

# THE PENTAGON

*A Mathematics Magazine for Students*

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## Fiftieth Anniversary Issue

Harold L. Thomas, National President

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The 1990-91 school year is a significant milestone in the history of Kappa Mu Epsilon. This special edition of *The Pentagon* is Volume 50 Number 1 and represents the golden anniversary of the publication of the official journal of our Mathematics Honor Society. In addition, in the spring of 1991, we will celebrate the sixtieth anniversary of the founding of Kappa Mu Epsilon.

The following article by Mary S. Elick, our national historian, details much of the history behind *The Pentagon*. Several editors and business managers have played an important role in the success of our journal over the past fifty years. The magazine continues to meet an important objective – to be a publication for students of mathematics. Many of the articles are written by students for students.

Kappa Mu Epsilon was founded on 18 April 1931 by Dr. Emily Kathryn Wyant at Northeastern Oklahoma State Teachers College in Tahlequah, Oklahoma. Professor L. P. Woods assisted Dr. Wyant in establishing the Society to recognize outstanding achievement in the study of mathematics by students whose emphasis is primarily at the undergraduate level. *KME* was incorporated as a non-profit organization for educational purposes on 14 May 1951 in the State of New York and was admitted to membership in the Association of College Honor Societies in 1968. The Society has received 53,658 members through June 1990 and includes one hundred-six active Chapters located in thirty-one states.

Kappa Mu Epsilon has traditionally held national conventions in odd numbered years and regional conventions in even numbered years. The sixtieth anniversary celebration of *KME* will take place at the 28th Biennial Convention on 11-13 April 1991 at the University of North

Alabama at Florence, Alabama. We plan to honor as many first place paper presenters from previous conventions as possible at this time. Additional information about this meeting appeared in the announcement on pages 65-67 of the Spring 1990 issue of *The Pentagon*.

Kappa Mu Epsilon proudly dedicates this commemorative issue of *The Pentagon* to all of the outstanding students of mathematics who have joined *KME* in the past as well as those who will join in the future, and to the dedicated faculty sponsors, corresponding secretaries and national officers who provide the leadership that allows the organization to grow and flourish. Kappa Mu Epsilon remains committed to fulfilling the objectives of the Society as detailed in the Constitution. These culminate in the recognition of superior scholarship in mathematics by undergraduate students and in the development of an appreciation for the beauty of mathematics.

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### *The Pentagon: The First Fifty Years*

Mary S. Elick, National Historian

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Missouri Southern State College  
Joplin, Missouri 64801

The first issue of *The Pentagon*, just off the press in the fall of 1941, cost one dollar for two years or thirty cents for a single copy. Records do not clarify to whom these first *Pentagons* were mailed, but certainly the recipients included the twenty-six existing chapters of Kappa Mu Epsilon. The first printing consisted of five hundred copies.

Authorization for an official journal of Kappa Mu Epsilon had been established in April 1941 at the fifth biennial convention held at Central Missouri State College in Warrensburg. Dr. O. J. Peterson, who was then

national president, said, "The publication of such a journal is probably the most significant project ever undertaken by the fraternity." A primary objective of the project was the establishment of a vehicle for the publication of student papers.

The task of determining the format of the journal and formulating the editorial policies was entrusted to Dr. Carroll V. Newsom of the University of New Mexico, Albuquerque. Newsom, national president of *KME* from 1939 to 1941, served as *Pentagon* editor from its inception until other duties, occasioned by the national emergency, demanded his attention in 1943. Later he would serve as editor of *The American Mathematical Monthly* (1947-1951) and become president of New York University (1956-1962). He died in February of 1990 as the *KME* journal he founded was nearing its fiftieth year of publication.



*Courtesy of the New York University Archives*

Dr. Carroll V. Newsom, founding editor of *The Pentagon*.

During those fifty years, the journal has been published semi-annually with the exception of the period from fall 1941 to spring 1945, when it was published once a year. During that time, as the war effort intensified, membership in the organization decreased and chapters were unable to contribute much to *The Pentagon*. In fact, in the spring 1943 *Pentagon*, Missouri Beta reported in the Chapter News section: "All the members of Missouri Beta except two are now in some branch of the service." Other chapters wrote sadly of members having been killed in action.

Nonetheless, by the spring of 1947, Dr. Harold D. Larsen of the University of New Mexico, Albuquerque, who succeeded Newsom as editor, reported that over one thousand copies of the journal were being printed. At that time, to encourage more and better student papers, Larsen announced a paper competition; for the best paper published in the journal during the next biennium, the student author would be awarded \$25 or a free trip to the national convention. Whether due to the contest (\$25 was much more money then than now), an increase in the membership of Kappa Mu Epsilon, or other motivating factors, by the the early fifties there were substantially more student papers being published and *The Pentagon* was flourishing. Indeed, during the summer of 1954 the journal was honored by being chosen by a committee of mathematicians to be one of the American publications on display at the International Congress of Mathematicians in Amsterdam.

Larsen, during his time of involvement with *The Pentagon*, had served as both editor and business manager of the publication. As the journal grew and the work load increased, it became necessary to separate these two positions. Therefore, when Dr. Carl Fronabarger of Southwest Missouri State College, Springfield, took over from Larsen, Prof. Dana Sudborough of Central Michigan University, Mt. Pleasant, assumed the position of business manager. Then, as now, two important tasks of the business manager were keeping an updated file of addresses of subscribers and getting *The Pentagon* into the mail twice a year. The first task would be simplified if student addresses weren't so subject to change and if subscribers would alert *The Pentagon* office of address changes. Sudborough once made the statement that were he to be adopted by a tribe of American Indians, his new name would be "He-Who-Wants-Your-Correct-Address." In describing the actual mailing of the journals before the time of computerized labels and with no *KME* funds earmarked to hire help, Sudborough recalls that twice a year his wife and teenaged sons were pressed into service, his home became a temporary office, and for hours at a time "the counter between kitchen and dining room, as well as the dining room table, was covered with

magazines, labels, and envelopes.”

Succeeding Sudborough in 1957 was Dr. Wilbur Waggoner, also of Central Michigan University, who, despite the struggles associated with the semi-annual mailings, would remain as business manager a record twenty years, serving under four different editors, each of whom “was great to work with.” During that time interval (1957-1977), the number of copies printed varied from two thousand to over three thousand four hundred and, at some time during that period, the journal was mailed to every state in the union and to many foreign countries including Argentina, Taiwan, Holland, New Zealand, Venezuela, Canada, British West Indies, Tunisia, Syrian Arab Republic, England, Hong Kong, and Germany. In summing up his years of service to the journal, Dr. Waggoner states, “I can not say the actual process of mailing *Pentagons* was fun but I certainly did enjoy my years as an officer of Kappa Mu Epsilon.”

Dr. Douglas Nance of Central Michigan University, Waggoner’s successor, was the first to computerize the process of getting the labels ready, thus simplifying considerably the mailing procedure. Today approximately three thousand *Pentagons* are mailed to subscribers in this and foreign countries. Each new member of Kappa Mu Epsilon automatically receives a two year subscription. For others, the subscription rate is \$5.00 for two years if mailed to a US address and \$5.00 for one year if mailed abroad.

Through the years, not all changes have been external in nature; the journal has also experienced changes in internal structure from time to time. “The Book Shelf,” a book review section, was added in the fifties and “The Cursor,” a section dealing with the interface between the disciplines of mathematics and computer science was added in the eighties. While these sections were popular for awhile and then dropped, student interest in other sections such as “The Problem Corner” has been constant. Another constant has been the mission of the journal: To publish articles of interest to undergraduate students of mathematics, to encourage and provide a medium for the publication of student papers, and to serve as a means of communication among the chapters.

In its fifty years of existence the journal has had eight editors and eight business managers, all striving to fulfill this mission. Not all have been mentioned here, but all have shown the same dedication and devotion to the publication. In addition, numerous individuals have served as associate editors or have contributed to the journal. The history of *The Pentagon*, more than anything else, is the history of all these people who gave so generously and effectively of their talents, their time

and their efforts to keep it going and make it better. Many of those who work with *KME* today were inspired by one of these individuals. Dr. Robert Moreland, corresponding secretary of Texas Alpha, Texas Tech University, Lubbock, writes of the encouragement he received from Dr. Emmett Hazlewood as an undergraduate, a graduate, and finally a colleague: "Emmett Hazlewood was probably the most influential man in my life other than my father." Hazlewood was an associate editor of *The Pentagon* during its first years of publication.

Certainly the "significant project" undertaken by Kappa Mu Epsilon in 1941 has been successful. We are grateful to all those who through their careful attention and guidance contributed to that success. We trust that the hundredth anniversary of *The Pentagon* will find it equally prosperous.

*Acknowledgements.* Information for this article came from many sources including historian files, biennial reports of the editors and business managers, and copies of the *The Pentagon*. Special thanks are due the following individuals who responded to a request for information for this article: Dr. Wilbur Waggoner, Prof. Dana Sudborough, Dr. F. Max Stein, Dr. J. M. Sachs and Dr. Robert Moreland.

- Editors -

1941-43	C. V. Newsom	New Mexico Alpha
1943-53	Harold D. Larsen	New Mexico Alpha and Michigan Alpha
1953-59	Carl V. Fronabarger	Missouri Alpha
1959-65	Fred W. Lott, Jr.	Iowa Alpha
1965-71	Helen Kriegsman	Kansas Alpha
1971-79	James K. Bidwell	Michigan Beta
1979-89	Kent Harris	Illinois Eta
1989-	Andrew M. Rockett	New York Lambda

- Assistant Editors -

1954-59	Edward H. Matthews	Missouri Alpha
1980-89	Iraj Kalantari	Illinois Eta

## - Associate Editors -

*The Scrapbook*

1941-42	E. A. Hazlewood	Texas Alpha
1942-46	Merle Mitchell	New Mexico Alpha
1946-55	Harold D. Larsen	New Mexico Alpha
1955-65	Jerome M. Sachs	Illinois Gamma
1965-69	George R. Mach	California Gamma
1969-86	Richard Barlow	Nebraska Beta

*Student Papers*

1941-43	E. R. Sleight	Michigan Alpha
1943-45	Fred W. Sparks	Texas Alpha
1946	Sister Helen Sullivan	Kansas Gamma

*News Editor*

1941-43	E. Marie Hove	Nebraska Alpha
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*Kappa Mu Epsilon News*

1941-43	Orpha Ann Culmer	Alabama Beta
1943-47	Sister Helen Sullivan	Kansas Gamma
1947-51	C. C. Richtmeyer	Michigan Beta
1951-55	Laura Greene	Kansas Delta
1955-59	Frank Hawthorne	New York Alpha
1959-63	Frank C. Gentry	New Mexico Alpha
1963-67	J. D. Haggard	Kansas Alpha
1967-71	Eddie Robinson	Missouri Alpha
1971-75	Elsie Muller	Iowa Gamma
1975-79	Sister Jo Ann Fellin	Kansas Gamma
1979-85	Harold L. Thomas	Kansas Alpha
1985-89	M. Michael Awad	Missouri Alpha
1989-	Mary S. Elick	Missouri Iota

*The Problem Corner*

1947-52	Judson W. Foust	Michigan Beta
1953-57	Frank C. Gentry	New Mexico Alpha
1957-63	J. D. Haggard	Kansas Alpha
1963-65	F. Max Stein	Colorado Alpha
1966-68	H. Howard Frisinger	Colorado Alpha
1968-73	Robert L. Poe	Texas Alpha
1974-	Kenneth M. Wilke	Kansas Delta

*Installation of New Chapters*

1949-51	Laura Greene	Kansas Delta
1952-55	Jerome M. Sachs	Illinois Gamma
1955-61	Mabel Barnes	California Beta
1961-70	Sister Helen Sullivan	Kansas Gamma
1970-83	Loretta K. Smith	Connecticut Alpha

*The Book Shelf*

1950-52	Carl V. Fronabarger	Missouri Alpha
1953-55	Frank Hawthorne	New York Alpha
1955-56	Rex D. Depew	Alabama Beta
1957-59	R.H. Moorman	Tennessee Alpha
1960-66	Harold E. Tinnappel	Ohio Alpha
1967	James P. Burling	New York Gamma
1967-68	John C. Biddle	Michigan Beta
1969-71	James Bidwell	Michigan Beta
1971-73	Elizabeth T. Wooldridge	Alabama Beta
1973-80	Oscar Beck	Alabama Beta

*The Cursor*

1984-1989	Jim Calhoun	Illinois Eta
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*The Hexagon*

1981-	Iraj Kalantari	Illinois Eta
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*- Business Managers -*

1941-43	C. B. Barker	New Mexico Alpha
1943-45	Frank C. Gentry	New Mexico Alpha
1945-53	Harold D. Larsen	New Mexico Alpha and Michigan Alpha
1953-57	Dana Sudborough	Michigan Beta
1957-77	Wilbur J. Waggoner	Michigan Beta
1977-85	Douglas W. Nance	Michigan Beta
1985-89	Gerald L. White	Illinois Eta
1989-	Sharon Kunoff	New York Lambda

## A Piece of Pie

Scott Steubing, *student*

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Presented at the 1989 National Convention.

The number  $\pi$  is the ratio of the circumference to the diameter of a circle. Cultures as early as the Babylonians and Hebrews used this ratio; usually approximating it by 3. The Egyptians approximated  $\pi$  more accurately to 3.16. Archimedes bounded  $\pi$  between  $3\frac{10}{71}$  and  $3\frac{10}{70}$  by determining the areas of 96-sided regular polygons that are enclosing and enclosed by a circle. William Jones was the first to use the symbol  $\pi$  for the ratio in his *Synopsis Palmariorum Matheseos: or, A New Introduction to the Mathematics* in 1706. This paper will concern itself with the formula for  $\pi/4$  discovered by Leibniz and the formula for  $\pi^2/6$  discovered by Euler.

Leibniz discovered his formula

$$(1) \quad \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

in an ingenious way. (A formal proof of Leibniz's formula will follow.) The circular sector in Figure One with radius 1 obviously has an area of  $\pi/4$ . The isosceles triangle  $\triangle OMT$  has an area of  $1/2$ , so we need to calculate the area  $A$  of the section cut off by chord  $OT$ . We estimate  $A$  by integrating the sliver like triangles  $\triangle POQ$ , where the arc  $PQ$  is so small as to be virtually straight. The point  $S$  on the  $y$ -axis is determined by extending  $PQ$  and then  $R$  is such that  $\angle ORS$  is a right angle. The point  $N$  has  $PN$  parallel to the  $x$ -axis and  $QN$  parallel to the  $y$ -axis, so that  $\angle NQP = \angle RSO$ . For the sliver  $\triangle POQ$ , we consider  $PQ$  to be the

base of length  $ds$  and the height to be  $OR$ , and then the similar right triangles  $\triangle PNQ$  and  $\triangle ORS$  have

$$\frac{ds}{dx} = \frac{OS}{OR} \quad \text{or} \quad OR \, ds = OS \, dx.$$

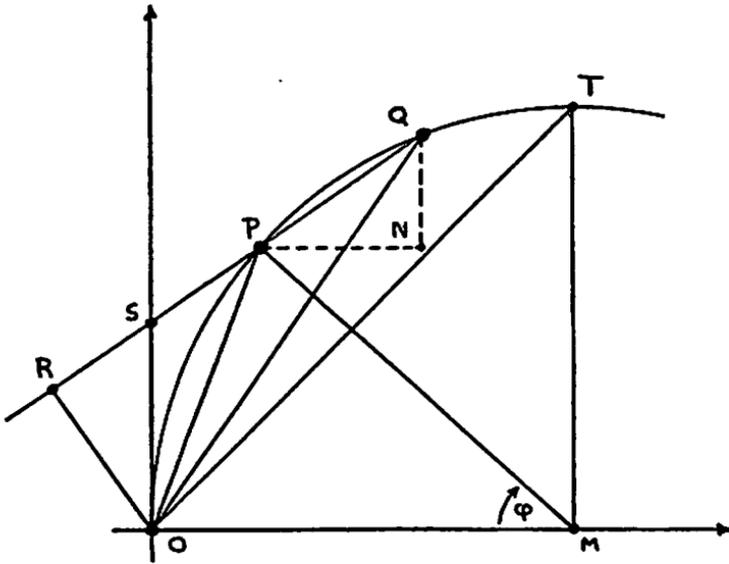


Figure One

So the area  $dA$  of  $\triangle POQ$  is

$$dA = \frac{1}{2} OR \, ds = \frac{1}{2} OS \, dx = \frac{1}{2} z \, dx,$$

where  $z$  denotes  $OS$ . As  $\triangle POQ$  sweeps across the segment in question,  $x$  goes from 0 to 1, so

$$A = \int dA = \frac{1}{2} \int_0^1 z \, dx.$$

To be able to integrate with respect to  $z$ , we integrate by parts to reverse the roles of  $x$  and  $z$  (as  $x$  goes from 0 to 1, the limits of  $z$  are 0 and 1):

$$A = \frac{1}{2} xz \Big|_0^1 - \frac{1}{2} \int_0^1 x \, dz$$

and so

$$A = \frac{1}{2} - \int_0^1 \frac{x}{2} \, dz.$$

Since  $dx$  is very small,  $\triangle SPM$  is almost a right triangle congruent to  $\triangle SOM$  and so  $\angle SMO = \phi/2$  where  $\phi = \angle PMO$ . Thus  $z = \tan(\phi/2)$  and  $\cos(\phi) = 1 - x$ , so that  $x = 1 - \cos(\phi) = 2 \sin^2(\phi/2)$ . We also need the following trigonometric identities:

$$\begin{aligned} \tan^2(\phi/2) &= \frac{\sin^2(\phi/2)}{\cos^2(\phi/2)} = \sin^2(\phi/2) \sec^2(\phi/2) \\ &= \sin^2(\phi/2) (1 + \tan^2(\phi/2)) \end{aligned}$$

which yield

$$\sin^2(\phi/2) = \frac{\tan^2(\phi/2)}{(1 + \tan^2(\phi/2))}$$

in order to get

$$(3) \quad \frac{x}{2} = \frac{z^2}{(1 + z^2)}.$$

The geometric series

$$\frac{1}{(1 + t^2)} = 1 - t^2 + t^4 - t^6 + \dots$$

(where  $t^2 < 1$ ) enables use to write (3) as

$$\frac{x}{2} = z^2 (1 - z^2 + z^4 - z^6 + \dots) = z^2 - z^4 + z^6 - z^8 + \dots.$$

Using this, equation (2) becomes

$$\begin{aligned}
 A &= \frac{1}{2} - \frac{1}{2} \int_0^1 (z^2 - z^4 + z^6 - z^8 + \dots) dz \\
 &= \frac{1}{2} - \left( \frac{1}{3} z^3 - \frac{1}{5} z^5 + \frac{1}{7} z^7 - \frac{1}{9} z^9 + \dots \right) \Big|_0^1 \\
 &= \frac{1}{2} - \left( \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9} + \dots \right) \\
 &= \frac{1}{2} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots
 \end{aligned}$$

Adding the  $\frac{1}{2}$  for the area of the isosceles triangle, we get

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots,$$

which is Leibniz's ingenious discovery.

To actually prove Leibniz's formula, we start with the series

$$(4) \quad \frac{1}{(1+t^2)} = 1 - t^2 + t^4 - t^6 + \dots - t^{4n-2} + \frac{t^{4n}}{(1+t^2)}$$

which is valid for all  $t$  and any  $n > 0$ , and can be checked by multiplying by  $1 + t^2$ . Integrating (4) over  $0 \leq t \leq x$ ,  $0 \leq x \leq 1$  gives

$$\begin{aligned}
 \tan^{-1}(x) &= \int_0^x \frac{dt}{(1+t^2)} \\
 &= \left( t - \frac{1}{3} t^3 + \frac{1}{5} t^5 - \frac{1}{7} t^7 + \dots - \frac{1}{4n-1} t^{4n-1} + R_n(t) \right) \Big|_0^x
 \end{aligned}$$

so

$$(5) \quad \tan^{-1}(x) = x - \frac{1}{3} x^3 + \frac{1}{5} x^5 - \frac{1}{7} x^7 + \dots - \frac{1}{4n-1} x^{4n-1} + R_n(x)$$

with  $R_n(x)$  being the remainder

$$R_n(x) = \int_0^x \frac{t^{4n}}{(1+t^2)} dt .$$

Obviously  $1 \leq 1 + t^2$ , so

$$0 \leq R_n(x) \leq \int_0^x t^{4n} dt$$

$$0 \leq R_n(x) \leq \left( \frac{1}{4n+1} t^{4n+1} \right) \Big|_0^x$$

$$(6) \quad 0 \leq R_n(x) \leq \frac{1}{4n+1} x^{4n+1} .$$

Because  $0 \leq x \leq 1$ , (6) shows that

$$0 \leq R_n(x) \leq \frac{1}{4n+1} .$$

Since  $R_n(x) \rightarrow 0$  as  $n \rightarrow \infty$ , (5) becomes

$$\tan^{-1}(x) = x - \frac{1}{3} x^3 + \frac{1}{5} x^5 - \frac{1}{7} x^7 + \dots$$

where  $0 \leq x \leq 1$ . Since  $\tan^{-1}(1) = \frac{\pi}{4}$ , when we set  $x = 1$  we have

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots ,$$

which is Leibniz's formula.

Euler's formula is

$$(7) \quad \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

To understand how Euler discovered this relation, we need to begin with some simple algebra. (Again, the formal proof will follow.) The roots of

$$(8) \quad \left(1 - \frac{x}{a}\right)\left(1 - \frac{x}{b}\right) = 0 \quad (\text{where } ab \neq 0)$$

are  $a$  and  $b$ . Equation (8) can also be written as

$$(9) \quad 1 - \left(\frac{1}{a} + \frac{1}{b}\right)x + \frac{1}{ab}x^2 = 0$$

As can be seen, the sum of the reciprocals of the roots is the negative coefficient of  $x$ . Replacing  $x$  by  $x^2$ ,  $a$  by  $a^2$  and  $b$  by  $b^2$  in (8) and (9), we have

$$(10) \quad \left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{x^2}{b^2}\right) = 0 \quad (\text{where } ab \neq 0)$$

and

$$(11) \quad 1 - \left(\frac{1}{a^2} + \frac{1}{b^2}\right)x^2 + \frac{1}{a^2b^2}x^4 = 0.$$

The roots of (11) are  $\pm a$  and  $\pm b$ . We see in (11) that the sum of the reciprocals of the squares of the roots is the negative coefficient of  $x^2$ . Moving on to a higher degree equation:

$$\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{x^2}{b^2}\right)\left(1 - \frac{x^2}{c^2}\right) = 0 \quad (\text{where } abc \neq 0)$$

$$1 - \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)x^2 + \left(\frac{1}{a^2b^2} + \frac{1}{a^2c^2} + \frac{1}{b^2c^2}\right)x^4 - \frac{1}{a^2b^2c^2}x^6 = 0,$$

we see that the same pattern holds.

If we now consider the equation

$$\sin(x) = 0,$$

or

$$x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 - \frac{1}{7!} x^7 + \dots = 0,$$

the expression can be thought of as an "infinite degree polynomial" with roots  $0, \pm\pi, \pm 2\pi, \pm 3\pi, \dots$ . We divide by  $x$  to remove the root of  $0$ , which gives

$$\frac{\sin(x)}{x} = 0$$

$$(12) \quad 1 - \frac{1}{3!} x^2 + \frac{1}{5!} x^4 - \frac{1}{7!} x^6 + \dots = 0,$$

with roots  $\pm\pi, \pm 2\pi, \pm 3\pi, \dots$ . The previous discussion on roots suggests that (12) can be written as an infinite product:

$$(13) \quad \frac{\sin(x)}{x} = \left(1 - \frac{x^2}{\pi^2}\right) \left(1 - \frac{x^2}{4\pi^2}\right) \left(1 - \frac{x^2}{9\pi^2}\right) \dots$$

The analogy further suggests that

$$\frac{1}{3!} = \frac{1}{\pi^2} + \frac{1}{4\pi^2} + \frac{1}{9\pi^2} + \dots$$

so that

$$\frac{\pi^2}{6} = 1 + \frac{1}{4} + \frac{1}{9} + \dots$$

The crux of the matter lies in (13), Euler's infinite product for the sine. As infinite products are beyond the scope of this paper, the formal proof of Euler's formula will take a different approach.

To begin the proof of Euler's formula, we define the functions  $f_n(x)$  by

$$(14) \quad f_n(x) = \frac{1}{2} + \cos(x) + \cos(2x) + \dots + \cos(nx).$$

We also need the closed form of  $f_n(x)$  given by

$$(15) \quad f_n(x) = \frac{\sin\left(\frac{2n+1}{2}x\right)}{2 \sin\left(\frac{x}{2}\right)}$$

for  $x \neq 2k\pi$ . This can be proved using the trigonometric identity  $2\cos(\theta)\sin(\phi) = \sin(\theta+\phi) - \sin(\theta-\phi)$ , which gives

$$\begin{aligned} 2 \cdot \frac{1}{2} \sin\left(\frac{x}{2}\right) &= \sin\left(\frac{x}{2}\right) \\ 2 \cos(x) \sin\left(\frac{x}{2}\right) &= \sin\left(\frac{3}{2}x\right) - \sin\left(\frac{x}{2}\right) \\ 2 \cos(2x) \sin\left(\frac{x}{2}\right) &= \sin\left(\frac{5}{2}x\right) - \sin\left(\frac{3}{2}x\right) \\ &\vdots \\ 2 \cos(nx) \sin\left(\frac{x}{2}\right) &= \sin\left(\frac{2n+1}{2}x\right) - \sin\left(\frac{2n-1}{2}x\right). \end{aligned}$$

If we add these up, we obtain (15).

Now we define a number  $E_n$  using (14):

$$E_n = \int_0^{\pi} x f_n(x) dx.$$

Substituting and integrating by parts yields

$$\begin{aligned} E_n &= \int_0^{\pi} \left( \frac{x}{2} + x \cos(x) + x \cos(2x) + \cdots + x \cos(nx) \right) dx \\ &= \frac{x^2}{4} \Big|_0^{\pi} + \int_0^{\pi} x \cos(x) dx + \int_0^{\pi} x \cos(2x) dx + \cdots + \int_0^{\pi} x \cos(nx) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{\pi^2}{4} + \left( x \sin(x) \Big|_0^\pi - \int_0^\pi \sin(x) \, dx \right) + \left( \frac{x}{2} \sin(2x) \Big|_0^\pi - \frac{1}{2} \int_0^\pi \sin(2x) \, dx \right) + \\
&\quad \cdots + \left( \frac{x}{n} \sin(nx) \Big|_0^\pi - \frac{1}{n} \int_0^\pi \sin(nx) \, dx \right) \\
&= \frac{\pi^2}{4} + \cos(x) \Big|_0^\pi + \frac{1}{2^2} \cos(2x) \Big|_0^\pi + \cdots + \frac{1}{n^2} \cos(nx) \Big|_0^\pi
\end{aligned}$$

and so

$$(16) \quad E_n = \frac{\pi^2}{4} + \sum_{k=1}^n \frac{(-1)^k - 1}{k^2}.$$

Since the even terms of (16) are zero, we can write it as

$$(17) \quad \frac{1}{2} E_{2n-1} = \frac{\pi^2}{8} - \sum_{k=1}^n \frac{1}{(2k-1)^2}.$$

If we can show that  $\lim_{n \rightarrow \infty} E_{2n-1} = 0$ , it will establish that

$$(18) \quad \frac{\pi^2}{8} = \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2},$$

which will be used to prove the final result.

Integrating (15) by parts and defining the function  $g(x)$  by

$$g(x) = \begin{cases} \frac{d}{dx} \left( \frac{x/2}{\sin(x/2)} \right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases},$$

we get

$$\begin{aligned}
 E_n &= \int_0^\pi x \frac{\sin\left(\frac{2n+1}{2} x\right)}{2 \sin\left(\frac{x}{2}\right)} dx \\
 &= \lim_{a \rightarrow 0^+} \left[ \frac{-2}{2n+1} \frac{x}{2} \cos\left(\frac{2n+1}{2} x\right) \right]_a^\pi \\
 &\quad + \frac{2}{2n+1} \int_0^\pi \cos\left(\frac{2n+1}{2} x\right) g(x) dx \\
 &= \frac{2}{2n+1} + \frac{2}{2n+1} \int_0^\pi \cos\left(\frac{2n+1}{2} x\right) g(x) dx
 \end{aligned}$$

because

$$\lim_{x \rightarrow 0} \frac{x/2}{\sin(x/2)} = 1.$$

Setting  $n = 2k-1$  gives

$$\begin{aligned}
 E_{2k-1} &= \frac{2}{4k-1} + \frac{2}{4k-1} \int_0^\pi \cos\left(\frac{4k-1}{2} x\right) g(x) dx \\
 &= \frac{1}{4k-1} \left( 2 + 2 \int_0^\pi \cos\left(\frac{4k-1}{2} x\right) g(x) dx \right).
 \end{aligned}$$

Using the Extreme Value Theorem, since  $g(x)$  is bounded by  $1/2$  and the cosine is bounded by  $1$ , the integral is bounded and this shows that  $E_{2k-1} \rightarrow 0$  as  $k \rightarrow \infty$ .

To complete the proof, we divide the positive integers into the evens and the odds and use (18) to write

$$\begin{aligned}\sum_{k=1}^{\infty} \frac{1}{k^2} &= \sum_{k=1}^{\infty} \frac{1}{(2k)^2} + \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} \\ &= \frac{1}{4} \sum_{k=1}^{\infty} \frac{1}{k^2} + \frac{\pi^2}{8}\end{aligned}$$

yielding

$$\left(\frac{3}{4}\right) \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{8}$$

and so

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{4}{3} \frac{\pi^2}{8} = \frac{\pi^2}{6},$$

which is Euler's formula.

Neither Leibniz's nor Euler's formula converges very fast. Leibniz's formula gives a value of 3.01418... after ten terms while Euler's formula gives 3.0494... after ten terms, compare to the actual value of 3.14159... for  $\pi$ . Better approximations of  $\pi$  can be found by using the power series

$$\tan^{-1}(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \dots,$$

where  $-1 \leq x \leq 1$ , and the formulas

$$\frac{\pi}{4} = \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$$

and

$$\frac{\pi}{4} = 4 \tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right).$$

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*Editor's Comment.* Further material on Leibniz and the construction of an appropriate quadratrix may be found in *Leibniz in Paris 1672-1676: his growth to mathematical maturity* by Joseph E. Hofmann (New York: Cambridge University Press, 1974), particularly pages 53-61.

## Kansas Gamma Golden Anniversary

The Kansas Gamma Chapter of Kappa Mu Epsilon at Benedictine College in Atchison, Kansas, celebrated its fiftieth anniversary this past spring. The Chapter was installed on 26 May 1940 with Sister Helen Sullivan as founding faculty sponsor at what was then Mount St. Scholastica College.



Shown above are (seated left to right) Sisters Jo Ann Fellin and Helen Sullivan with (standing left to right) student members Nancy Sheble, Matthew McIntosh (holding the Chapter charter), Ty Anderson and Julie Stenger. Sister Helen Sullivan also served as national historian and editor of the *Kappa Mu Epsilon News* section of *The Pentagon* during 1943-47.

A Multi-Dimensional Study of the  
Divergence Theorem

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Presented at the 1989 National Convention.

Although few people think about what happens when they turn on a light, something interesting does happen: an electric circuit is completed and electricity begins to flow along the wire. This flow of electricity can also be described with the aid of some formulas. This paper will begin with such a formula called the "Divergence Theorem." This result, as it was developed, works for three dimensional objects or spaces. For simplicity, I use the unit cube, based at the origin, (0,0,0), as my object. I then use the same argument to develop this theorem in two dimensions using the unit square as my object. The two dimensional analog of the Divergence Theorem is similar to Green's Theorem and the reasons for this are explored. These ideas can also be easily extended to higher dimensions; I will work explicitly in the fourth dimension with the "unit hypercube" or "tesseract" as my object of interest.

We will begin with a statement and proof of the Divergence Theorem for three dimensions. This result can be written:

$$\iint_S \mathbf{F}(x,y,z) \cdot \mathbf{n} \, dS = \iiint_V \operatorname{div} \mathbf{F}(x,y,z) \, dV.$$

This theorem says that the flux of a vector function  $\mathbf{F}$  through a closed surface  $S$  is equal to the triple integral of the divergence of the function taken over the volume of the object.

Before proceeding further, the language just used must be explained.

The term "flux" means a continual flow of a fluid or an electrical charge. The left hand side of the equation can be written as

$$\iint_S \left( F_x \cos(\alpha) + F_y \cos(\beta) + F_z \cos(\gamma) \right) dS$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are the angles which are normal (or perpendicular) to the surface and directed outward from it. This explains the "n" in the formula; it is the normal component for each vector. For our purposes,  $F_x = P$ ,  $F_y = Q$  and  $F_z = R$ . The divergence of a vector field can be expressed in terms of the partial derivatives of the function. Thus,  $\text{div } F = \nabla \cdot F$  where the symbol  $\nabla$ , called "del," is the operator defined by the equation

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k .$$

Assuming that our function has the form  $F = Pi + Qj + Rk$ , we have that

$$\nabla \cdot F = \left( \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) \cdot (Pi + Qj + Rk)$$

and thus

$$\text{div } F = \nabla \cdot F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} .$$

Therefore, the divergence of a vector field  $F$  is equal to the sum of the partial derivatives of each vector component of  $F$ .

The Divergence Theorem can now be written as

$$\begin{aligned} \iint_S \left( P \cos(\alpha) + Q \cos(\beta) + R \cos(\gamma) \right) dS \\ = \iiint_V \left( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dV . \end{aligned}$$

Because we will be only dealing with the unit cube,  $\alpha$ ,  $\beta$  and  $\gamma$  will always be either 0 or  $\pi$  and so, because  $\cos(0) = 1$  and  $\cos(\pi) = -1$ , each face of the cube will have either a positive or negative normal component. Now, because our cube can only have two values each for the

x, y and z directions and each direction must have a positive and negative component, the theorem can be written as

$$\begin{aligned} & \iint_S (P(X_1, y, z) - P(X_2, y, z)) dS + \iint_S (Q(x, Y_1, z) - Q(x, Y_2, z)) dS \\ & + \iint_S (R(x, y, Z_1) - R(x, y, Z_2)) dS \\ & = \iiint_V \left( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dV \end{aligned}$$

and it is this form of the Divergence Theorem which I shall prove. Let us begin by looking at a picture of the unit cube (see Figure One). The arrows designate the directions of the normal vectors, which are perpendicular to the faces of the cube. Also, each of the eight corners of the cube have been labeled.

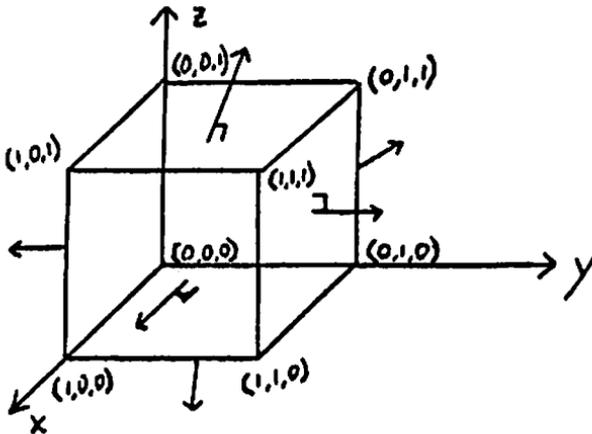


Figure One

Now, by plugging in our endpoints  $X_1$  and  $X_2$ ,  $Y_1$  and  $Y_2$ , and  $Z_1$  and  $Z_2$ , our equation becomes:

$$\int_0^1 \int_0^1 P(1, y, z) dy dz - \int_0^1 \int_0^1 P(0, y, z) dy dz$$

$$\begin{aligned}
& + \int_0^1 \int_0^1 Q(x,1,z) \, dx dz - \int_0^1 \int_0^1 Q(x,0,z) \, dx dz \\
& + \int_0^1 \int_0^1 R(x,y,1) \, dx dy - \int_0^1 \int_0^1 R(x,y,0) \, dx dy \\
& = \int_0^1 \int_0^1 \int_0^1 \left( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dx dy dz.
\end{aligned}$$

But this can be rewritten as

$$\begin{aligned}
& \int_0^1 \int_0^1 \left( P(1,y,z) - P(0,y,z) \right) dy dz \\
& + \int_0^1 \int_0^1 \left( Q(x,1,z) - Q(x,0,z) \right) dx dz \\
& + \int_0^1 \int_0^1 \left( R(x,y,1) - R(x,y,0) \right) dx dy \\
& = \int_0^1 \int_0^1 \int_0^1 \left( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dx dy dz,
\end{aligned}$$

and, because

$$P(1,y,z) - P(0,y,z) = \int_0^1 \frac{\partial P}{\partial x} \, dx$$

by the First Fundamental Theorem of Calculus, we can rewrite the first expression as

$$\int_0^1 \int_0^1 \left( \int_0^1 \frac{\partial P}{\partial x} \, dx \right) dy dz$$

and similarly for the expressions in Q and R. Thus we obtain

$$\begin{aligned} \int_0^1 \int_0^1 \left( \int_0^1 \frac{\partial P}{\partial x} dx \right) dydz + \int_0^1 \int_0^1 \left( \int_0^1 \frac{\partial Q}{\partial y} dy \right) dx dz + \int_0^1 \int_0^1 \left( \int_0^1 \frac{\partial R}{\partial z} dz \right) dx dy \\ = \int_0^1 \int_0^1 \int_0^1 \left( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dx dy dz, \end{aligned}$$

and we have established our special case of the Divergence Theorem. We should mention that if  $\text{div } F$  is positive throughout the cube then we say that it is a "source" and something (the fluid, the electrical charge, etc.) is being created inside the cube and flows out of it. Similarly, if  $\text{div } F$  is negative then we say it is a "sink" and whatever flows into the cube is being destroyed and nothing flows out.

With this method of proof in mind, let us move back to two dimensions. We will use the unit square as our object of study and we can expect that our equation will be

$$\int_S F(x,y,z) \cdot n \, dS = \int_A \text{div } F(x,y) \, dA.$$

This says that the flux of a two dimensional vector field, taken over the sides of a square, is equal to the double integral of its divergence. By rewriting this formula as before, we arrive at

$$\int_0^1 P(x,y) \, dy + \int_0^1 Q(x,y) \, dx = \int_0^1 \int_0^1 \left( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dx dy$$

and this relation is extremely similar to Green's Theorem, which states that

$$\int_C (P(x,y) \, dx + Q(x,y) \, dy) = \int_0^1 \int_0^1 \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy.$$

If we let  $(P_i + Q_j)$  be the parallel vector field, and by realizing that Green's Theorem is true for summing the line integrals along each face of the square, this works out rather quickly. Let us look at a picture of our unit square showing the direction of our line integrals and the direction

of the vector  $P_i + Q_j$  (see Figure Two).

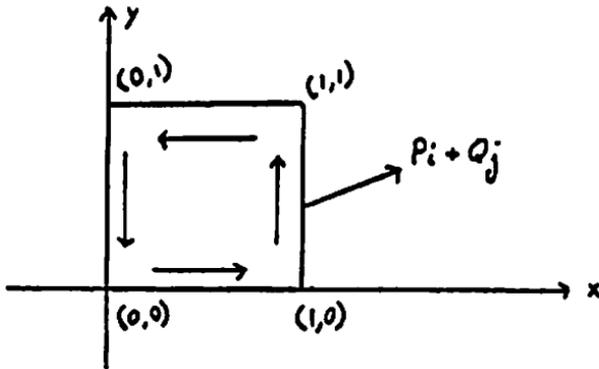


Figure Two

By first working with the right most side of the square, we take the line integral in the direction of the arrows. By noticing that  $x$  is constant at positive one, we get

$$\int_0^1 Q(1,y) dy$$

and for this side the  $P$  component does not affect our calculation. Following this around the rest of the square results in the expression

$$\int_0^1 Q(1,y) dy + \int_1^0 P(x,1) dx + \int_1^0 Q(0,y) dy + \int_0^1 P(x,0) dx.$$

By switching the limits of integration and combining, we soon have

$$\int_0^1 (Q(1,y) - Q(0,y)) dy - \int_0^1 (P(x,1) - P(x,0)) dx$$

and then as before, we obtain

$$\int_0^1 \int_0^1 \frac{\partial Q}{\partial x} dx dy - \int_0^1 \int_0^1 \frac{\partial P}{\partial y} dx dy = \int_0^1 \int_0^1 \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy,$$

as we wished to show.

This last quantity is called the "curl" of the vector function. Green's Theorem thus finds that the sum of the line integrals of a vector field is equal to the double integral of its curl. This result is also called the "work" done on the object. In order to find the flux, we have to use the perpendicular component of the vector and not the parallel component (see Figure Three). Again looking at our picture and starting on the right side of the square, we see that our equation becomes

$$\int_0^1 P(1,y) dy + \int_0^1 Q(x,1) dx - \int_0^1 P(0,y) dy - \int_0^1 Q(x,0) dx.$$

The last two sides are negative because their normals are  $-i$  and  $-j$ , respectively. As before, we can now combine terms to obtain

$$\begin{aligned} & \int_0^1 (P(1,y) - P(0,y)) dy + \int_0^1 (Q(x,1) - Q(x,0)) dx \\ &= \int_0^1 \int_0^1 \left( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dx dy. \end{aligned}$$

And this, finally, is the divergence of the vector field. Therefore, we see that the curl of one vector field is equal to the divergence of the perpendicular vector field.

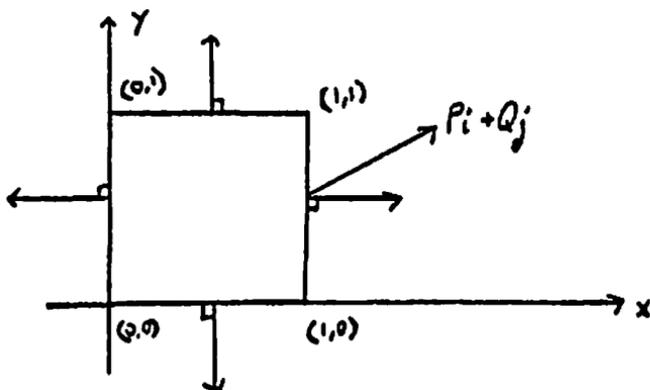


Figure Three

By using what we've observed in both two and three dimensions, we can have an idea as to what we want to accomplish in the fourth dimension. We would like to see a formula that shows the flux of a four dimensional object equaling the fourfold integral of its divergence.

Although the proof of this proposed formula is quite similar to the proof of the three dimensional Divergence Theorem, visualizing what is occurring can be difficult. The best representation of a four dimensional tesseract is a cube within a cube. It would look something as shown in Figure Four.

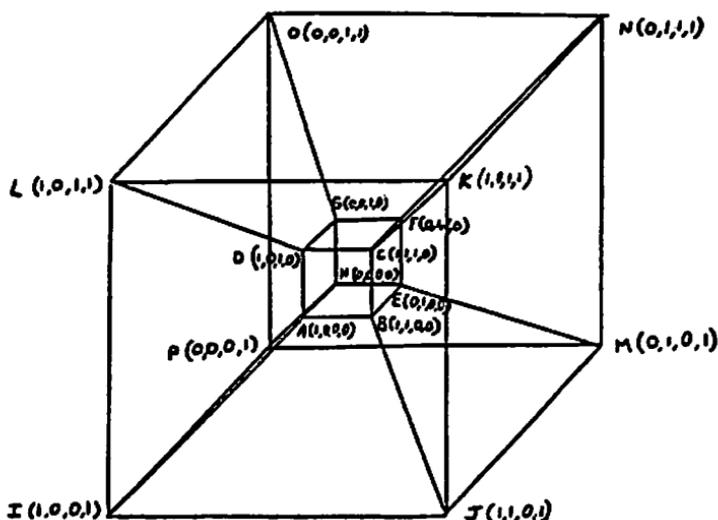


Figure Four

Although Figure Four is interesting to look at, theoretically it does not look so good. As this picture is described, remember that it is a four dimensional object and should always be viewed as such. Also, it is the theory that is more sensible than the picture and not the opposite. To begin, the coordinates of the inner cube's corners are the same as the usual three dimensional cube except that a fourth dimension,  $w$ , has been added. On the inner cube, the  $w$ -axis is at 0. The four coordinates for each point are written in the order  $(x,y,z,w)$  to preserve as much continuity from before as possible. The outer cube is the same as the inner cube except that it exists at  $w = 1$  (only the fourth dimension coordinate has been changed). This is now where the problems with the picture begin.

Noticing that all of the edges of the inner cube have unit length, the usual "three-dimensional" volume of the cube must be one; the length of an edge can be found using the four dimensional version of the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 + (w_2 - w_1)^2}$$

so the distance between points A and B can be calculated by plugging in the coordinates (1,0,0,0) and (1,1,0,0) to find

$$\sqrt{(1 - 1)^2 + (1 - 0)^2 + (0 - 0)^2 + (0 - 0)^2} = 1,$$

and the volume can be found using the usual three dimensional formula ("length · width · height") since it is a three dimensional object. Going one step further, each of the edges of the outer cube has length one and so it too has a volume of one. Thus, even though the cubes do not appear to be the same size, they are indeed. But, without looking at the picture, we should expect to achieve this result. Considering the fourth dimension as "time," the inner cube is the same as the outer cube except that it has been moved in time. I have drawn the cubes in such a relationship so that the connection between the adjacent vertices is easily seen.

Let us look at another aspect of the hypercube. The point A on the inner cube as coordinates (1,0,0,0) and the point I on the outer cube has coordinates (1,0,0,1). Therefore, by the distance formula, the length of  $\overline{AI}$  is one. By generalizing this observation to the rest of the picture, we see that each line segment connecting the inner cube to the outer cube also has a length of one. This means that the region ABCDIJKL is also a cube. By extending this observation, we realize that there are six cubes which are part of both dimensions. It must also be stated that these cubes do not have a three dimensional volume of one because they are a part of both dimensions. Therefore, they have a three dimensional volume of zero. Taking these six cubes together with the inner and outer cubes, we have a total of eight cubes forming the "faces" of our four dimensional tesseract.

Our formula should look something like

$$\iiint_S \mathbf{F}(x,y,z,w) \cdot \mathbf{n} \, dS = \iiint_V \operatorname{div} \mathbf{F}(x,y,z,w) \, dV.$$

Since we also know that  $F(x,y,z,w) = P(x,y,z,w) \cdot i + Q(x,y,z,w) \cdot j + R(x,y,z,w) \cdot k + L(x,y,z,w) \cdot h$ , this formula also would be

$$\begin{aligned} & \iiint_S P \, dx \, Q \, dy + R \, dz + L \, dw \\ &= \iiint_V \left( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} + \frac{\partial L}{\partial w} \right) dx dy dz dw. \end{aligned}$$

If we call cube ABCDIJKL our positive  $x$  direction, then the normal for this face is  $+i$ . Likewise, cube EFGHMNOP is our negative  $x$  direction and so has a normal of  $-i$ . Furthermore, the first cube has  $x$  held constant at 1 and the second cube has  $x$  held constant at 0. By working with just these two of the eight faces, we obtain

$$\begin{aligned} & \int_0^1 \int_0^1 \int_0^1 P(1,y,z,w) \, dy dz dw - \int_0^1 \int_0^1 \int_0^1 P(0,y,z,w) \, dy dz dw \\ &= \int_0^1 \int_0^1 \int_0^1 \left( P(1,y,z,w) - P(0,y,z,w) \right) dy dz dw \end{aligned}$$

and, again by the definition of a partial derivative we have

$$P(1,y,z,w) - P(0,y,z,w) = \int_0^1 \frac{\partial P}{\partial x} \, dx,$$

and so we obtain

$$\int_0^1 \int_0^1 \int_0^1 \left( \int_0^1 \frac{\partial P}{\partial x} \, dx \right) dy dz dw.$$

By extending this calculation over the remaining three pairs of faces of the hypercube, we arrive at the total expression

$$\int_0^1 \int_0^1 \int_0^1 \left( \int_0^1 \frac{\partial P}{\partial x} dx \right) dydzdw + \int_0^1 \int_0^1 \int_0^1 \left( \int_0^1 \frac{\partial Q}{\partial y} dy \right) dx dz dw$$

$$+ \int_0^1 \int_0^1 \int_0^1 \left( \int_0^1 \frac{\partial R}{\partial z} dz \right) dx dy dw + \int_0^1 \int_0^1 \int_0^1 \left( \int_0^1 \frac{\partial L}{\partial w} dw \right) dx dy dz$$

which is the same as the simpler expression

$$\int_0^1 \int_0^1 \int_0^1 \int_0^1 \left( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} + \frac{\partial L}{\partial w} \right) dx dy dz dw,$$

as was first proposed.

By noticing how similar the proof for the Divergence Theorem in three dimensions is to its four dimensional analog, it is fairly obvious that it could now be extended and proved in five or more dimensions. For that matter, the Divergence Theorem will be true in any given dimension. So, in order to generalize to these higher dimensions, we will have to notice the patterns that are forming. The first thing we will observe is that in three dimensions, a face of a cube is a two dimensional square. We can see that the theorem stated that the sum of the double integrals, taken over each square (or face of the cube) was equal to the triple integral of the divergence of the vector function. In four dimensions, we found that the face of the tesseract was a three dimensional cube and we took the sum of the triple integrals over the eight faces and obtained the fourfold integral of the divergence of the vector function.

In any dimension  $n$ , a "face" will be an object of dimension  $n-1$  and there will be  $2n$  of them. Let us write an element of integration as  $dx_1 \omega_1 = dx_1 dx_2 \cdots dx_n$  so that  $\omega_1 = dx_2 \cdots dx_n$  and, similarly, for  $dx_j \omega_j = dx_1 dx_2 \cdots dx_n$ . For example, in three dimensions we have  $\omega_1 = dx_2 dx_3$ ,  $\omega_2 = dx_1 dx_3$  and  $\omega_3 = dx_1 dx_2$ . These expressions cover the different elements of integration that appear on the left hand side of the Divergence Theorem. We then have the generalized formula

$$\sum_{j=1}^n \int_{(n-1)} \cdots \int \Delta P_j \omega_j = \int_{(n)} \cdots \int \operatorname{div} F dx_1 \cdots dx_n$$

where there are  $n-1$  integrations in each expression on the left and  $n$  integrations on the right and

$$\Delta P_j = P_j(x_1, \dots, x_{j-1}, 1, x_{j+1}, \dots, x_n) - P_j(x_1, \dots, x_{j-1}, 0, x_{j+1}, \dots, x_n)$$

with  $P_j$  being the  $j$ -th component of the vector function  $F$ .

Another interesting observation is that our regions of study do not have to be the unit cube for the given dimension. For example, in two dimensions we could approximate a region (see Figure Five) as a sum of many squares. On each square we can use the Divergence Theorem to find the flux and then the total flux on the area can be found by a limiting argument. This works because the adjoining sides of any two squares "cancel" each other out, leaving only the perimeter of all the squares combined. Thus, by the continued addition of more smaller squares, the flux on the outer boundary can be found. The only stipulation that needs to be placed on this idea is that the region be rectifiable (if the region is an infinite space, this will not work).

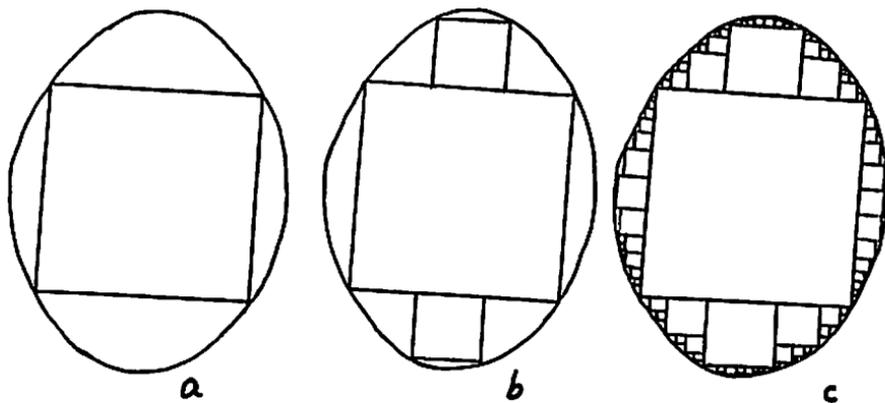


Figure Five

Using the premises of the Divergence Theorem, we have seen that they will work in higher dimensions and an analog of the theorem can be developed for all dimensions  $n \geq 2$ . Further, the shape and size of the region of interest is not limited to the unit cube in the given dimension since, by a limiting argument, expressions for both the flux and

divergence of a vector function can easily be found.

This paper is based on part of my honor thesis prepared under the direction of Dr. James V. Peters during the Fall 1989 semester.

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## Abstracts of Papers Presented at the 1990 Region I Convention

Edited by James C. Pomfret, Region I Director

The 1990 *KME* Region I Convention was hosted by the Pennsylvania Theta Chapter at Susquehanna University on April 27-28 (a report on the meeting appears on page 73). The following abstracts of the student papers not only provide a record of the meeting but may also serve as resources for future student projects.

Michelle CREOLA, *Pascal's Triangle Revisited*. Pennsylvania Lambda (Bloomsburg University of Pennsylvania, Bloomsburg, Pennsylvania 17815).

Pascal's triangle contains many patterns and numerical correspondences. Examples are used to demonstrate that each entry represents the number of paths to that position from the apex of the triangle. Adding the entries or alternately adding and subtracting the entries in any horizontal row in Pascal's triangle leads to interesting patterns as does adding diagonals of finite length.

The harmonic triangle of Leibniz is introduced and its recursion is defined and compared to that found in Pascal's triangle. A correspondence between entries in the two triangles is shown. Any entry in Leibniz's triangle is the reciprocal of the product of the corresponding entry in Pascal's triangle and one plus the row in which the number is located. Some properties of Leibniz's triangle are compared to some of those previously examined in Pascal's triangle.

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Janice M. Gessner, *Breaking the Code*. Pennsylvania Theta (Susquehanna University, Selinsgrove, Pennsylvania 17870).

Cryptography is the art of breaking secret codes. Enciphering the plaintext and deciphering the ciphertext (or "disguised message") are goals of individuals interested in cryptography. There are several ways in which secret codes are enciphered and deciphered. The shift transformation, affine transformation and matrix transformation of both a single letter message unit and digraph are some of those methods.

A secret message has two parts: a general system, which is usually a mathematical formula, and a specific key. The biggest challenge in cryptography is learning the value of this second part.

The shift transformation technique is the easiest to use and comprehend. Secret message are simple to encipher and decipher, given the appropriate tools. The general system consists of the equations  $C = P + b \text{ mod } N$  and  $P = C - b \text{ mod } N$ , where  $C$  is the value of each ciphertext message unit,  $P$  is the value of the plaintext message unit,  $N$  is the size of the alphabet being used and  $b$  is the second piece of information needed, or the specific key. Usually  $b$  is determined by frequency analysis. Once  $b$  is known, the message units can be substituted into the appropriate equation, and the ciphertext or plaintext can be determined.

The other methods are similar, but are more difficult to use and the messages remain secret to all but the intended recipient.

Samuel P. Gross, *Quasicrystals*. Pennsylvania Theta (Susquehanna University, Selinsgrove, Pennsylvania 17870).

This talk described the Penrose tiling and related tilings of the plane with a few basic shapes in such a way that the tiling is not periodic but has statistical symmetries. The Penrose tiling uses two shapes and five-fold rotational statistical symmetry. It has five sets of parallel lines, the Ammann lines, with special properties. Also shown were patterns with seven, nine and thirteen-fold rotational statistical symmetry. Rules were given for automatically constructing the tilings.

Lauren J. HENNEBERGER, *An Analysis of Diophantine Equations*. New York Lambda (C. W. Post / Long Island University, Brookville, New York 11548).

One passage in the "Palatine Anthology" of Greek mathematical problems written in poetic form describes the tomb of Diophantus of Alexandria (c. AD 250) and contains the known details of his private life. Interpreted as statements about the length,  $x$ , of Diophantus' life, the tomb inscription becomes a linear equation in one unknown having the integral solution  $x = 84$ . "Diophantine problems" are concerned with integral solutions of algebraic equations.

The linear equation  $ax + by = c$ , where  $a, b$  (not both zero) and  $c$  are real numbers has an entire line of solutions  $(x,y)$  in the Cartesian plane  $\mathbb{R}^2$  while the linear Diophantine equation  $ax + by = c$ , where the coefficients now are integers, has at most a discrete set of integral solutions  $(x,y)$  in the lattice  $\mathbb{Z}^2 \subset \mathbb{R}^2$ . I prove that a necessary and sufficient condition for the existence of solutions for this linear Diophantine equation is that the greatest common divisor of  $a$  and  $b$  be a divisor of  $c$ . Further, given any particular solution  $(x_0, y_0)$  of  $ax + by = c$ , I prove that every other solution  $(x,y)$  is of the form  $x = x_0 + bt, y = y_0 - at$  where the parameter  $t$  ranges over the integers.

While the "Euclidean algorithm" gives a completely general method of calculating particular solutions of solvable linear Diophantine equations, I conclude with the solutions of two word problems that demonstrate the method of "fractional parts" used by Diophantus in the solution of several problems in his *Arithmetica*.

Michelle LEUSER, *The {1} Condition Generalized Inverse*. Pennsylvania Theta (Susquehanna University, Selingsgrove, Pennsylvania 17870).

In recent years, the need for a generalized inverse of a rectangular or singular matrix has been felt in many areas of applied mathematics. By a generalized inverse of a matrix  $A$ , we mean a matrix  $X$  associated in some way to  $A$  such that: (a)  $X$  exists for a class of matrices larger than the class of nonsingular matrices, (b)  $X$  has some of the properties of the usual inverse, and (c)  $X$  reduces to the usual inverse when  $A$  is nonsingular.

In 1955, Roger Penrose showed that for every finite matrix  $A$ , of real or complex elements, there is a unique matrix  $X$  that satisfies these four equations: (1)  $AXA = A$ , (2)  $XAX = X$ , (3)  $(AX)^* = AX$  and (4)  $(XA)^* = XA$ , where  $( )^*$  denotes the conjugate transpose.

The "{1}-condition generalized inverse," which is used to solve singular linear equations, is a weaker inverse than the one described by Penrose and satisfies only the first equation. As a result, this inverse is not unique.

The construction of a {1}-condition generalized inverse for an arbitrary matrix  $A \in C_r^{m \times n}$  is simplified by first putting  $A$  into "row echelon" form such that (i) each of the first  $r$  rows contains at least one nonzero element and the remaining rows contain only zeros, and (ii) the first  $r$  columns of the identity matrix appear in the columns  $c_1, c_2, \dots, c_r$ . This can be done by performing elementary row operations on the matrix.

Once the matrix is in row echelon form, the Marcus and Minc definition states that by a suitable permutation of the columns, the matrix can be brought into the partitioned form

$$R = \left[ \begin{array}{c|c} I_r & K \\ \hline O & O \end{array} \right]$$

where  $I_r$  is the  $r \times r$  identity matrix,  $K \in C^{r \times (n-r)}$  and  $O$  denotes the null matrix. After the matrix is in its partitioned form, we define the {1}-condition generalized inverse to be the  $m \times n$  matrix

$$S = \left[ \begin{array}{c|c} I_r & O \\ \hline O & L \end{array} \right] \quad \text{where } L \in C^{(m-r) \times (n-r)}.$$

Francis D. THOMPSON, *Applications of Elementary LISP Functions to Computational Linguistics*. New York Lambda (C. W. Post / Long Island University, Brookville, New York 11548).

LISP (an acronym for list processing) is a programming language widely used in artificial intelligence applications. Some of the specialized functions and data types in LISP make its application to computational linguistics particularly attractive.

One of the basic data types in LISP is the "cons cell." A cons cell consists of two pointers: a CAR pointer and a CDR pointer. The CAR pointer refers to the memory location of a piece of data. The CDR pointer refers either to the CAR pointer of another cons cell or to the symbol NIL (which for our purposes just signifies the end of a sequence of cons cells). The idea of a cons cell can be represented pictorially by a cons cell diagram.

A sequence of linked cons cells constitutes another data type in LISP, the "list." Since a sentence can be thought of as a sequence of words, sentences can be represented in LISP as lists. Each CAR pointer in the list refers to a word of the sentence.

LISP contains specialized functions which operate on lists. Four of these functions are the "car," "cdr," "cons" and "list" functions. The car function returns the first element of a list. The cdr function returns the list with the first element deleted. The cons function inserts a given element into a list. The list function creates a list out of a set of elements.

These functions can help to parse sentences. Parsing, in computational linguistics, refers to assigning each word of a sentence its proper syntactic category (i.e., "noun," "verb," "adjective," etc.). For example, the sentence "He runs fast" would be parsed into "noun verb adverb." Car, cdr, cons and list can be written into a parsing algorithm to assist in sequentially processing the words of a sentence; that is, to determine the category of the first word, then the second and so on.

In addition to these functions and data types, LISP contains many other interesting features. For those interested in learning LISP, there are a number of excellent texts. *Essential LISP*, by Anderson, Corbett and Reiser, is an introductory text which assumes little or no programming experience. *A Programmer's Guide to Common LISP*, by Deborah Tatar, assumes a background in computer programming, but not an extensive one. *Common LISP*, by Gary Steele, is the source book for advanced programmers. For those interested in computational

linguistics, *Computational Linguistics: an introduction*, by Ralph Grishman, provides a comprehensive overview of the field.

Alice SZYMCZAK, *Finite Element Analysis*. Pennsylvania Lambda (Bloomsburg University of Pennsylvania, Bloomsburg, Pennsylvania 17815).

Finite element analysis is a relatively new area of engineering with emphasis in structural analysis that emerged with the advent of appropriate computer technology. The finite element analysis lecture was approached in one dimension using linear springs. The simplest case of a one spring system was used to introduce terminologies, such as element and node, and to generate the element stiffness equation,  $[k][u] = [X]$ , where  $[k]$  is the element stiffness matrix,  $[u]$  is the element displacement vector, and  $[X]$  is the element force vector. The case of a two spring system was used to illustrate the direct stiffness procedure, which is a method of combining the element stiffness matrices to produce the system stiffness matrix,  $[K]$ , which is related to the system force vector  $[P]$  by the equation  $[K][u] = [P]$ . The mathematics developed for the spring was then used to introduce the horizontal truss element. The mathematics for the horizontal truss is the same as the linear spring if  $k = E \cdot A/L$ , where  $E$  is the elastic modulus,  $A$  is the area, and  $L$  is the length of the truss. The stiffness equations for a two dimensional truss were then developed. A numerical example of a three member truss was discussed. When a force was applied to the nodes, the displacements of the vertices of the truss were found by solving  $[K][u] = [P]$  once boundary conditions were substituted into the equation. Due to the time consuming procedures used to solve the system of equations, the need for computer technology becomes obvious. The finite element analysis programs ANSYS 4.3 and PAL/2 were discussed. Because of computer technology, finite element analysis becomes a powerful tool in preventing structural collapse.

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## The Problem Corner

Edited by Kenneth M. Wilke

*The Problem Corner* invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August 1991. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1991 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

### IN MEMORIAM

Charles W. Trigg (1898-1989).

Born 7 February 1898, Charles W. Trigg began his career as a chemist, teaching that subject in 1927. He taught mathematics and physics at Los Angeles City College from 1938 to 1943. After service with the Navy, he returned to teaching at Los Angeles City College until he retired in 1963.

As one of this country's foremost problemists, he served as the editor of the "Problems and Questions" department of *Mathematics Magazine*, edited book reviews for *The Journal of Recreational Mathematics* and published his own anthology of problems entitled *Mathematical Quickies*. He was well known for his numerous articles on properties of numbers, topics in recreational mathematics and other mathematical subjects. He authored numerous problems and solutions involving a variety of mathematical topics, including algebra, plane geometry, solid geometry, number theory, cryptarithms and magic squares.

His problem proposals were simply stated; his solutions were always elegant. For a solution to be elegant, it had to be brief, concise, clear, logical and yet contain an element of surprise which often reflected the simplicity of the solution. Brevity could not be achieved by omitting steps essential for understanding or by relying on the mathematical crutch "obviously." He certainly followed his own philosophy because his solutions established the benchmark!

Charles W. Trigg died on 28 June 1989. His works constantly revealed the beautiful and witty side of mathematics. As a problemist, he had few peers. As a mathematician and friend, we shall miss him greatly. This issue of *The Problem Corner* is dedicated to his memory.

#### PROBLEMS 440-444.

**Problem 440.** Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. Let

$$I_1 = \int_0^1 \frac{1}{\sqrt{1-x^2}} dx \quad \text{and} \quad I_2 = \int_0^1 \sqrt{1-x^2} dx.$$

Without determining the value of either  $I_1$  or  $I_2$ , prove that  $I_1 = 2I_2$ .

**Problem 441.** Proposed by Albert White, Saint Bonaventure University, Saint Bonaventure, New York. Al has a batting average of .333 while Bob, who follows Al in the batting order, has a batting average of 0.222. What is the probability that Al gets his next hit before Bob does? Assume that they alternate taking turns at bat and that the performance of one player does not affect the performance of the other player.

**Problem 442.** Proposed by the editors. Find all primes  $p$  such that each nonzero element of  $\mathbb{Z}_p$  is its own multiplicative inverse.

**Problem 443.** Proposed by the editor. The integer 253 has the property that  $253^4 = 4097152081$  and  $253 = 81 + 20 + 15 + 97 + 40$ . Find all three and four digit integers  $N$  which have this property where  $N$  is a

perfect square or higher power. Note that digits are paired starting from the right.

**Problem 444.** Proposed by the editor. (This problem is dedicated to the 50th anniversary of *The Pentagon*.) Let  $r_n$  be defined by  $r_n = 1941r_{n-1} + 1940$  where  $r_0$  is an integer. Find an integer  $r_0$  such that  $r_{50}$  is divisible by 1991 or prove that none exist.

### 50th ANNIVERSARY PROBLEMS.

This issue of *The Problem Corner* commemorates the 50th anniversary of *The Pentagon* and over 40 years of *The Problem Corner*! The first *Problem Corner* appeared in the Fall 1947 issue of *The Pentagon*. To illustrate the types of problems which appeared in these early issues, this anniversary issue includes five additional problems (oldies but goodies) taken from early columns for which solutions also may be submitted.

**OBG 1.** Among grandfather's papers a bill was found:

72 turkeys \$ -67.9- .

The first and last digits of the number that obviously represented the total price of these fowls are replaced here by blanks, for they have faded and are now illegible. What are the two faded digits and what was the price of one turkey?

**OBG 2.** A city with a circular wall has two gates, one at each end of the north and south diameter. From the north gate a road leads directly north and from the south gate a road leads directly east. What is the diameter of the city if from a point 3 miles north of the north gate it is just possible to see past the wall to a point 9 miles east of the south gate?

**OBG 3.** The graph of a traffic count past a certain point is found to resemble a sine curve with a minimum of 30 at midnight and a maximum of 900 at noon. Write an equation which will give the traffic count at any time of the day. Also determine how many cars passed the point between 11:00 a.m. and noon.

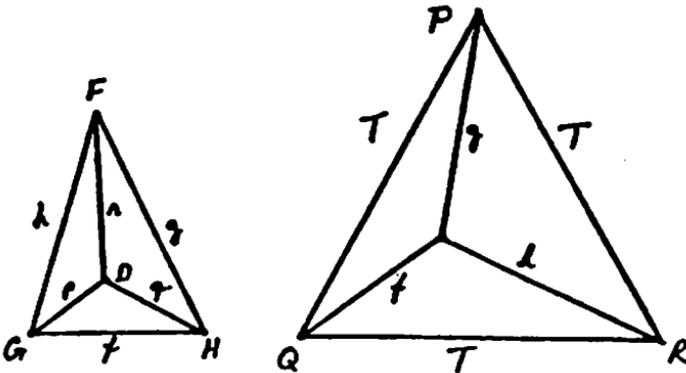
**OBG 4.** To number the pages of a bulky volume, the printer used 1890 digits. How many pages does the volume have?

**OBG 5.** Three numbers are in arithmetic progression. Three other numbers are in geometric progression. Adding the corresponding terms of these two progressions successively, we obtain 85, 76 and 84, respectively; adding all three terms of the arithmetic progression, we obtain 126. Find the terms of both progressions.

*Please help your editor by submitting problem proposals.*

**SOLUTIONS 422, 425 and 427-429.**  
(Problem 426 remains open.)

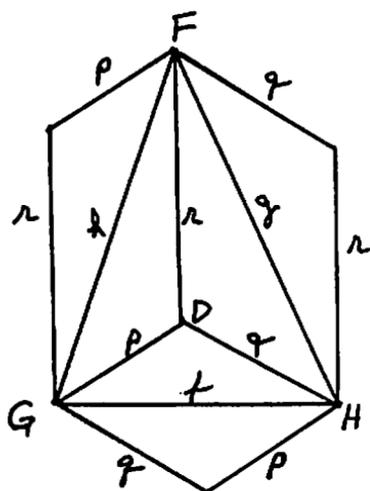
**Problem 422.** Proposed by the editor. Consider the two triangles  $\triangle FGH$  and  $\triangle PQR$  shown below with  $\angle FDH = \angle FDG = \angle GDH = 120^\circ$ . Let the line segments be denoted as marked. Prove that  $T = p+q+r$ . (Third USA Mathematical Olympiad 1974.)



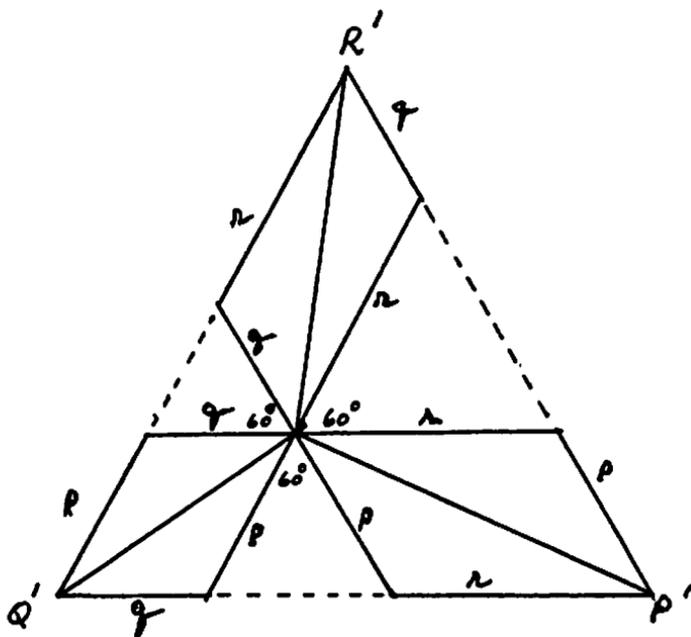
*Note by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. The following solution to this problem appears in the September 1975 issue of Eureka (now Cruz Mathematicorum) 1 No. 7, pp. 63-64.*

In the plane of triangle  $\triangle FGH$  construct three parallelograms

having a common vertex at D as shown below. Each parallelogram has angles of  $60^\circ$  and  $120^\circ$ .



Rearrange the parallelograms so that the vertices of their  $60^\circ$  angles meet at a point with an angle of  $60^\circ$  between each pair of parallelograms as shown below.



Rename the outer vertices of the parallelograms  $P'$ ,  $Q'$  and  $R'$ . Clearly the outer sides of the parallelograms coincide with the sides of triangle  $\triangle P'Q'R'$ . Clearly triangle  $\triangle P'Q'R'$  is equilateral because  $\angle P' = \angle Q' = \angle R' = 60^\circ$  due to the method of constructing the parallelograms. Finally, the length of each side of triangle  $\triangle P'Q'R'$  equals  $p+q+r$  as was to be shown.

*Editor's Comment.* The symmetry utilized in this solution is simple and aesthetically pleasing. For other solutions, see *The Mathematics Teacher*, 68 No. 1, (January 1975), pp. 4-9.

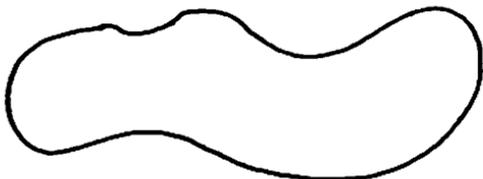
**Problem 425.** Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. Find all primes  $p$  such that  $p^2 + 3$  is a prime number.

*Solution by the Drake University Problem Solving Group, Drake University, Des Moines, Iowa.*

If  $p = 2$ , then  $p^2 + 3 = 7$ . If  $p$  is an odd prime, say  $p = 2k + 1$  for some integer  $k$ , then  $p^2 + 3 = (2k + 1)^2 + 3 = 4(k^2 + k + 1)$  which is never a prime number. Thus  $p = 2$  is the only prime number satisfying the condition.

*Also solved by* Charles Ashbacher, Mount Mercy College, Hiawatha, Iowa; Dr. Michael W. Ecker, Penn State - Wilkes-Barre Campus, Lehman, Pennsylvania; the Alma College Problem Solving Group, Alma College, Alma, Michigan; Robert L. Bailey, Niagara University, Niagara University, New York; the New York Lambda Problem Solvers, Long Island University, C. W. Post Campus, Brookville, New York; and the proposer.

**Problem 426.** Proposed by Dmitry P. Mavlo, Moscow, U.S.S.R. Prove that an arbitrary plane closed curve of length  $L$  as shown in the figure below can be completely placed into a pentagon having perimeter  $P$  where  $P$  is not greater than  $(\sqrt{5} - 1) \cdot L$ . Consider all cases in which the equality  $P = (\sqrt{5} - 1) \cdot L$  holds.



Since no correct solutions have been received, this problem will remain open for another issue.

**Problem 427.** Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. Let  $x$  and  $y$  be real numbers such that

$$x + y = \frac{\pi}{3\sqrt{2}} \quad \text{and} \quad xy = \frac{\pi^2}{144}.$$

Determine the value of  $\sin(x-y)$ .

**Solution by Michael White, Portville High School, Portville, New York.**

Given the values of  $x + y$  and  $xy$ , we have

$$(x - y)^2 = (x + y)^2 - 4xy = \frac{\pi^2}{36}$$

so  $x - y = \pm \pi/6$  and  $\sin(x - y) = \pm 1/2$ .

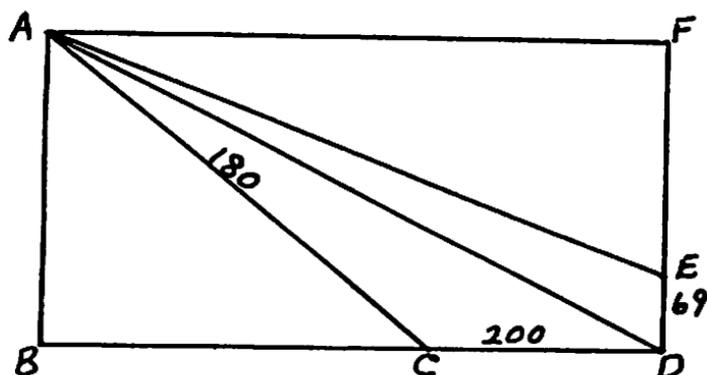
*Also solved* by the Alma College Problem Solving Group, Alma College, Alma Michigan; the Drake University Problem Solving Group, Drake University, Des Moines, Iowa; George Lasecki, Central Michigan University, Michigan; Robert L. Bailey, Niagara University, Niagara University, New York; the New York Lambda Problem Solvers, Long Island University, C. W. Post Campus, Brookville, New York; Richard A. Gibbs, Fort Lewis College, Durango, Colorado; and the proposer.

**Editor's Comment.** While the problem does not ask for the value of  $x - y$ , solvers and the proposer note that  $x - y = 2k \pm \pi/6$  with  $k$  an integer and the choice of sign corresponding to the sign in  $\pm 1/2$ .

**Problem 428.** Proposed by the editor. The figure at the top of the next page shows the paths found in the garden of an eccentric millionaire who required the length of each path to be an integral number of feet. Given that  $AC = 180$ ,  $CD = 200$  and  $DE = 69$ , find the lengths of  $AB$ ,  $AD$ ,  $BC$ ,  $AE$  and  $EF$ .

**Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.**

Denote the equation  $x^2 + y^2 = 5^2$  by (1) and denote the equation  $x^2 + y^2 = 6^4 \cdot 5^2$  by (2). If  $(x_0, y_0)$  is a positive integer solution of (1), then  $(36x_0, 36y_0)$  is a positive integer solution of (2).



It is known that if  $u^2 + v^2 = w^2$  where  $u$ ,  $v$  and  $w$  are positive integers, then either 3 divides  $u$  or 3 divides  $v$ . The same result holds when 3 is replaced by 2. It then follows that if  $x_1^2 + y_1^2 = 6^4 \cdot 5^2$  where  $x_1$  and  $y_1$  are positive integers, then 3 divides both  $x_1$  and  $y_1$ . By applying this result twice, we see that  $(x_1/36)^2 + (y_1/36)^2 = 5^2$ .

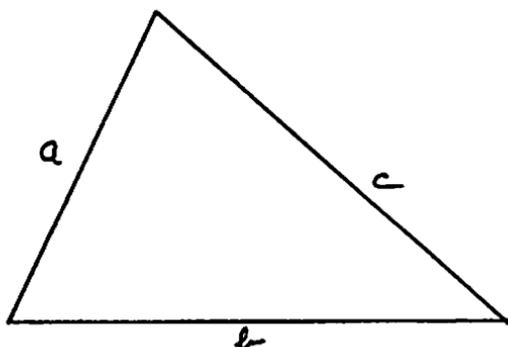
Taken together, the preceding discussion establishes that  $(a,b)$  is a solution of (1) if and only if  $(36a,36b)$  is a solution of (2). The only positive integer solutions of (1) are  $(3,4)$  and  $(4,3)$ . If  $AB = 108$  and  $BC = 144$ , then  $AD = 10\sqrt{13}$ , which is not the square of an integer. If  $AB = 144$  and  $BC = 108$ , then  $BD = 308$  and, by the Pythagorean Theorem,  $AD = 340$ . Hence  $EF = 144 - 69 = 75$  and  $AE = 317$ .

*Also solved by* Fred A. Miller, Elkins, West Virginia; Robert L. Bailey, Niagara University, New York; the New York Lambda Problem Solvers, Long Island University, C. W. Post Campus, Brookville, New York; and Charles Ashbacher, Mount Mercy College, Hiawatha, Iowa.

**Problem 429.** Proposed by the editor. Find an acute angled Heronian triangle such that each side of the triangle is a three-digit number and such that each of the nine positive digits is used in forming the integers representing the sides of the triangle. A Heronian triangle has the property that each of its legs and its area are all integers.

*Solution by* the New York Lambda Problem Solvers, Long Island

University, C. W. Post Campus, Brookville, New York.



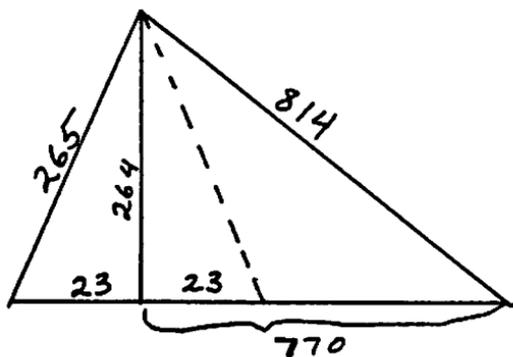
The New York Lambda Problem Solvers devised a computer program to generate three three-digit numbers using each of the nine positive digits exactly once. Then these numbers were used as sides of a triangle and then Heron's Formula was used to find integral areas. Their program found exactly 11 triangles whose sides utilize the nine positive digits and which have integral areas. Of these triangles, seven are obtuse while only four are acute (as required by the statement of the problem) as shown in the table below.

a	b	c	area	(type)
841	572	369	86130	obtuse
468	327	159	14310	obtuse
834	725	169	50040	obtuse
612	549	387	104490	acute
819	675	234	60840	obtuse
819	765	432	163296	acute
814	793	265	104676	acute
945	783	216	61236	obtuse
814	693	275	91476	obtuse
715	692	483	159390	acute
981	762	543	205740	obtuse

Also solved by Charles Ashbacher, Mount Mercy College, Hiawatha, Iowa.

*Editor's Comment.* Each of the triangles found in our featured solution can be obtained by "juxtaposing" two right triangles having a common side using proportionality factors where necessary. Using the notation  $K(h, l, s)$  to denote multiplication of the sides of a triangle by a constant

$K$ , where  $h$ ,  $l$  and  $s$  denote the hypotenuse, leg and side of a primitive Pythagorean triangle, and the notation  $K(h, l, s) \pm J(h, l, s)$  to denote the juxtaposition of two right triangles along a common leg or side, one can construct all the triangles  $(a, b, c)$  given in our featured solution; here the  $+$  (or  $-$ ) indicates juxtaposition without (or with overlapping), i.e. the addition (or subtraction) of areas. For example, the triangle  $(814, 793, 265) = 22(37, 35, 12) + (265, 23, 264) = (814, 770, 264) + (265, 23, 264)$  as shown below.



The triangles from our featured solution can be expressed as follows:

$$(a, b, c) = K(h, l, s) \pm J(h, l, s)$$

$$(841, 572, 369) = \frac{1}{13} \left( (10933, 3915, 10208) - (4797, 3915, 2772) \right)$$

$$(468, 327, 159) = 36(13, 5, 12) - (109, 91, 60)$$

$$(834, 725, 169) = 5(145, 143, 24) - (169, 119, 120)$$

$$(612, 387, 549) = 36(17, 15, 8) + (61, 11, 60)$$

$$(819, 675, 234) = 35(13, 5, 12) - 2(65, 33, 56)$$

$$(819, 765, 432) = 63(13, 5, 12) + 9(85, 84, 13)$$

$$(814, 793, 265) = 22(37, 35, 12) + (265, 23, 264)$$

$$(945, 783, 216) = 189(5, 3, 4) - 27(29, 21, 20)$$

$$(814, 693, 275) = 22(37, 35, 12) - 11(25, 7, 24)$$

$$(715, 692, 483) = 55(13, 5, 12) + 4(173, 165, 52)$$

$$(981, 762, 534) = 9(109, 91, 60) - 3(181, 19, 180)$$

## Kappa Mu Epsilon News

Edited by Mary S. Elick, Historian

News of chapter activities and other noteworthy *KME* events should be sent to Mary S. Elick, Historian, Kappa Mu Epsilon, Mathematics Department, Missouri Southern State College, Joplin, MO 64801.

### INSTALLATION OF NEW CHAPTERS

#### *Oklahoma Delta Chapter*

Oral Roberts University, Tulsa, Oklahoma

The installation of the Oklahoma Delta Chapter of Kappa Mu Epsilon was held on April 10, 1990, in the Regents Dining Room of the Graduate Center on the campus of Oral Roberts University. Dr. Harold L. Thomas, National President of Kappa Mu Epsilon, conducted the installation ceremony. Dr. Thomas was assisted by Professor Mary Elick, National Historian of Kappa Mu Epsilon. Dr. Roy Rakestraw and Dr. Roger Hartman, Kappa Mu Epsilon members who are currently on the ORU faculty also participated in the ceremony. Fifteen students and four faculty in addition to Dr. Rakestraw and Dr. Hartman constituted the founding group of the new chapter at Oral Roberts University. Those initiated were:

*Students:* Steven Adams, Joel Arnold, Kimberly Barker, Bunmi Folayan, Melissa Fulbright, Paul Hanson, Ben Hobmaier, Dave Largent, Chris Lomont, Mike McKinley, Bill Orth, Mike Sawyer, Margaret Schultz, Tina Walker, and Leah Watson.

*Faculty:* Prof. Tom Bomholt, Prof. Georganne Haney, Prof. Debra Oltman, and Dr. Verbal Snook.

Following the installation ceremony, the group enjoyed a banquet

with chicken kiev highlighting the menu. After dinner, Dr. Rakestraw welcomed those present. Dr. Thomas spoke on "KME: History and Organization." Dr. Verbal Snook, chairman of the ORU mathematics department, reminisced about mathematics at ORU. Dave Largent presented the student response.

Officers installed during the ceremony were: Dave Largent, president, Bill Orth, vice-president; Margaret Schultz, secretary; and Melissa Fulbright, treasurer. Faculty members Debra Oltman and Roy Rakestraw accepted the responsibilities of the corresponding secretary and faculty sponsor, respectively.

### *Colorado Delta Chapter*

Mesa State College, Grand Junction, Colorado

The installation of the Colorado Delta Chapter of Kappa Mu Epsilon was held on April 27, 1990 in the Boettcher Room of the W. W. Campbell Student Center on the campus of Mesa State College. The new initiates were welcomed and congratulated by Dr. James Rybak, Vice President of Academic Affairs, Dr. James Johnson, Acting Dean of the School of Natural Sciences and Mathematics, and Mr. Edwin C. Hawkins, Chair of Computer Science, Mathematics, and Engineering. Dr. Richard A. Gibbs, Director of Kappa Mu Epsilon Region V, conducted the installation ceremony. Professor John T. Marshall, a Kappa Mu Epsilon member on the MSC faculty, participated as Conductor. Twelve students and four faculty in addition to Dr. Marshall constituted the founding group of the new chapter at Mesa State College. Those initiated were:

*Students:* Gariann M. Castleton, Karl J. Castleton, Leah L. Crawford, Frank C. Crenshaw, Michael E. Goodwill, Tanya E. Henderson, Kristina L. Holderness, Dana V. Manthos, Rocky L. Ramsey, Christopher R. Riser, Lona D. Shockley, and Duncan T. Thompson.

*Faculty:* Prof. Clifford C. Britton, Prof. Harold Davenport, Prof. Edwin C. Hawkins, and Prof. Carl M. Kerns.

Officers installed during the ceremony were: Gariann M. Castleton, president; Leah Clifford, vice-president; Christopher R. Riser, recording secretary; and Karl J. Castleton, treasurer. Faculty members Clifford C.

Britton and Harold Davenport accepted the responsibilities of faculty sponsor and corresponding secretary respectively.

Several parents and friends of the initiates were present at the 3 p.m. installation ceremony and the reception which followed. At 6:30 p.m. approximately 50 people enjoyed a delicious buffet dinner at the Grand Junction Hilton Hotel. After dinner the group was welcomed by Dr. Ray Kieft, Acting President of Mesa State College. Dr. Gibbs gave the banquet talk "Organizing Your *KME* Chapter."

*North Carolina Gamma Chapter*  
Elon College, Elon College, North Carolina

The installation of the North Carolina Gamma Chapter of Kappa Mu Epsilon was held on May 3, 1990, in the Fine Arts Recital Hall on the campus of Elon College. Dr. James E. Lightner, professor of mathematics at Western Maryland College and past National President of Kappa Mu Epsilon, conducted the ceremony. Twenty-nine students and eight faculty constituted the charter members for the new chapter at Elon College. Those initiated were:

*Students:* Lori Blanchard, Alexander Brenner, Cheri Brown, Bradley Cokendolpher, Jeffrey Davis, Dawn Edwards, Francis Fish, Robert Hughes, Autumn Jordan, Michele Joyce, Timothy Keenan, Jennifer Lee, James McCarley, Kristen McMillan, Ari Miller, Eric Moore, Julia Morris, Michele Mueller, Edwin Neagle, Scott Oakes, Valerie Owen, Trevor Power, Diane Rodden, Aileen Scott, Jacqueline Shelton, Jon Theiss, Daniel Worrell, Matthew Wright, and Leslie Zeb.

*Faculty:* Prof. J. Wesley Alexander, Prof. William H. Barbee, Prof. J. Albert Carpenter, Dr. Jeffrey Clark, Prof. Ann Crawford, Dr. Rosaline Reichard, Prof. Helen Walton, and Prof. Patricia Whitaker.

Following the installation ceremony, Dr. Lightner spoke briefly on the nature and history of honor societies in America and more specifically on the history and development of Kappa Mu Epsilon. After the ceremonies were completed, the group adjourned for refreshments and a brief social hour.

Officers installed during the ceremony were: Valerie Dawn Owen, president; Timothy F. Keenan, vice-president; Kristen Kay McMillan, recording secretary; and Dawn Marie Edwards, treasurer. Faculty members Rosalind Reichard and Jeffrey Clark accepted the responsibilities of the corresponding secretary and faculty sponsor, respectively.

## CHAPTER NEWS

### *Alabama Zeta*

Birmingham-Southern College, Birmingham

Chapter President - Mark Kent

35 actives

Chad Jones, an employee of South Central Bell and an alumnus of Alabama Zeta, spoke to the club in the fall about employment opportunities in industry. Ashita Tolwani received the *KME* pin award for service to the chapter on Honors Day. Other 1990-91 officers: Pamela Brantley, vice president; Laura Francis, secretary/treasurer; Lola F. Kiser, corresponding secretary; Shirley Branan, faculty sponsor.

### *California Gamma*

California Polytechnic State University, San Luis Obispo

Chapter President - Andrew Skrylov

40 actives, 13 initiates

California Gamma held two fundraisers: its annual booksale in January and its annual tee-shirt sale during the Poly Royal festivities which occurred on April 27-28, 1990. The net profit from both these activities will be set aside to send representatives to next year's *KME* Biennial Convention. Thirteen pledges were initiated on May 4, 1990 at the annual banquet held this year at Marie Callender's Restaurant in Pismo Beach. At this time the new officers were introduced. The guest speaker was professor Gaye Benson of the Cal Poly Political Science Department. Also in attendance were Prof. George Mach, who presented the Founders' Award and W. Boyd Judd who presented this year's recipient of the scholarship which bears his name. Theresa Bly, the 1989-1990 President of California Gamma, who has been on co-op during the spring 1990 Quarter, returned to Cal Poly to preside over the banquet. She accepted the Founders' Award on behalf of Kathy Perino who was out of town and the W. Boyd Judd Scholarship on behalf of Karen Van der Veer who was on co-op and could not be present. Jeff Stought, a representative of the Arthur Andersen Company, presented the Arthur

Andersen Award to Andrew Schaffner who will be the 1990-1991 California Gamma Vice President. Other 1990-91 officers: Scott Langfeldt, secretary; Derek Bernhardt, treasurer; Julie Smeltzer, pledgemaster; Cindy Walter, social chairperson; Jason Kastner, curriculum committee representative; Leo Flores, representative to the SOSAM Council; Raymond D. Terry, corresponding secretary/faculty sponsor.

*Colorado Gamma*

Fort Lewis College, Durango

Chapter President - David Beazley  
30 actives, 6 initiates

Colorado Gamma held two meetings in the spring and two initiation ceremonies. Short video programs were shown at each meeting. Officers for the coming year were elected and plans begun for travel to the biennial meeting in Alabama. Other 1990-91 officers: Jeff Johnson, vice president; Todd Sehnert, secretary; Chris Kerlin, treasurer; Richard A. Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

*Illinois Delta*

College of St. Francis, Joliet

Chapter President - Donna Guderyahn  
25 actives, 16 initiates

Spring activities for Illinois Delta included a presentation by guest speaker Sue Eddins, 1989 recipient of the presidential award for excellence in teaching of mathematics, and a tour of the Midwest Stock Exchange and Board of Trade in Chicago. Other 1990-91 officers: Mary Houston, vice president; Amy Brockmiller, secretary; Jennifer Rogers, treasurer; Sister Virginia McGee, corresponding secretary/faculty sponsor.

*Illinois Zeta*

Rosary College, River Forest

Chapter President - Glenn Jablonski  
15 actives, 6 initiates

Other 1990-91 officers: Joseph Pignataro, vice president; Patti Rubio, secretary; Anna Lazenby, treasurer; Mary T. O'Malley, O.P., corresponding secretary/faculty sponsor.

*Illinois Theta*

Illinois Benedictine College, Lisle

Chapter President - Rafi Ali

15 actives, 9 initiates

The Illinois Theta chapter met about once every three weeks. It co-sponsored a mathematics contest for high school students on February 24. Members participated in a formal tutoring program for students taking college algebra, trigonometry, finite mathematics, and calculus. Chapter members also helped plan the annual mathematics and computer science awards dinner on campus. Other 1990-91 officers: Beth Trupiano, vice president; Rich Giza, secretary; Sandra Lepsi, treasurer; James M. Meehan, corresponding secretary; Lisa Townsley Kulich, faculty sponsor.

*Indiana Gamma*

Anderson University, Anderson

Chapter President - Melanie Dähler

7 actives, 5 initiates

A reception in honor of Dr. Gloria Olive was hosted by the members of Kappa Mu Epsilon in December. Dr. Olive was the founding faculty sponsor of Indiana Gamma and recently retired from 17 years of service to the mathematics department at the University of Otago, Dunedin, New Zealand. She was visiting professor at Anderson University during the first semester of 1989-90. Other 1990-91 officers: Linda Timmeman, vice president; Cyril Joudieh, secretary/treasurer; Stanley Stephens, corresponding secretary/faculty sponsor.

*Iowa Alpha*

University of Northern Iowa, Cedar Falls

Chapter President - Bill Pothoff

41 actives, 5 initiates

Iowa Alpha student members Mark Bohan, Lynn Cairney, Mike Collins, Mike Hirsch, and Bill Pothoff, along with faculty John E. Bruha and John S. Cross, attended the *KME* Region IV convention March 16, 17, 1990 in Atchison, KS. Mike Hirsch was awarded first place in the student competition for his paper on "Generalized Matrix Inverses." Students presenting papers at Iowa Alpha *KME* meetings were: Bill Pothoff on "The Parking Lot Problem," Lori Stenberg on "Entropy and its Applications," and Dina VanLent on "Automata Theory." Ben Schafer addressed The April *KME* initiation banquet on "Fractal Geometry" and included computer generated illustrations assisted by Mike Hirsch. Other 1990-91 officers: Mark Bohan and Mike Hirsch, co-vice presidents; Rachel Britson, secretary; Ben Schafer, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

**Iowa Delta**

Wartburg College, Waverly

Chapter Resident - Daniel Nettleton

44 actives, 10 initiates

The New Year activities began with a presentation by Dr. Jon H. Lemke from the Department of Preventative Medicine at the University of Iowa. He spoke on "A Biostatistician's Perspective on AIDS: Assessment and Screenings." A pizza party followed by bowling occurred on January 21. Faculty presented facts and opportunities about "Careers in Mathematics" during the February meeting. On March 10, the Chapter sponsored the thirteenth annual Wartburg Math Field Day with over one hundred competitors from fourteen schools. On March 31, Dr. Marvin Ott presented "More About Liberal Arts Education for Mathematicians" as the initiation and election banquet speaker. On April 30, the Chapter held its traditional end of the year picnic. Other 1990-91 officers: Todd Letsche, vice president; Stephanie Hurley, secretary; Jerrod Staack, treasurer; August W. Waltmann, corresponding secretary/faculty sponsor.

**Kansas Alpha**

Pittsburg State University, Pittsburg

Chapter President - Jason Williams

40 actives, 18 initiates

The spring semester activities started with a pizza party and initiation in February for eighteen new members. Following the initiation ceremony, Mike Wille presented the program. Mike demonstrated computer graphics as applied to items sewn on articles of clothing. The March program focused on the solution of problems published in *The Pentagon*. Two students and two faculty attended the Region 4 convention which was hosted by Kansas Gamma at Benedictine College. Several excellent papers were presented as well as a guest speaker presentation from Midwest Research. The students who made the trip to Benedictine College reported on the convention at the April meeting. The program for this meeting was given by Dr. Elwyn Davis, PSU departmental chairman, on finite geometry. The chapter assisted the Mathematics Department faculty in administering and grading tests given at the annual Math Relays, April 24, 1990. Several members also worked on the Alumni Association's Annual Phon-o-thon. They received third prize for amount of money raised by student organizations. In addition, Tamala Nation received 5th place in individual competition among all PSU students that participated. The final meeting of the semester was a social event held at Professor McGrath's home. Homemade ice cream and cake were served to those attending. Officers

for the 1990-91 school year were elected. The annual Robert M. Mendenhall awards for scholastic achievement were presented to Mike Wille, Lori Oneal, Tamala Nation, and Lora Woodward. They received *KME* pins in recognition of this honor. Those in attendance watched the videotape, "Chaos," to close out a busy semester. Other 1990-91 officers: Mark Stewart, vice president; Brenda Beat, secretary; Lori Bruns, treasurer; Harold L. Thomas, corresponding secretary; Gary McGrath, faculty sponsor.

### *Kansas Beta*

Emporia State University, Emporia

Chapter President - Brenda Korte

30 actives, 6 initiates

Kansas Beta sold teachers at the annual Math Day for high school students. A pizza party was also held for the new initiates. Other 1990-91 officers: Rachel Nichols, vice president; Sara Gleason, secretary; Angie Carlson, treasurer; Connie Schrock, corresponding secretary; Larry Scott, faculty sponsor.

### *Kansas Gamma*

Benedictine College, Atchison

Chapter President - Matthew McIntosh

8 actives, 12 initiates

Two students, Ty Anderson and Ken VanSpeybroeck, and faculty member Linda Herndon, OSB, were initiated into Kansas Gamma on February 7. Prior to initiation students profited from an informative presentation given by guest speaker Wendi Good from Blue Cross & Blue Shield of Kansas City. She talked about opportunities in the actuarial profession and her own experiences with the actuarial exams. Kansas Gamma hosted the Region IV Convention on March 17. Member Ty Anderson took 2nd place with his talk and demonstration on minimal surfaces. The chapter honored him with its Hypatian award. Many students and faculty were active on April 21 in the annual Math Tournament for area high school students sponsored by the chapter. Kansas Gamma celebrated its 50th anniversary this spring. The chapter, installed May 26, 1940, held a reception on 2nd floor of Westerman Hall on the afternoon of April 26 at which time all the memorabilia on the chapter was on display. Sister Helen Sullivan, foundress of the chapter, residing in Dooley Center, the convent retirement home, was in attendance. That evening active members and several alums of the chapter shared a meal at the Drury-Pennell house. Following the meal all assembled walked rapidly through the 50 years by way of a slide

presentation. Officers for 1990-91 will be elected in the fall. Jo Ann Fellin, OSB, is corresponding secretary and faculty sponsor.

**Kansas Delta**

Washburn University, Topeka

Chapter President - Scott McFarland

15 actives, 6 initiates

On May 12, 1990, Kansas Delta hosted a picnic for all university mathematics students. Over seventy people attended. Other 1990-91 officers: Jody Whitaker, vice president; Jonette Oestreich, secretary/treasurer; Allan Riveland, corresponding secretary; Ronald Wasserstein, faculty sponsor.

**Kentucky Alpha**

Eastern Kentucky University, Richmond

Chapter President - Kathy Ponder

14 actives, 24 initiates

Eight students, accompanied by Dr. Costello, attended the regional convention at Winthrop College. Three of the students presented talks. On the return trip, the group stopped briefly in Charlotte and took the "scenic" route over the mountains. Other semester activities included volleyball and softball games with the faculty. A group of students and Dr. Costello went ice skating in February. This year's initiation ceremony included an interesting talk by Dr. Bill Janeway on "The Pidgeon-Hole Principle." The traditional party following the initiation was held in the student center. Other 1990-91 officers: Jim Justice, vice president; Karen Hugel, secretary; Lisa Whitis, secretary; Patrick Costello, corresponding secretary; Bill Janeway, faculty sponsor.

**Maryland Alpha**

College of Notre Dame of Maryland, Baltimore

Chapter President - Celine Burque

16 actives, 6 initiates, 5 pledges

Six students were initiated at the annual pot luck dinner. The speaker for the occasion was Dr. David Elkin of the Space Telescope Institute, John Hopkins University. He spoke on "The Hubble Space Telescope." Other 1990-91 officers: Cheryl Gates, vice president; Ann Marie Webster, secretary; Marta Blotny, treasurer; Sister Marie A. Dowling, corresponding secretary; Joseph DiRienze, faculty sponsor.

**Maryland Beta**

Western Maryland College, Westminster

Chapter President - Tamara Mahan

13 actives, 1 initiate

Maryland Beta celebrated its twenty-fifth anniversary this spring. The chapter conducted several fund raisers to enable continued contributions to chapter scholarship funds. An end-of-year picnic concluded semester activities. One new member was initiated. Other 1990-91 officers: Andrea Pinkham, vice president; Laura Balakir, secretary; Deanna Dailey, treasurer; James E. Lightner, corresponding secretary; Linda Eshleman, faculty sponsor.

**Maryland Delta**

Frostburg State University, Frostburg

Chapter President - Wayne Squillari

40 actives, 21 initiates

On Feb. 25, 1990, Maryland Delta Chapter of *KME* inducted the following new members: Jennifer Barnhart, Robert Boore, Heather Cameron, Timothy Cooper, Charles Dodge, Patricia Frost, Daniel Jones, Dr. Mohammad Khavanin, David Klepitch, Jerry Kuhn, Amy LaParle, Duane McCormick, Eric Miller, Carla Saville, Maurice Singley, Martin Slodzinski, Wayne Squillari, Beth Stallings, Teresa White, Greg Wilburn and Dr. Richard Winton. During the semester the chapter held a puzzlefest and also co-sponsored the Frostburg State University Mathematics Symposium. Other 1990-91 officers: Carla Saville, vice president; Brenda Moore, secretary; Andrew Kaylor, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

**Michigan Beta**

Central Michigan University, Mount Pleasant

Chapter President - Deidra McClelland

40 actives, 21 initiates

Initiation was held on Feb. 19 with a meal followed by the initiation ceremony and a talk by Yuri Ionin, visiting professor from the Soviet Union. Programs for meetings included video tapes from the PBS show, "For All Practical Purposes," and a talk by Agnes Hausbeck, who spoke on her current student teaching experiences. In addition to *KME*, the mathematics department has a new organization, Students Make Mathematics Matter, for elementary education majors with an interest in mathematics. This organization is affiliated with NCTM. Members of Michigan Beta continue with mathematics help sessions for undergraduate courses. The year ended with a picnic enjoyed by *KME*,

SM3, and departmental faculty. Other 1990-91 officers: Laurie Raven, vice president; Tom DeClark, secretary; Mary Langeveld, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

*Mississippi Gamma* University of Southern Mississippi, Hattiesburg  
Chapter President - Theresa Kelly  
30 actives, 8 initiates

Other 1990-91 officers: Lisa Carroll, vice president; Jane Blackledge, secretary; Alice Essary, treasurer/corresponding secretary; Barry Piazza and Karen Thrash, faculty sponsors.

*Missouri Alpha* Southwest Missouri State University, Springfield  
Chapter President - David McWilliams  
35 actives, 17 initiates

Four monthly meetings were held with a presentation at each. Eight members, under the supervision of Dr. Kubicek, attended the regional convention at Benedictine College in Atchison, Kansas, on March 16-17. Graduate student, Pete Egler, presented a paper entitled "On Complex Quasimonotone and Complex Monotone Sequences." The chapter requested and received a state proclamation from Governor John Ashcroft for Mathematics Awareness Week, April 22-28. During that week Missouri Alpha set up booths each day with a different theme, organized a panel discussion on career opportunities in mathematics, and heard a faculty guest speaker. A successful year was brought to a close with sixty-six members, faculty, staff, and guests attending the banquet held at Raphael's restaurant on May 10. Other 1990-91 officers: Mark Gerke, vice president; Marc Meyer, secretary; Rhonda Crites, treasurer; Vera B. Stanojevic, corresponding secretary; M.M. Awad, faculty sponsor.

*Missouri Beta* Central Missouri State University, Warrensburg  
Chapter President - Michael Prock  
15 actives, 17 initiates

Missouri Beta chapter held monthly meetings with invited speakers at each meeting. Speakers included an Army recruiter, a representative from the placement office and several faculty. A delegation attended the Region IV Convention at Benedictine College, Atchison, Kansas, March 16-17. Other 1990-91 officers: Jan Finley, vice president; William West,

secretary; Sarah Moss, treasurer; Rhonda McKee, corresponding secretary; Debbie Detrick, Gerald Schrag, and Larry Dilley, faculty sponsors.

*Missouri Gamma*

William Jewell College, Liberty

Chapter President - Kevin Tanner

14 actives, 9 initiates

Other 1990-91 officers: James Mathis, vice president; Catherine Pagacz, secretary; Joseph T. Mathis, treasurer/corresponding secretary/faculty sponsor.

*Missouri Epsilon*

Central Methodist College, Fayette

Chapter President - Mary Ann Callaway

11 actives, 6 initiates

Other 1990-91 officers: John Slovensky, vice president; Jeff Wilcox, secretary/treasurer; William D. McIntosh, corresponding secretary/faculty sponsor; Linda O'Lambke, faculty sponsor.

*Missouri Eta*

Northeast Missouri State University, Kirksville

Chapter President - Julie Ridlen

23 actives, 6 initiates

Nine schools from Illinois, Iowa, and Missouri attended a Math Expo for high school students sponsored by the chapter on Feb. 17, 1990. During Math Awareness Week in April the organization set up an information table and viewed two math movies. Other 1990-91 officers: Ann Novitske, vice president; Lisa Aukee, secretary; Rhonda Gibler, treasurer; Mary Sue Beersman, corresponding secretary; Mark Faucette, faculty sponsor.

*Missouri Theta*

Evangel College, Springfield

Chapter President - Ralph Russo

11 actives, 5 initiates

Other 1990-91 officers: Greg Hayden, vice president; Jonathan Raines, secretary/treasurer; Don Tosh, corresponding secretary/faculty sponsor.

*Missouri Iota*

Missouri Southern State College, Joplin

Chapter President - Wayne Cripps

13 actives, 7 initiates

Seven Missouri Iota members attended the Region IV Convention at Benedictine College in Atchison, Kansas. Ana Witt presented a paper entitled, "The Unfaithful Wives Problem," which won third place in the paper competition. The annual initiation banquet was held at Wilkinson's Restaurant on April 3, 1990. Other 1990-91 officers: Melissa Burken, vice president; Terri Findley, secretary/treasurer; Mary Elick, corresponding secretary; Linda Hand, faculty sponsor.

*Nebraska Alpha*

Wayne State College, Wayne

Chapter President - Brenda Spieker

34 actives, 14 initiates

Throughout the spring semester Nebraska Alpha monitored the Mathematics-Science Building evenings as a money-making project. Susan Sorenson of Wayne, Nebraska was named outstanding freshman mathematics student as result of her performance on an exam administered by the chapter. The award includes the recipient's name being engraved on a permanent plaque, payment of *KME* national dues, and one year honorary membership in the local chapter. Keith Spiels and Mike Emory were awarded the \$25.00 book scholarships which are given to *KME* members each year by the club. Members Julie Gottschalk, Susan Sorenson, Gary Nelson, Annette Eilers, Keith Spiels, Lee Emanuel, and faculty members Fred Webber and Jim Paige attended the regional convention at Benedictine College in Atchison, Kansas on March 16-17, 1990. Keith Spieh's paper was selected as an alternate paper for presentation at the convention. In other activities, club members participated in the Wayne State College College-Bowl, assisted the mathematics faculty with the Sixteenth Annual W.S.C. Mathematics Contest, kept the bulletin board current, and sponsored a social function at the bowling alley for club members and guests. Other 1990-91 officers: Rory Rut, vice president; Julie Gottschalk, secretary/treasurer; Monte Gilliland, historian; Fred Webber, corresponding secretary; Jim Paige, faculty sponsor.

*Nebraska Beta*

Kearney State College, Kearney

Chapter President - Ann Gibson

24 active, 18 initiates

Other 1990-91 officers: James Nissen, vice president; Dawn James, secretary; Laura Howe, treasurer; Charles Pickens, corresponding secretary; Lutfi Lutfiyyer, faculty sponsor.

**Nebraska Gamma**

Chadron State College, Chadron

Chapter President - Lanelle Henderson

11 actives, 7 initiates

The following students were inducted during spring initiation ceremonies: June Brown, John Freudenberg, Gene Frerichs, Teresa Martin, Scott Jones, Michael Mooney and Tricia Schmunk. Other 1990-91 officers: Marla Soester, vice president; Laura Dooley, secretary; Maya Leicht, treasurer; James Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

**Nebraska Delta**

Nebraska Wesleyan University, Lincoln

Chapter President - Mary Rose Philpot

18 actives, 11 initiates

Other 1990-91 officers: Shelley Bolduan, vice president; Michele Spale, secretary; Halcyon Foster, treasurer; Muriel Skoug, corresponding secretary/faculty sponsor.

**New Mexico Alpha**

University of New Mexico, Albuquerque

Chapter President - Lisa Garcia

40 actives, 21 initiates

Other 1990-91 officers: William Grover, vice president; David Black, secretary; Marjorie Bond, treasurer; Richard C. Metzler, corresponding secretary/faculty sponsor.

**New York Alpha**

Hofstra University, Hempstead

Chapter President - Karm Grossu

13 actives, 7 initiates

During the spring semester New York Alpha sponsored a volleyball game and viewed the film "Stand and Deliver." Other 1990-91 officers: Christopher Rosenblatt, vice president; Diana Beaudette, secretary; Deanna De'Liberto, treasurer; Aileen Michaels, corresponding secretary/faculty sponsor.

*New York Eta*

Niagara University, Niagara University

15 actives, 2 initiates

The annual banquet and initiation were held on March 31. Dr. Wendy Duignan of the faculty spoke on "Some thoughts on Language and Logic in the Writings of Lewis Carroll." Officers for 1990-91 have not yet been elected.

*New York Lambda*

C.W. Post Campus/Long Island University, Brookville

Chapter President - Lauren Henneberger

26 actives, 8 initiates

The highlight of the spring semester was the the regional convention at Susquehanna University, Selingsgrove, Pennsylvania. Four students and two faculty attended; two students presented papers. The chapter enjoyed a very successful spring initiation attended by 40 members and friends. President Lauren Henneberger gave a preview of the paper which she later presented at the regional convention. Faculty officers: Sharon Kunoff, corresponding secretary; Andrew Rockett, faculty sponsor. Student officers for 1990-91 have not yet been elected.

*Ohio Alpha*

Bowling Green State University, Bowling Green

Chapter President - Jenifer Laveglia

57 actives, 17 initiates

A special program was held January 24, 1990, in the BGSU Planetarium on "Celestial Coordinate Systems." The guest speaker for the initiation banquet held March 6 was Dr. Arjun Gupta who spoke on "Statistics: Flaws and Fallacies." The entertainment for the event was provided by "The Logarithms," the faculty barbershop quartet consisting of Dr. Cliff Long, Dr. Dean Neumann, Dr. Charles Holland, and Dr. Herbert Hollister. The *KME* Excellence in Teaching Mathematics Award was presented to Dr. John Gresser and to Dr. Edsel Peña. Other 1990-91 officers: Tracie Wedell, vice president; Malcolm Shrimplin, secretary; Travis Doom, treasurer; Waldemar C. Weber, corresponding secretary; Thomas Hern, faculty sponsor.

*Ohio Gamma*

Baldwin-Wallace College, Berea

Chapter President - Gerald Hwasta

15 actives, 10 initiates

Other 1990-91 officers: Laura Evans, vice president; Tom Serdinak,

secretary; Mike Pavlick, treasurer; Robert Schlea, corresponding secretary/faculty sponsor.

### *Ohio Zeta*

Muskingum College, New Concord

Chapter President - Kristi Pritchett

23 actives, 7 initiates

Semester activities began with presentations by new chapter members. Five students and two faculty attended the Region II Convention in Milwaukee. Jennifer Suschil presented a paper entitled "Circulant Matrices." Student talks were also given by Mark Habibi, Krisiti Pritchett, Doug Miller, Jon Ransom, and Jen Suschil at the Ohio Section MAA meeting in Cincinnati. In other activities, seven new members were initiated in February and the organization sponsored a Careers Symposium in April. Major speaker for the symposium was C. Lynn Myers, NASA Goddard Space Center, who spoke on "Space Communications." Symposium coordinator was Monica Gibson. Other 1990-91 officers: Jon Ransom, vice president; Kim Forgrave, secretary; Tom Myers, treasurer; James L. Smith, corresponding secretary; Javad Habibi, faculty sponsor.

### *Ohio Eta*

Ohio Northern University, Ada

Chapter President - Emma Sturm

58 actives

Ohio Eta inducted 25 new members in January. Other 1990-91 officers: Lora Cowgill, vice president; Michelle Moore, secretary; Brad Meyer, treasurer; Carol O'Dell, corresponding secretary; Conduff G. Childress, Jr., faculty sponsor.

### *Oklahoma Alpha*

Northeastern State University, Tahlequah

Chapter President - Monique Harrison

50 actives, 12 initiates

This spring the Oklahoma Alpha chapter sponsored a talk by Dr. Darryl McCullough from University of Oklahoma on the topic "Sequences of Equidistant Points." The spring '90 initiation ceremonies for 12 students were held in the banquet room of the Western Sizzlin' restaurant in Tahlequah. Mike Reagan was honored with a plaque for serving many years as sponsor of the Oklahoma Alpha chapter. Many KME members served as presidors for the sessions of the Oklahoma

Council of Teachers of Mathematics spring meeting held on Northeastern State University campus. The annual ice cream social was held in April. Other 1990-91 officers: Lisa Singer, vice president; Rebecca Smith, secretary; Lori Austin, treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

*Oklahoma Gamma* Southwestern Oklahoma State University, Weatherford  
Chapter President - Jeanna Day  
20 actives, 15 initiates

Oklahoma Gamma heard speakers from Oklahoma University and Oklahoma State University. Other 1990-91 officers: Melicia Kirland, vice president; Karen Cochran, secretary; Kristen Casebeer, treasurer; Wayne Hayes, corresponding secretary; Robert Morris, faculty sponsor.

*Pennsylvania Alpha* Westminster College, New Wilmington  
Chapter President - Kimberly Hoener  
20 actives, 10 initiates

Pennsylvania Alpha continued to offer weekly tutoring services for students in the mathematics department. In addition, they sponsored a career night and invited speakers to campus to address the area of job opportunities in mathematics, computer science, and computer information systems. This was a joint venture with the local chapter of The Association for Computing Machines. Other 1990-91 officers: Christy Heid, vice president; Lori Metsger, secretary; Jeannette Huczko, treasurer; J. Miller Peck, corresponding secretary; Warren D. Hickman, faculty sponsor.

*Pennsylvania Gamma* Waynesburg College, Waynesburg  
Chapter President - Angela Stewart  
9 actives, 5 initiates

Other 1990-91 officers: Ron Shaffer, vice president; Jennifer Thyreen; Monica Mertes, treasurer/corresponding secretary/faculty sponsor; A. B Billings, faculty sponsor.

*Pennsylvania Delta* Marywood College, Scranton  
Chapter President - Diane Harvey  
12 actives, 8 initiates

Pennsylvania Delta participated in a student symposium at Moravian College on February 17. The written and oral math contests for high school students were held on February 24 and March 25, respectively. Initiation of new members took place on April 22. Other 1990-91 officers: John Zaums, vice president; Teresa Larkin, secretary; Kathy Tenelly, treasurer; Sister Robert Ann von Ahnen, corresponding secretary/faculty sponsor.

***Pennsylvania Kappa***

Holy Family College, Philadelphia

Chapter President - Monica Magilton

12 actives, 8 initiates

Pennsylvania Kappa members provided free tutoring for Holy Family College students. At the last meeting the organization set March 25, 1991, as the next initiation date. Former *KME* members and graduates of HFC will be invited as guest speakers, the theme to be selected at the October meeting next fall. Other 1990-91 officers: Paul Hiller, vice president; David McCabe, secretary/treasurer; Sister M. Grace Kuzawa, corresponding secretary; Linda Czajka, faculty sponsor.

***Texas Alpha***

Texas Tech University, Lubbock

Chapter President - Jennifer Ragland

10 actives, 24 initiates

Spring Semester activities centered around the initiation of 24 new members. Other 1990-91 officers: Charles Scholl, vice president; Darrell Taylor, secretary; Keisha Thomas, treasurer; Robert Moreland, corresponding secretary/faculty sponsor.

***Texas Eta***

Hardin-Simmons University, Abilene

Chapter President - Charles Reed

10 actives, 7 initiates

The Texas Eta Chapter of Kappa Mu Epsilon held its sixteenth annual induction banquet March 9. There were seven members inducted: David Haralson from Austin, Texas; Tondi Jeter from Jayton, Texas; Kristen Knebel from College Station, Texas; Al Onley from Greenville, Texas; and Charles Reed, Bessie Whitting, and John Zellner from Abilene, Texas. With the induction of these members, membership in the local chapter stands at 125. Dr. Mark Farris, Associate Professor of Mathematics at H-SU, addressed the local chapter on the subject,

"Mathematics Anxiety." Leading the induction ceremonies were Randal Schwindt, president; Stephen Cody, vice president; and Tina E. Hill, secretary and treasurer. Other 1990-91 officers: Tondi Jeter, vice president; Kristen Knebel, secretary/treasurer; Mary Wagner-Krankel, corresponding secretary; Edwin Hewett and Charles Robinson, faculty sponsors.

*Texas Iota*

McMurry University, Abilene

Chapter President - Rusty Teeter  
18 actives, 11 initiates

Other 1990-91 officers: Charles Converse, vice president; Randy McCarble, secretary; Jacqueline Bryan, treasurer; Diannne Dulin, corresponding secretary; Bill J. Dulin, faculty sponsor.

*Virginia Gamma*

Liberty University, Lynchburg

Chapter President - Guy Tarnstrom  
15 actives, 4 initiates

Other 1990-91 officers: Jim Ward, vice president; John Kurtz, secretary; Brian Renshaw, treasurer; Glyn Wooldridge, corresponding secretary; Robert Chasnov, faculty sponsor.

*Wisconsin Alpha*

Mount Mary College, Milwaukee

Chapter President - Lauri Malisch  
5 actives, 5 initiates

The *KME* Region II Convention was hosted by Wisconsin Alpha on March 16, 17. The meeting began with a buffet centered on a Wisconsin theme. Talks were given by students from Michigan Beta, Ohio Zeta, and Wisconsin Alpha. In other chapter activities, pledges Sandra Erickson and Mary Porter gave presentations on the topic of "Eminent Women Mathematicians." Other 1990-91 officers: Cyndi Heim, vice president/treasurer; Lauri Malisch, secretary; Sister Adrienne Eickman, corresponding secretary/faculty sponsor.

*Wisconsin Gamma*

University of Wisconsin-Eau Claire, Eau Claire

Chapter President - Julie Wineinger  
40 actives

During the spring semester the monthly meetings were highlighted

by six student presentations. Also, during this time, the organization sponsored three fund raisers: a bake sale, a popcorn sale, and a book sale. The book sale was by far the most successful of the three fund raisers. The organization participated in a week of events put on by the University to give recognition to the various honor societies. Besides having members serving on honor week committees and attending a banquet, the club sponsored a movie on mathematics. The social activities of the semester concluded with a picnic with the Chemistry and Physics clubs. Other 1990-91 officers: Jim Kelley, vice president; Kim Anderson, secretary; Julia Folsom, treasurer; Tom Wineinger, corresponding secretary/faculty sponsor.

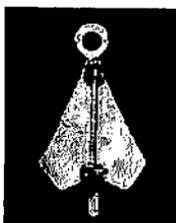
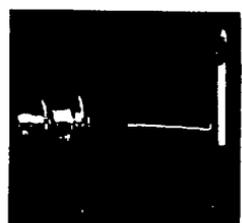
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## Kappa Mu Epsilon Jewelry

The Kappa Mu Epsilon key (shown below) is available as a charm, pin or tie-tac. Manufactured by J. O. Pollack & Company in gold plate, sterling silver or 10K gold, key charms currently are priced at \$4.75, \$7.50 and \$42.00 (respectively) while key pins and tie-tacs are \$5.25, \$8.00 and \$49.50.



front

#805  
charm back#806  
Pin back#807  
tie - tac back

Orders for Kappa Mu Epsilon jewelry **MUST** be placed through your Chapter's corresponding secretary.

## 1990 Regional Conventions

Edited by Arnold D. Hammel, President-Elect

There were four *KME* Regional Conventions during the Spring of 1990. The following reports were prepared from materials submitted by the regional directors.

### Report of the 1990 Region I Convention

The Region I Convention was held April 27-28 at Susquehanna University, Pennsylvania Theta. Four Chapters (Maryland Alpha, College of Notre Dame of Maryland; New York Lambda, C. W. Post/Long Island University; Pennsylvania Theta, Susquehanna University; and Pennsylvania Lambda, Bloomsburg University) were represented with a total registration of forty-one students and faculty. The meeting began with an induction dinner at which Mike Reid became a member of the Pennsylvania Theta Chapter. Following the induction ceremony, Ben Franklin (as portrayed by Physics Professor Fred A. Grosse of Susquehanna University) paid the convention a most entertaining and informative visit.

Saturday morning was devoted to the following student presentations:

*Applications of Elementary LISP Functions to Computational Linguistics*  
Francis D. Thompson (C. W. Post/Long Island University)

*Finite Element Analysis*  
Alise Szymczak (Bloomsburg University)

*The {1} Condition Generalized Inverse*  
Michelle Leuser (Susquehanna University)

*An Analysis of Diophantine Equations*  
Lauren J. Henneberger (C. W. Post/Long Island University)

*Quasicrystals*

Sam Gross (Susquehanna University)

*Breaking the Code*

Janice Gessner (Susquehanna University)

*Pascal's Triangle Revisited*

Michelle Creola (Bloomsburg University)

The convention concluded with an awards luncheon at which Mathematics Professor Dr. Ken Brakke of Susquehanna University spoke on "Soap Films and the Opaque Cube Problem." The Judging Committee (consisting of two faculty members and four students) announced the following awards: 1st Prize - Alise Szymczak, 2nd Prize - Janice Gessner and 3rd Prize - Michelle Creola.

## Report of the 1990 Region II Convention

The Region II Convention was held March 16-17 at Mount Mary College, Wisconsin Alpha. Three Chapters (Michigan Beta, Central Michigan University; Ohio Zeta, Muskingum College; and Wisconsin Alpha, Mount Mary College) were represented with a registration of about thirty-five students, faculty and alumni. The buffet on Friday evening celebrated many Milwaukee and Wisconsin traditions, including fish fry, brat roast, potato salad, baked beans, french fries and dairy. Another Milwaukee tradition - bowling - followed. There were three student papers:

*The Buffon Needle Problem*

George Lasecki (Central Michigan University)

*The Scrambler Transformation*

Lauri Malisch (Mount Mary College)

*Jordan Form of a Circulant Matrix*

Jennifer Suschil (Muskingum College)

Each presenter received a poster describing one of the summer festivals of Milwaukee. Time also was devoted to discussion groups for faculty and students.

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### Report of the 1990 Region III Convention

The Region III Convention was held March 30-31 at Winthrop College, South Carolina Gamma. Three current Chapters (Alabama Beta, University of North Alabama; Kentucky Alpha, Eastern Kentucky University; and South Carolina Gamma, Winthrop College) and the Mathematics Club from Elon College (which soon will be installed as a *KME* Chapter) were represented with a total registration of thirty. On Friday afternoon, we were treated to a fascinating talk on "Game Theory" by Dr. Harold Reiter from the University of North Carolina at Charlotte. Then followed a picnic supper at the college lake, a paper plane contest, and a human topological knot problem. There were five student papers:

*A Comparison of Simple Linear Extrapolation to Akima's Method*  
Becky Boltin (Winthrop College)

*History of Pi*  
Valerie Owen (Elon College)

*Legal Applications of Probability*  
Kathy Ponder (Eastern Kentucky University)

*Embedding of Some Graphs into Hypercubes*  
Tom Cheung (Eastern Kentucky University)

*Using DERIVE in Calculus*  
Lisa Whitis (Eastern Kentucky University)

The students speakers were all awarded glasses engraved with the *KME* initials.

### Report of the 1990 Region IV Convention

The Region IV Convention was held March 17 at Benedictine College, Kansas Gamma. There were thirteen Chapters represented (Iowa Alpha, University of Northern Iowa; Kansas Alpha, Beta, Gamma and Delta from Pittsburgh State University, Emporia State University, Benedictine College and Washburn University; Missouri Alpha, Beta, Eta, Theta and Iota from Southwest Missouri State University, Central

Missouri State University, Northeast Missouri State University, Evangel College and Missouri Southern State College; Nebraska Alpha, Beta and Gamma from Wayne State College, Kearney State College and Chadron State College) plus observers from Missouri Western College at St. Joseph. There were eighty-two in attendance: fifty-seven students and twenty-five faculty. There were seven student papers:

*Magical Minimal Mania*

Ty Anderson (Benedictine College)

*Generalized Inverses of Rectangular Matrices and Applications*

Michael Hirsch (University of Northern Iowa)

*Lagrange Multipliers*

Kevin Anderson (Chadron State College)

*The Unfaithful Wives Problem*

Analee Witt (Missouri Southern State College)

*Shake, Rattle and Roll: The Mathematics of an Earthquake*

Maria Soester (Chadron State College)

*Prime Numbers*

Matthew McIntosh (Benedictine College)

*On Complex Quasimonotone and Complex Monotone Sequences*

Peter Egler (Southwest Missouri State University)

Each speaker received a certificate of participation with awards of \$10, \$20 and \$30 given to the best three papers. Provisions were made for discussion groups for both faculty and students. The luncheon speaker was Karen Bauer, a statistician from Midwest Research Institute.

## Kappa Mu Epsilon National Officers

- Harold L. Thomas *President*  
Department of Mathematics  
Pittsburg State University, Pittsburg, Kansas 66762
- Arnold D. Hammel *President-Elect*  
Department of Mathematics  
Central Michigan University, Mt. Pleasant, Michigan 48859
- Robert L. Bailey *Secretary*  
Department of Mathematics  
Niagara University, Niagara University, New York 14109
- Jo Ann Fellin *Treasurer*  
Mathematics and Computer Science Department  
Benedictine College, Atchison, Kansas 66002
- Mary S. Elick *Historian*  
Department of Mathematics  
Missouri Southern State College, Joplin, Missouri 64801

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

## Active Chapters of Kappa Mu Epsilon

*Listed by date of installation.*

Chapter	Location	Installation Date
OK Alpha	Northeastern Oklahoma State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State College	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State College, Upper Montclair	21 April 1944
IL Delta	College of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949

IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	Kearney State College, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri - Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood College, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin - River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Northeast Missouri State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel College, Springfield	12 Jan 1971

PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop College, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	West Georgia College, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin - Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Illinois Benedictine College, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C. W. Post Center of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry College, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990

*The Pentagon* is prepared with the assistance of the  
Instructional Media Center at Long Island University.