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Can You Hear the Shape of a Tambourine?

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Presented at the 1989 National Convention and
awarded **FOURTH PLACE** by the Awards Committee.

Introduction.

Can you hear the shape of a tambourine? According to Mark Kac [1, p. 3], "Personally I believe that one cannot hear the shape of a tambourine but I may well be wrong and I am not prepared to bet large sums either way." In his paper "Can you hear the shape of a drum?" Kac [1] uses information known about certain regions to determine characteristics regarding their shape. The following discussion makes use of Kac's conclusions, as well as discoveries made by Hermann Weyl. The regions to be studied are a one dimensional circle of radius r and its circumference, and a two dimensional disk to radius r and its area. Not all of the equations are derived, but if rigorous proofs are desired, the articles by Kac [1] and by Weyl [2] should be inspected. Therefore, given the two regions above, the circle and the disk, and certain information about them, can their circumference and area, respectively, be determined?

The One Dimensional Case.

Consider a circle or radius $r = 1$. The circumference of the circle is then $2\pi r = 2\pi$. Take the circle, cut it, and bend it out into a straight line. The line will be of length 2π . Consider the wave equation of the circle (which is now a straight line) such that some wave is passing through the boundary of the circle (or down the line). Note that the wave equation evaluated at 0 and at 2π on the circle must be equal because on

a circle 2π is just another way of representing 0. The eigenvalues of the wave equation on a circle when $r = 1$ are:

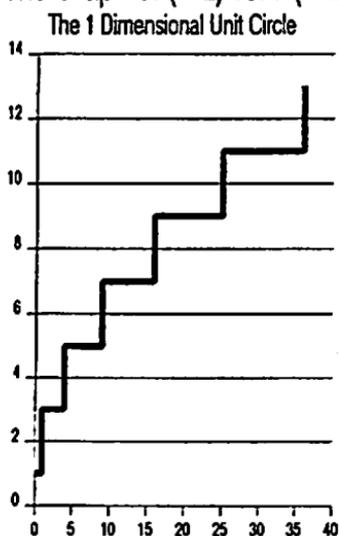
$$(-n^2, \dots, -4, -1, 0, -1, -4, \dots, -n^2).$$

Now define a function $N(-z)$ such that $N(-z)$ is equal to the number of eigenvalues of the solution, with multiplicity, that are in absolute value less than or equal to $-z$, where z is some eigenvalue. The following values for $N(-z)$ result:

$-z:$	0	1	4	9	16	25	...
$N(-z):$	1	3	5	7	9	11	...

When the values of $-z$ and $N(-z)$ are graphed with the $-z$ values on the x-axis and the $N(-z)$ values on the y-axis, a curve resembling $y = \sqrt{x}$ results (see Figure One).

The Graph of $(-z)$ vs. $N(-z)$



The Graph of $(-z)$ vs $2(\text{square root of } -z) + 1$

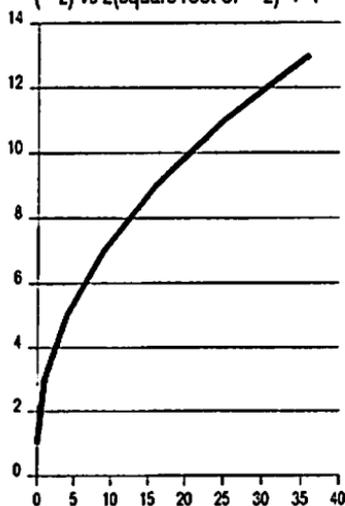


Figure One

Considering that $N(-z)$ grows in a similar fashion as the function $y = \sqrt{x}$, $N(-z)$ can be described as a function in terms of $\sqrt{-z}$. Suppose that

a is the coefficient of $\sqrt{-z}$ such that

$$N(-z) \sim a\sqrt{-z} \text{ as } z \rightarrow -\infty.$$

This equation says that $N(-z)$ is approximately equal to $a\sqrt{-z}$ as z approaches negative infinity. Or,

$$\lim_{z \rightarrow -\infty} \frac{N(-z)}{a\sqrt{-z}} = 1.$$

Consider $N(-z) = 2\sqrt{-z} + 1$. The coefficient a is equal to 2, which is the jump or slope of the graph and when $z = 0$, $N(-z) = 1$. This equation describes the function $N(-z)$ in terms of $\sqrt{-z}$.

$$N(-z) \sim 2\sqrt{-z} \text{ as } z \rightarrow -\infty$$

and

$$\lim_{z \rightarrow -\infty} \frac{N(-z)}{2\sqrt{-z}} = \lim_{z \rightarrow -\infty} \frac{2\sqrt{-z} + 1}{2\sqrt{-z}} = 1.$$

Now consider the case when the radius of the circle is equal to a value r that is greater than one. The circumference of the circle is $2\pi r$ and thus the length of the line (the circle stretched out onto a line) is $2\pi r$ also. The eigenvalues for the wave equation on a circle of radius r are

$$\left(\frac{-n^2}{r^2}, \dots, \frac{-9}{r^2}, \frac{-4}{r^2}, \frac{-1}{r^2}, 0, \frac{-1}{r^2}, \frac{-4}{r^2}, \frac{-9}{r^2}, \dots, \frac{-n^2}{r^2} \right)$$

and solving for $N(-z)$ in terms of these new z values gives a similar table to the one above. Again, the graph of $-z$ vs. $N(-z)$ is similar in shape to the graph of $\sqrt{-z}$. The actual equation relating $N(-z)$ to $y = \sqrt{-z}$ can be found in the same way used previously. In this case, with an arbitrary radius r ,

$$N(-z) = 2r\sqrt{-z} + 1$$

where the coefficient 2 is the jump or slope of the graph and $N = 1$ when $-z = 0$. Note the role the radius plays in the above equation.

In the case where the dimension of the shape is equal to one, as with

the circle, the counting function of the eigenvalues is a multiple of the squares of the chosen eigenvalue. Note that by this equation, if the eigenvalues of a particular boundary are known then the circumference can easily be calculated. Now let us consider the case of a disk or a membrane in the shape of a circle.

The Two Dimensional Case.

Suppose a membrane M of an elastic material is stretched across a region with boundary B and let the region be the unit circle. The area of the membrane M then πr^2 and, since $r = 1$, the area is just π . If the membrane is set into motion by applying some force to the surface of the membrane, its displacement is perpendicular to the original plane upon which the surface was stretched. The behavior of the membrane obeys the wave equation and it vibrates in response to the force applied.

The following is an analogy:

Consider a drum. The top surface of the drum is a circle. Now suppose the top surface of the drum is not only in the shape of a circle but the area of the top surface is $\pi r^2 = \pi$ then the radius of the drum is equal to one. The surface of the drum is made of an elastic material and when hit with a drumstick vibrates in response to the striking action. This striking action produces a sound in the case of an ordinary drum.

The surface considered above is more like a tambourine in that it has no volume but merely a top surface.

The wave equation is

$$\frac{d^2 F}{dt^2} = c^2 \nabla^2 F$$

where c^2 is a conductivity constant that depends upon the type of membrane involved and the amount of tension with which it is held. The solutions of the wave equation are of the form

$$F(p;t) = U(p)e^{i\omega t}$$

where U is a function of the position where the force was applied and the exponential part determines the shape of the surface due to the motion.

Note that the boundary condition that $U = 0$ on the boundary B of the surface must be fulfilled and the limit as p approaches the boundary must equal 0.

Consider a surface, like the tambourine described, with a radius equal to c , which vibrates under some tension T . The displacement of the surface of the membrane at any particular point depends only upon the time t and the distance r from the center. The displacement is caused by radial vibrations centered at the position where the force was applied and covering the region. As the motion approaches the boundary of the disk, the vibrations cease due to the fixed shape of the object. The following equation describes the displacement of the membrane at some position r and time t :

$$\frac{d^2U}{dt^2} = a^2 \nabla^2 U = a^2 \left(\frac{d^2U}{dr^2} + \frac{1}{r} \frac{dU}{dr} \right).$$

The variable a is a constant that depends upon the tension with which the membrane is stretched and the density of the material used. For simplification let $a = 1$ and then the equation becomes

$$\frac{d^2U}{dt^2} = \left(\frac{d^2U}{dr^2} + \frac{1}{r} \frac{dU}{dr} \right).$$

Since c is on the boundary of the disk, the displacement must equal zero because the boundary is fixed and so $U(c,t) = 0$. Consider a solution of the following form:

$$U(r,t) = R(r) \sin(\omega t).$$

Then

$$-\omega^2 R(r) \sin(\omega t) = R''(r) \sin(\omega t) + \frac{1}{r} R'(r) \sin(\omega t)$$

$$-\omega^2 R(r) = R''(r) + \frac{1}{r} R'(r)$$

$$0 = R''(r) + \frac{1}{r} R'(r) + \omega^2 R(r)$$

and so

$$(1) \quad 0 = r^2 R''(r) + r R'(r) + w^2 r^2 R(r).$$

$U(c,t) = R(c)\sin(wt) = 0$ must be true because c is on the boundary and therefore the displacement at c must equal zero. Therefore, $R(c) = 0!$

Equation (1) is the Bessel equation of order 0 with parameter w and its only nontrivial continuous solution is

$$R(r) = J_0(wr).$$

Since $R(c) = 0$, $R(c) = J_0(wc) = 0$ so wc must be one of the positive roots.

By use of integral equations, for a region M with a specified boundary B , the eigenvalues of the wave equation can be found. These numbers

$$z_1 \leq z_2 \leq z_3 \leq \dots$$

each correspond to some eigenfunction E such that the wave equation evaluated at E_n and z_n is equal to zero:

$$c^2 \nabla^2 E_n + z_n E_n = 0$$

and $E \rightarrow 0$ as the boundary is approached.

With the eigenvalues, a function similar to $N(-z)$ defined earlier also can be defined now. Let $N(z)$ be the function that when evaluated at some z is equal to the number of eigenvalues (with multiplicity) less than or equal to z . The following relation was found relating $N(z)$ to the area of the region under study:

$$N(z) \sim \frac{Az}{2\pi} \text{ as } z \rightarrow \infty$$

and

$$\lim_{z \rightarrow \infty} \frac{N(z)}{z} = \frac{A}{2\pi}$$

where A is the area of the membrane M . This amazing discovery was formulated by Hermann Weyl [2]. Thus, by this equation, if the eigenvalues for a specific region M with boundary B are known, the area of the region can easily be calculated. So, one can hear the area of M given the eigenvalues in the two dimensional case.

References.

- [1] Kac, Mark. "Can one hear the shape of a drum?" *Amer. Math. Monthly* 73 No. 4, Part II ("Papers in Analysis") (April 1966), 1-23.
- [2] Weyl, Hermann. "Über die asymptotische Verteilung der Eigenwerte," 368-375 in *Gesammelte Abhandlungen (Band I)*. Berlin: Springer-Verlag, 1968.

A Note about $\sum_{k=1}^n k \cdot k!$

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Introduction.

The well known result

$$(1) \quad \sum_{k=1}^n k \cdot k! = (n+1)! - 1$$

is frequently used in combinatorics. This formula has many important applications, among which is the definition of a new counting system. This method (due to Johnson [1], §3) is the basis of a famous scheme (see [3]) for generating all the permutations on any set of n distinct elements. By an analogy to the usual b -base systems in which

$$\sum_{k=0}^n (b-1) \cdot b^k = b^{n+1} - 1,$$

formula (1) allows us to represent decimal numbers in a new form as follows:

0 = 0	9 = 111	18 = 300
1 = 1	10 = 120	19 = 301
2 = 10	11 = 121	20 = 310
3 = 11	12 = 200	21 = 311
4 = 20	13 = 201	22 = 320
5 = 21	14 = 210	23 = 321
6 = 100	15 = 211	24 = 1000
7 = 101	16 = 220	25 = 1001
8 = 110	17 = 221	etc.

Another application of (1) is the (immediate) solution of the following problem:

Assume that a computer printer prints at 80 characters per second. How long will it take to print all the possible permutations of all the subsets of {1,2,3,4,5,6,7,8}?

In this note, we shall generalize (1) to higher powers of k by showing the following

THEOREM. For any positive integers m and n ,

$$\sum_{k=1}^n k^m k! = (n+1)! P_m(n) + a_m \sum_{k=1}^n k! + b_m$$

where a_m and b_m are integers and $P_m(n)$ is a polynomial of degree $m-1$ with integer coefficients.

For example, for $m = 2$ we have

$$(2) \quad \sum_{k=1}^n k^2 k! = (n+1)! n - \sum_{k=1}^n k!$$

and for $m = 3$ we have

$$\sum_{k=1}^n k^3 k! = (n+1)! (n^2 - 2) + \sum_{k=1}^n k! + 2.$$

These formulae are not only of pure mathematical interest; formula (2), for instance, is needed in my proof (see [2]) of a new result on complexity order in computer science.

Proof.

The proof is by induction. For $m = 1$, the claim holds by formula (1) with

$$a_1 = 0, b_1 = -1 \text{ and } P_1(n) = 1;$$

for completeness, in the trivial case $m = 0$ we set

$$a_0 = 1, b_0 = 0 \text{ and } P_0(n) = 0.$$

Assume that

$$\sum_{k=1}^n k^m k! = (n+1)! P_m(n) + a_m \sum_{k=1}^n k! + b_m$$

and let

$$F_m(n) = \sum_{k=1}^n k^{m+1} k! + \sum_{k=1}^n k^m k! .$$

We find that

$$\begin{aligned} F_m(n) &= \sum_{k=1}^n (k+1)! k^m \\ &= \sum_{k=1}^n (k+1)! \left((k+1)^m - \sum_{r=0}^{m-1} \binom{m-1}{r} k^r \right) \\ &= \sum_{k=1}^n (k+1)^m (k+1)! - \sum_{k=1}^n (k+1)! \sum_{r=0}^{m-1} \binom{m-1}{r} k^r \\ &= \sum_{k=1}^n k^m k! + (n+1)^m (n+1)! - 1 - \sum_{k=1}^n (k+1)! \sum_{r=0}^{m-1} \binom{m-1}{r} k^r, \end{aligned}$$

which, by the assumption, is

$$\begin{aligned} &= (n+1)! \left(P_m(n) + (n+1)^m \right) + a_m \sum_{k=1}^n k! + b_m \\ &\quad - 1 - \sum_{k=1}^n (k+1)! \sum_{r=0}^{m-1} \binom{m-1}{r} k^r. \end{aligned}$$

Now,

$$\begin{aligned}
 \sum_{k=1}^n (k+1)! \sum_{r=0}^{m-1} \binom{m}{r} k^r &= \sum_{k=1}^n (k+1) \sum_{r=0}^{m-1} \binom{m}{r} k^r k! \\
 &= \sum_{k=1}^n \sum_{r=0}^{m-1} \binom{m}{r} k^{r+1} k! + \sum_{k=1}^n \sum_{r=0}^{m-1} \binom{m}{r} k^r k! \\
 &= \sum_{r=0}^{m-1} \binom{m}{r} \sum_{k=1}^n k^{r+1} k! + \sum_{r=0}^{m-1} \binom{m}{r} \sum_{k=1}^n k^r k!
 \end{aligned}$$

which, again by the assumption, is

$$\begin{aligned}
 &= \sum_{r=0}^{m-1} \binom{m}{r} \left((n+1)! P_{r+1}(n) + a_{r+1} \sum_{k=1}^n k! + b_{r+1} \right) \\
 &\quad + \sum_{r=0}^{m-1} \binom{m}{r} \left((n+1)! P_r(n) + a_r \sum_{k=1}^n k! + b_r \right) \\
 &= (n+1)! \sum_{r=0}^{m-1} \binom{m}{r} \left(P_{r+1}(n) + P_r(n) \right) \\
 &\quad + \sum_{r=0}^{m-1} \binom{m}{r} \left(a_{r+1} + a_r \right) \cdot \sum_{k=1}^n k! + \sum_{r=0}^{m-1} \binom{m}{r} \left(b_{r+1} + b_r \right) \\
 &= (n+1)! Q_m(n) + A_m \sum_{k=1}^n k! + B_m
 \end{aligned}$$

where $\deg Q_m(n) = \deg P_m(n)$ and A_m and B_m are integers. Thus

$$F_m(n) = (n+1)! \left(P_m(n) + (n+1)^m - Q_m(n) \right) \\ + (a_m - A_m) \sum_{k=1}^n k! + (b_m - 1 - B_m).$$

Since

$$\sum_{k=1}^n k^{m+1} k! = F_m(n) - \sum_{k=1}^n k^m k!$$

we have that

$$\sum_{k=1}^n k^{m+1} k! = (n+1)! \left((n+1)^m - Q_m(n) \right) \\ + (-A_m) \sum_{k=1}^n k! + (-1 - B_m). \\ = (n+1)! P_{m+1}(n) + a_{m+1} \sum_{k=1}^n k! + b_{m+1}$$

as required.

Open Questions and Unsolved Problems.

For $m = 4, 5$ and 6 we have

$$\sum_{k=1}^n k^4 k! = (n+1)! (n^3 - 3n + 3) + 2 \sum_{k=1}^n k! - 3,$$

$$\sum_{k=1}^n k^5 k! = (n+1)! (n^4 - 4n^2 + 6n + 4) - 9 \sum_{k=1}^n k! - 4,$$

and

$$\sum_{k=1}^n k^6 k! = (n+1)! (n^5 - 5n^3 + 10n^2 + 5n - 30) + 9 \sum_{k=1}^n k! + 30.$$

A careful look at the coefficients of the polynomials $P_m(n)$,

$$\begin{array}{r} 1 \\ 1 \ -3 \\ 1 \ -4 \ 6 \\ 1 \ -5 \ 10 \ 5 \\ \dots \end{array}$$

reminds one of Pascal's triangle. Is this systematic? If not, is it still possible to find direct (that is, not by identification) formulae for calculating a_m , b_m and $P_m(n)$ as functions of m ?

Can the Theorem be extended to any positive real number x by using gamma functions? In other words, given a positive integer m , does

$$\int_1^x \Gamma(k+1) k^m dk = \Gamma(x+2) P_m(x) + a_m \int_1^x \Gamma(k+1) dk + b_m ?$$

Here a_m , b_m are real numbers and $P_m(x)$ is a polynomial of degree m with real coefficients. A finite (but very lengthy!) computer approximation seems to indicate an affirmative answer.

References.

- [1] Selmer M. Johnson, "Generation of permutations by adjacent transposition," *Math. Comp.* 17 (1963), 282-285.
- [2] David Naccache de Paz, "On the generation of permutations," *The South African Computer Journal*, to appear.
- [3] Dennis Stanton and Dennis White. *Constructive Combinatorics*. Addison-Wesley, 1986.

Pattern Duplication and Reflection: A Bouncing Ball Problem

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If a particle (or a beam of light) were set into motion inside a two dimensional box (assuming no friction), it would ricochet off of the wall indefinitely. It even might happen that after a finite number of bounces the particle will rebound from a wall at the exact point from which the particle first bounced off of that wall. If this happens, then from that point on the path that the particle traces out will be exactly the path (or pattern) previously traced out up to the point (see Figure One). Why this happens is a simple geometry problem, and rests on the fact that the angle of incidence is always the same as the angle of departure. So that each departure from a given wall will be at the same angle as the first. This then guarantees that if the particle revisits a point, it will do so along the same trajectory as it had previously.

Patterns generated in this manner we will refer to as "repeating patterns" and examples of these repeating patterns are those seen in Figure One. You will notice though that there is a slight difference between patterns 1a,b and 1c,d. In patterns 1a,b the particle motion along the path is always in the same direction, whereas in patterns 1c,d the direction of motion along the path alternates. These two different types of patterns we will refer to as "cyclic" and "noncyclic," respectively, and for this paper we will work exclusively with the cyclic repeating patterns. In particular, the purpose of this paper is to develop a theorem relating cyclic patterns generated in boxes of dimension $h \times L$ to patterns generated in boxes of dimension $mh \times L$.

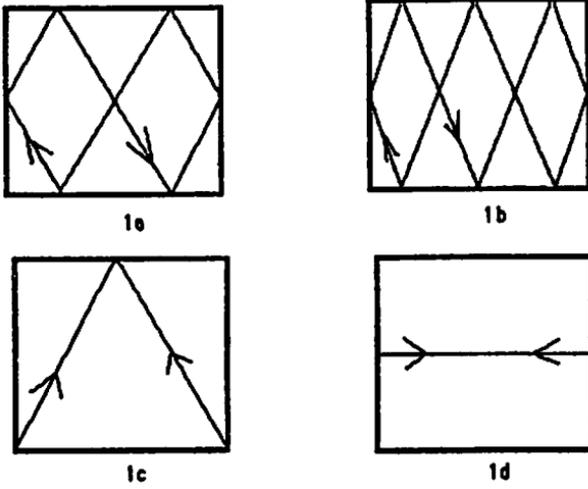


Figure One

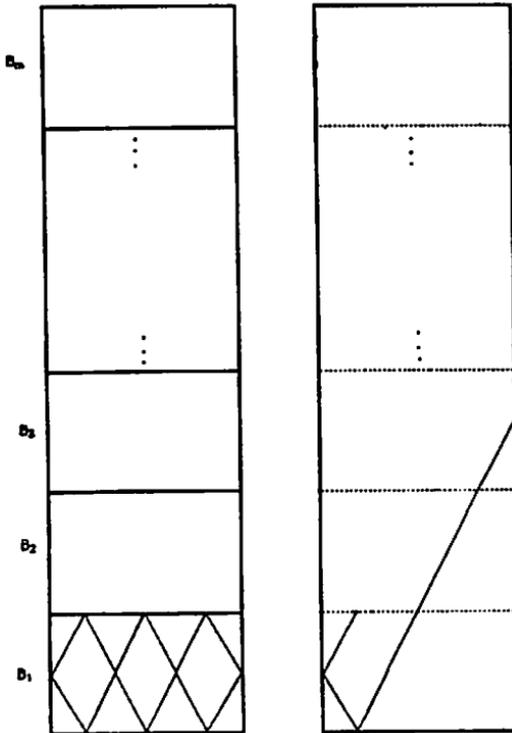


Figure Two

The problem which we will consider can be described as follows: We have a box of dimensions $mh \times L$ (as in Figure Two) partitioned into m boxes of dimension $h \times L$ labeled B_1, \dots, B_m . In B_1 we have a particle in motion tracing out a cyclic repeating pattern which from this point on we will refer to as C . The question then is: if we were to remove all of the separating walls and allow the particle to bounce around freely in the larger box what would the resulting pattern look like? Could we predict anything about this new pattern? Well, as it turns out, it can be shown that the resulting pattern will also be a cyclic repeating pattern, which is in some sense a derivative of C . By this I mean that if you examine each of the larger patterns in Figure Three, you will see that the partial paths that exist within each box B_i ($i = 1, 2, \dots, m$) are really just pieces of the original pattern C and (in some cases) vertical (or "upwards") reflections of those pieces. More specifically, if you were to take C and superimpose it or its vertically reflected image on each box B_i , the paths traced out in each B_i would be completely covered.

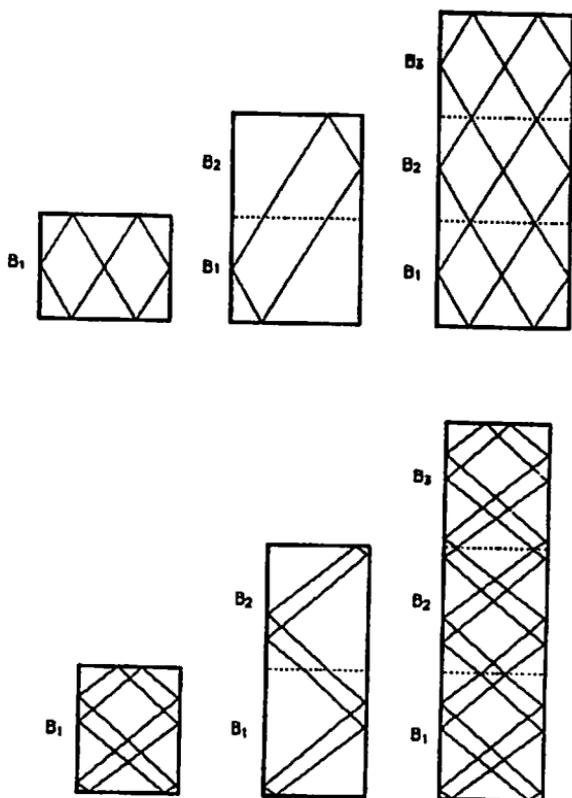


Figure Three

In some cases, you will even notice that the image or reflected image of C is completely duplicated in each B_i . This is not happenstance. There is a beautifully simple yet strong relationship which will guarantee that C or its vertically reflected image is duplicated in each and every B_i .

THEOREM. Given that in B_1 the particle bounces off the bottom wall n times then alternately C or its vertically reflected image will be duplicated in each box B_i ($i = 1, 2, \dots, m$) if and only if $(m, n) = 1$.

To make the proof more coherent, we first will set up some notation and do a little work on the side.

The notation.

From this point on we will consider C to consist of a series of consecutive *circuits* where a circuit is defined to be the partial path traced out by the particle between any two consecutive bounces off the top wall in B_1 (for an example of this definition, see Figure Four (a)). Now, since with this definition there is a one to one correspondence between circuits and bounces off the bottom wall in B_1 , we know that C consists of n consecutive circuits, which we will label C_0, \dots, C_{n-1} . The labeling of these circuits is as in Figure Four (b): along the bottom wall of B_1 with C_0 chosen arbitrarily.

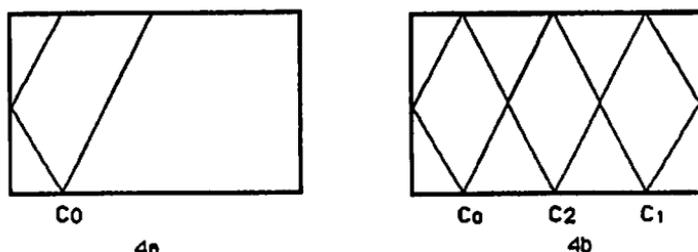


Figure Four

We also define at this time the "half-circuit" which is the partial path traced out by the particle in one pass from the top wall to the bottom wall in B_1 or vice versa. Note also that each circuit is then made up of two half-circuits.

The aside.

We will assume that the separating walls are removed just as the particle is tracing out the circuit C_0 in B_1 . We can assume this without loss of generality since C_0 was chosen arbitrarily. And with this assumption we claim that upon the particle's next reentry into B_1 , it will trace out precisely the circuit $C_{m \bmod n}$ and not a reflection of it. The rationale goes as follows.

It can be shown using a little geometry that the first half-circuit to be complete in B_2 will be a vertical reflection of the first half of C_1 . You should also be able to convince yourself that this is true through a quick study of Figure Three. Similarly, the first half-circuit completed in B_3 will be an exact duplicate of the second half of C_1 only displaced vertically two boxes. So that, in general, for every two boxes that the particle traverses (not counting B_1), it in effect displaces yet another consecutive circuit. Which means by the time the particle gets to the top of the rectangle, it will have displaced $(m-1)/2$ circuits, starting with C_1 . And by the time the particle has traveled back down the rectangle to the dividing line separating B_1 and B_2 , it will have displaced yet another $(m-1)/2$ circuits; in all, a total of $m-1$ circuits.

So that now upon reentry into B_1 , the particle will trace out the m^{th} circuit, the count starting with C_1 . Thus the circuit traced out would be C_m , or more specifically, since there are only n circuits (C_0, \dots, C_{n-1}), the circuit $C_{m \bmod n}$. In addition, it can be further concluded from the diagrams that this circuit will most certainly be a duplication of the original and not a reflection.

Now, before starting the proof, we have one more generalization to make. That is that upon the particle's k^{th} traversal of the rectangle (both up and back), the circuit that will be completed in B_1 will be $C_{km \bmod n}$. And this follows from the above argument, which in effect states that upon each reentry into B_1 the particle will trace out the circuit which is m circuits from the last circuit which was completed in B_1 .

Proof.

(1) Let us first assume $(m,n) = 1$. We then want to show that alternately C and its vertically reflected image will be duplicated in B_1, \dots, B_m . We will first show that C is duplicated in B_1 and to do this we are going to show that each circuit C_0, \dots, C_{n-1} is completed in B_1 . Since we already know that upon the particle's k^{th} traversal of the rectangle,

the circuit $C_{km \bmod n}$ is completed in B_1 , it only remains to show that if k is allowed to range over the integers (or at least a subset of them), $km \bmod n$ will take on the values $0, 1, \dots, n-1$.

Consider $k = 1, 2, \dots, n$. The claim is that $km \bmod n$ takes on the values $0, 1, \dots, n-1$. Well, since there are only n possible values that $km \bmod n$ can take (namely, $0, 1, \dots, n-1$), if we can show that no two k values between 1 and n (inclusive) generate the same value $km \bmod n$, we will be done. So let $1 \leq a, b \leq n$ and $a \neq b$. Then if $am \equiv bm \pmod n$, $n \mid (a-b)m$. But since $(m, n) = 1$, $n \mid (a-b)$. However, $1 \leq a, b \leq n$ implies that $-n + 1 \leq a-b \leq n - 1$ which implies that $|a-b| \leq n-1$. That combined with $n \mid (a-b)$ gives us that $a - b = 0$ or that $a = b$ and we are done; C_0, \dots, C_{n-1} are indeed completed in B_1 , and so C is duplicated in B_1 .

Now as to the rest of the boxes: given that a circuit C_p is traced out in B_1 , then by the previous argument, a reflection of the first half of the circuit C_{p+1} will be completed in B_2 . Similarly, for C_p to have been completed in B_1 , a reflection of C_{p-1} would have had to first been completed in B_2 . This combined with the fact that C is duplicated in B_1 is enough to guarantee that C will be duplicated in B_2 . Then to guarantee that alternately C or its vertically reflected image is duplicated in the remaining boxes, this last argument is used repeatedly $m-2$ more times.

(2) Let us now assume that C was alternately duplicated and reflected in each box. We must show that $(m, n) = 1$. Since C was duplicated in B_1 , we know that C_1 was completed in B_1 . Which implies that there exists a k such that $C_{km \bmod n} = C_1$, or that $1 \equiv km \pmod n$. This then implies that there exists an integer a such that $km - 1 = an$ or $km - an = 1$. But then if $(m, n) = d$, $d \mid 1$ and therefore $d = 1$.

The proof is finished.

Axiomatic Structure of the Integral

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In many situations and in widely separate fields, the graph of some function can yield some significant information concerning that function. Often the area bounded by several functions has some significance. In physics, for example, if a force (constant or varying continuously with position) acts along a line from a to b , the work done is the area beneath this line and between the shown boundaries.

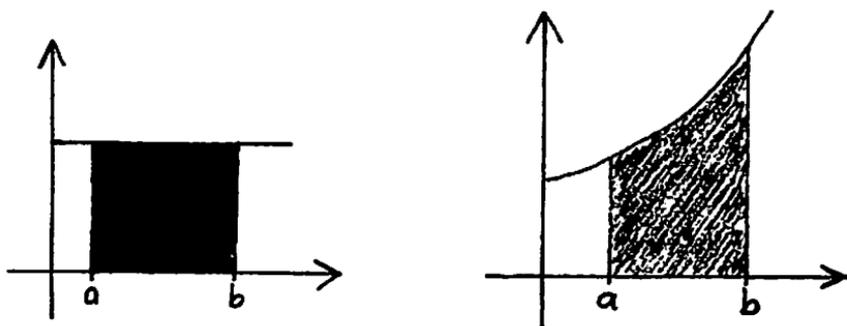


Figure One

In a business situation, let S represent the supply function of some product and D represent the demand function for that product. The point of intersection of these two functions is called the equilibrium point while the area bounded (as shown in Figure Two) above this point is the consumer surplus and that below is the producer surplus.

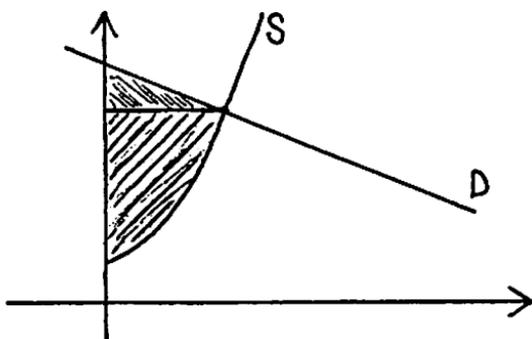


Figure Two

Although the area bounded by different functions could be quite complicated to determine, with the development of integral calculus we have an efficient and neat way to determine such areas. This is the main reason why integral calculus was developed before differential calculus. Leonard Gillman and Robert H. McDowell in their book [1] put forth an interesting development of the basis for integral calculus which we will consider here.



Figure Three

We will start with two obvious but fundamental properties concerning area. Take, for example, the graph of the function $f(x) = x^3 - 9x^2 + 24x - 15$ (see Figure Three). It is obvious that any definition of area must be additive and satisfy the betweenness property. To illustrate these two properties, we can see from the graph of our function that $A_1^3 + A_3^5 = A_1^5$, where A_a^b denotes the area from a to b . This is the additivity property of area. Also note that

$$(5 - 1) \min_{[1,5]} f \leq A_1^5 \leq (5 - 1) \max_{[1,5]} f.$$

This is the betweenness property of area.

Now, because the use of area was one of the motivations in the development of the integral, it is necessary to define the integral as a function that satisfies both the additivity and betweenness properties of area. That is, for our function $f(x)$ we need

$$\int_1^3 f + \int_3^5 f = \int_1^5 f,$$

so that additivity holds, and

$$(5 - 1) \min_{[1,5]} f \leq \int_1^5 f \leq (5 - 1) \max_{[1,5]} f,$$

so that betweenness holds.

Or more generally, we would like to define the integral as a function that satisfies additivity and betweenness. That is, we wish to have for f a continuous function over $[a, b]$ with $a \leq u \leq v \leq b$,

$$\int_a^u f + \int_u^v f = \int_a^v f,$$

and

$$(v - u) \min_{[u,v]} f \leq \int_u^v f \leq (v - u) \max_{[u,v]} f.$$

One of the milestones of mathematics was the realization that integration and differentiation were inverses of each other. Hence, integrals and antiderivatives are practically the same thing. This allows us to state that: (1) if f has an antiderivative then it has an integral, namely,

$$(1) \quad \int_u^v f = F(v) - F(u),$$

where F is any antiderivative of f , and (2) if f has an integral then it has an antiderivative, that is to say,

$$(2) \quad \frac{d}{dx} \int_a^x f = f(x).$$

Now we will prove these two properties using the two fundamental properties of area.

To prove property (2), we will start with the additive property:

$$\int_a^u f + \int_u^v f = \int_a^v f,$$

so

$$\int_u^v f = \int_a^v f - \int_a^u f.$$

Restating the betweenness property, we have

$$(v - u) \min_{[u,v]} f \leq \int_u^v f \leq (v - u) \max_{[u,v]} f.$$

Then by substitution,

$$(v - u) \min_{[u,v]} f \leq \int_a^v f - \int_a^u f \leq (v - u) \max_{[u,v]} f.$$

Now we will let $F_0(x) = \int_a^x f$. Hence,

$$(v - u) \min_{[u,v]} f \leq F_0(v) - F_0(u) \leq (v - u) \max_{[u,v]} f,$$

$$\min_{[u,v]} f \leq \frac{F_0(v) - F_0(u)}{(v - u)} \leq \max_{[u,v]} f.$$

Now we will write $x = u$ and $\Delta x = v - u$ so that $v = x + \Delta x$. Then we

have

$$\min_{[x, x+\Delta x]} f \leq \frac{F_0(x+\Delta x) - F_0(x)}{\Delta x} \leq \max_{[x, x+\Delta x]} f.$$

By the Mean Value Theorem,

$$\frac{F_0(x+\Delta x) - F_0(x)}{\Delta x} = F'_0(x^*)$$

where $x \leq x^* \leq x + \Delta x$. Now as $\Delta x \rightarrow 0$, the two outside quantities approach $f(x)$ and $F'_0(x^*)$ approaches $F'_0(x)$. Hence, by the Sandwich Theorem, $F'_0(x) = f(x)$ and thus

$$F'_0(x) = \frac{d}{dx} \int_a^x f = f(x).$$

Before we can prove the other property mentioned, we need to consider the uniqueness of the integral. It must be unique intuitively because area is unique. However, we will show this uniqueness of the integral.

We need to show first that for f a function that has an antiderivative on an interval I , for each point $a \in I$ and for every number b , there is one and only one antiderivative F for f for which $F(a) = b$. To do so, we will choose any antiderivative and adjust it if necessary by adding a constant to make the value at a equal to b . Thus, take F_0 an antiderivative and F the one we are after. Then, $F(x) = F_0(x) + C$. There is one and only one number C such that $F(a) = F_0(a) + C = b$ (namely, $C = b - F_0(a)$). Thus F is uniquely determined.

We have shown that $F_0(x) = \int_a^x f$ is an antiderivative of f . Now we will show that every integral leads to the same function $F_0(x)$. Take $F_0(a) = \int_a^a f$. By additivity, $\int_a^v f = \int_a^u f - \int_u^a f$. Then by substitution, $\int_u^a f = F_0(v) - F_0(u)$. Hence, $F_0(a) = \int_a^a f = F_0(a) - F_0(a) = 0$. This shows that $F_0(a)$ has the same value at a no matter what integral we started with. As proven above, there is one and only one antiderivative for which $F_0(a) = b$. Hence, the function $F_0(x)$ is determined uniquely and thus the integral is unique also.

We still need to show that $\int_a^v f = F(v) - F(a)$ where F is any antiderivative of f . Because we have defined $\int_a^v f$ to satisfy additivity and betweenness and have shown it to be unique, to prove property (1) we must only verify that $F(v) - F(a)$ satisfies additivity and betweenness.

To show additivity, we must check that $(F(u) - F(a)) + (F(v) - F(u)) = F(v) - F(a)$, which is obvious. Now we must verify that

$$(v - a) \min_{[a,v]} f \leq F(v) - F(a) \leq (v - a) \max_{[v,a]} f.$$

By the Mean Value Theorem,

$$\frac{F(x_i) - F(x_{i-1})}{(x_i - x_{i-1})} = f(w_i) \quad x_{i-1} \leq w_i \leq x_i$$

where our interval $[a,b]$ has been partitioned so that $a = x_1$ and $b = x_n$. Then

$$F(x_i) - F(x_{i-1}) = (x_i - x_{i-1})f(w_i)$$

$$\sum_{i=1}^n F(x_i) - F(x_{i-1}) = \sum_{i=1}^n (x_i - x_{i-1})f(w_i)$$

$$F(x_n) - F(x_1) = \sum_{i=1}^n (x_i - x_{i-1})f(w_i).$$

By the definitions of maximum and minimum,

$$\min_{[x_{i-1}, x_i]} f \leq f(w_i) \leq \max_{[x_{i-1}, x_i]} f,$$

$$(x_i - x_{i-1}) \min_{[x_{i-1}, x_i]} f \leq (x_i - x_{i-1})f(w_i) \leq (x_i - x_{i-1}) \max_{[x_{i-1}, x_i]} f,$$

$$\sum_{i=1}^n (x_i - x_{i-1}) \min_{[x_{i-1}, x_i]} f \leq \sum_{i=1}^n (x_i - x_{i-1})f(w_i) \leq \sum_{i=1}^n (x_i - x_{i-1}) \max_{[x_{i-1}, x_i]} f.$$

As the size of the interval increases, the $\min f$ must either decrease or remain the same and the $\max f$ must either increase or remain the same. Thus we can say

$$\sum_{i=1}^n (x_i - x_{i-1}) \min_{[a,v]} f \leq \sum_{i=1}^n (x_i - x_{i-1})f(w_i) \leq \sum_{i=1}^n (x_i - x_{i-1}) \max_{[a,v]} f.$$

By substituting in for the middle term and since the two outside terms are collapsing sums, we have

$$(x_n - x_1) \min_{[a,v]} f \leq F(x_n) - F(x_1) \leq (x_n - x_1) \max_{[a,v]} f$$

which is

$$(v - a) \min_{[a,v]} f \leq F(v) - F(a) \leq (v - a) \max_{[a,v]} f.$$

This verifies that betweenness is also satisfied by $F(v) - F(a)$ and, hence, we have shown that $\int_a^v f = F(v) - F(a)$.

Notice that our defined integral is the same as the classical Riemann integral. Notice also that we have proven that if f is a continuous function on $[a,b]$ and has an integral then it is a unique integral,

$$\frac{d}{dx} \int_a^x f = f(x)$$

and

$$\int_a^v f = F(v) - F(a),$$

which are the components of the Fundamental Theorem of Calculus. We have proven these components using our two basic properties of area.

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- [2] Kalmanson, Kenneth, and Patricia C. Kenshaft. *Calculus: A Practical Approach*. Worth Publishers, Inc., 1978.

The Hexagon

Edited by Iraj Kalantari

This department of *The Pentagon* is intended to be a forum in which mathematical issues of interest to undergraduate students are discussed in length. Here by "issue" we mean the most general interpretation. Examination of books, puzzles, paradoxes and special problems (all old and new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in *The Hexagon*. Address all correspondence to Iraj Kalantari, Mathematics Department, Western Illinois University, Macomb, IL 61455.

There are two engaging aspects of calculus: its applicability and its foundation. Naturally, a correct foundation for any subject justifies and sweetens the applicability of it.

When calculus was invented, the immediate speed of discovering fruitful applications for it, together with the inherent complexity of its mathematical rigor, delayed the satisfaction of finding a foundation. In the middle of the nineteenth century, Newton's approach to calculus was made rigorous by Weierstrass through the concept of "limit." About a hundred years later, Leibniz's approach to calculus was also made rigorous by Robinson through the concept of "nonstandard entities."

Metamathematicians' search for a foundation for rigorous mathematics has come to fruition through "set theory" which is capable of reflecting both of the approaches mentioned above. However, the description of nonstandard elements in ordinary set theory requires machinery of a highly complex nature and, therefore, is not

easily accessible. The following paper discusses "internal set theory," which is only a slight variant of ordinary set theory yet equipped to handle nonstandard elements with ease.

I.K.

What is Internal Set Theory?

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Introduction.

Internal set theory is yet another product of mathematicians' continuing endeavor to provide their discipline with a foundation that is not only sound and efficient, but also capable of inducing simplicity and elegance in the edifice they build upon it. It is a new framework for mathematics in which it is possible to rigorously define the ideas of "infinitely close" and "infinitely far," and use them in the development of various mathematical theories. The specialization of these concepts to the real number system yields the notions of "infinitely small" and "infinitely large" numbers, which can be used to develop calculus in the

same manner as intended by Leibniz and other mathematicians of the seventeenth and eighteenth centuries. A brief description of the origins of this subject follows.

The beginnings of the ideas of infinitely small (infinitesimal) and infinitely large numbers may be sought in the method of "exhaustion" used by Archimedes and his followers to determine such quantities as the area of a circular disc, the length of a segment of a curve, or the slope of its tangent line at some point. As applied, for example, to the calculation of the area of a circular disc (see Figure One), the method of exhaustion consists in inscribing and circumscribing regular polygons in the disc; as the number of the sides of these polygons increases, the areas of the polygons produce better and better approximations of the area of the disc. The error can be made smaller than any given amount by making the number of sides of the circumscribing and inscribing polygons sufficiently large.

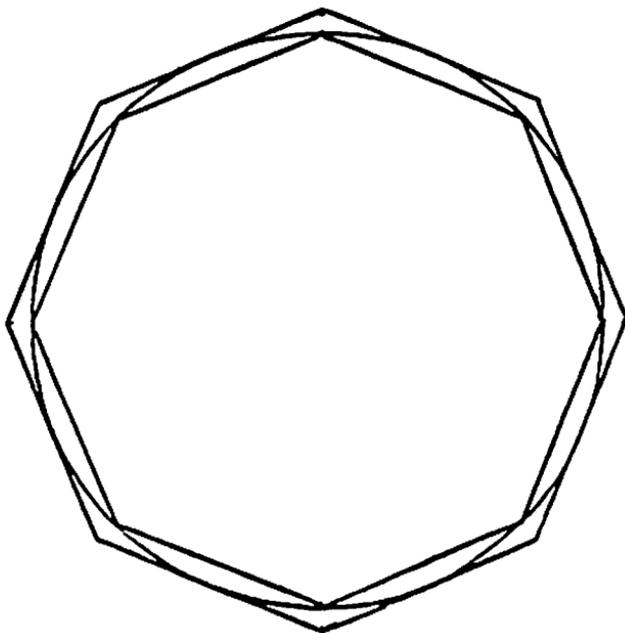


Figure One

This was the way the early Greek mathematicians argued to justify their formulas such as $A = \pi r^2$ for the area of a circular disc with radius

r. Today we derive the same formulas using the methods of the theory of differential and integral calculus, which are, by a large measure, more powerful and more efficient than the method of exhaustion. The so-called "epsilon-delta arguments" generally used today to justify the methods of calculus are very close to those used by Archimedes in the method of exhaustion. It was not, however, through this type of argument that the methods of calculus were first discovered. It was rather through thinking in terms of infinitely small and infinitely large quantities that Isaac Newton (1642-1727) and, somewhat later but independently, Gottfried Wilhelm Leibniz (1646-1716) were led to this seminal achievement of modern mathematics. These authors and their followers would calculate area by summing areas of rectangles of infinitesimal width, and define slopes as ratios of infinitely small quantities. They thought of infinitesimals as quantities less than any finite quantity, yet not zero. There is an inherent fallacy in this thought, and the philosophers of the time made sure everyone noticed it. If x is a positive number less than all positive numbers, it must, in particular, be less than itself. How can such a number exist? In 1734 Bishop George Berkeley in an essay addressed to mathematicians [1] referred to infinitesimals as "... ghosts of departed quantities." Voltaire [9] described calculus as "... the art of numbering and measuring exactly a thing whose existence cannot be conceived" Leibniz was aware of the foundational problems associated with these concepts as early as the 1670's when he formulated the rules of calculus. Nevertheless, he viewed them as ideal numbers, rather like imaginary numbers, with the same properties as ordinary numbers, whose existence was only of fictitious form with the purpose of bringing technical ease to arguments and calculations. Furthermore, he believed that all arguments and calculations involving infinitely small and infinitely large numbers could be replaced by the method of exhaustion if we ever desired to do so.

Indeed, abstract mathematical concepts of all sorts are our mental tools for analyzing problems and discovering the properties of our physical world. The ideas of infinitely small and infinitely large numbers are appealing to our physical intuition and their use does have a significant effect in reducing the logical complexity of arguments. Because of this, they became indispensable tools of mathematicians of the seventeenth and eighteenth centuries enabling their users to make, at an astonishingly rapid pace, great mathematical discoveries important for physical applications.

But mathematicians have never been content with just the practical success of their discipline. Much of the development of mathematics has been the result of efforts to bring soundness to its foundation and

elegance to the rest of its structure. Unfortunately, neither Leibniz nor his followers were ever able to precisely formulate a consistent set of rules governing infinitesimals which was sufficient to justify Leibniz's claims mentioned above. It took mathematicians about two centuries to bring rigor to the foundation of calculus. But it was done at the price of total rejection of infinitesimals as unsound and their replacement by the "epsilon-delta method" of Weierstrass. As a result, the use of infinitesimals waned and until 1960 there was little success in providing a consistent framework for the development of differential and integral calculus by means of infinitely small and infinitely large numbers. In 1948 Edwin Hewitt [2] constructed an extension of the real number system called the "hyper real field" which contained infinitely small and infinitely large numbers. But he did not attempt to apply this theory to the development of calculus. Another such enlargement of the real number system was given in 1958 by C. Schmieden and D. Laugwitz. But as far as a complete solution to Leibniz's problem is concerned, a breakthrough did not come until Abraham Robinson's 1960 discovery of what is called "nonstandard analysis." Using the methods of modern mathematical logic, Robinson was able to completely realize Leibniz's dream of developing the entire theory of differentiation and integration by means of infinitely small and infinitely large numbers. Robinson first reported his discovery in a seminar talk at Princeton University (November 1960) and later in a paper published in the proceedings of the Royal Academy of Sciences of Amsterdam [6]. Soon after these events, Robinson and his followers showed that nonstandard analysis is more than a solution to Leibniz's problem. It is a powerful new tool of mathematical research which has enabled its practitioners, over the past thirty years, to find solutions to unsolved problems in many areas, including functional analysis, probability theory, mathematical physics and mathematical economics.

The basic framework of nonstandard analysis as presented, for example, in [3], [7], or [8], involves the application of mathematical logic at quite a sophisticated level; and for this reason, these expositions are rather cumbersome. This has spurred attempts to introduce simpler approaches to nonstandard analysis. Among these, Edward Nelson's *internal set theory* (or "IST") [4] clearly stands out as the simplest. IST is an axiomatization of a large fragment of Robinson's nonstandard analysis, and a brief exposition of its basic framework is the purpose of the present article.

Internal Set Theory (IST).

All of the objects of conventional (ordinary) mathematics such as the numbers 2 and π , the set of all real numbers \mathbb{R} , the operation of addition $+$ on \mathbb{R} , the order relation $<$ on \mathbb{R} , and the like, can be defined by means of the notions and axioms of ordinary set theory (specifically, Zermelo-Fraenkel Set Theory with the axiom of choice). IST is an extension of ordinary set theory. This means that IST contains all of the notions and axioms of conventional mathematics plus some additional ones, which are not conventional. All of the unconventional notions of IST are defined in terms of a single undefined notion called *standard*. In addition to the axioms of ordinary set theory, IST has three new axioms which are called the *transfer principle*, the *idealization principle* and the *standardization principle*.

To get a picture of the form of the statements of IST, let us look at the following two examples:

$$(1) \quad (\exists y \in \mathbb{R})(\forall x \in \mathbb{R})[x < y]$$

and

$$(2) \quad (\exists y \in \mathbb{R})(\forall x \in \mathbb{R})[\text{If } x \text{ is standard then } x < y].$$

Statement (1) does not involve the new term "standard." It is a statement belonging to conventional mathematics. In the context of IST, such statements are called *internal formulas*. These formulas are governed by the axioms of ordinary set theory. Statements that do involve the term "standard" are called *external formulas*. Statement (2) is an example of an external formula. Statements of this type do not belong to conventional mathematics and the axioms of conventional mathematics do not apply to them. Failure to be aware of this fact can lead to inconsistencies. Here is a potential pitfall in applying the subset axiom. This is an axiom of ordinary set theory asserting that

If \mathcal{A} is a set and Ψ is a formula, then there is a set \mathcal{B} , denoted $\{x \in \mathcal{A} | \Psi(x)\}$, such that, for all x , x belongs to \mathcal{B} if and only if $x \in \mathcal{A}$ and $\Psi(x)$.

Since, in IST, the subset axiom can only be applied to internal formulas, you cannot apply this axiom, for example, to the formula "x is standard" and claim that there is a set \mathcal{B} , denoted $\{x \in \mathbb{R} | x \text{ is standard}\}$. There is no set in IST consisting of all of the standard members of \mathbb{R} .

We next use the term “standard” to introduce some new notions.

DEFINITIONS. Let x be a member of \mathbb{R} .

(1) We say that x is *infinitesimal* (“infinitely small”) if $|x| < \epsilon$ for all standard $\epsilon > 0$. We write $x \simeq 0$ if x is infinitesimal.

(2) We say that x is *limited* if $|x| < r$ for some standard $r > 0$; x is called *unlimited* (“infinitely large”) if it is not limited. We write $x \simeq \infty$ if x is unlimited.

(3) Two numbers x and y in \mathbb{R} are called *infinitely close*, written $x \simeq y$ if their difference $x - y$ is infinitesimal.

The set \mathbb{R} , as viewed through IST, has the “picture” shown in below.

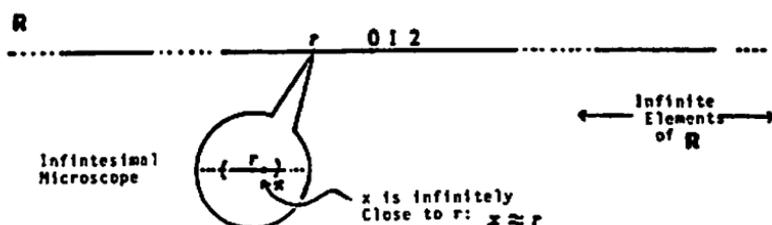


Figure Two

Axioms of IST.

We have used the term “standard” to state precisely what infinitely small and infinitely large numbers are. But we do not yet know what properties these numbers have or even whether they exist or not. Recall that the term “standard” is an undefined notion. This means that we cannot answer any questions involving this term unless we make some assumptions about it. Our first assumption is called the “transfer principle.” This principle guarantees that the new numbers will have the same properties as the ordinary numbers. To state this axiom precisely, let’s recall that a variable x in a formula is called a *bound variable* if it is quantified by either \forall or \exists ; otherwise, it’s called a *free variable*. For example, in the formula $(\forall \epsilon \in \mathbb{R})[x \simeq y]$, x is a bound variable while y is a free variable. In what follows, the notation \forall^{st} means “for all standard” and \exists^{st} means “there exists a standard.”

The Transfer Principle. Let ϕ be an internal formulas whose only free variables are x, y_1, \dots, y_n ($n \geq 0$). Then

$$(T) \quad (\forall^{st}y_1) \cdots (\forall^{st}y_n)[(\forall^{st}x)\phi \leftrightarrow (\forall x)\phi].$$

In words, this principle asserts that if all of the parameters y_1, \dots, y_n in ϕ are standard, then ϕ holds for all x if and only if it holds for all standard x . The dual form of this principle is

$$(T') \quad (\forall^{st}y_1) \cdots (\forall^{st}y_n)[(\exists^{st}x)\phi \leftrightarrow (\exists x)\phi].$$

An important implication of this principle is that all of the objects of ordinary mathematics such as the numbers 2 and π , the set of all natural numbers \mathbb{N} , the functions \sin, \cos, \log and the like are standard. Take \mathbb{R} , for example, and suppose that ψ is the formula that uniquely describes it within ordinary mathematics. This means that (a) ψ is an internal formula about \mathbb{R} , (b) if we replace \mathbb{R} by x , the only free variable of ψ will be x , and (c) the formula $(\exists!x)\psi$ is a theorem of IST. (The notation $\exists!$ means "there exists a unique.") Thus by (T') , we have $(\exists^{st}x)\psi$. By uniqueness, it follows that \mathbb{R} is standard. Thus every specific object of ordinary mathematics is standard. The natural question now is whether there are any nonstandard sets in IST. The answer to this question is given by means of the "idealization principle" which is our second axiom about the term "standard."

The Idealization Principle. The notation \forall^{stfin} means "for all standard finite sets" and \exists^{stfin} means "there exists a standard finite set such that." Let ϕ be an internal formula. Then

$$(I) \quad (\forall^{stfin}z)(\exists y)(\forall x \in z)\phi \leftrightarrow (\exists y)(\forall^{st}x)\phi.$$

The dual form of this principle is

$$(I') \quad (\exists^{stfin}z)(\forall y)(\exists x \in z)\phi \leftrightarrow (\forall y)(\exists^{st}x)\phi.$$

As an application of the Idealization Principle, we prove the following important

THEOREM. Let $(\mathcal{A}_x)_{x \in \mathbb{N}}$ be a sequence of sets. If for each standard n in \mathbb{N} there is y belonging to \mathcal{A}_k for all $k = 1, \dots, n$ then there exists y belonging to \mathcal{A}_x for all standard x in \mathbb{N} .

Proof. Substitute the formula $y \in \mathcal{A}_x$ for ϕ in the Idealization Principle (I) with x ranging over \mathbb{N} , y ranging over the set $\bigcup_{x \in \mathbb{N}} \mathcal{A}_x$ and z ranging over the subsets of \mathbb{N} . Then the right-hand side of (I) becomes

$$(\exists y)(\forall^{st}x)[y \in \mathcal{A}_x],$$

which is exactly the conclusion of our theorem. Hence we need only show that under the hypothesis of our theorem, the left-hand side of (I) holds. That is,

$$(\forall^{st/in}z)(\exists y)(\forall x \in z)[y \in \mathcal{A}_x].$$

Let z be any standard finite subset of \mathbb{N} and let n be the largest number in z . Then n is a standard member of \mathbb{N} , and so, by hypothesis, there is y belonging to \mathcal{A}_k for all $k = 1, \dots, n$. Since $z \subseteq \{1, \dots, n\}$, it follows that y belongs to \mathcal{A}_x for all x in z . \square

COROLLARY. The set \mathbb{R} , viewed through IST, contains both infinitely large and infinitely small elements.

Proof. For infinitely large elements, we need only show that there is an element ω in \mathbb{N} larger than every standard element of \mathbb{N} . Consider the sequence of sets $(\mathcal{A}_x)_{x \in \mathbb{N}}$ where \mathcal{A}_x is the set of all n in \mathbb{N} larger than x . This sequence satisfies the hypothesis of the previous theorem. By the conclusion of the theorem, there is an element ω in \mathbb{N} belonging to \mathcal{A}_x for all standard x . Thus ω is larger than all standard natural numbers. By the Transfer Principle (T) and the Archimedean Property of the reals, each standard real number is smaller than some standard natural number. Hence ω is larger than all standard real numbers. The number $1/\omega$ is infinitely small. \square

The Standardization Principle. This is our last axiom concerning the term "standard." Let ϕ be any formula (external or internal) not containing the variable y . Then the Standardization Principle states that

$$(S) \quad (\forall^{st}x)(\exists^{st}z)(\forall^{st}z)[z \in y \leftrightarrow z \in x \ \& \ \phi].$$

By the Transfer Principle (T), two standard sets are equal if they have the same standard elements. Therefore the standard set given by (S) is

unique. We denote this set by ${}^s\{z \in x | \phi\}$. Note that this is a standard subset of x and that only its standard members satisfy the formula ϕ . Any property of its nonstandard elements (which may or may not exist) must be derived from what we know about its standard elements. The set ${}^s\{z \in x | \phi\}$ may have a nonstandard member u satisfying the formula ϕ , but we cannot infer from this that u belongs to ${}^s\{z | \phi\}$. For example, let $\mathcal{A} = {}^s\{x | x \simeq 0\}$. Then $\mathcal{A} = \{0\}$ since 0 is the only standard number that is infinitely close to zero. Although \mathbb{R} has other infinitesimal numbers, they do not belong to ${}^s\{x | x \simeq 0\}$.

Thus, by adding the new term "standard" and three new axioms to ordinary mathematics, we have obtained a new framework in which it is possible to formulate and prove, in terms of infinitely small and infinitely large numbers, statements about sequences of numbers, real-valued functions of a real variable, subsets of \mathbb{R} and other mathematical objects that occur in calculus. To see some examples, let $(x_n)_{n \in \mathbb{N}}$ be a standard sequence of real numbers, f a standard real-valued function on \mathbb{R} and p a standard real number. Then

- (a) The sequence (x_n) converges to p if and only if for all infinitely large n in \mathbb{N} , x_n is infinitely close to p .
- (b) The sequence (x_n) is Cauchy if and only if $x_n \simeq x_m$ whenever m and n are infinitely large.
- (c) The function f is continuous at p if and only if $f(x) \simeq f(p)$ whenever $x \simeq p$.
- (d) The function f is uniformly continuous on \mathbb{R} if and only if $f(x) \simeq f(y)$ whenever $x \simeq y$.
- (e) The function f is differentiable at p if and only if there is a standard real number to which the quotient $(f(x) - f(p))/(x - p)$ is infinitely close whenever $x \simeq p$.
- (f) A standard subset \mathcal{K} of \mathbb{R} is compact if and only if for all $x \in \mathcal{K}$ there is a standard q in \mathcal{K} such that $x \simeq q$.
- (g) A standard subset \mathcal{G} of \mathbb{R} is open if and only if for all standard $p \in \mathcal{G}$ and all x in \mathbb{R} , if $x \simeq p$ then $x \in \mathcal{G}$.
- (h) A standard subset \mathcal{F} of \mathbb{R} is closed if and only if for all $x \in \mathcal{F}$ and all standard p in \mathbb{R} , if $x \simeq p$ then $p \in \mathcal{F}$.

For proofs of these assertions, we refer the reader to Nelson's original paper [4]. For a more detailed treatment of elementary analysis within IST, see [5].

References.

- [1] G. Berkeley (1734), "The Analyst," in *Collected Works*, Vol. 4, edited by A. A. Luce and T. E. Jessop, London, 1951.
- [2] E. Hewitt (1948), "Rings of real-valued continuous functions, I," *Transactions of the American Mathematical Society* 64, 54-99.
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- [4] E. Nelson (1977), "Internal set theory," *Bulletin of the American Mathematical Society* 83 (No. 6), 1165-1198.
- [5] A. Robert (1988), *Nonstandard Analysis*. New York: Wiley.
- [6] A. Robinson (1961), "Nonstandard analysis," *Proceedings of the Royal Academy of Sciences (Amsterdam)* (A) 64, 432-440.
- [7] A. Robinson (1966), *Nonstandard Analysis*. Amsterdam: North-Holland.
- [8] K. D. Stroyan and W. A. J. Luxemburg (1976), *Introduction to the Theory of Infinitesimals*. New York: Academic Press.
- [9] F. M. A. Voltaire (1733), *Letters Concerning the English Nation*. London.

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 January 1991. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 1991 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROBLEMS 430-434.

Problem 430. Proposed by the editor. John and his brother Bill have ages which when added together produce a perfect cube. Furthermore when John was half as old as Bill is now, Bill's age equalled the square of John's age when Bill was born. Find their current ages.

Problem 431. Proposed by the editor. In a high school math contest, the answer sheet stated that the equation

$$63x^2 - 11x - 4 = 11x^2 - 3x - 4$$

had only the solution $x = 4$. Prove or disprove the truth of this statement when x is a real number.

Problem 432. Proposed by the editor. Let $G(x)$ be a function over the real numbers such that

$$(x^2 + 3) \cdot G(x) - x^2 \cdot G(2-x) = 4x^3 - x^2 - 6.$$

Determine $G(x)$.

Problem 433. Proposed by the editor. Consider the family of hyperbolas

$$\frac{x^2}{a_i^2} - \frac{y^2}{b_i^2} = 1$$

where a_i and b_i satisfy the relation

$$\frac{1}{a_i^2} - \frac{1}{b_i^2} = 5$$

for $i = 1, 2, \dots, n$. Find all points which the hyperbolas have in common or prove that none exist.

Problem 434. Proposed by the editor. Let r be a positive rational number. Prove that $(8r + 21)/(3r + 8)$ is a better approximation to $\sqrt{7}$ than r is.

Please help your editor by submitting problem proposals.

SOLUTIONS 420-421 and 423-424.
(Problem 422 remains open.)

Problem 420. Proposed by the editor. Berwick once proposed a classic arithmetic restoration problem in which only seven 7's appear. For our version find the smallest cube of a positive integer which ends in 777777; i.e. seven 7's.

Solution by Drake University Problem Solving Group, Drake University, Des Moines, Iowa.

Let $K = a + b(10) + c(10^2) + d(10^3) + e(10^4) + f(10^5) + g(10^6)$ where a, b, c, d, e, f and g are digits to be determined. Then

$$\begin{aligned}
K^3 = & a^3 + (3a^2b)(10) + (3a^2c + 3b^2a)(10^2) \\
& + (3a^2d + 6abc + b^3)(10^3) \\
& + (3a^2e + 6bda + 3c^2a + 3b^2d)(10^4) \\
& + (6acd + 6abe + 3a^2f + 3c^2b + b^2d)(10^5) \\
& + (3a^2g + 6abf + 6ace + 6bcd + 3d^2a + 3b^2e + c^3)(10^6) \\
& + (6abg + 6acf + 6ade + 6bce + 3b^2f + 3d^2b + 3c^2d)(10^7) \\
& + (6acg + 6adf + 6bcf + 6bde + 3e^2a \\
& \quad + 3b^2g + 3d^2c + 3c^2e)(10^8) \\
& + (6adg + 6afe + 6bcg + 6bdf + 6cde \\
& \quad + 3e^2b + 3c^2f + d^3)(10^9) \\
& + (6age + 6bgd + 6bef + 6dcf + 3f^2a \\
& \quad + 3c^2g + 3e^2c + 3d^2e)(10^{10}) \\
& + (6afg + 6beg + 3f^2b + 6cdg + 6ecf + 3d^2f + 3e^2d)(10^{11}) \\
& + (3g^2a + 3f^2c + 3d^2g + 6def + 6ceg + 6bgf + e^3)(10^{12}) \\
& + (3g^2b + 6cfg + 6deg + 3f^2d + 3e^2f)(10^{13}) \\
& + (3g^2c + 6dgf + 3e^2g + 3f^2e)(10^{14}) \\
& + (3g^2d + 6feg + f^3)(10^{15}) + (3g^2e + 3f^2g)(10^{16}) \\
& + (3g^2f)(10^{17}) + (g^3)(10^{18}).
\end{aligned}$$

This expression can be solved systematically for a, b, c, d, e, f and g . Since K^3 must end in 7, we must have $a = 3$. Thus, taking into consideration the 2 which would be carried over to this position from computing a^3 , the coefficient of 10 would be $27b + 2$ which must be congruent to 7 modulo 10. By [1], since 27 and 10 are relatively prime, the congruence $27b + 2 \equiv 7 \pmod{10}$ has only one solution, namely $b = 5$. Proceeding in a similar manner, we find $c = 7, d = 0, e = f = 6$ and $g = 9$. Hence $K = 9660753$ and $K^3 = 901639512372747777777$.

Also solved by Charles Ashbacher, St. Marys College, Hiawatha, Iowa, and Tom Hok-Mo Cheung, Eastern Kentucky University, Richmond, Kentucky.

[1] M. J. Weiss and R. Dubisch, *Higher Algebra for the Undergraduate* (2nd. Edition). Wiley (1962), p. 20 (Theorem 17).

Editor's Comment. The featured solution is equivalent to defining the two sequences

$$x_{k+1} = 10^k t_k + x_k \text{ and } x_k^3 \equiv 7_k \pmod{10^k}$$

for integral $k > 0$ where 7_k denotes the number composed of k 7's. Then

$$x_{k+1}^3 \equiv (10^k t_k + x_k)^3 \equiv 3 \cdot 10^k t_k x_k^2 + x_k^3 \equiv 7_{k+1} \pmod{10^k}.$$

Solving for t_k , we have

$$t_k \equiv \frac{7_{k+1} - x_k^3}{3 \cdot 10^k \cdot x_k^2} \pmod{10}$$

which reduces to

$$t_k \equiv \frac{3(7_{k+1} - x_k^3)}{3 \cdot 10^k} \pmod{10}$$

since $9 \cdot x_k^2 \equiv 1 \pmod{10}$ for $k > 0$ and $x_1 = 3$. This approach is equivalent to our featured solution and eases the computations for hand held calculators.

Problem 421. Proposed by the editor. While working on her homework, a student noticed the following peculiar relationship between two sets of consecutive squares:

$$\frac{600^2 + 601^2 + 602^2 + 603^2 + 604^2}{144^2 + 145^2 + 146^2 + 147^2 + 148^2} = 17.$$

She wants to know if this relationship is unique or does 17 have other such representations?

Solution by Mark Young, Drake University, Des Moines, Iowa.

This representation of 17 is not unique. Other representations of 17 are listed as ordered pairs (x,y) that satisfy the equation

$$(1) \quad \frac{(x-2)^2 + (x-1)^2 + x^2 + (x+1)^2 + (x+2)^2}{(y-2)^2 + (y-1)^2 + y^2 + (y+1)^2 + (y+2)^2} = 17.$$

With a little simplification, equation (1) reduces to

$$(2) \quad x^2 = 17y^2 + 32$$

Other solutions are $(x,y) = (7,1), (10,2), (58,14), (95,23), (367,89),$

$(602,146),$
 $(3818,926),$
 $(6263,1518),$
 $(24215,5873),$
 $(39722,9634),$
 $(251930,61102),$
 $(413263,100231),$
 $(1597823,387529),$
 $(2621050,635698),$
 $(16623562,4031806),$
 $(27269095,6613727),$
 $(105432103,25571041),$
 $(172949578,41946434),$
 $(1096903162,266038094),$
 $(1799347007,436405751),$
 $(6956920975,1687301177),$
 $(11412051098,2767828946),$
 $(72378985130,17554482398),$
 $(118729633367,28796165389),$
 $(459051352247,111336306641)$ and
 $(753022422890,182634764002).$

By symmetry, any solution (x,y) can be replaced by $(-x,-y)$ because the equation being modeled is parabolic.

Also solved by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin and Gregory Hunt, University of Montevallo, Montevallo, Alabama.

Editor's Comment. Our featured solver suggests that our problem has an infinite number of solutions. This is indeed correct. Equation (2) in the featured solution has an infinite number of solutions. It is well known [1] that if x_1, y_1 is a solution of equation (2), and if X_k, Y_k is a solution of $X^2 - 17Y^2 = 1$, then

$$x = x_1X_k + 17y_1Y_k \text{ and } y = x_1Y_k + y_1X_k$$

is also a solution of equation (2). Here

$$X_k + Y_k\sqrt{17} = (X_0 + Y_0\sqrt{17})^k$$

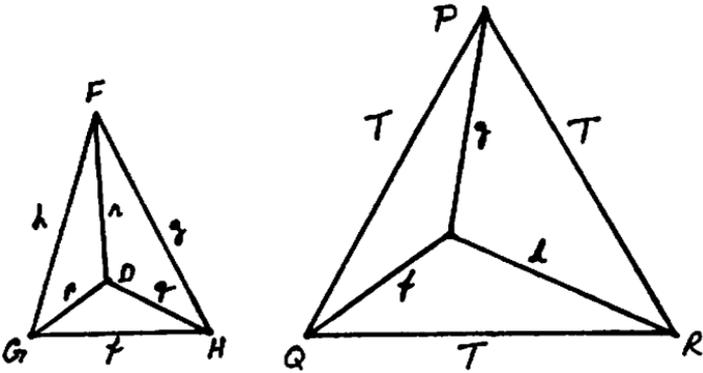
where X_0, Y_0 is the fundamental solution of the equation $X^2 - 17Y^2 = 1$. See [2]. In our problem, there are four infinite families of solutions derived from $(x_1, y_1) = (7,1), (10,2), (58,14)$ and $(95,23)$ respectively.

Other integers also have this property. For example, using the method of our featured solution, $(x,y) = (146,44)$ produces a representation of 11. Similarly $(x,y) = (58,41)$ produces a representation of 2; $(x,y) = (316,129)$ produces a representation of 6; $(x,y) = (52,30)$ produces a representation of 3; etc. Additional representations in these cases require integer solutions of $X^2 - 11y^2 = 1$, $X^2 - 2y^2 = 1$ and $X^2 - 6Y^2 = 1$, respectively.

[1] Adams and Goldstein, *Introduction to Number Theory*. Prentice Hall Inc., Englewood Cliffs, New Jersey (1976), p. 182.

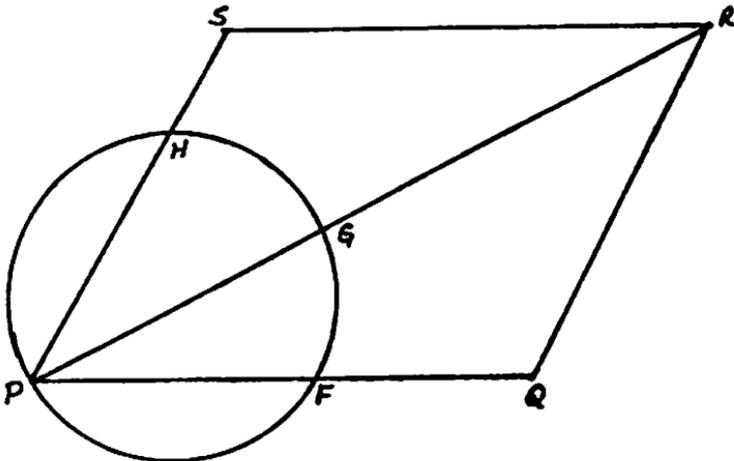
[2] Underwood Dudley, *Elementary Number Theory*. W. H. Freeman and Co., San Francisco (1969), p. 153.

Problem 422. Proposed by the editor. Consider the two triangles $\triangle FGH$ and $\triangle PQR$ shown below with $\angle FDH = \angle FDG = \angle GDH = 120^\circ$. Let the line segments be denoted as marked. Prove that $T = p+q+r$. (Third USA Mathematical Olympiad 1974.)



Since no correct solutions have been received, this problem will remain open. The problem's stated result holds regardless of whether or not $\triangle FGH$ is equilateral.

Problem 423. Proposed by the editor. Let a circle cut two adjacent sides and a diagonal of parallelogram PQRS at points F, G and H as shown below. Prove that $\overline{PF} \cdot \overline{PQ} + \overline{PH} \cdot \overline{PS} = \overline{PG} \cdot \overline{PR}$.



Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Connect points F, G and H to form $\triangle FGH$. Let $m(\angle ABC)$ denote the measure of angle ABC. Since $m(\angle GFH) = m(\angle GPH) = m(\angle RPS) = m(\angle PRQ)$ and $m(\angle FHG) = m(\angle FPG) = m(\angle QPR)$, $\triangle FGH$ is similar to $\triangle RPQ$. Hence

$$\frac{\overline{GH}}{\overline{PQ}} = \frac{\overline{FG}}{\overline{QR}} = \frac{\overline{FH}}{\overline{PR}} = k$$

so that

$$(1) \quad \begin{cases} \overline{GH} = k\overline{PQ} \\ \overline{FG} = k\overline{QR} = k\overline{PS} \\ \overline{FH} = k\overline{PR} \end{cases}$$

Next, "Ptolemy's Theorem" states that "if a quadrilateral is inscribed in a circle, the sum of the products of the lengths of the two pairs of opposite sides is equal to the product of the lengths of the diagonals." Thus $\overline{PF} \cdot \overline{GH} + \overline{FG} \cdot \overline{PH} = \overline{PG} \cdot \overline{FH}$ and it now follows from (1) that $\overline{PF} \cdot \overline{PQ} + \overline{PH} \cdot \overline{PS} = \overline{PG} \cdot \overline{PR}$.

Problem 424. Proposed by the editor. Completely factor the number $2^{34} + 1$ using only a pencil and paper. (No computers please!) This number has four distinct prime factors.

Solution by Bob Prielipp, University of Wisconsin Oshkosh, Oshkosh, Wisconsin.

By adding and subtracting 2^{18} , we obtain $2^{34} + 1 = (2^{17} + 1)^2 - (2^9)^2 = (2^{17} + 2^9 + 1)(2^{17} - 2^9 + 1) = 5 \cdot 26317 \cdot 130561$. Next we use the following theorem of Legendre: "every prime divisor of $a^n + 1$ is either of the form $2nk + 1$ for some integer k or divides $a^w + 1$ where w is the quotient of n by an odd factor" (see [1]). Thus every prime divisor of $2^{34} + 1$ other than $2^2 + 1 = 5$ must be of the form $68k + 1$ where k is an integer. The smallest prime of this type is 137 and $130561 = 137 \cdot 953$. It is given that $2^{34} + 1$ has four distinct prime factors. Hence the complete factorization of $2^{34} + 1$ is $5 \cdot 137 \cdot 953 \cdot 26317$.

[1] Leonard E. Dickson, *History of the Theory of Numbers*. Chelsea

Publishing Company, New York (reprinted 1966), Vol. I, p. 382.

Editor's Comment. In view of the use of Legendre's Theorem concerning the form of the prime divisors of $2^{34} + 1$, the primality of 26317 follows immediately from the fact that $26317 < 163^2$ and the fact that the first two primes of the form $68k + 1$ are 137 and 409. Hence the statement concerning the number of prime divisors of $2^{34} + 1$ is unnecessary.

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Kappa Mu Epsilon News

Edited by Mary S. Elick, Historian

News of chapter activities and other noteworthy *KME* events should be sent to Mary S. Elick, Historian, Kappa Mu Epsilon, Mathematics Department, Missouri Southern State College, Joplin, MO 64801.

CHAPTER NEWS

Alabama Gamma University of Montevallo, Montevallo
Chapter President - Lisa Watlington
6 actives, 6 initiates

The fall semester began with a general meeting for all chapter members. Three additional organizational meetings were held by the officers and corresponding secretary. A special lecture entitled "Being Critical About Thinking" was given by Dr. Don Alexander from the University's own mathematics department. The fall initiation ceremony was followed by a reception. Other 1989-90 officers: Lori Sims, vice president; Julie Higgins, secretary; Lisa Land, treasurer; Gene G. Garza, corresponding secretary; Angela Hernandez, faculty sponsor.

Arkansas Alpha Arkansas State University, State University
Chapter President - Teresa Culbertson
14 actives, 6 initiates

Arkansas Alpha heard various faculty members speak at regular monthly meetings. The organization sponsored a Halloween party for faculty and members and also a luncheon and Christmas party for the faculty. Other 1989-90 officers: Johnny Moore, vice president; James Chastain, secretary/treasurer; Jerry L. Linnstaedter, corresponding secretary; Roger Abernathy, faculty sponsor.

California Gamma California Polytechnic State University, San Luis Obispo
Chapter President - Theresa Bly
40 actives, 9 pledges

During the Fall Quarter California Gamma was active both socially and officially. With two exceptions, either the officers or the club as a whole met each Monday evening at 8:00 p.m. At one meeting Anne Patton and Athan Spiros reported on their activities while on co-op during the spring and summer of 1989. An effort was made to involve more faculty in *KME* activities. Dr. H. Arthur DeKleine addressed the club (Oct. 30, 1989) on certain famous problems in game theory and Dr. Estelle Basor spoke (Nov. 6, 1989) on solved and unsolved problems in operator theory with special emphasis on Toeplitz operators, her own area of specialization for many years. On October 2, 1989, nine potential pledges were introduced to the club at its fall ice cream social. These nine were recognized as pledges at the Pledge Induction Ceremony held on November 20, 1989, at Marie Callendar's Restaurant in scenic Pismo Beach, CA. The guest speaker was Kelly Abott, former Treasurer of California Gamma (1987-1988), who addressed the group on topics in applied/finite mathematics with which he has been involved during his employment at Daniel H. Wagner & Associates, a consulting firm in Sunnyvale, California. Fair Isaac hosted a dinner which was attended by *KME* members interested in working in the area of customer rating. Chevron U.S.A., Inc. (San Rafael, CA) hosted a dinner at Sinfully Delicious (Shell Beach, CA) attended by faculty advisors of on-campus honor societies. California Gamma delivered a Christmas tree to the Village (a retirement home in San Luis Obispo, CA), decorated it and put on a one-act play based on the story of the "Night Before Christmas." Dr. H. Arthur DeKleine portrayed Santa Claus. Other 1989-90 officers: Rachel Jeffries and Chris Lucke, co-vice presidents; Nicholas Braitto, secretary; Anne Patton, treasurer; Raymond D. Terry, corresponding secretary/faculty sponsor; Donald Priest and John Yip, pledgemasters; Beth Patton, social chairperson; Donald Priest and Chris Lucke, representatives to the Poly Royal Board; Darrell Dalke, publicity chairperson; Kathy Perino, representative to the school council; Athan Spiros, representative to the Mathematics Department Curriculum Committee.

Colorado Gamma

Fort Lewis College, Durango

Chapter President - Larry Hansen

27 actives

The fall meetings featured video programs on fractals and "The Search for Solutions" series. Chapter members participated in the college Alumni Phone-a-Thon and raised about \$100 for the chapter. Other officers: Terry Sherfey, vice president; David Beazley, secretary; Kevin Marushack, treasurer; Richard A. Gibbs, corresponding secretary/faculty

sponsor.

Georgia Alpha

West Georgia College, Carrollton

Chapter President - Vicki Shackelford

25 actives

For the second consecutive year, Georgia Alpha sponsored a Canned Food Drive for the needy, with the food collected being given to the Community Food Bank. On November 15, 1989, the organization enjoyed a social held at a local steak house. Other 1989-90 officers: Luqman Thayyil, vice president; Stephanie Edge, secretary; Doug Teate, treasurer; Thomas J. Sharp, corresponding secretary/faculty sponsor.

Illinois Beta

Eastern Illinois University, Charleston

Chapter President - Jacqi Sheehan

45 actives

The program for the first meeting of the fall semester was a panel discussion presented by an actuarial intern, a student teacher, and graduate students. Other programs included a talk by Shirley Stuart from the Placement Office and a presentation on problem solving skills. The chapter also enjoyed a fall picnic and a Christmas party. Other 1989-90 officers: Cecile Knizner, vice president; Tammie Traub, secretary; Jason Smith, treasurer; Lloyd Koontz, corresponding secretary; Allen Davis, faculty sponsor.

Illinois Delta

College of St. Francis, Joliet

Chapter President - Dave Laketa

Illinois Delta participated in the college Homecoming Fest and organized a taffy apple sale as fund raisers. The profit realized was \$250 and will be used for chapter projects during the spring semester. Other 1989-90 officers: Debra Becker, vice president; Michelle Safiran, secretary; Donna Gundergahn, treasurer; Sister Virginia McGee, corresponding secretary/faculty sponsor.

Iowa Alpha

University of Northern Iowa, Cedar Falls

Chapter President - Lori Stenberg

48 actives

The *KME* Homecoming Coffee was held October 21, 1989, at the home of Professor Emeritus and Mrs. E. W. Hamilton. The event was well attended by students, alumni, and faculty. Iowa Alpha hosted a pizza supper in honor of Past *KME* National President, Fred W. Lott. Dr. Lott was the recipient of the George Mach Award at the 1989

National Convention. Two students presented papers at local *KME* meetings: Melana Clark spoke on "Magic Squares" and Michael Hirsch addressed the December initiation banquet on "Generalized Matrix Inverses." Other 1989-90 officers: Lynn Cairney, vice president; Jody Barrick and Mark Bohan, secretaries; Bill Pothoff, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

Iowa Delta

Wartburg College, Waverly

Chapter President - Kaaren Hemmingson

32 actives

At the first meeting of the year, two Iowa Delta Chapter members presented information concerning their summer internships. The program for the October meeting was the film, "Stand and Deliver," the story of Jaime Escalanta, the AP calculus teacher. In November, David Neve, actuary and head of the Financial Management Department at Principal Financial Group in Des Moines and also a former officer of the Iowa Delta Chapter, spoke about actuarial careers and his experiences in the field. The traditional Christmas supper and party was held in December. Other 1989-90 officers: Diane Waltmann, vice president; Kent Hicok, secretary; Susan Olson, treasurer; August Waltmann, corresponding secretary; Glenn Fenneman, faculty sponsor.

Kansas Alpha

Pittsburg State University, Pittsburg

Chapter President - Mala Renganathan

35 actives, 6 initiates

The chapter held monthly meetings in October, November and December. Fall initiation was held at the October meeting. Six new members were initiated at that time. This meeting was preceded with a pizza party. The October program was presented by Matt Mayfield, instructor in the PSU Mathematics Department. He discussed and demonstrated "Hyper-card Software." Dr. Joel Haack, guest lecturer from Oklahoma State University Mathematics Department gave the November program. His presentation was "Mathematical Aspects of Escher Prints." In December, a special Christmas meeting was held at the home of Dr. Harold Thomas, *KME* National President and PSU's Corresponding Secretary. Lora Woodward presented a printing technology video for the program. Other 1989-90 officers: Lora Woodward and Mike Wille, co-vice presidents; Tamala Nation, secretary; Lori Oneal, treasurer; Harold L. Thomas, corresponding secretary; Gary McGrath, faculty sponsor.

Kansas Gamma

Benedictine College, Atchison

Chapter President - Matthew McIntosh

5 actives, 8 initiates

Soon after the beginning of the fall semester, Kansas Gamma held a "come-make and eat-pizza party" at the home of faculty member Richard Farrell. The Sunday afternoon event proved to be a fun way to welcome new students to the group as well as to welcome new faculty member Sister Linda Herndon. This year the traditional Christmas Wassail Party was hosted by senior Jeanne Chaloupka at her residence near the college. Other 1989-90 officers: Julie Stenger, vice president; Sister Jo Ann Fellin, corresponding secretary/faculty sponsor.

Kansas Delta

Washburn University, Topeka

Chapter President - Mary Jane Wilson

15 actives

Kansas Delta only had two active student members during the fall semester and so had no organized activities. Student initiation will be held in March. Other 1989-90 officers: G. Michael Poliquin, vice president; A. Allen Riveland, corresponding secretary; Ronald L. Wasserstein, faculty sponsor.

Kansas Epsilon

Fort Hays State University, Hays

Chapter President - Sharon Richards

17 actives

Kansas Epsilon fall activities included monthly meetings and a Halloween party. Other 1989-90 officers: Janet Ryan, vice president; Jodi Miller, secretary/treasurer; Charles Votaw, corresponding secretary; Mary Kay Schippers, faculty sponsor.

Kentucky Alpha

Eastern Kentucky University, Richmond

Chapter President - Kathy Ponder

23 actives

Fall semester started off with a faculty/*KME* picnic in September at Dr. Costello's house. Before the Homecoming game, *KME* hosted an alumni reunion in the Wallace building. Also in October, the students squeaked out a victory over the faculty in softball. Thirteen students took the Virginia Tech Math Exam and four students took the Putnam. The semester ended with a faculty/*KME* Christmas party. There were two talks presented to the chapter during the semester. Elisa Heinricher gave a talk on her experiences in the defense industry entitled "Do You Want to Play Thermonuclear War?" Dr. Mary Fleming gave a talk on employment opportunities for math majors and shared some insights on what employers look for. Other 1989-90 officers: Harry Collins, vice

president; Karen Hugle, secretary; Cathy Mason, treasurer; Patrick Costello, corresponding secretary/faculty sponsor.

Maryland Beta Western Maryland College, Westminster

Chapter President - Debbi Camara

14 actives, 2 initiates

Other 1989-90 officers: Tammy Mahan, vice president; Laura Baliker and Deanna Dailey, secretary; Lisa Brown, treasurer; James Lightner, corresponding secretary; Linda R. Eshleman, faculty sponsor.

Maryland Delta Frostburg State University, Frostburg

Chapter President - Andrew Kaylor

23 actives

Maryland Delta chapter met several times during the fall semester, and in October sponsored a presentation of the video "Ramanujan: The Man Who Loved Numbers." The semester ended with a pre-exam week pizza party. We look forward to inducting new members in the spring. Other 1989-90 officers: Brenda Moore, vice president; James Stegmaier, secretary; Paul Duty, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

Michigan Beta Central Michigan University, Mount Pleasant

Chapter President - Nancy Haskell

35 actives

Michigan Beta Chapter conducted math help sessions for freshman/sophomore mathematics classes on Wednesday evenings. Several of our members worked with Professor William Lakey in a problem session seminar in preparation for the Putnam exam. Also, plans were made to start a problem solving group that would work on problems in *The Pentagon*. We again hosted our annual Homecoming Punch and Doughnut Hour for faculty and alumni. For years we have had a *KME* initiation each semester. However, we have now decided to change to having an initiation only during the winter semester. Student talks at fall meetings were given by Agnes Hausbeck and Nancy Haskell. Many chapter members will be student teaching during winter, 1990. Other 1989-90 officers: David Richmond, vice president; Sandy Schmoldt, secretary; Karen Walmsley, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

Missouri Alpha Southwest Missouri State University, Springfield

Chapter President - Paul Scott

36 actives

The Missouri Alpha Chapter began the 1989 fall semester with the annual picnic for all mathematics faculty, staff, and students at Phelps Grove Park. Three regular monthly meetings were held during the semester. Programs presented at these meetings were: "Ovine Monochromatism" by Dr. Peter Detre, "The Cantor Set is a Fractal" by Rick Kapalko, and "Hexa-Flexagons" by Matt Thomas. In other activities, nine new members were initiated and the organization co-sponsored a presentation on "Mathematics - Career and Job Outlook" by Allan MacDougall. The chapter designed and sold several shirts to interested parties and ended the semester with a holiday pizza party at Soup-n-Sandwich restaurant. Other 1989-90 officers: Ann Schlemper, vice president; Deanna Wasman, secretary; D. Anne Watters, treasurer; Vera Stanojevic, corresponding secretary; Michael Awad, faculty sponsor.

Missouri Gamma

William Jewell College, Liberty

Chapter President - Ty Abbott

17 actives

Missouri Gamma held regular monthly meetings. At their September meeting they enjoyed a report by corresponding secretary Joseph Mathis on his 1989 trip to Peru, visit with Maria Reiche, and flight over lines in the desert in Nazca. Other 1989-90 officers: Kevin Tanner, vice president; Katherine Pagacz, secretary; Joseph T. Mathis, treasurer; Joseph T. Mathis, corresponding secretary/faculty sponsor.

Missouri Epsilon

Central Methodist College, Fayette

Chapter President - Erving Crowe

13 actives

Other 1989-90 officers: Shelia Tuley, vice president; Joy Powell, secretary/treasurer; William D. McIntosh, corresponding secretary/faculty sponsor; Linda O. Lambke, faculty sponsor.

Missouri Eta

Northeast Missouri State University, Kirksville

Chapter President - Wes Clifton

24 actives, 22 initiates

Other 1989-90 officers: David Smead, vice president; Julie Ridlen, secretary; John DeKeersgieter, treasurer; Mary Sue Beersman, corresponding secretary; Mark Faucette, faculty sponsor.

Missouri Iota

Missouri Southern State College, Joplin

Chapter President - Bill Elliott

12 actives

During the fall semester Missouri Iota worked the concession stands at all home football games as a money making activity. Subsequently all those who worked were rewarded with a pizza party. The organization held regular monthly meetings. Programs at these meetings included a talk by Mary Miller of the MSSC Math Department on the frequency of occurrence of alnipollenities and the probability of the existence of oil. Mrs. Miller had done research on this subject using equipment provided by Amoco. Ana Witt and Bill Elliott met with Dr. Tran Thuong for numerous problem solving sessions in preparation for the Putnam Exam. They also presented solutions to many Putnam problems at the *KME* meetings. The semester ended with a Christmas party and "gag gift" exchange at the home of Dr. Larry Martin. Other 1989-90 officers: Hsiao-Hui Lin, vice president; Vince Sprengle, secretary/treasurer; Mary Elick, corresponding secretary; Linda Hand, faculty sponsor.

Missouri Kappa

Drury College, Springfield

Chapter President - Scott Steubing

7 actives

The first activity of the semester was a chili supper at Dr. Allen's house. The students played the faculty in a game of Trivial Pursuit and the faculty just squeaked by with a win. The winners of the Annual Campus Math Contest were David Larkin (Calculus I and below) and Scott Steubing (Calculus II and above). Prize money was awarded at a pizza party held for all contestants. At a luncheon for the chapter, Scott Steubing gave a talk on his research: Series representations of Pi. The end of the semester was celebrated with a Christmas party. Other 1989-90 officers: Monty Towe, vice president; Laura DeNouden, secretary; Jim Rutan, treasurer; Charles Allen, corresponding secretary; Ted Nickle, faculty sponsor.

Mississippi Gamma

University of Southern Mississippi, Hattiesburg

Chapter President - Elizabeth Page

17 actives, 5 initiates

Other 1988-90 officers: Patsy Hughes, vice president; Theresa Kelly, secretary; Alice Essary, corresponding secretary; Barry Piazza and Karen Thrash, faculty sponsors.

Nebraska Alpha

Wayne State College, Wayne

Chapter President - Keith Spiehs

29 actives

Throughout the semester, club members monitored the Math-Science Building in the evenings to earn money for the club. The club participated in the college homecoming activities by painting and erecting a billboard. Club members also manned a "Fish for Suckers" booth at the Homecoming Carnival. With a grant from the Wayne State College Student Senate, *KME* and Computer Club Purchased a laser printer which is housed in the Student Placement Office and is available to all WSC students. At Christmas time the club purchased two gifts for the Toys for Tots Campaign. Social activities included a fall picnic with the Math-Science faculty and the other clubs in the building and a pizza-movie party at Dr. Paige's home. Other 1989-90 officers: Lee Emanuel, vice president; Paula Gustafson, secretary/treasurer; Brenda Spieker, historian; Fred Webber, corresponding secretary; Jim Paige and Hilbert Johs, faculty sponsors.

Nebraska Gamma

Chadron State College, Chadron

Chapter President - Kim Sedlacek

17 actives, 4 initiates

Twelve members of Nebraska Gamma attended the NCTM Regional Conference in Rapid City, South Dakota. Fall initiates were Lanelle Henderson, Dan Hof, Maya Leicht, and Kebere Tewahade. The semester ended with a chapter Christmas party. Other 1989-90 officers: Michelle Dodd, vice president; Pat Reilly, secretary; Betty Rudnick, treasurer; James A. Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

Nebraska Delta

Nebraska Wesleyan University, Lincoln

Chapter President - Cheryl Olsen

18 actives

Activities for the fall semester included a get acquainted pizza party, a talk by an alum about his experiences as a graduate computer science student, and a Christmas party. Other 1989-90 officers: Michael Mead, vice president; Mary Rose Philpot, secretary; Terry Bierman, treasurer; Muriel Skoug, corresponding secretary/faculty sponsor.

New Mexico Alpha

University of New Mexico, Albuquerque

Chapter President - David Anderson

40 actives, 20 initiates

Other 1989-90 officers: Lisa Garcia, vice president; Mark Andrews, secretary; Joe McCanna, treasurer; Richard Metzler, corresponding secretary/faculty sponsor.

New York Alpha

Hofstra University, Hempstead

Chapter President - Margo McCullagh

6 actives

New York Alpha enjoyed a talk by Dr. H. Hastings, entitled "Fractile Game," and a discussion of "Careers in Actuarial Science." Other 1989-90 officers: Karen Grossu and Deanna De Liberto, vice presidents; Michael Belluci, secretary; Michelle Lisi, treasurer; Aileen Michaels, corresponding secretary/faculty sponsor.

New York Eta

Niagara University, Niagara

Chapter President - Christine Carbone

10 actives

Other 1989-90 officers: Laura Plyter, vice president; Amy Potter, secretary; Theresa Toenniessen, treasurer; Robert L. Bailey, corresponding secretary; Kenneth Bernard, faculty sponsor.

New York Lambda

C. W. Post/Long Island University, Brookville

Chapter President - Lauren Henneberger

28 actives

Since the formation of a new problem solving group, students have been busily working on problems from *The Pentagon*. Chapter members are being encouraged to prepare papers for presentation at the spring regional meeting and plans are underway for the spring initiation and banquet. Other 1989-90 officers: Ahmed Samatar, vice president; Cynthia Ferro, secretary; Christopher Kelly, treasurer; Sharon Kunoff, corresponding secretary; Andrew M. Rockett, faculty sponsor.

Ohio Gamma

Baldwin-Wallace College, Berea

Chapter President - Gerald Hwasta

17 actives

Other 1989-90 officers: Steve Szatmary, vice president, Cassandra Reed, secretary; Sandi Holt, treasurer; Robert Schlea, corresponding secretary/faculty sponsor.

Ohio Zeta

Muskingum College, New Concord

Chapter President - Toni St. Clair

24 actives, 7 initiates

During the fall semester, members of Ohio Zeta heard Dr. Richard Laatsch of Miami University, Oxford, Ohio, speak on "Crystallography of Subring Lattices of Z_n " and Dr. Fred Gass, also of Miami University,

speak on "Geometric Probability." Dr. Hollingsworth was the speaker for the October initiation. The semester ended with a Christmas party/snack supper at Smith's. Other 1989-90 officers: Monica Gibson, vice president; Jennifer Suschil, secretary; Cari Fusco, treasurer; James L. Smith, corresponding secretary; Javad F. Habibi, faculty sponsor.

Oklahoma Alpha Northeastern State University, Tahlequah
Chapter President - Shelli Phillips
38 actives, 8 initiates

Oklahoma Alpha fall initiation ceremonies were held in the banquet room of the Western Sizzlin' Restaurant in Tahlequah. The December meeting was a Christmas pizza party with a prize offered for the best math joke. Mathematics professors again donated textbooks for the *KME* book sale. Other 1989-90 officers: Albert Peters, vice president; Mike O'Keefe, secretary/treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

Oklahoma Gamma Southeastern Oklahoma State University, Weatherford
Chapter President - Glen Mitchell
15 actives

Oklahoma Gamma Chapter published a student directory as a fund raising project during the semester. Other 1989-90 officers: Kathy Hogan, vice president; Kristin Casebeer, secretary; Darin Puritan, treasurer; Wayne Hayes, corresponding secretary; Robert Morris, faculty sponsor.

Pennsylvania Alpha Westminster College, New Wilmington
Chapter President - Matthew Mrozek
21 actives, 1 initiate

Wednesday night tutoring continued to be provided for students seeking help with mathematics. The organization sponsored an annual "career night" on Nov. 16, 1989, at which event, speakers, including representatives from Armco Corporation and USX, spoke on career opportunities in mathematics and computer science. The chapter is also working to install a student chapter of MAA, which they hope will be active by this spring. Other 1989-90 officers: David Chapnell, vice president; Christy Heid, secretary; Kimberly Hoener, treasurer; J. Miller Peck, corresponding secretary; Warren Hickman, faculty sponsor.

Pennsylvania Beta La Salle University, Philadelphia
Chapter President - Leonard Wisniewski
21 actives

Other 1989-90 officers: Megan Donnelly, vice president; Virginia Boyd, secretary; Judi Ann Drobile, treasurer; Hugh N. Albright, corresponding secretary; Carl McCarty, faculty sponsor.

Pennsylvania Delta

Marywood College, Scranton

Chapter President - Mary Frances Zelenak

12 actives

Pennsylvania Delta held monthly meetings to work on plans for the spring math contest for high school students. In addition, two chapter members attended the NCTM convention in Philadelphia, Pennsylvania. Other 1989-90 officers: Joseph Perri, vice president; Anne Undercoffler, secretary; Vittoria Mercaldo, treasurer; Sister Robert Ann von Ahnen, corresponding secretary/faculty sponsor.

Pennsylvania Epsilon

Kutztown University, Kutztown

Chapter President - Pat Dyson

8 actives

Other 1989-90 officers: Kelly Boyer, vice president; John Katchmin, secretary; Angela Laffredo, treasurer; Cherry C. Mauk, corresponding secretary/faculty sponsor.

Pennsylvania Zeta

Indiana University of Pennsylvania, Indiana

Chapter President - John Limbacher

17 actives, 2 initiates

Pennsylvania Zeta met four times during the fall semester. Topics covered at the meetings included "Mystery of Irrational Numbers," "How Math is Incorporated into Accounting," and Proficiency in Mathematics." Chapter members also spent some time on problem solving. Other 1989-90 officers: Bill Davie, Mary Lou Husband and Jill Anderson, vice presidents; Kathy Smith, secretary; Diane Zahurak, treasurer; George Mitchell, corresponding secretary/faculty sponsor.

Pennsylvania Eta

Grove City College, Grove City

Chapter President - Kim Coltrin

25 actives, 11 initiates

The annual Christmas party was held at Mr. Schlossnagel's place on Tuesday, December 5, 1989. Other 1989-90 officers: Phil Runninger, vice president; Chad Gregory, secretary; Donna Day, treasurer; Marvin Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Iota Shippensburg University, Shippensburg
Chapter President - John Thompson
25 actives, 9 initiates

Other 1989-90 officers: Tina Flory, vice president; Judy Morningstar, secretary; Howard Bell, treasurer; Lenny Jones, corresponding secretary; Rick Ruth, faculty sponsor.

Pennsylvania Kappa Holy Family College, Philadelphia
Chapter President - Vincent Frascotore
12 actives, 5 pledges

Pennsylvania Kappa held monthly meetings to solve and discuss challenging problems. Tutoring was provided for freshman students. Two field trips were taken, one to the regional NCTM meeting in Philadelphia and one to the Franklin Institute. Other 1989-90 officers: Frances Capponi, vice president; Karen McDowell, secretary/treasurer; Sister M. Grace Kuzawa, corresponding secretary/faculty sponsor.

Pennsylvania Lambda Bloomsburg University, Bloomsburg
Chapter President - Tom Rogers
35 actives

Members of the chapter conducted help sessions each Wednesday and Thursday evening during the semester. Other 1989-90 officers: Karen Cressman, vice president; LeAnn Schrann, secretary; Chris Case, treasurer; Jim Pomfret, corresponding secretary; John Riley, faculty sponsor.

Pennsylvania Mu Saint Francis College, Loretto
Chapter President - Kelly Subasic
19 actives, 8 initiates

Induction activities began with a mass, followed by dinner and highlighted by initiation of eight new members. Marie Sumner was presented a book as an award for attaining the highest score on the senior comprehensive exam. Dr. Robert Clickner, a senior statistician for Westat, Inc., of Rockeville, MD, visited campus and presented a lecture, "Survey and Sample Design in Theory and Practice." Seven students and four faculty attended a micro-computer conference at Penn State. A total of ten students and faculty attended a math club meeting at neighboring University of Pittsburg at Johnstown and heard a presentation about "The Fourth Dimension." Members also attended a play on the Penn State-Altoona campus entitled "The Human Adding Machine." Senior Mary Morrisard is conducting help sessions for both freshman calculus

and finite math. The resumé service provided by the math club has blossomed, easily producing enough revenue to buy a new laserjet printer. Picnics were held both in the spring and fall semesters. Other 1989-90 officers: Chris Blumenthal, vice president; Antonine Gatto, secretary; Kristine Miller, treasurer; Peter Skoner, corresponding secretary/faculty sponsor.

Tennessee Alpha Tennessee Technological University, Cookeville
 Chapter President - Doug Talbert
 25 actives

The organization is helping to implement a tutoring program for remedial math students at a local junior high school. We are planning to have a representative from the Navy speak at one of our meetings and our president is planning to speak to math classes at a local high school. Steven Wilson, a 1982 Tennessee Tech alumnus and a Tennessee Alpha member, was awarded the Pi Tau Sigma gold medal of the American Society of Mechanical Engineers during its winter meeting December 10-15 in San Francisco. This medal is awarded to the young engineering graduate who has demonstrated outstanding achievement in mechanical engineering within ten years of graduation. Other 1989-90 officers: Pete Howard, vice president; Usha Munukutta, secretary; Allen Hunt, treasurer; Frances Crawford, corresponding secretary; Barbara Briggs, faculty sponsor.

Tennessee Beta East Tennessee State University, Johnson City
 Chapter President - Dottie McCray
 10 actives

A fall social was held at the Firehouse Restaurant, at which time officers were elected. Also, a group picture was made for inclusion in the Buccaneer yearbook. Other 1989-90 officers: Jamie Whittimore, vice president; Charlene Rose, secretary/treasurer; Lyndell Kerley, corresponding secretary/faculty sponsor.

Tennessee Delta Carson-Newman College, Jefferson City
 Chapter President - Jenny Crutchfield
 22 actives

Fall semester activities included a picnic at Panther Creek State Park; an address on statistics by Dr. David Allen, Chair, Department of Statistics, University of Kentucky; and a Christmas dinner at the home of Dr. Verner Hansen. Other 1989-90 officers: Kim Caldwell, vice president; Jeff Holmes, secretary; Keith Repass, treasurer; Verner Hansen,

corresponding secretary; Carey Herring, faculty sponsor.

Texas Alpha

Texas Tech University, Lubbock

Chapter President - Nancy Lacey
22 actives

Fall semester activities included monthly meetings with invited speakers. Other 1989-90 officers: Steve Wester, vice president; David Watson, secretary; Jennifer Ragland, treasurer; Robert Moreland, corresponding secretary/faculty sponsor.

Texas Eta

Hardin-Simmons University, Abilene

Chapter President - Randal Schwindt
10 actives

At a Get-Acquainted Party for students interested in mathematics, the purpose and activities of the Texas Eta Chapter of *KME* were explained and professional opportunities in the mathematical sciences were discussed, as well as degree programs and awards available in the mathematics area of HSU. Also, shingles were presented to those members inducted in the spring. Other 1989-90 officers: Stephen Cody, vice president; Tina E. Hill, secretary/treasurer; Mary Wagner-Krankel, corresponding secretary; Charles Robinson and Ed Hewett, faculty sponsors.

Virginia Beta

Radford University, Radford

Chapter President - Patches Johnson
5 actives, 11 initiates

Other 1989-90 officers: Faye Smith, vice president; Beth Hale, secretary/treasurer; Stephen Corwin, corresponding secretary; J. D. Hansard, faculty sponsor.

Virginia Gamma

Liberty University, Lynchburg

Chapter President - Lisa Barwick
15 actives

In addition to regular meetings and a cook-out held prior to the homecoming football game, Virginia Gamma hosted a colloquium featuring Dr. Chimenti, Chairman of Computer Science, who spoke on "Games and Pascal's Wager," and a colloquium featuring Guy Farnstrom, an honors student, whose topic was "Vector Field Analysis of Complex Functions." Other 1989-90 officers: Guy Tarnstrom, vice president; Sarah Wu, secretary; Mary Beth Grayson, treasurer; Glyn Wooldridge, corresponding secretary; Robert Chasnov, faculty sponsor.

Wisconsin Alpha

Mount Mary College, Milwaukee

Chapter President - Lauri Malisch

6 actives, 3 initiates

Wisconsin Alpha's fall activities included sponsorship of the annual mathematics contest for junior and senior women from area high schools. Top prize for this contest is a partial renewable scholarship to Mount Mary College. Initiation took place on November 5, 1989, for Cyndi Heim, Lauri Malisch, and Janis Schmanske. Other 1989-90 officers: Cyndi Heim, vice president/treasurer; Lauri Malisch, secretary; Sister Adrienne Eickman, corresponding secretary/faculty sponsor.

Wisconsin Gamma

University of Wisconsin-Eau Claire, Eau Claire

Chapter President - Renee Wagner

55 actives, 16 initiates

The fall semester began with the induction of sixteen new members. Monthly meetings highlighted by one or more student speakers were the mainstay of the semester. Other 1989-90 officers: Debra Strauch, vice president; Emily Larsen, secretary; Leanne Johnson, treasurer; Tom Wineinger, corresponding secretary.

Announcement of the Twenty-Eighth Biennial Convention

The Twenty-Eighth Biennial Convention of Kappa Mu Epsilon will be hosted by the Alabama Beta Chapter and will be held 11-13 April 1991 at the University of North Alabama in Florence, Alabama. Since this convention will mark the sixtieth anniversary of the founding of Kappa Mu Epsilon, each attending chapter will receive \$60 in addition to the usual travel expense reimbursement from the national funds as described in Article VI, Section 2, of the Kappa Mu Epsilon constitution.

A significant feature of this convention will be the presentation of papers by student members of Kappa Mu Epsilon. The mathematical topic selected by each student speaker should be of interest to the author and of such scope that it can be given adequate treatment in a timed oral presentation. Student speakers will be chosen by the Selection Committee on the basis of written papers submitted prior to the convention. At the convention, the Awards Committee (composed of four students and four faculty members representing as many chapters as possible) will judge the speakers on both content and presentation and will select the prize winners.

Who may submit a paper?

Any undergraduate or graduate student member of Kappa Mu Epsilon may submit a paper for use on the convention program. A paper may be co-authored. If selected for presentation at the convention, the paper must be presented by one (or more) of the authors. Graduate students will not compete for prizes with undergraduates.

Presentation topics.

Papers submitted for presentation at the convention should discuss material understandable by undergraduate mathematics majors, preferably those who have completed differential and integral calculus.

The Selection Committee naturally will favor papers within this limitation and which can be presented with reasonable completeness within the time allotted.

Presentation time limits.

The presentation of the paper must take at least 15 minutes and no more than 25 minutes.

How to prepare a paper.

Five copies of your paper, together with a description of any charts, models or other visual aids you plan to use during the presentation, must be submitted. The paper should be typewritten in the standard form of a term paper. It should be written as it will be presented, including length. A long paper (such as an honors thesis) must not be submitted with the idea that it will be shortened later when you present it! Appropriate references and bibliography are expected.

The first page of your paper must be a "cover sheet" giving the following information: (1) title, (2) author (your name must not appear elsewhere in the paper), (3) your student status ("undergraduate" or "graduate"), (4) both your permanent and school addresses, (5) the name of your *KME* Chapter and school, (6) a signed statement giving your approval that your paper be considered for publication in *The Pentagon*, and (7) the signed statement of your Chapter's Corresponding Secretary that you are indeed a member of Kappa Mu Epsilon.

How to submit a paper.

You must send the five copies of your paper to:

Dr. Arnold D. Hammel
KME National President-Elect
c/o Department of Mathematics
Central Michigan University
Mt. Pleasant, Michigan 48859

no later than 21 January 1991.

Selection of papers for presentation.

The Selection Committee will review all papers submitted to the National President-Elect and will choose approximately fifteen papers for presentation at the convention; all other papers will be listed by title and author in the convention program and will be available as "alternates." The National President-Elect will notify all authors of the status of their papers after the Selection Committee has completed its deliberations.

Criteria used by the Selection and Awards Committees.

The paper will be judged on (1) topic originality, (2) appropriateness to the meeting and audience, (3) organization, (4) depth and significance of the content, and (5) understanding of the material. The presentation will be evaluated on (1) style of presentation, (2) maintenance of interest, (3) use of audio-visual materials (if applicable), (4) enthusiasm for the topic, (5) overall effect, and (6) adherence to the time limits.

Prizes.

All authors of papers presented at the convention will be given two-year extensions of their *Pentagon* subscriptions and prizes of \$60 in honor of the sixtieth anniversary of the founding of *KME*. Authors of the four best papers presented by undergraduate students, as determined by the Awards Committee, will be awarded additional cash prizes of \$60, \$40, \$30 and \$20, respectively. If enough papers are presented by graduate students then one or more prizes will be awarded in this category.

Publication.

All papers submitted to the convention are considered as submitted for publication in *The Pentagon* (see page 2 for further information). Prize winning papers will be published after any necessary revisions have been completed and all other papers will be considered for publication. All authors are expected to schedule brief meetings with the Editor during the convention to review their manuscripts.

Kappa Mu Epsilon National Officers

- Harold L. Thomas *President*
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- Mary S. Elick *Historian*
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Missouri Southern State College, Joplin, Missouri 64801

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation.

Chapter	Location	Installation Date
OK Alpha	Northeastern Oklahoma State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State College	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State College, Upper Montclair	21 April 1944
IL Delta	College of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949

MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	Kearney State College, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri - Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood College, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin - River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Northeast Missouri State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970

MO Theta	Evangel College, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop College, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	West Georgia College, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin - Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Illinois Benedictine College, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C. W. Post Center of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry College, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987

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