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Contents

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Announcements

New Dates for National Convention

The dates for the 32nd Biennial Convention of Kappa Mu Epsilon, to be hosted by Florida Beta, Florida Southern University, in Lakeland, Florida, have been changed to:

April 22–24, 1999

This is one week later than previously announced. Please mark your calendars accordingly! See pages 74–76 of the Spring 1998 issue for a complete announcement, including deadlines for proposals for talks.

New National Homepage URL

The URL for the national KME homepage has changed to:

www.cst.cmich.edu/org/kme_nat/

Please update all links to the national homepage that may appear on local chapter or faculty homepages, and don't forget to update your bookmarks!

This also means that the URL for the Cumulative Subject Index for *The Pentagon* has also changed. That new URL is:

www.cst.cmich.edu/org/kme_nat/indpent.htm

Please update links to this index also!

Search for New Editor and Business Manager of *The Pentagon*

Individuals interested in becoming the new editor or business manager of *The Pentagon*, beginning June 1, 1999, should contact national president Patrick Costello, Department of Mathematics, Statistics, and Computer Science, Eastern Kentucky State University, Richmond, KY, 40475, or e-mail matcostello@acs.eku.edu, as soon as possible. Appearance of this notice does not necessarily guarantee that the positions are still open, but they have not been filled as of the time this issue went to press.

Hyperbolic Tilings

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Presented at the 1997 National Convention and
awarded "top four" status by the Awards Committee.

When one first encounters the hyperbolic plane, one is struck by its strangeness and unfamiliarity. Many "truths" of Euclidean geometry, once held dear, are no longer usable. Thus, the question often arises, "We know P about the Euclidean plane. Does P hold in the hyperbolic plane?" And since tilings in the Euclidean plane ($\mathbf{E}^2$) are a source of interest and great beauty, it is natural to ask what we can discern about tilings in the hyperbolic plane ($\mathbf{H}^2$).

The hyperbolic realm of tiling is possibly even more interesting than its Euclidean counterpart, and anyone who has seen M. C. Escher's *Circle Limit* pieces has already encountered the fascinating patterns that arise in hyperbolic tilings (or, more precisely, in Euclidean representations of hyperbolic tilings). While it is true that there are many parallels between properties of tilings in $\mathbf{E}^2$ and $\mathbf{H}^2$, there are also many things one can do in the hyperbolic plane that are impossible in the Euclidean plane, highlighting the fundamental uniqueness of hyperbolic geometry. In this paper we shall see examples of both sorts of properties: those which we recognize from the plane we know and love, and those that are unique to $\mathbf{E}^2$'s strange cousin.

There are two different ways to generate a tiling of the hyperbolic plane; both ways are similar to methods of tiling in the Euclidean plane. One may either use subgroups of the group of isometries (transformations that preserve distance) of $\mathbf{H}^2$ to generate a *fundamental region* of that subgroup, a region whose symmetry group is that subgroup; or one may begin with one or more shapes to use as *prototiles* (one or more tiles to which all other tiles are congruent) and then apply symmetries and isometries to those prototiles to cover the plane ([6], p. 31). In the Euclidean plane, the first method is that which arises from the use of the 17 so-called *crystallographic* or *wallpaper* groups which generate all periodic tilings of the plane. The parallel in the hyperbolic plane is the Fuchsian groups. An example of an application of the second method is the tiling of bricks that one gets when one builds a wall: the face of the wall is a tiling of the Euclidean plane whose prototile is the face of a single brick.

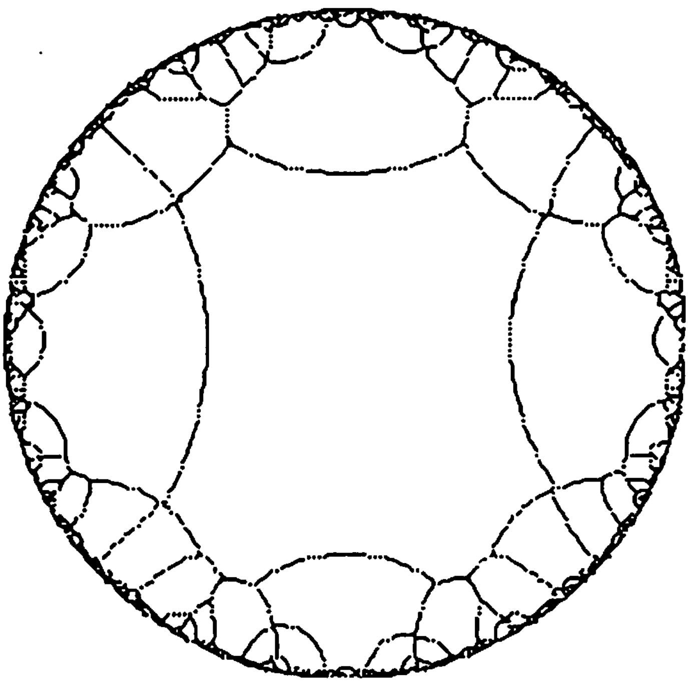

Figure 1. The "gamma" tiling generated with Fuchsian groups [3]. Used by permission.

Using the Fuchsian groups is unwieldy and complicated, and for this reason I will not go into great detail about this method. The general idea, though, is this: pick a point in the hyperbolic plane and find all of its images under a subgroup of the group of symmetries of $\mathbf{H}^2$. Then join the original point with each of its images by line segments. The fundamental region of this subgroup is the union of the half-planes formed by the perpendicular bisectors of these line segments ([6], pp. 16–17). John Mount has created several different fundamental regions and "Fuchsian tilings" and displayed them on his World Wide Web pages [3]. Four of these tilings are reproduced in figures 1–4. Notice how unconventional and interesting these shapes are; unfortunately, the tilings we consider henceforth will not have such unique prototiles (but they will have other interesting properties). Those readers who are already familiar with hyperbolic geometry recognize that the modular tiling (figure 2) provides a good example of the isometry (unique to the hyperbolic plane) called a *parallel displacement*, since one of the vertices of the fundamental region is an ideal point.

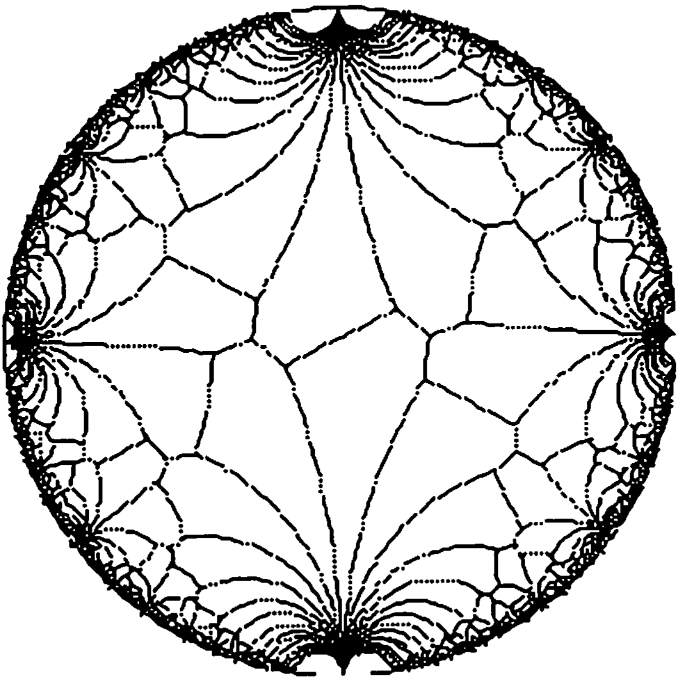

Figure 2. The "modular" tiling generated with Fuchsian groups [3]. Used by permission.

Since working with Fuchsian groups is so complicated, we seek an easier method by which to generate tilings. As stated previously, such a method is to choose a plane figure and then apply isometries and symmetries to it until it covers the plane. Now, since translations in the hyperbolic plane do not preserve lines, this transformation will not be as useful to us here as it is in Euclidean tiling. We will, however, make much use (exclusive use, actually) of reflections. Let us turn now to consider what sort of tilings we can get with this method.

Because it is the simplest plane figure, we begin by tiling with triangles. Tilings with triangles are called *triangulations*. We wish to cover $\mathbf{H}^2$ without overlap using only triangles. The easiest way to do this is to first place the triangle and reflect across its sides; next we reflect across the sides of the new triangles, and then reflect across the sides of those new triangles,

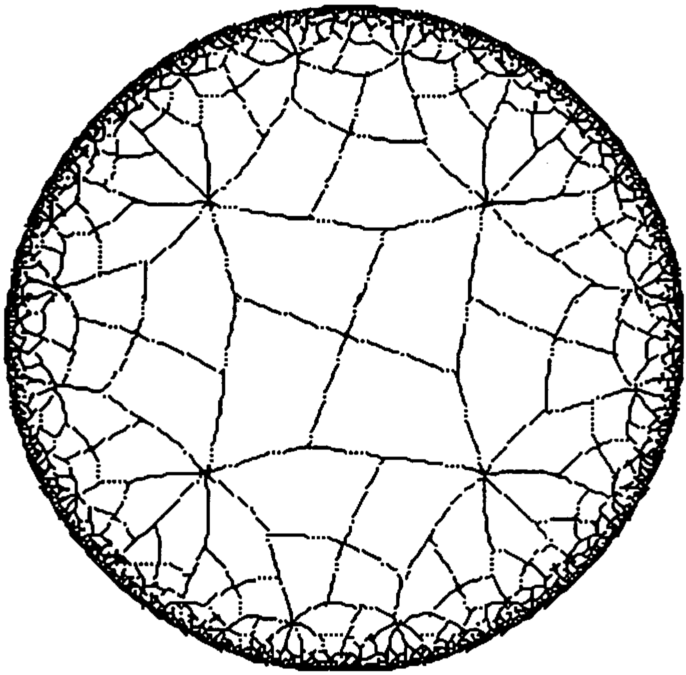

Figure 3. The "square 7" tiling obtained using Fuchsian groups [3]. Used by permission.

and so on and so forth. If the triangles are not isosceles (in general, they are not) there will necessarily be an even number of triangles around each vertex. Further restrictions on our choice of triangles are that we must (1) avoid overlap and (2) sum to 2π radians around each vertex. Given these facts, we can conclude that the three angles must be of the form π/l, π/m, and π/n, where l, m, and n are integers ([4], p. 106). If we are tiling on the sphere, we have the added requirement that $\pi/l + \pi/m + \pi/n > \pi$; in the Euclidean plane, we must have $\pi/l + \pi/m + \pi/n = \pi$. On the sphere this restriction gives us a small class of possible triangulations, and in $\mathbf{E}^2$ only three triangles work as prototiles. However, the requirement in $\mathbf{H}^2$ is that $\pi/l + \pi/m + \pi/n < \pi$, and there are an infinite number of triangles that satisfy this requirement. Snowden proves rigorously that every choice of l, m, and n leads to a triangulation ([6], pp. 22–33); thus, there are an infinite number of triangulations in the hyperbolic plane.

Some notes about these triangles are in order at this point. In hyperbolic geometry, similar triangles do not exist and the area of a triangle is directly proportional to its angular defect (defined as π minus the sum of the angles of the triangle). It follows, then, that specifying the three angles

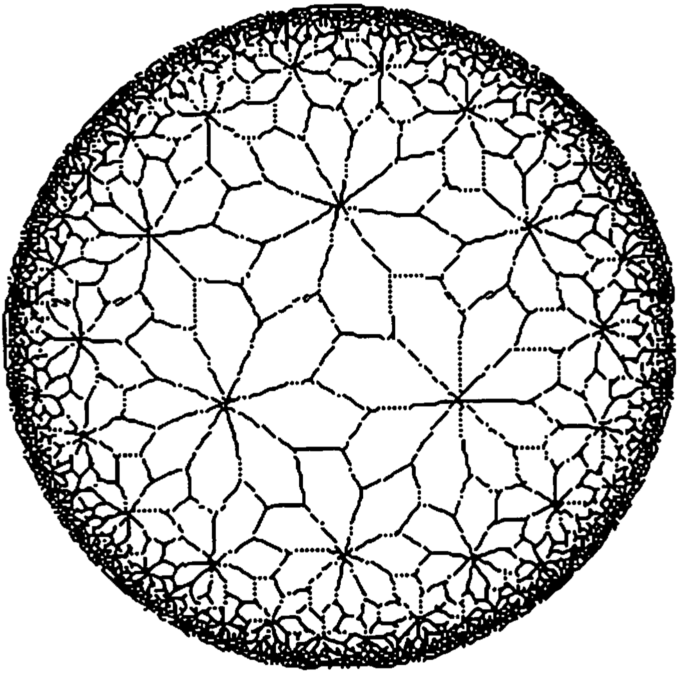

Figure 4. The "triangle 4" tiling created with Fuchsian groups [3]. Used by permission.

of a triangle determine a unique triangle with a determined area. Thus it makes sense to ask which is the smallest triangle which will tile the hyperbolic plane. Now, Euclidean triangles have angular defect of zero, so if they existed in the hyperbolic plane they would have zero area. Therefore, to answer our question it is sufficient to find the smallest perturbation of a prototile of a Euclidean triangulation which will generate a hyperbolic triangulation. The three triangles which tile the Euclidean plane are the triangles with angle measures $\pi/2$, $\pi/4$, $\pi/4$; $\pi/3$, $\pi/3$, $\pi/3$; and $\pi/2$, $\pi/3$, $\pi/6$. It is clear that the smallest change we can make to any of these triangles which creates a hyperbolic triangulation is to increase the angle $\pi/6$ in the third triangle to $\pi/7$. So the smallest triangle which will tile the hyperbolic plane is $(l, m, n) = (2, 3, 7)$, with angular defect of $\pi/42$.

Can we determine the largest triangle which will tile the hyperbolic plane? Remembering that area is proportional to angular defect, we see that the upper limit for areas of hyperbolic triangles is given by the triangle with angular defect π, the maximum possible angular defect. This is a triangle with all three vertices on the "boundary" of the hyperbolic plane, i.e., a triangle whose vertices are all ideal points. (True, these aren't really triangles, since their "sides" are limiting parallel rays; nevertheless it is useful and somewhat meaningful to refer to them as such.) Snowden proves that these triangles do indeed tile the plane ([6], pp. 34–36). One such tiling (figure 5) has been created in *Geometer's Sketchpad*, as well as a tiling of an "ideal quadrilateral" (figure 6). Note that the choice of the locations of the ideal vertices is completely arbitrary and motivated only by aesthetic concerns; any three (or four) ideal points will do.

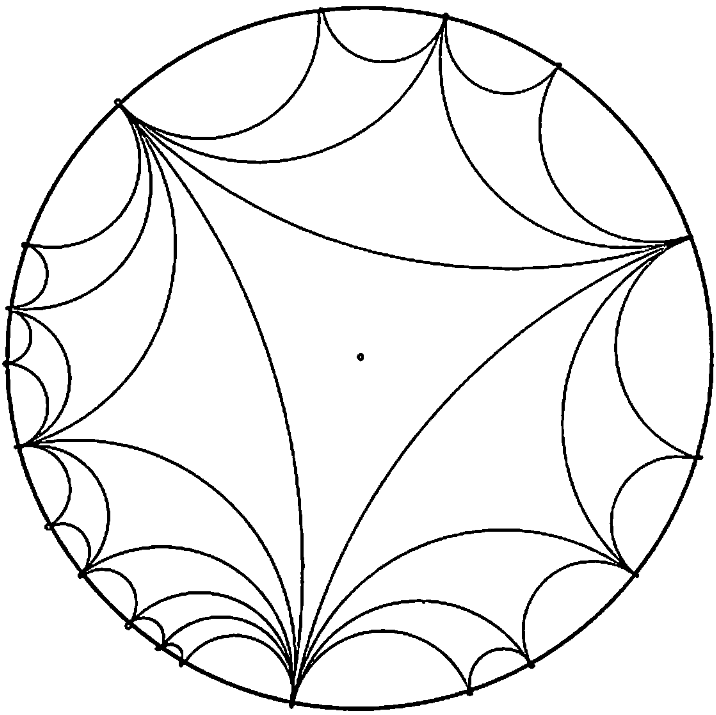

Figure 5. A tiling of the hyperbolic plane with "ideal triangles." The prototile is an unbounded region with finite area.

Before we continue, we need one more term. A *regular* tiling is a tiling whose prototile is a regular polygon. The notation $\{p,q\}$ refers to the regular tiling of faces with p sides where q faces meet at a vertex. Examples of regular tilings can be found in figures 7, 8, and 9. The next result is one which was not found directly stated in the sources, so it is stated and proved as a theorem.

Theorem. *The regular tiling* $\{p,q\}$ *is generated by a triangulation of* $(l,m,n)=(2,p,q)$.

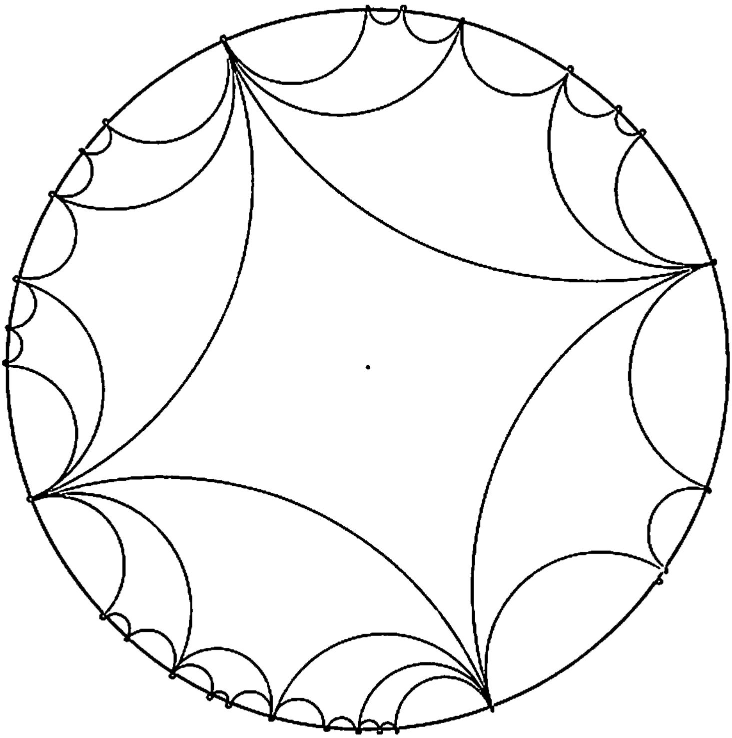

Figure 6. A tiling using "ideal quadrilaterals" as tiles.

Proof. Pick one triangle in the triangulation and all those triangles obtained by reflecting repeatedly across the sides of the angle whose measure is π/p. Take as the prototile of a new tiling the polygon formed by the outer edges of this group of tiles (more specifically, the set of edges that are adjacent to both an angle of measure π/q and an angle of measure $\pi/2$). We must show that this region is a regular polygon with p sides and that q faces meet at each vertex of the new tiling.

The interior angles of the new polygon will be determined by the angles in the original triangles of measure $\pi/2$ and π/q. Each angle of measure $\pi/2$ will sit next to another angle of measure $\pi/2$ from an adjacent reflected triangle; these two angles will add to give an interior angle of π. But an interior angle of π means that the sides that meet at that vertex actually join to make a single line segment which forms one side of the new polygon. Now, there will be $2p$ triangles around the center of each of these polygons, and each of these triangles contributes one of its sides to the new polygon. Since these line segments will "join" together at the interior angles of π as mentioned above, there are $2p/2 = p$ total sides in the new prototile.

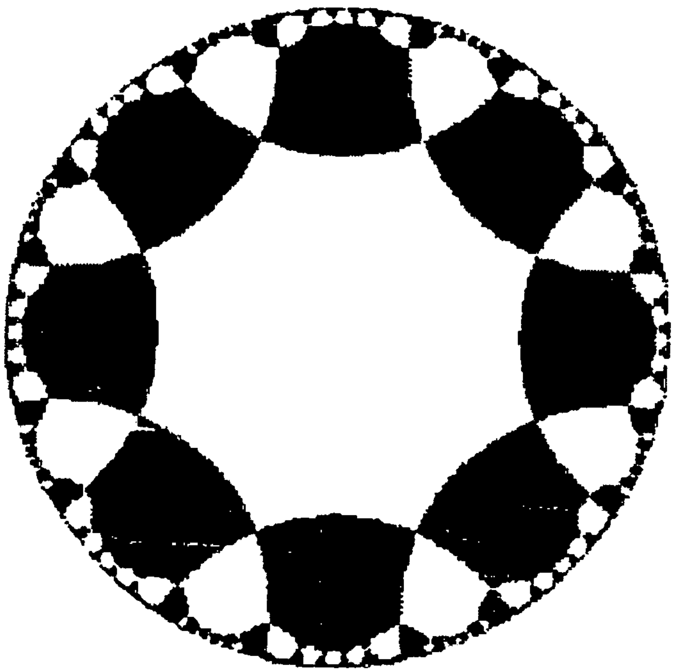

Figure 7. The regular $\{8,4\}$ tiling of the hyperbolic plane [2]. Used by permission.

The length of each side of this new polygon is equal to twice the length of the side of the original triangle between the angles of measure $\pi/2$ and π/q. The measures of the interior angles of this polygon are each given by $2\cdot\pi/q$, because two angles of measure π/q from the original triangles meet at each vertex. Thus, all sides and all angles are congruent, so we have a regular polygon. Also, since the interior angles have measure $2\pi/q$, there are q faces around each vertex of the new tiling.

So now we have an easy method by which we can generate any regular tiling we wish. This theorem shows us two additional things: any n-gon can tile the hyperbolic plane (if we choose a suitable value for q; if n is greater than 6, any q will do), and for a given number of sides n, we can tile the hyperbolic plane in several fundamentally different ways (e.g., we can construct a tiling $\{4,8\}$ and a tiling $\{4,9\}$ and a tiling $\{4,10\}$, etc.). Both of these results are at variance with the Euclidean plane, where the only regular shapes that tile are squares, triangles, and hexagons, and then one can only do it one way for each. It seems the hyperbolic plane is a bit more "flexible" than the Euclidean.

We can generate many other interesting tilings by modifying the regular

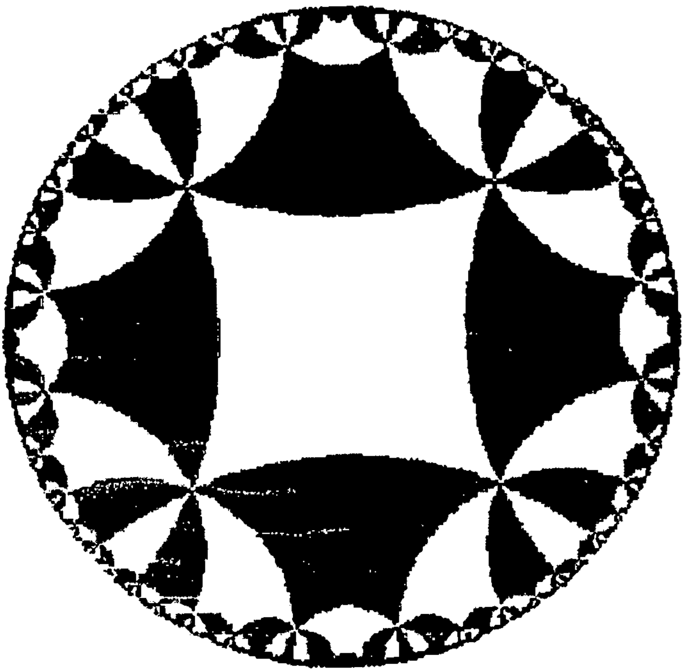

Figure 8. The regular $\{4,8\}$ tiling of the hyperbolic plane, dual of the tiling in figure 7 [2]. Used by permission.

tilings. The first way of doing so is to create the *dual* of a tiling. The dual is produced by joining with line segments the centers of the tiles around a vertex. The polygonal regions formed by these line segments are faces of the new tiling, and the centers themselves are vertices. In this way, the roles of face and vertex are interchanged [2]. For a regular tiling $\{p,q\}$, there will be q tiles around each vertex and hence the new tiling's faces will have q edges. Likewise, since there will be p line segments going out of each tile's center (one for each side), the new tiling will have p faces per vertex. Therefore, the dual of $\{p,q\}$ is $\{q,p\}$ [2]. The dual of the tiling in figure 7 is presented in figure 8.

We can also take a regular tiling $\{p,q\}$ and form a quasi-regular tiling (denoted quasi-$\{p,q\}$). This is done by joining the midpoints of the edges of the tiles around a vertex [2]. In this way, a tiling is created with two prototiles: one with p sides, and one with q sides. For an example, see figure 10. Quasi-regular tilings are also often called semiregular.

Another way to generate new tilings is by using *perfect colorings* of the regular tilings, an idea explored by John Rigby in his fascinating article [5]. A perfect coloring is a coloring of the tiling that has the same symmetries

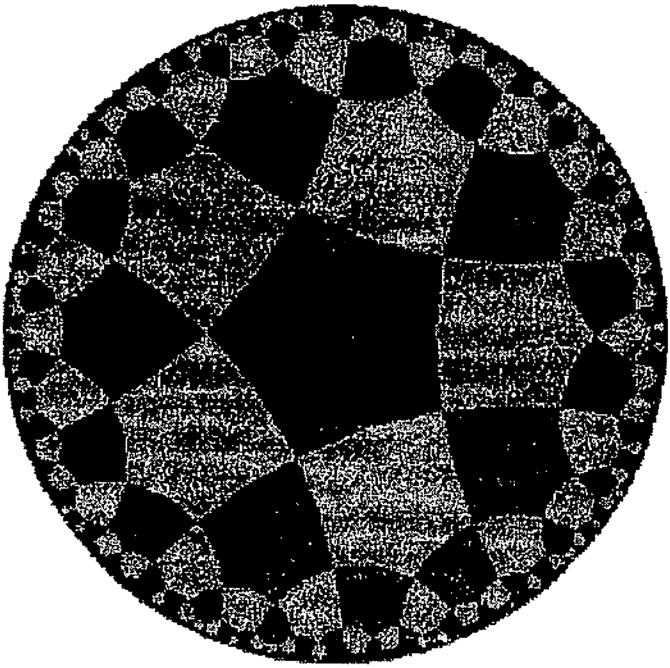

Figure 9. The regular $\{5,4\}$ tiling of the hyperbolic plane [2]. Used by permission.

as the tiling itself (one can also use *chirally perfect colorings*, where only the direct symmetries of the tiling apply to the coloring [5]). When one connects the centers of all the like-colored faces of a perfectly-colored tiling (e.g., connecting the centers of all the black faces), one can get some pretty interesting tiles. Rigby christens his prototiles with the fanciful names of "backbones" and "dendroids," noting their resemblance to spinal columns and branching trees, respectively. Besides their intriguing shapes, the tiles have another interesting property: some of their vertices are ideal points, as with the "ideal triangles" we encountered earlier. Interested readers are referred to Rigby's article for further elucidation (and pictures).

We have seen many techniques by which one can generate tilings of the hyperbolic plane: Fuchsian groups, triangulations, duals, quasi-regulars, and perfect colorings. Along the way we have seen that hyperbolic tilings can provide patterns of equal and surpassing interest to those of the Euclidean plane, and we have noted some of the strange properties and tiles that arise with hyperbolic tessellations. One might ask if these results prove useful outside of aesthetic and mathematical interest. Actually, hyperbolic tilings do have applications outside of art and geometry: they can be used

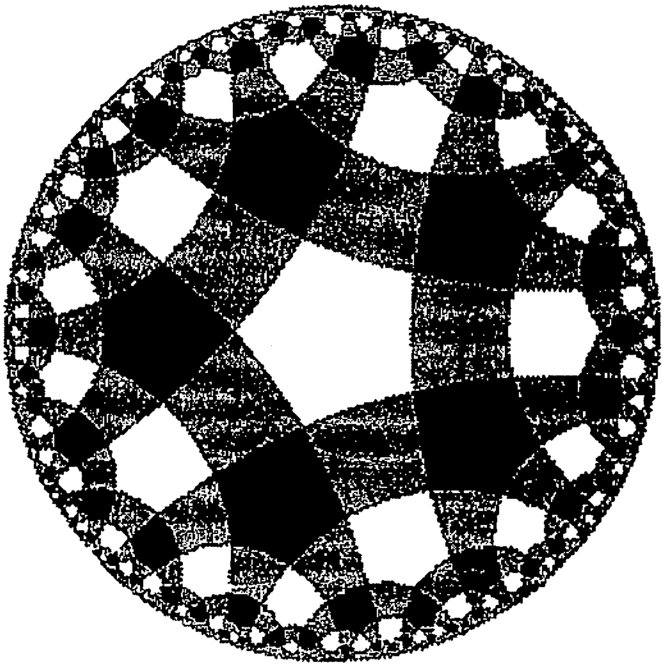

Figure 10. The quasi-regular tiling of the plane obtained from the tiling in figure 9, denoted quasi-{5, 4} [2]. Used by permission.

to represent surfaces of constant negative curvature that cannot be embedded in the Euclidean plane, such as the torus with two holes ([1], p. 343 and [7]). In fact, recent research has been done at the Geometry Center in Minnesota to try to extract a visualization of a Riemannian surface from a hyperbolic tiling. At any rate, it is clear that hyperbolic tilings provide a rich source of interesting mathematics and dazzling beauty.

Acknowledgements. I would like to thank Dr. Lynn Olson for the inspiration and assistance he gave me throughout the process of writing and preparing this paper.

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Kepler Conjecture Solved

Fermat's Last Theorem is not the only old problem that has recently been solved. Earlier this year, Thomas C. Hales of the University of Michigan announced that he has solved Kepler's conjecture, a nearly 400-year-old problem in geometry. That conjecture states that no packing of congruent spheres can have a density greater than the density of the face-centered cubic packing (the "normal" way everyone stacks spheres). Although no one had been able to imagine a better way to do it, and nearly everyone's intuition said the conjecture was true, no one had ever proven that the method was the most efficient possible.

A complete solution, including computer programs and data, appears in a web site at the following URL:

www.math.lsa.umich.edu/~hales/countdown/

A complete history of the problem, as well as other pertinent information, also appears on the site. Be sure to check out this piece of mathematical history!

Kepler Quote

The orbit of the earth is a circle; round the sphere to which this circle belongs, describe a dodecahedron; the sphere including this will give the orbit of Mars. Round Mars describe a tetrahedron; the circle including this will be the orbit of Jupiter. Describe a cube round Jupiter's orbit; the circle including this will be the orbit of Saturn. Now inscribe in the earth's orbit an icosahedron; the circle inscribed in it will be the orbit of Venus. Inscribe an octahedron in the orbit of Venus; the circle inscribed in it will be Mercury's orbit. This is the reason of the number of the planets.

—Kepler

Thankfully, not all of Kepler's reasoning went that way.

Two New Proofs of the Pythagorean Theorem

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There are many proofs of the Pythagorean Theorem found in the past three thousand years. In Dr. Elisha Loomis' book *The Pythagorean Proposition* [1], there are 370 proofs. In this note we give two new ones.

Proof 1. As in figure 1, let ABC and BED be two congruent right triangles in the position such that $\overline{AB}$ is perpendicular to $\overline{BD}$ and vertex C is on $\overline{BD}$. Let the intersection of $\overline{BE}$ and $\overline{AC}$ be F.

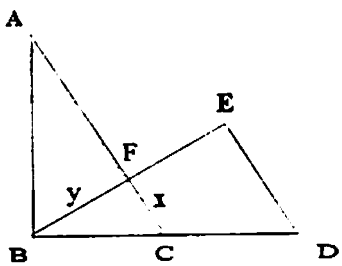

Figure 1

It is easily seen that $\overline{AC}$ is perpendicular to $\overline{BE}$. Let $BC = DE = a$, $AB = BE = b$, $AC = BD = c$, $CF = x$ and $BF = y$.

Since triangle ABC is similar to AFB, we have $AF/AB = AB/AC$ and $BF/AB = BC/AC$, or $(c-x)/b = b/c$ and $y/b = a/c$. Therefore $x = c - b(b/c)$ and $y = ab/c$.

Noticing that the area of triangle AFB is equal to the area of trapezoid $CDEF$, we have

$$\frac{y(c-x)}{2} = \frac{(x+a)(b-y)}{2},$$
$$yc - xy = xb + ab - xy - ay,$$

$$\begin{aligned}(c+a)y &= b(a+x),\\ (c+a)(ab/c) &= b(a+c-b(b/c)),\\ ac+a^2 &= ac+c^2-b^2,\end{aligned}$$

and

$$a^2+b^2=c^2.$$

Proof 2. Put two congruent right triangles ABC and EDA in the position as in figure 2, where $\overline{DE}$ is perpendicular to $\overline{AC}$ and $\overline{AE}$ is parallel to $\overline{BC}$. Construct the line perpendicular to $\overleftrightarrow{BC}$ through E. Let G be the foot of this perpendicular on $\overleftrightarrow{BC}$ and F its intersection with $\overleftrightarrow{AC}$.

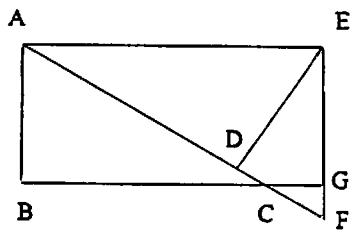

Figure 2

Let $AB = ED = a$, $BC = DA = b$ and $AC = EA = c$. It is easily checked that the three triangles EDA, FDE and FGC are similar. Therefore, $DF/ED = ED/DA$, or $DF/a = a/b$, and $DF = a(a/b)$; also $FG/GC = ED/DA = a/b$, or $FG = (a/b)GC$. Notice that $GC = BG - BC = EA - BC = c - b$, and we have $FG = (a/b)(c-b)$.

It is obvious to see that

$$\text{area } ABC + \text{area } EDA + \text{area } FED = \text{area } ABGE + \text{area } FGC.$$

Now we have the following calculation:

$$\begin{aligned}\frac{ab}{2}+\frac{ab}{2}+\frac{ED\cdot DF}{2} &= ac+\frac{FG\cdot CG}{2},\\ ab+ab+a\left(\frac{a^2}{b}\right) &= 2ac+\left(\frac{a}{b}\right)(c-b)^2,\\ 2b+\frac{a^2}{b} &= 2c+\frac{c^2+b^2}{b}-2cb,\\ 2b^2+a^2 &= c^2+b^2,\end{aligned}$$

and

$$a^2+b^2=c^2.$$

Acknowledgements. The authors thank Professor Jingcheng Tong for his advice.

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An Old Pythagorean Triple Problem

The Fall 1951 issue of *The Pentagon* reports the following in its "Mathematical Scrapbook" section:

> From the Editor's mailbag: "In the Spring issue of the PENTAGON under 'Topics for Chapter Programs,' there is mentioned a problem of finding three equal rational-sided right triangles. I have found three triangles which fit the description, namely, $(15, 112, 113)$, $(24, 70, 74)$, and $(40, 42, 58)$. ... I have also found another set, but I am not revealing them at present." Harvey Fiala, high school student.

It should be noted that in the above quote, "equal" means same area. Harvey Fiala later wrote a few articles that appeared in *The Pentagon*.

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Another Proof of the Pythagorean Theorem

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There are several hundred proofs of the Pythagorean Theorem, but we can still add some new ones. Here is an example.

Let ABC and AED be two congruent right triangles as in figure 1, where E is on $\overline{AB}$. Then $\overline{AD}$ is perpendicular to $\overline{AC}$. Extend $\overline{CB}$ to meet the extension of $\overline{AD}$ at point F. Draw $\overline{DG}$ perpendicular to $\overline{BF}$. It is immediate that triangles DFG and ADE are similar.

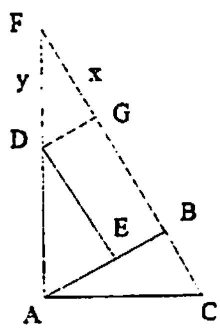


Figure 1

Let $AE = a$, $DE = b$, and $AD = c$. Then $BC = a$, $AB = b$, and $AC = c$. Let $FG = x$ and $DF = y$. Since $FG/DG = DE/AE$, we have $x/(b-a) = b/a$ and $x = (b/a)(b-a)$. Since $DF/DG = AD/AE$, we have $y/(b-a) = c/a$ and $y = (c/a)(b-a)$.

There are two ways to calculate the area of triangle AFC. The first way is $\frac{1}{2}AF \cdot AC = \frac{1}{2}(c+y)c$. The second way is area $ADE +$ area $ABC +$ area of trapezoid $DEBF = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}(b+(b+x))(b-a) = ab + \frac{1}{2}(2b+x)(b-a)$. Since the two ways give the same result, the following equality must be true:

$$\begin{aligned}
(c+y)c &= 2ab + (2b+x)(b-a), \\
c^2 + cy &= 2ab + 2b^2 + bx - 2ab - ax, \\
c^2 + \frac{c^2}{a}(b-a) &= 2b^2 + \frac{b(b-a)^2}{a}, \\
\frac{bc^2}{a} &= 2b^2 + \frac{b(b^2 - 2ab + a^2)}{a}, \\
\frac{c^2}{a} &= 2b + \frac{b^2}{a} - 2b + a,
\end{aligned}$$

and

$$c^2 = a^2 + b^2.$$

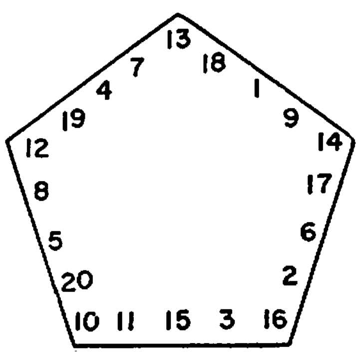
The proof is completed.

Magic Pentagon

Editor's comment: The following paragraphs are from the Fall 1951 issue of *The Pentagon*.

From the Editor's mailbag: "Magic squares have fascinated me greatly, but of late I have tackled formations other than squares. I couldn't help thinking that a pentagon design would not be inappropriate for the PENTAGON. My foreign language teacher of nearly fifty years ago harped continually on the ability of Caesar to form hollow squares, hollow triangles, hollow pentagons. These lay dormant in my mind until quite recently. I have (constructed) them up to decagons.

"The ideal pentagon would have: five sides all equal; five numbers in each side; the hollow pentagon a multiple of five; the series a multiple of five. [Here is an example.]"



—Ira G. Wilson

Princess Diana, Paul Revere, and Group Theory???

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Presented at the 1998 Region IV Convention and awarded "top three" status by the Awards Committee.

What do Princess Diana's funeral, Paul Revere, and group theory have in common? Oddly enough, such a connection does exist. It is that of change ringing. One might immediately think of songs, hymns, or melodies played by bells. Those particular types of bells are known as carillons, and are struck by hammers controlled by a carillonneur. In England, however, church bells are not rung in tunes, but in permutations (often referred to as changes in ringing and can be defined as a rearrangement of the elements of a set). This unique use of mathematics, generally associated with churches, helps announce important events such as weddings, coronations, calls to service, and even funerals such as Princess Diana's. Change ringing is a team effort which depends on huge bronze bells ranging in weight from a few hundred pounds to several tons. There can be anywhere from four to twelve bells in a "ring of bells" ([2], p. 1).

In tenth-century England, any Saxon possessing 500 acres, a church, and a belltower was granted the title of thane. This easily explains why there are more than 5000 belltowers in present-day England, with one in practically every town and village ([1], p. 1). At the dawn of change ringing, control over the bells was greatly limited. Before the fourteenth century, the bells were hung on a spindle with a rope or lever attached. Several men pulled on the lever, the bell would swing, the clapper hit the side, and thus a note would sound. Some bells, such as the three ton bell at Canterbury, required as many as twenty men! There was much development throughout the next two centuries to improve control. The bell was soon mounted on a quarter wheel, followed by a half wheel, and finally on a full wheel, which is what we have today. The full wheel enables the bell to swing 360° each time it rings. One very unusual advancement arrived in the form of the slider and stay, which allowed for the setting of the bell in the "mouth-up" position. Although somewhat strange, this position gives the ringer more control over the bell, allowing for the precise stopping and starting of the chimes ([5], p. 1). The full wheels and stout frames, along with the other improvements, work to achieve such incredible balance that even children can easily control the largest of bells. However, the concentration required of the ringers is still immense, and one bell entirely consumes the attention of one ringer. Robert Palgrave, an English tower captain, explained, "What a ringer needs most is not strength but the ability to keep time. Everybody must be dead on with their pulls. Nobody may be uneven. You must bring these two together in your mind, and let them rest there forever — bells and time, bells and time." Thus change ringing with four bells requires four people, six bells requires six people, etc. These people are usually arranged in a circle functioning as a team to create the beauty of change ringing. As Ben Johnson said, "Bell ringing is the poetry of the steeples" ([2], p. 1).

The timing of bells, much like that of a pendulum, cannot be easily altered. There must be some outside force pushing a bell faster or holding it back. This is one reason these enormous bells cannot be used for songs. Instead, they are rung one after another with each bell ringing exactly one time before the first rings again. As someone points out in the article "What is change ringing?," "Ringing bells in a precise relationship to one another is the essence of change ringing" ([2], p. 2).

The simplest ringing consists of ringing the bells in order from the lightest, highest-pitched bell to the heaviest, lowest-pitched bell. They strike one after another forming a sequence of sounds known as "rounds." This can be denoted as $(1234\ldots n)$ where n is the number of bells. This beautiful cascading sound often accompanies English brides and grooms down the aisle after the ceremony. "Changes" are created when bells change positions with adjacent bells in the row. Some examples include (213456), (124356), or even (214365). One can see in this last example that more than one swap took place. When this occurs, we refer to it as "cross changes." Ringers have since devised orderly systems of changing pairs. Each method begins in rounds and returns to rounds with no repeats along the way. Because there is no repetition, the more bells involved, the longer the bells can be rung. This is often referred to as a change. Four bells can have 24 changes $(1\cdot 2\cdot 3\cdot 4)$. Twenty-four changes is all the permutations possible and takes less than a minute to ring. Five bells allow 120 changes $(1\cdot 2\cdot 3\cdot 4\cdot 5)$. As seen here, the numbers increase very quickly, with six bells yielding 720 changes, and seven bells 5,040 changes. Any method exceeding 5,000 changes is described as a peal, which usually lasts about three hours. The first peal was not rung until 1715. Now, peals have even reached North America, with an average of 80 rung each year for the past decade ([2], p. 3).

Although change ringing is predominately an English practice, it has spread. In the eighteenth century, change ringing was introduced to the American Colonies by the British. Bells were installed in Boston, Charleston, New York, and Philadelphia. Paul Revere was a ringer from as early an age as fifteen. His experience at change ringing is what gave him the opportunity to use the tower for lantern signals on that fateful midnight ride. There are currently 32 operable rings in North America and over 300 ringers ([2], pp. 3–4).

Before discussing more of the mathematics involved in change ringing, here is an opportunity to actually listen to an example of this English art. [*Editor's Note*: This article was designed as a presentation, and cannot easily be revised without changing the nature of the article.] A few hints on what to listen for might make it more enjoyable: "First, the rhythm should not vary from row to row. The rhythm provides the steady framework within which the complex changes are heard. Listen for two rows rung in precise tempo, followed by a pause equal to the stroke of one bell, followed by two more rows and so on. The pause will help you determine which bell rings first. Second, listen for the bell that strikes the lowest note. This is the tenor. Sometimes it always strikes last, even when the other bells are changing. Listen for the highest bell, the treble, as it makes its way through the rows. Listen also for the rows in which large bells alternate with small bells throughout the row. These are considered particularly musical, and composers strive to include as many such rows as possible" ([2], p. 2).

The following method is actually the one rung at Princess Diana's funeral. Before the service, the tenor was rung in full circle one direction, balanced for 60 seconds, full circle the opposite direction, balanced for 60 seconds, etc. To make sure that the bell actually remained in balance, there was someone in the belfry throughout the service. This is extremely dangerous and normally not done, however the situation was obviously exceptional. After the pre-service tolling, the same man placed the "half-muffle" on the tenor clapper, which was used in the ringing after the service. The method Stedman Caters was rung with the bells alternating full voice and muffled voice, due to the somber occasion ([6], p. 1). Although not rung in half-muffle, here is Stedman Caters: (listen to CD).

After having the opportunity to listen to an example of change ringing, let us move on and examine a simpler composition to discover the group theory involved. Plain Bob Minimus is rung with four bells and thus has 24 rows $(4 \cdot 3 \cdot 2 \cdot 1)$, which also happen to correspond to the elements of S_4; see figure 1 ([4], p. 774). The figure lists all the rows in three columns with each giving one lead of the full extent (extents ring the entire $n!$ on n bells). As with all methods, it must return to rounds as seen at the bottom of the third column. Other than rounds, no other row is repeated.

To "compose" two permutations, we begin on the right side and work our way left. Example: $ab = (123)(23)$. We begin with 1 in the right

1234	1342	1423
2143	3124	4132
2413	3214	4312
4231	2341	3421
4321	2431	3241
3412	4213	2314
3142	4123	2134
1324	1432	1243
		1234

Figure 1

permutation. Since 1 is not present, we move to the left permutation and see $1 \to 2$. Now examine 2 starting at the right. In the right permutation $2 \to 3$ and in the left $3 \to 1$, thus $2 \to 1$. We now follow 3 starting at the right. In the right permutation $3 \to 2$ and in the left $2 \to 3$, thus $3 \to 3$. Because 4 is not mentioned and we are working with four positions, we assume $4 \to 4$. We can summarize our findings as follows: $1 \to 2 \to 1$ and $3 \to 3$ and $4 \to 4$. This can also be written as (12)(3)(4). If a number is sent to itself, it is not necessary to mention it if the number of positions is understood. We know there are four positions, therefore $(123)(23) = (12)$.

Let $a = (12)(34)$ denote the cross change that swaps bells 1 and 2 as well as bells 3 and 4 ($1 \to 2 \to 1$ and $3 \to 4 \to 3$). Let $b = (23)$ changing the middle pair ($2 \to 3 \to 2$). Using a and b, Arthur T. White explains the composition ab as follows: "$ab = (12)(34)(23) = (1243)$, composing right to left, which we interpret as ringing in position 1 bell 2, in position 2 bell 4, in position 4 bell 3, and in position 3 bell 1; that is the row 2413. This extends in a natural manner to a full correspondence between rows and permutations" [5].

Following this composition method, a and b generate the following rows:

$$
\begin{aligned}
e \text{ (identity)} &= 1234 \\
a &= (12)(34) = 2143 \\
ab &= (12)(34)(23) = (1243) = 2413 \\
aba &= (12)(34)(23)(12)(34) = (14) = 4231 \\
(ab)^2 &= (1243)(1243) = (14)(23) = 4321 \\
(ab)^2a &= (14)(23)(12)(34) = (13)(24) = 3412 \\
(ab)^3 &= (14)(23)(1243) = (1342) = 3142 \\
(ab)^3a &= (1342)(12)(34) = (23) = 1324
\end{aligned}
$$

These rows make up the first row of Plain Bob Minimus as well as the dihedral group D_4, which is the group of symmetries of a square as labeled

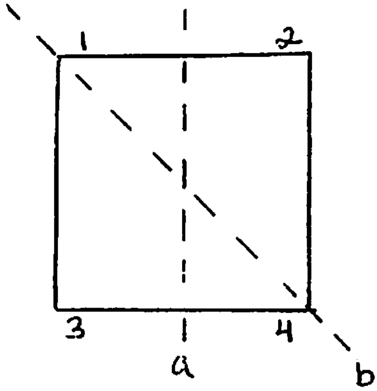

Figure 2. Permutations a and b as symmetries of a square.

in figure 2 ([4], p. 772).

Next, consider $c = (34)$ and thus column two $\{w, wa, wab, waba, w(ab)^2, w(ab)^2a, w(ab)^3, w(ab)^3a\}$ where $w = (ab)^3ac$. An interesting observation can be made regarding cosets at this time. First, a coset can be defined as follows (see [3], pp. 126–127): Let G be a group, and H a subgroup of G. For any element a in G, the symbol aH denotes the set of all products ah, as a remains fixed and h ranges over H. Then aH is called a left coset of H in G. In similar fashion, Ha denotes the set of all products ha, as a remains fixed and h ranges over H. Then Ha is called a right coset of H in G. In looking at the second column of Plain Bob Minimus, we can see it is simply the left coset wD_4! Finally, by using c a second time, we obtain the third column, which is in fact the left coset w^2D_4 ([4], p. 774). However, not forgetting rounds, we use c a third time. Voila — rounds and thus a complete composition!

White explains that in a composition such as this, six conditions must be met ([4], p. 774):

(1) The extent must begin and end with rounds. (This follows from $[(ab)^3ac]^3 = e$.)

(2) No other row is repeated. (The coset decomposition guarantees this.)

(3) From one row to the next, no bell moves more than one position. (This is forced by our choice of $a = (12)(34)$, $b = (23)$, and $c = (34)$.)

(4) No bell rests in the same place for more than two successive rows. (The alternation of $a = (12)(34)$ moves every bell appropriately.)

(5) The working bells (here, all but the treble) should all do the same work. (This is guaranteed by $w = (234)$, so that what

bell 2 does in the first lead, bell 3 does in the second and bell 4 does in the third, etc.)

(6) Each lead should be palindromic in its changes. (Examine $(ab)^3a$.)

The cosets have been discovered in the columns of Plain Bob Minimus, but what about the rows? By looking at the composition as an 8×3 matrix, we can easily see that the rows are the right cosets of the subgroup $\{e, w, w^2\}$ of S_4 (the set of all permutations of 4 elements). Further examination proves interesting as rows 1 and 8 give the subgroup $(S_4)_1$. An unusual characteristic of this subgroup is its similarity to the group S_3. The two are said to be isomorphic to one another. By definition, a bijective function exists from $(S_4)_1 \to S_3$ with the property that for any two elements a and b in $(S_4)_1$, $f(ab) = f(a)f(b)$ ([3], p. 93). Basically, they have the same structure, and are essentially the same group. The other right cosets consist of rows 2 and 7, rows 3 and 6, and rows 4 and 5.

Each permutation can be written as a product of transpositions, which are cycles of length two. For example, (1234) = (12)(23)(34). Because (1234) is a product of an odd number of transpositions it is said to be an odd permutation. Notice (12345) = (12)(23)(34)(45) and is even. In Plain Bob Minimus, rows 1, 2, 5, and 6 give the subgroup A_4 of S_4, which is the subgroup made up of all the even rows. Therefore, 3, 4, 7, and 8 consist of all the odd rows ([4], p. 775).

One can see all the mathematics and group theory at practice in a simple composition such as Plain Bob Minimus taking less than a minute to ring. Imagine, for a minute, the complexity of a more difficult method! Did Fabian Stedman, considered one of the "fathers of bell ringing," realize what was going on here mathematically back in 1677? Group theory arrived on the scene in the late eighteenth and nineteenth centuries! The second condition of the required six is the most difficult to prove: no row is repeated and each appears exactly once ([4], p. 776). Mathematically, we know that the cosets form a partition, meaning any two cosets are either disjoint or identical, with all elements accounted for. But how could a seventeenth-century ringer with no knowledge of group theory prove such compositions? This is a question causing much debate, one whose answer we may never know, but whose end result we will forever enjoy!!

Acknowledgements. Special thanks to Dr. Cynthia Woodburn for all her help.

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Base 8 Baseball

Many of you, our readers, have likely done some arithmetic in other bases, especially binary (base 2), octal (base 8) and hexadecimal (base 16). But, have you ever stopped to think how the world would be different if we all used base 8 instead of base 10? In all examples below, numbers are given in base 8 unless otherwise stated.

Consider, for instance, this year's historic home run race. A giant celebration was held when Mark McGwire broke Roger Maris' single-season home run record. But, if the world were running on octal instead of decimal, two more homers later and Mark reached number 100 (64 in base 10)! Sammy Sosa also would have reached that milestone. No doubt, there would have been fantastic celebrations of those home runs as well; in fact, it is possible that the celebration of breaking Maris' record might not have been as big as a result.

The question is, why would the numbering system matter so much? That's because of the psychological importance we place on nice "round" numbers. We use numbers to represent things; accomplishments in all areas of life are no exception. Because it is easier to remember "round" numbers, we use them as "milestones" to represent various levels of achievement. To some degree, those milestones are placed arbitrarily, but generally in the neighborhood of some number that is difficult to reach. An example would be career hits in baseball. We use 3000 base 10 to represent that milestone, but in octal we would likely use 6000 (that is, 3072 in base 10). Another would be career wins for a pitcher; instead of 300 base 10, we might use 500 (320 in base 10).

Of course, baseball would not be the only thing affected. I claim that the entire economy of the United States and the world would have been somewhat different as a result of the (once mandatory) retirement age being one year earlier at 100 (64 base 10). Think about it!

—the editor.

Checkers

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The computer-aided mathematical exploration of strategy board games began in the 1950's with chess, and soon included checkers. One important project was initiated in Alberta, Canada, by Jonathan Schaeffer, who designed the master checkers-playing computer Chinook (see [1]). His goal was to build a machine which could beat a master level player, and hopefully in the long term to solve the game of checkers (i.e. determine the ultimate winning strategy behind the game).

Schaeffer and the Chinook team went so far in achieving their first goal, that in 1992 Chinook began playing Marion Tinsley. Tinsley, at the time a mathematics professor from Florida State University, was considered the best checker player in history. At the first meeting Tinsley played Chinook to four victories, two losses, and 33 draws. Tinsley and Chinook would face off during the next five years, and at each time Tinsley would be the winner. This however changed in 1994; the two champions met for the Man vs. Machine World Checker Championship, where after playing 6 games to a draw Tinsley withdrew for health reasons giving the title of World Champion to Chinook.

The first goal was achieved by deliberately working towards the second one. In doing this Chinook used a database that contains information on possible positions and their eventual outcomes. These stored outcomes were generated by playing backwards, or taking a result and determining all the possible ways in which it could result. This massive database is housed on a dedicated mainframe which holds the eventual end game for confrontations of eight checkers or fewer. The information on these 400 billion positions allows Chinook to know the ending of any board arrangement with eight or fewer checkers on the board. To produce this feat, Chinook uses much hardware and time.

The number of possible positions of checkers on a game board is a large number. The board contains thirty-two spaces, which can hold either a dark piece, a light piece, or remain empty. The pieces can number between zero and twelve inclusive for each color. The pieces can also have the value of either a pawn or a king, and the relationship of the pieces depends upon which player has the turn. These are the criteria used to determine the total number of possible positions on a checker board. In calculating this amount, one must take the binomial coefficient

$$\frac{32!}{B!W!E!},$$

where B is the number of dark pieces, W is the number of light pieces, and E is the number of empty spaces. Also $B + W + E = 32$, $B \leq 12$ and $W \leq 12$. This would give the number of positions without regard to turn or type of pieces. To include the factor of turn the coefficient should be multiplied by the number of players, resulting in

$$\frac{32!(2)}{B!W!E!}.$$

To now include the two types of pieces, pawn and king, the above coefficient is multiplied by 2 to the power of the sum of the pieces, giving

$$\frac{32!(2)(2^{B+W})}{B!W!E!}.$$

This results in the number of positions for a single value of B and a single value of W. To achieve the total number of possible positions, this function must be summed for B going from zero to twelve and for W likewise going from zero to twelve, or more simply

$$\sum_{B=0}^{12}\sum_{W=0}^{12}\frac{32!(2)(2^{B+W})}{B!W!E!}.$$

This function will result in 3 696 129 032 684 478 521 090 or $3.7 \cdot 10^{21}$ total possible positions.

Since many of these positions will not occur in actual play, this figure is an upper bound for the total number of positions needing consideration. This figure differs from other previously published estimates.

The checkers project at Bowling Green State University began in the early 1990's as an effort between Thomas Hileman and Waldemar Weber. The main goal of this project was to write a software program for a personal computer to play a decent game of checkers. The project focused originally on "teaching" the computer to play by the American Checker Federation rules. It then primarily focused its efforts on trap detection and defense (see [2]).

The program reasoned hypothetically to simulate reasons by looking several moves ahead on a limited game tree. A game tree is a collection of possible moves, the possible responses to those moves, and the player's possible reactions to those moves. The program would look at the current position of the board and determine which would be its best move. It would then determine what the opponent's responses would be to that move and calculate which would do the most damage, and then determine the best move it could achieve in response to that course of action. The limited game tree would look similar to the diagram in figure 1.

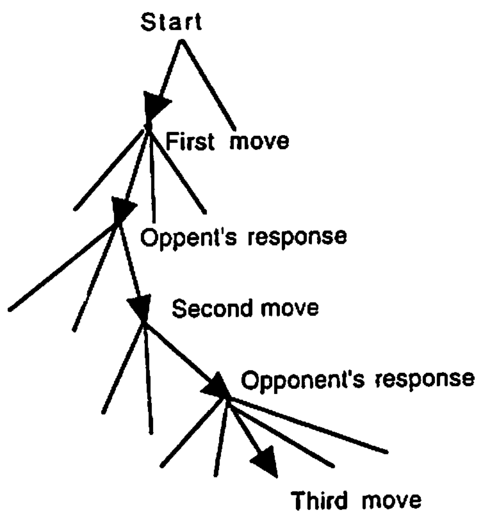


Figure 1. Limited game tree.

Once an optimal move was chosen, only the responses of that action were evaluated. The rest of the tree was not utilized. Those positions would not even be evaluated. See figure 2.

The improvements to the program for this project began with enhancements to the efficiency of the code resulting from the introduction of arrays of fixed length for lists of variable length whenever possible. Although these improvements did not alter the mathematical logic behind the game, they did however allow for more execution time and memory space for improved logic. Some of this logic improved the previous evaluation criteria by expanding it as a recursive definition to the entire game tree. The improvement in the efficiency of the code allows for the evaluation and storage of many more positions than was previously possible.

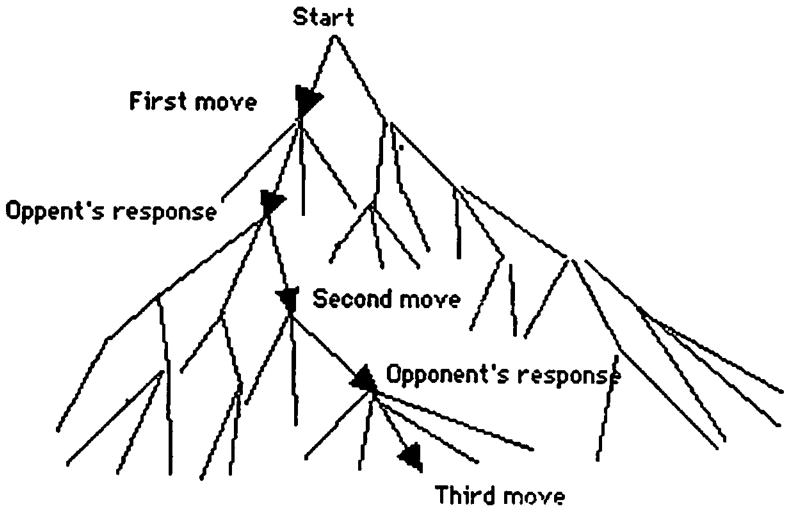


Figure 2. Complete game tree.

The program now at the beginning of the turn looks at all possible moves, then all possible retaliatory action, and then all feasible responses, etc., for up to six moves, or three turns. Since some sacrifices in pieces could be necessary, these additional considerations were essential for the improvement of the hill-climbing function.

A hill-climbing function is one which reduces the amount of trial and error required to achieve a goal, i.e. winning a game of checkers or a subgoal, such as taking more pieces while losing fewer. Indeed, a hill-climbing function reduces the number of guesses required to achieve the goal. Hill-climbing functions are monotonic functions that help to develop strategy by using evaluation functions to assign values to every state including the goal state. Once these values have been assigned, the function chooses the action which will bring it closest to the goal state. For example, suppose the goal state was evaluated at 0 and the function has a choice of two actions valued at 12 and 7. The function will select the action valued at 7, for it is closer to 0. See figure 3.

The above example is a single-step, or one-dimensional, hill-climbing function. After performing the function on the first decision, the result is 5. By only looking one action ahead, the goal value of zero was not achieved because it required actions which at first did not appear to result in the goal. If, in the above example, a two-dimensional technique was used, one in which the function looked not just at the first action, but also the next as well, the hill-climbing value would have reached the desired goal. This is a very significant reason for looking at the complete game tree.

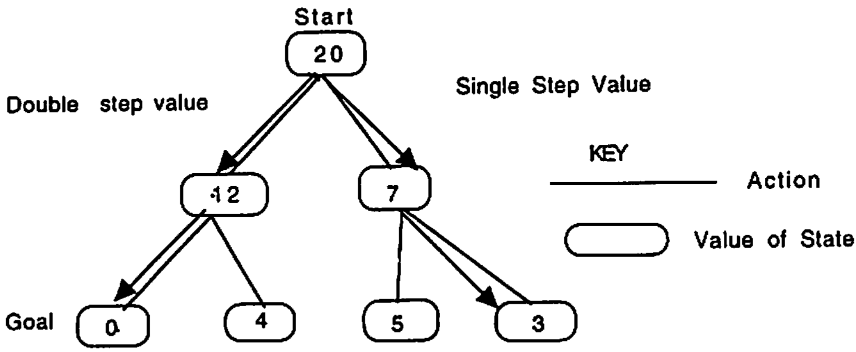


Figure 3. Hill-climbing function diagram.

In programming checker strategy with a limited game tree, it is easier to implement defensive strategies than it is to implement offensive ones. If the program could look at more of the game tree, it could realize the offensive advantage of losing one piece in order to take two or more of the opponent's pieces. The current program uses not a one or two-dimensional function, but a six-dimensional hill-climbing function. It determines the best possible move to make by considering the state value of up to three turns (both the move and the opponent's response) into the future. This alleviates some problems of the previous hill-climbing function.

One problem was the one-dimensionality of its choices. The program looked at one move and one move only and then determined the possible responses. It could chose that move even if a more effective one was present on a different branch of the tree. Another difficulty with only choosing one action to consider is the evaluation of the opponent's response. The program determined which move the other player could make by choosing the response that would be most damaging to its strategies. As the strategy of the other player is an unknown, the choice of the opponent's move was an approximation at best. With the additional memory space and execution time, the program is now able to look at the consequences of all of the opponent's possible moves. This action removes the approximation, and makes the decision more precise.

Even with the improvement upon the previous hill-climbing function, the current version is not perfect. It likewise encounters the same problems as the first, the restriction of space and time on the personal computer. The storage capacity of a personal computer will only allow for the storage of a finite number of moves, and only a certain number of evaluation states can be calculated in the five minutes of time allowed per turn by the American Checker Federation. Any evaluation state of actions beyond the sixth dimension is not calculated or considered. Just as in the example, the goal state would not have been reached by just considering the first action, if the desired winning move occurred down a branch of the game tree beyond the sixth move which is different than the one chosen just by looking at six layers. The program tries to lessen this limitation by incorporating end-game strategy.

The end game occurs during the point of play when each player is left with only a few pieces on the board. With the fewer number of pieces, the opportunities to use one's pieces to force your opponent into positions which would allow a double jump or another tactical advantage lessens. Instead, a different strategy must be utilized. End-game strategy revolves around "the Move" and parity.

"The Move" in checkers refers to the final move. In checkers one wins when one's opponent either has no pieces remaining on the board or cannot complete a turn. In either situation the player can complete the final move and win the game. The Move can be determined many moves into the future. This is done by calculating the parity.

To determine the parity, and thus the Move, a player must first know how many pieces are in his or her "system." A player's system consists of all the squares which are contained in the columns which run perpendicular to the playable squares in his or her king's row. See figure 4.

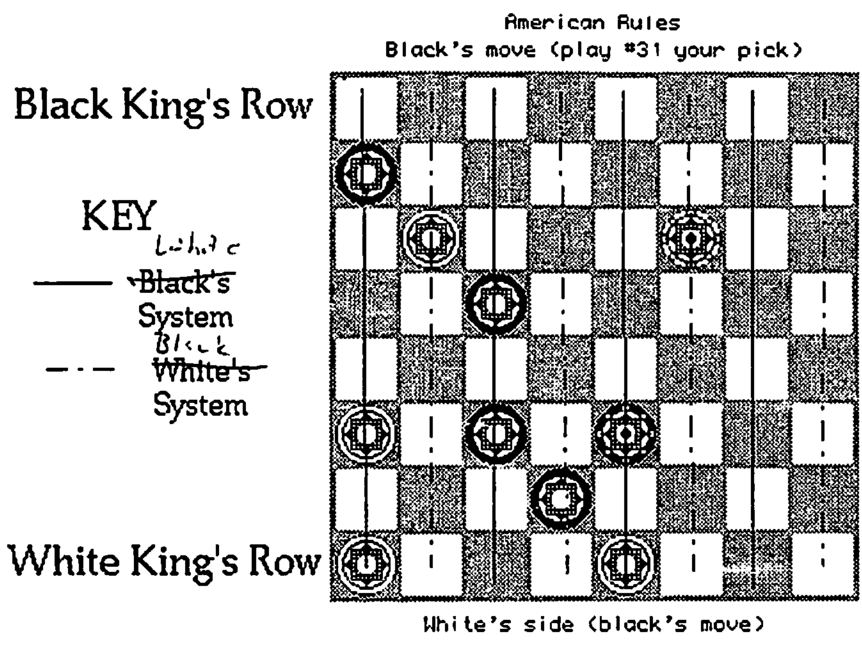


Figure 4. A player's system.

At the beginning of the turn, if the number of pieces in the player's system is odd then he controls the Move, else his opponent controls the Move. If we allow p to designate the player, q to designate the opponent, and e and o to designate even and odd parities, respectively, two observations can be made.

The first observation involves looking at the initial parity pairs (p, q) and determining each player's move and the defensive response. This results in the following matrix:

starting patterns	(o, o)	(e, e)	(o, e)	(e, o)
a step by p (non-jump)	(e, e)	(o, o)	$(e.o)$	(o, e)
p jumps from system p	(o, e)	(e, o)	(o, o)	(e, e)
p jumps from system q	(e, o)	(o, e)	(e, e)	(o, o)

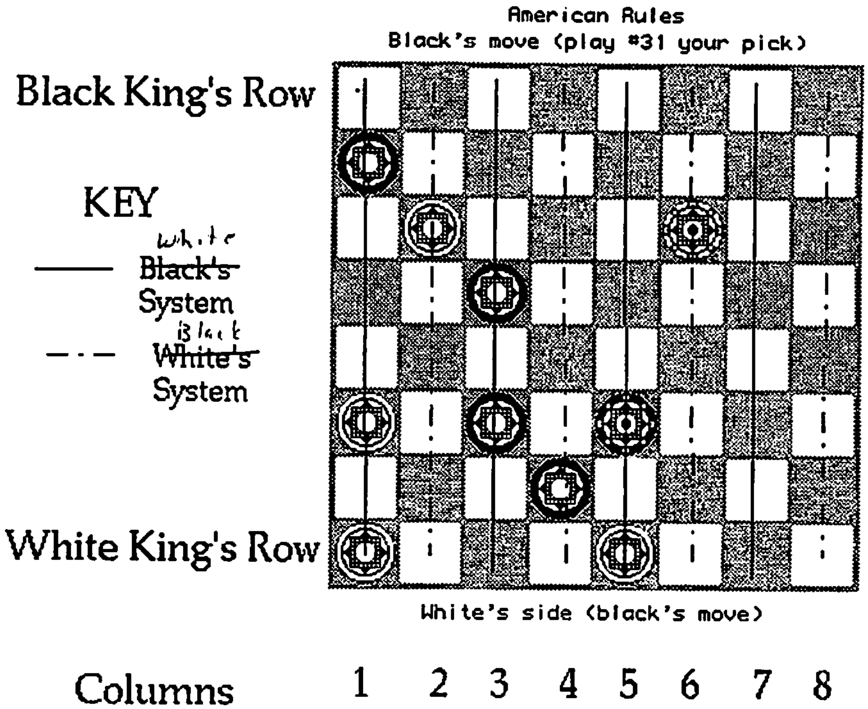
Each row and column of the preceding matrix contains some permutation of the starting parity patterns $(p, q) = (o, o), (e, e), (o, e), (e, o)$. The matrix is symmetrical to either diagonal, making it a dihedral group $C_2 \times C_2 = D_4$ in which each player always changes a starting pattern in one move. Also, since multiple jumps in one turn preserve the parity of the same count as the initial jump while altering the parity of the other player, a double jump or a one-for-one exchange transfers a given parity pattern to the other player.

It is also observed that consecutive steps will not alter the parity. The parity of a player's system cannot be altered without involving a jump by the player or the opponent. Also if a player starts with an even parity he or she is at a tactical disadvantage.

Ideally it is most advantageous to receive an odd parity at the beginning of one's turn, however it is also advantageous if one can return an even parity to his or her opponent. During the turn, a player cannot completely control the parity which he will receive, but he can take measures to control the parity of his opponent. This can be done by always giving the opponent not only an even system but also a balanced one. A balanced system is one in which the player can easily return an even system to his opponent at the end of his next turn. This can be done by looking at the number of pieces in each individual column of the opponent's system and taking the binary representation of each number. See figure 5.

If each column of these figures are summed the result is 031. Because the column totals contain at least one odd number it is unbalanced. The only way to make a balanced system is to increment or decrement only one of the columns on the board in his opponent's system by one.

To balance the system consider each row of the binary representation and the digits in the columns which sums are odd, in this case the two right-most digits. In order to create a balanced system, one of the rows and corresponding column on the board must be decremented so that the digits in the columns containing the odd sum are swapped from their current



White's System

Column 1 011
Column 3 010
Column 5 010
Column 7 000

031

Figure 5. Calculation of the balance of a system.

value (i.e. if it was a one it would be a zero and if it was a zero it would be a one). Since the player can only decrement or increment by one, altering the row with the value of three (011) will either change both digits to zero and also alter the first column's digit, or only alter a single digit to zero. Consideration must remain on the two rows containing the value of two. In this case if either one of them is decremented by one, the value of one is achieved with the binary value of 001, this will alter the two digits and allow the sums of the columns to be even. By removing one of the pieces in either of the columns of the board corresponding to those binary rows will make this system balanced.

By continuing to give an opponent an even parity, a player can ensure that he will always have the Move at the beginning of his opponent's turn. If the opponent is not able to jump, he will return an unbalanced system to the player. The only way a player is unable to return a balanced and even system to the opponent, is if he receives one at the beginning of his turn.

Due to this a further calculation was added to the program's evaluation function. The starting parities were ranked and given a value according to one the player's parity being odd, and second the opponent's parity being odd. This consideration resulted in the following ranking:

First (best)	(o, o)
Second	(o, e)
Third	(e, o)
Fourth	(e, e).

This addition to the evaluation function applied to hill-climbing has improved the performance of the Checkers program. It now plays a better game than before; however, some limitations still remain. The continual problem of the program will be limited execution time and storage space. As the goal of the Checkers Project was to write a program for a personal computer, additional logic improvements are dependent upon efficient code writing and advancements in technology and processor speed.

Acknowledgements. During the past year and a half, it has been my pleasure to work on this project with my advisor Dr. Waldemar Weber. Without his motivation and programming skills this second version would not have been possible.

References

1. "Chinook," http://www.cs.ualberta.ca/~chinook/. Verified March 25, 1997.
2. Hileman, Thomas, "Playing Checkers with Mathematical Logic," *The Pentagon* 55 No. 2 (Spring 1996), 30–36.

Palindromic Squares

There are many palindromic squares with an odd number of digits; for example, $121^2 = 14641$. But palindromic squares with an even number of digits are much harder to find. Here are two: $836^2 = 698896$, $798644^2 = 637832238736$.

—*The Pentagon*, Fall 1951, p. 41

Statistics and Extreme Values

Kathyrn Hickman, *student*
Steven Wall, *student*

Wisconsin Gamma

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Eau Claire, WI 54702

Presented at the 1997 National Convention

Last year we engaged in a research project focusing on extreme value estimation under the supervision of Dr. Marc Goulet. Our research was based on time series, but more specifically, predicting extreme values within a time series.

Our research goal was to understand and apply the theory of extreme values. In pursuing our goal, it was important to first identify what a time series was, since it was a new concept for us. Thus, we defined a time series as a sequence of numbers observed over time. Numerous types of time series exist including the Dow Jones Industrial Average, the unemployment rate in the United States over 3 years, and the average monthly air temperatures in a city in Brazil. These graphs are measured over a period of time; thus they are examples of time series.

As stated earlier, we are attempting to predict extreme values of time series. The purpose of extreme value theory, according to the mathematician Emil Gumbel, "is to explain observed extremes arising in samples of given sizes, ... and to forecast extremes that may be expected to occur within a certain sample size" ([1], p. 1). Of course, this forecast cannot predict the largest value to be obtained at a specific time, but rather it gives the most probable largest values to be obtained within a period of time. We will be applying this forecast to predict extreme values of a randomly generated time series. Extreme value theory provides limits within which these extreme values may be expected to lie by using the theory of probability and statistics.

Our research was based on the following problem: suppose we are given n data points from a selected time series. We want to predict the extreme value for the next $2n$ data points of the time series. The results of our research can be applied to many situations. These could include: 1) stock market investors who want to predict what the highest value of a stock will be in the next $2n$ days, given the highest value of the stock for the current n days; and 2) ocean researchers who need to be able to estimate the maximum wave height over a period of time, so that ocean structures can be built to withstand the resulting energy. The motivation behind our research was based on this aforementioned scientific problem. More specifically, we were informed, by our advisor, that engineers were in fact building oil rigs off the coast of Norway, and needed information about the extreme height of these waves to estimate at what height these rigs could withstand the energy created by these waves.

To apply ideas from probability theory and to make use of computer simulations, we made the assumption that our time series was normally distributed. We began by generating a dependent time normal time series (X_n) of length N. Rather than using this time series directly, we would like to consider an independent time series which would enable us to make a 'working' equation for the expected maximum of a time series (see appendix A). It should be emphasized that this working equation will give a theoretical value, as we will later be comparing this theoretical value against an empirical value.

In order for us to use our working equation, we needed to make the dependent time series model an independent time series. To do this it was necessary to find a scaling factor, which we will call ρ, to help model an independent time series. We computed this scaling factor by setting $\rho = n/N$, where n is the number of independent points and N is the length of the time series. This ratio is the effective number of independent points per point. This, in essence, says that n independent points model N dependent points. The difficulty in this process was finding n. To do this, we used the first m points of the time series. We then partitioned these points into sub-intervals and found $\rho = y/Y$, where y was the number of independent points in the sub-interval of length Y. We then set $y/Y = n/N$ since our ρ is constant throughout the time series. We set these ratios equal to one another in order to solve for n, which could easily be found after we find y through the following process.

To assist in solving our basic problem, we wrote a computer program in the computer language *Mathematica*, to help find y. In this program we first generated a time series, normalized it, and then searched through it to find the extreme value. We the studied the first one-third of the time series to help in predicting the remaining two-thirds of the time series. We then partitioned the first one-third of the time series into k sub-intervals and found the local extreme in each sub-interval. We chose k so that we could get enough intervals for a good average, but not too many that the length of the intervals would be too small. Finally we found the average of the k local extremes and called it the average maximum or avgmax. This program can be found in appendix B.

Since the theoretical expected maximum, which is what our working equation found, should be close to the empirical value (avgmax) that we found through our program, we can set them equal to each other to find y. After finding y, we could compute a value for $\rho = y/Y$, or the scaling factor. Now that we found ρ, we could predict the expected extreme value over the next $N - m$ points. The reason that we can do this so easily is that after finding ρ, we can take any time series of any length and make it act like an effective number of independent points by multiplying it by ρ. The scaling factor ρ will vary in value from 0 to 1 according to how closely a given time series resembles a dependent versus an independent time series. The more independent a time series is the closer ρ will be to 1. Thus, the more dependent a time series is, the closer ρ will be to 0. Finally, after re-scaling, we can use our working equation to find the extreme value within the next $N - m$ points.

In review, our general process was to first model a dependent time series with an independent one. Second, we split the first m points of the dependent time series into subintervals and averaged the local extremes of each subinterval. Third we set that average equal to the 'working' equation and solved for y so that we could determine the scaling factor, ρ. Finally we applied ρ to the remaining $N - m$ points and used the working equation to predict the expected maximum for the remaining $N - m$ points.

Early results of this process are encouraging. We have found relative errors of 13.5% for a time series of length 3000 points. Relative error is found by dividing the difference of the expected and the observed extremes by the expected extreme. The length of our time series was limited due to the power of the computers to which we had access. This error is expected to decrease for time series of longer duration.

Acknowledgements. We would like to thank Dr. Marc Goulet for his guidance and support throughout this independent study.

References

1. Goulet, Marc. R, Burton, Robert M., and Yim, Solomon, *On the Extreme Value Estimation,* Report No. OE–90–3, Oregon State University, 1990.
2. Gumbel, Emil J., *Statistics of Extremes,* Columbia University Press, New York, 1958.

Appendix A

Assume that a time series consists of independent and identically distributed random variables, $X_1, X_2, \ldots, X_n$. Assume that the distribution function is given by

$$F(x) = P(X \leq x).$$

Now define

$$M_n = \max_{1 \leq i \leq n} (|X_i|).$$

We now find $M_n's$ distribution:

$$\begin{aligned} G_n(x) &= P(M_n \leq x) \\ &= P(|X_1| \leq x, |X_2| \leq x, \dots, |X_n| \leq x) \\ &= P(|X_1| \leq x)P(|X_2| \leq x) \cdots P(|X_n| \leq x) \\ &= (F_{|X|}(x))^n \\ &= F^n_{|X|}(x). \end{aligned}$$

Since our time series are assumed normally distributed, we may write

$$F_{|X|}(x) = P(|X| \leq x) = \int_{-x}^{x} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du,$$

and

$$F^n_{|X|}(x) = (\int_{-x}^{x} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du)^n.$$

Since $|M_n| > 0$ we use as a definition of expected value

$$\begin{aligned} E[M_n] &= \int_0^\infty P(M_n > x)\, dx \\ &= \int_0^\infty (1 - P(M_n \leq x))\, dx \\ &= \int_0^\infty (1 - F^n_{|X|})\, dx \\ &= \int_0^\infty (1 - (\int_{-x}^{x} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du)^n)\, dx. \end{aligned}$$

This is our 'working' equation.

Appendix B

```
// This is the program we wrote in
//Mathematica.  In this program, F0 is a
//dominant frequency, Damp is a dampening
//factor, DT is a sampling rate, PHI1 and
//PHI2 are coefficients used in determining
//the time series.
```

```
F0=.05
Damp=.01
DT=2
SeedRandom[68]
PHI1=2*Exp[-2*Pi*F0*DAMP*DT]*Abs[Cos[2*Pi*F0]]
PHI2=-Exp[-4*Pi*F0*DAMP*DT]
V=Table[1, {1000}]
Sm=0
SmSq=0
For[i=3, i<3000, i++,
   x=Random[]*2*Pi;
   y=Sqrt[-2*Log[10, Random[]] ];
   RVN=Cos[x]*y;
   V[[i]]=PHI1*V[[i-1]]+PHI2*V[[i-2]]+RVN;
   Sm=Sm+V[[i]];
   SmSq=V[[i]]*V[[i]]+SmSq ]

Mx=0
For[i=3, i<3000, i++,
   V[[i]]=(V[[i]]-(Sm/2997)) / Sqrt[SmSq/2997-(Sm/2997)^2];
   If[Abs[V[[i]]]>Mx, Mx=Abs[V[[i]]] ]

Hi=Table[0,{9}]
For[i=1, i<10, i++,
   For[j=1, j<112, j++, k=111i-111+j;
      If[Abs[V[[k]]]>Hi[[i]],
         Hi[[i]]=Abs[V[[k]]] ] ] ]

premax=0
For[i=1, i<10, i++,
   premax=Hi[[i]]+premax]

avgmax=premax/9
g[s_]:=(1/Sqrt[2*Pi])*Exp[-(s^2)/2]
f[n_]:=NIntegrate[1 - (NIntegrate[g[s], {s,-t,t}]^n,{t,0,5}]
```

Strictly for Squares

The number 49 is a square. If we put 48 in the center of 49 we get 4489, which is 67^2. Another 48 gives 444889, which is a square. This can continue indefinitely; 44448889, 4444488889, etc. Can you find another pair of consecutive numbers that have this same property?

—*The Pentagon*, Spring 1966 pp. 126–127

C++ Christmas Carol

to the tune of *Jingle Bells*

Dashing to the lab
My assignment in my hand
Boot up the machine
And give it the command
Keys on the keyboard click
The screen it is so bright
Oh, what fun it is to code
Long into the night — Oh!

Chorus:
C++, C++, Program all the way
Oh what fun it is to code
Until the night is day
C++, C++, Program all the way
Oh what fun it is to code
Until the night is day

A day or two ago
I wrote a matrix class
But something went quite wrong
I think I went too fast
The code was mean and rank
Misfortune seemed its lot
It got into an infinite loop
And me, I got upsot — Oh!

(Chorus)

Now the screen is white
And sparks fly out the back
The keyboard's melting down
Oh, how I love to hack!
So fix that code tonight
Run it while you're young
Just program 'til your brains fall out
Then sing this awful song — Oh!

(Chorus)

— Anne Cable, Eastern New Mexico University

Chapter Web Sites

Send additions or corrections to Arnold Hammel at a.hammel@cmich.edu.

Alabama Zeta, Birmingham-Southern College:
www.bsc.edu/dept/math/kme.htm
California Gamma, California Polytechnic State University:
www.calpoly.edu/~jdgordon/kme
Colorado Beta, Colorado School of Mines:
www.mines.edu/all_about/events/orgs/honor/kme.htm
Illinois Delta, College of St. Francis:
www.stfrancis.edu/ma/honor.htm
Indiana Delta, University of Evansville:
cedar.evansville.edu/~mathweb/kme.html
Iowa Alpha, University of Northern Iowa:
www.math.uni.edu/CS/KME/KME.html
Kansas Alpha, Pittsburg State University:
www.pittstate.edu/math/kme.html
Kansas Gamma, Benedictine College:
www.benedictine.edu/math-cs.html
Kentucky Alpha, Eastern Kentucky University:
eagle.eku.edu/faculty/pjcostello/kme
Kentucky Beta, Cumberland College:
cc.cumber.edu/acad/math/kme.htm
Mississippi Beta, Mississippi State University:
www.math.msstate.edu/~pearson/kme-maa.htm
Missouri Alpha, Southwest Missouri State University:
science.smsu.edu/math
Missouri Beta, Central Missouri State University:
153.91.1.112/~kme/kme.html
Missouri Zeta, University of Missouri—Rolla:
www.umr.edu/~kme/
Ohio Alpha, Bowling Green State University:
www.bgsu.edu/departments/math/KME
Ohio Zeta, Muskingum College:
pluto.bsc.muskingum.edu/~kandhari/kme/kme.html
Oklahoma Alpha, Northeastern State University:
www.nsuok.edu/students/kme
Oklahoma Gamma, Southwestern Oklahoma State University:
www.swosu.edu/student/stdorg/kme
Wisconsin Gamma, University of Wisconsin—Eau Claire:
www.uwec.edu/Academic/Curric/gouletmr/kme/kmehome.htm

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before July 1, 1999. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1999 issue of *The Pentagon*. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: xxwilke@acc.wuacc.edu).

PROBLEMS 520–524

Problem 520. Proposed by the editor.

Let R_x denote the continued fraction

$$\cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x+\cdots}}}.$$

Find all solutions in positive integers a and b, if any, of the equation

$$5R_a - 2R_b = 1.$$

Problem 521. Proposed by the editor.

Sam is delivering a large rectangular box of pizza in a dormitory. He must take the pizza down a hallway whose width is 40 inches. Unfortunately this hallway intersects another hallway whose width is 30 inches at right angles and the room which ordered the pizza lies down this second hallway. Assuming that he always carries the pizza box in a horizontal plane, what are the dimensions of the pizza box having maximum area which he can deliver? For the purposes of this problem, consider the pizza box as being a horizontal plate.

Problem 522. Proposed by Bryan Dawson, Union University, Jackson, Tennessee.

A particular gradebook allows 30 students to be listed on a page. Suppose a class consists of 33 students, with 30 names listed on one page, and 3 on the next, and that the pages must be turned from one set of students to the other. If I record grades as I grade an assignment, in random order of students, and turn the page only when necessary, what is the expected number of page turns needed, not counting turning to the correct page of the gradebook for the first student? Assume that all students turned in the assignment.

Problem 523. Proposed by the editor.

A customer stops in at a local convenience store and purchases four items. The customer was told that the cost was \$7.70. The clerk had inadvertently multiplied together the prices of the four individual items. The customer protested that the four prices should have been added together instead of being multiplied. The clerk said that that was OK with him, but, the result was still the same: exactly \$7.70.

What were the prices of the four items?

Problem 524. From the "Mathematical Scrapbook" section of *The Pentagon*, Spring 1943.

How much of the earth's surface would a man see if he were raised to the height of the radius above it?

Please help your editor by submitting problem proposals.

SOLUTIONS 510, 512–514

Problem 510. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Eva and Al are going to run a 5K race. Eva's speed is proportional to the fourth root of the distance she has run. Al's speed is proportional to the distance left in the race. Eva reaches the midpoint of the race after 11.9 minutes while Al reaches the midpoint of the race after 13 minutes. Who will win the race and how long will it take for each runner to run the 5K?

Solution by the Alma College Problem Solving Group, Alma College, Alma, Michigan.

In order to find the time required for Eva to complete the 5K race, we must find her speed. We solve the differential equation

$$dx/dt = k(x)^{.25},$$

where k is a proportionality constant and x is the distance Eva has run. Since the equation is separable, we integrate

$$\int x^{-.25}\,dx = \int k\,dt$$

so that

$$\frac{4}{3}x^{.75} + c = kt.$$

Note that $c = 0$ because $x = 0$ when $t = 0$. At the halfway point, k is given by the relation $(4/3)(2.5)^{.75} = 11.9k$ or $k = 4(2.5)^{.75}/35.7$. Substituting the value of k into the original equation, we find Eva's time to be $t = 11.9(2)^{.75} = 20.01$ minutes.

Likewise, taking r as the proportionality constant for Al, we must solve the differential equation $dx/dt = r(5-x)$. Integrating, we have

$$\int (5-x)^{-1}\,dx = \int r\,dt$$

or

$$-\ln(5-x) + c = rt.$$

Since $5 - x > 0$, $\ln(5/(5-x)) = rt$, again since $c = 0$ because $x = 0$ when $t = 0$. Then at the halfway point, $\ln(5/(5-2.5)) = 13r$ so that $r = (\ln 2)/13$. Thus the equation for Al's time is given by $t = 13\ln(5/(5-x))/(\ln 2)$. To find Al's time, we take the limit

$$\lim_{x\to 5^-}((13/\ln 2)\ln(5/(5-x))) = (13/\ln 2)\lim_{x\to 5^-}\ln(5/(5-x)) = \infty.$$

Thus poor Al loses the race — it seems that he is still running!

Also solved by: Clayton Dodge, University of Maine, Orono, Maine and the proposer.

Editor's comment. Yes, the proposer intended for Al to still be running. As our featured solvers pointed out, if Al's speed was inversely proportional to the distance he had run, the differential equation would have been $dx/dt = r/(5-x)$ and we would have $r = 75/104$ so that Al would have had a winning time of 17.333 minutes. One wonders if Al was one of the proposer's hapless students.

Problem 511. Proposed by the editor.

Let $F(n) = \sum_{i=1}^{n} [\![\sqrt[3]{i}]\!]$, where $[\![\cdot]\!]$ denotes the greatest integer function and n is a positive integer. Find an infinite set of integers k for which $nk = F(n)$ for some n. (This delightful problem surfaced at the last convention!)

Editor's comment. Since no complete solution has been received, this problem will remain open for another issue.

Problem 512. Proposed by the editor.

For every positive integer k, show that 3^{2k} can be represented as the sum of 3^k consecutive integers.

Solution by Aaron Kerr, Alma College, Alma, Michigan.

Consider the required 3^k integers arranged in ascending order from left to right with the largest term at the right. Since the sum of the 3^k consecutive integers must be 3^{2k}, the average of these integers must be 3^k which is also the middle term of the series of these consecutive integers. Thus there are $x = (3^k - 1)/2$ integers on either side of the middle term 3^k. The sum of these terms is given by

$$\begin{aligned}
&[3^k - x] + [3^k - (x-1)] + \cdots + [3^k - 1] + 3^k + [3^k + 1] + \cdots \\
&\qquad + [3^k + (x-1)] + [3^k + x] \\
&= 3^k(3^k) + [-x - (x-1) - \cdots - 1 + 1 + \cdots + x - 1 + x] \\
&= 3^k(3^k) \\
&= 3^{2k}
\end{aligned}$$

since all the terms inside the square brackets add to zero. This demonstrates the desired result.

Also solved by: Alma College Problem Solving Group, Alma College, Alma, Michigan; Charles Ashbacher, Hiawatha, Iowa; Clayton Dodge, University of Maine, Orono, Maine; Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri; Mark Hoskins, Eastern Kentucky University, Richmond, Kentucky; Carl Libis, University of Alabama, Tuscaloosa, Alabama; Aaron Peters, Liberty University, Forest, Virginia: Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin; Alex Shaumyan, Eastern Kentucky University, Richmond, Kentucky; J. Sriskandarajah, University of Wisconsin Center—Richland, Richland Center, Wisconsin; and Matt Anderson, Greg Bowden, Heather Grant, Matt Hudson, Andrew Swihart, Cheril Lin Abeel-Wescoat and Nate Woodhams all of Alma College, Alma Michigan.

Editor's comment. Many of the solutions submitted were similar to the featured solution while others utilized standard summation formulas.

The featured solution was chosen arbitrarily by the editor from all of the excellent solutions submitted.

Problem 513. Proposed by the editor.

Consider the infinite sequence of integers $3_k1 = 333\cdots31$ in which there are k consecutive threes preceding the last digit. It is well known that 3_k1 is prime for $k = 1, 2, \ldots, 7$. Prove that this sequence contains an infinite number of composite terms and find one infinite sequence of composite terms.

Solution by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri.

We will prove by mathematical induction that 17 divides $3_{16n+8}1$ for all nonnegative integers n. For $n = 0$, $3_81 = 17 * 19607843$. Now assume that 17 divides $3_{16k+8}1$ for some integer $k > 1$. It remains to show that 17 divides $3_{16k+24}1$. But

$$3_{16k+24}1 = 3_{16}0_{16k+9} + 3_{16k+8}1. \tag{1}$$

By the induction hypothesis, 17 divides the second term on the right side of (1). Also since $3_{16} = 17 * 196078431372549$, 17 divides $3_{16}0_{16k+9}$. So by identity (1), 17 divides $3_{16k+24}1$. As a result, each term of the sequence $3_{16n+8}1$, where n is a nonnegative integer, is composite.

Also solved by: Alex Shaumyan, Eastern Kentucky University, Richmond, Kentucky; Pat Costello, Eastern Kentucky University, Richmond, Kentucky; Clayton Dodge, University of Maine, Orono, Maine; and Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

Problem 514. Proposed by Charles Ashbacher, Hiawatha, Iowa.

Given any integer $n \geq 1$, the value of the Pseudo-Smarandache function $Z(n)$ is the smallest integer m such that n evenly divides $\sum_{k=1}^{m} k$. Consider the Smarandache Pierced Chain Sequence 101, 1010101, 10101010101, 101010101010101, 1010101010101010101, ... or $c(n) = 101 \cdot (1000)_{n-1}1$ for $n \geq 1$. Here $(1000)_{n-1}1$ denotes the number in which there are $n - 1$ consecutive copies of 1000 followed by a 1. Prove the sequence $c(n)/101$ contains no prime numbers.

Solution by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

The terms of the sequence

$$c(n)/101 = (1000)_{n-1}1$$

are 1, 10001, 100010001, 1000100010001, 10001000100010001, ... Because 1 is not a prime number and $10001 = 73 \cdot 137$, the first two terms of the sequence are not prime numbers. Hence in the remainder of this solution, we will consider only the other terms of the sequence. These terms may be written in the form $1+10^4+10^8$, $1+10^4+10^8+10^{12}$, ..., $1+10^4+10^8+\cdots+10^{4j}$, ... for some integer $j>1$. We shall use the more general sequence $1+x^4+x^8$, $1+x^4+x^8+x^{12}$, ..., $1+x^4+x^8+\cdots+x^{4j}$, ... where x is an integer greater than 1.

If j is odd, then $j=2m+1$ for some positive integer m, and

$$\begin{aligned}1+x^4+x^8+\cdots+x^{4(2m+1)} &= (1+x^4)+x^8(1+x^4)+\cdots+x^{8m}(1+x^4)\\ &= (1+x^4)(1+x^8+\cdots+x^{8m}),\end{aligned}$$

which is clearly not a prime number because $m>0$.

If j is even, then $j=2m$ for some positive integer m and

$$\begin{aligned}&1+x^4+x^8+\cdots+x^{4(2m)}\\ &\quad=[1-(x^4)^{2m+1}]/(1-x^4)\\ &\quad=[1-(x^2)^{2m+1}]/(1-x^2)\cdot[1+(x^2)^{2m+1}]/(1+x^2)\\ &\quad=(1+x^2+x^4+\cdots+(x^2)^{2m})\cdot(1-x^2+x^4-\cdots+(x^2)^{2m}),\end{aligned}$$

which also is clearly not a prime number because $m>0$. Taking $x=10$ above establishes the desired result.

One incorrect solution was received.

Online Problem Database

MathPro Press has developed a free, searchable online database of problems from a wide variety of sources. The host for their database is the University of Missouri — Rolla (MO Zeta). Over 20,000 problems from approximately 60 sources are currently online. This includes problems from *The Pentagon* from 1975 to 1989 (that is a fairly common range of times available for most sources). The URL is:

problems.math.umr.edu

Searches can be conducted by keywords in problem statement, subject, source, year, proposer, solver, title, unsolved problems, by comments, or by other similar criteria. The database includes contest problems as well as problems from journals. The site is currently operated on a volunteer basis, and anyone wishing to volunteer time can contact Mark Bowron (bowron@compuserve.com). They plan to continue to add additional problems to the database as it is developed.

I (the editor) personally tried my favorite subject (Pythagorean triples), and got 47 matches!

Report of the North Central Regional Convention

Prepared by Mary Sue Beersman, MO Eta Chapter and Regional Director

The North Central Region of Kappa Mu Epsilon held a convention April 3–4, 1998, which was hosted by the MO Gamma chapter at William Jewell College, Liberty, MO. There were thirteen chapters present, which included 58 students and 24 faculty representing chapters in four states.

While the observatory could only be toured and not used on Friday night because of lots of clouds, students toured the White Science Center and our hosts even got the Holts machine up and running for us. Saturday, after being welcomed by the academic dean and provost of WJC, Dr. Nina Pollard, eight students gave their presentations and the top three were recognized and received an award. The following is a list of these presenters; the * indicates the award winners.

The Theory of Time Projection Gaseous Simulations
Mark V. Albert, Kansas Alpha, Pittburg State University

Distribution Plots
Laurel Berner*, Missouri Eta, Truman State University

Boundary Value Problems in Hollow Rectangular Beams
Jeffery D. Blanchard*, Kansas Gamma, Benedictine College

Princess Diana, Paul Revere, and Group Theory???
Kathy Denney*, Kansas Alpha, Pittsburg State University

Tetris for Mathematicians: A Study of Linear Algebra within Tetris
Mandy Fritz, Kansas Alpha, Pittsburg State University

"A" is NOT for Achievement
Beth Koch, Suzanne Shontz, and Gary Spieler,
Iowa Alpha, University of Northern Iowa

The Beginning of Mathematics
Melanie Kurtz, Kansas Beta, Emporia State University

Cubik Math
Allison L. Willson, Kansas Alpha, Pittsburg State University

All the presenters did a marvelous job and set a great example for those wanting to give presentations at next years national convention in Florida. If I can convince a couple of our members, I hope to get them started on ideas so they can do some of the work over the summer!

After lunch, the day concluded with a speaker: Richard Delaware, mathematics coordinator at the UMKC Mathematics and Physics Institute, whose topic was Fermats Little Theorem. Dr. Delawares talk was followed by presentations of awards.

For those chapters in attendance, I hope you had an enjoyable time. For those that could not come I would just say that we missed you and hope you will be able to attend the convention next year.

You May Be a Mathematician If ...

... you have ever constructed a regular 17-gon with compass and straightedge only.

... you videotaped the NOVA presentation on Fermat's Last Theorem, and watched it more than once.

... you've ever tried to prove Fermat's last Theorem.

... you know whether your date of birth 6-digit number (mmddyy) is a prime.

... you have ever spent more than two days working on the same problem.

... you have ever extracted a cube root with paper and pencil.

... you look at Pringle's potato chips and think of hyperbolic paraboloids.

... you can't tell the difference between a donut and a coffee cup.

... you've ever proved theorems on napkins during an after-dinner speech.

... you know who Paul Erdős was.

... you know the prime factorization of your social security number.

... you cringe every time you hear a sportscaster say "he's due."

... you've ever picked π when asked to choose a number between 1 and 10.

... you've ever worn the number $\sqrt{2}$ on your softball uniform.

... you've ever made up a cubic equation and solved it using Cardano's equations — just for the fun of it.

... you've ever tried to calculate the number of blades of grass in your yard.

... you've ever looked at a word problem and said "cool!"

... you know *why* July 4 is on Sunday next year when it was on Saturday this year.

... your clothes never match.

—the editor.

Report of the Great Lakes/New England Regional Convention

Prepared by Peter R. Skoner, PA Mu Chapter

The Great Lakes and New England Regions jointly held a convention on April 18, 1998 at Shippensburg, Pennsylvania, hosted by PA Iota at Shippensburg University. Located at the western edge of the New England Region and not far from the eastern edge of the Great Lakes Region, Shippensburg served as a central location for KME members from both regions. Coordinating the KME activities were Michael Seyfried, PA Iota corresponding secretary, Doug Ensley, PA Iota faculty member, Carol Harrison, New England regional director from PA Theta, and Peter Skoner, Great Lakes regional director from PA Mu.

The convention was held in conjunction with the Spring 1998 meeting of the EPADEL (Eastern Pennsylvania and Delaware) section of the Mathematical Association of America. KME members and others students attending were invited to two interesting MAA presentations including: "Strategic Aspects of Fair Division," by Alan D. Taylor of Union College; and "Puzzles and Paradoxes in Game Theory," by Ed Packel of Lake Forest College.

Student papers were presented in two sessions, one in the morning and one in the afternoon. The following KME student papers were given:

Tetris: A New Game, Possibly?
Kathleen Kelly, PA Iota, Shippensburg University

Mathematics is an MRI
Michael Wolfe, PA Iota, Shippensburg University

Music, the Harmony of Numbers
Joe Kloiber, NY Eta, Niagara University

A graphing calculator donated by Carol Harrison of PA Theta was awarded to Joe Kloiber of NY Eta for the best paper presented. The paper judging committee was chaired by Carol Harrison of PA Theta and included Amy Miko and Jason Green of PA Mu. Twelve KME members, including six students and six faculty, attended from four chapters (NY Eta, PA Theta, PA Iota, and PA Mu).

A special thank you is extended to PA Iota for hosting the event. Future opportunities for joint hosting will be pursued within the two regions.

Report of the South Central Regional Convention

Prepared by Dick Gibbs, CO Gamma Chapter

The 1998 South Central Regional KME Convention was held at Fort Lewis College in Durango, Colorado February 27–28. We had an excellent turnout, with the following attending chapters: CO Delta from Mesa State College, New Mexico Alpha from the University of New Mexico, Ohio Zeta from Muskingum College, Oklahoma Gamma from Southwestern Oklahoma State University, and Texas Iota from McMurry University.

The seniors in the Colorado Gamma chapter of Fort Lewis College participated in a poster session, with poster topics ranging from "The Golden Section and Mozarts Sonatas" to "A Continuous Version of Newtons Method." Jim Smith, a past national KME president, gave an exciting talk on integrating mathematics. The following student paper was also presented:

Predictive Factors for Final Grade Outcome:
Case Study of a UNM Math Course
Jennifer Gill, New Mexico Alpha, University of New Mexico

The members of the Colorado Gamma chapter would like to thank the participating chapters for attending the 1998 South Central Regional Convention.

Modulo Fermat

Editor's Comment: The following note appeared in the Fall 1966 issue of *The Pentagon*. It was submitted to the "Mathematical Scrapbook" column of that issue by R. S. Luthar, Waterville, Maine.

A specialized solution of Fermat's last theorem,

$$x^n + y^n = z^n,$$

is found if we consider the set $S = \{0, 1, 2, 3, 4, 5\}$ with multiplication and addition defined on it as ordinary multiplication and addition reduced modulo 6. It can now be verified that $3^n + 1^n = 4^n$, for $n \in S$ and $n \neq 0$.

Report of the Southeastern Regional Convention

Prepared by Pat Costello, KY Alpha and Gayle S. Kent,
FL Beta and Regional Director

The Southeastern Regional Convention was held February 27, 1998 in conjunction with the 14th Annual Symposium in Mathematics, Statistics and Computer Science at Eastern Kentucky University, Richmond, KY. Four KME chapters were in attendance: KY Alpha, KY Beta, TN Gamma and FL Beta. Over 115 people were present at the talks listed below. Unless indicated otherwise, all speakers are undergraduates and KME undergraduate participants are identified by chapter.

Smiths ala Yates
Lynne Brosius (graduate speaker), KY Alpha,
Eastern Kentucky University

The Driven Diode Resonator
Juraj Topolancik

Multiple Angle Formulae
Mike Carpenter

Distance Education and the Internet
Jason Nichols

Scout — A Java-based Infrastructure for Robots, Proxies, and Agents
A. Borchers (graduate speaker), R. Finkel, V. Marek,
J. Oldham, M. Truszczynski

The Challenge of Non-linear Dynamics
Dr. Grzegorz Swiatek (faculty speaker)

Intelligent Computation of Presentation Documents
Joseph D. Oldham (graduate speaker), Victor W. Marek,
Miroslaw Truszczynski

Phigits
Danny Thorne (graduate speaker, KME member)

Imaging and Multimedia: Concepts, Algorithms and Applications
Dr. W. Brent Seales (faculty speaker)

Motion Estimation from Compressed-Video Streams
Michael S. Brown (graduate speaker) and William Brent Seales (faculty)

The number "5"
Jet Chaw

Mathematical Solutions of the Rubiks Cube
Allison Smith, TN Gamma, Union University

The Formation of Rainbows
Jennifer Murrah, TN Gamma, Union University

Simple Programming on the HP48G Calculator
Gerald F. Calkin (faculty speaker)

The Continuum Hypothesis — Enfant Terrible in the Household of Mathematical Foundations
Tom M. Hughes (faculty speaker)

Awards were presented to student participants as follows:

First Place Graduate ($50): Anthony Borchers
Second Place Graduate ($25): Lynne Brosius
First Place Undergraduate ($50): Jennifer Murrah
Second Place Undergraduate ($25): Juraj Topolancik

Back Issues

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The Pentagon Business Manager
Division of Mathematics and Computer Science
Emporia State University
Emporia, KS 66801 USA

Math Joke

"Pa, how do you find the greatest common divisor?"
"Great scot! Ain't that been found yet?"

—*The Pentagon*, Fall 1950, p. 44

Kappa Mu Epsilon News

Edited by Don Tosh, Historian

News of chapter activities and other noteworthy KME events should be sent to Don Tosh, Historian, Kappa Mu Epsilon, Mathematics Department, Evangel College, 1111 N. Glenstone, Springfield, MO 65802, or to toshd@evangel.edu.

INSTALLATION OF NEW CHAPTERS

Alabama Eta

University of West Alabama, Livingston

The installation of the Alabama Eta chapter of Kappa Mu Epsilon was held on May 4, 1998 in the Livingston Community Center near the campus of The University of West Alabama. Dr. Eddy Joe Brackin, corresponding secretary of the Alabama Beta chapter, University of North Alabama, served as installation officer with Dr. Julia Massey, member of the mathematics faculty of the University of West Alabama, serving as conductor. Fourteen students and four faculty will be charter members of the new Alabama Eta Chapter. Those initiated were:

Students: Chris Bedwell, Carlos Chang, William Rex Edwards, III, George Genjei, Jonathan Germany, Leroy Griffith, Evelyn B. Jones, Kenny Middlebrooks, Jason Overstreet, Edward Senu-Oke, Jamie L. Shutt, Justin L. Smith, Craig P. Williams, and James E. Zimlich.

Faculty: Louise M. Boyd, Julia E. Massey, G. Michael Reekie, and Micky Smith.

Preceding the installation ceremony, Dr. Brackin gave a brief history of Kappa Mu Epsilon and shared experiences he and his students have enjoyed in attending many national conventions of the society.

Officers installed during the ceremony were: Justin L. Smith, president; James E. Zimlich, vice president; Jamie L. Shutt, recording secretary; Jason L. Overstreet, treasurer; G. Michael Reekie, corresponding secretary; Julia E. Massey, sponsor.

Georgia Beta

Georgia College and State University, Milledgeville

The installation of the Georgia Beta chapter of Kappa Mu Epsilon was held on April 25, 1998, in the Maxwell Student Union on the campus of Georgia College and State University. Dr. Joe Sharp, corresponding secretary of the Georgia Alpha chapter of KME at the State University of West Georgia, conducted the installation ceremony. Dr. Hugh Sanders served as the conductor during the ceremony. There were 28 charter members of Georgia Beta:

Initiates: Gerald W. Adkins, Michelle C. Blay, Wilfred Boykin, Mae Carpenter, David J. DeVries, Keith M. Ellenberg, Elizabeth Farmer, Russell D. Herron, Deborah Hutcherson, Peter Jarvis, Mohammad Kahkesh, Joy Keegan, Robert Keegan, Zhe Li, Stacy L. McCoy, Lynne Melder, Robert J. Mitchell, Jonathan Potter, James Powers, Mary Pratt-Cotter, Johnny Preyer, Jr., John S. Robertson, Hugh A. Sanders, Robert Savona, Paul Schuette, Craig Turner, R. Scott Wynn, and J. F. Yao.

Following the 6:30 p.m. installation ceremony, a banquet was held in honor of the charter members of the Georgia Beta chapter. After the banquet, Dr. Hugh Sanders, the faculty sponsor of Georgia Beta, and Dr. John S. Robertson, chairman of the Department of Mathematics and Computer Science at GC&SU, both made some brief remarks to the guests. Afterwards, Dr. Joe Sharp gave a brief history of the founding of Kappa Mu Epsilon on the national level as well as some of his experiences as the corresponding secretary and faculty co-advisor of the Georgia Alpha chapter of KME at the State University of West Georgia in Carrollton for the last 22 years.

Officers installed during the ceremony were: Robert Savona, president; Robert Mitchell, vice president; Jon Potter, recording secretary; Lynne Melder, treasurer; Craig Turner, corresponding secretary; Hugh Sanders, faculty sponsor.

Kansas Zeta

Southwestern College, Winfield

The installation of the Kansas Zeta chapter of Kappa Mu Epsilon took place on Tuesday, April 14, 1998 in the mathematics lab in the Christy Administration Building on the campus of Southwestern College. Bryan Dawson, Kansas Beta, editor of *The Pentagon*, served as the installing officer. Mehri Arfaei, a faculty member of Southwestern College, served as the conductor. Six students and two faculty constituted the charter members of the new chapter at Southwestern College. Those initiated were:

Students: Jennifer Hannah Benevento, Travis Wayne Ethridge, Carol Black, Thyrza Loyde Mucambe, Jana Wallace, and Deanna Pennington.

Faculty: Reza Sarhangi and Mehri Arfaei.

Two additional students and one more faculty member were unable to attend and will be initiated at a later date.

Following the 11:00 a.m. installation ceremony, Dr. Dawson gave a brief history of Kappa Mu Epsilon and *The Pentagon*. Much enjoyable discussion followed afterwards. Officers installed during the ceremony were Travis Ethridge, president; Thyrza Mucambe, vice president; and Carol Black, secretary. Faculty member Mehri Arfaei accepted the responsibility of corresponding secretary, and Mehri Arfaei and Reza Sarhangi jointly accepted the responsibility of faculty sponsor.

Michigan Epsilon

Kettering University, Flint

The installation of the Michigan Epsilon chapter of Kappa Mu Epsilon was held on March 28, 1998, on the campus of Kettering University. Prior to its name change in January, 1998, Kettering University was known as General Motors Institute. Welcoming remarks were given by Professor David Green, Jr., Mathematics Department head. Dr. Arnold Hammel, past national president of Kappa Mu Epsilon, conducted the installation ceremony. Professor Brian McCartin of Kettering University served as the conductor. Professor McCartin and other members of the Mathematics Department were responsible for arranging the ceremony. Forty-six students and sixteen faculty constituted the founding group of the new chapter at Kettering University. Five of the faculty are members of MKE from their student days. Those initiated included:

Students and faculty: Craig Andres, Joel Austin, Rachael Ayotte, Aziz Bakar, Steven Bates, Kevin Beardsley, Mark Brummel, Andrew Buske, Srinivas Chakravarthy, Patrick Chapman, Stefanka Chukova, Mark Colwell, Tzvetelina Dimitrova, Boyan Dimitrov, Matthew Drane, Derek Fisackerly, Michael Fisackerly, Todd Foster, Stefano Giacomantonio, Daniel Girdwood, David Green Jr., Jonathan Griggs, James Hendrickson, Danny Holmquist, Sheri Houston, John Hume, Katie Jiang, Thomas Joseph III, Erin Kennedy, Ilya Kudish, Suzanne Labadie, Brian McCartin, Duane McKeachie, Robert Mitchell, Jeffrey Morris, Daryl Nolt, Joel Pfauth, Jeremy Plenzler, Martin Przyjazny, Jamie Quaderer, Brian Rathgeb, Philip Richard Jr., Robert Riley, Mark Sander, Roy Schweitzer, Jason Sobick, Grzegorz Slota, Corey

Stefanczak, Michael Szczesniak, Kevin TeBeest, Roy Thorsell, James Tornincasa, Robin Vacha, Eric White, Robert White, Joshua Zets, and Jason Zink.

Faculty initiated earlier at other chapters were G. Reginald Bell, Richard Bolander, Doyle McOwen, Jo Smith, and William Webster.

Since Kettering University students alternate between school and co-oping through successive terms, two sets of local officers were installed during the ceremony. One set was presently on campus but leaving for co-op that day (these officers participated in the installation ceremony) while the second set returned to campus from their co-op that day. Officers installed during the ceremony were Michael Fisackerly and Suzanne Labadie, president; Joel Pfauth and Joel Austin, vice president; Derek Fisackerly and Rob Riley, secretary; Jeremy Plenzler and Dan Holmquist, treasurer. Faculty member Jo Smith accepted the responsibilities of corresponding secretary and Brian McCartin will serve as faculty sponsor.

After the installation ceremony, Dr. Hammel gave an introduction to and brief history of Kappa Mu Epsilon. A delicious buffet luncheon was then served. Following the luncheon, an interesting, entertaining and much enjoyed keynote lecture was given by Professor Ed Aboufadel of Grand Valley State University. His title was "A Mathematician Catches A Baseball." Professor Brian McCartin concluded the ceremony with closing remarks.

Missouri Mu

Harris-Stowe College, St. Louis

The installation of the Missouri Mu chapter of Kappa Mu Epsilon was held on Saturday, April 25, 1998 in the cafeteria of Harris-Stowe College in St. Louis. Don Tosh of Missouri Theta and national historian, served as the installing officer. Jack Behle, department chairman at Harris-Stowe and a member of KME, served as conductor. Twenty students and two faculty members were initiated, giving Missouri Mu a solid base of support for its program. Ann Podleski deserves credit for the substantial time and effort she spent organizing the installation. After the installation, refreshments were served and the audience offered congratulations. Those initiated included:

Students: Becky Boehmer, Russell Buchek, Karla Conrey, Lynn Conway, Karen Foley, David Fox, Scott Goodrich, Matthew Haack, Patricia Hose, Deborah Kentch, Art McCoy, Cheryl Moonier, Michael Orr, Jamar Scott, Tjuannia Seals, Glover Stewart, Amy Swartman, Linda Thatcher, Arthur Thomas, and Rega Wesley.

Faculty: Ann Podleski and Terry Werner.

New York Xi

Buffalo State College, Buffalo

The installation of the New York Xi chapter of Kappa Mu Epsilon was held on May 12, 1998, at the Blackthorn Restaurant in Buffalo, New York. Dr. Robert L. Bailey, national president-elect of Kappa Mu Epsilon, conducted the installation ceremony. Dr. Robin Sanders of the Department of Mathematics at Buffalo State served as conductor during the ceremony. Twenty-one students and one faculty constituted the founding group of the new chapter at Buffalo State College. Those initiated were:

Students: Heather S. Krysty, Julie A. Zybert, Craig M. Dana, Brian M. Fischer, Kimberly A. Smyntek, Thomas Donofrio, Michelle Miszewski, Christopher M. Watson, Shannon L. Wise, Paul R. Hosking, Jamie Dimitri, Andy Bowen, Marie L. Pluchino, Scott M. Gizzi, Andy Cudy, Laura L. Zlotek, Holly Mascia, Janine L. Jakubowshi, Christina M. DeMunda, Sadeq A. Elbaneh, and Timothy M. Frawley.

Faculty: Robin S. Sanders.

The installation ceremony was preceded by a banquet at 6:00 p.m. Officers installed during the ceremony were Chris Watson, president; Michelle Miszewski, vice president; Jamie Dimitri, secretary; and Andy Bowen, treasurer. Faculty member Robin Sanders accepted the responsibilities of both corresponding secretary and faculty sponsor.

Tennessee Epsilon

Bethel College, McKenzie

The installation of the Tennessee Epsilon chapter of Kappa Mu Epsilon was held on April 16, 1998, in the Oasis Room of the Student Center Building on the campus of Bethel College. Dr. Matt Lunsford, corresponding secretary of Tennessee Gamma, served as the installation officer and Professor Russell Holder of Bethel College served as the conductor for the installation ceremony. Six students and three faculty members constituted the charter members of the new chapter at Bethel College. Those initiated were:

Students: Jennifer M. Dowdy, Christina Hill, Jonathan Lankford, Melody S. Rush, Belinda Thompson, and James Wiggleton.

Faculty: Russell Holder, Roger C. Johnson, and David Lankford.

The evening began with a banquet at 6:00 p.m. The installation ceremony began at 6:45 p.m. Following the ceremony, Dr. Lunsford made some brief remarks about Kappa Mu Epsilon and its role at his institution.

Officers installed during the ceremony were Jennifer M. Dowdy, president; Jonathan Lankford, vice president; Melody S. Rush, recording secretary; Jamie Wiggleton, treasurer. Faculty members Russell Holder and David Lankford were installed as the chapter's corresponding secretary and faculty sponsor, respectively.

CHAPTER NEWS

AL Beta Chapter President — Caacie Brown

University of North Alabama, Florence 24 actives, 8 associates

Other 1998–99 officers: Adrienne Hackworth, vice president; Corey Tays, secretary; Eddy Joe Brackin, corresponding secretary; Patricia Roden, faculty sponsor.

AL Gamma Chapter President — Dorothy Gearheart

University of Montevallo, Montevallo 15 actives, 11 associates

Other 1998–99 officers: John Woodruff, vice president; Pauline Kennard, secretary; Genger Hand, treasurer; Larry Kurtz, corresponding secretary; Michael Sterner, faculty sponsor.

AL Zeta Chapter President — Melissa Boren

Birmingham Southern College, Birmingham 28 actives

A program was presented on Applied Statistics in Graduate School by faculty from the University of Alabama. Members helped with the "Math Derby" at Woodrow Wilson Elementary School. Melissa Boren received the KME award at the Honors Day Convocation. Former KME member Rajiv Gala, now in medical school, gave a talk on applications of math to medicine. Other 1998–99 officers: Melanie Styers, vice president; Shelley Moor, secretary/treasurer; Mary Jane Turner, corresponding secretary; Shirley Branan, faculty sponsor.

CO Gamma Chapter President — Heather Duncan

Fort Lewis College, Durango 15 actives, 2 associates

Other 1998–99 officers: Cyndie Hilliker, vice president; David Crawford, secretary; Travis Kirkpatrick, treasurer; Deborah Berrier, corresponding secretary; Debbie Berrier, faculty sponsor.

CO Delta
Chapter President — John D. Bright

Mesa State College, Grand Junction
32 actives, 12 associates

A chapter meeting was held at Old Chicago Restaurant in January to discuss plans for attending the 1998 regional convention. No students were interested in going. The corresponding secretary did attend the regional convention at Fort Lewis College in February. Three members (Robin O'Connor, Adam Furst, and John Bright) participated as a team in the COMAP Mathematical Modeling Contest. Both Dan Carroll and Adam Furst presented sessions at the college Brown Bag Seminar series in mathematics. The annual banquet and initiation ceremony were held on April 9, 1998. Thirty-five members, initiates, and guests were in attendance. New initiate Phillip Upton received the Tom Mourey Computer Science/Mathematics Award. Other 1998–99 officers: Amanda H. Widel, vice president; Sarah L. Kennedy, secretary; David G. Wing, treasurer; Donna K. Hafner, corresponding secretary; Kenneth S. Davis, faculty sponsor.

FL Beta
Chapter President — Cindy Chastain

Florida Southern College, Lakeland
25 actives, 5 associates

Florida Beta Chapter members attended the regional KME meeting at Eastern Kentucky University. Other 1998–99 officers: Chris Scofield, vice president; Danny Koury, secretary; Susan Rinker, corresponding secretary; Susan Rinker, faculty sponsor.

GA Alpha
Chapter President — Nancy Bryson

State University of West Georgia, Carrollton
20 actives, 8 associates

On May 25, 1998, the Georgia Alpha chapter met and initiated 8 new members. We elected chapter officers for 1998–1999. Following the initiation, a reception was held in honor of the new members and publicity photos were taken of the new members. Other 1998–99 officers: Tonya McElwaney, vice president; Nancy Boyette, secretary; Roger Huffstetler, treasurer; Joe Sharp, corresponding secretary; Mark Faucette and Joe Sharp, faculty cosponsors.

IL Theta
Chapter President — Kris Frees

Benedictine University, Lisle
22 actives, 15 associates

At this year's Math Banquet, 14 students and one faculty member were inducted into Illinois Theta. Faculty Dr. Wangler and Dr. Cicero gave an impressive presentation on GPS, the Global Positioning Satellite system. In addition to helping with department activities such as placement exams and a high school contest, KME members were active in a local Calculus Competition and gathered to watch the NOVA special on Andrew Wiles and Fermat's Last Theorem. Other 1998–99 officers: Geoffrey Pacana, vice president; Teresa Barlow, secretary; Lisa Townsley Kulich, corresponding secretary/faculty sponsor.

IA Alpha

Chapter President — Suzanne Shontz

University of Northern Iowa, Cedar Falls — 36 actives

Students presenting papers at local KME meetings incuded Christopher Geerts on "Probability and the NBA Draft Lottery" and Giao Vu on "Fractal Curves." Mary Noga addressed the spring intiation banquet on April 30, 1998 on "Bigger, Better Power Ball." Eight new initiates joined KME that evening. KME students Suzanne Shontz and Marc Pedersen along with faculty members John Cross and Mark Ecker attended the Region IV KME Convention at William Jewell College in Liberty, MO on April 3–4 where Suzanne presented her paper on "A is NOT for Achievement." Other 1998–99 officers: Marc Pedersen, vice president; Sarah Lacox, secretary; Erin Blaine, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

IA Delta

Chapter President — Emily Bailey

Wartburg College, Waverly — 59 actives

Our January meeting was a pizza party at the home of faculty sponsor Dr. Glenn Fenneman. The February meeting involved preliminary planning for attending the regional meeting at Western Illinois University. For our program, Andy Miller reported about his Fall Term in Budapest, Hungary, as a student in the Budapest Semesters in Mathematics Program. On March 13 and 14, the chapter co-sponsored the Explorations in Mathematical Sciences event on our campus for high school juniors and seniors. At our initiation banquet on March 14, we received thirteen new members and heard Pat Glawe, a 1989 Wartburg graduate, KME member, and Fellow in the Society of Actuaries speak on the topic "Actuarial Science: One Mathematical Possibility." Our last event of the school year was a picnic on May 14. Other 1998–99 officers: Christine Morrissey, vice president; Joel Nelson, secretary; Keith Cummer, treasurer; August Waltmann, corresponding secretary; Mariah Birgen, faculty sponsor.

KS Alpha

Chapter President — Mark Albert

Pittsburg State University, Pittsburg — 45 actives, 8 associates

The spring 1998 semester activities began in January with a pizza party, initiation, and special speaker Professor Larry Claypool, head of the Statistics Department at Oklahoma State University. The second meeting of the semester included the viewing of the film "The Proof." At the next meeting, KME member Kathy Denney gave a trial presentation of her paper "Princess Diana, Paul Revere and Group Theory?," which was awarded one of the top three papers at the regional convention. Nine students and two faculty from Kansas Alpha attended the regional convention held at

William Jewell College in Liberty, MO. Four of the students presented papers: Mark Albert, Kathy Denney, Mandy Fritz, and Allison Willson. In April, the chapter assisted the Mathematics Department faculty at the annual Pitt State Math Relays by administering and grading exams. Also, several members called for the PSU Foundation annual phonathon to help raise money. The final meeting of the semester was an ice cream social held at the home of Professor Gary McGrath. Officers for the 1998–99 school year were elected. Also, Professor Harold Thomas was presented with a plaque in appreciation for his many years of dedicated service to the Kansas Alpha chapter of KME. Other 1998–99 officers: Kathleen Denney, vice president; Kari Hamm, secretary; Lisa Swain, treasurer; Cynthia Woodburn, corresponding secretary; Yaping Liu, faculty sponsor.

KS Beta Chapter Co-Presidents — Rae Ann LeValley, Kristen Goetz
Emporia State University, Emporia 15 actives, 5 associates

Our spring semester initiation was held on March 16, 1998. We initiated five new members and gave out an annual high school math and/or computer science teacher of the year award. This award is sponsored by our mathematics department. This semester several of our members attended conferences. Some went to the regional KME conference in Missouri where Melanie Kurtz presented a paper on the history of mathematics. Also, that same weekend several members attended the national NCTM meeting in Washington, D.C. Officer elections for the 1998–99 school year were held at our end-of-the-year barbecue April 20. Other 1998–99 officers: Megan Little, treasurer; Connie Schrock, corresponding secretary; Larry Scott, faculty sponsor.

KS Gamma Chapter President — Kevin Slattery
Benedictine College, Atchison 8 actives, 15 associates

Spring semester culminated in a "banner year" for KS Gamma. Three CS students placed in the top five with their homepage contest entry called "Rarities in Our Midst" which gives information on the rare books in our campus library. Three math students received a "meritorious" rating for their entry on "Grade Inflation" in the COMAP Modeling Contest. Jeff Blanchard placed in the top three for his presentation at the regional KME meeting at William Jewell on 4 April. Finally, at the 22 April Honors Convocation moderator Jo Ann Fellin, OSB was named Distinguished Educator of the Year. That same evening Kevin Slattery was presented the Sister Helen Sullivan Scholarship by KS Gamma. At the spring MAA Kansas Section Meeting in Topeka, Sister Jo Ann gave a 30 minute presentation on "A Mathematical Journey: The History of the Math Department at Mount St. Scholastica College 1923–71" based upon the 105-page account which she completed during her sabbatical last year. KS Gamma archives provided a rich source for this history. On 30 April, KS Gamma honored the math/CS graduates with a steak picnic at the home of Richard Farrell. The chapter extends thanks to Richard Farrell for his hospitality. The chapter earlier extended sympathy to Farrell on the sudden death of his wife on 5 March. Other 1998–99 officers: Jo Ann Fellin, corresponding secretary/faculty sponsor.

KS Delta Chapter President — Laurie Payeur

Washburn University, Topeka 30 actives

Kansas Delta met twice during the semester for luncheon meetings in conjunction with the department math club, Mathematica. There were short student presentations. Attending the April regional KME convention at MO Gamma, William Jewell College, with faculty Al Riveland and Donna LaLonde, were students Doug Appenfeller, Stephanie Adelhardt, Mandy Chester, Milorad Felbapov, Justin Freeby, Elena Kanaeva, Laurie Payeur, and Chung-Fei Tang. Donna Lalonde and Doug Appenfeller were official judges for the student presentations. Other 1998–99 officers: Stephanie Lambert, vice president; Justin Freeby, secretary/treasurer; Allan Riveland, corresponding secretary; Donna LaLonde and Ron Wasserstein, faculty sponsors.

KS Epsilon Chapter President — Mariam Riazi

Fort Hayes State University, Hayes 20 actives, 5 associates

The chapter meets monthly. Usually a speaker or film is shown, refreshments are served, and business is conducted. Other 1998–99 officers: B. J. Peterson, vice president; Emily McDonald, secretary/treasurer; Chenglie Hu, corresponding secretary; Linda Kallam, faculty sponsor.

KY Alpha Chapter President — Elizabeth Barrett

Eastern Kentucky University, Richmond 43 actives, 18 associates

The spring semester began with floppy disk sales (together with the ACM chapter) to students in the computer literacy class and the Mathematica class. A meeting in January made plans for publicizing and hosting a regional convention. On February 27, three other chapters were able to attend the day of talks at the Symposium/Regional Convention. Florida Beta was able to stay and join us at Arlington House for a banquet. Dr. Costello gave a brief talk entitled "Puzzles and Some People Who Solve Them." On March 9, there were eighteen students initiated as members. Dr. Kirk Jones gave an interesting talk with lots of pictures entitled "Continuous, Nowhere Differentiable Functions." A meeting in April included new officer nominations and plans for an end-of-the-year picnic in the Ravine. On the day of the picnic, when rain threatened, the picnic was moved to the lobby of Sullivan Hall where everyone got plenty to eat. During Math Awareness

Week, there was a bake sale. Other 1998–99 officers: David Curd, vice president; Tina Jordan, secretary; Jeremy Miller, treasurer; Patrick Costello, corresponding secretary.

KY Beta Chapter President — Story Robbins

Cumberland College, Williamsburg 36 actives

In February, the Kentucky Beta chapter participated in a campus spirit contest at a basketball game, and won! On February 27, 1998, five student members and two faculty members attended the Kappa Mu Epsilon regional convention at Eastern Kentucky University. On March 6, the Kentucky Beta chapter held an initiation and banquet at the Atrium for nine new student members. Members inducted last year and graduating seniors were also recognized during the banquet, presided over by outgoing president, Story Robbins. Jointly with the Mathematics and Physics Club, the Kentucky Beta Chapter hosted Dr. Carroll Wells from Western Kentucky University on April 30. He spoke on topology. On May 1, members also assisted in hosting a regional high school math contest, held annually at Cumberland College. On May 4, the entire department, including the Math and Physics Club and the Kentucky Beta chapter, held the annual spring picnic at Briar Creek Park. Other 1998–99 officers: Candace Osborne, vice president; Laura Thompson, secretary; Melynda Hazelwood, treasurer; Jonathan Ramey, corresponding secretary; John Hymo, faculty sponsor.

MD Alpha Chapter President — Judith Simon

College of Notre Dame of Maryland, Baltimore 7 actives, 3 associates

At the Annual Induction Dinner held on May 4 five students and one associate were inducted. The speaker was Dr. Margaret Sullivan, who spoke on the topic "Qualitive Graphs: Confronting Misconceptions." Other 1998–99 officers: Marie Morrow, vice president; Michelle Yeager, secretary; Laura Bopp, treasurer; Sister Marie Augustine Dowling, corresponding secretary; Joseph DiRienzi, faculty sponsor.

MD Beta Chapter President — Jason Barr

Western Maryland College, Westminster 33 actives, 1 associate

Five members were inducted at a Career Night dinner for all mathematics majors. Three math alumni came to discuss job opportunities in mathematics and computer science. We had a record attendance of 30 people. A dessert reception was held to honor Dr. James Lightner, past national president of KME, who founded the Maryland Beta chapter, and is retiring after teaching here for 35 years. We also held an end-of-year picnic for all mathematics majors. Our members participated in the college's Spring Fling Weekend by selling pretzels. Other 1998–99 officers:

Robert Newman, vice president; Julie Brown, secretary; Fred Butler, treasurer; Linda Eshleman, corresponding secretary; James Lightner, faculty sponsor.

MD Delta Chapter President — Heidi Femi

Frostburg State University, Frostburg 36 actives

The semester began with an orientation meeting for new initiates and initiation was held March 8. At the initiation, university provost Christine Grontkowski gave a talk on "Plato's Mathematical Imagination." Later in the semester the members designed and ordered KME t-shirts, and also set up a booth at the university's "Spring Splash" featuring activities for children from the local community. Other 1998–99 officers: Steven Fairgrieve, vice president; Sean Carley, secretary; Andrew Adam, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

MA Alpha Chapter President — Tammy Ives

Assumption College, Worcester 11 actives

Nine new members were initiated on April 30, 1998. Following a dinner in honor of the new initiates, student member Jonathan Jankowski spoke on "Explorations of the Ramsey Number $R(3,3,3,3)$." Other 1998–99 officers: Shelley Kijek, vice president, Cara Lambert, secretary; Charles Brusard, corresponding secretary/faculty sponsor.

MS Gamma Chapter President — Jason Haight

University of Southern Mississippi, Hattiesburg 20 actives, 5 associates

Other 1998–99 officers: Paula Thigpen, vice president; Adrienne Davis, secretary; Alice Essary, treasurer/corresponding secretary; Barry Piazza, faculty sponsor.

MS Epsilon Chapter President — Ashley Riley

Delta State University, Cleveland 16 actives

Other 1998–99 officers: Ken Byars, vice president; Chad Huff, secretary/treasurer; Paula Norris, corresponding secretary; Rose Strahan, faculty sponsor.

MO Alpha Chapter President — Lisa Burger

Southwest Missouri State University, Springfield 12 actives, 5 associates

During the spring semester the Missouri Alpha chapter initiated 5 new members and held three meetings, which included two student speakers. Two students and one faculty attended the regional convention at William Jewel College. A faculty and a student participated in the judging of the papers presented at that meeting. Other 1998–99 officers: Mirian Ligon, vice president; Jessica McDonnell, secretary; Katherin Puetz, treasurer;

John Kubicek, corresponding secretary; Yungchen Cheng, faculty sponsor.

MO Beta — Chapter President — Dennis Wise

Central Missouri State University, Warrensburg — 25 actives, 8 associates

The Missouri Beta chapter of KME met monthly during the spring semester. In January, Dr. McKee gave a presentation titled "Some Simple Math Behind Computer Animation." In February, Dr. Cooper gave a talk on "Sums of Large Digit Functions and Digital Sum Functions." Inititation was held in March with five people being initiated. At that meeting, the group also watched half of the video "The Proof," about Andrew Wiles and his proof of Fermat's Last Theorem. Elections, a pizza party and the rest of "The Proof" were on the agenda in April. Many students volunteered to help with Math Relays in March and four students and one sponsor attended the regional convention at William Jewell College in Liberty, MO in April. Other 1998–99 officers: Tammy Surfus, vice president; Aaron Shaefer, secretary; Cassie Young, treasurer; Melissa Elliott, historian; Rhonda McKee, corresponding secretary; Scotty Orr, Larry Dilley, and Phoebe Ho, faculty sponsors.

MO Gamma — Chapter President — Jennifer Puls

William Jewell College, Liberty — 23 actives, 8 associates

The officers and members of the Missouri Gamma chapter held regular Wednesday evening help sessions for students the entire year. More than ten of the members participated at one time during the year. Sign-up sheets were posted for members to fill in, showing who would be responsible for the help session on a given night. The chapter sponsored the Region IV convention of Kappa Mu Epsilon, April 3 and 4, 1998. Thirteen chapters were represented. The Spring Banquet and initiation was held on March 24, 1998. Our speaker at the banquet was Dr. Larry Campbell of Southwest Missouri State University. Eight new members were inducted. The chapter obtained the video "The Proof" from WGBH-TV, and viewed it, along with having pizza, for its last meeting of the year. Other 1998–99 officers: Allison Cooper, vice president; James Brochtrup, secretary; Joseph Mathis, treasurer/corresponding secretary/faculty sponsor.

MO Epsilon — Chapter President — David Bates

Central Methodist College, Fayette — 12 actives, 3 associates

Other 1998–99 officers: Christina Miller, vice president; Sheryll Rector, secretary; William McIntosh, corresponding secretary; Linda Lembke and William McIntosh, faculty sponsors.

MO Zeta — Chapter President — Tanya Peters

University of Missouri—Rolla, Rolla — 7 actives, 5 associates

Other 1998–99 officers: Joe Morton, vice president; Andrew Van Brunt, secretary; Gretchen Schmeling, treasurer; Roger Hering, corresponding secretary; Ilene Morgan, faculty sponsor.

MO Eta Chapter President — Laurel Berner

Truman State University, Kirksville 10 actives, 4 associates

Meetings are held on the first Monday of every month. In January we made preparations for the expo. In February we hosted a Math Expo for local high schools. In March we had a bake sale. In April we attended the regional conference in Liberty, MO, and had elections and a picnic. Other 1998–99 officers: Amanda Nixon, vice president, Angela Kell, secretary; Ann Herberholt, treasurer; Mary Sue Beersman, corresponding secretary; Jay Belanger, faculty sponsor.

MO Theta Chapter President — Jeremy Osborne

Evangel University, Springfield 8 actives, 2 associates

Meetings were held monthly. New officers were elected in January. Don Tosh hosted a party in his home in March. Three members and one pledge were able to attend the regional convention on April 3rd and 4th at William Jewell. Attendees were Christie Tosh DeArmond, Mandy Wilson, Amanda Wachsmuth, and faculty member Don Tosh. Other 1998–99 officers: Mandy Wilson, vice president; Don Tosh, corresponding secretary/faculty sponsor.

MO Iota

Missouri Southern State College, Joplin 20 actives

Eight new members were inducted into the chapter in early April. A delegation of five, including two of the new members, attended the regional convention hosted by MO Gamma at William Jewell College in Liberty. In late April a demonstration was given by chapter officers Chris Baker and Agdon Brister on physical interpretations of solutions of differential equations. A volleyball game and pizza party at the home of Mrs. Mary Elick closed out a successful semester. Chapter officers will be elected in the fall. Other 1998–99 officers: Mary Elick, corresponding secretary; Charles Curtis, faculty sponsor.

MO Lambda Chapter President — Perriann McCoppin

Missouri Western State College, St. Joseph 37 actives

The MO Lambda chapter initiated seven new members on March 1. Guest speaker for the occasion was Dr. Tim Miller. Other spring activities included attending the Region IV Convention held at William Jewell College in Liberty, assisting with the Missouri Academy of Science meeting held on campus, conducting a bake sale, celebrating the end of the semester with a cookout held at Dr. Atkinson's home, and electing officers for the '98–99 academic year. Other 1998—99 chapter officers: Stephanie Tingler, vice president; William Slabaugh, secretary; Sean Hutto, treasurer; John Atkinson, corresponding secretary; Jerry Wilkerson, faculty sponsor.

NE Alpha Chapter President — Stacy Olmer

Wayne State College, Wayne 44 actives, 19 associates

On April 23 the club hosted the math/science banquet, and we awarded a $25 book scholarship to Stacy Olmer. Other 1998–99 officers: Emily Negus, vice president; David Pease, secretary/treasurer; Matt Jansen, historian; John Fuelberth, corresponding secretary; Jim Paige, faculty sponsor.

NE Gamma Chapter President — Andy Boell

Chadron State College, Chadron 12 actives, 4 associates

Other 1998–99 officers: Shaun Daugherty, vice president; Craig Bruner, Jr., secretary; Erin Johnson, treasurer; Jim Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

NE Delta Chapter President — Ryan Shanesy

Nebraska Wesleyan University, Lincoln 22 actives, 9 associates

Other 1998–99 officers: Jacquie Neuhaus, vice president; Ben Darbro, secretary/treasurer; Gavin LaRose, corresponding secretary/faculty sponsor.

NM Alpha Chapter President — Jason Strauch

University of New Mexico, Albuquerque 80 actives, 12 associates

Other 1998–99 officers: Jennifer Gill, vice president; Walter Kehowski, secretary; Merlin Decker, webmaster; Archie Gibson, corresponding secretary/faculty sponsor.

NY Alpha Chapter President — Vinod Gulani

Hofstra University, Hempstead 7 actives, 8 associates

We had a faculty/student volleyball game and sponsored a talk "Polyhedra and Art" by Professor George W. Hart. Other 1998–99 officers: Angela Boccio, vice president; JoAnne Taormina, secretary; Bill D'Angelo, treasurer; Aileen Michaels, corresponding secretary/faculty sponsor.

NY Eta Chapter President — Stacey Lauricella

Niagara University, Niagara 35 actives, 9 associates

A Career Day for students was held on April 16. Rebecca Bauer, a graduate of Niagara and recent recipient of a master's degree from Rochester Insitute of Technology, spoke to the group about job opportunities in the field of industrial engineering. A good discussion concerning students' future plans followed the talk. The KME Combined Great Lakes and New England Regional Convention held at Shippensburg University, PA, April

18, was attended by Joseph Kloiber, student, and Robert Bailey, corresponding secretary. Joe gave a paper on music and mathematics and was judged to have given the best presentation as a KME participant. Joe's prize was a graphing calculator. Other 1998–99 officers: Jennifer Egan, vice president; Lara Brown, secretary; Leslie Good, treasurer; Robert Bailey, corresponding secretary; Kenneth Bernard, faculty sponsor.

NY Kappa — Chapter President — Kyle Hill

Pace University, New York — 20 actives, 6 associates

Other 1998–99 officers: Geraldine Taiani, corresponding secretary; Wesley Jordan, faculty sponsor.

NY Lambda — Chapter President — Loriann Loraia

C.W. Post Campus of Long Island University, Brookville — 21 actives

Five new student members were initiated by the chapter officers during our annual banquet at the Greenvale Town House restaurant on the evening of March 26th, bringing the chapter membership to 182. Our guest of honor was Dr. Sharon Kunoff, the founder of our chapter and the former business manager of *The Pentagon*, who retired last summer. The evening began with remarks by Dr. Neo Cleopa, who presented gifts from the department to Dr. Kunoff and to George Schiro (who retired at the end of the Fall 1997 semester) and also introduced Dr. Jozsef Losonczy, who will join our department next fall. After dinner, Dr. James Peters introduced Myleen Rojano, member #114, who spoke on "Life After Graduation" and recounted her career so far at New York Life Insurance. Our evening concluded with the announcements by Dean Paul Sherwin of the departmental awards for 1997–98 (the Lena Sharney Memorial Award to Suzann Weaver, the Joseph Panzeca Memorial Award to Loriann Laraia, the Claire Adler Award to Loriann Laraia and Suzann Weaver, and the Hubert Huntley Memorial Award to Jill Kahan) and by Dr. Maithili Schmidt-Raghavan of the Dean Maithili Schmidt Scholarhip Award for the outstanding junior in the College of Arts and Sciences (to Jill Kahan). Alumni Colin Grimes (member #141) and Jean Jerome (member #58) also attended. Other 1998–99 officers: Jill Kahan, vice president; Nicole Garofalo, secretary; Joseph Sprague, treasurer; Andrew Rockett, corresponding secretary.

OH Gamma — Chapter President — Amy Booth

Baldwin-Wallace College, Berea — 26 actives, 5 associates

Mathematics luncheon videos with pizza/pop for sale were held biweekly during the Winter and Spring quarters. We sponsored a talk by a B-WC alum who is now completing a Ph.D. at the University of Chicago. Our initiation was held on 5/9/98. Other 1998–99 officers: Cassandra Kirby, vice president; Margot Mailloux, secretary; Anila Xhunga, treasurer; David Calvis, corresponding secretary; David Calvis and Chungsim Han, advisors.

OK Alpha

Chapter President — Josh Baker

Northeastern State University, Tahlequah — 36 actives, 4 associate

The initiation of our eight students and one mathematics faculty member was held in the banquet room of Roni's Pizza. It was well attended by faculty and students. We continue to have joint activities with NSU's student chapter of the MAA and participate in "The Problem Solving Competition" sponsored by the MAA. We met regularly to solve mathematics problems. Melinda Weigle and Tina Wolfe represented our chapter in the Campus Christian Fellowship Canoe Race Challenge down the Illinois River. We celebrated Math Awareness Week with several activities. Brandy Johnson, a member of the Oklahoma Alpha chapter, gave a preview of her presentation at the National Conference on Undergraduate Research to students and faculty at NSU. Her research was on "Optically Stimulated Ionic Conductivity in Glass." We also sponsored the annual KME "Pre-finals Ice Cream Social" for faculty and math majors. Other 1998–99 officers: Dan Sisk, vice president; Tera McGrew, secretary; Tracey McCutchen, treasurer; Joan Bell, corresponding secretary/faculty sponsor.

OK Delta

Chapter President — Daniel Gregory

Oral Roberts University, Tulsa — 5 actives, 10 associates

Other 1998–99 officers: Vidar Ligard, vice president; Angela Golberg, secretary/treasurer; Vincent Dimiceli, corresponding secretary/faculty sponsor.

PA Alpha

Chapter President — Shannon Mack

Westminster College, New Wilmington — 8 actives, 13 associates

Other 1998–99 officers: Stephanie Tangora, vice president; Dena Streit, secretary; Michael Leiper, treasurer; Warren Hickman and Carolyn Cuff, corresponding secretaries.

PA Gamma

Chapter President — Angela Colinet

Waynesburg College, Waynesburg — 16 actives, 4 associates

Other 1998–99 officers: Lisa Janicki, vice president; Kristien Fox, secretary; Douglas Schmitt, treasurer; A. B. Billings and Rick Leipold, corresponding secretaries/faculty sponsors.

PA Delta

Chapter President — Jennifer Snyder

Marywood University, Scranton — 4 actives

Jennifer Snyder and Maura Regan made presentations at Moravian College's Student Math Conference on February 21. Several members attended the NCTM annual conference in Washington, D.C. on April 1–4. The written portion of Marywood's annual High School Math Contest was February 28, and the oral portion was April 26. Other 1998–99 officers: Maura Regan, vice president; Brenda Rudzinski, secretary/treasurer; Sr. Robert Ann Von Ahnen, IHM, corresponding secretary/faculty sponsor.

PA Iota Chapter President — Abby Todd
Shippensburg University, Shippensburg 18 actives, 4 associates

Other 1998–99 officers: Peter Burnett, vice president; Nycole Miller, secretary; Stacey Lytle, treasurer/historian; Michael Seyfried, corresponding secretary; Cheryl Olsen, faculty sponsor.

PA Kappa Chapter President — Cheryll Stone-Schwendimann
Holy Family College, Philadelphia 6 actives, 2 associates

The Pennsylvania Kappa chapter inducted two new members on March 27. Dr. Thomas Bartlow from Villanova University (also a KME member) gave the keynote address on the topic of Parliamentary Power. The chapter sponsored its 4th Annual Grade School Math Competition on Saturday, April 25. Ten local elementary schools participated. During the summer, chapter members will begin working on a math newsletter for college students. Other 1998–99 officers: Brian Minster, vice president; Sr. Marcella Louise Wallowicz, corresponding secretary/faculty sponsor.

PA Mu Chapter President — Jen Gibbons
Saint Francis College, Loretto 23 actives, 6 associates

The PA Mu chapter held induction ceremonies on February 12. A formal dinner preceded the actual initiation ceremony for six new student members. Officers were chosen for the 1998–99 academic year. Five members attended the Great Lakes Regional Meeting on April 18 at Shippensburg University. Dr. Peter Skoner helped organize the meeting as regional director. Ms. Amy Miko and Jason Greene were members of the judging committee. Jennifer Gottshall and Kourosh Barati-Sedeh also attended. KME members picked litter in April along a two-mile stretch of highway near the college as part of Pennsylvania's Adopt-A-Highway program. Other 1998–99 officers: Brad Offman, vice president; Ernie Pagliaro, secretary; Ryan Howard, treasurer; Pete Skoner, corresponding secretary; Amy Miko, faculty sponsor.

SC Gamma Chapter President — Leslie Hogan
Winthrop University, Rock Hill 12 actives

Other 1998–99 officers: Karen Wade, vice president; Kortnee Barnett, secretary; Stephanie Boswell, treasurer; Donald Aplin, corresponding secretary; James Bentley, faculty sponsor.

SD Alpha Chapter President — Kristy Schuster

Northern State University, Aberdeen 12 actives

We put together algebra handbooks for a fundraising activity. These handbooks cover the material used in college algebra at Northern State University. Other 1998–99 officers: Margo Maynard, vice president; Becky Hanson, secretary; Stacy Garrels, treasurer; Lu Zhang, corresponding secretary; Raj Markanda, faculty sponsor.

TN Alpha Chapter President — Jonathan Sprinkle

Tennessee Technological University, Cookeville 30 actives

Other 1998–99 officers: Russell Watts, vice president; Andy Adams, secretary; Deborah Watkins, treasurer; Frances Crawford, corresponding secretary; Allen Mills, faculty sponsor.

TN Gamma Chapter President — Jennifer Murrah

Union University, Jackson 13 actives, 8 associates

Student members Jennifer Murrah and Allison Smith presented papers at the KME Regional Convention at Eastern Kentucky University. Allison presented "Mathematical Solutions of the Rubik's Cube" and Jennifer was awarded Best Undergraduate Talk Award for her presentation "The Formation of Rainbows." Lori Davis, Mandy Davidson, and faculty members Matt Lunsford and Chris Hail also attended. Other 1998–99 officers: Lori Davis, vice president; Mandy Davidson, secretary/treasurer; Matt Lunsford, corresponding secretary; Troy Riggs, faculty sponsor.

TN Delta Chapter President — Robert John

Carson-Newman College, Jefferson City 9 actives, 6 associates

We held a Spring Initiation Banquet and a picnic. Other 1998–99 officers: Raymona Pedigo (Fall 98) and Melissa Holland (Spring 99), vice president; Wendy Williams (Fall 98) and Sarah S. Montgomery (Spring 99), secretary; Brian Renninger, treasurer; Catherine Kong, corresponding secretary/faculty sponsor.

TX Eta Chapter President — Wendy James

Hardin-Simmons University, Abilene 19 actives, 6 associates

Our only major activity this year was our 24th annual banquet for the induction of new members, held on April 18 at the Embassy Suites. Six new members were inducted, bringing the membership in the local chapter to 193. Leading the induction ceremonies were President Sylvia Cantu, vice president Wendy James, secretary Stephanie Helbert, and treasurer Eric Macy. Following the induction ceremony, pens and shingles were presented to the 1997 new members, and the Julius Olsen Award for the outstanding senior mathematics student was presented to Eric Macy. To conclude the program, Dr. James Ochoa, assistant professor of mathematics, gave a talk titled "Math and Music." Other 1998–99 officers: Gareth Jenkins, vice president; Sarah McCraw, secretary; Micah Lindstrom, treasurer; Frances Renfroe, corresponding secretary; Ed Hewett, Andrew Potter, James Ochoa, faculty sponsors.

TX Kappa Chapter President — Seleta Daniell
University of Mary Hardin-Baylor, Belton 12 actives, 5 associates

Other 1998–99 officers: Jennifer Key, vice president; Cheyanna Orsag, secretary; Peter Chen, corresponding secretary; Maxwell Hart, faculty sponsor.

VA Alpha Chapter President — Talvus Lucas
Virginia State University, Petersburg 36 actives, 11 associates

VA Alpha joined the Mathematics Department in hosting the regional meeting of the Mathematical Association of America in March. Members assisted in registration, campus guides and introduction of presentors for speakers in various sessions. Dr. Christopher Barat presented a research paper at the meeting. Toni Piper, a junior physics major and KME member, received numerous awards during the 1997–98 academic year. She was also the recipient of the annual Louise Stokes Hunter Endowed Scholarship Award at the annual initiation and banquet on April 24. Four members of Virginia Alpha chapter were interns in a research program sponsored by the National Aeronautics and Space Administration and Howard University, beginning in the summer of 1997. The results of their research were presented at various conferences, including poster sessions at the MAA National Convention, MathFest, a Student Research Symposium at Hampton University, and the Virginia Alpha Chapter Research Symposium. Other 1998–99 officers: Amirah Cutts, vice president; Jacqueline Payton, secretary; Emma Smith, treasurer; Elinor Poarch-Wall, corresponding secretary; Azzala Owens, faculty sponsor.

VA Gamma Chapter President — Tim Dunevant
Liberty University, Lynchburg 27 actives, 11 associates

The Virginia Gamma chapter held several special events during the spring semester to keep its members active and non-members interested. On February 24 we had two of our own faculty members, Dr. Skoumbourdis, professor of physics and mathematics, and Dr. Kester, professor of mathematics, tell of the experiences that led them to Liberty University and that prepared them for their profession. On March 26, we heard from Liberty University graduate Jim Ward, an actuary. He spoke about the nature of his job and gave advice to students on how to prepare for the actuarial field. Finally, on April 14, we held the Kappa Mu Epsilon induction and banquet at Ryan's restaurant. The usual procedure for the induction of new members and election of new officers was followed. Other 1998–99 officers: Valerie Chase, vice president; Leah Witham, secretary; David Schweitzer, treasurer; Glyn Wooldridge, corresponding secretary; Sandra Rumore, faculty sponsor.

Prosthaphaeresis

The following paragraphs are from the Fall 1950 issue of *The Pentagon*, on page 45. They constitute an interesting piece on some mathematical history. Apparently, some of it came from an article by J. W. L. Glaisher in the October 10, 1889 issue of *Nature*.

Logarithms owe their importance in arithmetic to the fact that they permit replacing certain operations by simpler ones; multiplication is replaced by addition, division is replaced by subtraction, etc. But logarithms are not the only quantities which permit such simplifications. Indeed, a table of natural sines can be used to replace a multiplication by a subtraction. Such a method was decised by Wittich of Breslau. For a short time Wittich was an assistant to Tycho Brahe and they used the method in their calculations in 1582. The method disappeared upon the invention of logarithms in 1614.

The method of Wittich was called *prosthaphaeresis* and was based on the identity

$$\sin\alpha\sin\beta = \frac{1}{2}[\sin(90^\circ - (\alpha-\beta)) - \sin(90^\circ - (\alpha+\beta))].$$

Four entries in a table of natural sines are required to compute a product with the aid of this formula. The following example in which a five-place table was used will illustrate the method.

Example: Find 0.8695×0.3170. We have $\alpha = \arcsin 0.8695 = 60^\circ\, 24.1'$, $\beta = \arcsin 0.3170 = 18^\circ\, 28.9'$, $90^\circ - (\alpha - \beta) = 48^\circ\, 4.8'$, $90^\circ - (\alpha+\beta) = 11^\circ\, 7.0'$, $\sin 48^\circ\, 4.8' = 0.74408$, $\sin 11^\circ\, 7.0' = 0.19281$, $0.74408 + 0.19281 = 0.55127$, and $0.55127 \div 2 = 0.27563$. Hence, $0.8695 \times 0.3170 = 0.27563$.

Obviously, in applying this method, each factor must be reduced to a number between 0 and 1 before a table of sines can be used. Thus,

$$869.5 \times 3.170 = (0.8695 \times 10^3) \times (0.3170 \times 10^1) = 0.27563 \times 10^4 = 2756.3.$$

We see here an analogy to the characteristics of common logarithms. It has been speculated that prosthaphaeresis may have given Napier the suggestion for his logarithms.

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Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation.

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959

IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984

CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College	12 May 1998

KME Website

The national KME website can be found at:

www.cst.cmich.edu/org/kme_nat/

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- How to start a KME chapter
- Information on KME conventions
- The cumulative subject index of *The Pentagon*
- Lists of KME chapters
- How to contact national officers
- KME History

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