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The Orbit of Hale-Bopp

Michelle D. Biggers-Beach, *student*

Missouri Kappa

Drury College
Springfield, MO 65802

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Looking into the night sky, thousands of dazzling lights litter the sky like broken glass. Among these lights are stars and planets. On some nights, an observer might see one of these many stars move. Stars don't move, but comets do. Everyone has heard of a comet, whether it is Halley's Comet which visits the earth every seventy-six years or the most recent media fascination, Comet Hyakutake. People of ancient times believed that these burning balls of gas moving through the sky were souls of kings and emperors traveling to heaven while others thought that these tailed stars brought disease and death to those on Earth ([1], p. 206). So what do comets have to do with the study of mathematics? More than one might think at a first glance. The subject of this research project is to determine the motion of a chosen comet given only Newton's law of gravity and a position and velocity vector at a particular time.

So how does one start such a project? First, we start off with Newton's law of gravity:

$$\overline{F} = -\left(\frac{Gm_1m_2}{r^2}\right)\widehat{r}, \tag{1}$$

where G is the universal gravitational constant, m_1 and m_2 are the masses of the two objects, r is the distance between the two objects, and $\widehat{r}$ is the unit vector of distance between the two objects.

Newton's law of gravity indicates the force that each object exerts on the other. Object 1 will exert a force on object 2 and vice versa. This is too general for the purpose of the comet. We are interested in the forces that are occurring between the comet and the sun. Assuming the mass of the comet is much, much less than the mass of the sun, we are able

to fix the sun's position, allowing the formation of a coordinate system. Through this assumption, the two-body problem that was originally dealt with in Newton's law of gravity becomes a one-body problem. A two-body problem adds the complication of having to worry about the sun's movement with respect to the comet which causes an increased level of difficulty for the distance calculation between the comet and the sun. It is through our assumption that we become concerned only with the comet's motion. Before proceeding further, a comet needs to be chosen. For the purpose of this research, the comet Hale-Bopp was chosen. The primary reason for this choice is because this comet will be closest to the earth in the spring of 1997, providing a fantastic opportunity to view it.

Returning back to equation (1), in order to determine the force that the sun is exerting on Hale-Bopp, a coordinate system needs to be established. We have already mentioned that the sun will be fixed, and so we will fix it at $(0,0)$, the origin of the coordinate system. The axes that run through the origin will be at ninety degree angles with one another just like those of the Cartesian coordinate system. Now that we have a coordinate system, we need to note that the unit vector of the distance between the two objects is now the unit vector of position of the comet. Therefore, the Newtonian law of gravitation can now be written as

$$\overline{F} = -\left(\frac{GmM_{\odot}}{r^3}\right)\mathbf{r}, \tag{2}$$

where m is the mass of the comet, $M_{\odot}$ is the mass of the sun, and $\mathbf{r} = \hat{r}/r$ is the comet's position vector.

Newton's second law of motion states that the force on an object is related to the acceleration as follows:

$$\overline{F} = m\left(\frac{d^2\mathbf{r}}{dt^2}\right).$$

Therefore by substituting this into equation (2) we now have

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right) = -\left(\frac{GmM_{\odot}}{r^3}\right)\mathbf{r}.$$

The comet's mass cancels leaving a second-order differential equation,

$$\frac{d^2\mathbf{r}}{dt^2} = -\left(\frac{GM_{\odot}}{r^3}\right)\mathbf{r}. \tag{3}$$

If we let $\mu = GM_{\odot}$, equation (3) can be written in a simplified manner:

$$\frac{d^2\mathbf{r}}{dt^2} = -\left(\frac{\mu}{r^3}\right)\mathbf{r}. \tag{4}$$

To solve this second-order differential equation, the rectangular coordinates need to be converted to polar coordinates. Assuming that the comet's motion is taking place in the xy-plane, then

$$\mathbf{r} = x\hat{i} + y\hat{j} \tag{5}$$

and

$$r\cos\theta = x \text{ while } r\sin\theta = y. \tag{6}$$

Assuming that both x and y are functions of time, the second derivative of equation (5) is

$$\frac{d^2\mathbf{r}}{dt^2} = \left(\frac{d^2x}{dt^2}\right)\hat{i} + \left(\frac{d^2y}{dt^2}\right)\hat{j}. \tag{7}$$

We now need to find the second derivatives of x and y from equation (6). The first derivatives are given by

$$\frac{dx}{dt} = -r(\sin\theta)\frac{d\theta}{dt} + (\cos\theta)\frac{dr}{dt}$$

and

$$\frac{dy}{dt} = r(\cos\theta)\frac{d\theta}{dt} + (\sin\theta)\frac{dr}{dt},$$

which then allows us to find the second derivatives as follows:

$$\frac{d^2x}{dt^2} = -2(\sin\theta)\frac{dr}{dt}\frac{d\theta}{dt} - r(\cos\theta)\left(\frac{d\theta}{dt}\right)^2 - r(\sin\theta)\left(\frac{d^2\theta}{dt^2}\right) + (\cos\theta)\frac{d^2r}{dt^2}$$

and

$$\frac{d^2y}{dt^2} = 2(\cos\theta)\frac{dr}{dt}\frac{d\theta}{dt} - r(\sin\theta)\left(\frac{d\theta}{dt}\right)^2 + r(\cos\theta)\left(\frac{d^2\theta}{dt^2}\right) + (\sin\theta)\frac{d^2r}{dt^2}.$$

Therefore by substituting these values into equation (7) we have

$$\begin{aligned}\frac{d^2\mathbf{r}}{dt^2} = &\left[-2(\sin\theta)\frac{dr}{dt}\frac{d\theta}{dt} - r(\cos\theta)\left(\frac{d\theta}{dt}\right)^2 - r(\sin\theta)\left(\frac{d^2\theta}{dt^2}\right) + (\cos\theta)\frac{d^2r}{dt^2}\right]\hat{i} \\ &+ \left[2(\cos\theta)\frac{dr}{dt}\frac{d\theta}{dt} - r(\sin\theta)\left(\frac{d\theta}{dt}\right)^2 + r(\cos\theta)\left(\frac{d^2\theta}{dt^2}\right) + (\sin\theta)\frac{d^2r}{dt^2}\right]\hat{j}.\end{aligned} \tag{8}$$

Equating equation (8) with equation (4), we now have the following relationship:

$$-\left(\frac{\mu}{r^3}\right)\mathbf{r} = \left[-2(\sin\theta)\frac{dr}{dt}\frac{d\theta}{dt} - r(\cos\theta)\left(\frac{d\theta}{dt}\right)^2 - r(\sin\theta)\left(\frac{d^2\theta}{dt^2}\right) + (\cos\theta)\frac{d^2r}{dt^2}\right]\hat{i} + \left[2(\cos\theta)\frac{dr}{dt}\frac{d\theta}{dt} - r(\sin\theta)\left(\frac{d\theta}{dt}\right)^2 + r(\cos\theta)\left(\frac{d^2\theta}{dt^2}\right) + (\sin\theta)\frac{d^2r}{dt^2}\right]\hat{j}$$

or

$$-\frac{\mu}{r^3}\left[r(\cos\theta)\hat{i} + r(\sin\theta)\hat{j}\right] = \left[-2(\sin\theta)\frac{dr}{dt}\frac{d\theta}{dt} - r(\cos\theta)\left(\frac{d\theta}{dt}\right)^2 - r(\sin\theta)\left(\frac{d^2\theta}{dt^2}\right) + (\cos\theta)\frac{d^2r}{dt^2}\right]\hat{i} + \left[2(\cos\theta)\frac{dr}{dt}\frac{d\theta}{dt} - r(\sin\theta)\left(\frac{d\theta}{dt}\right)^2 + r(\cos\theta)\left(\frac{d^2\theta}{dt^2}\right) + (\sin\theta)\frac{d^2r}{dt^2}\right]\hat{j}.$$

Since we have two vectors equalling one another, we can therefore equate the $\hat{i}$ and $\hat{j}$ components to obtain the following:

$$(9) \qquad \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right](\cos\theta) - \left[r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right](\sin\theta) = -\frac{\mu}{r^2}(\cos\theta)$$

and

$$(10) \qquad \left[r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right](\cos\theta) + \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right](\sin\theta) = -\frac{\mu}{r^2}(\sin\theta).$$

We need to note that from here on, the notation will be changed as follows:

$$\frac{dr}{dt} = \dot{r} \text{ and } \frac{d\theta}{dt} = \dot{\theta},$$

while

$$\frac{d^2r}{dt^2} = \ddot{r} \text{ and } \frac{d^2\theta}{dt^2} = \ddot{\theta}.$$

Looking at equations (9) and (10), we notice that we have two equations and two unknowns. Solving for the coefficients of sine and cosine we discover the following:

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2} \tag{11}$$

and

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0. \tag{12}$$

Hence we have developed two differential equations with r and θ. We now want to find an equation for r in terms of θ in order to have an equation which will give us the path of the comet. To do this, the two differential equations, equations (11) and (12), must be solved. First let us look at equation (12). This can be rewritten as

$$\frac{1}{r}\left[\frac{d}{dt}\left(r^2\dot{\theta}\right)\right] = 0,$$

which allows us to arrive at the conclusion that

$$r^2\dot{\theta} = h \text{ or } \dot{\theta} = \frac{h}{r^2}, \tag{13}$$

where h is a constant. Using this fact, substitute equation (13) into equation (11) to obtain the following:

$$\ddot{r} - r\left(\frac{h}{r^2}\right)^2 = -\frac{\mu}{r^2}$$

or

$$\ddot{r} - \frac{h^2}{r^3} = -\frac{\mu}{r^2}. \tag{14}$$

Now substitute $z = 1/r$ into equation (14) and express the second derivative of r with respect to time as the second derivative of z with respect to θ. First, we will find the first derivative:

$$\begin{aligned}
\frac{d}{dt}(r) = \frac{d}{dt}\left(\frac{1}{z}\right) &= -\frac{1}{z^2}\left(\frac{dz}{dt}\right) \\
&= -\frac{1}{z^2}\left(\frac{dz}{d\theta}\frac{d\theta}{dt}\right) \\
&= -\frac{1}{z^2}\left(\frac{dz}{d\theta}\frac{h}{r^2}\right) \\
&= -h\left(\frac{dz}{d\theta}\right). \qquad (15)
\end{aligned}$$

The second derivative is given by

$$\begin{aligned}\frac{d^2r}{dt^2} &= \frac{d}{dt}\left(-h\frac{dz}{d\theta}\right)\\ &= -h\left(\frac{d^2z}{d\theta^2}\frac{d\theta}{dt}\right)\\ &= -h\left(\frac{h}{r^2}\right)\left(\frac{d^2z}{d\theta^2}\right)\\ &= -h^2z^2\left(\frac{d^2z}{d\theta^2}\right). \qquad (16)\end{aligned}$$

Next we will substitute equations (15) and (16) into equation (14) which gives

$$-h^2z^2\left(\frac{d^2z}{d\theta^2}\right) - h^2z^3 = -\mu z^2$$

or

$$\frac{d^2z}{d\theta^2} + z = \frac{\mu}{h^2}.$$

The general solution to this differential equation is

$$z = A(\sin\theta) + B(\cos\theta) + \frac{\mu}{h^2}, \qquad (17)$$

and substituting $z = 1/r$ back into equation (17) we have

$$\frac{1}{r} = A(\sin\theta) + B(\cos\theta) + \frac{\mu}{h^2}$$

or

$$r = \frac{1}{A(\sin\theta) + B(\cos\theta) + \frac{\mu}{h^2}}. \qquad (18)$$

We can then write $A(\sin\theta) + B(\cos\theta)$ as just a cosine function from the identity

$$\cos(\theta - \omega) = (\sin\theta)(\sin\omega) + (\cos\theta)(\cos\omega)$$

where $\sin\omega = A$ and $\cos\omega = B$. Using this idea we have

$$A\sin\theta + B\cos\theta = \frac{\sqrt{A^2+B^2}}{\sqrt{A^2+B^2}}(A\sin\theta + B\cos\theta)$$

or

$$\sqrt{A^2+B^2}\left[\frac{A}{\sqrt{A^2+B^2}}\sin\theta + \frac{B}{\sqrt{A^2+B^2}}\cos\theta\right]. \qquad (19)$$

Using equation (19), let

$$\sin\omega = \frac{A}{\sqrt{A^2+B^2}}$$

so that

$$\cos\omega = \frac{B}{\sqrt{A^2+B^2}},$$

giving

$$\sqrt{A^2+B^2}\,[\sin\omega\sin\theta + \cos\omega\cos\theta] = k\cos(\theta-\omega) \tag{20}$$

where

$$k = \sqrt{A^2+B^2}.$$

Therefore by making the substitution using equation (20) into equation (18), we have a relationship between r and θ which is as follows:

$$r = \frac{1}{\frac{\mu}{h^2} + k\cos(\theta-\omega)}. \tag{21}$$

Applying algebra techniques to equation (21), we can simplify it into the following:

$$r = \frac{\frac{h^2}{\mu}}{1+\left(\frac{kh^2}{\mu}\right)\cos(\theta-\omega)}.$$

Now let $p = h^2/\mu$ and $e = kh^2/\mu$, which gives

$$r = \frac{p}{1+e\cos(\theta-\omega)}. \tag{22}$$

From Calculus III studies, we recognize that equation (22) is an equation for the conic sections in polar coordinates. The constant e is defined to be the eccentricity whose value will determine the type of conic section. The values for e and the corresponding types of the conic section are $e = 0$, circle; $0 < e < 1$, ellipse; $e = 1$, parabola; and $e > 1$, hyperbola. In order for Hale-Bopp to be seen again, the path of its orbit must be circular or elliptical, which will allow it to move in a closed path instead of shooting off into space.

Now that we have found an equation with r in terms of θ, we need to establish a coordinate system which applies to the real-life situation of the comet in space. The coordinate system that was established previously was done so in order to solve the differential equation in two-dimensional space. What needs to be done now is to translate our two-dimensional physics problem into the three-dimensional world that we live in. The earth and

all of the bodies in the heavens are moving in an xyz-coordinate system. In our solar system, all of the planets are orbiting the sun, which is essentially fixed in the system because of its massive size compared to all of the other planets. Therefore, the sun will be the origin, fixed at the point $(0,0,0)$, in the coordinate system that we are going to establish. The x-axis will be established by taking the position of the earth at vernal equinox and running a line from the sun to the earth at this position. The y-axis will then be established by rotating the x-axis ninety degrees in the direction of the orbit of the earth around the sun. The z-axis is established by using the right-hand rule. The system that we have established is the system used by both the National Aeronautics and Space Administration (NASA) and the Jet Propulsion Laboratory (JPL). The units of the system are astronomical units (AU), where 1 AU is defined to be the distance the earth is from the sun.

The comet's motion with respect to this coordinate system is the next thing that we need to establish. At a given time, the comet will have a position and velocity vector which will be calculated using the sun as the origin of this coordinate system in space. The plane of the orbit of the comet is the plane in which all of our work so far has taken place in. We now need to superimpose the two-dimensional problem into three dimensions. Therefore, we will run our x-axis for the plane of the comet's orbit from the origin of the coordinate system through a fixed point, r_o, at a given time. We will define θ to be increasing with the comet's motion. In order to find the comet's plane of motion, we will take the cross product of the position and velocity vector at a given time. Using the resultant vector (known as the normal vector) and the point $(0,0,0)$ which lies in the plane, the equation of the plane of the comet's orbit is

$$N_x(x) + N_y(y) + N_z(z) = 0,$$

where N_x, N_y, and N_z are the components of the normal vector.

Now that the comet's orbit plane has been established, we need to find expressions for the constants found in equation (22). The first constant that will be determined is p. Our constant p was related to h and μ by $p = h^2/\mu$. Since μ is the product of the gravitational constant and the mass of the sun, we are left with finding h, which was given in equation (13). In this equation, r was the magnitude of the position vector and $\dot{\theta}$ was the first derivative of θ with respect to time. Therefore, the value of the first derivative of θ needs to be found in order to find h, which will allow p to be found.

If we look at the comet at a particular time, it has a position and velocity vector as shown in figure 1. Taking the dot product of the two vectors, the angle α between them can be found as follows:

$$\mathbf{r} \cdot \mathbf{v} = \|\mathbf{r}\|\|\mathbf{v}\| \cos\alpha,$$

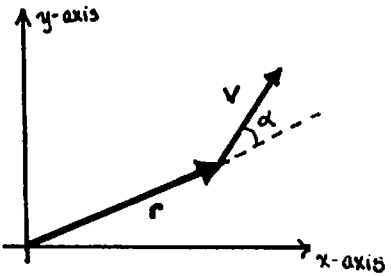


Figure 1

or

$$\alpha = \cos^{-1}\left(\frac{\mathbf{r}\cdot\mathbf{v}}{\|\mathbf{r}\|\|\mathbf{v}\|}\right). \tag{23}$$

Now the velocity vector in the plane of the orbit, $\mathbf{v}$, has both a radial component and an angular component as shown in figure 2, where $\mathbf{v} = (r_{\text{comp}})\widehat{r} + (\theta_{\text{comp}})\widehat{\theta}$ and $\widehat{r}$ is the unit vector of position while $\widehat{\theta}$ is the unit vector in the direction of increase of the component θ.

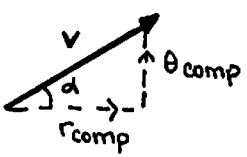


Figure 2

From figure 2, the coefficients for each of the components are

$$r_{\text{comp}} = \|\mathbf{v}\|\cos\alpha$$

and

$$\theta_{\text{comp}} = \|\mathbf{v}\|\sin\alpha,$$

which allows $\mathbf{v}$ to be written as

$$\mathbf{v} = (\|\mathbf{v}\|\cos\alpha)\widehat{r} + (\|\mathbf{v}\|\sin\alpha)\widehat{\theta}. \tag{24}$$

Likewise, if we look at the position vector, $\mathbf{r}$, and take its derivative we develop another expression for the velocity vector:

$$\begin{aligned}
\mathbf{r} &= r\widehat{r} = r(\cos\theta\widehat{i} + \sin\theta\widehat{j}) \\
\dot{\mathbf{r}} &= \dot{r}(\cos\theta\widehat{i} + \sin\theta\widehat{j}) + r(-(\sin\theta)\dot{\theta}\widehat{i} + (\cos\theta)\dot{\theta}\widehat{j}) \\
\dot{\mathbf{r}} &= \dot{r}\widehat{r} + r\dot{\theta}\widehat{\theta}. \qquad (25)
\end{aligned}$$

The two equations for the velocity vector, equations (24) and (25), may be combined to give

$$\dot{r} = \|\mathbf{v}\| \cos\alpha \tag{26}$$

and

$$r\dot{\theta} = \|\mathbf{v}\| \sin\alpha. \tag{27}$$

From equation (27) we have another expression for $\dot{\theta}$ which can be set equal to equation (13), which gives us the following relationship:

$$\frac{\|\mathbf{v}\| \sin\alpha}{r} = \frac{h}{r^2}$$

or

$$h = r\|\mathbf{v}\| \sin\alpha. \tag{28}$$

Now that we have found h, p can be expressed as

$$p = \frac{(r\|\mathbf{v}\| \sin\alpha)^2}{\mu}, \tag{29}$$

where r is the magnitude of the position vector and $\|\mathbf{v}\|$ is the magnitude of the velocity vector.

The next constant that will be solved is ω. Using equation (22), we have

$$r = p(1 + e\cos(\theta - \omega))^{-1} \tag{30}$$

and taking the derivative of equation (30) yields

$$\dot{r} = \frac{r^2 e\dot{\theta} \sin(\theta - \omega)}{p}. \tag{31}$$

In order to evaluate equation (31) for ω, we need to set some boundary conditions. Looking at initial conditions of the three-dimensional coordinate system, at position r_0, our $\theta = 0$ at that point. Therefore, to evaluate equation (31), let $\theta = 0$, giving us

$$\dot{r} = \frac{r^2 e\dot{\theta} \sin(-\omega)}{p}$$

or

$$-\frac{p\dot{r}}{r^2\dot{\theta}} = e\sin\omega. \tag{32}$$

We need to note that r has become fixed at this point, meaning $r = r_0$. Therefore, we will now take equation (22) and solve for cosine and apply the boundary conditions of $\theta = 0$ and $r = r_0$, giving

$$\frac{p}{r} - 1 = e\cos\omega. \tag{33}$$

Dividing equation (32) by equation (33), we have an expression for ω as follows:

$$\omega = -\tan^{-1}\left(\frac{p\dot{r}}{r\dot{\theta}(p-r)}\right). \tag{34}$$

The last constant that we need to solve for is e. Using equation (33), we find e as

$$e = \frac{1}{\cos\omega}\left(\frac{p}{r} - 1\right). \tag{35}$$

We have now found expressions for all the constants involved in the equation for the motion of Hale-Bopp. Before calculating physical numbers, the physical significance of the constants needs to be discussed. The eccentricity, e, corresponds to the type of conic section that the comet will travel in. The constant ω corresponds to the needed rotation of the coordinate system to reduce equation (22) to the canonical equation for conic sections involving just θ and r. The constant p relates the angular momentum of the comet and the universal gravitational constant as well as the mass of the sun to the comet's motion.

We are now at a point to begin using data points. Dr. Donald Yeomans at the Jet Propulsion Laboratory in California sent three different data sets. Each set had the position and velocity vector at a given time for the comet Hale-Bopp in the coordinate system which was previously described. The position and velocity vectors used in the calculation of all the constants are

$$r_0 = \langle 7.05527569318466 \cdot 10^{-1}, -3.20466371711724, \\ -5.36947018805670 \cdot 10^{-1}\rangle$$

and

$$\mathbf{v} = \langle -2.68691205191263 \cdot 10^{-3}, 9.67312656398213 \cdot 10^{-3}, \\ 8.70026498513763 \cdot 10^{-3}\rangle.$$

Note that the units for the position are in AU and the units for the velocity are AU/day. The magnitudes of each of the vectors are

$$\|r_0\| = 3.3250489909 \text{ AU}$$

and

$$\|\mathbf{v}\| = .01328470868 \frac{\text{AU}}{\text{day}}.$$

Before calculating the values for the constants, we need to have the value for μ in AU's and days. The value is

$$\mu = 2.959244628 \cdot 10^{-4} \frac{\text{AU}^3}{\text{day}^2}.$$

Now, taking each constant separately, we will begin by finding p. The value for p is given in equation (29). Therefore calculating the values for α, from equation (23), and h, from equation (28), we have

$$\alpha = 148.260773281^\circ$$

and

$$h = .023237019915 \frac{\text{AU}^2}{\text{day}}.$$

The constant p was related to h and μ by equation (29), so by using the calculated values for h and μ we have

$$p = 1.82465176903 \text{ AU}.$$

Next, we will calculate the value for ω, whose value is given in equation (34). To calculate ω, we need to find the values for $\dot{r}$ and $\dot{\theta}$ in addition to using the values of p and r_0 which we have already found. We find that $\dot{r}$, using equation (26), is

$$\dot{r} = -.011297995822 \text{ AU/day}$$

and using equation (13) we find that $\dot{\theta}$ is

$$\dot{\theta} = .002101765859 \text{ radian/day}.$$

Therefore by substituting these values into equation (34) as well as noting the signs of sine and cosine which result from equations (32) and (33) and the fact that e is positive, we have

$$\omega = 116.959534546^\circ.$$

With the value of ω, we can now find the eccentricity. Using equation (35) we find that

$$e = .995322774245.$$

The eccentricity value indicates that Hale-Bopp is moving in an elliptical orbit. It is because of this elliptical nature that we will see Hale-Bopp again. Therefore the equation for the elliptical motion of the comet Hale-Bopp is

$$r = \frac{1.82465176903 \text{ AU}}{1 + .995322774245 \cos(\theta - 116.959534546°)}.$$

Taking the cross product of r_0 and v we have the normal vector with the following coefficients:

$$\mathrm{N} = \langle -0.022687467056, -0.004695547391, -0.001785992091 \rangle,$$

where the units of the normal vector are AU^2/day. This allows us to calculate the plane in which Hale-Bopp is orbiting in as

$$0.022687467056x + 0.004695547391y + 0.001785992091z = 0.$$

Now that we have found the equation of motion and the plane of orbit for Hale-Bopp, there are further areas that could be pursued. With more work, not to mention time, further information about the comet can be obtained. From the equation of motion, the perihelion and the aphelion points in the orbit can be calculated as well as the period of Hale-Bopp's orbit. We found r in terms of θ and could then extend this equation in order to find r in terms of t and θ in terms of t. There is also the idea of looking at Jupiter's effect on Hale-Bopp which would introduce a perturbation into the problem.

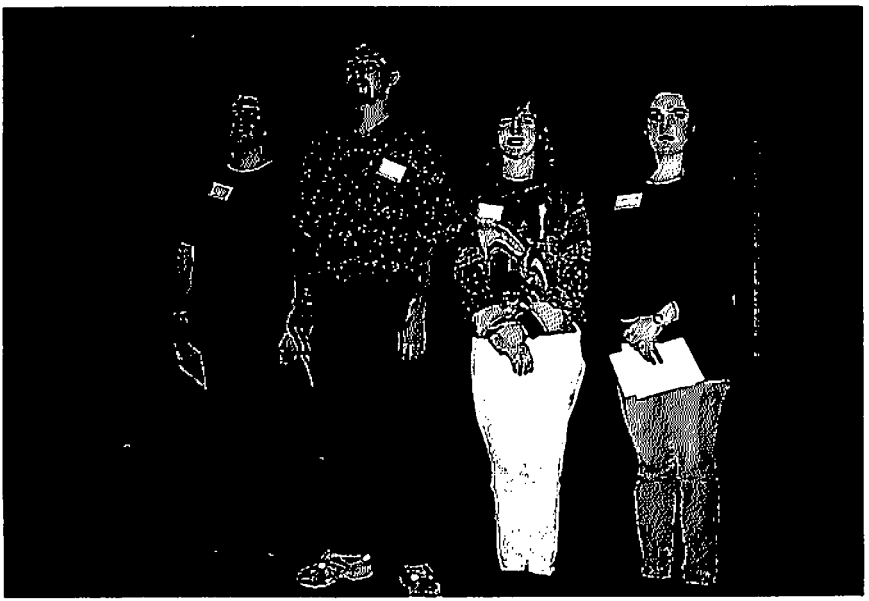
In this paper, we have taken a two-dimensional force problem and expanded the idea so that it would apply to a three-dimensional real life problem. From the Newtonian law of gravity and a position and velocity vector for the comet Hale-Bopp, we have developed a working equation which relates the position and the angle made with the origin of the coordinate system that our solar system moves in. By using the home page for the Jet Propulsion Laboratory (http://www.encke.jpl.nasa.gov/hale_bopp_info.html), we are able to locate Hale-Bopp's position in the sky. On the dates of March 26 through April 12, if you look into the northwest sky (around the constellation Lacerta), at about a forty-five degree angle above the horizon after sunset, you will find Hale-Bopp traveling on its elliptical orbit which we have found.

Acknowledgements. I would like to thank Dr. Charles Allen for all of his "advising" and support throughout this research project. He has been the best advisor a young mathematician could work with. I would also like to thank Dr. Bruce Callen for informing me of Hale-Bopp long before I or any other non-astronomy buff knew about it. To my new husband, David — thank you for putting up with the long hours and continual typing noises. To Alan Hale and Thomas Bopp: thank you for your discovery!!

References

1. Gallant, Roy A., *National Geographic Atlas of Our Universe*, National Geographic Society, Washington D.C., 1980.

Convention Winners



Top four presenters at the 31st Biennial Convention (left to right): Kelli Polotaye, New York Lambda; Michelle Biggers-Beach, Missouri Kappa; Andrew Miller, Iowa Delta; and Suzanne Shontz, Iowa Alpha.

Cumulative Subject Index

The Cumulative Subject Index for *The Pentagon* is up and running! Check it out at www.cst.cmich.edu/org/kme/, the national KME homepage, or directly at **www.cst.cmich.edu/org/kme/indpent.htm**.

Mostly organized by standard course titles, there are 25 topics to choose from. This can be a great resource for your courses, whether you are a student or faculty! Literally hundreds of articles are listed, on an incredible variety of fascinating topics. (Do I sound like a car salesman, or what?) Check it out today!

T^3T^2: Three Tantalizing Teasers — Times Two?

Larry Campbell, *faculty*

Missouri Alpha

Southwest Missouri State University
Springfield, MO 65804

Based on the Banquet Address at the 1997 National Convention

This article represents an attempt to capture and roughly reproduce some of the remarks I made at the banquet talk of the KME 31st Biennial Convention, held in Springfield, Missouri, this past April. My motivation for trying to do this is not necessarily what one might suspect at first glance. It is true that I had a great deal of fun preparing and giving that talk, and if the feedback I received after the talk is any indication, many in the audience shared that fun. However, since I'm aware that immediate feedback from such banquet talks is often, shall we say, more polite than truthful, and since the remarks were prepared with minimal notes for a live audience, the thought of having someone read the comments had not even crossed my mind.

Instead, it was the feedback I received later, over the next few days, that birthed the idea for this article. Several in the audience, it seemed, had enjoyed one or more of the crazy problems and challenges I pitched out. More than one communication mentioned (without cursing, I might add) losing several hours of sleep after the banquet working on one or more of the problems. Now THAT I considered a compliment! And, I must confess, an irresistible call of sorts. Perhaps, I thought, others would like tackling the problems as well.

So, I've collected three of the more popular problems from the talk, each with its corresponding challenges (hence the "Times Two?" in the title), polished them a little, edited out the rest of the talk (you're welcome) and am offering the result again in this slightly different format. My purpose is primarily to see if others might have just as much fun working on these questions. But who knows? Perhaps there will also be other original and/or beautiful solutions that surface as a result, maybe even in a future issue of this journal . . . ?

Confession and Warning: Since most readers probably weren't at

the banquet, you may wish to know the following: there's nothing terribly deep in what follows. In fact, as it turns out, there's nothing too terribly original either. Quite honestly, I had never really intended otherwise on either score, but if you're hoping to find a review of some deep or even classic problems of mathematics, you can stop here.

On the other hand, that appears to be what contributed to some of the fun. These particular problems and challenges, besides being somewhat new to many of us, are the type that apparently can grab you by the proverbial throat and not let you go for awhile — even threatening your sleep, it appears. (I take no credit/blame for that, since none of these problems originated with me!) The fact that they also appear to be imminently "doable" is even more of an attraction. So there's your warning. You, too, may get caught as some of us did. Proceed at your own risk.

Problem #1: The "Seven Questions" Game

A variation of what must be one of the most boring games in the world ("I've got a number from 1 to 10 — guess my number"), this problem/game has a twist that has made it one of my personal favorites for quite some time. Person A picks a positive integer from 1 to 16 inclusive. Person B may ask any seven YES/NO questions (i.e., Person A must be able to answer the questions YES or NO) he/she wishes to help determine the number. On the surface this is not hard — four questions and a binary search should normally do it — but here's where the fun begins: Person A is allowed to lie up to once during his/her seven answers! The object, of course, is to try to weed through the possible misinformation to determine the secret number within the seven-question limit.

Other notes and hints: 1) Person B gets to ask all seven questions and hear the answers before making any public declarations about the number. 2) It is highly advisable to write down all seven questions and answers to minimize post-game arguments over the number of lies. 3) Note that the rules allow ANY seven yes/no questions. It is, therefore, perfectly legal to ask a question twice. This last bit of information, provided somewhat reluctantly, should be a huge strategy hint.

During the actual talk, I mentioned some of the unexpected fun I've had with this game over the years (including the beauty of the question "Have you lied yet" and the student who once asked as the first question "Do you plan to lie?" — a wonderful example of a paradox that may make your head swim), but at this point, I proceed directly to the challenges connected to the mathematics of this game.

Challenge(s) #1 (Related to Seven Questions)

1A. Develop a foolproof strategy for guessing the number in seven tries. From the first moment I heard of this problem, I also heard

that such a strategy existed, but I never quite got around to discovering one myself (I once thought I had it, but got confused by my own strategy!). After the banquet, Dr. David Craft from Muskingham College e-mailed me a beautiful strategy that I'm convinced works, so it indeed can be done, but I'll continue to leave this one on the table for your own fun. Part of the joy is discovering a strategy on your own[1].

1B. Articulate the relationship between the number of questions allowed/needed (in this case 7) and the number of numbers in the list (in this case 16). I've always wondered about this one, but again, had never taken the time (partly on purpose)[1] to pursue it very far. What if there are, say, 25 numbers in the list — how many questions will it take? Or, what if you only have 5 questions available — what's the maximum number of numbers in the list that will still allow you to guess the number with a good strategy?

I suggested at the banquet that nailing down a foolproof strategy (see 1A) will undoubtedly go a long way towards answering this one. After examining Dr. Craft's particular strategy, I am now convinced that is true. But again, have at it yourself. Maybe there are short, elegant relationships worth discovering.

1C. Is there a best strategy for the person with the number? It's always been interesting to me that this game is almost as fun and challenging for the person with the number as it is for the person trying to guess that number (a non-strategic answer by the number-holder can greatly reduce the guesser's work). So, even if assuming, in the best game-theoretic sense, that you're playing with a person who knows all the subtleties of the game him/herself, how can you play so as to minimize the other person's chances of guessing your number?

Problem #2: Pascal's Triangle

This is obviously a misnomer of sorts, since Pascal's Triangle is not really a "problem" seeking any type of solution, but I list it here because I used it to set up my next set of challenges.

By way of review, Pascal's Triangle is the infinite array of numbers listed in the pattern in figure 1. Each number in each row after the first can be derived by adding the two numbers above and on either side of it. There is evidence that the triangle has been known since the time of the early Chinese[2] and contains seemingly countless numbers of beautiful patterns (try to find some yourself and see Challenge 2A below)[3]. It is also incredibly rich in its connections to various branches of mathematics.

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[1] Refer also to comments under Final Perspective Check.

[2] The triangle has born Pascal's name ever since he wrote about it in 1653.

[3] The fact that there are nearly 48 quadrijillion patterns in Pascal's Tri-

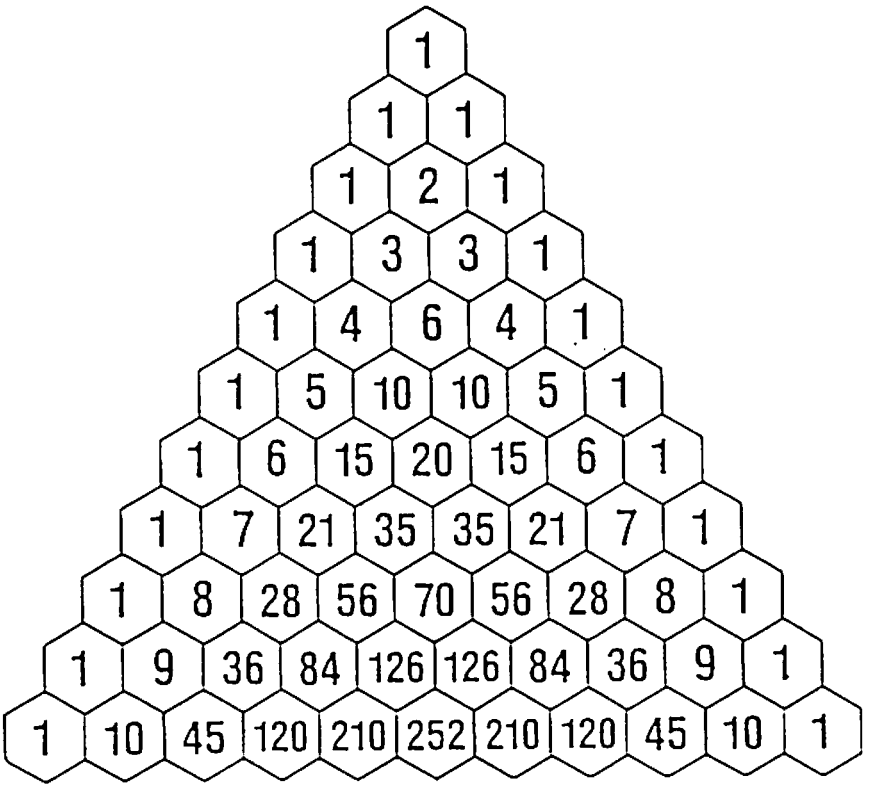

Figure 1. A portion of Pascal's Triangle.

Challenge(s) #2 (Related to Pascal's Triangle)

2A. Find a "new" pattern in Pascal's Triangle. I put "new" in quotation marks, because the pattern only has to be new to you to make the challenge fun (in this regard, the less you've seen of the Triangle, the better!). Search until you've found all the patterns you think you can, and then search some more. Finding something you've never noticed before and then wondering if anyone else has ever seen it is one of those rare but wonderful joys in mathematics.

I once discovered such a "new" pattern for myself: sum the squares of each entry in any given row of Pascal's Triangle (recall that the top "1" in the Triangle is called Row 0). If that row is, say, Row n, then the sum of those squares will be the middle number in Row $2n$. This pattern was

---

angle can be seen by perusing the popular books *Pascal's Triangle* (Green and Hamburg) and *Visual Patterns in Pascal's Triangle* (Seymour), both published by Dale Seymour Publications.

excitingly new to me, and appeared to always work (a dangerous statement of course), but the nature of the Triangle made it difficult to verify for lots of cases, and a proof seemed at the very least time-consuming.

2B. Find a(nother) proof of the pattern above. During the banquet, I had presented this challenge in a "prove or disprove" format and was still selfishly and hopefully calling it "Campbell's 'new' pattern." I suggested that perhaps a proof could be cranked out with brute force by using the fact that the ith entry (counting the first 1 in each row as the 0th entry) in row n is also the same as $C(n, i)$ (one of the combinatoric notations for "n-choose-i" or "all the combinations of n things taken i at a time").

Since the banquet, I have discovered that the pattern is indeed true, and, alas, relatively well-known. Dr. Craft even provided one reference (as did a reviewer of this article later). I have now also seen at least two different proofs. One of these, an original proof by Dr. Don Tosh of Evangel College in Springfield, Missouri, follows this article. Another, sketched to me right after the banquet, is an even shorter, more visual proof focusing on the fact that from the top of the triangle to any entry in the triangle, the number of paths to that entry is the same number as the entry itself. (**Mini-challenge:** finish the proof with just that information.) I have also since been somewhat embarrassed to discover that both the pattern and the shorter proof appear in the first work referenced in footnote #3.

Problem #4: Four Fours

Yes, I know. This should be Problem #3. But doesn't it just seem right that any problem which deals with four fours should be labeled #4? If large hotels can go from the 12th to the 14th floors, we can go from Problem #2 to #4.

Perhaps you've seen this one before. The idea is to try to represent each whole number using exactly four fours. Four is the only number which can be used and there must be exactly four of them used. Some examples, designed to also showcase hints, are:

$$0 = 44 - 44 \qquad = 4 - 4 + 4 - 4 \qquad = [.4444]^4$$

$$1 = 44/44 \qquad = \left[\sqrt{\sqrt{\sqrt{\sqrt{4444}}}}\right]$$

$$3 = (4 + 4 + 4)/4$$

$$85 = (4/.\bar{4})^{\sqrt{4}} + 4$$

$$100 = 4 \cdot 4! + \sqrt{4} + \sqrt{4}$$

$$200 = (4 \cdot (4! - 4))/.4$$

[4] (A footnote, not an exponent!) Recall that $[x]$ is the greatest integer less than or equal to x.

As can be deduced, any operation or notation may be used (though some seem more like cheating than others), with creative uses of factorials, square roots, and the greatest integer function being particularly useful.

Challenge(s) #4 (Related to Four Fours)

4A. Extend the four fours list well past 250 ... or 300 ... or ??? Currently, I have a rather fat file, gathered over many years of using this problem, of representations for all integers from 0 to 200, with many more representations at random after that. Often, especially early on, there are several representations for each integer. My list begins to taper at 200 only because that's where I usually stop on the "pizza challenges" I frequently give classes (Note to Campbell: start asking for more!), but one begins to wonder just how far this list can or can't go.

4B. Find as many representations as possible for each integer and/or find a "new" representation for a given integer. I have nearly a dozen different representations for some integers in my own files. And, again, there is great satisfaction in discovering a beautiful "new" representation for an integer, especially the larger integers. Perhaps we should petition the Guinness Book of Records for a category "Integer with the most four-fours representations" (see 4C-b below).

4C. Assorted Other Challenges, ranging from "easy enough" to "possibly quite difficult."

a) Demonstrate that every integer from 1 to 50 (or 100 or 200 or ...) has more than 1 (or 2 or 3 or ...) representation(s).

b) Find the integer(s) with the most representations. This record would constantly be changing of course and could require a definition of what we mean by "different."

c) Prove that the set of integers with representations is infinite (think factorials).

d) Is the set of integers with more than one representation infinite?

e) Do all non-negative integers have representations? On the surface, this doesn't seem likely. Are there integers, then, (clearly large?) which could somehow be proven don't have representations? Good luck.

f) Others?

Final Perspective Check And Bonus Challenge

Since the banquet talk, I have rediscovered that being both a mathematician and a teacher provides a bittersweet tension on these and other problems, and helps better explain my occasional admissions that I hadn't solved a particular problem yet. Like any mathematician, I LOVE both the irresistible lure of the unsolved-yet-seemingly-reachable problem and the joy of solving those problems. As a professor trying to share those same loves, however, it is often even more enjoyable to be able to truthfully

say aloud, "I wonder if ... " or "I don't have that answer yet." This delicate balance, by the way, is apparently not unique to me. Dr. Craft's first e-mail message to me politely asked if I wanted to see his solutions, since I could then no longer truthfully claim to not know the answer!

So while all of me is ecstatic at the beauty of several of the newfound (at least to me) results above, part of me is nevertheless sad I didn't get to discover them myself, and yet even more of me is sad that I can no longer say "I don't know that answer yet."

Clearly then, the most satisfying answer to this dilemma is to move on and refocus on the next set of new "favorite problems" which have the same power to reach out and grab us, and for which "we don't know that answer yet." I have my own collection waiting in the wings to move up (Cori the Camel, The Consecutive Integer Problem, and others — pique your interest?), but those are for another time and perhaps another sharing of some kind. And there are obviously many others as well. Who will help us along by providing the next beautiful impediments to sleep?

Newest Record Prime

Yes, again! Gordon Spence, using software written by George Woltman, discovered the newest record, announced August 24, 1997. The largest known prime is now $2^{2976221} - 1$, which has 895932 digits. Can the million-digit mark be far off? If you wish to participate in the Great Internet Mersenne Prime Search yourself, see the WWW site below:

ourworld.compuserve.com/homepages/justforfun/prime.htm

KME Quiz Answers (see p. 40)

1. Pennsylvania
2. New York and Missouri
3. Illinois
4. Pennsylvania
5. Kansas
6. Eta
7. 8 (in 1965)
8. Teacher's College (10 to 6)
9. New York Alpha
10. Colorado School of Mines
11. Springfield, Missouri
12. New Mexico Alpha
13. South Dakota Alpha
14. Massachusetts Alpha

Pascal's Triangle Revisited

Don Tosh, *Faculty*

Missouri Theta

Evangel College
Department of Science and Technology
Springfield, MO 65802

Theorem. $\binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i}^2 = \binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2.$

This paper was prompted by a challenge Dr. Larry Campbell gave during his talk at the banquet during the national KME convention in Springfield on May 4, 1997. His talk consisted of posing several open questions and challenging the listeners to find an answer. I didn't get much sleep that night — I stayed up working on the problem above until into the morning — but I found the answer. I thank Dr. Campbell for a very entertaining talk, and I do love a challenge.

The first few rows of Pascal's triangle are as given in figure 1.

```
                                  ↙ Column 0
                  Row 0 →     1     ↙ Column 1
                Row 1 →     1   1     ↙ Column 2
              Row 2 →     1   2   1     ↙ Column 3
            Row 3 →     1   3   3   1     ↙ Column 4
          Row 4 →     1   4   6   4   1     ↙ Column 5
        Row 5 →     1   5  10  10   5   1     ↙ Column 6
      Row 6 →     1   6  15  20  15   6   1     ↙ Column 7
    Row 7 →     1   7  21  35  35  21   7   1
```

Figure 1. Pascal's triangle

Two of the most commonly known facts about the triangle are that a given entry is the sum of the two entries above it and that the entry in

row i and column j is $\binom{i}{j}$. The pattern that Dr. Campbell noticed was that the sum of the squares of the entries in row n is the middle entry of row $2n$. For example, the sum of the squares of row 3 is $1^2 + 3^2 + 3^2 + 1^2$ which equals 20 and is the middle entry of row 6. He hypothesized that the pattern would continue and that the proof of this result, which I have stated in combinatorial format in the theorem, would involve messy algebra involving factorials. My challenge was to find a non-messy proof of the conjecture.

My solution involves finding the pattern involved in writing a given entry in terms of the sum of elements more than one row above. We write $P_{i,j}$ as element j of row i. We start by developing a recursive formula for $P_{6,3}$. The usual identity is

$$P_{6,3} = P_{5,2} + P_{5,3} = 1 \cdot P_{5,2} + 1 \cdot P_{5,3}.$$

Each of $P_{5,2}$ and $P_{5,3}$ can be written as the sum of elements above them, so

$$\begin{aligned} P_{6,3} &= P_{5,2} + P_{5,3} \\ &= (P_{4,1} + P_{4,2}) + (P_{4,2} + P_{4,3}) \\ &= 1 \cdot P_{4,1} + 2 \cdot P_{4,2} + 1 \cdot P_{4,3}. \end{aligned}$$

We now write each of these as the sum of the two elements above them and get

$$\begin{aligned} P_{6,3} &= 1 \cdot P_{4,1} + 2 \cdot P_{4,2} + 1 \cdot P_{4,3} \\ &= 1 \cdot (P_{3,0} + P_{3,1}) + 2 \cdot (P_{3,1} + P_{3,2}) + 1 \cdot (P_{3,2} + P_{3,3}) \\ &= 1 \cdot P_{3,0} + 3 \cdot P_{3,1} + 3 \cdot P_{3,2} + 1 \cdot P_{3,3}. \end{aligned}$$

At this point you should recognize the coefficients on the right-hand side as the elements of row 3. Replacing these coefficients by their symbolic representations gives

$$P_{6,3} = {P_{3,0}}^2 + {P_{3,1}}^2 + {P_{3,2}}^2 + {P_{3,3}}^2.$$

The pattern is

$$\begin{aligned} P_{2n,n} &= 1 \cdot P_{2n-1,n-1} + 1 \cdot P_{2n-1,n} \\ &= P_{1,0}P_{2n-1,n-1} + P_{1,1}P_{2n-1,n} \\ &= 1 \cdot P_{2n-2,n-2} + 2 \cdot P_{2n-2,n-1} + 1 \cdot P_{2n-2,n} \\ &= P_{2,0}P_{2n-2,n-2} + P_{2,1}P_{2n-1,n-1} + P_{2,2}P_{2n-2,n} \\ &= P_{3,0}P_{2n-3,n-3} + P_{3,1}P_{2n-3,n-2} + P_{3,2}P_{2n-3,n-1} + P_{3,3}P_{2n-3,n} \\ &= \ldots. \end{aligned}$$

In retrospect, it is not surprising that since elements in Pascal's triangle are sums of elements in previous rows, each element could be reasonably seen to be a linear combination of previous entries with coefficients from the triangle itself.

A rigorous proof is done by induction and goes as follows. The induction hypothesis is that for all $N \geq 0$ and $0 \leq k \leq n$,

$$P_{N,n} = \sum_{i=0}^{k} P_{k,i} P_{N-k,n-k+i}, \tag{1}$$

with the understanding that $P_{i,j} = 0$ if $j < 0$ or $j > i$. The case for $k = 0$ is trivial and the case $k = 1$ is the usual triangle identity that

$$P_{N,n} = P_{N-1,n-1} + P_{N-1,n}. \tag{2}$$

We assume the hypothesis (1) is true for all $k \leq K - 1$ and then try to show it is true for $k = K$. To do this we use (1) on each of the terms in (2) giving

$$\begin{aligned} P_{N,n} &= P_{N-1,n-1} + P_{N-1,n} \\ &= \sum_{i=0}^{K-1} P_{K-1,i} P_{N-K,n-K+i} + \sum_{i=0}^{K-1} P_{K-1,i} P_{N-K,n-K+i+1}. \end{aligned}$$

Adjusting the limits of the second summation above gives

$$\begin{aligned} P_{N,n} &= \sum_{i=0}^{K-1} P_{K-1,i} P_{N-K,n-K+i} + \sum_{i=1}^{K} P_{K-1,i-1} P_{N-K,n-K+i} \\ &= \sum_{i=0}^{K} (P_{K-1,i} + P_{K-1,i-1}) P_{N-K,n-K+i} \\ &= \sum_{i=0}^{K} P_{K,i} P_{N-K,n-K+i} \text{ (using (2))}. \end{aligned}$$

This proves the induction hypothesis (1) to be true for all $N \geq 0$ and $0 \leq k \leq n$. The special case we are dealing with is when $N = 2n$ and $k = n$, yielding the desired result

$$P_{2n,n} = \sum_{i=0}^{n} P_{n,i} P_{2n-n,n-n+i} = \sum_{i=0}^{n} P_{n,i}^2.$$

This result is not original, as pointed out by a referee. An alternate combinatorial proof can be found in Brualdi's "Introductory Combinatorics." Also, this result is similar to Carl Libis' solution to Problem 477 in The Problem Corner in the Spring 1996 issue of *The Pentagon*. For more unsolved problems in combinatorics, Stanley's "Enumerative Combinatorics" gives a tantalizing assortment.

Tilings and the Art of M. C. Escher

Jennifer D. Larson, *student*

Illinois Theta

Benedictine University
Lisle, IL 60532

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M. C. Escher was a Dutch artist who, despite the lack of an extensive mathematical background, used concepts from geometry and algebra in his works. Many of Escher's works were tessellations. Tessellation is another name for a tiling. This paper examines three of Escher's works (see figures 8–10) and identifies the symmetry groups associated with each, and also describes the tiling structure of each. In addition, this paper classifies the three Escher works as wallpaper groups.

Because of the symmetry apparent in Escher's works, we are interested in the group of symmetry operations which preserve the initial picture. Such operations are called isometries. An isometry is a one-to-one and onto function that preserves shape and size. There are four types of planar isometries. The first of these isometries is a translation. A translation is achieved by taking any ordered pair and adding to it a fixed second ordered pair, which shifts the point left or right and/or up or down. Figure 1 shows an example of a translation. The second isometry is a rotation. A rotation is achieved by taking an ordered pair and rotating it through a specified angle θ by multiplying it by the rotation matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

Figure 1 also shows an example of a rotation.

The third isometry is a reflection, which was used in very few of Escher's works. A reflection can occur across the y-axis (by negating the x-coordinate), the x-axis (by negating the y-coordinate), or the origin (by negating both coordinates). Figure 2 gives examples of all three reflections of the point $(1, 1)$. The fourth and final isometry is a glide reflection. A

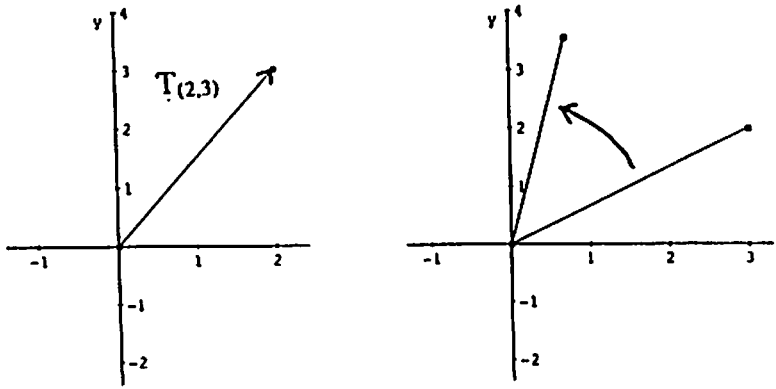

Figure 1. Left: translation by $(2,3)$; $T_{(2,3)}(0,0) = (0,0)+(2,3) = (2,3)$. Right: Rotation by $\pi/4$ about the origin; $(x,y) = (3,2)$, $\theta = \pi/4$.

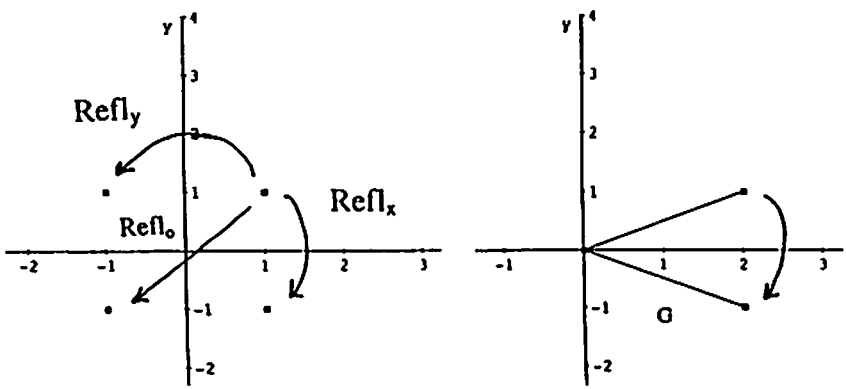

Figure 2. Left: reflection. Right: glide reflection; $G(0,0) = (2,1) \circ (x,-y)$.

glide reflection is the composition of a translation and a reflection. Figure 2 also shows an example of a glide reflection.

Next, we verify that the isometries form a group under composition. First, composition is closed: the composition of two isometries is another isometry. For example, the composition of two translations yields another translation, the composition of a translation and a reflection yields a glide reflection, and the composition of a glide reflection and a translation yields a glide reflection (see figure 3 for illustrations).

Secondly, we see that the identity of the group is the identity map id:

$$\mathrm{id} \circ f(x) = f \circ \mathrm{id}(x) = f(x) \text{ for all } x, \text{ for all } f.$$

Next, we look at some inverses. The inverse of a translation is a trans-

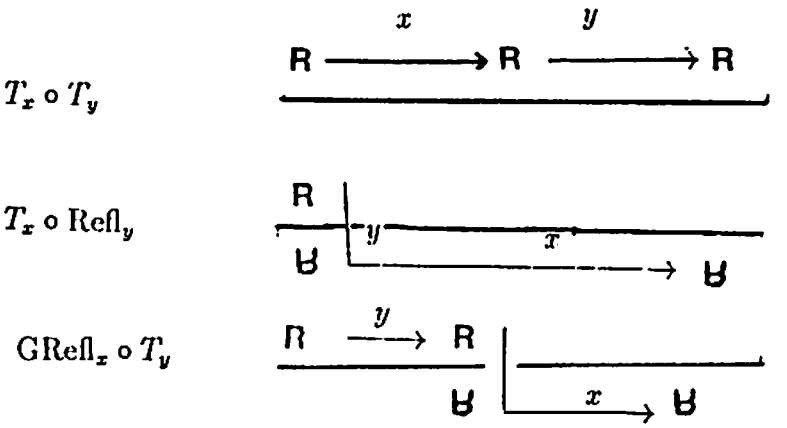

Figure 3. Examples of compositions of isometries.

lation and the inverse of a glide reflection is a glide reflection (see figure 4 for illustrations). Reflections are self-inverses, and the inverse of a rotation of angle θ is a rotation of angle $2\pi - \theta$.

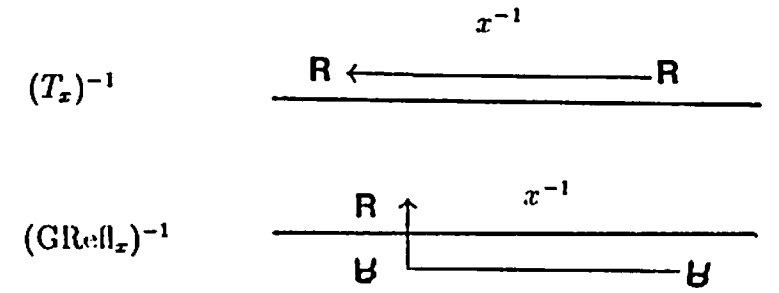

Figure 4. Examples of inverses of isometries.

Finally, function composition is well-known to be associative (cf. Gallian [1], p. 26). Thus, the isometries form a group under composition.

Let us now examine how tilings of the plane are produced. We begin with an underlying lattice. The underlying lattice of a tiling is formed by one of the six regular divisions of the plane which are parallelograms, rectangles, squares, 60° rhombi, triangles, and regular hexagons. There are only six regular divisions because in order for a figure to be a regular division, its vertex angle must divide 360°. For example, the vertex angle of a regular hexagon is 120° and $360/120 = 3$. Therefore, a regular hexagon is a regular division. Consider the pentagon with a vertex angle of 108°. Since $360/108 = 3.33$, the pentagon is not a regular division. Attempted

tilings with pentagons yield overlaps or gaps.

Tilings may be classified as dihedral or monohedral and/or isohedral. A monohedral tiling is a tiling in which the figures are directly congruent, that is, they are the exact same size and shape. Figures 5 and 6 show examples of monohedral tilings. An isohedral tiling is a tiling in which all of the figures are congruent and for which there exists an element in the symmetry group which sends one of the tiles into each of the others. This means that there exists an isometry which places one tile directly onto all of the others. Figure 5 shows an example of an isohedral tiling. The isometry is a rotation. Finally, a dihedral tiling is a tiling in which the figures are not congruent. Figure 7 shows an example of a dihedral tiling. The fish facing to the right have tails which point up and those facing left have tails which point down. Therefore, the fish are not congruent.

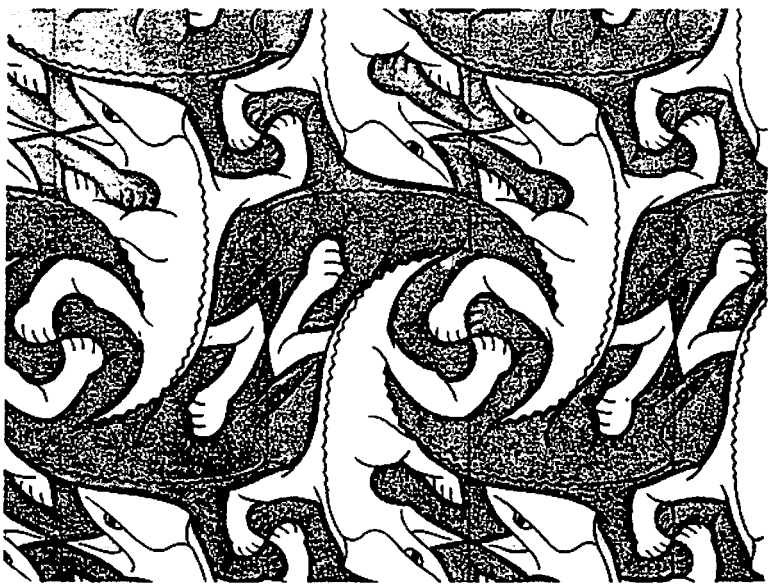

Figure 5. Symmetry Drawing E-15.

Figures 8, 9 and 10 show the three Escher works that we will categorize. We will identify the symmetry operations, the lattice of the tiling and the type of tiling that it is. Finally, we will see what kind of group arises by comparing to a list of symmetry groups for all planar tilings, or wallpaper groups.

In figure 8, the underlying lattice is square. The isometries present are reflections, translations, rotations, and glide reflections. A rotation

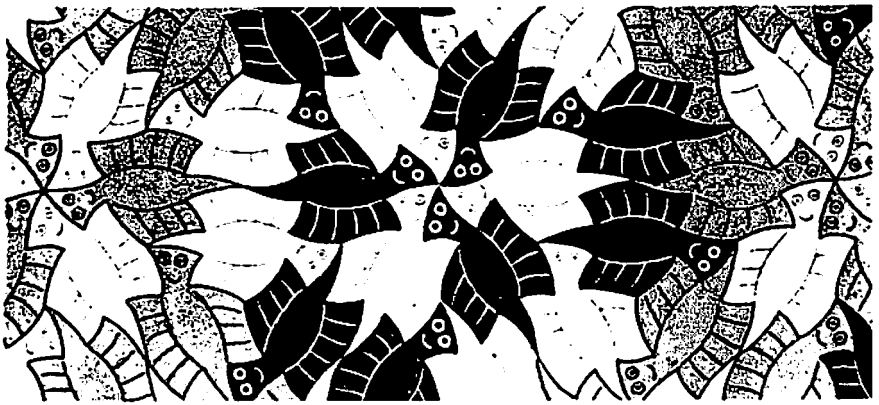

Figure 6. Symmetry Drawing E–137.
M. C. Escher's Symmetry Drawings ©1996 Cordon Art — Baarn Holland. All rights reserved. Used by permission.

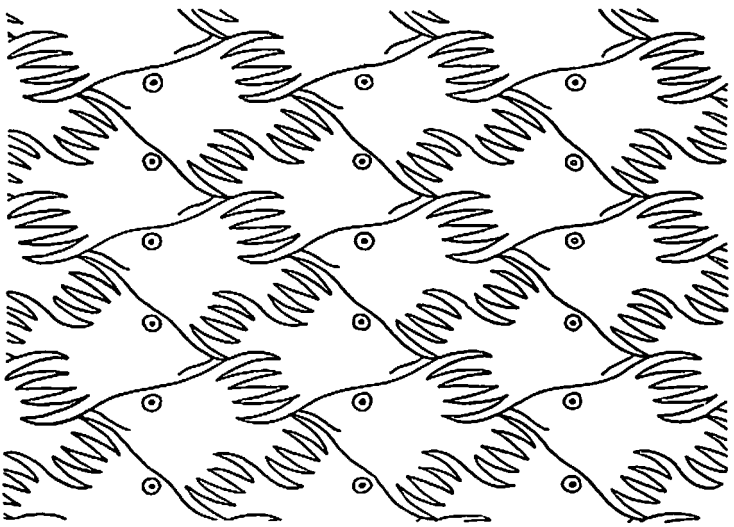

Figure 7. Symmetry Drawing E–93.
M. C. Escher's Symmetry Drawings ©1996 Cordon Art — Baarn Holland. All rights reserved. Used by permission.

of 90° places a man sitting vertically onto a man sitting horizontally. A glide reflection places two horizontal men facing the opposite way onto each other. The highest order of rotation is 4 because after 4 rotations, the figure ends up where it started. The 90° rotations do not occur on a reflection

axis. This work is monohedral and isohedral, as the figures are congruent and there exists an isometry which takes one figure onto every other figure. The group with these isometries and properties is a wallpaper group, *p4g* (cf. Gallian [1], p. 395).

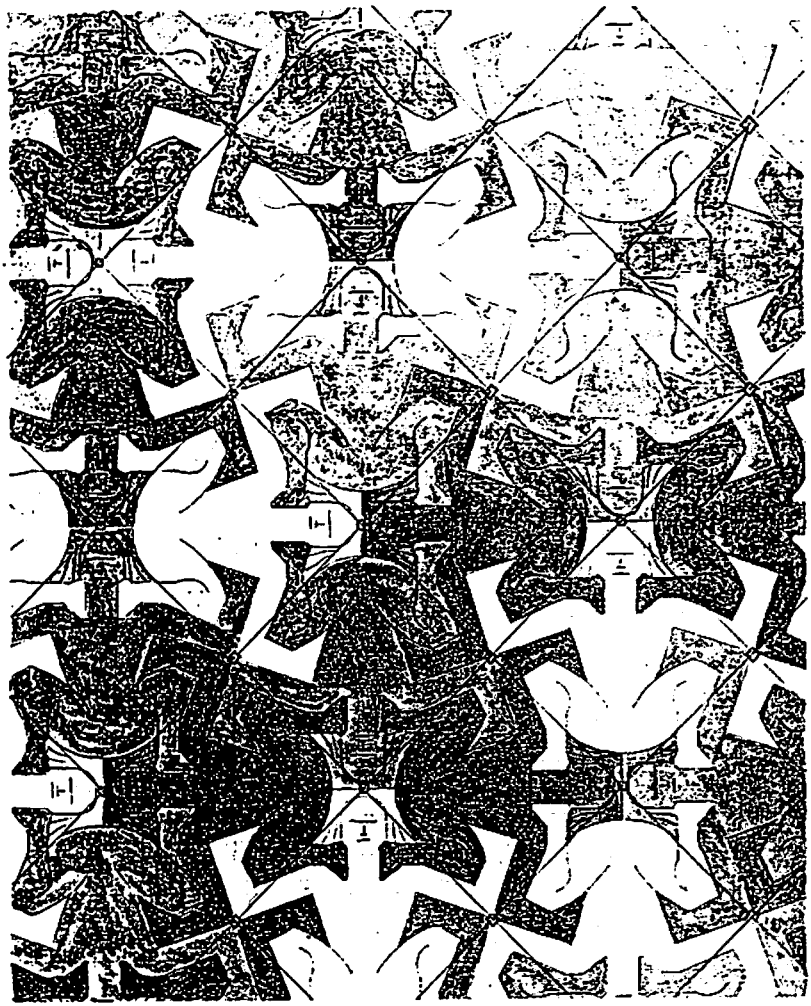

Figure 8. Symmetry Drawing E-3.

In figure 9, the underlying lattice is hexagonal. The isometries present are rotations and translations. A rotation of 120° places a white elf onto a black elf. The highest order of rotation is 3. This work is monohedral and isohedral. The group with these isometries and properties is also a

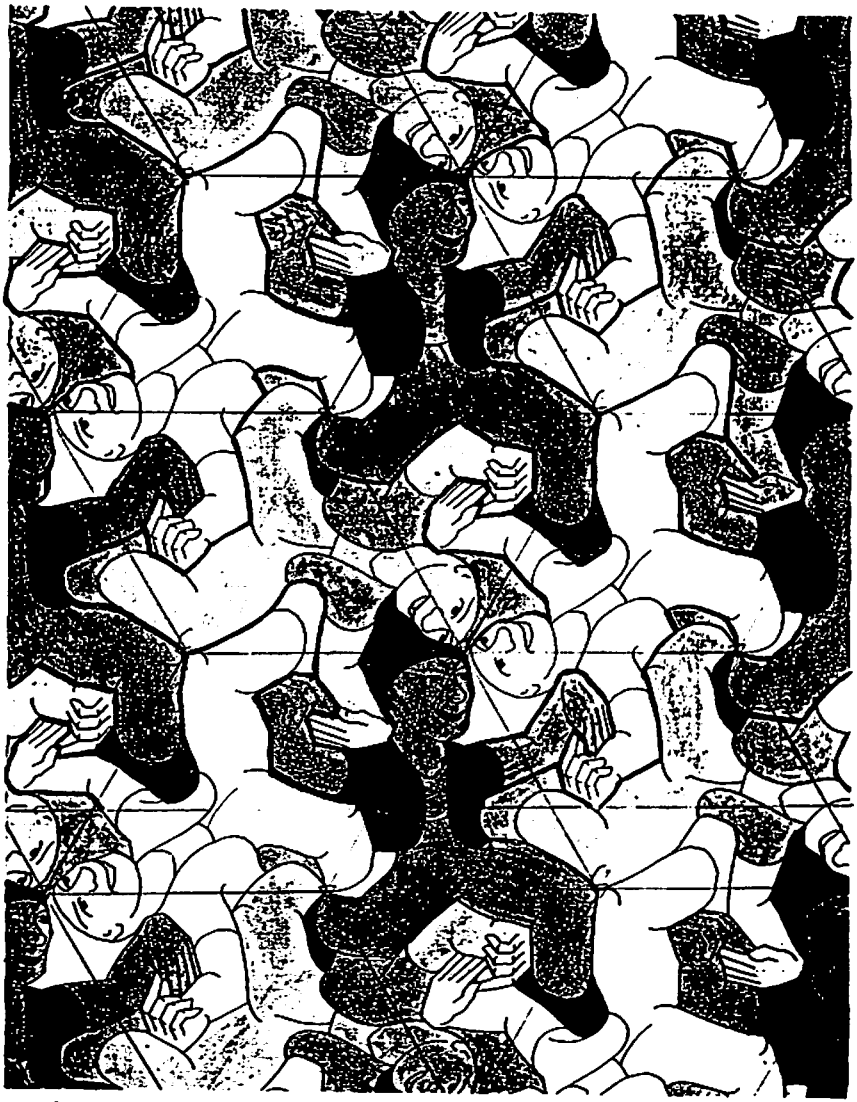

Figure 9. Symmetry Drawing E–21.

wallpaper group, $p3$ (cf. Gallian [1], p. 395).

Lastly, in figure 10, the underlying lattice is hexagonal. The isometries present are rotations and translations. A rotation of 120° places one lizard onto another lizard. The highest order of rotation is 3. This work is monohedral and isohedral. The group with these isometries and properties

Figure 10. Symmetry Drawing E–25.
M. C. Escher's Symmetry Drawings ©1996 Cordon Art — Baarn Holland. All rights reserved. Used by permission.

is also a wallpaper group, $p3$ (cf. Gallian [1], p. 395).

The research of symmetry groups as used in art is an interesting and enjoyable supplement to the study of abstract algebra. The ability to use visual examples makes the mathematics involved much easier to understand. Our appreciation of the art of M. C. Escher is enriched by an understanding of the underlying symmetry operations.

References

1. Gallian, Joseph, *Contemporary Abstract Algebra*, Second ed., DC Heath, Lexington, Massachusetts, 1990.

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before July 1, 1998. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1998 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: xxwilke@acc.wuacc.edu).

PROBLEMS 510–514

Problem 510. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Eva and Al are going to run a 5K race. Eva's speed is proportional to the fourth root of the distance she has run. Al's speed is proportional to the distance left in the race. Eva reaches the midpoint of the race after 11.9 minutes while Al reaches the midpoint of the race after 13 minutes. Who will win the race and how long will it take for each runner to run the 5K?

Problem 511. Proposed by the editor.

Let $F(n) = \sum_{i=1}^{n} [\![\sqrt[3]{i}]\!]$, where $[\![\cdot]\!]$ denotes the greatest integer function and n is a positive integer. Find an infinite set of integers k for which $nk = F(n)$ for some n. (This delightful problem surfaced at the last convention!)

Problem 512. Proposed by the editor.

For every positive integer k, show that 3^{2k} can be represented as the sum of 3^k consecutive integers.

Problem 513. Proposed by the editor.

Consider the infinite sequence of integers $3_k1 = 333\cdots31$ in which there are k consecutive threes preceding the last digit. It is well known that 3_k1 is prime for $k = 1, 2, \ldots, 7$. Prove that this sequence contains an infinite number of composite terms and find one infinite sequence of composite terms.

Problem 514. Proposed by Charles Ashbacher, Hiawatha, Iowa.

Given any integer $n \geq 1$, the value of the Pseudo-Smarandache function $Z(n)$ is the smallest integer m such that n evenly divides $\sum_{k=1}^{m} k$. Consider the Smarandache Pierced Chain Sequence 101, 1010101, 10101010101, 101010101010101, 1010101010101010101, ... or $c(n) = 101 \cdot (1000)_{n-1}1$ for $n \geq 1$. Here $(1000)_{n-1}1$ denotes the number in which there are $n-1$ consecutive copies of 1000 followed by a 1. Prove the sequence $c(n)/101$ contains no prime numbers.

Please help your editor by submitting problem proposals.

SOLUTIONS 500–504

Problem 500. Proposed by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

The nth triangular number is given by $t_n = n(n+1)/2$ where n is a positive integer. Prove that there are an infinite number of triangular numbers which can be expressed as a sum of two distinct triangular numbers.

Solution by Cindy Chastain, student, Florida Beta, Florida Southern College, Lakeland, Florida.

Let $t_n = n(n+1)/2$ for any nonnegative integer n. Then

$$t_n - t_{n-1} = n. \tag{1}$$

Then if n is a triangular number, say $n = t_p = p(p+1)/2$, we have for any integer $p > 0$,

$$\begin{aligned} t_{t_p} - t_{t_p - 1} &= \frac{\frac{p(p+1)}{2}\left(\frac{p(p+1)}{2}+1\right)}{2} - \frac{\left(\frac{p(p+1)}{2}-1\right)\frac{p(p+1)}{2}}{2} \\ &= p(p+1)/2 = t_p. \end{aligned}$$

Solution by Clayton W. Dodge, University of Maine, Orono, Maine.

Let $t_n = n(n+1)/2$ for any nonnegative integer. Then we shall show that the equation $t_n + t_k = t_r$ has at least one solution for every positive integer $n > 1$. Consider $2t_n = 2t_r - 2t_k$, which can be rewritten as

$$(2) \qquad n(n+1) = r(r+1) - k(k+1) = (r-k)(r+k+1).$$

If the right side of equation (2) can be written as a product pq with $p < q$, then taking $p = r - k$ and $q = r + k + 1$ then we find $r = (p+q-1)/2$ and $k = (q-p-1)/2$. For k and r to be positive integers, p and q must be of opposite parity and $q > p+1$. It suffices to take $p = 1$ and $q = n(n+1)$ for $n > 1$ when $r = n(n+1)/2$ and $k = (n(n+1)/2) - 1$ so that $t_{t_n} = t_{t_n - 1} + t_n$ as found above.

If $n(n+1)$ has more than one odd prime factor, then there are other solutions; e.g. if $n = 8$, then in addition to $(p,q) = (1,72)$ we may also use $(3,24)$, which yields $(k,r) = (10,13)$ so that $t_8 + t_{10} = t_{13}$. Taking $p = n$ and $q = n+1$ leads to $k = 0$, which is not allowed.

Also solved by: Charles Ashbacher, Hiawatha, Iowa; Clayton W. Dodge, University of Maine, Orono, Maine; Russell Euler, Northwestern Missouri State University, Maryville, Missouri; Carl Libis, University of Alabama, Tuscaloosa, Alabama; Cindy O. Tosh, student, Missouri Theta, Evangel College, Springfield, Missouri and the proposer.

Editor's comment. The proposer gave and proved the relation $t_{4+5k} + t_{9+12k} = t_{10+13k}$.

Problem 501. Proposed by Charles Ashbacher, Cedar Rapids, Iowa.

Given any integer $n > 1$, the Smarandache function $S(n)$ is the smallest integer m such that n divides $m!$. The formula for the determinant of a 2×2 matrix is well known. Prove that there exists an infinite set of 4-tuples (a_1, a_2, a_3, a_4) such that:

a) all a_i are composite and greater than 2;

b) $a_i \neq a_j$ for $i \neq j$; and

c) we have

$$\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix} = \begin{vmatrix} S(a_1) & S(a_2) \\ S(a_3) & S(a_4) \end{vmatrix}.$$

Editor's comment. Since no solution has been received, the problem will remain open for another issue.

Problem 502. Proposed by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

Prove that the following procedure always yields a Pythagorean triangle. (1) Add together either the reciprocals of two consecutive even integers

or the reciprocals of two consecutive odd integers. (2) In the reduced form of the resulting fraction, the numerator and denominator are the legs of a Pythagorean triangle and the hypotenuse is 1+ the denominator if the denominator is even and the hypotenuse is 2+ the denominator if the denominator is odd.

Solution by Andy Miller, student, Iowa Delta, Wartburg College, Waverly, Iowa.

We consider two different cases depending upon whether we have two consecutive even integers or two consecutive odd integers.

Case A. Consecutive even integers. Consider the sum $(1/m) + (1/n)$ where m and n are the consecutive even integers $2k$ and $2k+2$ respectively. Then

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{2k} + \frac{1}{2k+2} = \frac{2k+1}{2k(k+1)}. \tag{1}$$

Since $\gcd(2k+1, 2k(k+1)) = 1$, the fraction on the right side of (1) is reduced to lowest terms. Then taking $a = 2k+1$, $b = 2k(k+1)$ and $c = 2k^2 + 2k + 1 = b + 1$, we have

$$\begin{aligned} a^2 + b^2 - c^2 &= a^2 + b^2 - (b+1)^2 = a^2 - (2b+1) \\ &= (2k+1)^2 - (2(2k(k+1)) + 1) = 0, \end{aligned}$$

which verifies that a, b and c are the sides of a Pythagorean triangle.

Case B. Consecutive odd integers. Consider the sum $(1/m) + (1/n)$ where m and n are the consecutive odd integers $2k+1$ and $2k+3$ respectively. Then

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{2k+1} + \frac{1}{2k+3} = \frac{4k+4}{4k^2+8k+3}. \tag{2}$$

Since $\gcd(4(k+1), (2k+1)(2k+3)) = 1$, the fraction on the right side of (2) is reduced to lowest terms. Then taking $a = 4k+4$, $b = (2k+1)(2k+3)$ and $c = (2k+1)(2k+3) + 2 = b + 2$, we have

$$\begin{aligned} a^2 + b^2 - c^2 &= a^2 + b^2 - (b+2)^2 = a^2 - (4b+4) \\ &= (4k+4)^2 - (4(2k+1)(2k+3) + 4) = 0, \end{aligned}$$

which verifies that a, b and c are the sides of a Pythagorean triangle. Combining the results of Cases A and B completes the proof of the desired result.

Also solved by: Charles Ashbacher, Hiawatha, Iowa; Cindy Chastain, student, Florida Beta, Florida Southern College, Lakeland, Florida; Clayton W. Dodge, University of Maine, Orono, Maine; Carl Libis, University of Alabama, Tuscaloosa, Alabama and the proposer.

Problem 503. Proposed by C. Bryan Dawson, Emporia State University, Emporia, Kansas.

Using only a compass and an unmarked straightedge, construct the orthocenter, circumcenter, centroid, and the nine-point circle of an arbitrary triangle using the compass six or fewer times. The drawing of the nine-point circle is included as one of the uses of the compass.

Editor's comment. Since no solution has been received, the problem will remain open for another issue.

Problem 504 (corrected). Proposed by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

If A, B, and C are the angles of a triangle, prove that

$$2\cos A\cos B\cos C = 1 - \cos^2 A - \cos^2 B - \cos^2 C.$$

Solution by J. Sriskandarajah, University of Wisconsin Center—Richland, Richland Center, Wisconsin.

Since A, B and C are the angles of a triangle, $A + B + C = 180°$; thus $\cos C = -\cos(A + B)$. Then

$$\begin{aligned} \cos A\cos B\cos C &= \cos A\cos B(-\cos(A+B)) \\ &= \cos A\cos B\sin A\sin B - \cos^2 A\cos^2 B. \end{aligned} \tag{1}$$

Also

$$\begin{aligned} \cos^2 C &= \cos^2(A+B) = (\cos A\cos B - \sin A\sin B)^2 \\ &= \cos^2 A\cos^2 B - 2\cos A\cos B\sin A\sin B + (1-\cos^2 A)(1-\cos^2 B) \\ &= 2\cos^2 A\cos^2 B - 2\cos A\cos B\sin A\sin B + 1 - \cos^2 A - \cos^2 B. \end{aligned} \tag{2}$$

Combining (1) and (2) we have

$$\cos^2 C = -2\cos A\cos B\cos C + 1 - \cos^2 A - \cos^2 B$$

or

$$2\cos A\cos B\cos C = 1 - \cos^2 A - \cos^2 B - \cos^2 C.$$

Also solved by: Clayton W. Dodge, University of Maine, Orono, Maine; Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri and the proposer.

Editor's comment. Both Clayton W. Dodge and the proposer derived the additional identity $4\cos A\cos B\cos C = -1 - \cos 2A - \cos 2B - \cos 2C$.

KME Quiz Results

The winners of the KME Quiz contest are Vicki Nelson and Shari Brand of Missouri Iota (Missouri Southern State College). Congratulations! Although their entry was not perfect, it contained the greatest number of correct answers (nine) from among those received. They each have received a two-year extension on their subscriptions. If you missed it, the quiz is repeated below; **the answers appear on page 23.** Can you beat Vicki and Shari?

1. Name the only state with active Kappa, Mu, and Epsilon chapters of KME.

2. Name the states with active chapters with names of exactly two of Kappa, Mu, and Epsilon.

3. Which state had the first Delta chapter?

4. Which state has held the record for having the most chapters for the longest period of time?

5. Which state with five or more chapters has gone the longest without an installation of a new chapter?

6. Name the first letter of the Greek alphabet for which more active chapters of KME are named than are named for the preceeding letter.

7. What is the greatest number of currently active chapters that happened to be installed in the same year?

8. Fifty years ago, was "University" or "Teacher's College" more commonly in the name of an institution with a KME chapter?

9. What chapter is credited with writing "The Math Student Blues," a song which appeared in *The Pentagon* some time in the 1940's?

10. Name the only institution with an active chapter of KME whose name does not include either "University" or "College."

11. Name the only city with three active chapters of KME.

12. Which chapter has initiated the most total members into KME?

13. Which active chapter is located the furthest north?

14. Which active chapter is located the furthest east?

Kappa Mu Epsilon News

Edited by Don Tosh, Historian

News of chapter activities and other noteworthy KME events should be sent to Don Tosh, Historian, Kappa Mu Epsilon, Mathematics Department, Evangel College, 1111 N. Glenstone, Springfield, MO 65802, or to toshd@evangel.edu.

INSTALLATION OF NEW CHAPTERS

Pennsylvania Omicron

University of Pittsburgh at Johnstown, Johnstown

The installation of the Pennsylvania Omicron Chapter of Kappa Mu Epsilon was held on April 10, 1997, in the Irving J. Whalley Chapel on the campus of the University of Pittsburgh at Johnstown. Dr. Peter Skoner, region two director of Kappa Mu Epsilon, conducted the installation ceremony. Ms. Nina Girard, assistant professor of mathematics education, arranged the ceremony and served as conductor. Thirty-five students and eight faculty constituted the founding group of the new chapter at the University of Pittsburgh at Johnstown. Those initiated were:

Students: Melissa Adamy, Jennifer Barner, Maryann Cernansky, Daniel Coleman, Lori Duncan, Heather Eckenrode, Wendy Fry, Nikki Gerba, Elizabeth Heggi, Daniel Heisig, Alice Hetz, Barbara Hoelzeman, Kelley Hurst, Sarah Leach, Misty Litzinger, Nathan Lloyd, Ryan McNeal, Hope Meyers, David Myers, John Neff, Corey Norris, Melissa Owens, Becky Piscitella, David Rosendale, Erin Sikora, Nicholas Sikora, Shannon Speelman, Robert Speicher, Jessica Stainback, Michael Troyan, Damon Vaccaro, Aaron Verhovsek, Allen Wilson, Robert Yanosky, and Scott Young.

Faculty: Stephen Curran, Ildefonso Cruz, Nina Girard, Boris Kushner, Marc Mehlman, Theresa Shustrick, Thomas Sigmund, and Florence Warfel.

Also in attendance were administrators from the University of Pittsburgh at Johnstown including Dr. James Alexander, interim vice president for Academic Affairs, Dr. William Brice, division head of Natural Sciences, and the parents and friends of many of the inductees.

Dr. Skoner began the evening ceremony with an introduction to and a brief history of Kappa Mu Epsilon. Officers installed during the ceremony were Daniel Coleman, president; Melissa Owens, vice president; Lori Duncan, recording secretary; Nikki Gerba, treasurer; and Sarah Leach, historian. Faculty member Nina Girard accepted the responsibilities of corresponding secretary and faculty sponsor. After the ceremony, Ms. Girard offered a few acknowledgments and Dr. Florence Warfel, Mathematics Department chair, offered concluding remarks. A reception was held following the ceremony.

Michigan Delta

Hillsdale College, Hillsdale

The installation of the Michigan Delta Chapter of Kappa Mu Epsilon was held on April 30, 1997, on the campus of Hillsdale College. Dr. Arnold Hammel, national president of Kappa Mu Epsilon, conducted the installation ceremony. Dr. Waldemar Weber, national secretary of Kappa Mu Epsilon, served as the conductor. Mark Watson, professor of mathematics, Hillsdale College, arranged the ceremony. Twelve students and four faculty constituted the founding group of the new chapter at Hillsdale College. Those initiated included:

Students: Stephanie Boll, Debra Eible, Kary Hackbarth, Kelly Harnish, Matthew Hunt, Rajeev Karki, Jeffrey Khoubesser, Kurt Knueve, Douglas Martindale, Jason Osborne, Nicholas Peters, and Joseph Pietraszewski.

Faculty: Kathy Andrews, John Reinoehl, Mark Watson, and Reinhardt Zeller.

Officers installed during the ceremony were Jeffrey Khoubesser, president; Matthew Hunt, vice president; Douglas Martindale, secretary; and Debra Eible, treasurer. Faculty member Kathy Andrews accepted the responsibilities of corresponding secretary and faculty sponsor.

After the installation ceremony, Dr. Hammel gave an introduction to and brief history of Kappa Mu Epsilon. Dr. Weber also offered congratulatory remarks. A reception was held following the ceremony.

CHAPTER NEWS

AL Gamma

Chapter President — Cheryl Coley

University of Montevallo, Montevallo — 13 actives, 4 associates

Other 1997–98 officers: Ava Putman, vice president; Carrie Chickerell, secretary; David Taylor, treasurer; James Ochoa, corresponding secretary; Karolyn Morgan, faculty sponsor.

AL Zeta

Chapter President — Scott A. Matthews

Birmingham-Southern College, Birmingham — 27 actives

Other 1997–98 officers: James M. Corder, vice president; Melissa Boren, secretary/treasurer; Mary Jane Turner, corresponding secretary; Raju Sriram, faculty sponsor.

CA Gamma

Chapter President — Tom Sanborn

California Polytechnic State University, San Luis Obispo — 17 actives

Other 1997–98 officers: Susyn Heidenrich, vice president; Judy Fetcho, secretary/treasurer; Kent Morrison, corresponding secretary/faculty sponsor.

CO Gamma

Chapter President — Heather Duncan

Fort Lewis College, Durango — 20 actives, 12 associates

In the spring semester we had a meeting to plan our trip to the biennial convention. We also had a pizza party initiation where twelve new members were inducted. The convention was attended by Heather Duncan, Cynthia Hilliker, Mary Middleton-Short, and Professor Dick Gibbs. After the convention, we had a meeting to report on the convention and to elect 1997–98 chapter officers. Other 1997–98 officers: Travis Kirkpatrick, vice president; Cindy Hilliker, secretary; David Crawford, treasurer; Richard A. Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

CO Delta

Chapter President — Adam Furst

Mesa State College, Grand Junction — 27 actives, 14 associates

Natisha Littlejohn and Donna Hafner attended the national convention. Clinton Thompson was unable to attend the convention, but his paper "On the Algebraic Nature of Cantor Set Construction" was submitted for publication in *The Pentagon* and presented at Mesa State's Brown Bag Seminar series in mathematics. Other 1997–98 officers: Christopher Day, vice president; Michelle McGarry, secretary; Saori Okamura, treasurer; Donna Hafner, corresponding secretary; Kenneth Davis, faculty sponsor.

GA Alpha

Chapter President — Tonja Davis

State University of West Georgia, Carrollton — 28 actives, 12 associates

The chapter met on May 28, 1997, and initiated 12 new members, our largest initiation in years. At the reception honoring the new members, the following KME students were announced as being awarded scholarships for 1997–98: Tonja Davis, Andrea Galinat, Kimberly Hannah, Laura Henkelman, Stephanie Parker, and John Pickering. Other 1997–98 officers: Stephanie Parker, vice president; Lisa Farmer, secretary; Marta Valentic, treasurer; Thomas J. Sharp, corresponding secretary; Thomas J. Sharp and Mark Faucette, faculty sponsors.

IL Beta

Chapter President — Justin Large

Eastern Illinois University, Charleston — 42 actives, 15 associates

The chapter's activities included several meetings, a math contest, a book sale, an initiation, a banquet and a picnic. Other 1997–98 officers: Brett Swanson and Valerie Kleinfeld, vice presidents; Gwen Fuchs, secretary; Nikki Blickenstaff, treasurer; Lloyd Koontz, corresponding secretary/faculty sponsor.

IL Eta

Chapter President — Bethany Webb

Western Illinois University, Macomb — 10 actives, 12 associates

Chapter activities for the spring included talks on cryptology, statistics in medical research, "If Copernicus Had a Computer," and mathematics graphics in *Word Perfect*. Other activities were a reception for new math majors, hosting a high school math contest, KME initiation, and a field trip to State Farm's home office. Other 1997–98 officers: Thomas T. Johnson, vice president; Katherine Fijalkowski, secretary; Andrea L. Crary, treasurer; Larry J. Morley, corresponding secretary/faculty sponsor.

IN Alpha

Chapter President — Jennifer Bowman

Manchester College, North Manchester — 13 actives, 5 associates

The mathematics department hosted a banquet that combined several activities. Four students and one faculty were initiated into KME and elections were also held for new officers. Sigma Pi Sigma also took part in the banquet, and graduating seniors in math, physics, and computer science were honored. Other 1997–98 officers: Jennifer Kunkle, vice president; Heather Bailey, secretary; Aaron Richardson, treasurer; Stanley Beery, corresponding secretary; Andy Rich, faculty sponsor.

IN Beta

Chapter President — Jessica Goldsand

Butler University, Indianapolis — 15 actives, 13 associates

Initiation was held on April 18, 1997. Other 1997–98 officers: Lee Duncan, vice president; Catherine Tischio, secretary; Yuzhen Ge, corresponding

secretary.

IN Gamma Chapter President — Rhonda Merrill
Anderson University, Anderson 12 actives, 6 associates

Other 1997–98 officers: Julia Oliver, vice president; Michelle Muhlenkamp, secretary; Stanley L. Stephens, corresponding secretary/faculty sponsor.

IN Delta Chapter President — David M. Zimmer
University of Evansville, Evansville 38 actives, 23 associates

Other 1997–98 officers: Dennis E. Goodman, vice president; Jaclyn E. Cron, secretary; Mohammad K. Azarian, treasurer/corresponding secretary/faculty sponsor.

IA Alpha Chapter President — Suzanne Shontz
University of Northern Iowa, Cedar Falls 39 actives

Students who presented papers at local KME meetings included Erin Blaine, "The Odds of a DNA Match;" Marc Pedersen, "A Casual Look at the Sierpinski Gasket;" Eric Shields, "Predicting Conditions Necessary for Linear Absorptive Thin Transmission Holograms;" and Angela Koos, "Game Theory." Five members were initiated into KME. Sarah Lacox, Andy Shafer, Suzanne Shontz and faculty John Bruha and John Cross attended the national convention. Suzanne's paper "Molecules and Their Symmetries" placed in the top four. Locally, KME members assisted with the telethon, Science and Technology Day, and Mathematics Awareness Week. Three professors retire this year. John Bruha, Diane Baum, and Augusta Schurrer have for decades been friends of Iowa Alpha and we will miss their generous support. We hope they will join us as faculty emeriti. Other 1997–98 officers: Amber Grotjohn, vice president; Sarah Lacox, secretary; Erin Blaine, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

IA Delta Chapter President — Shilah Lybeck
Wartburg College, Waverly 38 actives, 9 associates

Our chapter continued the once-a-month Sunday evening meals begun in the fall term as socials. The February business meeting included final plans for the trip to Chicago's Science and Industry Museum (15 went), forming a planning committee for Math Awareness Week, planning for Explorations in Mathematical Sciences (which was held in March), and shared experiences from Andy Miller's presentation at the joint MAA and AMS meetings in San Diego. Eric Stahlberg was the banquet speaker at which 21 members were initiated into KME. In March, three members shared their experiences at the Mathematical Contest in Modelling. In May we

again had a picnic. Other 1997–98 officers: Christopher Judson, vice president; Joshua Nelson, secretary; Emily Bailey, treasurer; August Waltmann, corresponding secretary; Glenn Fenneman, faculty sponsor.

KS Alpha

Chapter President — Mark Albert
Pittsburg State University, Pittsburg — 52 actives, 14 associates

Spring activities began with a pizza party and initiation in February. Fourteen members were initiated. KME member Shannon Wilkinson spoke on "Three Greek Impossibilities" in the March meeting. Professor David Surowski from Kansas State University presented "Cyclic Polynomials over the Rational Field" at the next meeting. Six students and five faculty attended the national convention. Two students presented papers: Kari Hamm, "Newton to Chaos," and Shannon Wilkinson, "Three Greek Impossibilities." In April, Dr. Yaping Liu of PSU spoke on fallacies, paradoxes, games, and puzzles. The final meeting in May combined a Mexican meal, ice cream social, announcing winners of awards to outstanding math graduates, and election of officers. Other 1997–98 officers: Kathy Denney, vice president; Kari Hamm, secretary; Lisa Swaim, treasurer; Cynthia Woodburn, corresponding secretary; Yaping Liu, faculty sponsor.

KS Beta

Chapter Co-Presidents — Rae Ann Levalley, Kristen Goetz
Emporia State University, Emporia — 25 actives, 2 associates

Other 1997–98 officers: Rae Ann Levalley and Kristen Goetz, vice presidents; Shannon Decker and Megan Little, treasurers; Connie S. Schrock, corresponding secretary; Larry Scott, faculty sponsor.

KS Gamma

Benedictine College, Atchison — 15 actives, 1 associate

Jason Suelter was initiated into KS Gamma on March 18. Following the initiation, Jeff Blanchard presented his talk "Choices, Choices, Choices" on the Axiom of Choice, which he also presented at the national convention. Beth Gilbert presented "Tournaments and Graph Theory." Jeff Blanchard, Chad Eddins, Erik Kurtenbach, Kevin Slattery and faculty Richard Farrell and Jo Ann Fellin attended the national convention. Chad Eddins and Jeff Blanchard received the Sister Helen Sullivan Scholarship. A steak and chicken cookout honoring seniors was held at Richard Farrell's home in April. George Blodig retires this year after 38 years at St. Benedict's College. He will be missed as our unofficial chapter photographer. Student officers will be elected in the fall. Other 1997–98 officers: Jo Ann Fellin, corresponding secretary/faculty sponsor.

KS Delta

Chapter President — Douglas Appenfeller
Washburn University, Topeka — 30 actives, 10 associates

On February 17, 1997, KS Delta initiated 10 new members at the an-

nual initiation banquet. The eight members who traveled to the national convention were Doug Appenfeller, Mandy Chester, Justin Freeby, Laurie Payeur, and faculty Donna LaLonde, Al Riveland, and Ron Wasserstein. Ken Wilke, editor of the Problem Corner in *The Pentagon*, also attended. Other activities throughout the semester included several picnics held in conjunction with the university math club, Mathematica. Other 1997–98 officers: Laurie Payeur, vice president; Chung-Fei Tang, secretary; Justin Freeby, treasurer; A. Allan Riveland, corresponding secretary; Donna LaLonde and Ron Wasserstein, faculty sponsors.

KS Epsilon

Fort Hays State University, Hays — 27 actives, 5 associates

KS Epsilon participated with a float in the homecoming parade and won the top award. The float theme was mathematics through history. Other activities included a picnic with current members and retired faculty and an initiation banquet in April with an invited speaker. Approximately 30 attended the banquet in spite of the blizzard that cancelled afternoon classes. Student officers will be elected in the fall. Other 1997–98 officers: Ellen Veed, corresponding secretary; Linda Kallom, faculty sponsor.

KY Alpha

Eastern Kentucky University, Richmond — 23 actives, 21 associates

The spring semester began with floppy disk sales (together with the ACM chapter). A January meeting made plans for the national convention. On March 3, 21 students were initiated, and Dr. Ken Nelson gave a talk "The Period Doubling Route to Chaos." Fourteen students and Pat Costello attended the national convention. Elizabeth Barrett presented "The Language of Mathematics: Teaching in the Proximal Zone" and C. E. Davis presented "An Algorithmic Method for the Construction of a 4×4 Magic Square Consisting of Only Prime Numbers." Besides attending meetings, the students had an interesting Friday evening at The Class Act. On Saturday morning, Pat Costello was installed as president of KME. The semester ended with a student/faculty picnic at Lake Reba Park. Student officers will be elected in the fall. Other 1997-98 officers: Patrick Costello, corresponding secretary.

KY Beta

Chapter President — Story Robbins

Cumberland College, Williamsburg — 34 actives

KY Beta initiated 14 students and one faculty on February 21 during a banquet at the Atrium. Members initiated last year and graduating seniors were also recognized during the banquet, presided over by outgoing president Timothy Wilson. On April 7 the chapter had an informal pizza party at Dr. Ramey's house. On April 24, the chapter hosted, jointly

with the Mathematics and Physics Club, Dr. Carroll Wells from Western Kentucky University. He spoke on the "Magic of Topology." In April, members assisted in hosting a regional high school math contest. In May the entire department, including the Math and Physics Club and the KY Beta chapter, had a picnic at Briar Creek Park. Other 1997–98 officers: Candace Osborne, vice president; Laura Thompson, secretary; Melynda Hazelwood, treasurer; Jonathan Ramey, corresponding secretary; John Hymo, faculty sponsor.

MD Alpha

Chapter President — Judith Simon

College of Notre Dame of Maryland, Baltimore — 13 actives, 6 associates

In addition to the usual spring fund-raising event, the members had the opportunity to talk with and hear from Dr. Horace Russell, dean of engineering at the University of Maryland, on April 10. On May 5, three students were initiated and three became associate members. At this meeting Dr. Meg Urry from the Space Telescope Science Institute spoke on "Highlights from the Hubble Space Telescope." Other 1997–98 officers: Marie Morrow, vice president; Michelle Yeager, secretary; Laura Bopp, treasurer; Sister Marie Augustine Dowling, corresponding secretary; Joseph DiPienze, faculty sponsor.

MD Beta

Chapter President — Jason Barr

Western Maryland College, Westminster — 29 actives

MD Beta hosted a Mathematics Careers Night Banquet on April 29. Alumni speakers were Christopher Conklin, an actuary; Mary Beth Kepner from the FBI; and Terry Holland, a statistician with the National Agricultural Statistics Service. Other 1997–98 officers: Robert Newman, vice president; Julie Brown, secretary; Fred Butler, treasurer; Linda R. Eshleman, corresponding secretary; James Lightner, faculty sponsor.

MD Delta

Chapter President — Heidi Femi

Frostburg State University, Frostburg — 43 actives

MD Delta opened the spring semester with a reprise of last fall's mathematical treasure hunt at a get acquainted meeting for students about to join KME. The following weekend 10 new members were initiated. Brother Patrick Carney spoke at the initiation on "Mathematical Humor." In April Dr. Edward White presented a talk entitled "Some Mathematical Fallacies, or Ten Proofs That I Am Tom Cruise." At the final meeting in May former chapter president Jesse Siehler gave a presentation dealing with his experience studying math in Hungary during the fall semester. Other 1997–98 officers: Steven Fairgrieve, vice president; Sean Carley, secretary; Andrew Adam, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

MI Beta

Chapter President — Nicole Weitzel

Central Michigan University, Mount Pleasant — 25 actives

Time was spent early in the semester planning for attendance at the national convention. Six students and faculty member Arnie Hammel attended. It was a great experience for all and we thank the three KME chapters for doing a great job in planning and hosting. Meetings were held every three weeks. Guest speaker at our spring initiation was statistician Felix Famoye of the CMU Mathematics Department. Twelve members were initiated. We closed out the year with a picnic with department members and the other mathematical organizations in the department. Other 1997–98 officers: Courtney Stapleton, vice president; Sara Masko, secretary; Ben Vesper, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

MS Alpha

Chapter President — Patricia A. DiBlasi

Mississippi University for Women, Columbus — 13 actives, 2 associates

General meetings were held October 10 and April 4. An initiation was held November 1 and a joint initiation/election was held April 22. Other 1997–98 officers: Lani R. Crowder, vice president/treasurer; Patricia A. DiBlasi, secretary; Shaochen Yang, corresponding secretary; Jane Wenstrom, faculty sponsor.

MS Gamma

Chapter President — Craig Collier

University of Southern Mississippi, Hattiesburg — 20 actives, 4 associates

Other 1997–98 officers: Paula Thigpen, vice president; Michelle Hill, secretary; Alice W. Essary, treasurer/corresponding secretary; Barry Piazza, faculty sponsor.

MS Epsilon

Chapter President — Ashley Riley

Delta State University, Cleveland — 11 actives

Other 1997–98 officers: Ken Byars, vice president; Chad Huff, secretary; Paula A. Norris, corresponding secretary; Rose Strahan, faculty sponsor.

MO Beta

Chapter President — Dennis Wise

Central Missouri State University, Warrensburg — 25 actives, 6 associates

MO Beta had monthly meetings in February, March, and April. In February, David Clements spoke on "Looking for Mersenne's Primes." Dr. Sundberg gave a presentation on spatial abilities in March. Initiation was also held in March. In April, Jeff Hanna, an attorney and graduate of the Math and Computer Science Department, gave the Klingenberg Lecture. Eight students and two faculty attended the national convention in April. Other activities included a book sale, volunteering at Math Relays, and a

pizza party. A committee was formed to update the bylaws. The revised bylaws will be voted on in the fall. Other 1997–98 officers: Tammy Surfus, vice president; Aaron Shaefer, secretary; Cassie Young, treasurer; Melissa Elliott, historian; Rhonda McKee, corresponding secretary; Scotty Orr, Larry Dilley and Phoebe Ho, faculty sponsors.

MO Gamma

Chapter President — Jennifer Puls

William Jewel College, Liberty — 20 actives, 6 associates

Other 1997–98 officers: Allison Cooper, vice president; James Brochtrup, secretary; Joseph T. Mathis, treasurer/corresponding secretary/faculty sponsor.

MO Epsilon

Chapter President — Becky Grier

Central Methodist College, Fayette — 12 actives, 6 associates

Other 1997–98 officers: Jessica Simpson, vice president; Keith Ogle, secretary/treasurer; William D. McIntosh, corresponding secretary; Linda O. Lembke and William D. McIntosh, faculty sponsors.

MO Eta

Chapter President — Laurel Berner

Truman State University, Kirksville — 16 actives

We held our annual KME Math Expo for high school students. The competition consisted of nine tests which were written by KME members and given to high school students. This year we had nine schools competing in two different divisions. Other 1997–98 officers: Amanda Nixon, vice president; Russ Whiteford, secretary; Karen VanCleave, treasurer; Mary Sue Beersman, corresponding secretary; Joe Hemmeter and Gregory Boucher, faculty sponsors.

MO Theta

Chapter President — Christie Tosh

Evangel College, Springfield — 12 actives, 6 associates

Four new members were initiated in January, and new officers were elected for the year. The national convention was held at Evangel, with Drury (MO Kappa) and SMSU (MO Alpha) helping out. The convention went well and Don Tosh, sponsor and corresponding secretary, was elected national historian. Other 1997–98 chapter officers: Amy Lee and Stan Roberts, vice presidents; Don Tosh, corresponding secretary/faculty sponsor.

MO Iota

Missouri Southern State College, Joplin — 18 actives

Semester activities included a presentation by student member Mark Stamps concerning his internship with Argonne Labs in Chicago, and a talk by MO Iota president Vicki Nelson on "Art and Geometry." Nine

members were initiated in early March at the annual initiation banquet. Seven students and two faculty attended the national convention in April. Faculty sponsor Charles Curtis served on the convention awards committee and corresponding secretary Mary Elick completed her second four-year term as national historian. New student officers will be elected in the fall. Other 1997–98 officers: Mary Elick, corresponding secretary; Charles Curtis, faculty sponsor.

MO Lambda Chapter President — Perriann McCoppin
Missouri Western State College, St. Joseph 38 actives

MO Lambda initiated nine new members on March 9. Guest speaker for the program was Dr. Keith Brandt. A leadership workshop for officers and members of KME was held on February 28. Mr. Bill Huston, associate professor of mathematics, was workshop leader. A bake sale was held April 2 to raise funds for attending the national convention. Five members attended the national convention: Tanya Griffin, Perriann McCoppin, Cynthia Ready, William Slabaugh, and sponsor John Atkinson. MO Lambda members volunteered their services during the Missouri Section of the MAA meeting on April 11 and 12 at MWSC. Students assisted with details such as registration and hosting presentation sessions. Officers were elected at the annual Math Club/KME Spring Picnic at the home of past president Brian Bettis. Other 1997–98 officers: Stephanie Tingler, vice president; William Slabaugh, secretary; Sean Hutto, treasurer; John Atkinson, corresponding secretary; Jerry Wilkerson, faculty sponsor.

NE Alpha Chapter President — Rustin Slaughter
Wayne State College, Wayne 41 actives, 20 associates

On February 20 we elected officers and initiated 20 members. We held two general meetings during which we discussed fund raisers and socials for the fall. We are planning a pizza drive and many fun socials to unify our chapter. Five members attended the national convention: Tim Anglen, Chris Headley, Brian Steinkraus, Ben Wittler, and faculty Steven Deckelman. They enjoyed the presentations and maybe in the near future we will be able to take what they brought back and become more involved in the convention. The chapter also helped with the math department picnic at WSC in which we honored the favorite teacher. The chapter helped at the Math Contest on campus on May 12. Other 1997–98 officers: Karl Laursen, vice president; Ann Boes, secretary; Renee Fuhr, treasurer; John Fuelberth, corresponding secretary; Jim Paige, faculty sponsor.

NM Alpha Chapter President — Jason Strauch
University of New Mexico, Albuquerque 75 actives, 14 associates

Other 1997–98 officers: Larry Montaño, vice president; Shawn McCul-

lough, secretary/treasurer; Archie Gibson, corresponding secretary/faculty sponsor.

NY Alpha Chapter President — Norbert Lis
Hofstra University, Hempstead 15 actives, 12 associates

We had a talk by a recent Hofstra graduate about her experiences in graduate school. We also had a trip to the New York Hall of Science in Queens to see the first traveling museum show in the U. S. devoted to mathematics. Other 1997–98 officers: Adam Katz, vice president; Ophir Feldman, secretary; Lisa Fontana, treasurer; Aileen Michaels, corresponding secretary/faculty sponsor.

NY Eta
Niagara University, Niagara 12 actives, 15 associates

Student officers will be elected in the fall. Other 1997–98 officers: Robert L. Bailey, corresponding secretary; Kenneth Bernard, faculty sponsor.

NY Kappa Chapter President — Leigh Ann Gervis
Pace University, New York 10 actives, 4 associates

Other 1997–98 officers: Geraldine Taiani, corresponding secretary; Robert Cicenia, faculty sponsor.

NY Lambda Chapter President — Kelli Ann Polotaye
C. W. Post Campus of Long Island University, Brookville 28 actives

Eight students were initiated during the annual banquet at the Greenvale Town House restaurant on March 17. Kelli Polotaye spoke on "Elliptic Curves, Groups and Fermat" and the evening concluded with the announcement by Dr. Neo Cleopa of the departmental awards. Ms. Polotaye and Dr. Rockett attended the national convention. Other 1997–98 officers: Joseph D. Sprague, vice president; Robin M. Cancellieri, secretary; Jason R. Rand, treasurer; Andrew M. Rockett, corresponding secretary; Sharon Kunoff, faculty sponsor.

OH Gamma Chapter President — Amy Booth
Baldwin-Wallace College, Berea 37 actives

We had no one, the chapter was almost dormant, and then we initiated 37 on May 10. Other 1997–98 officers: Cassandra Kirby, vice president; Margot Mailloux, secretary; Anila Xhunga, treasurer; David Calvis, corresponding secretary/faculty sponsor.

OH Zeta Chapter President — Chetan Kandhari
Muskingum College, New Concord 20 actives, 7 associates

Eight members journeyed to the national convention: Melissa Frutig,

Chetan Kandhari, Jim Likovic, Chris Luzier, Sarah McConnell, emeritus professor Jim Smith, and professors David Craft and Rich Daquila. Other 1997–98 officers: Chris Luzier, vice president; Melissa Frutig, secretary/treasurer; David Craft, corresponding secretary; Richard Daquila, faculty sponsor.

OK Alpha Chapter President — Josh Baker

Northeastern State University, Tahlequah 35 actives, 1 associate

We continue to have joint activities with NSU's student chapter of the MAA and participate in "The Problem Solving Competition" sponsored by the MAA. Eleven students were initiated in the banquet room of Roni's Pizza. It was well attended by faculty and students. In April, Dr. Itrel Monroe, mathematics professor at the University of Arkansas, spoke on "The Weierstrass Theorem." His presentation was excellent and he also talked about graduate mathematics at the University of Arkansas. We celebrated Math Awareness Week with several activities. Bill King, instructor of math and computer science at NSU, gave a presentation on searching the Internet for math and computer science topics. Other professors in attendance shared their favorite bookmarks. We also sponsored the annual KME "Pre-Finals Ice Cream Social" for faculty and math majors. Other 1997–98 officers: Dan Sisk, vice president; Tera McGrew, secretary; Russell Cravens, treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

OK Gamma Chapter President — Marina Rameriz

Southwestern Oklahoma State University, Weatherford 12 actives

Other 1997–98 officers: Cara Scrum, vice president; Zoe Yau Yok, secretary; Jennifer Pibler, treasurer; Wayne Hayes, corresponding secretary; Rochelle Beatty, faculty sponsor.

PA Alpha Chapter President — Matt Reel

Westminster College, New Wilmington 16 actives, 6 associates

The spring semester was very busy for PA Alpha. We started by initiating six students on March 12. For Math Awareness Week in April we held a seventh grade mathematics competition for one of the local middle schools. The homeroom that did the best on the test won a pizza party. Also during the week we held our second annual Spring Banquet and Career Night. This event was co-sponsored by the local chapter of Upsilon Pi Epsilon, the computer science honor society. A math teacher, a computer programmer, and an actuary were the three speakers. The Spring Banquet is held for all math and computer science majors. Finishing Math Awareness Week, we held our first annual High School Mathematics Competition for six of the local schools. We had nine participants and hope that this competition will grow in the future. We finished the semester with pizza on reading day

and donuts and coffee every morning during finals. Other 1997–98 officers: Jason Levish, vice president; Erin Murray, secretary; Stephanie Tangora, treasurer; Carolyn Cuff, corresponding secretary/faculty sponsor.

PA Gamma Chapter President — Amanda Beisel
Waynesburg College, Waynesburg 18 actives, 8 associates

Other 1997–98 officers: Jennifer Baltes, vice president; Kristien Fox, secretary; Angela Colinet, treasurer; Anthony A. Billings, corresponding secretary/faculty sponsor.

PA Delta Chapter President — Jennifer Snyder
Marywood College, Scranton 2 actives, 4 associates

Jennifer Snyder gave a presentation at Moravian College's Student Conference on February 22. Some members attended the PCTM Annual Conference in March. Marywood's Annual High School Math Contest was held on April 5 and 20. Some members attended the NPCTM 10th Anniversay meeting on April 24. We initiated four students on May 7. Other 1997–98 officers: Maura Regan, vice president; Brenda Rudzinski, secretary/treasurer; Sr. Robert Ann Von Ahnen, IHM, corresponding secretary/faculty sponsor.

PA Eta Chapter President — Frederick W. Lamm
Grove City College, Grove City 28 actives, 3 associates

On March 18 we met to elect officers and initiate two members. Afterwards, Dr. Gary Thompson talked about the harmonic series and how slowly it diverges. KME annually administers a competitive math exam to select the outstanding freshman math student. The test was given on April 22 and 24 this year. KME members took turns staffing a "KME Study Room" one evening each week. The sessions were advertised to all math students and any who wished could come and get help on their math courses. Other 1997–98 officers: Greta E. Kessler, vice president; Sarah M. Lawhon, secretary; Diane L. Schnelbach, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

PA Iota Chapter President — Abby Todd
Shippensburg University, Shippensburg 25 actives, 7 associates

Other 1997–98 officers: Peter Burnett, vice president; Nycole Miller, secretary; Stacey Lytle, historian; Mike Seyfried, corresponding secretary; Gene Fiorini, faculty sponsor.

PA Kappa Chap. Pres.—Paul O'Connor, Cheryll Stone-Schwendiman
Holy Family College, Philadelphia 7 actives, 3 associates

Three members were initiated on April 3. The chapter celebrated Math

Awareness Month by hosting several events on campus, including the chapter's third annual Grade School Math Competition in which nine local elementary schools participated. Nick Gross, chapter president, gave a talk on his co-op experiences during the spring semester as a statistics intern with the Philadelphia 76ers. Other 1997–98 officers: Brian Minster, vice president; Sr. Marcella Louise Wallowicz, corresponding secretary/faculty sponsor.

SC Delta Chapter President — Craig Group
Erskine College, Due West 5 actives, 6 associates

Other 1997–98 officers: Vicky Williams, vice president; Margaret Houck, secretary/treasurer; Anne Bowe, corresponding secretary; Susan Patterson, faculty sponsor.

SD Alpha Chapter President — Kristi Schuster
Northern State University, Aberdeen 12 actives, 3 associates

Other 1997–98 officers: Margo Maynard, vice president; Becky Hanson, secretary; Stacy Garrels, treasurer; Lu Zhang, corresponding secretary; Raj Markanda, faculty sponsor.

TN Alpha Chapter President — Sharon Tyree
Tennessee Technological University, Cookeville 15 actives

Other 1997–98 officers: Ara Nazarian, vice president; Leigh Anne Smith, secretary; Stacey Hund, treasurer; Frances E. Crawford, corresponding secretary; Jeff Norden, faculty sponsor.

TN Beta Chapter President — Ellie Alevritis
East Tennessee State University, Johnson City 18 actives, 10 associates

Ten members were initiated at the annual spring initiation in the Culp University Center on April 4. The initiates were accompanied by friends and relatives. A presentation was made on "Giant Red Stars" by Dr. Don Luttermoser from the East Tennessee State University Physics Department. Other 1997–98 officers: Corey Williams, vice president; Brian Heaton, secretary; Chris Wallace, treasurer; Lyndell Kerley, corresponding secretary/faculty sponsor.

TN Gamma Chapter President — Jennifer Murrah
Union University, Jackson 23 actives, 4 associates

Other 1997–98 officers: Lori Davis, vice president; Mandy Davidson, secretary/treasurer; Matt Lunsford, corresponding secretary; Troy Riggs, faculty sponsor.

TN Delta Chapter President — Michael D. Kelley
Carson-Newman College, Jefferson City 9 actives, 2 associates

Other 1997–98 officers: Min D. Lee, vice president; Rebecca J. Gritman, secretary/treasurer; Catherine Kong, corresponding secretary/faculty sponsor.

TX Alpha Chapter President — Heather Crawford
Texas Tech University, Lubbock 53 actives, 26 associates

Other 1997–98 officers: Leo Mendoza, vice president; Ellen Strayhorn, secretary; Janet Webb, treasurer; Victor Shubov, corresponding secretary/faculty sponsor.

TX Eta Chapter President — Sylvia Cantu
Hardin-Simmons University, Abilene 12 actives

TX Eta sponsored a pizza/movie night on November 8. Twelve members were initiated at the 23rd annual Initiation Banquet on March 22 at the Embassy Suites. Total membership in the local chapter now stands at 187. Following the ceremony Ms. Terry Minami, secretary to the director of University Libraries, gave a presentation on Origami which included participation by KME members. Other 1997–98 officers: Stephanie Helbert, vice president; Wendy James, secretary; Eric Macy, treasurer; Frances Renfroe, corresponding secretary; Edwin Hewett, Dan Dawson, and Andrew Potter, faculty sponsors.

TX Iota
McMurry University, Abilene 25 actives, 13 associates

Student officers will be elected in the fall. Other 1997–98 officers: Dianne Dulin, corresponding secretary; Bill J. Dulin, faculty sponsor.

VA Alpha Chapter President — Trina Bailey
Virginia State University, Petersburg 30 actives, 5 associates

VA Alpha was busy this semester with several activities. During Black History Month KME cosponsored, along with other math organizations on campus, a lecture series. The lectures were given by African-American mathematicians including Dr. Melvin Currie, chief of the Mathematics Branch of the INFOSEC Research Division of the National Security Agency, and Dr. Arlie Peters, professor of mathematics at Princeton University. Five members were initiated at the 42nd Annual Initiation Ceremony held on April 24 in the Hunter McDaniel Building. The speaker was Dr. Christopher Barat, assistant professor of mathematics at VSU and a member of KME. He lectured on "Problems in the History of Probability." Several awards were given to deserving students for their academic achievement and to faculty for effective teaching. The Louise Stokes Hunter Scholarship (a

KME Award) was presented to chapter president Trina Bailey. Because of generous contributions, the award was for $1000, twice the usual amount. Other 1997–98 officers: Talvas Lucas, vice president; Jacqueline Payton, secretary; Emma B. Smith, treasurer; Joycelyn Josey-Harris, corresponding secretary; Azzala Owens, faculty sponsor.

WI Gamma Chapter President — Jennifer Deitte

University of Wisconsin—Eau Claire, Eau Claire 15 actives, 7 associates

Other 1997–98 officers: Jeremy Eppler, vice president; Patricia Rosek, secretary; Kendra Zillmer, treasurer; Marc Goulet, corresponding secretary/faculty sponsor.

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Report on the 31st Biennial Convention

As I pause and reflect upon the Thirty-First National Biennial Convention of the Kappa Mu Epsilon Mathematics Honor Society, a casual remark of one student caught my attention, because it reflected the community of interest that becomes available to all good students and their instructors as they become engaged with mathematics and one another beyond the classroom. According to my roll call, he had found himself among 114 students and 48 faculty, representing 30 chapters in 15 states, and consequently he had recognized a mathematical community perhaps for the first time. After learning that this convention was being cohosted by three neighboring chapters in Springfield, Missouri, I too marveled at the thought of the extra cooperation that would be required from them to welcome us into their community, and evidently here was a student who felt like me. Our warmest thanks certainly go to Donald Tosh of Missouri Theta at Evangel College, Charles Allen of Missouri Kappa at Drury College, and Yungchen Cheng of Missouri Alpha at Southwest Missouri State University, who set a wonderful example of what can be accomplished by working together, especially since Ed Huffman, the conference organizer and corresponding secretary for Missouri Alpha, died unexpectedly during the planning process.

After his widow, Shirley, attended the last day to thank the convention for a heartfelt contribution to a scholarship fund, recently established in his honor by Southwest Missouri State University, we realized how important inspirational leadership is to the national organization as well as the local chapters. Since the entire convention could have been canceled, if such leadership had not been shared with student officers and professional colleagues, who rallied for the common good, we joined the resolutions committee by extending our combined thanks to corresponding secretaries Don Tosh, Charles Allen, and Yungchen Cheng, as well as the student officers and faculty advisors of Missouri Theta, Missouri Kappa, and Missouri Alpha, for including us in their community over two days, April 3–5, 1997. Christine Tosh, the student president of Missouri Theta, may be the first daughter to join her father in exercising local leadership, and since Michelle Biggers and Catherine Montgomery, the student presidents of Missouri Kappa and Missouri Alpha, also received our thanks for their chapters, the continuing contribution of women in leadership positions at all levels of the society became more readily apparent, though it had always been important. Indeed, the society was founded by Emily Wyant of Oklahoma Alpha at Northeastern Oklahoma State Teachers College, on April 18, 1931, and Mississippi Alpha was installed at the Mississippi University for Women as early as

May 30, 1932.

Approximately 60,816 people have been initiated by 144 chapters of the Kappa Mu Epsilon National Mathematics Honor Society as of the current biennium, which ended March 10, 1997, and of these chapters 116 are currently active. Peter Skoner, a regional director, installed Pennsylvania Omicron at the University of Pittsburg at Johnstown on April 10, 1997, and by a show of hands the convention accepted the petition of Hillsdale College for the Michigan Delta chapter, which was installed by Arnold Hammel, our national president, on April 30, 1997. Nina Girard and Kathy Andrews are the first corresponding secretaries for Pennsylvania Omicron and Michigan Delta. Since the active chapters accepted the petition of Pennsylvania Omicron earlier this year by mail ballot, these two new chapters followed different constitutional provisions for orderly organizational growth.

Our sense of building community began to accelerate after Richard Poss, the national president of the Pi Mu Epsilon National Mathematics Honor Society, was introduced as a guest of the convention. Indeed, some members currently serving on the Pi Mu Epsilon Council were initiated by Kappa Mu Epsilon, and conversely some members currently serving on the Kappa Mu Epsilon Council were initiated by Pi Mu Epsilon. Situations such as these happen when students initiated by one society eventually join the faculty at institutions whose chapters were installed by the other society. Since both societies have similar goals and ways of reaching them, an ardent consensus for encouraging cooperation at national, regional, and local levels instinctively emerged. Constructive suggestions for such cooperation will be considered by our new national president, Patrick Costello of Kentucky Alpha at Eastern Kentucky University, and our new president-elect, Robert Bailey of New York Eta at Niagara University. Patrick Costello succeeds Arnold Hammel of Michigan Beta at Central Michigan University as our national president, and Donald Tosh of Missouri Theta at Evangel College succeeds Mary Elick of Missouri Iota at Missouri Southern State College as our national historian. Meanwhile, Allan Riveland of Kansas Delta at Washburn University of Topeka and I continue as the national treasurer and national secretary. Harold Thomas, the preceding national president, received the George Mach Service Award, and Cynthia Woodburn succeeded him as the corresponding secretary of Kansas Alpha at Pittsburg State University.

Student Participation

Everyone keenly enjoys the student presentations of their contributed papers at every convention, whether regional or national, and this year the audience enjoyed no fewer than twelve excellent presentations. Sometimes mathematics is called the science for young people, and certainly the history of mathematics includes many important advances that have been

made when young people ask the right questions at the right time. The names of the presenters and the titles of their papers appear among the details that follow, and some of their contributions will eventually be published by *The Pentagon* (the paper by Suzanne Shontz already appeared in the last issue). Though we certainly can be highly pleased with all of the honor students who represented their chapters on the stage, in the audience, and on the committees, it is difficult to recapture the atmosphere of the auditorium at Evangel College during the live presentations as I write. Nevertheless, I am able to recall that Benjamin F. Finkel joined the faculty of neighboring Drury College about a century ago. We remember him today primarily because he founded the *American Mathematical Monthly* in 1894 and published it there until 1913, but his story actually involved many people and evidently that spirit of cooperation still remains in Springfield, Missouri, as we continue to rediscover ourselves (read about "The human aspect in the early history of the *American Mathematical Monthly*," *American Mathematical Monthly* **38** (1931), pages 305–320, in the editor's own words).

The students — many of whom aspire to earn graduate degrees in mathematics — also offered some practical suggestions for future direction. For example, while they always knew that mathematics could be engaging and challenging, they began to realize that it also had some rewarding aspects, both personally and socially, that they had not experienced before. The prospect of greater cooperation between both national mathematics honor societies also implied additional respect, involvement, impact and growth. Though no one envisioned a campus with chapters of Kappa Mu Epsilon and Pi Mu Epsilon competing for student attention, the compatibility of a chapter of either one of these two distinct national honor societies with the student chapter concept, sponsored by the Mathematical Association of America, introduced some attractive possibilities. One chapter described an associate membership category for students as yet ineligible for membership in an honor society at the national level. This idea undoubtedly attracted a number of first-year students to their chapter activities, and according to their report nearly all of them eventually qualified for full honors.

Additional suggestions for future regional and national conventions included poster displays for students not wanting to make formal presentations, but who still had something that they wanted to contribute. Though such displays never would replace any formal presentations, they could provide superior opportunities for access to technology and interaction with the audience. Watch convention announcements for details as this supplementary format becomes developed. We're always open to suggestions from our local chapters, since many good things are happening in our society. Finally, on behalf of the National Council, I want to take this opportu-

nity to publicly thank the hosts, faculty, and students, who enthusiastically produced another memorable convention this year.

Program Details

Thursday evening, April 3. The conference began with a social mixer in the Findlay Student Center and a meeting of the National Council in the Trustees Board Room at Drury College. This meeting of the National Council was attended by Arnold Hammel, the national president; Patrick Costello, the president-elect; Mary Elick, the national historian; Waldemar Weber, the national secretary; Allan Riveland, the national treasurer; Carol Harrison, director of the New England Region; Peter Skoner, director of the Great Lakes Region; Mary Sue Beersman, director of the North Central Region; Richard Gibbs, director of the South Central Region; Bryan Dawson, editor of *The Pentagon*; Larry Scott, its business manager; and Richard Poss, president of the Pi Mu Epsilon National Mathematics Honor Society, as an invited guest of this convention. Gayle Kent of Florida Beta at Florida Southern College will succeed Patrick Costello of Kentucky Alpha at Eastern Kentucky University as the director of the Southeastern Region, and Donna Hafner of Colorado Delta at Mesa State College will succeed Richard Gibbs of Colorado Gamma at Fort Lewis College as the director of the South Central Region. Raymond Terry of California Gamma at California Polytechnic State University is the director of the Far Western Region.

Friday morning, April 4. The first general session of this convention was convened in the spacious chapel at Evangel College with Arnold Hammel, the national president, presiding. Robert Spence, the president of Evangel College, provided the welcome, and Patrick Costello, the national president-elect, provided the response. Christine Tosh, Michelle Biggers and Catherine Montgomery, the student presidents of Missouri Theta, Missouri Kappa and Missouri Alpha, added their own greetings. Each chapter present then filed official delegate and travel vouchers during the roll call, conducted by Waldemar Weber, the national secretary. This session also approved the minutes of the last national convention, hosted by Colorado Gamma of Fort Lewis College at Durango, Colorado, received a new chapter petition from Hillsdale College at Hillsdale, Michigan, set the agenda for old and new business, and received the report of the nominating committee, chaired by Harold Thomas of Kansas Alpha at Pittsburg State University. The committee nominated Peter Skoner of Pennsylvania Mu at Saint Francis College and Robert Bailey of New York Eta at Niagara University to succeed Patrick Costello of Kentucky Alpha at Eastern Kentucky University as president-elect. Rhonda McKee of Missouri Beta at Central Missouri State University and Donald Tosh of Missouri Theta at Evangel College were nominated to succeed Mary Elick of Missouri Iota at Missouri

Southern State College as the national historian. The following student papers were given and a group picture was taken this morning after the first general session recessed.

Newton to Chaos: An Unexpected Turn in Numerical Analysis
KARI HAMM, Kansas Alpha
Pittsburg State University

Statistics and Extreme Values
KATHRYN HICKMAN and STEVEN WALL, Wisconsin Gamma
University of Wisconsin — Eau Claire

The Orbit of Hale-Bopp
MICHELLE BIGGERS-BEACH, Missouri Kappa
Drury College

Hyperbolic Tilings
ANDREW MILLER, Iowa Delta
Wartburg College

Friday afternoon, April 4. After lunch in Springfield, five more student papers were presented in two groups that were separated by a student meeting with Christine Tosh, Michelle Biggers and Catherine Montgomery, the student presidents of the hosting chapters, presiding and a parallel faculty meeting with Arnold Hammel, the national president, presiding. Thus, the participants at this convention had an opportunity to review issues, discuss concerns, and make suggestions for consideration at the next meeting of the National Council. The student papers, presented this afternoon, were:

Choices, Choices, Choices
JEFFREY BLANCHARD, Kansas Gamma
Benedictine College

Checkers
STACY MOORMAN, Ohio Alpha
Bowling Green State University

The Language of Mathematics: Teaching in the Proximal Zone
ELIZABETH BARRETT, Kentucky Alpha
Eastern Kentucky University

Molecules and Their Symmetries: Determining the Hybridization of a Central Atom using Point Groups
SUZANNE SHONTZ, Iowa Alpha
University of Northern Iowa

Groups on Elliptic Curves
KELLI POLOTAYE, New York Lambda
C. W. Post Campus of Long Island University

Friday evening, April 4. The convention banquet was held in the Crusader Hall of Evangel College. After dinner, the George R. Mach Distinguished Service Award was presented to Harold L. Thomas, the preceding president (see citation on pages 66–67), and the guests were treated to several interesting brain teasers, presented by Larry Campbell from Southwest Missouri State University.

Saturday morning, April 5. Three more student papers preceded the second general session, which adjourned the 31st Biennial National Convention after finishing the business that was brought before the attending chapters and their delegates. The final student papers to be presented at this convention were:

Three Greek Impossibilities
SHANNON WILKINSON, Kansas Alpha
Pittsburg State University

Gravitational Pull of a Black Hole
JASYN VOSHELL, Pennsylvania Mu
St. Francis College

An Algorithmic Method for the Construction of a 4×4 Magic Square Consisting Only of Prime Numbers
CLARENCE DAVIS, Kentucky Alpha
Eastern Kentucky University

This convention contributed $100 to a scholarship fund at Southwest Missouri State University in memory of Ed Huffman. Reports were received from Larry Scott, the business manager, and Bryan Dawson, the editor of *The Pentagon*. To take advantage of discounted costs by using bulk rate, Larry needs to have addresses that meet postal regulations by including a postal box number or physical street address. Bryan is developing a cumulative subject index for *The Pentagon* and was thankful for student authors that contributed many feature articles. Mary Elick, the national historian, who edits the Chapter News for *The Pentagon*, thanked the eighty chapters that contributed material at least once during the last biennium. This feature of *The Pentagon* not only maintains a permanent record of local activities, it also provides a forum to help chapters share successful ideas with one another. Allan Riveland, the national treasurer, thanked Rhonda McKee of Missouri Beta at Central Missouri State University for the au-

dit committee report and Jo Ann Fellin of Kansas Gamma at Benedictine College, who kept excellent records as the preceding treasurer. Waldemar Weber, the national secretary, reviewed the growth of the organization and the status of its chapters during the last biennium. He also started to compile an electronic database of membership records for each chapter, but this project is still in progress. Selected reports follow this summary of the whole convention.

After Patrick Costello, the president-elect, and Arnold Hammel, the national president, reviewed the work and leadership of the Kappa Mu Epsilon National Mathematics Honor Society during the last biennium, thanking the chapters for their responsiveness, communication and participation, Carol Harrison of Pennsylvania Theta at Susquehanna University summarized the discussions of the faculty meeting and the student presidents of the hosting chapters summarized the discussions of the parallel student meeting. Rhonda McKee of Missouri Beta at Central Missouri State University chaired the audit committee and thanked Allan Riveland for introducing computer software into the record-keeping process of the national treasurer as well as helping to maintain and stabilize important assets. The audit committee accepted the financial records and analysis of the national treasurer.

Donna LaLonde of Kansas Delta at Washburn University, who chaired the resolutions committee, then reported the following resolutions, which were then adopted:

> Resolved: That the Thirty-First Biennial Convention of Kappa Mu Epsilon expresses its gratitude to Arnie Hammel and Mary Elick who have served as national officers. They have given so generously of their time and talent allowing our organization to prosper.
>
> Resolved: That this "double-double convention" was truly inspiring.
>
> Resolved: That the Thirty-First Biennial Convention of Kappa Mu Epsilon expresses its appreciation:
>
> 1. To Evangel College, Drury College, and Southwest Missouri State University faculty and students for their work in planning and implementing an outstanding convention.
>
> 2. To Christie Tosh, Michelle Biggers, and Catherine Montgomery, host presidents, for leading the three-college collaborative effort.

3. To Dr. Robert Spence, president of Evangel College, for welcoming us to the campus.

4. To the national officers, Patrick Costello, Bryan Dawson, Mary Elick, Arnold Hammel, A. Allan Riveland, Larry Scott, and Waldemar Weber, for their exemplary work throughout the year and during the convention.

5. To all the regional directors, faculty sponsors, and corresponding secretaries whose leadership of local Kappa Mu Epsilon chapters inspired the many faculty and student participants at the Thirty-First Biennial Convention.

6. To the Auditing, Awards, and Nominating Committees for their conscientious services.

7. To the Selection Committee who crafted such an exciting program of presentations.

8. To Dr. Larry Campbell for a stimulating keynote address.

9. To all the members of Kappa Mu Epsilon for providing a wonderful community in which to learn and discuss mathematics.

10. To all the student presenters for the unexpected turns as we Hale-Bopped our way to tiling our minds with extreme values. Making choices about checkers, we were dazzled by molecules and groups and amazed by black holes and Greek impossibilities. It was algorithmic magic!

The election of national officers then proceeded with each chapter having two votes if there were two delegates, but only one vote if there was only one delegate. Robert Bailey of New York Eta at Niagara University was elected to succeed Patrick Costello of Kentucky Alpha at Eastern Kentucky University as president-elect, and Donald Tosh of Missouri Theta at Evangel College was elected to succeed Mary Elick of Missouri Iota at Missouri Southern State College as the national historian. Next, the petition of Hillsdale College to become the Michigan Delta chapter was approved by a unanimous vote.

Chapters desiring to host the regional conventions in 1998 or the next national one in 1999 should contact the national president. Starting with the last national convention, the chapters hosting a national convention

can apply their travel allowance to the next biennium, and depending upon need, the National Council also assists the hosting chapters of the national or any regional conventions with some program expenses. These policies are intended to encourage smaller chapters as well as larger ones to become convention hosts.

The convention adjourned after Richard Gibbs of Colorado Gamma at Fort Lewis College, who chaired the awards committee, distributed calculators, provided by Hewlett-Packard and Texas Instruments, to the four undergraduate students who presented the best papers in the judgment of students and faculty. The jury of ten that recommended these papers for special recognition considered the delivery as well as content. The winners, who set the standard for this convention, were (in order of presentation) Michelle Biggers-Beach of Missouri Kappa, Andrew Miller of Iowa Delta, Suzanne Shontz of Iowa Alpha and Kelli Polotaye of New York Lambda.

Travel allowances were paid to the chapter delegates by Allan Riveland, the national treasurer. The convention was adjourned at 12:15 p.m. on Saturday, April 5, and as far as I know, everyone returned home safely.

Keep in touch,

Waldemar Weber

CITATION FOR DR. HAROLD L. THOMAS THE GEORGE R. MACH DISTINGUISHED SERVICE AWARD APRIL 4, 1997

When Dr. Harold Thomas answered "It is" and "I do" to initiation ceremony questions concerning his desire and willingness to support the ideals and purposes of Kappa Mu Epsilon, he had no idea to what extent his life would be devoted to this end. Chapter News lists Dr. Thomas as faculty sponsor of Kansas Alpha starting in 1966 and corresponding secretary starting in 1967. From that time on, he served his local chapter unfailingly until his phased retirement in the fall of 1996. Bryan Dawson, editor of *The Pentagon* and a student member of Kansas Alpha, paid tribute to Harold's contribution to his home chapter in these words:

> "During my time at Pittsburg State, KME was what everyone would imagine it should be. I always looked forward to the monthly meeting, where we would see a student or faculty presentation and drink Pepsi afterward. I still remember many of the talks given by other students. Involvement in KME was something no student

ever thought about — it was just 'normal,' something everybody did as a matter of course, just like taking calculus. To me, KME seemed as though it had a life of its own. I now realize that it takes dedicated faculty members to nurture that atmosphere."

Harold's commitment to Kappa Mu Epsilon, of course, extended far beyond the local level. In 1970 with the formation of the regional structure, he became the first regional director of Region IV. He remained in this position until 1979, at which time he became national historian. After six years as historian, he became president-elect in 1985 and president in 1989. He was particularly successful as president in promoting new chapters of Kappa Mu Epsilon; indeed eleven new chapters were installed during his tenure. Since leaving the presidency in 1993, he continues to serve on committees as needed and offers counsel when and where he can. In all he has done, his attention to detail, along with his knowledge of the history of Kappa Mu Epsilon and his faith in its ideals, have contributed greatly to the status of the organization. It is with much pleasure that we recognize his outstanding dedication and service to Kappa Mu Epsilon by presenting to him the George R. Mach Distinguished Service Award.

Citation prepared by Mary S. Elick, national historian, Kappa Mu Epsilon

Report of the President

My service during the second biennium as your national president is now nearing its close. Although more than just a little while ago, it doesn't seem like it has been eight years since my election as president-elect in 1989. Words can't totally describe the phenomenal experience it has been. It has been a joy to work with super KME members ranging from student members, faculty corresponding secretaries and local advisers, National Council members, *Pentagon* personnel, and regional directors. I know that I will miss being on the council; however, I eagerly look forward to future conventions where acquaintances can be renewed again.

A special thanks is extended to each of the faculty who serve as corresponding secretaries and faculty sponsors with our active chapters. I want you to know how much your time and effort on behalf of your local members is appreciated. Their interest in mathematics and collegiality found in a student organization can all be attributed to your careful guidance.

Keep up the good work. Students were able to attend this fine convention because of the efforts of our faculty. I know also that many of you played an important role in assisting your students in the preparation of the excellent papers we have on the convention program this year.

Furthermore, we all express our gratitude to each of the students who did the work, endured the stress and prevailed in submitting and presenting a paper at this convention. Without the student papers, the major focal point of the convention does not exist.

I also want to recognize and applaud the outstanding efforts put forth by the members of the National Council in their respective areas of responsibility. We have special indebtedness to these very capable and conscientious individuals who so unselfishly give of their time and efforts in making our honor society the very best it can possibly be. It is a joy to work with them. To illustrate, whenever I write a letter to the whole council, I try to describe what task each had been doing or planning to do. It is always interesting to see what projects they are working on. There is always something happening. Al will be receiving my pay orders, sending checks, creating bimonthly reports, filing our income tax forms, checking our income and expenses, etc. Mary will be asking for chapter reports that then need to be typed before going off to *The Pentagon* for the Chapter News section. She also keeps me up to date with interesting information about the history of KME in her historian files, etc. Pat will be coordinating regional conventions and assisting the regional directors in these fine meetings, preparing a request for the Mach Award nominations, writing a Call for Papers for the national convention and then coordinating that important aspect of our convention, etc. Waldemar will be receiving new initiate information from the local chapters, filing them and then sending appropriate information to the treasurer and jeweler. He updated the KME brochure and keeps us all supplied with cards, forms, etc. He has also undertaken the gigantic task of creating a list for each chapter of their initiate information thoughout the life of the chapter. Wow. Much e-mail, many phone calls, and lots of letters go back and forth between us. We also meet once each fall to conduct KME business, make plans for the organization, and discuss the future. It is always a fun and hard-working time. Often we have held these on the campus of a council member and that makes it just that much more interesting.

I next want to recognize the fantastic job which is being done by those who work with, manage, write for, and produce our journal, *The Pentagon*. We are most appreciative of the editorial leadership of Bryan Dawson (Kansas Beta) and the sound business management given to us by Larry Scott (Kansas Beta). Also big thank you's to associate editors Mary Elick (MO Iota) and Kenneth Wilke (KS Delta). Mary edits the Chapter News section and Ken the Problem Corner.

The National Council continues to support the regional structure of KME. Please refer to the report by President-Elect Patrick Costello for the summary of regional conventions held in 1996. With much gratitude we recognize the work and efforts of our regional directors. These people have served the society well and deserve the thanks of each of us. I would like to especially cite Carol Harrison, Region 1 director and PA Theta (Susquehanna University), Pat Costello, Region 3 director and KY Alpha (Eastern Kentucky University), and Dick Gibbs, Region 5 director and CO Gamma (Fort Lewis College), who have just completed four-year terms. Terms still continuing and doing great jobs in their regions are Peter Skoner, Region 2 director and PA Mu (Saint Francis College), Mary Sue Beersman, Region 4 director and MO Eta (Truman State University), and Raymond Terry, Region 6 director and CA Gamma (California Polytechnic State University at San Louis Obispo). The appointment of the next group of regional directors for regions 1, 3, and 5 will be made shortly after this convention.

We are further indebted to all of the individuals who did all of the work necessary to bring this convention to fruition and to those who have served on the convention committees. A big thank you goes to the members of the Missouri Theta, Kappa, and Alpha chapters at Evangel College, Drury College, and Southwest Missouri State University, respectively, for their fine work in hosting the convention. To all of you at these chapters, under the capable direction of Don Tosh, Charles Allen and Yungchen Cheng, please accept our sincere thanks for a job well done.

The cooperation of faculty and students in their willingness to serve on convention committees has been very impressive. Through the many phone calls, letters and e-mails I have had the neat opportunity to "chat" with many of you during the last several months. This kind of response, both at convention time, and throughout my tenure thus far, has made the privilege of being your president much easier. Thank you much.

During the past two years in this biennium we have unfortunately installed no new chapters. I am hopeful that this is just a lull moment in our history before a big rush of activity in the future on new chapter installation. As a matter of fact, I am pleased to report that the 117th chapter will be installed next week, April 10, 1997, at the University of Pittsburgh at Johnstown, in Johnstown, Pennsylvania. Peter Skoner of PA Mu will serve as the installing officer. In addition, two other institutions have returned completed forms and are awaiting approval from the council and the chapters. Also many other colleges and universities during the past two years have indicated an interest in Kappa Mu Epsilon and received information about our organization. Often the reason that a school finds out about KME is via the students and faculty at existing chapters. If you have friends and colleagues at schools that do not have a KME chapter, but are

interested, have them contact new president Pat Costello. I am noticing that the existence of the national KME World Wide Web Home Page has generated considerable interest in KME amongst faculty and students in mathematics departments around the country.

I am pleased to report that none of our chapters were placed on inactive status during the past biennium.

During the past biennium, I have represented Kappa Mu Epsilon once at the annual meeting of the Association of College Honor Societies (ACHS). It has been very helpful to meet with officers of the other 59 honor societies that are members of ACHS. Hopefully this exchange of ideas and gathering of suggestions can be put to good use in possibly improving on the programs we currently have in place.

During the past several years council members of the two national mathematics honor societies, Pi Mu Epsilon and Kappa Mu Epsilon, have met at council meetings, met at MAA conventions, and conducted correspondence in attempts to learn more about the respective organizations. I am pleased that PME national president Rick Poss of St. Norbert College is in attendance at our convention this weekend. Welcome, Rick. I hope that you are enjoying your visit.

In summary, I can honestly say that I have thoroughly enjoyed the privilege of serving Kappa Mu Epsilon at the national level the past several years. Under the capable leadership of Dr. Pat Costello and the rest of the National Council, I know that Kappa Mu Epsilon will continue to grow and occupy a position of pride in our hearts and minds because of our affiliation with this very special honor society. Best wishes to each of you. My sincere thanks to all for your participation in this 31st Biennial Convention.

Arnold D. Hammel

Report of the President-Elect

One responsibility of the president-elect is to serve as coordinator of regional activities of the society through the regional directors. During the spring of 1996, there were two regional conventions held. They were in:

Region IV at Kansas Delta, Washburn University,
April 26–27, Mary Sue Beersman, regional director.

Region V at Colorado Delta, Mesa State College,
April 19–20, Richard Gibbs, regional director.

Programs at the regional conventions included student papers, guest talks, and good social times. The Region IV convention was especially successful in that there were so many student talks submitted that paral-

lel sessions were required. We extend our sincere thanks to the two host chapters, their regional directors, and all who participated in this important regional activity. We also appreciate the efforts of all the remaining regional directors in attempting to have regional conventions in their regions.

A second duty of the president-elect is to coordinate nominations for the George R. Mach Service Award. In the fall of 1996 a letter went out requesting nominations for the award. I want to thank those corresponding secretaries and faculty sponsors who made nominations and the students and alumni who wrote letters in support of these nominations. At the national council meeting in October a decision was made to present the award to Harold L. Thomas at this convention.

It is another of the president-elect's responsibilities to publish announcements of the national convention. Besides an announcement that appeared in the Spring 1996 issue of *The Pentagon*, announcements of this meeting have appeared in both the AMS *Notices* and the MAA *Focus* magazines.

One of the most important responsibilities of the president-elect is to arrange for the presentation of student papers at the national convention. I am pleased to report that thirteen students, representing eleven chapters and nine states, submitted papers for this convention. Twelve papers were written by undergraduates and one by a graduate student. Two papers were submitted after the published deadline. The Paper Selection Committee read and ranked the eleven papers submitted prior to the published deadline. The committee indicated that it was difficult to rank the papers because they were all so good. These eleven papers will be judged by the Awards Committee at the convention. On behalf of the society, I want to extend special thanks to the members of the Paper Selection Committee who read and ranked the papers: Professors Jeffrey Clark (Elon College), Duane Broline (Eastern Illinois University), and Catherine Kong (Carson-Newman College). The rankings of the Paper Selection Committee will be combined with the rankings of the Awards Committee to determine a final ranking of the judged papers. The top four judged papers will each receive $100. The eleven papers ranked by the Paper Selection Committee plus the two late papers have been placed on the program for presentation at the convention. On behalf of the Society, I want to express our sincere thanks to all thirteen students who prepared and submitted papers. It is this work that makes for a truly successful convention.

Patrick J. Costello

Report of the National Secretary

During the last biennium, the Kappa Mu Epsilon National Mathemat-

ics Honor Society initiated 2,042 new members in 116 active chapters with a cumulative membership of 54,657. In addition, there are 28 inactive chapters with a cumulative membership of 6,159 for a combined membership of 60,816 at the end of the biennium. No new chapters were installed this biennium.

As national secretary, I maintain permanent records of the initiates of all local chapters, and I assist the corresponding secretaries any way that I can. For example, I order official keypin insignia and membership certificates from our jewelers, the J. O. Pollack Company of Chicago, Illinois, and I have begun compiling an electronic database of all members. I also take minutes at meetings of the National Council and at Biennial Conventions.

Waldemar Weber

Financial Report of the National Treasurer
1995–97 Biennium (March 21, 1995 through March 10, 1997)

The 95–97 biennium ended with an asset base of $34,594.12. This total included $29,366.96 in savings accounts and $5,227.16 in a checking account. The asset base was $45,455.94 at the beginning of the biennium. These figures reflect a $10,861.82 asset loss over the biennium. The 1995 national convention Audit Committee recommended, and the National Council concurred, the KME should maintain an asset base reserve of $30,000. The biennium cash flow is summarized below.

RECEIPTS		
Initiation fees received	$36140.00	
Installation fees received	75.00	
Interest income	3149.64	
Inventory income	190.30	
Overpayments received	220.00	
Total receipts		39774.94
EXPENDITURES		
Assoc. College Honor Soc. expenses	889.19	
Administrative expenses	5242.11	
National Convention expenses	13481.46	
Regional Convention expenses	551.79	
Council meeting travel	2206.02	
Certificates, jewelry, shipping	14246.15	
Inventory expenses	340.50	
Overpayments returned	215.00	
Pentagon expenses	12973.15	

Miscellaneous expenses	491.39	
Total Expenditures		50,636.76
Biennium cash flow		−10861.82

Al Riveland

Report of the National Historian

The files of the national historian are being maintained and continually updated with records received from chapters concerning their events; with information from regional directors concerning regional events; and with historically significant material received from the national officers.

Twice a year a request for chapter news is sent out to each chapter. Responses are then compiled and edited for publication in the Chapter News section of *The Pentagon*. It is worth remembering that when you respond to the chapter news request and have your chapter activities published in the KME journal, you have one more source of record should your own files be lost or destroyed.

During the past biennium, 80 chapters have responded at least once to this request. The following chapters are to be commended for responding to each of the four requests: CO Delta, GA Alpha, IL Delta, IA Alpha, IA Gamma, IA Delta, KS Alpha, KS Beta, KS Gamma, KS Delta, KY Alpha, KY Beta, MD Beta, MD Delta, MI Beta, MS Alpha, MS Epsilon, MO Beta, MO Gamma, MO Epsilon, MO Iota, MO Lambda, NE Alpha, NE Delta, NY Alpha, NY Eta, OK Alpha, PA Alpha, PA Gamma, PA Iota, and PA Kappa.

My thanks to all those with whom I have corresponded relative to this office. It has been a privilege and a pleasure to serve as your national historian. I am particularly grateful for the opportunity to have worked with the other members of the National Council and I appreciate no end, the help of the two *Pentagon* editors, Andy Rockett and Bryan Dawson, with whom I worked during my years in office. Finally, once again, I want to acknowledge the corresponding secretaries and the faculty sponsors of the chapters. From Chapter News reports, I am deeply aware that these are the people who most closely allow Kappa Mu Epsilon to serve its purpose. My thanks to all of you.

Mary S. Elick

Report of the Editor of *The Pentagon*

Nineteen student papers have appeared or will appear in volumes 55

and 56 of *The Pentagon*. Of these, all but one were presented at a KME convention. No faculty papers appeared. "The Problem Corner," "Kappa Mu Epsilon News," and convention reports also make up a large portion of the journal and are essential to its success.

In my first biennium as editor, I have attempted to keep the same great style, look and feel of *The Pentagon* as formed by the previous editor, Andrew Rockett. Only a couple of changes have been made, namely that typesetting is now accomplished using $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX and that "filler" material of various sorts has been included to fill out pages.

Unsolicited manuscripts are still refereed by faculty volunteers. The efforts of six such individuals were acknowledged in the Spring 1996 issue. Many other faculty have volunteered their time since then. They have been a great help both to myself and to the previous editor.

The next project for the editor is the creation of a cumulative subject index for *The Pentagon*, to be available on our WWW site. The index is about half complete at this time.

The continued efforts of associate editors Kenneth M. Wilke and Mary S. Elick have been greatly appreciated, as has their patience with my many unusual questions over the past two years. I am also grateful for the support and assistance of the previous editor, Andrew Rockett, the current business manager, Larry Scott, and the National Council, all of whom contributed greatly to the continued success of this journal.

Keep those manuscripts coming!

C. Bryan Dawson

Report of the Business Manager of *The Pentagon*

It is a pleasure to make my first business manager's report at this 31st Biennial Convention. As many of you know, the business manager's primary responsibility is to maintain a current list of subscribers, to oversee our mailings, and to assist the editor in managing *The Pentagon*.

All new initiates receive a two-year subscription to *The Pentagon* and are encouraged to continue their subscriptions for a modest fee of $5.00 per year. Issues are mailed in December and May of each academic year. Our mailing list includes subscribers in this country, Europe, Asia, and Africa. During the past biennium, we have serviced over 2500 subscribers per issue. Approximately 500 renewal notices will be mailed next week. Please watch the expiration date for your subscription and renew early.

Postal regulations for bulk mailings require that each address be validated at least once per year and that each address contain either a street address or post office box number. Please check your address and make

corrections if necessary. In December we paid almost $200 in penalties for forwarding issues and returning undeliverable issues. Please use a permanent address and make sure that your address is valid.

Complementary copies of *The Pentagon* are sent to the library of each college or university with an active chapter of Kappa Mu Epsilon. Anyone contributing an article for an issue will receive two free copies. Speakers at the 31st Biennial Convention will have their subscriptions extended for two years.

I am appreciative of the support and assistance given by the previous business manager, Sharon Kunoff, and the previous editor, Andrew Rockett. I would like to thank C. Bryan Dawson, editor of *The Pentagon*; Arnold D. Hammel, KME president; A. Allan Riveland, KME treasurer; and Waldemar Weber, KME secretary. Their cooperation and assistance have made things move smoothly. I gratefully acknowledge the assistance of my secretary, Sharon Brown.

Larry Scott

Harold L. Thomas (right) receives the George R. Mach Distinguished Service Award from Patrick Costello. Photograph courtesy of Cynthia Woodburn, KS Alpha.

Thirty-First Biennial Convention of Kappa Mu Epsilon, April 3–5, 1997, Springfield, Missouri, hosted jointly by Missouri Alpha, Missouri Theta, and Missouri Kappa. Picture in Evangel College Chapel. National officers are standing on the edge of the stage, in the center of the next-to-next-to-last row of the picture. From left to right: Waldemar Weber, Mary S. Elick, Patrick J. Costello, Arnold D. Hammel, A. Allan Riveland, C. Bryan Dawson, and Larry Scott.

Kappa Mu Epsilon National Officers

Patrick J. Costello *President*
Department of Mathematics, Statistics and Computer Science
Eastern Kentucky University, Richmond, Kentucky 40475
matcostello@acs.eku.edu

Robert Bailey *President-Elect*
Mathematics Department
Niagara University, Niagara University, New York 14109
rbail@niagara.edu

Waldemar Weber *Secretary*
Department of Mathematics and Statistics
Bowling Green State University, Bowling Green, Ohio 43403
kme-nsec@mailserver.bgsu.edu

A. Allan Riveland *Treasurer*
Department of Mathematics and Statistics
Washburn University, Topeka, Kansas 66621
zzrive@acc.wuacc.edu

Don Tosh *Historian*
Department of Science and Technology
Evangel College, 1111 N. Glenstone Ave., Springfield, Missouri 65802
toshd@evangel.edu

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation.

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	College of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959

IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood College, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel College, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984

CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997

Starting a KME Chapter

Complete information on starting a chapter of KME may be obtained from the National President. Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; students must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $50) and the expenses of the installing officer. The individual membership fee to the national organization is $20 per member and is paid just once, at that individual's initiation. Much of this $20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offerings and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.