THE PENTAGON

Volume XLIV(44) Fall, 1984 Number 1

CONTENTS

NATIONAL OFFICERS

IDA Z. ARMS....................................President

Indiana University of Pennsylvania
Indiana, Pennsylvania 15705

JAMES L. SMITH..........................President-Elect

Muskingum College
New Concord, Ohio 43762

GEORGE R. MACH................................Secretary

California Polytechnic State University
San Luis Obispo, California 93407

SISTER NONA MARY ALLARD.....................Treasurer

Rosary College
River Forest, Illinois 60305

HAROLD L. THOMAS...............................Historian

Pittsburg State University
Pittsburg, Kansas 66762

Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics; due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

EDMUND HALLEY ON LIFE TABLES

William L. Schaaf
Professor Emeritus, Brooklyn College

It is well known that the scientific basis of life insurance rests upon tables of mortality. These did not come into general use until the end of the 18th century, subsequent to the publication of the famous Northampton Table of 1783, which was rather unscientifically constructed. It was followed some thirty years later by the Carlisle Table (1815), which, although built upon limited statistical data, was nevertheless carefully compiled. We are interested here in the earlier attempts. The registration of deaths was begun in England by Henry VII as early as 1532. By 1630 the weekly Bills of Mortality issued in London had become fairly complete and well established. The first important published treatise along these lines was John Graunt's <u>Observations on the London Bills of Mortality</u> (1662). Soon thereafter a crude attempt was made by Jan de Witt (1671) to evaluate the purchase price of annuities. But only with the publication of a classic paper by the astronomer Edmund Halley in 1693, can modern actuarial science be said to have begun. This paper, which appeared in

Philosophical Transactions (XVII), and was later reprinted in Miscellanea Curiosa, vol. I (1726), is entitled An Estimate of the Degrees of the Mortality of Mankind, drawn from Curious Tables of Births and Funerals at the City of Breslaw; with an Attempt to ascertain the Price of Annuities upon Lives. It is from this famous paper that we cull a few excerpts. We read:

> The Contemplation of the Mortality of Mankind, has besides the Moral, its Physical and Political Uses, both of which have been some Years since most judiciously considered by the curious Sir William Petty, in his Natural and Political observations on the Bills of Mortality of London, owned by Captain John Craunt: And since in a like Treatise on the Bills of Mortality seemed even to their Authors to be defective: First, In that the Number of the People was wanting. Secondly, That the Ages of the People dying was not to be had. And Lastly, That both London and Dublin, by reason of the great and casual accession of Strangers who die therein, (as appeareth in both, by the great Excess of the Funerals above the Births) rendred them incapable

of being Standards for this purpose; which requires, if it were possible, that the People we treat of, should not all be changed, but die where they were born, without any adventitious Increase from abroad, or Decay by Migration elsewhere.

This Defect seems in a great measure to be satisfied by the late curious Tables of the Bills of Mortality at the City of Breslaw, lately communicated to this Honourable Society by Mr. Justell, wherein both the Ages and the Sexes of all that die, are monthly delivered, and compared with the Number of the Births, for five Years last past, viz. 1687, 88, 89, 90, 91, seeming to be done with all the Exactness and Sincerity possible.

This City of Breslaw is ... very far from the sea, and as much a Mediterranean Place as can be desired, whence the Confluence of Strangers is but small, and the Manufacture of Linnen employs chiefly the poor People of the Place, as well as of the Country round about;... for these Reasons, the people of this city seem most proper for a Standard; and the rather, for that the Births do a small matter exceed the Funerals. The only thing wanting, is the Number of the whole People, which in some

> measure I have endeavour's to supply, by the comparison of the Mortality of the People of all Ages, which I shall from the said Bills trace out with all the Accuracy possible.

At this point Halley explains specifically how he uses the data, and then continues:

> From these considerations I have form'd the adjoined Table, whose Uses are manifold, and give a more just Idea of the State and Condition of Mankind, than any thing yet extant that I know of. It exhibits the Number of People in the City of Breslaw of all Ages, from the Birth to extreme Old Age, and thereby shews the Chances of Mortality at all Ages, and likewise how to make a certain Estimate of the Value of Annuities for Lives, which hitherto has been only done by an imaginary Valuation: Also the Chances that there are that a Person of any Age proposed does live to any other Age given; with many more, as I shall hereafter shew. This Table does shew the Number of Persons that are living in the Age current annexed thereto, as follows: Thus it appears, that the whole People

of *Breslaw* does consist of 34000 *Souls*, being the Sum *Total* of the Persons of all Ages in the *Table*: The first Use hereof is to shew the Proportion of *Men* able to bear *Arms* in any *Multitude*, which are those between 18 and 56, rather than 16 and 60; the one being generally too weak to bear the *Fatigues* of *War*, and the Weight of *Arms*; and the other too crasie and infirm from *Age*, notwithstanding particular Instances to the contrary...The *Second Use* of this Table, is, to shew the differing Degrees of *Mortality*, or rather *Vitality*, in all Ages; for if the Number of Persons of any Age remaining after one Year, be divided by the difference between that and the number of the *Age* proposed, it shews the *Odds*, that there is, that a Person of that *Age* does not die in a year...So likewise for the *Odds*, that any Person does not die before he attain any proposed *Age*: Take the *Number* of the remaining Persons of the *Age* proposed, and divide it by the difference between it and the number of those of the *Age* of the Party proposed; and that shews the *Odds* there is between the Chances of the Party's living or dying.

Use III. But if it be enquired at what number of

> Years, it is an even Lay that a Person of any Age shall die, this Table readily performs it. ...

The author then proceeds to discuss "Uses IV, V, VI and VII", which deal with various problems in the determination of the price of annuities on one or more lives under various conditions. He concludes the paper with the following unique paragraph:

> Besides the Uses mentioned, it may perhaps not be an unacceptable thing to infer from the same Tables, how unjustly we repine at the shortness of our Lives, and think our selves wronged if we attain not old Age; whereas it appears hereby, that the one half of those that are born are dead in seventeen Years time, 1238 being in that time reduced to 616. So that instead of murmuring at what we call an untimely Death, we ought with Patience and Unconcern to submit to that Dissolution which is the necessary Condition of our perishable Materials, and of our nice and frail Structure and Composition: And to account it as a Blessing that we have survived, perhaps by many years, that Period of Life, whereat the one half of the whole Race of Mankind does not arrive.

A second Observation I make upon the said Table, is that the growth and Increase of Mankind is not so much stinted by any thing in the Nature of the <u>Species</u>, as it is from the cautious Difficulty most People make to adventure on the State of <u>Marriage</u>, from the Prospect of the Trouble and Charge of providing for a Family. Nor are the poorer sort of People herein to be blamed, since their Difficulty of subsisting is occasion'd by the unequal Distribution of Possessions, all being necessarily fed from the Earth, of which yet so few are Masters. So that besides themselves and Families, they are yet to work for those who own the Ground that feeds them: And of such does by very much the greater part of Mankind consist; otherwise it is plain, that there might well be four times as many Births as we now find. For by computation from the Table, I find that there are nearly 15000 Persons above 16, and under 45, of which at least 7000 are Women capable to bear Children. Of these notwithstanding there are but 1238 born yearly, which is but little more than a sixth part: So that about one in six of these Women do breed yearly; whereas were they all married, it would not appear strange or unlikely, that four of six should bring a Child every Year. The political Consequences hereof I shall not insist on; only the Strength and Glory of a King being in the multitude of his Subjects, I shall only hint, that above all things, Celibacy ought to be discouraged, as, by extraordinary Taxing and Military Service: And those who have numerous Families of Children to be countenanced and encouraged by such Laws as the *Jus trium Liberorum* among the Romans. But especially, by an effectual Care to provide for the Subsistence of the Poor, by finding them Employments, whereby they may earn their Bread, without being chargeable to the Publick.

In the century and a half following the publication of Halley's table, considerable developments ensued. The Carlisle Table (already mentioned) published in 1815 by Joshua Milne served as a model for later tables, one of the most prominent being the *Healthy English Table* devised by Dr. Farr about 1854. By this time a considerable number of life insurance companies had sprung up, and the records of these companies furnished a more effective basis for life tables. Since then, of course, many further refinements in mortality tables have been made by actuaries both in England and in the United States.

THE DIVISIBILITY OF SEVEN AND OTHER NUMBERS

Thomas S. Shannon
Student, University of Missouri - Rolla

One of the most ancient procedures for checking the validity of mathematical computations is the divisibility test; that is, a test to determine if a number n is divisible by a number m. Certain numbers have divisibility tests so simple that we tend to do them automatically without even thinking about them. Some of these numbers are: 2, 5, and 10.

For example, to determine if a number n is divisible by 2, we merely inspect its last digit. If that digit is 0, 2, 4, 6, or 8, we know that n is divisible by 2. Similarly, to test for divisibility by 5, we look at the last digit of the number; and if it is 0 or 5, then the number is divisible by 5. If the number's last digit is 0, then we know the number is divisible by 10.

Testing for divisibility by 4 is quite similar to testing for divisibility by 2, 5, or 10; but we must inspect the last two digits of the number being tested. If this two digit block is a multiple of 4, then the number is divisible by 4.

Some numbers, however, require some mathematical computation to determine their divisibility. To determine if a number is divisible by 3, we may add up all the digits of that number. If the resulting sum is a multiple of 3, then we know that the original number is divisible by 3.

Example: Is 8,562 divisible by 3?

$$8 + 5 + 6 + 2 = 21$$

21 is a multiple of 3. Therefore, 8,562 is divisible by 3.

The divisibility test for 9 is identical to the test for 3 except that the resulting sum must be a multiple of 9.

Of the one digit positive numbers, the most difficult for which to devise a divisibility test is the number 7. Leonardo Fibonacci proposed a divisibility test for 7 based on the theory of congruences, or modular arithmetic.[1] In the theory of congruence, two integers, a and b, are congruent for the modulus m when the difference a - b is divisible by m with remainder zero.[2] This congruence is written:

$$a \equiv b \pmod{m}$$

Any integer N can be written in the form:

$$N = a_n 10^n + a_{n-1} 10^{n-1} + \ldots + a_2 10^2 + a_1 10^1 + a_0 10^0.$$

Fibonacci's divisibility test for 7 requires obtaining the sum $\sum_{i=1}^{n} a_i x_i$ where $x_i = 10^i \pmod 7$. If the sum is divisible by 7, then the number N is divisible by 7.

Example: Is 1,508,213 divisible by 7?

$$10^0 = 1,\ 10^1 = 3,\ 10^2 = 2,\ 10^3 = -1,\ 10^4 = -3,$$
$$10^5 = -2,\ 10^6 = 1 \pmod 7)$$

Therefore $\sum_{i=1}^{n} a_i x_i = 1\cdot3 + 3\cdot1 + 2\cdot2 - 1\cdot8 - 3\cdot0 - 2\cdot5 + 1\cdot1 = -7$

-7 is divisible by 7, therefore, 1,508,213 is divisible by 7 also.

This method works, but it has the drawback that for every digit a_i in the number N being tested, the value $x_i = 10^i \pmod 7$ must be calculated.

I propose a different method for a divisibility test of 7 which requires only one calculation for one value of x. This method involves recognizing that $100 = 2 \pmod 7$. The number N to be tested for divisibility by 7 is broken up into groups of two digits beginning at the right. The most significant group of digits is then multiplied by 2 and added to the next most significant group. This results in a new number which is broken up into groups in the same manner. This process continues until only one or two digits remain. If the remaining number is divisible by

7, then the original number N is also divisible by 7.

Using the same example as with Fibonacci's method:

$$
\begin{array}{rrrr}
1 & 50 & 82 & 13 \\
\\
 & 50 & 82 & 13 \\
 & \underline{+2(1)} & & \\
 & 52 & 82 & 13 \\
\\
 & & 82 & 13 \\
 & & \underline{+2(52)} & \\
 & 1 & 86 & 13 \\
\\
 & & 86 & 13 \\
 & & \underline{+2(1)} & \\
 & & 88 & 13 \\
\\
 & & & 13 \\
 & & & \underline{+2(88)} \\
 & & 1 & 89 \\
\\
 & & & 89 \\
 & & & \underline{+2(1)} \\
 & & & 91
\end{array}
$$

91 is divisible by 7, therefore 1,508,213 is divisible by 7 also.

To prove that this method is correct, we must be aware of the following properties of congruences:

1. $a \equiv a \pmod{m}$
2. $a \equiv b \pmod{m}$ if and only if $a + k \equiv b + k \pmod{m}$
3. $a \equiv b,\ c \equiv d \pmod{m}$ if and only if $ac \equiv bd \pmod{m}$

What we need to show is that given a four digit number N where

$$N = (a_3 10^3 + a_2 10^2 + a_1 10 + a_0),$$

then $(a_3 10^3 + a_2 10^2 + a_1 10 + a_0) \equiv$

$$(a_3 10 + a_2)(2) + a_1 10 + a_0 \pmod{7}.$$

In other words, if N is divisible by 7, then the sum of twice the two most significant digits and the last two significant digits is also divisible by 7.

Proof: Let $N = (a_3 10^3 + a_2 10^2 + a_1 10 + a_0)$.

$(a_3 10^3 + a_2 10^2 + a_1 10 + a_0) =$
$\quad (a_3 10 + a_2)(100) + a_1 10 + a_0$

$(a_3 10^3 + a_2 10^2) + (a_1 10 + a_0) \equiv$
$\quad (a_3 10 + a_2)(100) + a_1 10 + a_0 \pmod{7}$

$(a_3 10^3 + a_2 10^2) \equiv (a_3 10 + a_2)(100) \pmod{7}$ by property 2.

But $100 \equiv 2 \pmod{7}$.

So $(a_3 10^3 + a_2 10^2) \equiv (a_3 10 + a_2)(2) \pmod{7}$ by property 3.

Therefore $(a_3 10^3 + a_2 10^2 + a_1 10 + a_0) \equiv$

$$(a_3 10 + a_2)(2) + a_1 10 + a_0 \pmod 7.$$

Q.E.D.

It is a simple matter of induction to expand this proof for values of N greater than four digits.

A variation of this method may be made to test for divisibility by 7 by dividing N into single digits and multiplying the most significant digit by 3 before adding it to the next most significant digit. This process is continued until only one digit remains. If that digit is divisible by 7, then N is divisible by 7 also.

Example: Is 182 divisible by 7?

$$\begin{array}{rrr}
1 & 8 & 2 \\
 & 8 & 2 \\
\multicolumn{2}{r}{\underline{+3(1)}} & \\
1 & 1 & 2 \\
 & 1 & 2 \\
\multicolumn{2}{r}{\underline{+3(1)}} & \\
 & 4 & 2 \\
 & & 2 \\
 & \multicolumn{2}{r}{\underline{+3(4)}} \\
 & 1 & 4 \\
 & & 4 \\
 & \multicolumn{2}{r}{\underline{+3(1)}} \\
 & & 7
\end{array}$$

Therefore 182 is divisible by 7.

The proof is similar to the preceding one.

We need to show that for a given two digit number N where

$$N = (a_1 10 + a_0)$$

then $(a_1 10 + a_0) = (a_1)(3) + a_0 \pmod{7}$.

Proof: Let $N = a_1 10 + a_0$.

$a_1 10 + a_0 = a_1(10) = a_0 \pmod{7}$

$a_1 10 = a_1(10) \pmod{7}$

But $10 = 3 \pmod{7}$,

so $a_1 10 = a_1 (3) \pmod{7}$.

Therefore $(a_1 10 + a_0) = (a_1)(3) + a_0 \pmod{7}$.

Q.E.D.

Once again, it is a simple matter of induction to expand this proof for values of N greater than two digits.

It is also a matter of induction to show that this method can be expanded to any power of ten by finding the appropriate value for $10^i \pmod{7}$.

Additionally, it can be shown that this method can be applied to test not only for divisibility of 7 but also for divisibility of any integer m.

What we need to show here is that for a given two-digit number N where

$$N = (a_1 10 + a_0)$$

and a given modulus m

then $(a_1 10 + a_0) \equiv (a_1)(x) + a_0 \pmod{m}$

where $10 \equiv x \pmod{m}$.

Proof: Let $N = a_1 10 + a_0$

and let $x \equiv 10 \pmod{m}$.

$a_1 10 + a_0 \equiv a_1(10) + a_0 \pmod{m}$

$a_1 10 \equiv a_1(10) \pmod{m}$

But $10 \equiv x \pmod{m}$,

so $a_1 10 \equiv a_1(x) \pmod{m}$.

Therefore

$(a_1 10 + a_0) \equiv (a_1)(x) + a_0 \pmod{m}$.

Q.E.D.

Once again, it is a simple matter of induction to expand this proof for values of N greater than two digits or to expand it for other powers of 10.

As a trivial example of an application of this, we will test a number for divisibility by 2 recognizing that $10 \equiv 0 \pmod{2}$.

Is 132 divisible by 2?

$$\begin{array}{rrr} 1 & 3 & 2 \end{array}$$

$$\begin{array}{rr} 3 & 2 \\ +0(1) & \\ \hline 3 & 2 \end{array}$$

$$\begin{array}{r} 2 \\ +0(3) \\ \hline 2 \end{array}$$

2 is divisible by 2, therefore 132 is divisible by 2 also.

Notice, in this case, how applying this method to test for divisibility by 2 results in examining only the last digit of the number being tested. This is the original test we had for divisibility by 2.

As another example, let us test a number for divisibility by 3, recognizing that 10 = 1 (mod 3).

Is 708 divisible by 3?

$$\begin{array}{rrr} 7 & 0 & 8 \end{array}$$

$$\begin{array}{rr} 0 & 8 \\ +1(7) & \\ \hline 7 & 8 \end{array}$$

$$\begin{array}{r} 8 \\ +1(7) \\ \hline 1\ \ 5 \end{array}$$

$$\begin{array}{r} 5 \\ +1(1) \\ \hline 6 \end{array}$$

6 is divisible by 3, therefore 708 is divisible by 3.

Notice in this example, how testing a number for divisibility by 3 is functionally equivalent to summing up all the digits of the number. This is the same as the original test for divisibility by 3 which we discussed earlier.

In fact, all of the divisibility tests we have discussed, with the exception of Fibonacci's divisibility test for 7, are nothing more than applications of this method. In the case of testing a number N for divisibility by m where $m = 2$, 5, or 10, the fact that $10 \equiv 0 \pmod{m}$ reduces the test down to inspecting the last digit of N. When $m = 3$ or 9, we recognize that $10 \equiv 1 \pmod{m}$ and the effect of the test is to merely sum up the digits of N. In the case where $m = 4$, we may recognize that $100 \equiv 0 \pmod{m}$ and the test reduces to inspecting the last two digits of N. Therefore, this method not only can be used to quickly create divisibility tests for any desired integer, but also shows how many of the commonly used divisibility tests, even though they appear to be different, are actually only applications of the same technique.

BIBLIOGRAPHY

[1]Oystein Ore, Number Theory and Its History (New York: McGraw-Hill, 1948), p. 228.

[2]Oystein Ore, Number Theory and Its History (New York: McGraw-Hill, 1948), p. 211.

THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August 1985. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1985 issue of The Pentagon, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 377: Proposed by Charles W. Trigg, San Diego, California.

The only square integer that is the concatenation of two two-digit squares is $1681 = 41^2$. Find a six digit square integer, N^2, that is the concatenation of three two-digit squares.

Problem 378: Proposed by Charles W. Trigg, San Diego, California.

Show that in every system of notation with an even positive base there is a repdigit triangular number.

Problem 379: Proposed by the editor.

Let $\langle\sqrt{5}\rangle$ denote the fractional part of $\sqrt{5}$. Then $\langle\sqrt{5}\rangle = \sqrt{5} - \sqrt{4}$. Also $(\langle\sqrt{5}\rangle)^2 = \sqrt{81} - \sqrt{80}$ and $(\langle\sqrt{5}\rangle)^2 = \sqrt{1445} - \sqrt{1444}$.

Prove that for each positive integer n, there is an integer T_n such that $(\langle\sqrt{5}\rangle^n = \sqrt{T_n} - \sqrt{T_n - 1}$ and show how T_n can be found.

Problem 380: Proposed by the editor.

A golden rectangle is one whose sides a and b satisfy the proportion a:b = b:(a-b). Suppose that this golden rectangle has been drawn on a coordinate axis with the longer side lying on the x axis. We mark off on the left side of the rectangle the largest possible square; a smaller golden rectangle remains. We then rotate the figure 90° counterclockwise so that the longer side of the smaller golden rectangle lies among the x axis. Again we mark off the largest possible square on the left side of this new rectangle. By repeating this process, continually smaller golden rectangles are constructed until only one point in the original rectangle remains. What is the location of this unique point?

Problem 381: Proposed by the editor.

Little Euclid has a toy box in the shape of a rectangular parallelopiped. The length of each side and the length of the diagonal of each lateral face is an integral number of inches. If the sum of the three side lengths is 401 inches, what are the dimensions of the box?

SOLUTIONS

367: Proposed by the editor.

Find the two smallest numbers in the decimal system such that each of the numbers is multiplied by 27 by placing the digit 1 on each side of the original number.

Solution by Charles W. Trigg, San Diego, California.

$27N = 1N1 = 10^k + 10N + 1$ where N has $k - 1$ digits. Then $17N = 10^k + 1$. Now $17(3) = 51$, so we divide 17 into 100... 00 until a remainder of 5 is obtained, whereupon the last digit 0 is replaced by the digit 1, and the division is complete. Thus $(10^8+1)/17 = 5882353$ is the smallest number sought.

If we await the second appearance of 5 as a remainder before replacing the last digit 0 with the digit 1, the second smallest number is found to be $(10^{24}+1)/17 = 5882329411764705882353$.

Also solved by: Fred A. Miller, Elkins, West Virginia and Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

368: Proposed by Charles W. Trigg, San Diego, California.

A square is divided into nine congruent square cells. In each corner cell, place a different odd digit. In each side cell place an even digit, so that

1) the digits along each side of the square and of its reflections about a side form a prime integer, and

2) the sum of the digits in the eight cells is a square integer.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

The only 3-digit primes which have distinct odd hundreds and units digits and even tens digits whose reverses are also prime are 107, 149, 167, 347, 389, 709, and 769.

Thus 1, 3, 7, and 9 must occupy the corner cells and only 0, 4, 6, or 8 can appear in the side cells. Since $1 + 3 + 7 + 9 = 20$, it follows that the entries in the side cells must sum to 16. (By the second condition of the problem, they can't sum to 5 or 29 because these entries are all even.)

Next we observe that there is no requirement that the entries in the side cells be different. Thus the possibilities for these entries are:

(1) 0, 0, 8, 8 (3) 0, 4, 4, 8

(2) 0, 4, 6, 6 and (4) 4, 4, 4, 4

Since 8 appears in only one prime pair and since 4 appears in only two prime pairs, (1) and (4) can be eliminated. If 8 is not an entry in a side cell, then there is only one prime pair having 3 as the units or hundreds digit; hence (2) is eliminated. Thus the pairs 389 - 983, 149 - 941, and 347 - 743 must appear. Also either 107 - 701 or 709 - 907 must appear. Thus the unique solution is

3	4	7
8		0
9	4	1.

Also solved by the proposer.

369: Proposed by Michael W. Ecker, Pennsylvania University, Worthington-Scranton Campus, Scranton, Pennsylvania.

Given a checkerboard which is n x n, find an explicit function of n which counts a) the total number of squares of all sizes in the checkerboard and b) the total number of rectangles of all sizes in the checkerboard.

Solution by the proposer.

a). For 1 x 1 squares there are n^2 such squares; for 2 x 2 squares there are $(n-1)^2$ such squares; in general, for $1 \leq j \leq n$ there are $(n-1)^2$ squares of size j x j. Thus, we have

$$\sum_{j=1}^{n} (n - j + 1)^2 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

For the regular checkerboard, n = 8, yielding a total of 204 squares.

b). The n x n checkerboard corresponds to an (n + 1) x (n + 1) grid of points. Since a rectangle is uniquely determined by two diagonally opposite vertices, it suffices to count the number of possible pairs of diagonally opposite vertices. There are $(n + 1)^2$ possible choices for the first point. Given a point which is the intersection of a horizontal and a vertical line, there are 2n + 1 points which must be excluded as a possible diagonally opposite vertex, the point itself and the other n points on either the horizontal or the vertical line. Thus there are $(n+1)^2 - (2n+1) = n^2$ possible choices for the second point on the diagonally opposite vertex. Since any single vertex of the rectangle can be used as the first point, each rectangle has been counted four times. Thus there are $n^2(n+1)^2/4$ rectangles of all sizes including squares. By subtracting the result found in (a) above, there are $n(n-1)(n+1)(3n+2)/12$ pure (non square) rectangles.

Also solved by: Fred A. Miller, Elkins, West Virginia and Kyle Bagwell, student at Stanford University, Stanford, California.

370: Proposed by the editor.

Find all positive integers n such that n^3+1 is exactly divisible by $dn - 1$ where d is a positive integer.

Solution by Steve Ligh, University of Southwestern Louisiana, Lafayette, Louisiana.

Suppose that n^3+1 is divisible by $dn - 1$ for some positive integer d. Then there exists a positive integer k such that $n^3+1 = k(dn-1)$. Then it follows that $n(n^2-kd) = -(k+1)$. Taking $k + 1 = ny$, we get

$$d = \frac{n^2 + y}{ny - 1}\,. \tag{1}$$

Next we must find n and y such that d is a positive integer.

Case 1. Suppose $n = 1$. Then d is a positive integer iff $y = 2$ or 3.

Case 2. Suppose $y = 1$. Then d is a positive integer iff $n = 2$ or 3.

Case 3. Suppose $d = 1$. Then the quadratic formula yields

$$n = \frac{y \pm \sqrt{(y - 2)^2 - 8}}{2}$$

Since $(y-2)^2-8$ must be a perfect square, that square must be 9 so that $y = 5$ and $n = 2$ or 3.

Case 4. Suppose that $n > 1$, $y > 1$, and $d > 1$. From (1) we have

$$n = \frac{dy \pm \sqrt{(dy)^2 - 4(y + d)}}{2} \qquad (2)$$

Let $d^2y^2-4(y+d) = w^2$. Then

$$(dy + w)(dy - w) = 4(y + d) = AB$$

for some integers A and B. From (2) we see that both factors on the left side of (3) are even; say $dy + w = A = 2a$ and $dy - w = B = 2b$ for some integers a and b with $a \geq b$. Hence $dy = a + b$ and $y+d = ab$. Since $dy \geq y + d \geq 4$, then $a + b \geq ab \geq 1$, so that $a \geq (a - 1)b$ and $2 > \frac{a}{a-1} \geq b$.

Hence $b = 1$ or 2. If $b = 2$, then $a = 2$ and $d = y = 2$. Thus $b = 1$ so that $dy - 1 = a = y + d$. Hence $d = (y + 1)/(y - 1)$ is an integer.

Finally $y = 2$ or 3 with corresonding values for n of 1 and 5 respectively. Thus the only solutions are:

$$1^3+1 = (2\times1-1)(3\times1-1)$$

$$2^3+1 = (1\times2-1)(5\times2-1) = (2\times2-1)(2\times2-1)$$

$$3^3+1 = (1\times3-1)(5\times3-1) \text{ and}$$

$$5^3+1 = (2\times5-1)(3\times5-1).$$

371: Proposed by the editor.

A long straight rod was dropped causing it to break into two pieces. If one of the two pieces is picked up at random. This piece is dropped and it breaks into two pieces. What is the probability that the three pieces of the original rod will form a triangle?

Solution by the proposer.

The three pieces of the original rod will form a triangle whenever no single piece is longer than one-half the length of the original rod. This condition is satisfied whenever the second cut is made on the larger piece resulting from the first cut. The probability of this event is .5.

Next, assuming that the piece of the rod chosen for the second cut has unit length, let t denote the length of the smaller piece resulting from the second cut and 1 - t denote the length of the larger piece resulting from the second cut. The probability that a triangle will occur equals

$$\frac{1}{2}\int_0^{1/2} \frac{2t}{(1-t)}\,dt = [(1-t) - \ln[1-t]]\Big|_0^{1/2} =$$

$\ln 2 - .5 = .1931.$

This problem appears in Mathematical Bafflers by Angela Dunn which is published by Dover Publications, Inc., New York, (1980), at pp. 137-138. It was chosen because of the subtle difference from the usual problem where all three cuts occur at the same time.

LATE SOLUTIONS

Solutions for problems 352, 353, 355 and 356 were received from Gregory J. Haas, student, Bowling Green University, Bowling Green, Ohio. Solutions for problems 352, 353, 354, 355 and 356 were received from Bill Olk, student, Carroll College, Waukesha, Wisconsin.

THE HEXAGON

EDITED BY IRAJ KALANTARI

This department of THE PENTAGON is intended to be a forum in which mathematical issues of interest to undergraduate students are discussed in length. Here by issue we mean the most general interpretation. Examination of books, puzzles, paradoxes and special problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to **Iraj Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.**

If you were asked whether a given statement is true or not, you would have to inquire where and how you are to interpret it. A cliche' example, which is over-used, is '2+2 = 4'. This statement is true when '+' is interpreted as the addition operation and is false if '+' is interpreted as addition modulo 3 (then 2+2 = 1). Hence in two different systems, a calculation which starts the same way might end differently. What might be a theorem in one system, might not be the same of another system. These ideas first emerged through discovery of geometries different from Euclid's. This issue's Hexagon examines a well-known theorem of Euclidean geometry for its absoluteness.

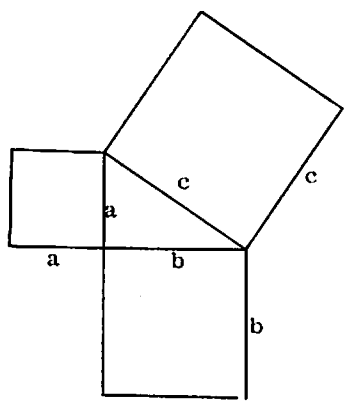
IK

DO YOU BELIEVE IN PYTHAGORUS?

Ross Wilkinson*

The aim of these few pages is to make you suspicious. In particular, you may become more suspicious of proofs offered to you by your teachers and by books.

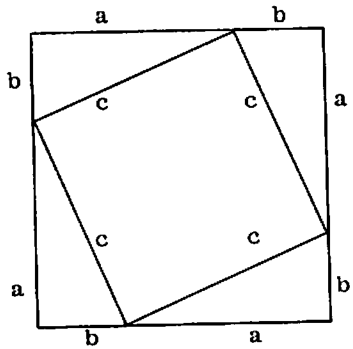
About 500 B.C. Pythagorus claimed that if a, b, and c were the lengths of a right angled triangle, the side of length c opposite the right angle, then $a^2+b^2=c^2$. In pictorial form we have



*Professor Wilkinson received his Ph.D. in mathematics from Monash University at Melbourne, Australia and is currently a member of the mathematics department of Western Illinois University. His interests include abstract algebra and formal language theory.

Should we believe him? Notice that he was talking about "real" triangles, not just about abstract mathematical notions. Thus, if I am on a ship that is travelling at 3 knots due west and I walk across the ship at 4 knots, my speed relative to the ocean is 5 knots.

To try to prove this claim of Pythagorus we can draw the following diagram.



The area of this box is $(a+b)^2$. Also it has area that is given by four triangles, each with sides of length a, b and c, and one square with sides of length c. Now the area of each triangle is $\frac{1}{2}ab$, so we see that

$$(a+b)^2 = 4(\frac{1}{2}ab) + c^2$$

so simplifying we get

$$a^2+2ab+b^2 = 2ab+c^2$$

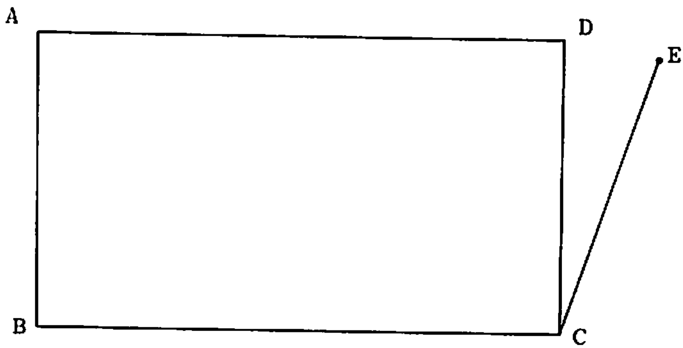
and cancelling 2ab from both sides we get

$$a^2+b^2 = c^2$$

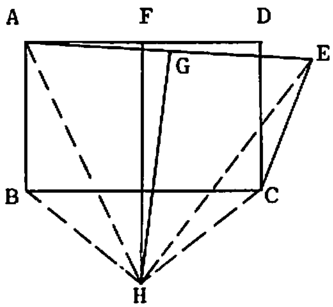
It looks as if Pythagorus was right!

Before believing this proof though, consider two more problems.

First let us construct a rectangle ABCD and let E be a point the same distance away from C as D is.



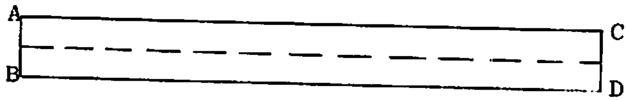
Now find the midpoint of AD, call it F, and draw the perpendicular bisector from F, through BC. Next find the midpoint of AE, call it G and draw the perpendicular bisector from G through BC. Now these two bisectors will intersect at another point, say H. Thus we now have the following diagram.



Now d(AB), the distance from A to B, is equal to d(EC) as d(EC) = d(DC). Also d(AH) = d(EH), since GH is a perpendicular bisector of AE. Also d(CH) = d(BH), since FH is a perpendicular bisector of BC. Thus the triangles ΔABH and ΔECH are congruent, and so the angle $\angle$ABH equals the angle $\angle$ECH. Now $\angle$CBH equals $\angle$BCH, as FH is a perpendicular bisector. Thus we can conclude that $\angle$ABC = $\angle$ABH - $\angle$CBH = $\angle$ECH - $\angle$BCH = $\angle$ECB. This means that a right angle $\angle$ABC is equal to an angle larger than a right angle $\angle$ECB. What is wrong? Perhaps you would like to pause and see if you can find the error in the argument.

Each of the steps of the argument looks all right, however the problem comes with the diagram. If drawn more carefully, you will see that EH passes <u>outside</u> C, so that the argument fails.

A second more simple problem also indicates that care should be taken when using knowledge of what is obvious. Suppose we take a long thin piece of paper and draw a line down the middle of it lengthways.

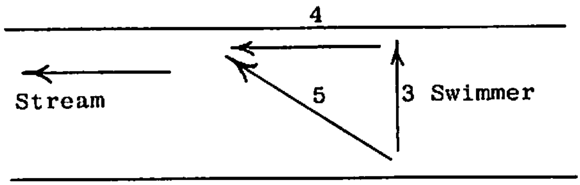


Next, join the two ends and cut along the line. How many pieces of paper do you end with? It seems obvious that the answer is two. Perhaps you would care to perform this experiment, connecting A to D and B to C and see what happens. I think you will agree with me that what seems "obvious" should be viewed a little more suspiciously!

How can we prove anything then? Well, Euclid in about 300 B.C. was concerned with this problem also and he gave five assumptions or axioms that he said could be used to prove all the other true geometric statements we might want to use in dealing with this world about us. Only these five axioms could be used. Diagrams and claiming that something was obvious was not allowed to be a part of the proof. As you may know, this was very successful and many theorems, or facts, could be proved, including the result of Pythagorus.

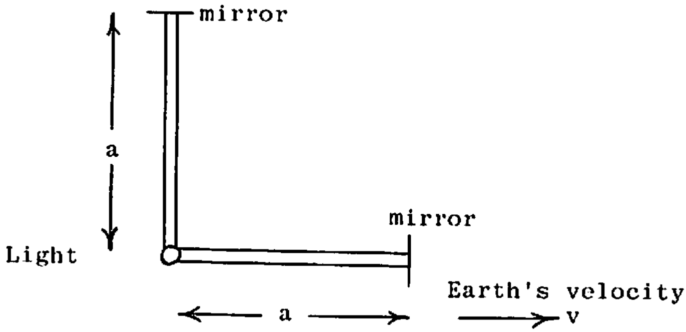
Does this mean we can believe Pythagorus? It does if we believe Euclid. If we treat the theory of Euclid as abstract mathematics with many applications to ordinary life, then we run into no problems. However, as statements about the real world, we need to view them more suspiciously.

One of the most important uses of Pythagorus' theorem occurred in 1887. To determine the velocity of a swimmer who is crossing a stream at 3 miles per hour while the stream is flowing at 4 miles per hour, we find the swimmer's velocity to be $\sqrt{3^2+4^2} = 5$



Michelson and Morley used similar computations in an experiment with light. At that time it was believed that light moved at a constant speed with respect to a substance that was everywhere called "ether". Call this speed c. Also at some particular time the earth is moving with speed v.

Michelson and Morley had a rather complicated piece of apparatus, but in essence it consisted of two rods of equal length that were joined end to end at right angles with a mirror at each non-connected end and a light source and a device to compare times at the connected ends.



Now imagine a flash of light occurs. Using the classical theory, it would take a/(c-v) seconds for the light to travel to the mirror and a/(c+v) to travel back to the source.

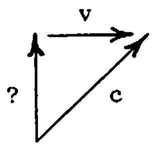
So

$$t = \frac{a}{c-v} + \frac{a}{c+v}$$

$$= \frac{a(c+v)+a(c-v)}{(c-v)(c+v)}$$

$$= \frac{2ac}{c^2-v^2}$$

For the light travelling along the other rod we need Pythagorus' theorem to calculate the velocity. Since light is travelling at speed c through the "ether", its speed along the rod is needed.



If w is that speed $w^2+v^2 = c^2$, so $w = \sqrt{c^2-v^2}$. This is the speed travelling out along the rod and back, so

$$t_2 = \frac{2a}{\sqrt{c^2-v^2}}$$

Now unless $v = 0$, t_1 is not equal to t_2. However in Michelson and Morley's remarkable experiment, it was found $t_1 = t_2$. Since this experiment was performed at different times of the year it couldn't be that the earth's velocity was zero, so the ether theory was badly battered.

As a result of this and other experiments, in 1905 Albert Einstein suggested an entirely different set of assumptions or axioms. These can be very simply stated:

(1) The laws of physics are the same in all systems with no external forces acting.

(2) The speed of light in free space has the same value c in all systems with no external forces acting.

Let us examine the consequences of these assumptions. Imagine we are both floating deep in outerspace. Anything that happens, happens somewhere which we can say has three space co-ordinates x_1, x_2 and x_3, and happens at some time t. So both of us will be able to specify with 4 numbers any event that occurs, although we might use different numbers if we don't use the same co-ordinates. Suppose that I am floating past you (or you are floating past me, which is after all just the same thing).

We both detect that a flash of light occurred. I think it occurred at (x_1,x_2,x_3,t). You think it occurred at (y_1,y_2,y_3,s). A moment later I detect that it is at (x_1',x_2',x_3',t') and you detect it at (y_1',y_2',y_3',s'). Thus from my point of view, the light travelled from (x_1,x_2,x_3) to (x_1',x_2',x_3') in $t'-t$ seconds. If $\Delta x_1 = x_1'-x_1$, $\Delta x_2 = x_2'-x_2$, $\Delta x_3 = x_3'-x_3$ and $\Delta t = t'-t$, the light travelled a distance of $\sqrt{\Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2}$ in Δt seconds. So by Einstein's second axiom,

$$\sqrt{\Delta x_1^2+\Delta x_2^2+\Delta x^3} = c\Delta t$$

So $$\Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2 = c^2 \Delta t^2$$

If $x_4 = ct$, and $\Delta x_4 = c \Delta t$ this says

$$\Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2 - \Delta x_4^2 = 0$$

Now by Einstein's first axiom, this statement shouldn't be true just for me it should be true for you too. So if $y_4 = cs$ and $\Delta y_4 = c \Delta s$, then

$$\Delta y_1^2 + \Delta y_2^2 + \Delta y_3^2 - \Delta y_4^2 = 0$$

Thus, if I want to change from my co-ordinates to yours, I have to make sure that this relationship is preserved. These changes have special names - Lorentz transformations. If you are travelling with speed v in the x_1 direction, the transformations can be given as

$$y_1 = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}}$$

$$y_2 = x_2$$

$$y_3 = x_3$$

$$s = \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}}$$

There are many remarkable consequences of this. You have probably heard of the most famous of these --

$$E = mc^2$$

Less well known is that you cannot use Pythagorus' theorem to add velocity vectors. If I observe a flash of light while moving at speed v perpendicular to that of the light, it will seem to me to be travelling at c, not $\sqrt{c^2+v^2}$, as the classical theory tells us. Thus, in some ways, Pythagorus was wrong.

FURTHER READING:

Ball and Coxeter, "Mathematical Recreations and Essays". University of Toronto Press, Toronto (1974).

Einstein, "The Meaning of Relativity". Princeton University Press, Princeton (1956).

Friedrichs, "from Pythagorus to Einstein". Random House, New York (1965).

THE CURSOR

Edited by Jim Calhoun

This is the second appearance of this department in THE PENTAGON. The topics presented here can be broadly classified as belonging to computer science but their emphasis is more narrowly defined. Like most of the applied sciences, computer science depends heavily upon a large body of mathematical theory. It is the aim of this department to present discussions which help to define the interface between the disciplines of mathematics and computer science. Specifically, it seeks to relate ideas from the theory of mathematics to an understanding of concepts in computer science. Readers are encouraged to submit articles on any topic which seems directed toward this goal. Address all correspondence to Jim Calhoun, Mathematical Systems Department, Sangamon State University, Springfield, Illinois 62708.

THE CURSOR was chosen as the name of this department because of the role that mathematics plays in pointing the way toward the understanding of many important concepts in computer science.

MATHEMATICAL INDUCTION, SETS AND PROGRAMS

INTRODUCTION

One of the more difficult ideas for programmers to comprehend is the concept of a procedure or subroutine which calls itself. Beginning programmers, and even some with experience, struggle for years attempting to understand the nature of programs which make use of this programming technique. Worse yet, many avoid it like the plague and criticize others for using it, since it seems impossible to fully understand. In observing programmers struggling with this idea, it becomes obvious that individuals with good mathematics backgrounds struggle less than those with less math experience. However, in spite of the truth of this observation, even students with substantial mathematics backgrounds (substantial in quantity at least) sometimes have difficulty too. The problem, it seems, is that they have been unable to establish a firm foundation upon which to base the needed understanding.

The discussion presented here attempts to look at a basis for an understanding of self recursive procedures (procedures that call themselves). It looks in turn at the two standard principles of mathematical induction, methods of defining sets and finally at examples of self recursion.

MATHEMATICAL INDUCTION (review)

The First Principle of Mathematical Induction:

> If P(K) is true for K = 0 and
> the implication
> P(K) ===> P(K+1)
> is true for every K
> then P(N) is true for every natural number N.

The first principle states that if it is known that the proposition is true for the smallest natural number and also known that whenever it is true for some natural number then it is true for the next, then the proposition is true for each natural number.

<u>The Second Principle of Mathematical Induction:</u>
If P(K) is true for K=0 and
the implication
[P(J)true for all $0\leq J\leq K$] ===> P(K+1)
is true for every K
then P(N) is true for every natural number N.

The second principle states that if it is known that the proposition is true for the smallest natural number and also known that whenever it is true for all natural numbers less than a given natural number then it is true for that number, then the proposition is true for each natural number.

<u>The Second Principle of Mathematical Induction</u> (extended):

If P(K) is true for K=0 & K=1 and

the implication

[P(J)true for all $0\leq J\leq K$] ===> P(K+1)

is true for every $K\geq 1$

then P(N) is true for every natural number N.

METHODS OF DEFINING SETS

There are a number of ways to define a set. For example, the set of integers between 7 and 13 can be defined in the following ways:

i) Enumeration - {8,9,10,11,12}

ii) Set builder notation - $\{J|7<J<13\}$

Each method of defining a set, enumeration and set builder, gives the necessary and sufficient conditions for an element to belong to the set. Enumeration simply lists the elements of the set and suggests the test for set membership should be a table look-up or a list search. In contrast, the set builder method gives a condition (i.e., a predicate or boolean function) which, when applied to an element, returns the proper result for a test of set membership. In the above example this function is:

```
function MEMBERSHIP(J): returns boolean;
   begin
       if (7<J) and (J<13) then
            return(TRUE)
       else
            return(FALSE)
   end
```

The important point is that a set can be defined in many ways, and if a programmer restricts his/her thinking to a single definition, it will directly affect the choice of an algorithm. Note that the enumeration method applies only to finite sets, but that the set builder method can be used to define (some) infinite sets as well as finite sets.

Another method which is also very useful in defining some infinite sets is the inductive definition method. This method is constructive in nature in that it describes how to build the set (i.e., construct new elements of the set), but gives little hint of how to test for set membership. As an example, we give one inductive definition of the set called LIST (of integers) (also defined are the list operations FIRST and TAIL).

Definition of LIST:

<u>Basis clause:</u> $\langle\ \rangle$ is a LIST of integers which contains zero integers

<u>Inductive clause</u>: If $L = \langle a_1, a_2, \ldots, a_n \rangle$ is a LIST of integers and a_0 is an integer then $L' = \langle a_0, a_1, a_2, \ldots, a_n \rangle$ is a LIST of integers with $FIRST(L') = a_0$ and $TAIL(L') = L$

<u>Extremal</u> <u>clause:</u> L is a LIST of integers if and only if it can be constructed through a finite number of applications of the basis and inductive clauses.

Note: FIRST(L) and TAIL(L) are undefined if L is the empty list.

This definition of LIST illustrates the constructive nature of inductive definitions. A new list is constructed from an existing list by adding an additional element to the front of the list. The element which was added is the first element of the newly constructed list and the original list is its tail. It is this inductive definition which serves as the basis for understanding the recursive function searchlist shown below. It is claimed that the function will search <u>any</u> LIST of integers for the value of KEY and return the correct response. An understanding of the correctness of this function and the construction of a proof of correctness can be obtained by applying the first principle of mathematical induction to the set LIST (of integers).

```
FUNCTION searchlist(L:LIST; KEY: INTEGER) returns
   boolean;
   begin
    if L = emptylist then
       return(FALSE)
    else
       if FIRST(L) = KEY then
          return(TRUE)
       else
             searchlist(TAIL(L))
   end;
```

Informal proof:

Searchlist searches an empty list correctly since it returns FALSE if L is empty (i.e., it works correctly on lists of length zero). If one assumes as an inductive hypothesis that it searches all lists of length K correctly then it only remains to be shown that it will search all lists of length K+1 correctly. Note that a list of length K+1 must be nonempty and therefore have a first element. Clearly if the first element of L is the same as KEY, then the search was successful; if not, then its success depends on the presence of KEY in the tail of L. Since the tail of L must be of length K and because the inductive hypothesis implies searchlist works correctly on such lists, we are guaranteed that the correct response will be returned.

For the discussion which follows we assume the following terms and functions have been defined (here we describe them informally):

INFORMAL DEFINITIONS

1) If $L_1 = \langle a_1, a_2, \ldots, a_n \rangle$ then

   i) FIRST(L_1) = a_1 and LAST(L_1) = a_n unless L is the empty list in which case both FIRST (L_1) and LAST(L_1) are undefined.

   ii) FIRSTHALF(L_1) = $\langle a_1, \ldots, a_{half(n)} \rangle$ and LASTHALF(L_1) = $\langle a_{half(n)}, \ldots, a_n \rangle$ (where half(4) = 2, half(3) = 1 and half(1) = 0).

   Note that half(n) < n and half(n) > 0 whenever n > 1 and in such cases FIRSTHALF(L_1) <u>and</u> LASTHALF(L_1) each have less elements than L_1.

2) INSERT(c,L_1) = $\langle a_1, a_2, \ldots, a_j, c, a_{j+1}, \ldots, a_n \rangle$ if c is greater than each a_i, $1 \le i \le j$

   and

   INSERT(c,L_1) = $\langle c, a_1, a_2, \ldots, a_n \rangle$ if $c \le a_1$.

3) i) If $L_1 = \langle a_1, a_2, \ldots, a_n \rangle$ and $L_2 = \langle b_1, b_2, \ldots, b_m \rangle$ then CONCATENATE(L_1,L_2) = $\langle a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_m \rangle$.

ii) LESSTHAN(c, L_1) is the list of elements from L_1 which are less than c.

iii) GREATER_or_EQUAL(c, L_1) is the list of elements from L_1 which are greater than or equal to c.

4) MERGE(L_1,L_2) = L_3 where L_3 is the usual ordered merge.

We now address the question of what it means for a list to be sorted (in ascending order). If one chooses to construct an inductive definition, the issue to be addressed is, "How can sorted lists be constructed from lists already known to be sorted?" That is, how does one build sorted lists from sorted lists. It is obvious that there may be different characterizations of sorted lists which are constructive in nature which could be used as the basis for an inductive definition. The design of a self recursive algorithm for the purpose of sorting lists is directly affected by the definition chosen. Four definitions (or characterizations) of the set of sorted lists (of integers) follows:

DEFINITION1: If the list $L = \langle a_1, a_2, \ldots, a_n \rangle$ and for each i, $1 \le i \le n-1$ we have $a_i \le a_{i+1}$ then L is a sorted list.

DEFINITION2:

Basis clause: The empty list is a sorted list.

Induction clause: If $L_1 = \langle a_1, a_2, \ldots, a_n \rangle$ is a sorted list then INSERT(c,L) is a sorted list.

Extremal clause: L is a sorted list of integers if and only if it can be constructed through a finite number of applications of the basis and inductive clauses.

DEFINITION3:

Basis clause: If L is the empty list then L is sorted.

Induction clause: If $L_1 = \langle a_1, a_2, \ldots, a_n \rangle$ & $L_2 = \langle b_1, b_2, \ldots, b_m \rangle$ are sorted lists and $\text{LAST}(L_1) \le \text{FIRST}(L_2)$ then CONCATENATE(L_1,L_2) is a sorted list.

Extremal clause: L is a sorted list of integers if and only if it can be constructed through a finite number of applications of the basis and inductive clauses.

DEFINITION4:

Basis clause: If L or TAIL(L) is empty then L is a sorted list (i.e., the empty list and all singleton lists are sorted).

Induction clause: If $L_1 = \langle a_1, a_2, \ldots, a_n \rangle$

& $L_2 = \langle b_1, b_2, \ldots, b_m \rangle$

are sorted lists then

MERGE(L_1,L_2) is a sorted list.

Extremal clause: L is a sorted list of integers if and only if it can be constructed through a finite number of applications of the basis and inductive clauses.

All of the above definitions are intended to define the same set; however, we will not prove that they do. Note that all the definitions except the first use the inductive method of set definition. Although each defines the same set, there are important differences between these inductive definitions. The construction of sorted lists as given by the second definition builds new sorted lists, each of whose length is one more than the list from which it was built. The third and fourth definitions, however, use two lists as the material from which they build a new list. The length of each newly constructed sorted list is the sum of the lengths of those two sorted lists. These differences imply that the first principle of mathematical induction, although it can be used to prove properties about sorted lists if the second definition of sorted lists is used, cannot be used when either the second or third definition is used. The functions SORT2, SORT3, and SORT4 reflect the corresponding inductive definitions: definition2, definition3 and definition4 respectively. These functions are self recursive and an understanding of their use of recursion, and ultimately the proof of correctness of each, requires application of the appropriate principle of mathematical induction to the corresponding inductive definition of sorted list. Each of these functions makes use of the terminology which was defined informally above.

```
function SORT2(L:LIST):returns LIST;
 begin
   if L is the empty list then
     return(empty list)
   else
     return(INSERT(FIRST(L), SORT1(TAIL(L)))
 end;
```

Informal proof: SORT2 sorts lists of length zero correctly since it returns the empty list and the empty list is sorted. Assume that SORT2 sorts all lists of length K correctly. Now if L is of length K+1 then TAIL(L) is of length K and by hypothesis SORT2 will sort it correctly. Inserting the first element of L into the sorted list resulting from the tail of L produces a sorted list containing the same elements as L. Hence, SORT2 sorts L correctly and by the first principle of mathematical induction it sorts all lists correctly.

```
function SORT3(L:LIST):returns LIST;
 begin
  if (L is the empty list) or (TAIL(L) is the empty
                                              list) then
    return(L)
  else
    begin
        L1:= SORT2(LESSTHAN(FIRST(L),TAIL(L)));

        L2:= SORT2(GREATERTHAN(FIRST(L), TAIL(L)));

        L3:=CONCATENATE(L1, <FIRST(L));

        return(CONCATENATE(L3,L2)
 end;
```

Informal proof: SORT3 sorts lists of length zero correctly since it returns the empty list. Now assume that SORT3 sorts all lists of length less than or equal to K correctly. Now if L is a list of length K+1 then the lists LESSTHAN(FIRST(L),TAIL(L)) and GREATERTHAN(FIRST(L), TAIL(L)) are of length less than or equal to K and hence the SORT3 sorts them correctly producing sorted list L_1 and L_2 as given in the algorithm. Note also that FIRST(L) together with L_1 and L_2 constitute all the elements of L and that FIRST(L) is greater than each element of L_1 but less than or equal to each element of L_2. Also note that the list returned was the result of concatenating the appropriate three list in the correct order. Hence the result returned is the correct. By the second principle of mathematical induction, SORT3 sorts all lists correctly.

```
function  SORT4(L:LIST):returns LIST;
 begin
  if L is the empty list then
    return(empty list)
  else
    if TAIL(L) is the empty list then
            return(L)
    else
            return(MERGE(SORT3(FIRSTHALF(L)),
                            SORT3(LASTHALF(L)))
 end;
```

Informal proof: Clearly SORT4 sorts lists of length zero and one correctly. Now assume that it sorts correctly all lists of length less than or equal to K(K≥1) and assume that L is a list of length K+1. Note that in this case K+1 is at least two and both FIRSTHALF(L) and LASTHALF(L) have fewer than K elements; thus SORT3 sorts each of these lists correctly. The list returned is a correct sorting of L provided the function MERGE applied to sorted lists produces a sorted list. By the (extended) second principle of mathematical induction, SORT4 sorts all lists correctly.

CONCLUSION

Many data structures such as lists and trees have a number of inductive definitions, each of which could serve as their characterization. If a programmer's thinking is limited to a single characterization, then the probability is small that he/she will be able to read with understanding someone else's recursive program. Both computer science and mathematics depend on abstractions to simplify and bring order to the problem solving process. Since a program represents a solution to a problem and because programs written by one set of individuals are often maintained by another set, it is important that both the writers and readers of the program use the same set of abstractions.

A final word concerning mathematical induction. Among the list of mathematical concepts which are important to computer science, mathematical induction is near the top. It is important not only to the understanding of recursion but also to the more general study of program correctness and the study of programming languages and their grammars.

KAPPA MU EPSILON NEWS

Edited by Harold L. Thomas, Historian

News of chapter activities and other noteworthy KME events should be sent to Dr. Harold L. Thomas, Historian, Kappa Mu Epsilon, Mathematics Department. Pittsburg State University, Pittsburg, Kansas 66762.

REPORT ON THE 1984 REGION 2 CONVENTION

Michigan Beta hosted the Region 2 meeting at Mt. Pleasant, Michigan on April 27, 1984. Regional Director J. Frederick Leetch reports that Michigan Beta showed lots of enthusiasm for KME and did a fine job of making convention arrangements. Student papers presented were: Greg Green, "Votes and a Half Binomial" and Paul O'Dea, "Problems with Functions."

Guest Speaker for the banquet was Professor Adras Kroo who is a researcher at the Mathematical Institute of the Hungarian Academy of Sciences and a Visiting Professor of Mathematics at Central Michigan University. He received his undergraduate degree from Moscow State University in 1976 and his Ph.D. from the Hungarian Academy of Sciences in 1979. Professor Kroo and the two student speakers were given books in appreciation for their presentations.

CHAPTER NEWS

Alabama Zeta, Birmingham-Southern College, Birmingham
Chapter President - Judy Tanquary
33 actives, 21 initiates

This spring, the Chapter initiated twenty-one candidates into its membership. Dr. Johnson, a professor of Mathematics at Birmingham-Southern College, spoke to the members at the ceremony. KME placed three of its members in the Scholar's Bowl competition held at Birmingham-Southern. Two luncheons were also sponsored for members and faculty. These allowed students and faculty an opportunity to get to know each other outside the classroom. Rita Crook, a new KME initiate, received the Louise Hall and Robert Echols Scholarship in mathematics. Donny Herring, outgoing president of Alabama Zeta, received the Acton Award for Achievement in Mathematics and the KME annual award for service to the chapter. Dr. Arthur Segal, a bio-statistician from the University of Alabama in Birmingham, spoke to the organization about "Math and Medicine." His presentation was very interesting and informative. Other 1984-85 officers: Keyna Warren, vice president; Karin Johnson, secretary; Richard Sturgeon, treasurer; Lola F. Kiser, corresponding secretary; Sarah E. Mullins, faculty sponsor.

Alabama Gamma, University of Montevallo, Montevallo
Chapter President - Debborah Ann Evans
7 actives, 5 initiates

On February 27, 1984, the chapter sponsored a lecture entitled "Sayre and Palmer ... The Process of Mathematics History in Alabama" given by Joe Albree, Professor of Mathematics at the University of Auburn in Montgomery. On April 10, 1984, Alabama Gamma supplied quiz questions and helped judge the Shelby County Public School math tournament. Other 1984-85 officers: Vicki Lynne Simmons, vice president; Laurie Virginia Stevens, secretary; Cathy Lynn Johnson, treasurer; Joseph Cardone, corresponding secretary; Angela Hernandez, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President - Harland Duncan
25 actives, 64 initiates

The chapter assisted the Mathematics Department with the annual Poly Royal Math Contest which attracted over 500 high school students to the campus. Weekly chapter meetings featured alumni and industry speakers. About 110 were present at the annual spring initiation and banquet. Harland Duncan was the third recipient of the annual Arthur Andersen & Co. Professional Performance Award. Other 1984-85 officers: Julie Justice & Damon Antos, vice president; Laura Melody, secretary; Charles Hughes, treasurer; George R. Mach, corresponding secretary; Adelaide T. Harmon-Elliott, faculty sponsor.

Colorado Alpha, Colorado State University, Fort Collins
Chapter President - Terri M. Woods
6 actives - 19 initiates

The chapter held initiation May 3, 1984 in conjunction with the undergraduate math club. Other 1984-85 officers: Curtis D. Bennett, vice president; Kim Pokorny, secretary and treasurer; Arne Magnus, corresponding secretary; Robert E. Gaines, faculty sponsor.

Connecticut Beta, Eastern Connecticut State University, Willimantic
Chapter President - Michael Rousseau
25 actives, 20 initiates

The Chapter held two colloquia on the Four-Color Problem. A banquet and initiation service was attended by 250. Other 1984-85 officers: Stephen Kenton, faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President - Maureen Ramey
22 actives, 9 initiates

The annual initiation meeting was held on May 28, 1984. Nine new members were received at that time. A reception was held in honor of the 1984 initiates after the initiation and election of officers. Yolanda Hasty and Patsy Spinks, both KME members, were announced as mathematics scholarship winners for 1984-85. Other 1984-85 officers: Sandra Hyde, vice president; Leanne Perry, secretary; Beth Hyde, treasurer; Joe Sharp, corresonding secretary and faculty sponsor.

<u>Illinois Beta</u>, Eastern Illinois University, Charleston
Chapter President - David Bryden
31 actives, 37 initiates

Spring semester activities included a February meeting with Dr. Michael Schmitz as guest and a March meeting with a Student Forum. April was a busy month with a Mixer, Honor Banquet, and a field trip to St. Louis. A Math Club - <u>KME</u> picnic was held May 1, 1984 for faculty and students. Other 1984-85 officers: Greg Oberlag, vice president; Pat Winkler and Jeff Nettles, secretary; Ken Mills, Treasurer.

<u>Illinois Zeta</u>, Rosary College, River Forest
Chapter President - Sheila Ruh-Schultze
8 actives, 5 initiates

Illinois Zeta chapter initiated five new members on March 20, 1984. Other 1984-85 officers: Mary Martins, secretary; Lisa Behnke, treasurer; Sister Nona Mary Allard, corresponding secretary and faculty sponsor.

<u>Illinois Eta</u>, Western Illinois University, Macomb
Chapter President - Tamara J. Lakins
10 actives, 3 initiates

The main activity for Spring 1984 was the initiation held on May 4. Three new members were initiated. The faculty was invited, and a dinner was held following the initiation. Other 1984-85 officers: Doug Faries, vice president; Lissa Pope, secretary and treasurer; Alan A. Bishop, corresponding secretary; Iraj Kalantari, faculty sponsor.

<u>Iowa Alpha</u>, University of Northern Iowa, Cedar Falls
Chapter President - Kande Hooten
28 actives, 4 initiates

The following students presented papers at Iowa Alpha meetings: Lisa Naxera on "Negative Bases," Mark Gross on "Hamilton's Quaternions," and John Holm on

"Mathematics of Interest." Professor Fred W. Lott, a past national president of KME and longtime supporter of Iowa Alpha, retires from UNI this year. He and Mrs. Lott were guests of Iowa Alpha at the spring initiation banquet. The chapter sold "Academic T-shirts" this semester and made a small profit. Other 1984-85 officers: Lisa Naxera, vice president; Kelly Donlin, secretary; Scott Kibby, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines
Chapter President - Scott Rothfus
16 actives, 2 initiates

Other 1984-85 officers: Tracy Parks, vice president; Sheryl Shapiro, secretary; Ruth Gornet, treasurer; Joseph Hoffert, corresponding secretary; Lawrence Naylor, faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President - Sue Pyles
44 actives, 10 initiates

The Spring Semester began with a banquet and initiation for the February meeting. Ten new members were initiated at that time. After the initiation Brad Averill gave a talk on "Integer Linear Programming." Dr. Elwyn Davis, Mathematics Department faculty member, spoke at the March meeting. His talk, "Ivory Tower Meets Real World," related some of his experiences he had at Phillips Petroleum Company while on sabbatical leave in the Spring of 1983. Two student papers were given for the April program. Hazel Coltharp discussed "A Program for the Surgical Watch Schedule at the U.S. Naval Hospital, San Diego, California" and Earlena Brownewell gave a historical talk on "The Development of Probability Theory." The chapter also assisted the Mathematics Department faculty in administering and grading tests given at the annual Math Relays, April 17, 1984. Several members worked for the Alumni Association's Spring phon-o-thon. Their efforts were well rewarded when it was announced that Kansas Alpha won the top prize of $100 for the most money raised by student organizations. David Pennington received a $25 prize for one of the individual awards given. The final meeting of the semester was a social event held at Professor Thomas' home. It was highlighted by election of officers for the 1984-85 school year. In addition, the annual Robert M. Mendenhall awards for scholastic achievement were presented to Brad Averill, Debbie Birney, and Mark Million. They received KME pins in recognition of this honor. Other 1984-85 officers: David Pennington, vice president; Earlena Brownewell, secretary; Tami Dodds, treasurer; Harold L. Thomas, corresponding secretary; Helen Kriegsman and Gary McGrath, faculty sponsors.

Kansas Gamma, Benedictine College, Atchison
Chapter President - Mary Jo Muckey
19 actives, 20 initiates

The first activity the chapter sponsored during the spring semester was a Computer Dance held on January 21. Everyone who attended had a good time and was provided with many opportunities to meet people with the same interests. In February, induction was held. There were eight new members inducted into Kansas Gamma. There were also 18 students who became Associate members. For a fun activity in March, the chapter organized a team to participate in a Volleyball tournament to support students who work in a Mexican orphanage during spring break. April was a busy month. On April 7, the 14th Math Tournament was conducted. Later in the month, a guest speaker, Larry D. Schultz, presented a lecture entitled, "Fourier Transforms: Applications and Extensions." The final activity for the semester, held on April 27, was a picnic in honor of the seniors. At the spring Honors Banquet, Beverly Weishaar was presented the Sister Helen Sullivan Scholarship Award for the 1984-85 academic year. Other 1984-85 officers: Rita Lundstrom, vice president; Patti Patterson, secretary and treasurer; Sister Jo Ann Fellin, corresponding secretary and faculty sponsor.

<u>Kansas Delta</u>, Washburn University, Topeka
Chapter President - Doug Bogia
22 actives

Ward Canfield presented a paper to the chapter on "Computing the Relative Standings of Teams Who Do Not Play Each Other in a Tournament or League." Other 1984-85 officers: Ward Canfield, vice president; Regina Estes, secretary and treasurer; Robert Thompson, corresponding secretary; Allan Riveland, faculty sponsor.

<u>Kansas Epsilon</u>, Fort Hays State University, Hays
Chapter President - Bev Musselwhite
26 actives, 8 initiates

Larry Burchett of Union Wire Rope, Kansas City, gave a talk to the chapter on March 5, 1984. A banquet was held April 5, 1984. Jeff and Barbara Barnett discussed their trips to the People's Republic of China. Other 1984-85 officers: Todd Deines, vice president; Michelle Ferland, secretary and treasurer; Charles Votaw, corresponding secretary; Jeffrey Barnett, faculty sponsor.

<u>Maryland Alpha</u>, College of Notre Dame of Maryland, Baltimore
Chapter President - Michele Ritter
8 actives, 4 initiates

In February, the Third Mathematics Olympiad was held. This is a contest conducted by the Society for High School Girls of the Junior and Senior Years. In May, the chapter had its annual dinner and initiation of new members into Maryland Alpha. There were four new inductees and eight temporary members were introduced to <u>KME</u>. After dinner, two alumnae discussed their respective careers with IBM and USF&G. Other 1984-85 officers: Donna Woods, vice president; Nikki Simmers, secretary; Lori Starnes, treasurer; Sister Marie Augustine Dowling, corresponding secretary.

<u>Maryland</u> <u>Beta</u>, West Maryland College, Westminster
Chapter President - Cliff Martin
21 actives

Other 1984-85 officers: Steve Coffman, vice president; Julie Winkler, secretary; Wendy Reeser, treasurer; James E. Lightner, corresponding secretary.

<u>Maryland</u> <u>Delta</u>, Frostburg State College, Frostburg
Chapter President - Kurt Lemmert
26 actives, 13 initiates

Chapter members aided the FSC Math Department in hosting the fourteenth annual mathematics symposium in April. In February and March, talks were presented to <u>KME</u> by math faculty members Mr. Revennaugh and Dr. White. Mr. Revennaugh introduced students to census bureau procedures for obtaining data on sensitive issues. His talk was titled "Sensitive Sampling." Dr. White's talk was titled "Perfect Code? An Application to Cryptography." Thirteen students entered MD Delta Chapter at a dinner-inducted ceremony in February. The semester activities terminated with the usual <u>KME</u>, ACS, Physics Club picnic. Other 1984-85 officers: Lisa McIntosh, vice president; Alana Fatkin, secretary; Theresa Neville, treasurer; Don Shriner, corresponding secretary; John Jones, faculty sponsor.

<u>Massachusetts</u> <u>Alpha</u>, Assumption College, Worcester
Chapter President - Roger Martin
7 actives, 3 initiates

Three new members were initiated on April 24, 1984. Following a dinner in honor of the new members, Professor Vincent Cioffari, of the Assumption faculty, spoke on "Calendars and the Determination of Easter." Other 1984-85 officers: Virginia Chazotte, vice president; Karen Higgins, secretary; Charles Brusard, corresponding secretary and faculty sponsor.

Michigan Beta, Central Michigan University, Mt. Pleasant
Chapter President - Larry Ludwig
60 actives, 24 initiates

Much of the spring semester was spent in preparation for the hosting of the regional convention at CMU on April 27. Many committees were formed and the members found the planning to be a worthwhile experience. Professor Don Fisk of CMU spoke at the Spring Initiation on a Random Walk Problem. The Regional Convention had Professor Andras Kroo as guest speaker. He is a Visiting Professor in Numerical Analysis at CMU from the Hungarian Academy of Sciences. Student members Paul O'Dea and Greg Green of CMU were the student speakers. During the semester, the chapter provided mathematics help for freshman-sophomore mathematics students. Other 1984-85 officers: Mike Vieau, vice president; Deidre Lentz, secretary; Lisa Burnell, treasurer; Arnold Hammel, corresponding secretary and faculty sponsor.

Mississippi Alpha, Mississippi University for Women, Columbus
Chapter President - Susan Furlow
17 actives

Other 1984-85 officers: Rebecca Flowers, vice president; Elizabeth Douglas, treasurer; Jean A. Parra, corresponding secretary; Carol Ottinger, faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President - John McDonald
30 actives, 10 initiates

Other 1984-85 officers: Twila Hendry, vice president; Deanne Caveny, secretary; Alice Essary, corresponding secretary.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President - Shari Birkenbach
55 actives, 16 initiates

Other 1984-85 officers: Joy Farr, vice president; Mary Smith, secretary; Mike Kimzey, treasurer; M. Michael Awad, corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President - Lynn Hitchcock
26 actives, 28 initiates

The chapter held 4 regular meetings with one initiation during the Spring Semester with guest speakers Dr. Cammack, Dr. Grimes, and Dr. Goodman from the CMSU Mathematics Department, and Dr. Patterson from the CMSU English Department. Other activities included an Honors Banquet, the Annual Klingenburg Lecture, and a field trip to Hallmark in Kansas City, MO. Other 1984-85 officers: Roberto Ribas, vice president; Cheryl Harris, secretary; Brenda Enke, treasurer; Homer F. Hampton, corresponding secretary; Larry Dilley and Gerald Schrag, faculty sponsors.

Missouri Gamma, William Jewell College, Liberty
Chapter President - Eric Conrad
14 actives, 9 initiates

The chapter met five times during the year plus a meeting at the annual initiation ceremony and banquet. Presentations were given by students in the chapter at each meeting. The banquet, held in April, had speaker Dr. Summerhill, Chairman of the Mathematics Department at Kansas State University. Other 1984-85 officers: Brian Wells, vice president; Laurie Honeyfield, secretary and treasurer; Joseph T. Mathis, corresponding secretary and faculty sponsor.

Missouri Epsilon Central Methodist College, Fayette
Chapter President - Cheryl Mathewson
8 actives, 7 initiates

Other 1984-85 officers: Stacy Garrett, vice president; Robin Hamil, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville
Chapter President - Bob Clark
40 actives, 9 initiates

The chapter conducted the Spring Math contest for area high schools with 800 entries. In addition, 80 invited high school seniors attended Math Expo, which included a guest lecturer. Volleyball games between KME and the mathematics faculty were held. Members also aided with Special Olympics and held a bake sale to raise money. Other Spring activities included initiation, picnic, and presentations by the six seniors. Other 1984-85 officers: Nancy Schmidt, vice president; Yvonne Hall, secretary; Rebecca Hutton, treasurer; Sam Lesseig, corresponding secretary; Mary Sue Beersman, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President - Annette Schmit
29 actives, 17 initiates

To make money throughout the spring semester, club members have monitored the Math-Science Building in the evenings. The club administered the annual test to identify the outstanding freshman majoring in mathematics. The award went to Tammy Strand whose home town is Hooper, Nebraska. The award includes the recipient's name being engraved on a permanent plaque, payment of KME National dues, one year honorary membership in the local KME chapter, and announcement of the honor at the annual spring banquet. At the annual spring banquet, sponsor Dr. Hilbert Johs of the mathematics department was named Outstanding Professor in the Mathematics-Science Division for the 1983-84 academic year. The selection of the Outstanding Professor is by secret ballot where students majoring or minoring in the sciences and mathematics are eligible to vote. Speaker at the annual banquet was Mr. John Wrenholt who is the director of the Big Apple Computer Club of Norfolk, Nebraska. Annette Schmit was awarded the $25.00 book scholarship which is given to a KME member each semester. The local KME chapter sponsored two College Bowl teams in April. One team was eliminated in the semifinals and the other was eliminated in the finals. Team participants were Doug Anderson, Kelli Goodner, Marlyn Roth, Jim Urbanec, Kurt Meisinger, Mike Rood, Val Collins and Annette Schmit. The chapter also contributed to the Juvenile Diabetes Foundation by auctioning off four car washes. Club members assisted the Wayne State College mathematics faculty with the Tenth Annual WSC Mathematics Contest on May 10, 1984, kept the KME bulletin board current, and sponsored some social functions for club members. Other 1984-85 officers: Lori Schulenberg, vice president; Sandy Sunderman, secretary and treasurer; Jerry Wiesler, historian, Fred Webber, corresponding secretary; James Paige and Hilbert Johs, faculty sponsors.

Nebraska Beta, Kearney State College, Kearney
Chapter President - Christine L. Moses
16 actives, 5 initiates

KME nominated Sharon Hostter as Outstanding Senior and she was selected as Kearney State College's Outstanding Senior Woman. Laura Issac was awarded the Lydia Butler Scholarship. In May, KME sponsored a Senior Tea for all graduating Math, Statistics and Computer Science seniors. Angela Vlasin was elected Membership Chairman and Shelly Black was elected Historian of the chapter. Other 1984-85 officers: Gloria Liljestrand, vice president; Mary Morris, secretary; Laura Issac, treasurer; Charles Pickens, corresponding secretary; Nelson Fong, faculty sponsor.

Nebraska Gamma, Chadron State College, Chadron
Chapter President - Annette Wiemers Stumf
16 actives, 3 initiates

The 1983-1984 Freshman Award in Mathematics was presented to Terri Scofield by KME during the Ivy Day ceremonies on May 3, 1984. A fund raising activity organized by KME raised $78.78 during Spring Daze Week at Chadron State College. Other 1984-85 officers: David Mundt, vice president; Steven Dent, secretary; Gene McDowell, treasurer; James Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

New York Eta, Niagara University, Niagara
Chapter President - Chris Reilly
18 actives, 4 initiates

The main spring activity was the annual banquet and initiation held this year in April. Students raised money selling raffle tickets, and part of the proceeds were used at a year-end party attended by KME members as well as math club members and faculty. Other 1984-85 officers: Ray Muller, vice president; Amy Davis, secretary; Mary Ertl, treasurer; Robert Bailey, corresponding secretary.

Ohio Gamma, Baldwin-Wallace College, Berea
Chapter President - Diane Kopacko
20 actives, 9 initiates

Other 1984-85 officers: Jim Kerr, vice president; Janelle Keberle, secretary; John Kirchner, treasurer; Robert Schlea, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President - Doug Walters
25 actives, 13 initiates

Spring semester activities included a guest speaker from Texas Tech, initiation of 13 new members, helping with math tests given to high school students and an end of school picnic. Other 1984-85 officers: Mike Ragan, vice president; Latrrica Anderson, secretary and treasurer; Wayne Hayes, corresponding secretary.

Pennsylvania Alpha, Westminster College, New Wilmington
Chapter President - Barry Hall
35 actives, 19 initiates

Spring activities included Math Flicks night, initiation banquet, a picnic for old and new members, and tutorial sessions held twice each week. Other 1984-85 officers: David Gore, vice president; Andrea Marttala, secretary; Nancy Niccolls, treasurer; Barbara Faires, faculty sponsor.

Pennsylvania Beta, LaSalle University, Philadelphia
Chapter President - Karen Bruno
20 actives, 5 initiates

The chapter held a meeting April 25, 1984 to elect officers and to initiate new members. Other 1984-85 officers: Mary McGee, vice president; Lisa Tresnan, secretary; Leon Weiner, treasurer; Hugh N. Albright, corresponding secretary; Carl McCarty, faculty sponsor.

Pennsylvania Delta, Marywood College, Scranton
Chapter President - Ellen Filicko
7 actives

The chapter and the Marywood Mathematics Club sponsored the fourth annual high school math contest. Sr. Robert Ann Von Ahnen is corresponding secretary and faculty sponsor.

Pennsylvania Zeta, Indiana University of PA, Indiana
Chapter President - Sue Garrett
24 actives, 6 initiates

The chapter held initiation of new members in February. Dr. Arlo Davis, member of the Mathematics Department faculty, spoke about graduate school opportunities. Members participated in the Career Night program of the Mathematics Department in March. Several IUP math alumni told about their work experiences. Students in a Computer Science seminar presented a talk on "Graphics" at the April meeting. The Annual Spring Banquet was held in May. Students helped prepare the food under the guidance of Mr. Raymond Gibson, Mathematics Department faculty member. Dr. James Reber, Department Chairperson, gave a talk entitled "Saints and Sinners in the History of Calculus." Other 1984-85 officers: Allan Williams, vice president; Kelly Orndorff, secretary and treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President - Jim Kimpel
38 actives, 11 initiates

The Annual Spring picnic was held at the Grove City Park on Sunday, May 6. Volleyball, softball and charcoal broiled hamburgers made for a very enjoyable afternoon. Other 1984-85 officers: Mark Snavely, vice president; Shelly Crevar, secretary; Spencer Krummenacher, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Lambda, Bloomsburg University, Bloomsburg
Chapter President - Wayne Wilker
40 actives, 23 initiates

Fifteen members attended the regional convention on April 6-7 at Shippensburg University. Four papers were presented. The chapter is now trying to decide how to raise money to go to Dallas next spring. Other 1984-85 officers: Craig Funt, vice president; Lisa Cummings, secretary; Lori Nelson, treasurer; Jim Pomfret, corresponding secretary; Joe Mueller, faculty sponsor.

Pennsylvania Mu, St. Francis College, Loretto
Chapter President - Ivan Jareb
3 actives, 7 initiates

Other 1984-85 officers: Lori A. Pavlekovsky, vice president; Susan Andrews, secretary; Gerard Weaver, treasurer; Rev. John Kudrick, corresponding secretary; Adrian Baylock, faculty sponsor.

South Carolina Gamma, Winthrop College, Rock Hill
Chapter President - Kenneth Peay
9 actives, 4 initiates

Two members of KME deserve special mention. Chuck Baldwin had an article published in INCIDER and Pam Garrett was an Academic All American in women's basketball. Other 1984-85 officers: Phil Blankstein, vice president; Angie Breland, secretary; Norma Jean Hightower, treasurer; Donald Aplin, corresponding secretary; Edward Guettler, faculty sponsor.

Tennessee Beta, Eastern Tennessee State University, Johnson City
Chapter President - Greg Comer
6 actives, 14 initiates

The annual initiation service was held in the D.P. Culp University Center on campus. Dr. Jim Frank gave a talk concerning his industrial work in South America. Gregory Comer conducted the initiation service in which fourteen students were initiated. Other 1984-85 officers: Candy Tsiao, vice president; Christina Hutchins, secretary; Lyndell Kerley, corresponding secretary.

Tennessee Delta, Carson-Newman College, Jefferson City
Chapter President - Jeff Knisley
16 actives, 11 initiates

The chapter sponsored a viewing of a film on the development of FORTRAN. The spring initiation banquet was held at Casey's Restaurant, and the spring picnic was at Panther Creek State Park. Other 1984-85 officers: Jeff Kinsler, vice president; Art Blevins, secretary; Susan Williams, treasurer; Albert Myers, corresponding secretary; Carey Herring, faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene
Chapter President - Linda Haire
23 actives, 9 initiates

The chapter held its tenth annual induction banquet February 24, 1984. There were nine members inducted. With these new members, membership in the local chapter stands at 87. Mr. Kent Hurst, Instructor of Mathematics a McMurray College, addressed the chapter on the subject, "Gaining Satisfaction from Intellectual Inquiry." Special guests included Dr. Lawrence Clayton, Dean of the College of Arts and Sciences at H-SU, and his wife, Sonja. Other 1984-85 officers: Ben Barris, vice president; Donna George, secretary; Mike Cagle, treasurer; Mary Wagner, corresponding secretary; Charles Robinson and Ed Hewett, faculty sponsors.

Virginia Beta, Radford University, Radford
Chapter President - Sharon Goad
20 actives, 17 initiates

Other 1984-85 officers: Tamara Altice, vice president; Susan Weeks, secretary; Kathleen Cargo, treasurer; Coreen Mett, corresponding secretary; J. D. Hansard, faculty sponsor.

West Virginia Alpha, Bethany College, Bethany
Chapter President - Kris Kuzma
12 actives, 19 initiates

New members were recognized at the college Honors Program. They were initiated at a ceremony on May 2, 1984. Other 1984-85 officers: Paula Stabler, vice president; Lynn Sengewalt, secretary and treasurer; James Allison, corresponding secretary and faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President - Linda Schmidt
9 actives, 3 initiates

Three pledges were initiated into Wisconsin Alpha on April 26, 1984. Prior to initiation, each pledge had given a talk to the members and pledges of the chapter. Members viewed and discussed the short film "Donald Duck in Mathmagic Land." Other 1984-85 officers: Lisbeth Zaborske, vice president and treasurer; Linda Schmidt, secretary; Sister Adrienne Eickman, corresponding secretary; Sister Petronia Van Straten, faculty sponsor.

Wisconsin Gamma, University of Wisconsin-Eau Claire, Eau Claire
Chapter President - Susan Kelly
35 actives, 19 initiates

Other 1984-85 officers: David Hasse, vice president; Sue Krueger, secretary; Erin Kelley, treasurer; Tom Wineinger, corresponding secretary; Wilbur Hoppe and Bob Langer, faculty sponsors.

IF YOUR SUBSCRIPTION HAS EXPIRED

We hope you have found **THE PENTAGON** both interesting and helpful. Your suggestions are always welcome and may be written on this form. They will be forwarded to the Editor.

If you wish to renew your subscription for two years, please send $5* to **THE PENTAGON**, Department of Mathematics, Central Michigan University, Mt. Pleasant, Michigan 48859.

Please give your name below in the manner in which you wish your mail addressed (Miss, Mrs., Prof., etc.). If your name has changed, please indicate also the name to which **THE PENTAGON** was sent.

Name____

Address____

City______________________State_________Zip__________

*Foreign subscriptions require an additional $1 per year for postage.

REMINDER

The 25th Biennial convention of Kappa Mu Epsilon will be held on April 11-13, 1985 at Southern Methodist University, Dallas, Texas. Each chapter that sends a delegation will be allowed some travel expenses from National Kappa Mu Epsilon funds. Travel funds are disbursed in accordance with Article VI, Section 2 of the KME constitution.