 & 0 & 0 \\ 1 & -1 & 0 & \sqrt{2} & 0 & 0 & 2 & 0 \\ 1 & -1 & 0 & \sqrt{2} & 0 & 0 & -2 & 0 \\ 1 & -1 & 0 & -\sqrt{2} & 0 & 0 & 0 & 2 \\ 1 & -1 & 0 & -\sqrt{2} & 0 & 0 & 0 & -2 \end{bmatrix} \qquad C := \begin{bmatrix} d_{00} \\ c_{00} \\ c_{10} \\ c_{11} \\ c_{20} \\ c_{21} \\ c_{22} \\ c_{23} \end{bmatrix}$$

y is the data vector; W has the complete set of functions used to transform the data as its columns. $y = WC$.

$$\text{augmen}(W, y) = \begin{bmatrix} 1 & 1 & 1.414 & 0 & 2 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1.414 & 0 & -2 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1.414 & 0 & 0 & 2 & 0 & 0 & 1 \\ 1 & 1 & -1.414 & 0 & 0 & -2 & 0 & 0 & 2 \\ 1 & -1 & 0 & 1.414 & 0 & 0 & 2 & 0 & 1 \\ 1 & -1 & 0 & 1.414 & 0 & 0 & -2 & 0 & 0 \\ 1 & -1 & 0 & -1.414 & 0 & 0 & 0 & 2 & -3 \\ 1 & -1 & 0 & -1.414 & 0 & 0 & 0 & -2 & 2 \end{bmatrix} \blacksquare$$

$$\text{rref}(\text{augmen}(W, y)) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -0.354 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0.354 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1.25 \end{bmatrix} \blacksquare$$

Therefore,

$$C = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.354 \\ 0.354 \\ 0.25 \\ -0.25 \\ 0.25 \\ -1.25 \end{bmatrix} \blacksquare$$

The data function, y, can be expressed as a linear combination of the scaling function and wavelets with these coefficients stored in C.

$$y = \frac{1}{2}\phi_{00} + \frac{1}{2}\psi_{00} + \frac{-1}{2\sqrt{2}}\psi_{10} + \frac{1}{2\sqrt{2}}\psi_{11} + \frac{1}{4}\psi_{20} + \frac{-1}{4}\psi_{21} + \frac{1}{4}\psi_{22} + \frac{-5}{4}\psi_{2}$$

For the first method of compression, some of these coefficients and the inverse of W will be used.

$$W^{-1} = \begin{bmatrix} 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 & -0.125 & -0.125 & -0.125 & -0.125 \\ 0.177 & 0.177 & -0.177 & -0.177 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.177 & 0.177 & -0.177 & -0.177 \\ 0.25 & -0.25 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & -0.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & -0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & -0.25 \end{bmatrix}$$

$y = WC$

$W^{-1}y = C$

$W^{-1}y' = C'$, where C' will be the coefficients chosen, and y' will be the compressed data.

   If all of the coefficients were used, the original data would be given back.

$$C' = \begin{bmatrix} \dfrac{1}{2} \\[2mm] \dfrac{1}{2} \\[2mm] \dfrac{-1}{2\sqrt{2}} \\[2mm] \dfrac{1}{2\sqrt{2}} \\[2mm] 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

These are the coefficients that are being chosen. In other words, only the scaling function, the Mother Wavelet, and the first level of Wavelets are being used.

$$\text{augment}(W^{-1}, C') = \begin{bmatrix} 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.5 \\ 0.125 & 0.125 & 0.125 & 0.125 & -0.125 & -0.125 & -0.125 & -0.125 & 0.5 \\ 0.177 & 0.177 & -0.177 & -0.177 & 0 & 0 & 0 & 0 & -0.354 \\ 0 & 0 & 0 & 0 & 0.177 & 0.177 & -0.177 & -0.177 & 0.354 \\ 0.25 & -0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & -0.25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & -0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & -0.25 & 0 \end{bmatrix} \blacksquare$$

$$\text{rref augment}(W^{-1}, C') = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1.5 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1.5 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -0.5 \end{bmatrix} \blacksquare$$

$$y' = \begin{bmatrix} 0.5 \\ 0.5 \\ 1.5 \\ 1.5 \\ 0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{bmatrix}$$ This is the compressed data obtained using the first method of compres

Compressed Data Against Original Data

Another method that can be used to transform the data is to use the transpose of W. The transpose will now be used to compress the data.

$$
W^T = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1.414 & 1.414 & -1.414 & -1.414 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.414 & 1.414 & -1.414 & -1.414 \\
2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & -2
\end{bmatrix}
$$

$W^T y" = C.$
Note that all of the original coefficients will be used this time.

$$
\text{augment}W^T, C = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0.5 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0.5 \\
1.414 & 1.414 & -1.414 & -1.414 & 0 & 0 & 0 & 0 & -0.354 \\
0 & 0 & 0 & 0 & 1.414 & 1.414 & -1.414 & -1.414 & 0.354 \\
2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 \\
0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & -0.25 \\
0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0.25 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & -1.25
\end{bmatrix}
$$

$$\text{rref augment } W^T, C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.125 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0.125 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0.25 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.125 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -0.375 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.25 \end{bmatrix} \blacksquare$$

By using the transpose of W, a completely different compressed vector, y'', is obtained.

$$y'' = \begin{bmatrix} 0.125 \\ 0 \\ 0.125 \\ 0.25 \\ 0.125 \\ 0 \\ -0.375 \\ 0.25 \end{bmatrix} \blacksquare$$

Compressed Data Against Original Data
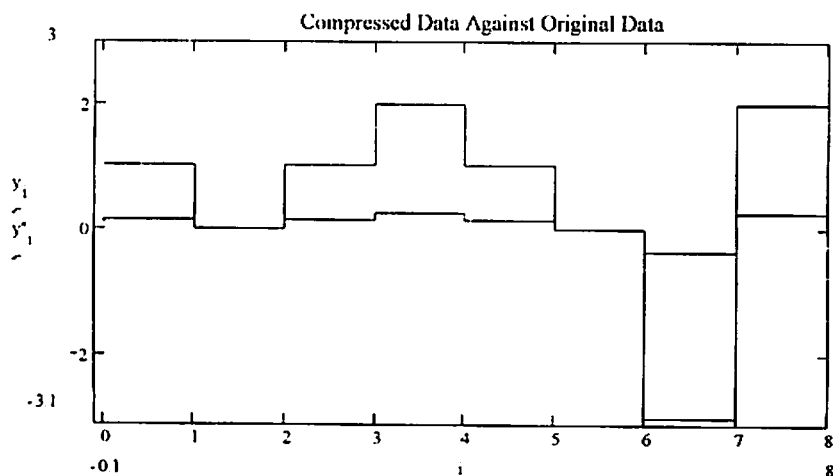
Now that some trivial wavelets with a small data set have been seen, it is interesting to observe how a larger data set can be transformed using a set of nontrivial wavelets. The Daubechies Wavelet will be used to "denoise" a noisy mass of tidal data from Bridgeport Harbor.
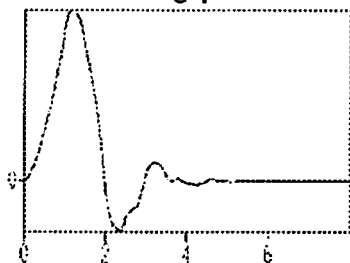
Above is the plot of Daubechies' scaling function. A plot of one of the Daubechies Wavelets, namely her four-coefficient wavelet, follows. This is the wavelet used by Mathcad to perform a wavelet transformation.

Hourly Tide Heights at Bridgeport Harbor
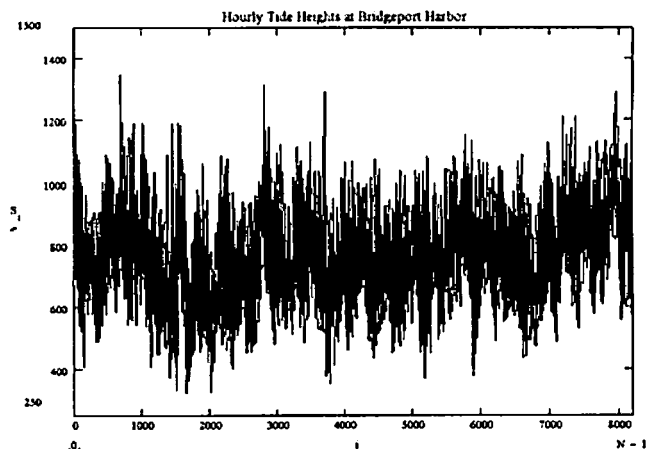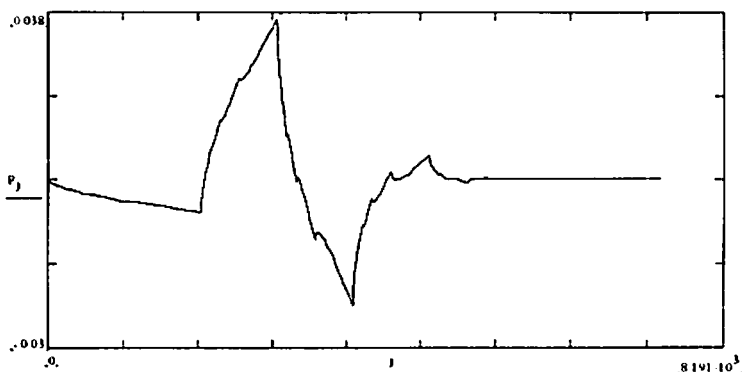
Mathcad's built-in "wave" function uses the Daubechies wavelet whose plot is shown above.

$$W := \text{wave}(S)$$

$$\text{Nlevels} := \frac{\ln(N)}{\ln(2)} - 1 \qquad \text{Nlevels} = 12$$

Notice that the only way that a wavelet transformation can be performed on a data vector is if the data vector is of size 2^k, where $k \in N$.

The above computation reveals that twelve levels of wavelets are used to transform this data. The following is what the data looks like at levels 4 through 8.
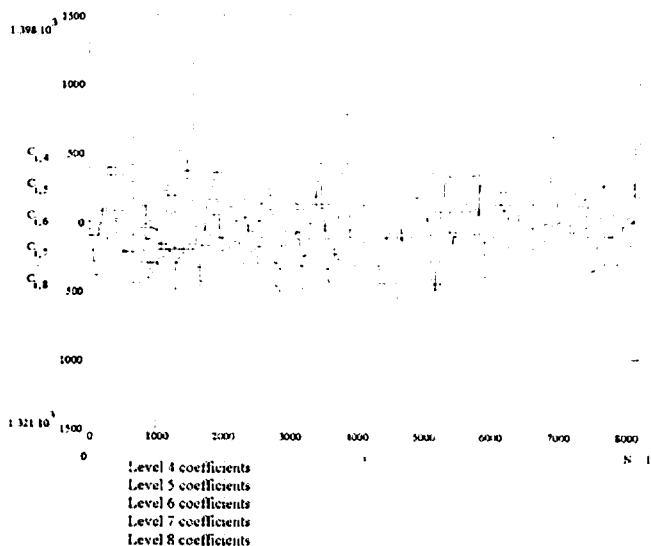
The "middle" levels are displayed below because they are the most practical ones of the set. The lower levels are rather boring, while the higher levels are too active to provide a clear picture.

First level at which coefficients are set to zero:

$$L := 6$$

$$j := 2^L .. N - 1 \qquad W_j := 0 \qquad S' := \text{iwave}(W)$$

Since $L = 6$, only the first five levels of wavelets are being used to denoise the data. These five levels give the following approximation or denoised data set. If one were to use an L less than 6, the approximation might not be acceptable. On the other hand, if one were to have an L greater than 6, there could still be too much noise in the data to obtain a clear picture.
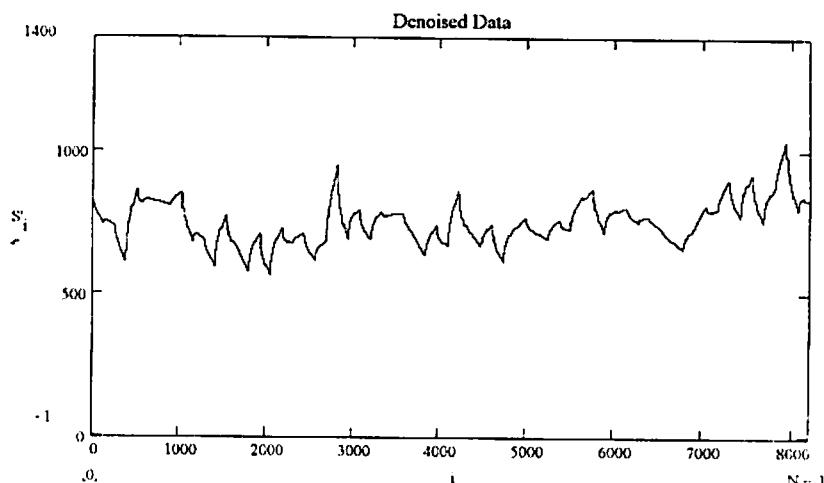
One agency that makes a great use of wavelets is the Federal Bureau of Investigation. When the FBI digitizes a single fingerprint, it occupies 10 MB of disk space. That is about seven high-density floppy disks. The FBI has approximately 200 million fingerprint cards. This means that the FBI's fingerprint database occupies 2000 terabytes of disk space. They use a process called Wavelet Scalar Quantization which uses a discrete wavelet transform, as seen earlier, to compress the data with a ratio of 20:1. Not only does this transform help them use only 5 % of the disk space that they would have used, but it also makes transmission of data much faster if sending via e-mail.

Wavelets can also be used to denoise a signal in signal processing and "clean up" images such as x-rays or magnetic-resonance images to give a clearer picture of what needs to be seen. Although wavelet compression is not widely used on the Internet yet, ".wif" files exist, along with plug-ins available on the Internet to help decode them.

Acknowledgments. I would like to give sincere thanks to two of my professors, Dr. John Stevenson and Dr. Andrew Rockett, from Long Island University-C.W. Post Campus for their time and consideration.

References

1. Vidakovic, Brani and Muller, Peter Wavelets for Kids: A Tutorial Introduction, Duke University, "unpublished".

2. "Historical Perspective", http://www.amara.com/IEEEwave/IW_history.html, verified on (11/24/98)

3. http://www.c3.lanl.gov/brislawn/FBI.FBI.html, verified on (12/10/98)

4. "Discovering Wavelets", http://www.gvsu.edu/mathstat/wavelets.htm verified on (12/02/98)

Starting a KME Chapter

Complete information on starting a chapter of KME may be obtained from the National President. Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; students must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $50) and the expenses of the installing officer. The individual membership fee to the national organization is $20 per member and is paid just once, at that individual's initiation. Much of the $20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offering and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before July 1, 2001. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 2001 issue of *The Pentagon*, with credit being given to the student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: xxwilke@acc.wuacc.edu).

PROBLEMS 540-545

Problem 540. Proposed by the Robert Rogers, SUNY College at Fredonia, Fredonia, New York.

Given a quintic polynomial f(x) with exactly one inflection point at x = 0, one maximum at x = -1, and one minimum at x = m, what is the maximum value the polynomial can attain? [Note: For a cubic polynomial, m = 1.]

Problem 541. Proposed by the Alma College Problem Solving Group, Alma College, Alma, Michigan.

Find a closed form for $\sum_{k=0}^{n} k^5 \binom{n}{k}$

Problem 542. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Instead of using the correct arc length formula, a student used the formula $\int_b^a \sqrt{1 + f''(x)}\,dx$ on a test and obtained the correct answer. Find all functions $f(x)$ such that this formula will produce the correct answer for arc length.

Problem 543. Proposed by Carol Browning, Drury College, Springfield, Missouri.

Let a_0 be a given positive integer and define the sequence a_k by $a_{k+1} =$

$\frac{a_k}{2}$ if $2|a_k$ and $a_{k+1} = 3a_k + 1$ otherwise. For example, if a_0 is 3, the sequence is $3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, \ldots$. Define the function $P(n)$ on the positive integers by $P(n) = j$ if in the sequence originated by $a_0 = n$, the first power of 2 to appear is 2^j.

For example, $P(3) = 4$. If no power of 2 appears in the sequence, we define $P(n)$ to be 0. Prove that $P(n)$ is odd exactly when n is an odd power of 2.

Problem 544. Proposed by Robert Stump, Richmond, Virginia.

Given triangle ABC with the lengths of $AB = c$, $AC = b$, and $BC = a$ respectively,

(a) Let $CM_1 = m_1$ be the median to AB in triangle ABC. Let $M_1M_2 = m_2$ be the median to AC in ACM_1. Continuing this process, let $M_{n-1}M_n = m_n$ be the median to AM_{n-2} in triangle $AM_{n-2}M_{n-1}$. In terms of a, b, and c find $\sum_{k=1}^{\infty} m_k$.

(b) Let $CH_1 = h_1$ be the altitude to AB (or AB extended) in triangle ABC. Let $H_1M_2 = h_2$ be the altitude to AC in triangle ACH_1. Continuing this process, let $H_{n-1}H_n = h_n$ be the altitude to AH_{n-2} in triangle $DAH_{n-2}H_{n-1}$. In terms of a, b, and c find $\sum_{k=1}^{\infty} h_k$.[1]

The editor wishes to acknowledge that a late solution for problem 525 was received from Justin Provchy and a late solution for problem 529 was received from Kim Goto. Both are students at California State University, Fresno, California.

| Please help your editor by submitting problem proposals. |

SOLUTIONS 528 and 530–534

Problem 528. Proposed by the editor.

Consider a paired number $p(n)$ to be formed by concatenating the same number twice; e.g. $p(1234) = 12341234$. What is the smallest integer n for which $p(n)$ is a perfect square? What is the next smallest integer nn for which $p(nn)$ is a perfect square and nn has more digits than n does? smallest square twin?

Solution by the editor.

Let $p(n) = RR$ where R has k digits. Then $p(n) = R(10^k + 1)$ where

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[1] This portion of the problem was submitted without a solution.

k is a positive integer and $10^k > R > 10^{k-1}$ in order for R not to have zero as a leading digit.

Suppose that $p(n) = N^2 = R(10^k + 1)$. Let $(R, 10^k + 1) = d$ for some integer d. Then

$$\left(\frac{N}{d}\right)^2 = \left(\frac{R}{d}\right)\left(\frac{10^k + 1}{d}\right) \tag{1}$$

where $(\frac{R}{d}, \frac{10^k+1}{d}) = 1$. Now since $\frac{R}{d} < \frac{10^k+1}{d}$, by equation (1) we must have $\frac{10^k+1}{d}$ is divisible by $\frac{R}{d}$ so that $\frac{10^k+1}{d} = \left(\frac{R}{d}\right) T$ for some integer T. Hence equation (1) becomes $\left(\frac{N}{d}\right)^2 = \left(\frac{R}{d}\right)^2 T$ so that T must be a perfect square.

Thus $10^k + 1$ must have a nontrivial square factor (i.e. a square factor > 1).

Thus we seek integers k such that $10^k + 1$ has a nontrivial square factor. Using a program like UBASIC, one can easily test numbers of the form $10^k + 1$ for nontrivial square factors. The smallest such k is 11.

Then $10^{11} + 1 = 11^2 * 23 * 4093 * 8779 = 121 * 826446281$ and R must be a square multiple of 826446281. The two smallest such multiples of 826446281 which exceed 10^{10} are $16 * 826446281 = 13223140496$ and $25 * 826446281 = 20661157025$. Hence the solutions for part (a) are

$$1322314049613223140496 = 36363636364^2$$

and

$$2066115702520661157025 = 45454545455^2$$

For part (b), the next k which yields a nontrivial square factor is $k = 21$ with $10^{21} + 1$

$$= 7^2 * 11 * 13 * 127 * 2689 * 459691 * 909091$$
$$= 49 * 20408163265306122449.$$

Proceeding as before, the next smallest appropriate multiples of

$$20408163265301122449 > 10^{20}$$

are

$$9 * 20408163265530122449 = 183673469387755102041$$

and

$$16 * 20408163265530122449 = 326530612244897959184$$

Finally numbers are

$$183673469387755102041183673469387755102041$$
$$= 428571428571428571429^2$$

and

$$3265306122448979591843265306122448897959184$$
$$= 571428571428571428572^2$$

Problem 530. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Start at the origin, move to the right one unit, move up $\frac{1}{2}$ unit, move to the right $\frac{1}{4}$ unit, move up $\frac{1}{8}$ unit, etc. Connect the limiting point of the path from the origin to the origin by drawing a straight line connecting these two points. What is the area of the figure enclosed by connecting the origin and the limiting point of the original path?

Solution by Bradley Sward, Benedictine University, Lisle, Illinois.(Revised by the editor.)

From the information given, we have an infinite sequence of similar right triangles connected in such a way that their hypotenuses form the line $y = x/2$. Considering the bases of the right triangles, we have an infinite series $1, \frac{1}{4}, \frac{1}{16}, \ldots$ with the base of the nth right triangle given by $2^{-2(n-1)}$. Hence the x coordinate of the limit point is $1 + \frac{1}{4} + \frac{1}{16} + \ldots = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$. Correspondingly considering the altitudes of the right triangles, we have the infinite series $\frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \ldots 2^{-(2n-1)}$. Hence the y coordinate of the limit point is $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \ldots = \frac{1}{2} \cdot \frac{1}{1-\frac{1}{4}} = \frac{2}{3}$. Hence the limiting point of the figure is given by $(x, y) = (\frac{4}{3}, \frac{2}{3})$.

Adding together the areas of the triangles, we have

$$A = \sum_{n=1}^{\infty} \frac{1}{2} \left(2^{-(2n-1)} 2^{-2(n-1)} \right) = \sum_{n=1}^{\infty} 2^{-(4n-2)} = \left[\frac{1}{4} \cdot \frac{1}{1-\frac{1}{16}} \right] = \frac{4}{15}$$

Also solved by the proposer. Partial solutions were received from the Alma College Problem Solving Group and Clayton W. Dodge, University of Maine. These solutions inadvertently omitted the coordinates of the limit point.

Problem 531. Proposed by Russell Euler and Jawad Sadek jointly, Northwest Missouri State University, Maryville, Missouri.

A bridge in the form of a circular arc spans a river. At a distance of A feet measured horizontally from the shore, the bridge is B feet above the surface of the water. At the center of the bridge, the bridge is C feet above the surface of the water. Assuming that the bridge rests exactly on

the shores, find the width of the river in terms of A, B, and C. For $B > C$, discuss the cases where there are zero, one or two solutions.

Solution by Clayton W. Dodge, University of Maine, Orono, Maine.

We shall use lower case letters to denote distances and upper case letters to denote points. As shown in Figure 1, the banks of the river are located at points S and T, the midpoint of the bridge is located at M, and O denotes the center of the circular arch. Let the distance across the river be $2u$ and the point on the bridge at which height b is measured be denoted by H. As shown in Figure 1, two right triangles are formed with hypotenuses OS and OH, each of length r, the radius of the circular arch. The legs of these right triangles are respectively u and $r - c$, and $u - a$ and $b + u - c$. The Pythagorean Theorem yields $r^2 = u^2 + (r - c)^2$ and $r^2 = (u - a)^2 + (b + r - c)^2$.
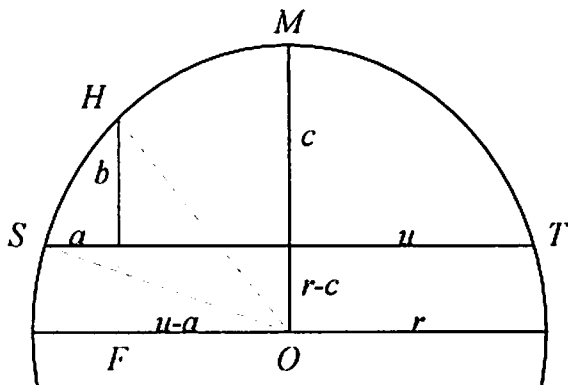
Figure 1

After expanding and simplifying each equation and subtracting the first equation from the second, the first equation and the difference become respectively, $0 = u^2 - 2cr + c^2$ and $0 = b^2 + 2br - 2bc - 2au + a^2$. Eliminating r from these equations yields $bu^2 - 2acu + (b^2c - bc^2 + a^2c) = 0$. The solution of this equation is given by $u = \frac{ac \pm \sqrt{c(c-b)(a^2+b^2)}}{b}$. The width of the river is $2u = \frac{2ac \pm 2\sqrt{c(c-b)(a^2+b^2)}}{b}$.

There is one other case to consider: when the distance a is measured from the other side of the river; i.e. from the point T rather than from the point S. Then the only change in the algebra is that the distance $u - a$ becomes $a - u$ in the triangle with hypotenuse OH. Hence the solution remains unchanged.

For a solution to exist we must have $c > b$. Then there is always a solution using the plus sign in the expression for u. For a second solution to exist, we must have $a^2c^2 > c(c-b)(a^2+b^2)$ which reduces to $a^2 + b^2 > bc$,

and in addition $u < a$, where the expression for u uses the minus sign, which reduces to $a^2 > 4c(c - b)$.

Also solve by the proposers.

Problem 532. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Join consecutively the points

$$(1,0), (\tfrac{1}{2}, (\tfrac{1}{2})^2), (\tfrac{1}{3}, 0), (\tfrac{1}{4}, (\tfrac{1}{4})^2), \ldots, (\tfrac{1}{2n}, (\tfrac{1}{2n})^2), (\tfrac{1}{2n+1}, 0), \ldots$$

with line segments, and include the point $(0,0)$ in the resulting graph. Use the x axis as a base of the graph which should look like an infinite series of triangles. Find the total area of the series of triangles. (This is a generalization of Pentagon problem 214.)

Solution by SUNY Fredonia Student Group, SUNY Fredonia, Fredonia, New York.

Let A denote the desired area. Then since the area of the ith triangle is given by $A_i = \frac{1}{2} \left(\frac{1}{(2i)^2} \right) \left(\frac{1}{2i-1} - \frac{1}{2i+1} \right)$, we have

$$A = \sum_{i=1}^{\infty} \frac{1}{2} \left(\frac{1}{(2i)^2} \right) \left(\frac{1}{2i-1} - \frac{1}{2i+1} \right) \tag{1}$$

$$= \frac{1}{4} \sum_{i=1}^{\infty} \frac{1}{i^2 (2i-1)(2i+1)} \tag{1}$$

Using partial fractions to decompose the right side of equation (1), we get $A = \frac{1}{4} \sum_{i=1}^{\infty} \left(\frac{-1}{i^2} + \frac{2}{2i-1} - \frac{2}{2i+1} \right) = -\frac{1}{4} \sum_{i=1}^{\infty} \frac{1}{i^2} + \frac{1}{2} \sum_{i=1}^{\infty} \left(\frac{1}{2i-1} - \frac{1}{2i+1} \right)$
$= -\frac{1}{4} \frac{\pi^2}{6} + \frac{1}{2} \left[(1 - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{5}) + (\frac{1}{5} - \frac{1}{7}) + \ldots \right] \approx 0.08776648.$

Also solved by the Alma College Problem Solving Group, Alma College, Alma, Michigan; Clayton W. Dodge, University of Maine, Orono, Maine and the proposer.

Problem 533. Proposed by the editor.

Which of the following quantities is larger
$$(31415926535!)^2 \ or \ 31415926535^{31415926535}?$$

Solution by Brock Kremer, Alma College, Alma, Michigan.

We prove that $(n!)^2 > nn$ for any integer $n \geq 3$.

$\frac{(n!)^2}{n^n} = \frac{1 \cdot n}{n} \cdot \frac{2 \cdot (n-1)}{n} \cdot \frac{3 \cdot (n-2)}{n} \cdot \ldots \cdot \frac{k \cdot (n-k+1)}{n} \cdot \ldots \cdot \frac{(n-1) \cdot 2}{n} \cdot \frac{n \cdot 1}{n}$. Each of the first and last factors each are equal to 1. It remains to determine the nature of the intermediate factors. Each of these factors has the form $\frac{k(n-(k-1))}{n}$ for integers n and k such that $2 \leq k < n-1$. Suppose that $\frac{k(n-(k-1))}{n} \leq 1$. Then $k(n-(k-1)) \leq n$ which reduces to $k(n-k) \leq (n-k)$. But since $2 \leq k < n-1$, $n-k$ is positive and we have $k \leq 1$, a contradiction. Hence $\frac{k(n-(k-1))}{n} > 1$ for all integers n and k such that $2 \leq k < n-1$. Therefore $(n!)^2 > nn$ for all integers $n \geq 3$.

In particular $(31415926535!)^2 > 31415926535^{31415926535}$.

Also solved by: Alma Problem Solving Group, Alma College, Alma , Michigan; Casey Barwell, Centre College, Danville, Kentucky; Clayton W. Dodge, University of Maine, Orono, Maine; Kensaku Umeda, Eastern Kentucky, University, Richmond, Kentucky; J. Spencer Wideman, Alma College, Alma, Michigan; Jessica Little, Alma College, Alma, Michigan; Mariah Grant, Alma College, Alma, Michigan; Robin Levere, Alma College, Alma, Michigan; Chris farmer, Northwestern Missouri State University, Maryville, Missouri.

Problem 534. Proposed by the editor.

The millennium is fast approaching. Whether it starts on January 1, 2000, as many people believe or January 1, 2001, as the purists argue is not material to this problem. Discover whether or not there is a prime p such that $p!$ ends in exactly 2000 zeroes. Is there a corresponding prime q such that $q!$ ends in exactly 2001 zeroes?

Solution by Clayton W. Dodge, University of Maine, Orono, Maine.

When looking at the number of zeroes in which $n!$ terminates, we observe that each zero represents a factor of $10 = 2 \cdot 5$. Since there are plenty of factors of 2 available, we need only count the factors of 5, one of which is gained each time we multiply by a multiple of 5. Thus $5! = 120$ is the first factorial which ends in a zero, $10!$ ends in two zeroes, and so forth. Multiples of 25 produce two more factors of 5, multiples of 125 produce three more factors of 5, and in general multiples of 5^k produce k extra factors of 5.

Then the number z of zeroes that a positive integer factorial $n!$ ends in equals

$$z = \left[\frac{n}{5}\right] + \left[\frac{m}{5^2}\right] + \left[\frac{n}{5^3}\right] + \left[\frac{n}{5^4}\right] + \ldots \qquad (1)$$

where $[x]$ denotes the greatest integer which does not exceed x. If one

approximates z by dropping the brackets, the problem requires that

$$z = \frac{n}{5} + \frac{n}{5^2} + \frac{n}{5^3} + \frac{n}{5^4} + \ldots = n\left(\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \ldots\right) = \frac{\frac{n}{5}}{1 - \frac{1}{5}} = \frac{n}{4}.$$

Thus $n \approx 8000$. For $n = 8000$, $z = 1600 + 320 + 64 + 12 + 2 = 1998$ zeroes. Since we need two more factors of 5, we examine the range from 8010 to 8014. We find that 8011 is prime and 8011! Ends in precisely 2000 zeroes. Similarly, examining the range from 8015 to 8019, we find that 8017 is prime and 8017! ends in precisely 2001 zeroes. Working downwards, we find that 8009! ends in 1999 zeroes, 7993! ends in 1994 zeroes, 7963! ends in 1987 zeroes and 7951! ends in 1985 zeroes. Working in the other direction, one finds that 8039! ends in 2010 zeroes, 8059! ends in 2011 zeroes and 8069! ends in 2013 zeroes.

Also solved by the Alma College Problem Solving Group, Alma College, Alma, Michigan and Steven F. Shearer, Winchester, Kentucky.

Editor's Comment: Steven Shearer is a 10 year old student who is being home schooled.

Chapter Web Sites
Send additions or corrections to Arnold Hammel at a.hammel@cmich.edu

Alabama Zeta Birmingham-Southern College
> http://www.bsc.edu/science/math/kme.htm

Arkansas Alpha Arkansas State University
> http://www.csm.astate.edu/students/kme/index.html

California Gamma California Polytechnic State University
> http://www.calpoly.edu/~kappamu/

Colorado Beta Colorado School of Mines
> http://magma.mines.edu/Stu_life/organ/kme/kme.html

Connecticut Beta Eastern Connecticut State University
> http://www.ecsu.ctstateu.edu/depts/mates/nhs.html#kappa

Illinois Delta College of St. Francis
> http://www.stfrancis.edu/ma/honor.htm

Illinois Eta Western Illinois University
> http://www.wiu.edu/users/mikme/

Indiana Alpha Manchester College
> http://www.manchester.edu/department/MathCptrSci_old/kme.htm

Indiana Delta University of Evansville
> http://www2.evansville.edu/mathweb/

Iowa Alpha University of Northern Iowa
> http://www.math.uni.edu/KME/KME.html

Kansas Alpha Pittsburg State University
> http://www.pittstate.edu/math/kme.html

Kansas Gamma Benedictine College
    http://www.benedictine.edu/math-cs/kme.html
Kentucky Alpha Eastern Kentucky University
    http://eagle.eku.edu/faculty/pjcostello/kme/
Kentucky Beta Cumberland College
    http://cc.cumber.edu/acad/math/kme.htm
Maryland Beta Western Maryland College
    http://wwwfac.wmdc.edu/HTMLpages/Academics/Math/KME.html
Mississippi Beta Mississippi State University
    http://www.math.msstate.edu/~pearson/kme-maa.htm
Mississippi Gamma University of Southern Mississippi
    http://www.math.usm.edu/organizations_html/kme.html
Missouri Alpha Southwest Missouri State University
    http://studentorganizations.smsu.edu/KME/
Missouri Beta Central Missouri State University
    http://153.91.1.112/~kme/kme.html
Missouri Zeta University of Missouri-Rolla
    http://www.umr.edu/~kme/
New Mexico Alpha University of New Mexico
    http://www.math.unm.edu/~kme/
New York Eta Niagara University
    http://www.niagara.edu/math/kme.html
New York Lambda C.W. Post Center-Long Island U. Brookville
    http://www.cwpost.liunet.edu/cwis/cwp/clas/math/kme.htm
New York Xi Buffalo State College
    http://math.buffalostate.edu/~kme/index.html
Ohio Alpha Bowling Green State University
    http://www.bgsu.edu/departments/math/kme/
Ohio Zeta Muskingum College
    http://pluto.bsc.muskingum.edu/~kandhari/kme/kme.html
Ohio Gamma Baldwin Wallace College
    http://www.bw.edu/~wwwkme/
Oklahoma Alpha Northeastern State University
    http://arapaho.nsuok.edu/~kme
Oklahoma Gamma Southwestern Oklahoma State University
    http://www.swosu.edu/student/stdorg/kme
Pennsylvania Xi Cedar Crest College
    http://www.cedarcrest.edu/academic/mat/saa.htm
Wisconsin Gamma University of Wisconsin–Eau Claire
    http://www.uwec.edu/Academic/Curric/gouletmr/kme/kmehome.html

Reports of the Regional Conventions

Report of the South Central Regional Convention

The Oklahoma Delta Chapter at Oral Roberts University hosted the South-Central Regional Conference on April 14 and 15, 2000. The conference included the chapter initiation, a banquet, featured speakers, and student presentations. In attendance were members from Colorado and Oklahoma. The chapter initiation on Friday evening welcomed twenty-two new members. A banquet of Mexican fare highlighted the evening. Saturday morning Dr. Dominic Halsmer delivered the Keynote Speech regarding his work on spinning spacecraft for NASA. Six student presentations followed. Topics included the Lorenz equations, linear transformations, and Markov chains. Regional director Dr. Donna Hafner, who also closed the conference with words of thanks to the OK Delta chapter sponsor, Dr. Vincent Dimiceli, presented awards.

Report of the North Central Regional Convention

The KME North Central Regional Convention was held on April 7 and 8, 2000, at Benedictine College in Atchison, KS. Ninety-four people attended from 17 different chapters. The host chapter, KS Gamma, celebrated its 60th Anniversary at the convention. Seven student papers were presented, with awards going to the top two papers. The award winners were Micah James from Iowa Delta and Lindsey Crain from Tennessee Gamma. Bryan Dawson gave an after-lunch talk titled "KME Student Scholarship - 1931 to the Present." At the closing session it was noted that three corresponding secretaries from our region are retiring at the end of this school year. They are: Mary Elick from MO Iota, Mary Sue Beersman from MO Eta, and John Atkinson, MO Lambda.

Report of the Great Lakes Regional Convention

The Great Lakes Region of the Kappa Mu Epsilon Mathematics Honor Society held a regional convention on March 24 and 25, 2000. The Ohio Zeta chapter at Muskingum College hosted the convention. Ohio Zeta President Jeff Shoemaker and Professor Andy McHugh organized the event.

Pizza and beverages were served at a reception on Friday evening in the Boyd Science Center. Registration was held on Friday evening and Saturday morning in the same location. A total of 24 members attended representing Michigan Beta, New York Eta, Ohio Alpha, Ohio Zeta, Pennsylvania Mu, and Gary Sherman who is now a faculty member at Rose-Hulman

Institute of Technology. Also in attendance was Professor Leo Schneider from John Carroll University who was representing Pi Mu Epsilon.
Five student papers were presented on Saturday morning including:

Entropy Properties of 2x2 Games
Benjamin Otto, Ohio Alpha, Bowling Green State University

A Method for Deriving the Principal Unit Normal Vector for Two-Space Vectors
Nicholaos John Jones, Pennsylvania Mu, St. Francis College

From Snowflakes to Lobsters
Jeff Shoemaker, Ohio Zeta, Muskingum College

A Content Analysis of Gender Representation in Algebra Textbooks
Katherine Wallace, Pennsylvania Mu, St. Francis College

Expected Value of Randomly Generated Triangles
Courtney Fitzgerald New York Eta, Niagara University

Jeff Shoemaker was selected for presenting the outstanding paper and awarded a TI-83 graphing calculator. A special presentation was also made by National President-Elect Robert Bailey to past National President Arnold Hammel. It was a Certificate of Distinction from the Association of College Honor Societies.
The guest speaker was Dr. Gary Sherman of the Rose-Hulman Institute of Technology. His talk entitled "How Long Does it Take to Shuffle A Deck of Cards?" was well received. A fine buffet lunch preceded the adjournment.

Kappa Mu Epsilon News

Edited by Don Tosh, Historian

News of chapter activities and other noteworthy KME events should be sent to Don Tosh, Historian, Kappa Mu Epsilon, Mathematics Department, Evangel College, 1111 N. Glenstone, Springfield, MO 65802, or to toshd@evangel.edu.

Installation of New Chapters

Georgia Gamma
Piedmont College, Demorest

The Installation of the Georgia Gamma Chapter of Kappa Mu Epsilon was held on April 7, 2000, in the Conference Room of Piedmont College's Corner Cafe. Dr. Joe Sharp, Corresponding Secretary of the Georgia Alpha Chapter of KME at the State University of West Georgia, served as the Installing Officer at the Installation Ceremony. Mr. Tony McCullers served as the conductor during the ceremony. There were 3 charter members of Georgia Gamma: Heather Knight, Tony McCullers, and Amie Mills. Following the 5pm Installation Ceremony, a reception was held in honor of the charter members of the Georgia Gamma Chapter.

Chapter News

AL Gamma   Chapter President—Chris Harmon
University of Montevallo, Montevallo   18 actives, 9 associates
Other spring 2000 officers: Tommy Fitts, vice president; Jared Phillips, secretary; Don Alexander, corresponding secretary.

AL Zeta   Chapter President—Melanie Styers
Birmingham Southern College, Birmingham   20 actives
Other spring 2000 officers: Kelly O'Donnell, vice president; Elizabeth White, secretary/treasurer; Mary Jane Turner, corresponding secretary; Shirley Brannan, faculty sponsor.

AR Alpha   Chapter President—Michael Mott
Arkansas State University, Jonesboro   7 actives, 5 associates
Other spring 2000 officers: Laura Firestone, secretary; Jacob Hamilton, treasurer; William Paulsen, corresponding secretary/faculty sponsor.

CA Gamma   Chapter President—Andrew Oster

Cal Poly, San Luis Obispo   14 actives, 5 associates
   Other spring 2000 officers: Jeff Mintz, vice president; Jonathan Shapiro, corresponding secretary/faculty sponsor.

CO Delta   Chapter President—Natalie Todd
Mesa State College, Grand Junction   23 actives
   Twenty-two members attended a pizza party at Big Cheese Pizza on April 10. Pins and certificates were presented to members initiated in the fall, and new chapter officers were elected for 2000-01. Other spring 2000 officers: Valerie Coniff, vice president; Richard Hasenauer, secretary; Sylvia Myhre, treasurer; Donna Hafner, corresponding secretary; Kenneth Davis, faculty sponsor.

GA Alpha   Chapter President—Karen Jones
State University of West Georgia, Carrollton   20 actives, 11 associates
   GA Alpha held its annual Initiation Meeting on April 19 and initiated 11 new members. New officers for 2000-2001 were then elected. Following the Initiation Ceremony, a reception was held in honor of the new initiates. At the reception, the names of the students who received mathematics scholarships and/or awards this year were announced (most of whom are KME members): Karen Jones won both the Boyd Award and one of the two Marion Crider Awards, Blake Smith won the Burson Calculus Award, Christin Phillips won the other Marion Crider Award, Bryan Crawford won the Whatley Scholarship, Mike Maycumber won the Cooley Scholarship, and Natalie Young won the Martin Scholarship. Other spring 2000 officers: Christin Phillips, vice president; Kaitlin Lewis, secretary; Daemon Whittenburg, treasurer; Joe Sharp, corresponding secretary; Mark Faucette and Joe Sharp, faculty sponsors.

GA Beta   Chapter President—Billie Jo Matkovitch
Georgia College and State University, Milledgeville   15 actives
   Other spring 2000 officers are Michelle Blay, vice president; Robert Stepowany, secretary/treasurer; Craig Turner, corresponding secretary; Hugh Sanders, faculty sponsor.

GA Gamma   Chapter President—Tony McCullers
Piedmont College, Demorest   4 actives
   The Georgia Gamma Chapter of Kappa Mu Epsilon was installed on April 7.
   Other spring 2000 officers: Tony McCullers, vice president; Heather Knight, secretary; Amie Mills, treasurer; Shahryar Heydari, corresponding secretary/faculty sponsor.

IL Theta   Chapter President—Rebecca Wagner

Benedictine University, Lisle 15 actives, 5 associates

Together with the Math Club, the KME chapter was very active this spring. There were several meetings with math videos/games and refreshments. Student speakers practiced their talks for local conferences. Other spring 2000 officers: Ripul Panchal, vice president; Natasha Brasic, secretary; Lisa Townsley Kulich, corresponding secretary/faculty sponsor.

IA Alpha   Chapter President—Gary Spieler
University of Northern Iowa, Cedar Falls 45 actives

Student member Brad Rolling presented his paper "Investigation into Buffon's Needle" at our February meeting. John Neely presented his paper "Impossible? Prove It! A Treatise on the Impossibility of Trisecting an Angle using a Compass and Straightedge" at the March meeting while Douglas Kinney presented "Dyscalculia, More Than Not Being Able to 'Do Math'" at the April meeting. Student member Teresa Grothus addressed the spring initiation banquet with "The Mobius Strip". In addition, we were privileged to have honorary guest Lester Artherholt, a 1931 Charter Member, at our April banquet where we initiated three new student members. Lastly, Douglas Kinney presented his paper "Dyscalculia, More Than Not Being Able to 'Do Math'" at the KME Regional Convention at Benedictine College in Atchison, KS on April 8. Other spring 2000 officers: Allysen Edwards, vice president; Kamilla Guseynova, secretary; Barbara Meyers, treasurer; Mark Ecker, corresponding secretary/faculty sponsor.

IA Gamma   Chapter President—A. G. Kruger
Morningside College, Sioux City 11 actives, 12 associates

Our only activity this semester was the initiation. Other spring 2000 officers: Mary Curry, vice president; Michelle Harvey, secretary; Kyle Kolander, treasurer; Doug Swan, corresponding secretary/faculty sponsor.

IA Delta   Chapter President—Paul Seberger
Wartburg College, Waverly 58 actives, 2 associates

The January meeting resulted in final selection of the chapter T-shirt design, plans for the Mathematical Sciences Explorations event co-sponsored by the Mathematics, Computer Science and Physics Department and our KME chapter, and recruitment of team members for the math modeling contest sponsored by the Iowa MAA. During the February meeting, T-shirts were distributed and plans were made for the Initiation Banquet and other future meetings. We initiated 19 new members into our chapter during our March meeting. Jerrod Staack, a mathematics teacher in the Waverly-Shell Rock school system and former KME officer, was our banquet speaker. Micah James, a senior member of our chapter, presented the

paper he had prepared for the KME Regional at a special meeting in April. Micah's paper was selected as one of the two top papers presented at the Regional KME Meeting in Atchison, Kansas on April 8. The year-end activity for our chapter was a picnic on May 15 where members participated in several yard games and enjoyed an evening picnic meal together with the computer science and physics clubs. Other spring 2000 officers: Robyn Brent, vice president; Janelle Young, secretary; Daniel Bock, treasurer; August Waltmann, corresponding secretary; Mariah Birgen, faculty sponsor.

KS Beta   Chapter President—Katrina Penner
Emporia State University, Emporia   16 actives, 7 associates
   Other spring 2000 officers: Leah McBride, vice president; Melinda Born, secretary; Thad Davidson, treasurer; Connie Schrock, corresponding secretary; Larry Scott, faculty sponsor.

KS Gamma   Chapter President—Lance Hoover
Benedictine College, Atchison   12 actives, 15 associates
   In early February three KS Gamma members participated in the COMAP Modeling Contest. In late February, for the North Central on-site visit, the chapter had a display showing the various chapter activities. Initiation of four national members and nine associate members took place on March 1. After the initiation, all shared in pizza and conversation. In mid-March Dr. Vern Ostdiek spoke in the Faculty Colloquium Series on his sabbatical work in atmospheric science. On March 29 Brett Herbers, Janelle Kroll, and Angela Shomin received Sister Helen Sullivan Scholarships at the Honors Convocation. Kansas Gamma celebrated its 60th anniversary this spring. This was highlighted at the luncheon during the North Central Regional Convention hosted by Kansas Gamma on April 7-8. Several alumni attended including two of the charter members. Five Kansas Gamma members presented during the April 12 Discovery Day activities. Lance Hoover and Davyeon Ross took second place in the Business Plan Competition with their project called "AlumConnect." On May 1, the faculty entertained the senior graduates with a dinner at Marywood, home of Sister Jo Ann Fellin. Other spring 2000 officers: Curtis Sander, vice president; Jo Ann Fellin, corresponding secretary/faculty sponsor.

KS Delta   Chapter President—Laurie Payeur
Washburn University, Topeka   28 actives
   On February 28, the chapter had its annual spring initiation banquet with President Laurie Payeur presiding. Nine new members were initiated into KME. On April 8 five students and three faculty attended the Regional KME Convention at Benedictine College in Atchison, Kansas.

Dr. Kevin McCarter served as a judge for the presented papers. Additionally, on three occasions we met with the university mathematics club, "Mathematica", for a speaker and lunch and/or picnic. Other spring 2000 officers: Stephanie Adelhardt, vice president; Milissa Mikkelsen, secretary/treasurer; Allan Riveland, corresponding secretary; Ron Wasserstein and Donna LaLonde, faculty sponsors.

KS Epsilon   Chapter President—Adam North
Fort Hays State University, Hays   15 actives, 10 associates

Other spring 2000 officers: Wendy Scott, secretary/treasurer; Chenglie Hu, corresponding secretary; Lance Young and Greg Force, faculty sponsors.

KY Alpha   Chapter President—Shannon Purvis
Eastern Kentucky University, Richmond   23 actives

The spring semester began with floppy disk sales (together with the ACM chapter) to students in the computer literacy class and the "Mathematica" class. At a meeting in early February we made plans for initiation and discussed travel plans. On March 15, there were twenty-three students initiated as national members. Dr. John Wilson from Centre College gave an interesting talk entitled "Mathematics with the Lights Out (Puzzle)." Tiger Electronics markets a game called Lights Out that has 25 push-button switches that can be set initially on or off. Then when a switch is flipped, all adjacent lights are changed. The object is to push a sequence of switches that turns off all lights. Dr. Wilson had students stand and sit to illustrate the 3x3 version of the puzzle. In late March, the KY MAA meeting was held at EKU and Shannon Purvis (KY Alphas President) gave a talk on "Linear Programming: From Steel to Wall Street." April is Math Awareness Month and every day lists of several interesting facts about that day's number were placed all over the Wallace Building. For example, 13 is prime, part of a twin prime pair, a Fibonacci number, a Wilson Prime, and a Lucky number. The number facts were also available on the department web site. At the meeting in May, Dr. Costello gave a talk on "Fibonacci numbers and the Golden Ratio." Included in the talk were a few minutes from the video, "Donald in Mathemagic Land." Other spring 2000 officers: Katy Fritz, vice president; Jennie Campbell, secretary; Kensaku Umeda, treasurer; Pat Costello, corresponding secretary.

KY Beta   Chapter President—Velma Birdwell
Cumberland College, Williamsburg   44 actives

On March 7 at the atrium, KY Beta held an initiation and a joint banquet with Sigma Pi Sigma, the physics honor society. The chapter inducted seven new student members. Members inducted last year and graduating

seniors were also recognized during the banquet, presided over by outgoing president, Velma Birdwell. The department gave Senior awards at the banquet. Jointly with the Mathematics and Physics Club, KY Beta hosted Dr. Carroll Wells from David Lipscomb University on April 13. He spoke on "Michelangelo to Japan by Way of Grandma's-The Trail of a Geometric Construction." On April 14, members also assisted in hosting a regional high school math contest, held annually at Cumberland College. On April 24, the entire department, including the Math and Physics Club, Sigma Pi Sigma, and the Kentucky Beta Chapter, held the annual spring picnic at Briar Creek Park. Other spring 2000 officers: Simeon Hodges, vice president; Amanda Kidd, secretary; Melanie Maxson, treasurer; Jonathan Ramey, corresponding secretary; John Hymo, faculty sponsor.

MD Alpha   Chapter President—Kristen Balster
College of Notre Dame of Maryland, Baltimore   12 actives, 5 associates

On May 7 the annual induction ceremony for new permanent members was held with a picnic lunch followed by a presentation by Dr. Melissa McGrath of the Space Telescope Science Institute. She gave a very interesting and informative talk entitled Ten Years of Science with the Hubble Telescope. Other spring 2000 officers: Francesca Palek, vice president; Jane Orcutt, secretary; Jennifer Crawford, treasurer; Sister Marie Augustine Dowling, corresponding secretary; Joseph Di Rienzi, faculty sponsor.

MD Beta   Chapter President—Christina Addeo
Western Maryland College, Westminster   11 actives

Five new members were inducted this spring. At a Career Night Dinner, which was open to all math majors, four alumni spoke about career opportunities in mathematics and computer science. The chapter provided free tutoring service for mathematics and computer science courses. For "Movie Night" we showed "Cube". We paid a visit to the National Cryptologic Museum at Ft. Meade. We also elected new officers and sponsored an end-of-year picnic for all math majors. Other spring 2000 officers: Kevin Worley, vice president; Amy Bittinger, secretary; Michael Morgan, treasurer; Linda Eshleman, corresponding secretary; Harry Rosenzweig, faculty sponsor.

MA Alpha   Chapter President—Laura Small
Assumption College, Worcester   7 actives, 6 associates

On May 2, 2000, the Massachusetts Alpha chapter held an initiation and dinner for 5 new student and faculty members. Dr. Malcolm Asadoorian, of the Assumption faculty, spoke on "Calculus and Economics: Applications to Environmental Issues." Other spring 2000 officers: Christie Gaulin, vice president; Meredith Tebbetts, secretary; Charles Brusard, cor-

responding secretary/faculty sponsor.

MI Epsilon   Chapter President—Martin Przyjazny
Kettering University, Flint   25 actives, 49 associates
    During the Winter Term of 1999 our chapter by-laws were approved
and there was a second showing of "The Proof", the movie about Fermat's
last theorem. We held our initiation ceremony and banquet in March, initi-
ating 49 new members at that time. Professor Gary Johns of Saginaw Val-
ley State University was the guest speaker. His talk, entitled "Taking the
High Road Through Garbage, Tobacco, and Politics", included a survey of
mathematics as applied to everyday life. The movie "John Von Neumann"
about the 20th century giant in applied mathematics was shown during
Summer Term. New A-section officers were elected and a pizza party was
held. Kettering Professor Boyan Dimitrov spoke at the party about his ca-
reer in mathematics as he moved from Bulgaria to Russia to Canada and
finally to the United States. Other spring 2000 officers: David Murphy,
vice president; Erik Poppe, secretary; Jamey Howard, treasurer; Jo Smith,
corresponding secretary; Brian McCartin, faculty sponsor.

MS Alpha   Chapter President—Chris Sansing
Mississippi University for Women, Columbus   10 actives, 1 associate
    We held the March meeting on the 12th, and had the Initiation on the
28th. On April 10 we sponsored "Kite Nite", and on April 28th we co-
sponsored "Science/Mathematics Game & Fame Day". Other spring 2000
officers: Mindy Hill, vice president; Jennifer Kimber, secretary; Kent
Smith, treasurer; Shaochen Yang, corresponding secretary; Beate Zimmer,
faculty sponsor.

MS Epsilon   Chapter President—Eric Carpenter
Delta State University, Cleveland   17 actives
    Mississippi Epsilon held an initiation ceremony on Sunday, April 2.
Five new members were initiated. Other spring 2000 officers: Audrey
Stewart, vice president; Sallie Bodiford, secretary/treasurer; Paula Norris,
corresponding secretary; Rose Strahan, faculty sponsor.

MO Alpha   Chapter President—Sam Blisard
Southwest Missouri State University, Springfield   20 actives, 12 associates
    During the Spring semester the Missouri Alpha Chapter held monthly
meetings. Presentations at the meeting included two faculty presentations
and a student presentation. The president of KME hosted the annual math-
ematics department banquet. Two students along with the faculty sponsor
attended the Regional Convention at Benedictine College. Other spring
2000 officers: Rachel Netzer, vice president; Erin Stewart, secretary; Sheri

Puestow, treasurer; John Kubicek, corresponding secretary/faculty sponsor.

MO Beta — Chapter President—Beth Hilbish
Central Missouri State University, Warrensburg — 25 actives, 5 associates

KME meeting programs for this semester included a video on chaos, a presentation by Drs. Cooper and Edmondson on the Great Internet Mersenne Prime Search, and a presentation by Steve Shattuck on RATS sequences. Students volunteered in the Math Clinic and at Math Relays. Six students and two faculty attended the North Central Regional Convention in Atchison, KS on April 7-8. Andrew Feist presented a paper. The April meeting consisted of a pizza party and election of officers for next year. The Claude H. Brown Mathematics Achievement Award for Outstanding Senior was presented to Andrew Feist. The last event of the semester was a trip to a Royals baseball game on April 28. Other spring 2000 officers: Briehan Larson, vice president; Becky Stafford, secretary; Jeff Callaway, treasurer; Beth Usher, historian; Rhonda McKee, corresponding secretary; Steve Shattuck, Phoebe Ho, and Larry Dilley, faculty sponsors.

MO Gamma — Chapter President—Laura Cline
William Jewell College, Liberty — 21 actives, 12 associates

The spring initiation and banquet were held on March 21 with 12 new members. In April, Chapter President Josh Stephenson and Faculty Sponsor Truett Mathis attended the Regional convention at Benedictine. Other spring 2000 officers: Shane Price, vice president; Joel Campbell, secretary; Truett Mathis, treasurer/corresponding secretary/faculty sponsor.

MO Epsilon — Chapter President—Sarah Moulder
Central Methodist College, Fayette — 6 actives, 6 associates

Other spring 2000 officers: Amy Ketchum, vice president; Beth Kurtz, secretary/treasurer; William McIntosh, corresponding secretary; Linda Lembke & William McIntosh, faculty sponsors.

MO Zeta — Chapter President—Sarah Taylor
University of Missouri, Rolla — 8 actives, 11 associates

Other spring 2000 officers: Matt Swenty, vice president; Suzanne Minier, secretary; Laura Edmonds, treasurer; Roger Hering, corresponding secretary; Ilene Morgan, faculty sponsor.

MO Theta — Chapter President—David Bush
Evangel University, Springfield — 5 actives, 1 associate

We had monthly meetings, and voted for and installed our new officers in our February Meeting. We initiated three members during a social at Dr. Tosh's home. Amanda Wachsmuth, past president, presented a paper

at the regional meeting in Atchison in April. John Gale, Joel Elliot, and Dr. Tosh also attended the convention. Other spring 2000 officers: John Gale, vice president; Don Tosh, corresponding secretary/faculty sponsor.

MO Iota   Chapter President—Douglas Osborne
Missouri Southern State College, Joplin   12 actives

In observance of 25 years as a chapter of KME, Missouri hosted two Anniversary events. The first, the 25th annual spring initiation banquet featured a presentation by the charter president of the chapter, Dr. Cynthia Carter Haddock, currently of University of Alabama, Birmingham. Her address, entitled "Time Flies," centered on lessons she has learned in the past 25 years. The second event was a Careers Seminar; eight Missouri Iota alumni returned to campus to share with students and faculty their education and career activities since leaving MSSC. Four students and three faculty attended the Regional Convention held at Benedictine College in Atchison, KS. One student and one faculty member served on the convention Awards Committee. Programs for the monthly meetings held throughout the semester were presented by faculty and students. After serving as corresponding secretary of Missouri Iota since its inception in 1975, Mary Elick will be retiring from full time teaching after this academic year. Dr. Charles Curtis will be assuming the roll of corresponding secretary at that time. Other spring 2000 officers: Christin Mathis, vice president; Dondi Mitchell, secretary; Ted Walker, treasurer; Mary Elick, corresponding secretary; Chip Curtis, faculty sponsor.

MO Lambda   Chapter President—Shawna Smith
Missouri Western State College, St. Joseph   37 actives

Nine new members were initiated on March 5. Dr. John Atkinson, the retiring Corresponding Secretary, presented the program on "The History of Kappa Mu Epsilon - Nationally and Locally". Other activities included a cookout at Dr. Atkinson's home. Other spring 2000 officers: Shane Taylor, vice president; Charisa Greenfield, secretary; Byron Robidoux, treasurer; Donald Vestal, corresponding secretary; Jerry Wilkerson, faculty sponsor.

MO Mu   Chapter President—Cheryl Moonier
Harris-Stowe State College, St. Louis   15 actives, 5 associates

Missouri Mu helps sponsor the Mathematics Club. At our most recent club meeting we looked at the mathematics behind several magic tricks and number puzzles. We are also in the process of setting up a web site. Our initiation ceremony was held on April 8. Five students were initiated. Other spring 2000 officers: Jack Behle, corresponding secretary;

Ann Podleski, faculty sponsor.

NE Beta   Chapter President—Brenna Knott
University of Nebraska at Kearney, Kearney   14 actives, 4 associates
    Craig Merihew was awarded a $200 scholarship. Six members of NE
Beta attended the regional convention in Atchison, KS. Other spring 2000
officers: Jenny Gier, vice president; Jenny Rutar, secretary; Scott Barber,
treasurer; Stephen Bean, corresponding secretary; Richard Barlow, faculty
sponsor.

NE Delta   Chapter President—Chad Parker
Nebraska Wesleyan University, Lincoln   15 actives, 6 associates
    Other spring 2000 officers: David Sovey, vice president; Thor Es-
bensen, secretary/treasurer; Gavin LaRose, corresponding secretary/faculty
sponsor.

NH Alpha   Chapter President—Nancy Peratto
Keene State College, Keene   18 actives, 8 associates
    KME sponsored a trip to the Hudson River Undergraduate Math Con-
ference at Vassar College on the 8th of April. Five students, Nancy Peratto,
Lisa Phillips, Rebecca Batchelder, Scott Price, and Tim Hall and two fac-
ulty members, Vincent Ferlini and Ockle Johnson, gave talks. KME also
co-sponsored, with the Math Club, a day trip to the Boston Science Mu-
seum and a screening of the PBS video "Life by the Numbers, Part II".
Other spring 2000 officers: Kate Doyle, vice president; Karrie Hibbard,
secretary; Kate Dorio, treasurer; Vincent Ferlini, corresponding secretary;
Ockle Johnson, faculty sponsor.

NM Alpha   Chapter President—William Tierney
University of New Mexico, Albuquerque   110 actives, 22 associates
    Information about the New Mexico Alpha Chapter may be found on the
WWW at http://math.unm.edu/~kme. Other spring 2000 officers: Jennifer
Gill, vice president; Tony Malerich, secretary/treasurer; Bill Stanton, web
master; Archie Gibson, corresponding secretary/faculty sponsor.

NY Alpha   Chapter President—Patricia Scavuzzo
Hofstra University, Hempstead   10 actives, 4 associates
    We had a student/faculty basketball game. Other spring 2000 officers:
Rosemary Escobar, vice president; Kimberly Bleier, secretary; Vincent
Perniciaro, treasurer; Aileen Michaels, corresponding secretary/faculty spon-
sor.

NY Eta   Chapter President—Chris Laden
Niagara University, Niagara University   20 actives, 10 associates
    Our chapter was represented at the Great Lakes regional convention

held March 24-25 at Muskingum College in New Concord, Ohio. One of our members, Courney Fitzgerald, presented a paper entitled "Expected Value of Randomly Generated Triangles." Other spring 2000 officers: Courtney Fitzgerald, vice president; Amanda Everts, secretary/treasurer; Robert Bailey, corresponding secretary; Eduard Tsekanovskii, faculty sponsor.

NY Kappa   Chapter President—Monica Mitrofanoff
Pace University, New York   20 actives, 5 associates

Our Spring induction dinner was held on May 1 at 1 Pace Plaza, Pace University. Prof. Roman Kossak of the CUNY system gave a talk on "Goldstein's Sequences". Other spring 2000 officers: Svetlana Kolomeyskaya, vice president; Ilya Kats, secretary; Tim Zihharev, treasurer; Geraldine Taiani, corresponding secretary; Robert Cicenia, faculty sponsor.

NY Lambda   Chapter President—Renee des Etages
C. W. Post Campus of Long Island University, Brookville   28 actives

Eleven students were initiated by the chapter officers during our annual banquet at the Greenvale Town House restaurant on the evening of March 27th, bringing the Chapter membership to 205. After dinner, Dr. Nicholas Ramer spoke on "Designing New Complex Ferroelectric Materials." Dr. Ramer recently completed his Ph.D. in chemistry at the University of Pennsylvania and is now an assistant professor of chemistry at C. W. Post. He graduated summa cum laude in 1994 with B.S.'s in both chemistry and mathematics, as well as a minor in art history, and is New York Lambda member number 112. The evening concluded with the announcement by Dean Paul Sherwin of the 1999-2000 department awards: the Claire Adler Award to Charissa Vercillo, the Lena Shamey Memorial Award to Rosella Viscome, the Joseph Panzeca Memorial Award to Charissa Vercillo, and the Hubert Huntley Memorial Award to Elizabeth Keating. Other spring 2000 officers: Stephanie Calzetta, vice president; Suzann Weaver, secretary; Steven McKinnon, treasurer; Andrew Rockett, corresponding secretary; John Stevenson, faculty sponsor.

NY Nu   Chapter President—Nathan Preston
Hartwick College, Oneonta   12 actives, 8 associates

The induction ceremony was held May 6. Other spring 2000 officers: Stephanie Schreckengost, vice president; Amanda Reed, secretary; Christopher Laidlaw, treasurer; Ronald Brzenk, corresponding secretary/faculty sponsor.

NC Gamma   Chapter President—Brooklyn Tester
Elon College, Elon College   20 actives, 11 associates

We held our annual induction ceremony on Thursday, April 20, at 7:00

p.m. We inducted 11 new members as well as installing the officers for next year. Dr. John Swallow from Davidson College gave a very good talk called "Fermat's Last Theorem: A Coat of Many Colors" describing some of the things about mathematics that makes it such an interesting profession or hobby. Other spring 2000 officers: Hilary Shannon, vice president; Jessica Pollard, secretary; Brian Neiberline, treasurer; Skip Allis, corresponding secretary/faculty sponsor.

OH Gamma   Chapter President—Anila Xhunga
Baldwin-Wallace College, Berea   14 actives, 8 associates

Again this semester our chapter sponsored Monday afternoon talks with pizza provided. These talks were quite well attended and much appreciated by the students. Other spring 2000 officers: Duke Hutchings, vice president; Jeff Smith, secretary; Corina Moise, treasurer; David Calvis, corresponding secretary/faculty sponsor.

OK Alpha   Chapter President—Aaron Lee
Northeastern State University, Tahlequah   38 actives, 2 associates

The initiation of 9 new members, including our Department of Mathematics chairman, was held in the banquet room of a local restaurant. For the third year in a row, we designed and sold new "original" KME T-shirts. The front of the shirts contained our KME and NSU logos. Calculus, computer science and physics equations were skillfully displayed on the back of the shirts. We sold over forty shirts this semester. One of our spring speakers was Dr. Bill Warde, Head of the Department of Statistics, Oklahoma State University. The title of his seminar was "How to sample if you must - problems with polling". Also presenting at our campus this spring was Dr. Mark Arnold, Department of Mathematics, University of Arkansas. He spoke of "The Life and Times of a Graduate Student". Dr. Joan Bell, Dr. Julia Sawyer, and Miranda Hale, our vice-president elect, attended the KME regional meeting at Oral Roberts University in April. We sponsored Math Awareness Month by wearing our awesome KME shirts at the annual Ice Cream Social. Other spring 2000 officers: Rhonda Cook, vice president; Chris Burba, secretary; Gregg Eddings, treasurer; Joan Bell, corresponding secretary/faculty sponsor.

OK Gamma   Chapter President—Kory Hicks
Southwestern Oklahoma State University, Weatherford   20 actives, 3 associates

Other spring 2000 officers: Christy Koger, vice president; Shelly Davenport, secretary; Wayne Hayes, corresponding secretary; Gerry East, faculty sponsor.

OK Delta   Chapter President—Brandon Randleman

Oral Roberts University, Tulsa   20 actives, 22 associates

The Oklahoma Delta Chapter at Oral Roberts University hosted the South Central Regional Conference on Friday, April 14 and Saturday, April 15. The conference included the chapter initiation, a banquet, featured speakers, and student presentations. In attendance were members form Colorado and Oklahoma. The chapter initiation on Friday evening welcomed twenty-two new members. A banquet of Mexican fare highlighted the evening. Saturday morning Dr. Dominic Halsmer delivered the keynote speech regarding his work on spinning spacecraft for NASA. Six student presentations followed. Topics included the Lorenz equations, linear transformations, and Markov chains. Awards were presented by regional director Donna Hafner, who also closed the conference with words of thanks to the OK Delta chapter sponsor, Vincent Dimiceli. Other spring 2000 officers: Jennifer Randleman, vice president; Arvid Ligard, secretary/treasurer; Dorothy Radin, corresponding secretary; Vincent Dimiceli, faculty sponsor.

PA Delta   Chapter President—Susan Carlo

Marywood University, Scranton   4 actives, 3 associates

Other spring 2000 officers: Susan Kulikowski, secretary/treasurer; Robert Ann Von Ahnen, corresponding secretary/faculty sponsor.

PA Kappa   Chapter Co-Presidents—Linda Bruce & Lindsay Janka

Holy Family College, Philadelphia   6 actives, 8 associates

On March 13 the members and pledges had a Pizza party to celebrate Pi Day (3/14). On March 17 the annual induction ceremony was held (jointly with the Tri-beta (Biology) honor society.) Dr. Duckyun Kim, a biologist/statistician from MCP Hahneman University gave a presentation on spinal cord injuries. Eight new inductees were initiated into KME. Sister Marcella Louise gave a brief biography of each of the new inductees. On April 8 the members hosted the sixth annual grade school math competition. Nine local elementary schools participated in individual mathlete events including arithmetic computation, algebra, geometry, problem solving and mathematical reasoning and two team events. A planning meeting for the 2000-2001 academic year is scheduled for the summer at Joe Coll's family home down the shore in Ocean City, NJ. Members also met for problem solving sessions and submitted solutions to problems in Math Horizons. The chapter continued publishing its monthly newsletter "KME News". Each Monday a "Problem of the Week" was posted. This activity was open to all faculty, staff and students. Those who correctly solved these problems were eligible for a drawing for a twenty-dollar gift certificate to Franklin Mills Mall. Other spring 2000 officers: Shannon Mar-

czely, secretary; Sister Benedykta Mazur, treasurer; Sister Marcella Louise Wallowicz, corresponding secretary/faculty sponsor.

PA Mu   Chapter President—Glenn Eckenrode
Saint Francis College, Loretto   30 actives, 9 associates
   Nine new members were inducted in a ceremony held February 15, bringing the total number of members to 180, including 30 active members. Dinner preceded the induction ceremony. Two students and corresponding secretary Pete Skoner attended the Great Lakes regional convention held on March 24 and 25 at Muskingum College in New Concord, Ohio. Five student papers were presented including two by Saint Francis College students: "A Method for Deriving the Principal Unit Normal Vector for Two-Space Vectors," by Nicholaos John Jones, senior philosophy major, and "A Content Analysis of Gender Representation in Algebra Textbooks," by Katherine Wallace, senior mathematics/education major. Other spring 2000 officers: Chrissy Petrarca, vice president; Jason Lowmaster, secretary; Kate Wallace, treasurer; Pete Skoner, corresponding secretary; Amy Miko, faculty sponsor.

PA Omicron   Chapter President—Andrew Stumpf
University of Pittsburgh at Johnstown, Johnstown   28 actives, 17 associates
   The annual induction ceremony for new KME initiates was held March 30 at the UPJ Whalley Chapel. Sixteen new student initiates and one new faculty initiate were added to our chapter membership. Outgoing officers (who were all graduating seniors) were honored and new officers were also elected and bestowed. Other spring 2000 officers: Todd McDowell, vice president; Christopher Wain, secretary; Chad Long, treasurer; Nina Girard, corresponding secretary/faculty sponsor.

SC Gamma   Chapter President—Sheri Alderman
Winthrop University, Rock Hill   12 actives, 4 associates
   Other spring 2000 officers: Allen Plyler, vice president; Andrew Dean, secretary; Andrew Lanier, treasurer; Frank Pullano, corresponding secretary; Jim Bentley, faculty sponsor.

TN Alpha   Chapter President—Ryan Fulkerson
East Tennessee State University, Johnson City   24 actives, 11 associates
   Other spring 2000 officers: Kristin Pierce, vice president; Jimmy Nelson, secretary; Amy Brown, treasurer; Jeff Norden, corresponding secretary; Michael Allen, faculty sponsor.

TN Beta   Chapter President—B. J. Smith
East Tennessee State University, Johnson City   24 actives, 11 associates
   The Tennessee Beta Chapter held its annual initiation service April 18.

The service was conducted by officers B. J. Smith, president, and Susan Hosler, secretary. There were 11 initiates. Following the initiation, a talk on the mathematics of space flight was given Dr. Jeff Knisley, Dept of Math, ETSU. The outstanding graduating senior was Susan Hosler. Susan will attend graduate school at ETSU. Jason Osborne and Justin Christian were recognized as the most promising mathematicians. Jason will attend graduate school at NC State University and Justin will attend graduate school at the University of Wyoming. Those being recognized for receiving scholarships were: Elizabeth Hyder and Lora Hart, Depew Scholarship; Austin Howey, Ed Stanley Scholarship; Jamie Howard, Ree'l Street, and Jason Lewis, Faber-Neal Scholarship. Other spring 2000 officers: Mark Taylor, vice president; Susan Hosler, secretary; Tabitha Taylor, treasurer; Lyndell Kerley, corresponding secretary.

TN Gamma   Chapter President—Lindsey Crain
Union University, Jackson   24 actives
     TN Gamma's first meeting of the semester was held on February 29. Dr. William Dembski, a Fellow of the Discovery Institute's Center for the Renewal of Science and Culture, spoke on his experiences and gave some remarks on the philosophy of mathematics. Three students (Lindsey Crain, Cathie Scarbrough, and Andy Nichols) and two faculty (Drs. Dawson and Lunsford) attended the KME North Central Regional Convention in Atchison, KS on April 7-8. Lindsey Crain presented her paper "The Mathematics of Music" and was awarded one of two "best paper" awards. Dr. Dawson was the luncheon speaker. His topic was "KME Student Scholarship - 1931 to the Present". Eight students were initiated this spring. The initiation banquet was held April 11 at the Casey Jones Old Country Store, with Mike Adams ('98) as speaker. An end-of-the-year celebration was held jointly with our student ACM chapter at the home of Dr. Jan Wilms, chair of the department. Other spring 2000 officers: Cathie Scarbrough, vice president; Melissa Culpepper, secretary; Sarah Shaub, treasurer; Bryan Dawson, corresponding secretary; Matt Lunsford, faculty sponsor.

TN Epsilon   Chapter President—Jennifer Dowdy
Bethel College, McKenzie   7 actives
     In addition to monthly meetings, the chapter gathered for special movie nights and participated with the Gamma Beta Phi honor society in campus events. Other spring 2000 officers: Belinda Thompson, vice president; Christina Hill, secretary/treasurer; Russell Holder, corresponding secretary; David Lankford, faculty sponsor.

TX Alpha   Chapter President—Charla Newlon

Texas Tech University, Lubbock   3 actives
Other spring 2000 officers: Collin McCurley, vice president; Jeffrey Hood, secretary; Thomas Mullen, treasurer; Victor Shubov, corresponding secretary.

TX Eta   Chapter President—Crystal Cooksey
Hardin-Simmons University, Abilene   11 actives, 5 associates
The 25th annual induction ceremony for Texas Eta was held March 22. There were five new members. With the induction of these members, membership in the local chapter stands at 202. Leading the induction ceremonies were Vice-President Sarah McCraw and Treasurer James Martin. Assisting them was KME member Crystal Cooksey. Following the induction ceremony, membership shingles and pins were presented to the 1999 inductees. In addition, the Burnam Award was presented to James Martin, an outstanding senior mathematics major. Changing the format of our induction ceremony, KME then adjourned, and the members, inductees, and chapter sponsors enjoyed pizza and cold drinks. Other spring 2000 officers: Brooke Motheral, vice president; Katie Smith, secretary/treasurer; Frances Renfroe, corresponding secretary; Edwin Hewett, Andrew Potter, and James Ochoa, faculty sponsors.

VA Gamma   Chapter President—Bobbi Heim
Liberty University, Lynchburg   28 actives, 6 associates
Other spring 2000 officers: Fan Shum, vice president; David Justamente, secretary; Derek Culp, treasurer; Glyn Wooldridge, corresponding secretary; Sandra Rumore, faculty sponsor.

Cumulative Subject Index

The Cumulative Subject Index for *The Pentagon* is up and running! Check it out at www.cst.cmich.edu/org/kme_nat/, the national KME homepage, or directly at www.cst.cmich.edu/org/kme_nat/indpent.htm.
Mostly organized by standard course titles, there are 25 topics to choose from. This can be a great resource for your courses, whether you are a student or faculty! Literally hundreds of articles are listed, on an incredible variety of fascinating topics. Check it out today!

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Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon
Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983

MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000

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