THE PENTAGON

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NATIONAL OFFICERS

Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

MERSENNE PRIMES, THE 27TH MERSENNE PRIME* AND THE CRAY-1 COMPUTER

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A perfect number is any number such that it is equal to the sum of its aliquot divisors. An example of such a number is 6, which is equal to 1+2+3. Another is 28, equal to 1+2+4+7+14. Leonhard Euler proved that all even perfect numbers (no odd perfect number has been found yet)must be of the form $2^{n-1}(2^n-1)$ where 2^n-1 is a prime number. Numbers of the form 2^n-1 are known as Mersenne numbers (denoted M_n). If n is composite, M_n must be composite. Thus for M_n to be prime, n must also be prime. Prime Mersenne numbers are naturally known as Mersenne Primes. Consequently, the search for even perfect numbers is narrowed down to searching for Mersenne Primes.

Mersenne numbers are named after Father Marin Mersenne (1588-1648), a French monk who made a provocative assertion concerning their primality. In 1644, he asserted that M_p is prime for p=2,3,5,7,13,17,19,31,67, 127,257 and composite for all other primes $p < 257$.[1] Mathematicians knew Mersenne couldn't have possibly tested the primality of all these numbers, but neither could they. In 1772, Euler determined M_{31} to be prime.

*A paper presented at the 1981 National Convention of KME and awarded first place by the Awards Committee.

Other than that, it was not until 1947, 303 years after Mersenne made his assertion, that he stood corrected. It was found that Mersenne made five mistakes: M_{67} and M_{257} were not prime and he excluded M_{61}, M_{89}, and M_{107}, which were prime, from his list.

It is rather astonishing that it took over three centuries to set the good friar straight. Back in Mersenne's time, the only way to test the primality of a number was to test every prime divisor of that number (because an integer is either prime or a product of primes). Cataldi later improved the test for a number n by testing only prime divisors up to $[\sqrt{n}]$. This is because $[\sqrt{n}]^2 \leq n$ and if the first divisor found is greater than $[\sqrt{n}]$, then the second must be also which implies that their product is greater than n--an impossibility. Finally, in 1876, Edouard Lucas developed the sequence $r_1=3$, $r_2=7$, $r_3=47$, $r_4=2207, \ldots, r_{m+1}=r_m^2-2$. This sequence allowed Lucas to develop a criterion for testing the primality of certain Mersenne numbers.

> <u>Lucas Criterion</u>: If prime $p=4n+3$, and $M_p=2^p-1$ is the corresponding Mersenne number, then M_p is prime if and only if $r_{p-1} \equiv 0 \pmod{M_p}$ and otherwise composite.[3]

The notation $r_{p-1} \equiv 0 \pmod{M_p}$ implies M_p divides

r_{p-1}-0. For example 15 ≡ 6 (mod 3) because 3 divides 15-6.

The Lucas test works only for M_p where p is of the form 4n+3. In 1930, D. H. Lehmer improved the test so that it would work for a prime p of any form. For the Lucas-Lehmer test a new sequence was used -- 4,14,194, ..., $r_{m+1}=r_m^2-2$. As before M_p is prime iff it divides r_{p-1}.

To present an example of the Lucas-Lehmer test, let us look at p=3. $M_3=7$ should divide term (p-1) or the 2nd term of the sequence 4,14,... . Since 7 divides 14, M_3 is prime.

Clearly, Mersenne primes as well as terms of the Lucas-Lehmer sequence explode into huge numbers. The tenth term of the sequence has 293 digits and the 388th term has approximately 1.8×10^{116} digits. Mersenne had stated that all eternity wouldn't suffice to tell if a 15 or 20 digit number is prime.[4] However, in 1953, the National Bureau of Standards Western Automatic Computer (SWAC) used the Lucas-Lehmer test to analyze 42 Mersenne numbers, the smallest of which had 80 digits. R. M. Robinson and D. H. Lehmer were stunned to see SWAC do in 48 seconds what had taken Lehmer over 700 hours, using a desk calculator (2 hrs. per day for a year), 20 years earlier: prove that

$2^{257}-1$ is composite. Consequently, Lehmer calculated that each minute with SWAC was equivalent to one year of work by a person with a desk calculator. Thus when SWAC took 13.5 minutes to determine $2^{1279}-1$ prime, it did a job that would have taken 125 years for a human.

Overall, SWAC found five new Mersenne primes. The 13th through the 17th Mersenne primes (M_p) are for values of p=521, 607, 1279, 2203, and 2281. Because the Lucas-Lehmer test was well suited for high-speed computers, large Mersenne primes were found rather quickly. In 1961, the Lucas-Lehmer test took 50 minutes on an IBM 7090 to find the 20th Mersenne prime, M_{4423}. Ten years later, an IBM System/360 took 40 minutes to find the 24th Mersenne prime, $2^{19937}-1$. In 1978, the 25th and 26th Mersenne primes, $2^{21701}-1$ (a 6533 digit no.) and $2^{23209}-1$, were found on a CDC Cyber 174. The respective CPU time for the Lucas-Lehmer test on these two new primes was 7 hours, 40 minutes and 8 hours, 39 minutes.[5]

Finally, David Slowinski went searching for the 27th Mersenne prime. His computer, commercially available at $7500 per hour, was the Cray-1. The arithmetic of the Lucas-Lehmer test is ideally performed on the Cray-1. To test a Mersenne number M_p usually requires about p^3 operations. The Cray-1 is a vector computer rather than a scaler computer. Scaler computers perform one operation at a time and then store the result in memory, necessitating a recall from memory. Vector computers perform multiple computations without submitting intermediate results to memory. This processor reduced the squaring portion of the Lucas-Lehmer test to a series of vector loads, multiplies, shifts, and adds, such that 80 million load-multiply-shift-adds can be done per second. The mod function is also fast on the Cray-1. For a binary number y (at most 2p bits long), y (mod M_p) is simply the sum of the low-order p bits of y and its high-order bits. For example, 12345678 mod (9999) is 6912, which is 1234 + 5678.[6]

Slowinski applied the Lucas-Lehmer test to about a thousand numbers and determined in April of 1979 that $2^{44497}-1$ is the 27th Mersenne prime. It has over 13,000 digits and its associated perfect number has over 26,000 digits.

Historically, it has required about four times as much computation to discover the next Mersenne prime as it would to rediscover all previously known Mersenne primes. Thus, searching for Mersenne primes has been an accurate measure of computing power for over 200 years.[6] Consequently, the Cray-1 must be a power to be reckoned with in the computer industry.

The Cray-1 has tremendous speed when compared with other computers. This was demonstrated when the Lucas-Lehmer test for M_{8191} was performed. The test took 100 hours on an Illiac I, 5.2 hours on an IBM 7090, 49 minutes on an Illiac II, 3.17 minutes on an IBM 360, and 10 seconds on the Cray-1. Also the 26th Mersenne prime, which was found in 8 hours, 39 minutes on a Cyber 174, was found on the Cray-1 in less than seven minutes.[6]

There are many engineering reasons why the Cray-1 is so fast. First, it is a vector processor rather than scaler. Secondly, all wires in the computer are straight so as to minimize signal time between arbitrary points. Electronic signals travel faster in a straight wire than they would in a wire that was bent or curved. The Cray-1 has an average speed of 80 million floating-point operations per second (80 MFLOPS). It has been pushed to 140 MFLOPS and clocked in short bursts at 250 MFLOPS. Comparatively, the IBM 360 and Cyber 176, each superfast by former standards, have a speed of only four to seven MFLOPS.[7]

The longest wires in the Cray-1 are four feet. The design for its successor, the Cray-2, will have one foot as the length of the longest wire. Thus, the Cray-2 would be twice as fast and four times as efficient as the Cray-1. This leaves little doubt, with computers like the Cray-2, that the search for new Mersenne primes will continue. Perhaps the 28th Mersenne prime is not far away.

NOTES

[1]Elementary Number Theory, David M. Burton, Allyn and Bacon, Inc., 1980, pp. 225-6.

[2]Solved and Unsolved Problems in Number Theory, Daniel Shanks, Chelsea Publishing Co., 2nd Ed., 1978.

[3]An Introduction to the Theory of Numbers, Hardy & Wright, Oxford Press, 4th Ed., 1960.

[4]Recreations in the Theory of Numbers,Albert H. Beiler, Dover Publications Inc., 1964, pp. 11-25.

[5]"The 25th and 26th Mersenne Primes", Mathematics of Computation, Oct. 1980, pp. 1387-90.

[6]"Searching for the 27th Mersenne Prime", David Slowinski, Journal of Recreational Mathematics, Vol. 11 (4), 1978-79, pp. 258-61.

[7]"Supercomputer", Popular Science, June 1979, pp. 86-9.

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THE EIGHT POINT CIRCLE*

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Student, University of Northern Iowa

Most geometrical properties, postulates and theorems are rooted in ancient times. Knowledge of practical geometry is evident in the designs of Egyptian pottery as early as 4000 B.C. Ancient Babylonians and Egyptians used practical geometry in their irrigation and construction projects. A Greek philosopher, Thales, was the first to use logic in studying geometric abstractions in about 600 B.C. Also Euclid combined the geometric knowledge of his time into a "postulate system" around 300 B.C. Advances in the field, however, did not terminate. One well-known theorem of the 19th century is that of the nine point circle, which says that there are nine points associated with a triangle which always lie on a circle. These points are the midpoints of the sides, the feet of the altitudes, and the midpoints of the segments joining the orthocenter to the vertices. The center of the circle is the midpoint of the segment connecting the orthocenter with the circumcenter (Figure 1).

An even more recent and much less well known theorem, however, is the Eight Point Circle Theorem dis-

*A paper presented at the 1981 National Convention of KME and awarded second place by the Awards Committee.

covered in 1944 by Lois Brand. The Theorem asserts that there are a certain eight points associated with any quadrilateral having perpendicular diagonals which lie on a circle. These eight points are the midpoints of each side and the feet of the perpendiculars from the midpoint of one side to the opposite side. To prove that the eight points are on a common circle the

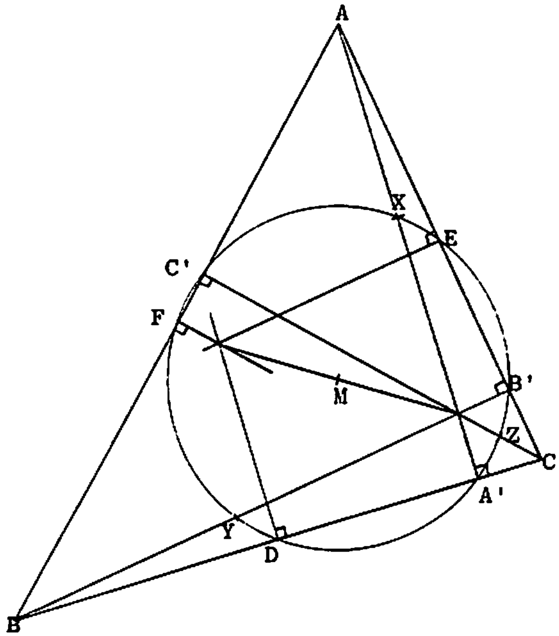

Figure 1

following theorems are needed.

1. Let ABCD be any convex quadrilateral; let E, F, G, and H be midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$; then EFGH is a parallelogram.
2. Given $\triangle ABC$ and midpoints D and E of $\overline{AB}$ and $\overline{BC}$, respectively, then $\overleftrightarrow{DE} \parallel \overleftrightarrow{AC}$ and $DE = \frac{1}{2}AC$.
3. Given $l \parallel m$ with transversal n intersecting l and m, also $n \perp l$, then $n \perp m$.
4. The diagonals of a rectangle bisect each other.
5. Parallelogram Property Theorem: Given parallelogram ABCD, then
   1) $AB = CD$ and $BC = AD$
   2) $\angle BAD \cong \angle BCD$ and $\angle ABC \cong \angle ADC$ and
   3) $\overline{AC}$ and $\overline{BD}$ bisect each other.
6. Side Angle Side Triangle Congruence: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, the triangles are congruent.
7. The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.

Starting with quadrilateral ABCD having perpendicular diagonals $\overline{AC}$ and $\overline{BD}$, P, Q, R, S, P', Q', R', S' should lie on a circle with center M (Figure 2). To

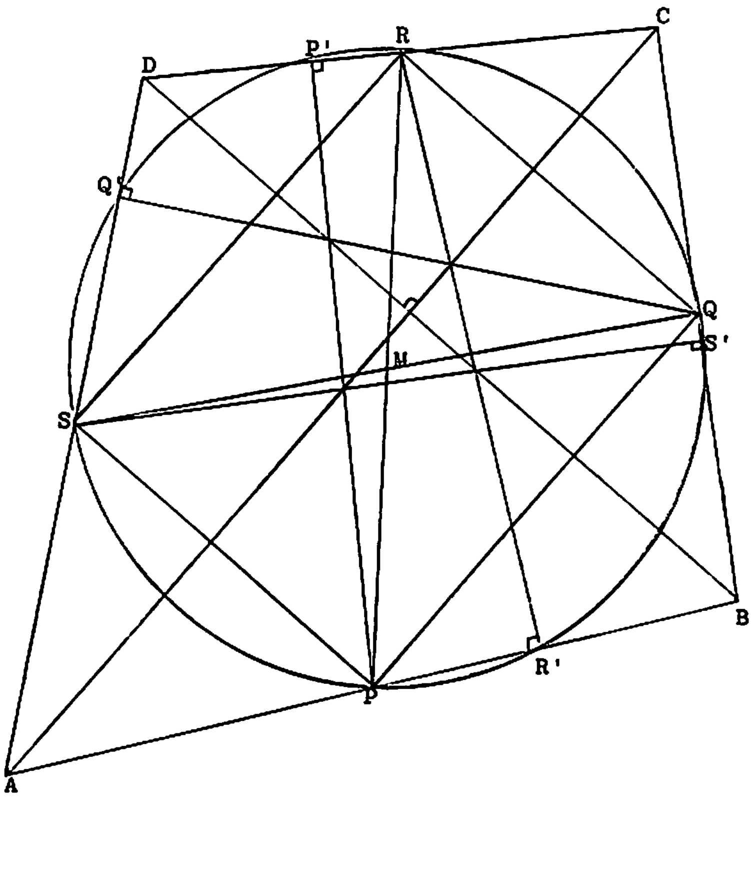

Figure 2

begin, connect the midpoints of the given quadrilateral to form PQRS which is a parallelogram by Theorem 1 stated above. Then $\overleftrightarrow{RQ}$ is parallel to $\overleftrightarrow{DB}$ because the segment joining the midpoints of two sides of a triangle is parallel to the third side (Theorem 2). Also $\overleftrightarrow{SP}$ is parallel to $\overleftrightarrow{DB}$. By applying the same theorem to triangles ACD and ACB it follows that $\overleftrightarrow{SR}$ and $\overleftrightarrow{PQ}$ are parallel to $\overleftrightarrow{AC}$. But $\overline{AC}$ is perpendicular to $\overline{DB}$. Therefore, $\overline{SR} \perp \overline{RQ}$, $\overline{RQ} \perp \overline{QP}$, $\overline{PQ} \perp \overline{SP}$, and $\overline{PS} \perp \overline{SR}$ because a line perpendicular to one of two parallel lines is perpendicular to the other (Theorem 3). As a result PQRS is a rectangle and the diagonals, $\overline{SQ}$ and $\overline{RP}$, bisect each other (Theorem 4). Then by the definition of a bisector, $\frac{1}{2}SQ = SM = MQ$ and $\frac{1}{2}PR = RM = MP$. In addition, applying the Parallelogram Property Theorem (5) reveals that SP = RQ. Since PQRS is a rectangle it has four right angles so angle SPQ is congruent to angle RQP and by the reflexive principle, PQ = QP. Therefore, triangle SPQ is congruent to triangle RQP because of the Side Angle Congruence Theorem (6). Thus SQ = PR because corresponding parts of congruent triangles are congruent. Then $\frac{1}{2}SQ = \frac{1}{2}PR$, so SM = RM = QM = PM and M is the center of a circle passing through P, Q, R, and S. Also triangle PRP' is a right triangle and M is the midpoint of $\overline{PR}$ which is the hypotenuse. As a result M is equidistant from the three vertices, P, R,and P' (Theorem 7). Consequently the circle with its center at M and passing through points P, Q, R, and S also passes through P'. Similarly, considering triangle QSQ', triangle RPR',and triangle SQS' it is evident that Q', R', and S' also lie on this circle. Therefore, the eight points P, Q, R, S, P', Q', R', and S' determine a circle with center at M which is the point of intersection of the segment joining the midpoints of the opposite side of the given quadrilateral.

The Eight Point Circle Theorem also holds true for nonconvex quadrilaterals. A particular case of such a quadrilateral, ABCH, can be found in a triangle with orthocenter H which has perpendicular diagonals $\overline{AC}$ and $\overline{BH}$ (Figure 3). The midpoints of the sides are A', C', X, and Z. The perpendicular from C' meets the opposite side, $\overline{CH}$ at F, from A' the perpendicular intersects $\overline{AH}$ at D, the perpendicular to $\overline{AB}$ from Z meets the side at F, and from X the perpendicular constructed to $\overline{BC}$ meets $\overline{BC}$ at D. Since F And D occur twice, the eight point circle with its center at the intersection of the diagonals of C'A'ZX, the parallelogram formed by connecting the midpoints of the four sides of quadrilateral ABCH, is reduced to a six point circle (A',C',X,Z,F,D).

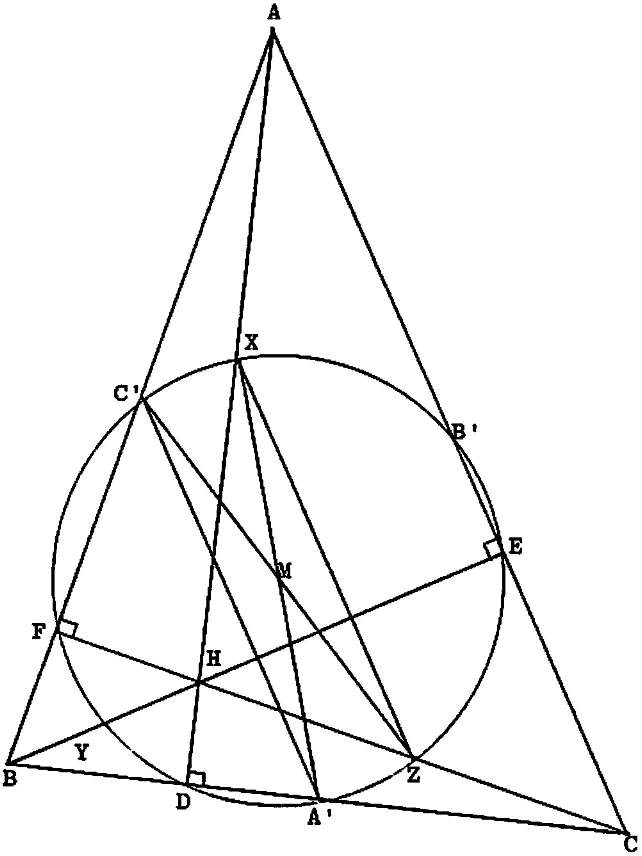

Figure 3

Now consider another convex quadrilateral ACBH in the same figure. Again the eight point circle is reduced to a six point circle. This circle has center M and passes through A', B', X, Y, D, and E. These two point circles have the points, A', X, and D, in common and since these points determine a circle, the two circles are the same. Therefore the famous nine point circle can be viewed as two common eight point circles of ABCH and ACBH.

Another consequence of the Eight Point Circle Theorem can be seen by looking back at quadrilateral ABCD in Figure 2. It is known that M is the midpoint of the diameter $\overline{PR}$. Also P and R are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively. Suppose a unit mass was hung at each of the four vertices, A, B, C, and D. If the two masses from A and B were suspended at the midpoint, P, and the equal masses from C and D were moved to its midpoint, R, the balance would be equivalent. Therefore, the center of gravity is the midpoint of $\overline{PR}$ and the center of the eight point circle, M.

Now consider quadrilateral ABCH in Figure 4. By applying the above result, the center of gravity of ABCH is M, the center of the associated eight point circle. G is the intersection of the medians of triangle ABC and is known as the centroid or center of gravity. Consequently, a unit mass at each of A, B, and C is equivalent to three units suspended at G. Now quadrilateral ABCH is balanced with one unit at H and three units at G and since M is the center of gravity, it

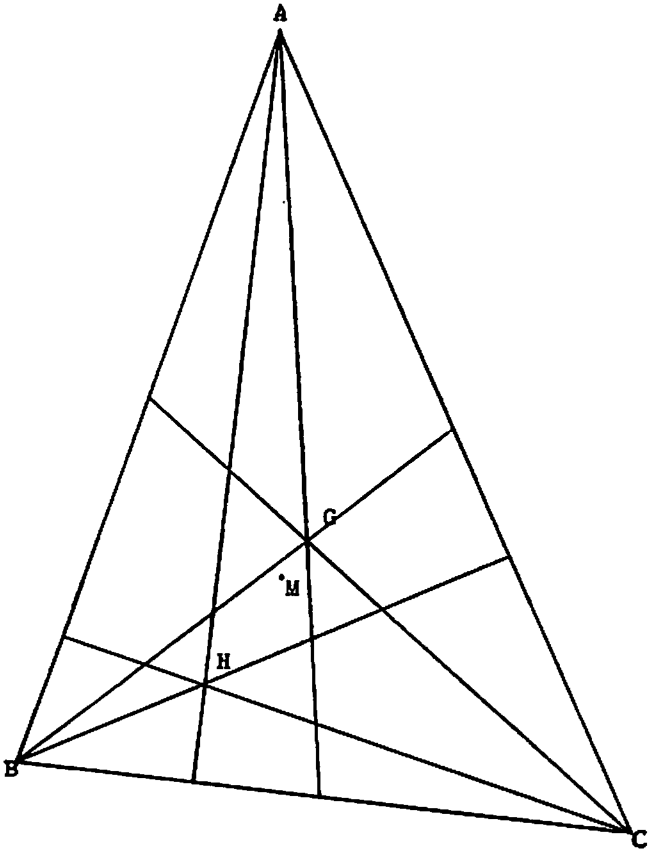

Figure 4

must lie on $\overline{HG}$. Recall that the distance from the suspended weight to the center of gravity times the weight at that point is equal to the product of the opposite weight and distance, $d_1w_1 = d_2w_2$ (Figure 5).

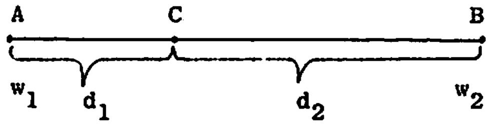

Figure 5

Using this in the situation in Figure 4 results in the equation, HM = 3MG. Then M divides $\overline{HG}$ into the ratio 3:1 or the center of the eight point circle is one-third of the distance from the orthocenter to the centroid.

For centuries mathematicians have been combining and applying ancient theorems to achieve new geometric results. The Eight Point Circle Theorem is one of the most recent of these fascinating discoveries which associates a standard figure with any odd shaped quadrilateral having perpendicular diagonals.

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THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 February 1982. The solutions will be published in the Spring 1982 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 332: Proposed by Charles W. Trigg, San Diego, California.

In the cyrptarithm

COOK+COOK+COOK=MEAL

each distinct letter represents a different digit, and MEAL is a permutation of consecutive digits. Find the unique restoration of this decimal addition.

Problem 333: Proposed by Charles W. Trigg, San Diego, California.

When asked the age of his brother,Ralph replied, "Jim's age, like mine, is one more than eight times the sum of its digits." How old is Jim?

Problem 334: Proposed by Charles W. Trigg, San Diego, California.

In the decimal scale, find a three-term arithmetic progression of three-digit primes in which the first and last terms are permutations of the same set of digits and the three digits of the middle term are distinct.

Problem 335: Proposed by the editor.

Given a point P on one side of triangle ABC, construct a line through P which divides the triangle into two regions of equal area.

Problem 336: Proposed by Fred A. Miller, Elkins, West Virginia.

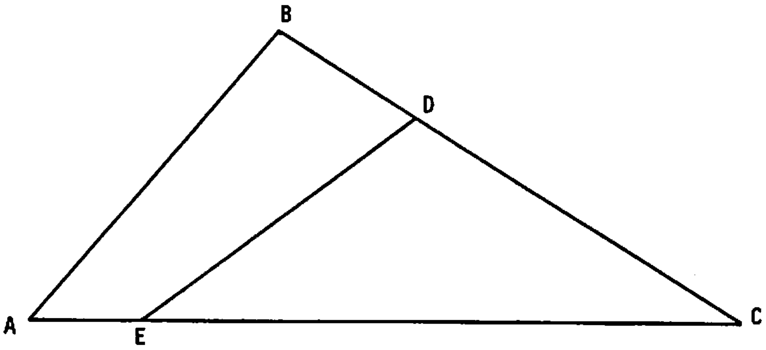
Let A, B, C and D be any four concyclic points such that C and D are separated by A and B. If P_1, P_2 and P_3 denote the lengths of the perpendiculars from D to the lines AB, BC and CA respectively, show that

$$\frac{AB}{P_1} = \frac{BC}{P_2} + \frac{CA}{P_3}$$

SOLUTIONS

322. *Proposed by John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico.*

In ΔABC, AB=25, AC=56, points D and E lie on BC and AC respectively. Also AE=8, BC=3BD and DE=26. Calculate the length of BC.



(Figure 1)

Composite of solutions submitted independently by Carole LaRoche, Bloomsburg State College, Bloomsburg, Pennsylvania, and Thomas E. Cosby, Tennessee Alpha Chapter, Tennessee Technological University, Cookeville, Tennessee.

Since BC=3BD, let BD=x so that DC=2x and BC=3x. Then since angle C is a common angle in each triangle, the law of cosines yields

in triangle ABC, $$\cos C = \frac{56^2-25^2+(3x)^2}{2(56)(3x)} \qquad (1)$$

and in triangle EDC, $$\cos C = \frac{48^2-26^2+(2x)^2}{2(48)(2x)}\,. \qquad (2)$$

Equating equations (1) and (2) yields

$$8x^2 = 1352 \text{ or } x = 13. \qquad (3)$$

Hence BC = 39.

Also solved by Simon Jin Jokwi, University of Alberta, Edmonton Alberta, Canada, James McCann, Bowling Green State University, Bowling Green, Ohio, Fred A. Miller, Elkins, West Virginia, Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin, Mark Schultz, University of Wisconsin-Marathon County, Wausau, Wisconsin, Robert A. Stump, Hopewell, Virginia, Charles W. Trigg, San Diego, California, Diana Wilson, University of Missouri-Rolla, Rolla, Missouri, and the proposer.

323. Proposed by Michael W. Ecker, Pennsylvania State University, Worthington-Scranton Campus, Scranton, Pennsylvania.

Let N be the set of natural numbers and define a function d(n) as follows:

$$d(n) = \det \begin{bmatrix} 1 & 2 & \cdots & n \\ n+1 & n+2 & \cdots & 2n \\ & & & \\ \cdots & & & n^2 \end{bmatrix}$$

the determinant of the nxn matrix shown. Find a formula for d(n).

Solution by Matt Maggio, Elon College, Elon College, North Carolina.

Let d(n) denote the value of the given determinant corresponding to a given value of n. It is easy to verify that d(1) = 1 and d(2) = -2. For n > 2 a formula for d(n) can be established by performing elementary row operations on the determinant as follows: by subtracting row 1 from row 2 the new row $[n \ \ n \ \ldots \ n]$ is formed.

By subtracting row 2 from row 3, the new row $[n \; n \; \ldots \; n]$ is formed. Now since the given determinant is equivalent to a new determinant formed by replacing the first two rows of the given determinant with the two rows found above, $d(n) = 0$ for all $n > 2$ because if two rows of a determinant are identical the value of the determinant is zero.

Also solved by James McCann, Bowling Green State University, Bowling Green, Ohio, Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin, Mark Schultz, University of Wisconsin-Marathon County, Wausau, Wisconsin, Robert A. Stump, Hopewell, Virginia, Charles W. Trigg, San Diego, California, and the proposer.

324. Proposed by Michael W. Ecker, Pennsylvania State University, Worthington-Scranton Campus, Scranton, Pennsylvania.

Let $x_1, x_2, \ldots, x_n$ be positive numbers whose sum is 1. What is the smallest possible value of the sum of the reciprocals $\sum_{i=1}^{n} \frac{1}{x_i}$?

Solution by Mark Schultz, University of Wisconsin-Marathon County, Wausau, Wisconsin.

Since the arithmetic mean of n numbers is not less than the harmonic mean of n numbers, we have

$$\frac{n}{\sum_{i=1}^{n} 1/x_i} \leq \frac{\sum_{i=1}^{n} x_i}{n} = \frac{1}{n} . \qquad (1)$$

But this is equivalent to

$$\sum_{i=1}^{n} 1/x_i \geq n^2 \quad . \tag{2}$$

Thus the smallest possible value of the given sum is n^2.

Also solved by James McCann, Bowling Green State University, Bowling Green, Ohio, Simon Jin Jokwi, University of Alberta, Edmonton, Alberta, Carole LaRoche, Bloomsburg State College, Bloomsburg, Pennsylvania, Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin, Robert A. Stump, Hopewell, Virginia, Charles W. Trigg, San Diego, California, and the proposer.

325. Proposed by the editor.

In the prison sits a prisoner who is sentenced to die. Fortunately the warden, an eccentric, offers the prisoner a chance to live. The warden gives the prisoner 12 black balls and 12 white balls. Next the warden gives the prisoner two boxes and tells the prisoner that tomorrow the executioner will draw one ball at random from one of the boxes. If a white ball is drawn, the prisoner will be freed; if a black ball is drawn, the sentence will be carried out. How should the prisoner arrange the balls in the boxes so as to maximize his chance for freedom?

Solution by Robert A. Stump, Hopewell, Virginia.

If the balls are arranged so that the probability of drawing a white ball out of one box is $\frac{1}{2}$, then the probability of drawing a white ball out of the other box is also $\frac{1}{2}$. Hence suppose that the first box contains m white balls and n black balls. Then the probability of drawing a white ball from the first box is $\frac{m}{m+n}$ and the corresponding probability for the other box is $\frac{12-m}{24-m-n}$. W.L.O.G. suppose $\frac{m}{m+n} > \frac{1}{2}$. Then $m > n$. Furthermore the fraction $\frac{12-m}{24-m-n}$ is maximized for all choices of m by taking $n = 0$ with the optimum value occurring with $m = 1$.

Hence the prisoner's best chance for freedom is to place a single white ball in one box and the remainder of the balls in the other box. His probability of freedom will be

$$\frac{1}{2}(1) + \frac{1}{2}\left(\frac{11}{23}\right) = \frac{17}{23} \approx 0.7391304 \text{ or approximately } 73.9\%.$$

Also solved by Michael W. Ecker, Pennsylvania State University, Worthington Scranton Campus, Scranton, Pennsylvania, James McCann, Bowling Green State University, Bowling Green, Ohio, Mark Schultz, University of Wisconsin, Marathon County, Wausau, Wisconsin, and Diana Wilson, University of Missouri-Rolla, Rolla, Missouri.

Editor's Comment

Ecker points out that the placing of a single white ball in one box produces a maximum probability of freedom is not obvious. Using the notation of the featured solution, the desired probability is

$$\frac{1}{2}\left(\frac{m}{m+n}+\frac{m'}{m'+n'}\right)=\frac{1}{2}\left(1+\frac{mm'-nn'}{(m+n)(m'+n')}\right)$$

where $m'=12-m$ and $n'=12-n$. Then since at least one box contains at most 12 balls, one may assume that $m+n \le 12$ and that neither box is empty. It remains to show that $n=0$ maximizes the value of the fraction

$$\frac{mm'-nn'}{(m+n)(m'+n')} \text{ at a value of } \frac{mm'}{m(12+m')} .$$

With $n=0$, $n'=12$, then $mm' \ge mm'-nn'$ is certainly true. Finally $m(12+m') \le (m+n)(m'+n')$ is equivalent to $0 \le n\,(m'+12-m-n)$ which in turn is true because $m+n \le 12 \le 12+m'$ regardless of the value of n since it was assumed that $m+n \le 12$. Hence $n=0$ maximizes the numerator of the fraction and minimizes its denominator at the same time, and the desired maximum probability is obtained by taking $m=1$ and $n=0$ as in the featured solution.

326. Proposed by the editor.

Solve the dual cryptarithm and give the retired Cockney sailor a hand.

```
 FIX     FIX
+ ME    - ME
BOAT    'ELM
```

Solution by Charles W. Trigg, San Diego, California.

From the addition, B = 1, O = 0, and F = 9. Then, from the subtraction, E = 8. The dual cryptarithm establishes the addition

$$\begin{array}{r} ELM \\ +ME \\ +ME \\ \hline BOAT \end{array}$$

From the units' and tens' columns, respectively,

$$M + 16 = T + 10k$$

$$2M + L + k = A + 20$$

The only non-duplicating solutions of the first equation are (M,T,k) = (6,2,2) and (7,3,2). The first set requires L = 7 to exceed 20 in the second equation, but the corresponding A = 1 is a duplicate. The only non-duplicating values based on the second set are L = 6, A = 2; whereupon X = 5 and I = 4. The unique reconstruction is

$$945 + 78 = 1023 \text{ and } 945 - 78 = 867.$$

Also solved by Michael W. Fox, Fredericksburg, Virginia, James McCann, Bowling Green State University, Bowling Green, Ohio, Fred A. Miller, Elkins, Virginia, Carole LaRoche, Bloomsburg State College, Bloomsburg, Pennsylvania, Mark Schultz, University of Wisconsin-Marathon County, Wausau, Wisconsin, and Robert A. Stump, Hopewell, Virginia.

Late solutions were received from H. O. Eberhart, Columbia, Maryland, for problems 324 and 326 and from Jeff Teeters, Wisconsin Beta Chapter, University of Wisconsin - River Falls, Wisconsin, for problems 322, 323 and 324.

The editor apologizes for the misspelling of Simon Jin Jokwi's name in the last issue.

THE MATHEMATICAL SCRAPBOOK

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material directly to the Scrapbook editor. Material will be used where possible and acknowledgment will be made in THE PENTAGON. Please send all materials to Dr. Richard Lee Barlow, Department of Mathematics, Kearney State College, Kearney, NE 68847

Recent articles in both the mathematical education publications and the popular magazines and newspapers have indicated that possibly the older mathematics textbooks are still valid today as mathematical textbooks in junior and senior high schools. One area of current mathematics education study is the differences in achievement for students using the current "modern" texts and the older traditional texts. Some research has indicated that students today achieve more when the older (pre 1960) mathematics textbooks are used. One argument is that the older texts teach more basic mathematical computational concepts which lend themselves to "deeper" mathematical thinking than most of the current "shallow" modern mathematical texts which tend to emphasize sets, logic, and calculator computations.

As an endeavor to further examine this argument, I have gone back to my great-grandfather's high school mathematics text: RAY'S NEW PRACTICAL ARITHMETIC published in 1877 by Van Antwerp, Bragg & Company. Topics covered in this text included the following: the Arabic and Roman systems of notation, addition, subtraction, multiplication, division, compound numbers, factoring, fractions, decimal fractions, the metric system, percentage, interest,discount, exchange, insurance, taxes, ratio, proportion, involution, evolution, mensuration, and progressions. As an example of the approach used in this text, consider the evolution problem of finding the cube root; the extraction of which is seldom considered today.

To find the cube root of a number, one must first determine the number of figures in the cube root. The text uses the following approach.

"The cube root of 1 is 1, and the cube root of 1000 is 10; between 1 and 1000 are all numbers consisting of one, two, or three figures, and between 1 and 10 are all numbers consisting of one figure; therefore, when a number consists of one, two, or three figures, its cube root consists of one figure." Similarly, the text states that numbers consisting of four, five, or six figures have cube roots of two figures; numbers consisting of seven, eight, or nine figures have cube roots of three figures; etc. Therefore, the following rules are stated.

(1) If a number is pointed off into periods of three figures each, the number of periods will be the same as the number of figures in the cube root.

(2) The cube of the units will be found in the first period,the cube of the tens in the second period, the cube of the hundreds in the third period, etc.

The following examples illustrate this procedure.

1. The point off of 876453921 into periods of three figures each is $87\dot{6}45\dot{3}92\dot{1}$.

2. Similarly, the point off of 37683.5624 is $3\dot{7}68\dot{3}.56\dot{2}40\dot{0}$.

As further explanation,the text gave the following: "Rule - Place a point over the order units, and then over every third order from units to the left and to the right. Rem. 1 - The first period on the left of the integral part of the number will often contain but one or two figures. Rem. 2 - When the first period on the right of the decimal part contains but one or two figures,ciphers must be annexed to complete the period." The cube root extraction process is illustrated as follows:

Example 1 - Extract the cube root of 13824.

Solution Steps.

$$\begin{array}{rl} 1\dot{3}82\dot{4} & (2 \\ \underline{8} & \\ 5824 & \end{array}$$

(1) Point off 13824 into periods of three figures each as $1\dot{3}82\dot{4}$.

(2) The largest cube in 13 is 8 which has as its cube root 2. Place the root 2 on the right, and subtract the cube 8 from 13, getting the remainder of 5. Now bring down the next period of 3 digits, namely 824.

$$\begin{array}{ll|ll} & & 1\dot{3}82\dot{4} & (24 \\ & & \underline{8} & \\ 2*2*300 = & 1200 & 5824 & \\ 2*4*30 = & 240 & & \\ 4*4 = & \underline{16} & & \\ & 1456 & & \\ 1456*4 = & & \underline{5824} & \\ & & 0 & \end{array}$$

(3) Square the root 2 and multiply it by 300;i.e., $2^2*300 = 1200$. This number is the trial divisor. Find how many times 1200 is contained in 5824. The result is 4. Place 4 in the root to the right of 2.

(4) Multiply 2 by 4 and by 30. Next square 4. Then add these products of 240 and 16 to 1200, getting the sum of 1456 which is called the complete divisor. Multiply 1456 by 4, and subtract the product 5824 from 5824. The remainder is 0. Therefore, 13824 is a perfect cube having a cube root of 24.

As a second example, the following is presented:

Example 2 - Extract the cube root of 413.5147.

Solution:

```
                                 413.514700 (7.45+
                                 343
      7*7*300 = 14700           | 70 514
      7*4*30  =   840           |
      4*4     =    16           |
                15556           | 62 224
   74*74*300=1642800            |  8 290700
   74*5*30  =   11100           |
   5*5      =      25           |
              1653925           |  8 269625
                                |    21075
```

Hence, $\sqrt[3]{413.5147} = 7.45.$

To justify the above approach in determining the cube root of a number, the text offered the following geometric explanation, which is quoted directly:

> *After finding that the cube root of the given number will contain two places of figures (tens and units), and that the figure in the tens' place is 2, form a cube, A, Fig. 1, 20 (2 tens) inches long, 20 in. wide, and 20 in. high; this cube will contain 20x20x20=8000 cu. in.; take this sum from the whole number of cubes, and 5824 cu. in. are left, which correspond to the number 5824 in the numerical operation.*

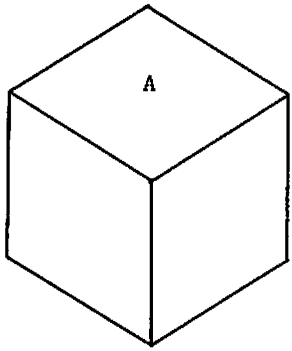


Figure 1

It is obvious that to increase the figure A, and at the same time preserve it a cube, the length, breadth, and height must each receive an equal addition. Then, since each side is 20 in. long, square 20, which gives 20x20 = 400, for the number of square inches in each face of the cube; and since an addition is to be made to three sides, multiply the 400 by 3, which gives 1200 for the number of square inches in the 3 sides. This 1200 is called the trial divisor, because, by means of it, the thickness of the additions is determined.

By examining Fig. 2 it will be seen that, after increasing each of the three sides equally, there will be required 3 oblong solids, C,C,C, of the same length as each of the sides, and whose thickness and height are each the same as the additional thickness; and also a cube, D, whose length, breadth, and height are each the same as the additional thickness. Hence, the solid contents of the first three rectangular solids, the three oblong solids, and the small cube, must together be equal to the remainder (5824).

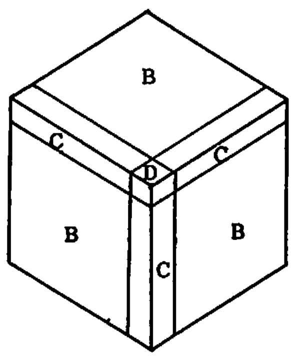


Figure 2

Now find the thickness of the additions. It will always be something less than the number of times the trial divisor (1200) is contained in the dividend (5824). By trial, we find 1200 is contained 4 times in 5824; proceed to find the contents of the different solids. The solid contents of the first three additions, B, B, B, are found by multiplying the number of sq.in. in the face by the thickness; there are 400 sq.in. in the face of each, and 400x3 = 1200 sq.in. in one face of the three; then, multiplying by 4 (the thickness) gives 4800 cu. in. for their contents. The solid contents of the three oblong solids, C, C, C, are found by multiplying the number of sq. in. in the face by the thickness; now there are 20x4 = 80 sq. in. in one face of each, and 80x3 = 240 sq. in. in one face of the three; then multiplying by 4 (the thickness), gives 960 cu. in. for their contents. Lastly, find the contents of the small cube, D, by multiplying together its length, breadth, and thickness; this gives 4x4x4 = 64 cu. in.

If the solid contents of the several additions be added together, as in the margin, their sum, 5824 cu. in. will be the number of small cubes remaining after forming the first cube, A. Hence, when 13824 cu. in. are arranged in the form of a cube, each side is 24 in.; that is, the cube root of 13824 is 24.

ADDITIONS	
BxBxB=4800	cu.in.
CxCxC= 960	cu.in.
D= 64	cu.in.
Sum, 5824	

Can you determine the cube root of 1029.6213504 using this extraction process?

THE HEXAGON

EDITED BY IRAJ KALANTARI

This department of THE PENTAGON is intended to be a forum in which mathematical *issues* of interest to undergraduate students are discussed in length. Here by *issue* we mean the most general interpretation. Examination of books, puzzles, paradoxes and special problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to *Iraj Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.*

WHAT IS A PICTURE WORTH?

Richard Fleissner*
Department of Mathematics
Western Illinois University
Macomb, IL 61455

We've all heard the old saying that a picture is worth a thousand words, but I know of an instance in which one wasn't. I had been given a two thousand word term paper assignment to do for an English class in high school and I attempted to get by with a picture and one thousand words. I would have gone for two pictures, but I didn't want to push my luck. My teacher gave me the alternative of coming up with the missing verbiage

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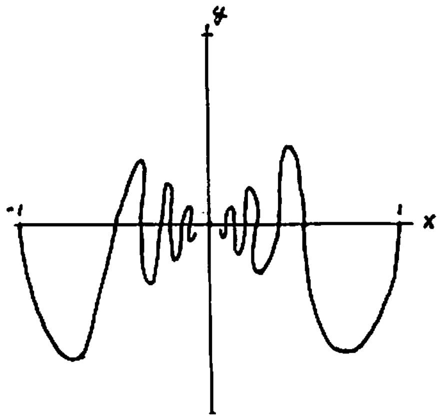
*This issue's article for this department is contributed by Richard Fleissner who received his Ph.D. from University of Wisconsin at Milwaukee in 1974. He has been with Western Illinois University since 1976 and his main area of interest is analysis.

or the moral equivalent of getting out of town by sundown.

I'm writing this article to share with you a picture that I think has some value. I hope that you will find it to be at least amusing. It was shown to me in graduate school by a longtime friend and colleague and although it exhibits no previously unknown mathematical properties, we have used the same technique in our research to create some functions that do.

First, consider the function $f(x) = x\sin(\pi/x)$ if $x \neq 0$ and $f(0) = 0$, the graph of which is sketched below.



This function is continuous at every point of the interval $(-1,1)$ and it has two slightly unusual properties. First, it is not differentiable at $x=0$ because

$(f(x)-f(0))/(x-0)$ equals 0 if x is of the form $1/n$ and it equals 1 if x is of the form $2/(4n+1)$. Consequently, the limit doesn't exist. Second, the equation $f(x)=0$ has infinitely many solutions in the interval $(-1,1)$, namely at $x=1/n$.

The function defined by our next picture will be continuous, will not be differentiable at <u>any</u> point and will assume <u>every</u> value in its range infinitely many times. At this point you might have some fun by setting aside this article and trying to draw the graph of a continuous function that assumes several values infinitely many times.

There is some history to the construction of nowhere differentiable functions that I'd like to relate before getting down to the business of describing one with pictures. During the first half of the last century, many mathematicians felt that a continuous function could not possibly exhibit such terrible behavior. They thought that functions could have some irregularity at a few isolated points, but in between those points they must be very regular. A possible cause for this belief would be that they viewed functions as being built up from the elementary functions like the ones you have studied in your calculus course (polynomials, rational functions, trigonometric functions, exponentials and so forth). Indeed, these functions are very well behaved.

Although there had been previous definitions of continuous nowhere differentiable functions (see [2, p. 954] for a list), the most famous was given by Karl Weierstrass in 1872. His example is

$$f(x) = \sum_{n=0}^{\infty} b^n \cos(a^n \pi x)$$

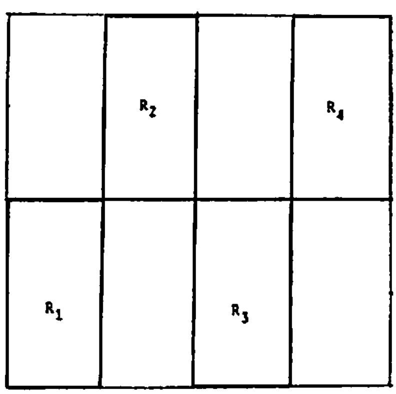
where a is an odd integer, $0 < b < 1$ and $ab > 1 + (3\pi/2)$. The details which show that this function is continuous and nowhere differentiable can be found in Titchmarsh [3, p. 351]. After slightly more than a page of calculations, Titchmarsh remarks, "The graph of the function may be said to consist of an infinity of infinitesimal crinkles; but it is almost impossible to form any definite picture of it which does not obscure its essential feature." I hope our picture can do a bit better.

Not everyone was pleased about the invention of functions like these. Hermite wrote, "I turn away with fright and horror from this lamentable evil of functions that don't have derivatives." Poincaré wrote, "Formerly when one invented a new function, it was for some practical purpose; today they invent them solely to put into default the reasoning of our fathers, and nothing else can be inferred from them."

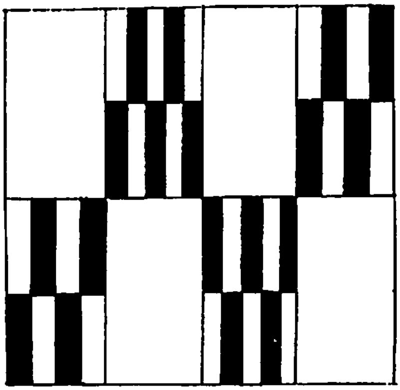
However, if such functions hadn't been discovered, calculus books might still contain erroneous proofs that continuity implies differentiability. I doubt that any bridges would fall because they were designed by people who thought that this is the case, but I'm certainly opposed to presenting phoney patterns of reasoning to students who often have some trouble with the patterns that I think are valid.

Before constructing the promised example, it should be noted that a continuous function which assumes every value in its range infinitely many times was constructed by J. Gillis in 1939 [1].

We first divide the unit square [0,1]x[0,1] into 8 rectangles as shown below and retain the four of them that are marked R_i along with their edges.

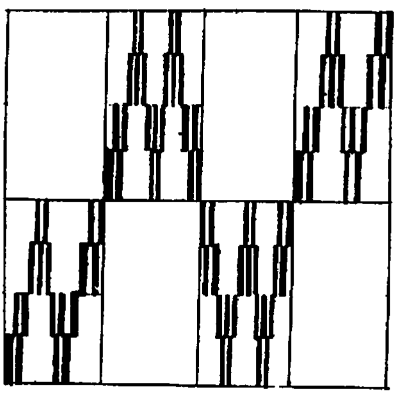


Let $S_1 = R_1 \cup R_2 \cup R_3 \cup R_4$. The set S_1 isn't the graph of a function of course. It's much too thick. So the next step is to subdivide each of the rectangles R_i as follows: cut heights in half and widths into four or five equal pieces depending on whether we want our new set S_2 to run from a lower corner to an upper corner of R_i or we want it to end up on the same level of R_i that it starts. We retain the shaded pieces (and their edges) as shown below.



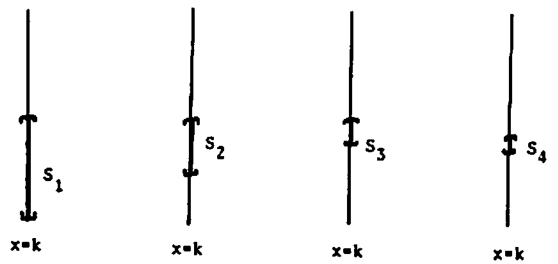
S_2 is the union of the shaded rectangles. It still isn't the graph of a function, but let's pause a moment to note a couple of things about the sets S_1 and S_2. Every horizontal line $y=k$ $0 \leq k \leq 1$ intersects at least two of the rectangles of S_1 ($y=\frac{1}{2}$ meets all four) and the number of intersections is either doubled or tripled when we look at S_2. If we can continue this pattern (and we will),our function will end up crossing each of these lines an infinite number of times. In other words, the function will assume every value in its range an infinite number of times. Moreover, S_1 and S_2 are connected sets, that is, they are unbroken. This is important because we want our function to be continuous and continuous functions have unbroken graphs.

We continue the process by breaking up each rectangle of S_2 into eight or ten parts by cutting heights in half and widths into four or five equal parts. We retain half of these new rectangles in such a way that the number of times each horizontal line is crossed is doubled or tripled and so that the set S_3 remains unbroken.



At this point we must begin using our imaginations and proceed by induction. We form the set S_n by subdividing the rectangles of S_{n-1} in precisely the same manner that we have been, heights in half and widths into four or five equal pieces. The set S_n is then formed so that the number of times each horizontal line is crossed doubles or triples (each one is now crossed at least 2^n times) and so that S_n is unbroken. We also shall insist that the point (0,0) be in each of the sets S_n so that there is no ambiguity about which of the new rectangles must be retained.

Let $S = \bigcap_{n=1}^{\infty} S_n$. Then S is a set of ordered pairs. To show that S is a function, we must demonstrate that it meets every vertical line x=k $0 \leq k \leq 1$ in precisely one point. Let's look at the intersection of such a line with the sets S_n.

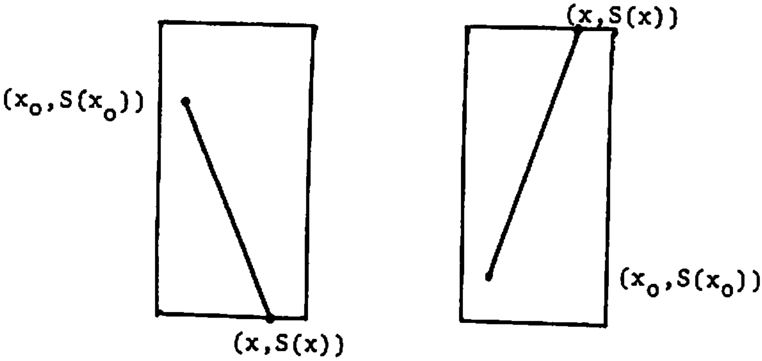


These intersections are closed intervals each of which is precisely half of the previous one. Since heights are continually being cut in half, it should be clear that the final intersection could not contain more than one point. That such an intersection contains exactly one point is equivalent to an axiom of the real number system called the Completeness Property. So S is indeed a function.

Why is it continuous? Well, because we were careful not to break its graph. I'd better elaborate on that though,since it's one of the most reckless statements I've made all day. First, any sort of limiting process is a powerful tool and things have been known to "break" under them. For example, the pointwise limit of continuous functions needn't be continuous. Second, the invention of these nowhere differentiable functions was one of the primary causes of mathematicians becoming much more suspicious of geometric intuition than they were in the last century. A mathematically valid argument that S is continuous may go a bit beyond your present level of training, but if so, you will probably run into these ideas soon in your upper division courses. A point (x,y) is called a limit point of a set Q in the plane if there are points in Q that are arbitrarily close to (x,y). A subset of the plane is said to be closed if it contains all of its limit points. Each of our sets S_n is a closed set since we included the boundaries of the rectangles. An elementary theorem of topology asserts that the intersection of any number of closed sets is a closed set. So S is closed. A somewhat less elementary idea called the Closed Graph Theorem will do the rest of the work. In the present setting it asserts that if the graph of a function is a bounded, closed subset of the plane, then the function is continuous. (I like the unbroken graph explanation better.)

To show that S is nowhere differentiable, it will be convenient to adopt function notation and write $y=S(x)$. We must show that if x_0 is any point [0,1], then the limit as x approaches x_0 of $(S(x)-S(x_0))/(x-x_0)$ does not exist. Since this expression measures the slope of secant lines on the graph of S, more pictures will be helpful. Since the point $(x_0,S(x_0))$ belongs to the set S, it must belong to each of the sets S_n and consequently to one of the rectangles that make up S_n. We need to make two observations about these rectangles and looking back at the pictures might be a good idea. First, there must be points of S on the top and bottom boundaries of each of the rectangles because at each subsequent stage of the construction, we are running from top to bottom to top to bottom...etc. of that rectangle. Second, each of the rectangles that make up the set S_n has the property that the ratio of its height to its width is at least 2^n. This is because when we subdivide a previous rectangle into 8 parts, we double the previous ratio and when we subdivide into 10 parts, we multiply the previous ratio by 2.5 and induction does the rest.

These observations enable us to show that there are points x on the graph such that x is as close as we wish to x_0 and such that the difference quotient $(S(x)-S(x_0))/(x-x_0)$ is enormous. To get close to x_0 we simply go far enough in the construction so that the rectangles of S_k are as thin as we wish. We then locate the rectangle $\overline{R}$ that contains $(x_0,S(x_0))$ and look at two cases. In the figure below and on the left, $(x_0,S(x_0))$ lies (anywhere) in the upper half of $\overline{R}$ and $(x,S(x))$ is any point on the graph that lies on the bottom boundary of $\overline{R}$. In the figure on the right, their positions are reversed. Note that $(x_0,S(x_0))$ must be in one half or the other and there are points of S on both the top and bottom boundaries.



Now consider the absolute value of the slope of the secant line that joins these points. It must travel up or down at least half the height of $\overline{R}$ and it cannot travel from left to right more than the width of $\overline{R}$. Since this rectangle came from S_k, the ratio of its height to width is at least 2^k. Consequently, the absolute value of the slope of our secant line must be at least 2^{k-1}. Increasing k just makes the point x we located closer to x_0 and makes these slopes larger and larger. Thus $S'(x_0)$ could not be a finite number. But it can't be infinite either! If we subdivide $\overline{R}$ once more, we see that the graph of S must cross the line $y=S(x_0)$ again inside the $\overline{R}$ at some point x distinct from x_0. For this value of x, $(S(x)-S(x_0))/(x-x_0)$ equals 0, so the limit can't be infinite. So S isn't differentiable at any point of [0,1].

A bit more can be said about the number of solutions of the equation $S(x)=k$, where k is any number in [0,1]. The solution set can actually be put into one-to-one correspondence with the set of all real numbers. In this sense the equation $S(x)=.77$ has more solutions than the equation $x\sin(\pi/x)=0$ that we looked at in the beginning of this article. To show this would require a discussion of an object called the Cantor Set. This set has many geometric and measure theoretic properties that I think are surprising and wonderful. Perhaps someone could be persuaded to write an article about it (and sets like it) for the Pentagon. (I hope he or she would include some pictures).

Well, what is a picture worth? I guess that will always be in the eye of the beholder.

References

1. J. Gillis, Note on a conjecture of Erdos, Quart. J. Math. Oxford 10(1939), 151-154.

2. M. Kline, Mathematical Thought from Ancient to Modern Times, Oxford University Press, (1972).

3. E. Titchmarsh, The Theory of Functions, Oxford University Press, Second Edition, (1939).

KAPPA MU EPSILON NEWS

News of chapter activities and other noteworthy KME events should be sent to Dr. Harold L. Thomas, Historian, Kappa Mu Epsilon, Mathematics Department, Pittsburg State University, Pittsburg, Kansas 66762.

CHAPTER NEWS

Alabama Gamma, University of Montevallo, Montevallo
Chapter President-James A. Richey
15 actives

The chapter met early in the semester. To publicize the organization, members purchased KME jerseys. Other meetings included plans for a society activity and sponsoring an area high school mathematics tournament. Other officers remain as published in the fall issue except Charlene Garrett is now secretary and Melody Acker is treasurer.

Alabama Zeta, Birmingham-Southern College, Birmingham
Chapter President-Billy Childress
28 actives

This is one of KME's brand new chapters. Installation for twenty-one students and seven faculty members was held February 18, 1981. Look for a detailed account of the installation in the "Installation of New-Chapters" section of the *Pentagon*. Other officers for 1980-81: Chris Canfield, vice president; Dale Gann, secretary; Carolyn Millican, treasurer; Lola F. Kiser, corresponding secretary; William J. Boardman, faculty sponsor.

Arkansas Alpha, Arkansas State University, State University
Chapter President-Ron Simpson
12 actives, 9 pledges

Other officers for 1980-81: Jody Carreiro, vice president; Dianne Watson, secretary; Kathy Steffy, treasurer; Jerry L. Linnstaedter, corresponding secretary; Robert P. Smith, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President-Cristi Strain
46 actives, 32 pledges

The Chapter held a fall faculty-student picnic. Monthly chapter meetings featured speakers from industry, the faculty, and the placement center. A Christmas social and pledge ceremony were held at the end of the quarter. Other officers remain as published in the fall issue. In addition, Dr. Dina Ng serves as faculty sponsor with Dr. Adelaide Harmon-Elliott.

California Delta, California State Polytechnic University, Pomona
Chapter President-Michael Wallace
8 actives, 2 pledges

Twelve members visited the Griffith Observatory in the fall quarter to see the planetarium and laserium shows. During last spring and summer, a committee evaluated applications for a book scholarship. The chapter awarded a $75 scholarship to Ernest Chiu. A display is also maintained by the group in the display case in the Science Building. Other officers for 1980-81: Jon Penner, vice president and treasurer; Mary Muggia, secretary; Dick Robertson, corresponding secretary; Cameron Bogue, faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President-Lisa G. Yates
20 actives

The chapter met jointly with the Physics Club three times during the fall quarter. Programs included: October 20, Dr. Dale Doering, NASA-Ames Laboratory, discussed "Catalysis Research"; November 17, Dr. Lucy Garmon, WGC chemistry and physics department, lectured on "Preparation and Examination of Thin Crystal Films"; November 20, Professor Roy Bogue, WGC mathematics department, spoke on "e^z" and Associates". Other officers for 1980-81 remain as published in the fall issue.

Illinois Beta, Eastern Illinois University, Charleston
Chapter President-Jeff Bivin
49 actives

Members organized a candy sale and a book sale to raise money for a spring field trip. The chapter is making plans to attend the NCTM national meeting in St. Louis in April. Other officers for 1980-81 remain as published in the fall issue except Mr. Lloyd Koontz is now faculty sponsor.

Illinois Zeta, Rosary College, River Forest
Chapter President-Sharon Holder
13 actives

The chapter held a plant sale during the fall semester in order to raise money for expenses to the National Convention in the spring. President Sharon Holder upheld the tradition of presenting problems at chapter meetings. Other officers for 1980-81 remain as published in the fall issue. In addition, Mark Siwek is now treasurer.

Illinois Eta, Western Illinois University, Macomb
Chapter President-Douglas Sorensen
9 actives

Chapter money raising projects included selling lunches to faculty and entering runners in a jog-a-thon. A variety of programs were presented at fall semester meetings. A WIU mathematics major graduate who is now employed by IBM presented a program. Honor students from surrounding high schools were guests at this meeting. Doug Sorensen, chapter president, presented a program on computer crimes. Faculty members, Dr. G. White and Dr.J. Stipanowich gave programs on the TRS-80 and basic language and history of mathematics, respectively. Other officers for 1980-81 remain as published in the fall issue with the exception of corresponding secretary, now filled by Joseph Stipanowich.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President-Jo Ann Vannini
37 actives

The following students presented papers at local chapter meetings: Darla Dettmann on "The Eight-Point Circle," Scott Pierce on "$e^{\pi i} = -1$," Doug Lourens on "Extended Precision Arithmetic," and June Day on "The Monster Group." Fall semester activities included a pizza supper KME organized for members of all the honor societies in the College of Natural Sciences. The annual KME homecoming breakfast, which continues to be quite popular, was held October 25 at the home of Professor and Mrs. Carl Wehner. Seventeen alumni were in attendance this year. The initiation banquet was held December 3 and the annual Christmas party was hosted by Professor and Mrs. Gregory Dotseth on December 12. An informal ski trip after final exams led to minor knee damage for V. P. Chizek. Other officers for 1980-81 remain as published in the fall issue.

Iowa Beta, Drake University, Des Moines
Chapter President-Mitch Adams
10 actives

A chapter picnic and two meetings were held during the fall semester. Spring semester activities opened with a party at the home of the faculty sponsor. Other officers for 1980-81: Brad Barks, vice president; Dave Reagor, secretary and treasurer; Wayne Woodworth, corresponding secretary; Alex Kleiner, faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President-Tim Alpers
14 actives, 5 pledges

Fall semester meetings and activities included: a presentation by Cooperative Education Students on summer work experiences; additional cooperative education reports plus some historical tidbits; "Careers in Statistics" given by Robert Woolson, University of Iowa; a Christmas party which included dinner and math games; films on computers and microelectronics. Other officers for 1980-81 remain as published in the fall issue except Glenn C. Fenneman is now corresponding secretary and faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President-Brenda Brinkmeyer
40 actives

The chapter held monthly meetings in October, November, and December. In addition, a fall picnic was hosted for all mathematics and physics students. Fall initiation for new members was held at the October meeting. Three new members were received at that time. The October program was given by Susan Spineto. She told about her experiences at the Argonne Laboratory. Linda McCracken gave the November program. She presented Math Circles anecdotes from Eve's book. In December, a special Christmas meeting was held at the home of Dr. Helen Kriegsman, Chairman, Dept. of Mathematics. Maeve Cummings discussed a history of algebra and Debbie Scheer spoke on, "Use of Graphs". Other officers for 1980-81 remain as published in the fall issue.

Kansas Gamma, Benedictine College, Atchison
Chapter President-Leo Vitt
14 actives, 9 pledges

The chapter sponsored a fall picnic for members and interested mathematics students. The traditional Wassail party was held at the home of Sister JoAnn Fellin on the evening of December 8. Other officers for 1980-81 remain as published in the fall issue with the addition of Jim Ewbank as faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President-Rebecca Hladky
20 actives

Monthly meetings were held with outside speakers on two occasions. The December meeting was a Christmas party at a faculty home with "seasonal" mathematical games being played. Other officers for 1980-81 remain as published in the fall issue with the addition of Billy Milner as faculty sponsor with Gary Schmidt.

Kansas Epsilon, Fort Hays State University, Hays
Chapter President-Geralyn Kraus
30 actives

The chapter has started publication of a student newsletter. As a money raising project members have sold pretzels. Other officers for 1980-81 remain as published in the fall issue.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President-Sally Fisher
26 actives

Chapter members conducted weekly problem sessions for calculus and pre-calculus students during the academic year. Meetings were held on a biweekly basis. Dr. Dorian Yeager hosted a chapter picnic and Kevin Preston hosted a Halloween party. Mr. Kurt Zimmerman from Career Development and Placement presented a talk on the services available through his office. Other officers for 1980-81 remain as published in the fall issue with the addition of Dr.Don Greenwell who is now faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President-Rebecca Weller
20 actibes, 3 pledges

Three new members were inducted in October. The initiation was followed by math games and challenges. The chapter sponsored a career night panel composed of four former students who were all KME members: Jahn Buhrman,'65; Deborah Simmons Tasky, '77; Tony W. Sager, '77; and Ted D. Tupper, '72. The panelists discussed their present employment and the role mathematics played in preparing them for this work. Members ran a raffle of luncheon for two at a local restaurant to raise money for expenses to the National Convention in April. Additional fund raisers are planned for January term and spring semester. Other officers for 1980-81 remain as published in the fall issue.

Maryland Delta, Frostburg State College, Frostburg
Chapter President-James Martens
19 actives

The chapter heard interesting talks by Miss Kathryn Snyder, a middle school mathematics teacher, and by Dr. Vadim Komkov, West Virginia University mathematics department head. Members enjoyed sampling the shoebox activities presented by Miss Snyder. The mathematics methods students also presented their projects to the group. Plans were made for a trip to the Smithsonian Institution early in the spring semester. The chapter will also initiate twelve new members in February. Other 1980-81 officers remain as published in the fall issue.

Michigan Beta, Central Michigan University, Mt. Pleasant
Chapter President-Margaret McNally
45 actives, 13 pledges

The fall semester began with a picnic for KME members and mathematics faculty members. Fall initiation was held October 16 with Dr. James Bidwell, past editor of the Pentagon, as guest speaker. He spoke on the history of KME and the importance of active membership. Student speakers for fall chapter meetings were Beth Babcock, reporting on summer job in computing, and Laurie Cooper and Donna Leslie, discussing mathematical games for high school students. The chapter also hosted their annual homecoming coffee hour for alumni. Members are also available several hours per week to tutor students in freshman and sophomore level mathematics courses. Other 1980-81 officers remain as published in the fall issue.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President-Ted Blaylock
25 actives, 9 pledges

Programs for monthly meetings included: Donna Pearce, graduate assistant, "Computerized X-ray Tomography"; Liz O'Neal, student, and Mrs. Alice Essary, faculty, "Rose Curve Sketching." The chapter held a barbeque for the new fall initiates. Other 1980-81 officers re-remain as published in the fall issue.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President-Richard Robertson
32 actives, 16 pledges

The semester activities began with a special fall picnic. There were three chapter meetings at which papers were presented. Sixteen new members were initiated. Other 1980-81 officers: Kim Birkenbauch, vice president; Donna Garoutte, secretary; Jayne Ward, treasurer; M. Michael Awad, corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President-Danny Baker
26 actives, 5 pledges

Fall activities included four regular meetings, a Christmas party,and initiation. Other 1980-81 officers: David Harris,vice president; Colleen Seiter, secretary; Dave Sample, treasurer; Homer F. Hampton, corresponding secretary; Alvin Tinsley, faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President-Janet Doll
2 actives

William D. McIntosh is corresponding secretary and faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President-Shawnee Plock
22 actives

To make money,the chapter sells floppy discs to students who use the Apple II computer. They also sold helium filled balloons at the W.S.C. homecoming football game. The chapter also won first place in the organization display competition for homecoming. A KME member, Joe Painter, was elected W.S.C. homecoming king in October. Several KME members were listed in *Who's Who Among American College Students* in the fall semester. An on going project of the club is proctoring and providing assistance for students who use the computer rooms in the evenings. This responsibility is shared with two other clubs. Other officers for 1980-81 remain as published in the fall issue.

Nebraska Beta, Kearney State College, Kearney
Chapter President-Annette Herz
15 actives, 19 pledges

The chapter had a very successful fund raising project in the fall semester. Approximately 700 T-shirts were sold for the Diamond Jubilee of Kearney State College. This is believed to be the largest money raising project in the history of Nebraska Beta. The profits will be used for a scholarship and for expenses to attend the National Convention in April. Members also gave a presentation to high school students that attended Senior Day. Other 1980-81 officers: Robert Gentzler, vice president; Joy Kenton, secretary; Patti Welch, treasurer; Charles Pickens, corresponding secretary; Marilyn Jussel, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque
Chapter President-William Short
30 actives

Other 1980-81 officers remain as published in the fall issue.

New York Alpha, Hofstra University, Hempstead
Chapter President-Dorothy Chappell
10 actives

The chapter sponsored a job opportunities program on The Actuary. Other 1980-81 officers: Susan Stack, vice president; Amy Zwarico, secretary; Deborah Adler, historian; Scott M. Jeffreys, treasurer; Stanley Kertzner, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara
Chapter President-Karen Young
15 actives

The chapter held meetings to discuss fund-raising activities for the National Convention and for some local events, one of which was a Christmas party held jointly with the Math Club and Math faculty. Other 1980-81 officers: John Michalek, vice president; Diane Smith, secretary; Michael Samson, treasurer; Robert L. Bailey, corresponding secretary; Sr. John Frances Gilman, faculty sponsor.

Ohio Gamma, Baldwin-Wallace College, Berea
Chapter President-Patricia Toth
32 actives

A practicing actuary gave a talk to the chapter. Other 1980-81 officers: Vickie Richards, vice president; Janet Gosche, secretary; Karen Brown, treasurer; Robert Schlea,corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President-Kevin McCaffrey
34 actives, 7 pledges

Fall semester programs included a talk on Archimedes by President Kevin McCaffrey. The chapter also hosted two excellent speakers, Dr. Dave Kullman, Miami University at Oxford, who spoke on "New Light on an Old Problem" and Dr. Bob Dieffenbach, Miami University at Oxford, who discussed "Games People Play." New members were initiated in October at the initiation banquet. The initiates gave talks on mathematicians. A Christmas party was held at Dr. Smith's home. Other 1980-81 officers: Cathy Roby, vice president; Cathy Harper, secretary; Gregory Adams, treasurer; James L. Smith, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President-Doris Pyles
35 actives, 20 pledges

The chapter had a speaker from the 3M company and also worked on a fund raising project. Other 1980-81 officers: Linda Bartel, vice president; Joyce Cox, secretary and treasurer; Wayne Hayes, corresponding secretary; Dave and Rosie Taylor, faculty sponsors.

Pennsylvania Beta, LaSalle College, Philadelphia
Chapter President-Theresa Gauder
32 actives, 17 pledges

Seventeen new members were initiated on December 2, 1980. At this meeting, Dr. Stephen Andrilli spoke about Galois theory and its application to ruler and compass constructions. Other officers remain as published in the fall issue.

Pennsylvania Gamma, Waynesburg College, Waynesburg
Chapter President-Kim Hemskey
22 actives, 1 pledge

The chapter entered a float in the 1980 Homecoming parade and won first place. Other officers remain as published in the fall issue with the exception that Patrick Hayes is now vice president.

Pennsylvania Epsilon, Kutztown State College, Kutztown
Chapter President-George Malafarino
21 actives, 1 pledge

Members enjoyed a picnic in September followed by five regular meetings. Student speakers presented interesting programs on topics such as "Geometry of the Front End of an Automobile" and "Fibonacci Series."

Other officers remain as published in the fall issue.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana
Chapter President-Kelly Barber
18 actives, 1 pledge

One new member was initiated at the October meeting. Dr. Donald Duncan, mathematics department faculty, gave an interesting talk at this meeting about the application of the binomial distribution for developing a breeding program for Shetland sheepdogs. Dr. Duncan's hobby is raising these dogs. For the November meeting, Debbie Mentch reported on her experiences in regards to several interesting job interviews she had had. She graduated in December 1980 and now works in Boston. Lorrie Kachline related her experiences of student teaching for the semester for the December program. Other 1980-81 officers: Mary Markert, vice president; Tracy Snyder,secretary; James Benner, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President-Valerie Olbrick
28 actives, 9 pledges

Meetings were held on September 16 and September 25 in South Lobby of Mary Anderson Pew dormitory. The yearbook picture was taken in November in Pew Fine Arts Center. The highlight of the semester was the Christmas party held in the home of Jack Schlossnagel, Mathematics Department Chairman. As a service to the College, KME members conduct free tutoring sessions in calculus and finite mathematics. Other 1980-81 officers: Deborah Smith, vice president; Georgia Rougas, secretary; Ruth

Radakovic, treasurer; Marvin C. Henry, corresponding secretary; Daniel Dean, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President-Suzanne Moriat
5 actives, 6 pledges

Fall activities included a problem-solving contest and discussions relating to the eligibility of mathematics majors for KME. April 15, 1981, was set as the date for induction of new members and election of new officers. Many former graduates and KME members will continue to offer free tutorial services to other mathematics students at the new facility, the Math Center. This program is under the supervision of instructor, Kathleen Kulesza. Other officers remain as published in the fall issue.

Pennsylvania Lambda, Bloomsburg State College, Bloomsburg
Chapter President-Earl Robinson
24 actives, 6 pledges

This chapter has been preparing for a college bowl type high school math contest in the spring. Several members are also working on papers to submit for the National Convention. A Christmas banquet was held in December. Other 1980-81 officers: Carol LaRoche, vice president; Ellen Ramsey, secretary; Kim Hellerman, treasurer; James Pomfret, corresponding secretary; Joseph Mueller, faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene
Chapter President-David Proctor
50 actives

Other 1980-81 officers: Debbie Smith, vice president; Nancy Chege, secretary and treasurer; Mrs. Anne Bentley, corresponding secretary; Charles Robinson and Edwin

Hewett, faculty sponsors.

West Virginia Alpha, Bethany College, Bethany
Chapter President-Bart Balint
16 actives

Other officers remain as published in the fall issue.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President-Mary Pat Ganzer
7 actives

The chapter sold doughnuts in September to raise money for students to attend the National Convention. In October, Wisconsin Alpha hosted the Milwaukee Area Mathematics Council. At one of the meetings each KME member gave a short report on some mathematician. On November 22, 1980, the annual Mathematics Contest for Young Women was held. Fifty students from seventeen different schools participated. George Polya's film, "Let's Teach Guessing" was viewed by the chapter in December. Other officers remain as published in the fall issue.

Wisconsin Beta, University of Wisconsin, River Falls
Chapter President-Gail Norderhaug
28 actives, 10 pledges

Other officers remain as published in fall issue.

Wisconsin Gamma, University of Wisconsin, Eau Claire
Chapter President-Beth Magnuson
20 actives, 4 pledges

Fall activities began with a picnic held jointly with the Chemistry and Biology Clubs. In October, the chapter helped with registration at the Wisconsin Mathematics Council's fall meeting. In conjunction with this, a bake sale was held to raise money. A Christmas Party for members and faculty was held in December.

Monthly programs given in September,October, and November were on the cooperative education program on campus and placement, the basis of the new Honeywell Computer System on campus, and the use of finger mathematics. Other officers for 1980-81 remain as published in the fall issue with the exception that Sandy Hop is now secretary.

REPORT ON THE TWENTY-THIRD BIENNIAL CONVENTION

The Twenty-third Biennial Convention of Kappa Mu Epsilon, celebrating the 50th anniversary of the Society, was held April 2-4, 1981 in Springfield, Missouri, on the campuses of Southwest Missouri State University and Evangel College, with Missouri Alpha and Missouri Theta, respectively, as co-host chapters.

On Thursday evening, April 2, following registration in Lewis Hall on the Evangel College campus, games and get-acquainted activities were held in Lewis Hall and the National Council met in the Seminar Room of Lewis Hall.

On Friday morning, April 3, registration continued in the lobby of Temple Hall on the Southwest Missouri State University campus. The first general session (business meeting) was held in Temple Hall L102 commencing at 8:45 a.m. with James E. Lightner of Maryland Beta, National President, presiding. Dr. Robert Spence, President of Evangel College, and Dr. Duane Meyer, President of Southwest Missouri State University, gave addresses of welcome and Ida Z. Arms of Pennsylvania Zeta, National Vice-President, responded for the Society. The roll call of the chapters was made by George R. Mach of California Gamma, National Secretary. Twenty nine chapters and about 180 members were in attendance. Delegate certification forms and travel vouchers were filed and delegate voting cards were issued. It was reported that the constitution and by-laws revisions approved at the Twenty-second Biennial Convention were ratified by the chapters on November 16, 1979 and that the documents were reprinted and distributed. It was also reported that the National Council voted to increase the initiation fee to $15.00 and the mileage allowance to 15¢ per mile, effective July 1, 1979.

Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

1. "Magic Squares, DeLa Loubere's Method," Jerome O'Brien, Pennsylvania Lambda, Bloomsburg State College

2. "Earthquake Analysis," Lisa Beverly, California Gamma, California Polytechnic State University

3. "The Eight Point Circle," Darla Dettmann, Iowa Alpha, University of Northern Iowa

4. "Catalan Numbers," Kimberly Yarnall, Pennsylvania Lambda, Bloomsburg State College

At noon, a group picture was taken in Briggs Stadium. Convention committees and the National Council met during lunch.

The convention reconvened at 1:30 p.m. in Temple Hall L102. Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

5. "Aliquot Divisors: An Expanded Look at Perfect Numbers," Mary Markert, Pennsylvania Zeta, Indiana University of Pennsylvania

6. "The Fibonacci Numbers,"Annette Herz, Nebraska Beta, Kearney State College

7. "On the Positive Elements of a Field," Mary Beuerlein, Kansas Beta, Emporia State University

At 2:45 p.m. a student section discussing program ideas met in Temple Hall L102 with Richard Robertson, President of Missouri Alpha, presiding; a student section discussing money-raising ideas met in Temple Hall L103 with Brenda Taylor, President Missouri Theta, presiding; and the faculty section met in Temple Hall L101 with James E. Lightner of Maryland Beta,National President, presiding.

The convention reconvened at 3:45 p.m. in Temple Hall L102. Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

8. "On Pigeons and Mathematics," James E. Benner, Pennsylvania Zeta, Indiana University of Pennsylvania

9. "The Characterization of a Finite Power Set as a Ring," Karen Young and John Michalek, New York Eta, Niagara University

10. "Computer Graphics: Three Dimensional Representation of Spheres," David Harris, Missouri Beta, Central Missouri State University

The convention banquet was held on Friday evening, April 3, in the Campus Union on the Southwest Missouri State University campus with Richard Robertson, President of Missouri Alpha, and Brenda Taylor, President of Missouri Theta, as masters of ceremonies. Musical entertainment was provided by the Evangel Trumpet Quartet. Guest speaker, Dr. Troy L. Hicks, University of Missouri-Rolla, gave the address, "A Potpourri of Mathematical Problems."

A 50th anniversary booklet of history and information about Kappa Mu Epsilon, compiled and edited by Harold L. Thomas of Kansas Alpha, National Historian, was presented to the Society. Copies were given to persons attending the banquet and they will also be offered to all convention delegates and mailed to all chapters.

James E. Lightner of Maryland Beta,National President, presented some reflections entitled, "Looking Back Over Fifty Years". This concluded with the announcement of the fifty recipients of the Distinguished Member Awards and the introduction of those recipients attending the banquet. All recipients will receive a certificate and their names and initiating chapters were included in the banquet program.

The convention reconvened at 8:30 a.m.on Saturday, April 4, in Temple Hall L102. Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

11. "e, π, i, and Their Relationship," Scott Pierce, Iowa Alpha, University of Northern Iowa

12. "History of the Fifth Postulate," Jayne Ward, Missouri Alpha, Southwest Missouri State University

13. "Mersenne Primes, the 27th Mersenne Prime, and the Cray-1 Computer," David L. Fox, Pennsylvania Lambda, Bloomsburg State College

14. "The Quaternion Group and Its Subgroups," Teresa Beye, Kansas Gamma, Benedictine College

15. "Metric Spaces and Generalized Metric Spaces," Julie Shipley (Graduate Student) Missouri Alpha, Southwest Missouri State University

The second general session (business meeting) was then held in Temple Hall L102. The following national officers presented reports (copies attached):

Douglas Nance	-	Business Manager THE PENTAGON Michigan Beta
Kent Harris	-	Editor THE PENTAGON Illinois Eta
Harold Thomas	-	National Historian Kansas Alpha
Wilbur J. Waggoner	-	National Treasurer Michigan Beta
George R. Mach	-	National Secretary California Gamma
Ida Z. Arms	-	National Vice-President Pennsylvania Zeta
James E. Lightner	-	National President Maryland Beta

James C. Pomfret of Pennsylvania Lambda reported for the auditing committee that the treasurer's books were examined and found to be in excellent order.

Mary Elick of Missouri Iota reported for the resolutions committee. The following resolutions were adopted:

<u>Resolved</u>: That the Twenty-third Biennial Convention of Kappa Mu Epsilon express its gratitude to James Lightner who has served as President of Kappa Mu Epsilon and to Ida Arms who has served as Vice President of Kappa Mu Epsilon, both of whom have given so generously of their time and talent.

<u>Resolved</u>: That the Twenty-third Biennial Convention of Kappa Mu Epsilon express its appreciation:

1. To Michael Awad and the members of Missouri Alpha and to Glenn Bernet and the members of Missouri Theta for their work in the expeditious planning of this, our Golden Anniversary Convention.

2. To Dr. Robert Spence, President of Evangel College, and to Dr. Duane Meyer, President of Southwest Missouri State University, for the gracious hospitality and the many services rendered the chapters and officers of the convention.

3. To Troy Hicks for his interesting and entertaining banquet talk.

4. To the Auditing, Nominating, Selection,and Awards Committees who gave so unselfishly of their time to the primary activity of Kappa Mu Epsilon.

5. To the students who prepared and submitted papers for the convention.

6. To the National Officers of Kappa Mu Epsilon for their diligent service during and preceding the Biennial Convention.

7. To the Evangel Trumpet Quartet who provided entertainment for us during the convention banquet.

8. To Harold Thomas, National Historian, for his work in compiling and editing the history of the first fifty years of Kappa Mu Epsilon in the Golden Anniversary Souvenir Booklet.

Resolved: That the National Council of Kappa Mu Epsilon:

1. Consider redefining the national requirements for membership in Kappa Mu Epsilon, in particular with regard to accepting computer courses and/or statistics courses as part of the mathematics requirement.

Sister Nona Mary Allard of Illinois Zeta reported for the nominating committee. Nominations were requested from the floor. There being none, nominations were closed and ballots were distributed to the voting delegates.

Invitations to host the Twenty-fourth Biennial Convention were extended by: Colorado Beta, Colorado School of Mines (mail invitation): Nebraska Beta, Kearney State College; Kansas Epsilon, Fort Hays State University; California Gamma, California Polytechnic State University; Wisconsin Alpha, Mount Mary College; Wisconsin Gamma, University of Wisconsin-Eau Claire.

Richard Robertson,President of Missouri Alpha, reported for the student section meeting which discussed program ideas. Brenda Taylor, President of Missouri

Theta, reported for the student section meeting which discussed money-raising ideas. Helen Kriegsman of Kansas Alpha reported for the faculty section meeting.

Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presented certificates to all students who had presented papers at the convention. James L. Smith of Ohio Zeta reported for the awards committee and announced the following student paper awards:

First Place	($60) -	David L. Fox Pennsylvania Lambda
Second Place	($40) -	Darla Dettmann Iowa Alpha
Third Place	($30) -	David Harris Missouri Beta
Fourth Place	($20) -	Annette Herz Nebraska Beta

The election results were announced. The following officers were elected for the next four years, 1981-85, and they were installed by James E. Lightner of Maryland Beta, retiring president:

President	- Ida Z. Arms Pennsylvania Zeta
President-Elect	- James L. Smith Ohio Zeta
Historian	- Harold L. Thomas Kansas Alpha

Travel allowances were paid to the delegates by Wilbur J. Waggoner of Michigan Beta,National Treasurer. Convention evaluation forms were collected by the host chapters. The convention adjourned at 12:00 noon.

George R. Mach

REPORT OF THE NATIONAL PRESIDENT

During the last biennium the Society has been very active. We have added four new chapters to our roster of active chapters, bringing the number to 100. The Illinois Theta Chapter at Illinois Benedictine College, Lisle, was installed on May 18, 1979, by Past National Historian Sister Jo Ann Fellin, assisted by Sister Nona Mary Allard. The Pennsylvania Mu Chapter at St. Francis College, Loretto, was installed by National Vice President Ida Arms on September 14, 1979. Both of these chapters were voted upon favorably at the last biennial convention. The Texas Theta Chapter at Southwest Texas State University, San Marcos, was installed by National Historian Harold Thomas on April 25, 1980; and I had the pleasure recently to install the Alabama Zeta Chapter at Birmingham-Southern College, Birmingham, on February 18, 1981. One other new chapter has been fully approved by the Council and the chapters and will be installed at Eastern Connecticut State College, Willimantic, on May 2, 1981, by Professor Loretta K. Smith, Associate Editor of The Pentagon.

As you heard from the Vice President, the regions still continue to function, although some not as actively as we could hope for. I know you look forward to next year and the opportunity to attend a meeting in your own geographical region at which additional student papers will be presented and the Kappa Mu Epsilon spirit and interest in evidence here at this national convention will be maintained and strengthened. In accordance with National Council Policy, directors for regions I, III, and V will be appointed at this convention for a four-year term.

At the last convention we studied the revised constitution which had been prepared by our ad hoc Constitutional Revision Committee. Following discussion and minor changes, the constitution was submitted to all the chapters for ratification. It was formally approved and went into effect on November 16, 1979. Copies have been sent to all chapters; additional copies are available from the National Secretary.

Following the last convention the action of the National Council raising the initiation fee to $15 also took effect and appears to have stemmed the tide of mounting inflation which had been forcing the Society almost to operate in the red, at least in regard to the services provided to student members. From the report of the National Treasurer, you can see that we are on solid footing, and no further increases appear to be needed at least for the next biennium. It should be noted that for societies of our type and size and which provide the programs and publications that we do, our one-time fee of $15 is slightly below the average. The fee increase, it might also be noted, did not in any way reduce the number of initiates over the last biennium; that number, over 2000, has held relatively constant for the past dozen or so years. Your National Council continually strives to provide the maximum program and service for the minimum cost to the student.

The past two years have been busy ones as the Council and our host chapters planned for this convention which marks the fiftieth anniversary of the founding of Kappa Mu Epsilon. I want to express my thanks again to National Historian Harold Thomas for his special effort in publishing our new history booklet, and to all those chapters, through their corresponding secretaries and advisors, who nominated members for the Distinguished Member awards. All of us who read and use this history in the future will remember this special moment in our Society's life; and the award recipients, I am sure, will feel honored by this recognition of their accomplishments, on the occasion of this golden anniversary. I also want to express appreciation to all those members who agreed to serve on the various convention committees. Much of their work goes on behind the scenes, either before or during the convention, so that things will go smoothly and we can accomplish our purposes.

As I leave the National Council, I want to thank all my colleagues on the Council and the Regional Directors (both past and present) for their support and help during the last biennium and indeed during the years I served as Vice President and President of the Society. Together, we weathered some turbulent times and emerged stronger and closer because of them. The experience of serving Kappa Mu Epsilon has been most rewarding, and the friendships I have made will always be cherished.

I know that the future will see Kappa Mu Epsilon even stronger than it is now, and I challenge each of you to make a contribution in one form or another, to the Society in the years to come. In this way we cannot only be proud of our past accomplishments but also be optimistic, enthusiastic, and confident about our future.

James E. Lightner

REPORT OF THE VICE PRESIDENT

One of the responsibilities of the Vice President is to serve as coordinator of regional activities of the Society through the Regional Directors. During the Fall of 1979 I compiled a summary of minutes from past National Council meetings that dealt with regional organization, along with a listing of the duties and responsibilities of Regional Directors, and suggested forms for reporting information about Regional Conventions. In the Spring of 1980 two Regional Conventions were held. Region I (Carol Harrison, Director) held a convention at Shippensburg State College (PA IOTA). Region IV (John Cross, Director) held its convention at Western Illinois University (ILL ETA).

On September 14,1979 I had the pleasure of serving as the installing officer for Pennsylvania Mu Chapter at St. Francis College, Loretto, PA.

It is the Vice President's responsibility to make arrangements for the presentation of student papers at the National Convention. I am pleased to report that twenty five students, representing twelve chapters and nine states, submitted papers. Fourteen undergraduate students and one graduate student will present papers at the convention. My special thanks to the members of the Paper Selection Committee who read and ranked the papers: Dr. Charles Pickens (Nebraska Beta), Professor Homer Hampton (Missouri Beta) and Professor Robert Bailey (New York Eta). I am particularly grateful to the twenty five students who prepared and submitted papers. These papers are very important in helping to make any Kappa Mu Epsilon Convention a success.

Ida Z. Arms

REPORT OF THE NATIONAL SECRETARY

During the last biennium 4 new chapters of Kappa Mu Epsilon were installed: They are: Illinois Theta at Illinois Benedictine College, installed on May 18, 1979; Pennsylvania Mu at Saint Francis College installed on September 14, 1979; Texas Theta at Southwest Texas State University, installed on April 25, 1980; and Alabama Zeta at Birmingham-Southern College, installed on February 18, 1981. Alabama Epsilon at Huntingdon College was reactivated on April 27, 1979. On November 9, 1979 the National Council declared the following chapters to be inactive: Illinois Delta at College of St. Francis, New York Delta at Syracuse University Utica College, and Texas Zeta at Tarleton State College. The Society now has 100 active chapters in 31 states.

During the last biennium 2,246 members were initiated. The 100 active chapters have a combined membership of 37,385 and the 22 inactive chapters have a combined membership of 4,401, making the total membership of Kappa Mu Epsilon 41,786 at the end of the biennium on February 28, 1981.

The amendments (revisions) of the constitution and bylaws approved at the Twenty-second Biennial Convention were ratified by the chapters on November 16, 1979. The new documents were reprinted and I distributed them to all active chapters. I also distributed Chapter Handbook revisions to all active chapters during the last biennium.

As National Secretary, I maintain permanent chapter files, including reports of all chapter initiations. I order membership certificates for all new members and I stock all supplies, including forms, invitations, and jewelry. I assist corresponding secretaries in any ways that I can and I take minutes of National Council meetings and biennial conventions.

George R. Mach

FINANCIAL REPORT OF THE NATIONAL TREASURER
Biennium April 1, 1979 to March 2, 1981

	<u>Receipts</u>			
1.	Cash on Hand April 1, 1979			11,739.05
2.	Receipts from Chapters			
	Initiates (2233)	28,870.00		
	Jewelry	1,059.34		
	Supplies	356.20		
			30,285.54	
3.	Miscellaneous			
	Interest	728.43		
	Chapter Installations	304.27		
	Royalties	36.69		
	Other	9.02		
			1,078.41	
4.	Total Receipts			31,363.95
5.	Receipts plus Cash on Hand			43,103.00
	<u>Expenditures</u>			
6.	National Officers Expense		3,655.90	
7.	Jewelry			
	Balfour and Pollack		1,070.78	
8.	Printing			
	Blake Printery, Herff Jones, Brochures		5,461.82	
9.	Pentagon (4 issues)		12,755.55	
10.	Biennial Convention-1979		3,758.17	

11.	Miscellaneous			
	Refunds	117.88		
	Legal Services	550.00		
	National Council Meeting	345.13		
	Installations	309.00		
	Other	11.28		
			1,333.29	
12.	ACHS		1,626.71	
13.	Total Expenditures			29,662.22
14.	Cash on Hand March 2, 1981			13,440.78
15.	Total Assets March 2, 1981			
	Checking	3,213.28		
	Savings Account	7,137.50		
	Certificate	3,090.00		
		$13,440.78		

Wilbur J. Waggoner

REPORT OF THE NATIONAL HISTORIAN

The files of the National Historian are being maintained and continually updated with the records received from the chapters about their events and activities; with information received from Regional Directors about regional conventions and items of interest related to the regions; and with material received from the National Officers which has historical significance.

News items have been solicited from the corresponding secretaries semi-annually, in January and in May. The responses are then edited for publication in the chapter news section of The Pentagon.

During the past biennium 80 of the 100 active chapters responded at least once to the chapter news request. Special mention goes to the following twenty-four chapters for their cooperation in responding to all four inquiries: CA Gamma, GA Alpha, IL Zeta, IL Eta, IA Alpha, IA Delta, KS Alpha, KS Gamma, KS Epsilon, MD Beta, MD Delta, MS Gamma, MO Alpha, MO Beta, MO Epsilon, NE Alpha, NM Alpha, OK Gamma, PA Zeta, PA Eta, PA Kappa, TX Eta, WI Alpha and WI Beta. I would urge chapters to reply to the requests for chapter news even if it is just to identify chapter officers. This would provide chapters with a permanent record of their local officers in the event they do not retain that information within their own chapter.

Regional convention reports from Regions I and IV were received and published in the Fall, 1980, issue of The Pentagon.

Two other significant events occurred in the Historian's office in the past biennium. In April, 1980, it was my pleasure to install the Texas Theta Chapter at Southwest Texas State University, San Marcos, Texas. It was with considerable pride and pleasure that I compiled and edited the Golden Anniversary edition of the KME History and Information booklet which was distributed at the banquet at this convention.

I want to extend thanks to all with whom I have corresponded relative to this office--the National Officers, the Regional Directors, the editors of The Pentagon, corresponding secretaries, and individual KME members. I have been most pleased to serve KME in this office.

Harold L. Thomas

REPORT OF THE EDITOR OF THE PENTAGON

Since the last national convention, one faculty paper and ten student papers have been published in THE PENTAGON. We would like all regional convention papers as well as national convention papers submitted for possible publication. Also, you are urged to submit material to any of the associate editors for possible publication in their respective sections of THE PENTAGON.

During the last two years, several editorial changes have occurred. Jim Bidwell stepped down after eight years as editor, and Oscar Beck stepped down as associate editor of THE BOOKSHELF after eight years. Their services are valued highly. My colleague, Iraj Kalantari,has replaced Oscar and has changed the format of this section (now THE HEXAGON) of THE PENTAGON in a way that we feel will be of more interest to our student readers.

Due to the recommendations of Jim Bidwell and Douglas Nance, Business Manager, THE PENTAGON is now being produced using a "camera-ready" process. This results in significant savings during these times of rising costs. Our technical typist, Linda McDonald, has been very helpful during this transition and we thank her for her suggestions and excellent work. I wish to thank my associates, Richard Barlow, Loretta Smith, Kenneth Wilke, Harold Thomas, Iraj Kalantari, and Douglas Nance. Their continued efforts contribute much to THE PENTAGON. The student papers form the heart of THE PENTAGON.

Consideration is now being given to the addition of a "computer corner" to THE PENTAGON, and your comments and suggestions will be welcomed. Again, I remind you that we would like to have your work submitted to the different departments of THE PENTAGON for possible publication.

Kent Harris

REPORT OF THE BUSINESS MANAGER OF THE PENTAGON

It is a pleasure to make my third Business Manager's report during this golden anniversary meeting. As many of you know, the Business Manager's primary responsibility is to see that THE PENTAGON gets mailed to members of Kappa Mu Epsilon who have current subscriptions. Due to recent changes, mailing dates are approximately June and December.

During this past biennium, we mailed an average of 2500 Pentagons per issue. The mailing list includes subscribers in forty-five states and sixteen foreign countries. States receiving the most copies of THE PENTAGON are, in descending order, Pennsylvania, Tennessee, Ohio, Illinois, Michigan and Missouri.

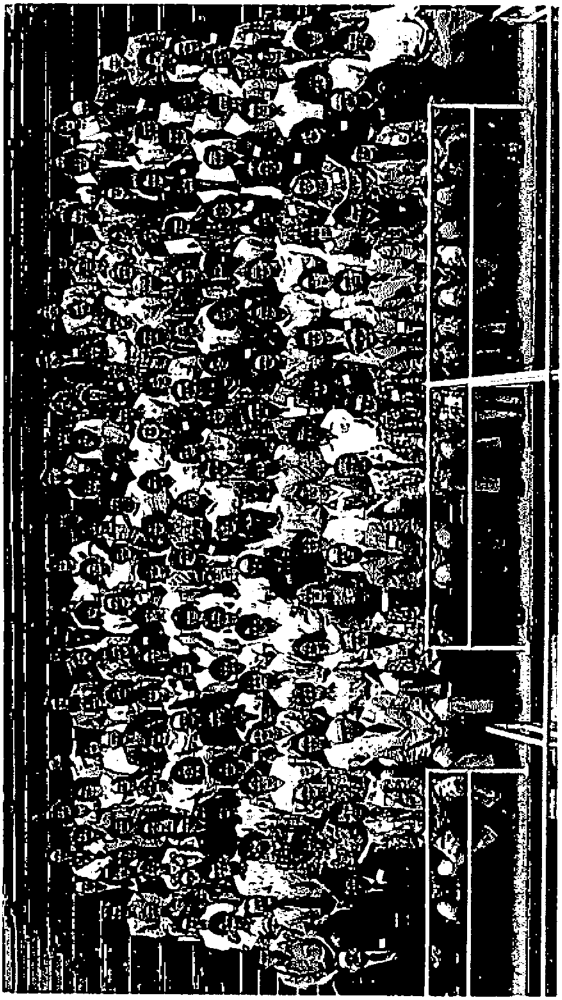
During each semi-annual mailing, approximately 50 Pentagons are returned to the Office of the Business

Manager by the postal service as undeliverable due to incorrect address. Please inform your chapter members that to receive their journal they must keep a current address on file with the Business Manager. If a subscriber has any problem with receiving THE PENTAGON, please contact the office of the Business Manager.

Complimentary copies of THE PENTAGON are sent to the library of each college or university with an active chapter of Kappa Mu Epsilon. Also, complimentary copies are sent to authors of articles in THE PENTAGON. Speakers at this convention will automatically have their subscriptions extended for two years.

During this past biennium, I have received cooperation and support from former business manager Wilbur Waggoner, editor Kent Harris, national secretary George Mach and the mathematics department at Central Michigan University. This cooperation is gratefully acknowledged.

Douglas W. Nance



Kappa Mu Epsilon, Twenty-Third Biennial Convention
April 2-4, 1981, Springfield, Missouri

The National Officers, Editors of the PENTAGON, and all members of Kappa Mu Epsilon take this opportunity to give a special THANK YOU to Dr. James Lightner who has devoted much time and effort in fostering the well being of Kappa Mu Epsilon. He has served as National Vice President from 1973-1977 and as National President from 1977-1981. He has been responsible for the planning of the last three biennial conventions, as well as providing the leadership for all other KME activities during the last four years.

A very special THANK YOU to Dr. James Lightner.

EXCERPTS FROM DR. LIGHTNER'S BANQUET ADDRESS

"A LOOK BACK OVER FIFTY YEARS"

Since we are thinking about our history a bit tonight, it seems appropriate to reflect briefly on the past 50 years, to appreciate better where we are today. In the fall of 1930 a new professor of mathematics arrived on the campus of Northeastern Oklahoma State Teachers College in Tahlequah. While a mathematics club called the Pentagon had been in existence on the campus since 1927,Dr. Emily Kathryn Wyant saw in it the basis for a more far-reaching organization of mathematics students -- a national mathematics fraternity. Dr. Wyant and Professor L.P. Woods, head of the mathematics department,together with 22 other faculty and students, became the charter members of Oklahoma Alpha Chapter of Kappa Mu Epsilon on April 18, 1931. But one chapter does not make a national society. Dr. Wyant had carried on extensive correspondence with faculty members at several other colleges as she planned for this new society, with the result that the second chapter was installed as Iowa Alpha at Iowa State Teachers College, Cedar Falls, thirty-nine days later on May 27. During the following year, Kansas Alpha at Kansas State Teachers College, Pittsburg, and Missouri Alpha at Southwest Missouri State College, Springfield, were installed.

From this start fifty years ago, Kappa Mu Epsilon began to grow steadily. By the end of 1940, there were 24 chapters. In the next decade 21 more chapters were added. Between 1951 and 1960, 20 additional chapters were installed. By the end of 1970, 34 more were added. And in the decade between 1971 and 1981, we have installed 23 more. While some of these chapters have for a variety of reasons, become inactive, what this statistical summary shows is that Kappa Mu Epsilon has shown consistent, steady growth even during periods of national distress, war, campus unrest, and changing societal values. Emily Wyant's dream has been fulfilled in great measure! Our one hundred chapters are distributed over 31 states, with Pennsylvania holding the record for the most chapters: 12, and all still active from the time of the installation. (New York is second with ten chapters, 6 of which are still active, and Missouri is third with 9 chapters, 8 of which are active.)

Ten years after the founding of the Society, the first issue of The Pentagon was published and was considered then to be the most significant project ever undertaken by the Society. The magazine continues to meet an important objective -- to be a publication for college students of mathematics which includes many papers and other articles written by college mathematics students. The journal also has expanded and grown over its forty year history and shares the spotlight tonight as we celebrate our heritage.

One can only wonder what Dr. Wyant would say if she were to be here tonight to see so many chapters represented, to read the history booklet, to read the recent issue of The Pentagon, and to see the progress we've made and the interest in mathematics which has been generated by Kappa Mu Epsilon on the 100 various campuses. I think she would be very pleased and proud that her dream had been so successfully realized -- that Kappa Mu Epsilon is a national honor society truly worthy of the name and one which has been admitted to the selective Association of College Honor Societies because of this stature.

But lest we get too pleased with ourselves tonight, we must remember that we are celebrating a golden anniversary tonight only because we happen to be members NOW. And any success we note in 1981 can, of course, be seen only in the light of the 50 years of work and dedication of all those officers and 40,000 members who have gone before us. We owe them a great debt of gratitude which we can never repay. Some of these people are with us tonight and it is my very special privilege to introduce them to you and to welcome them on behalf of you. They are the past national officers of Kappa Mu Epsilon:

1. Carl V. Fronabarger
   Editor, The Pentagon, 1953-1959
   President, 1959-1963, and Mrs. Fronabarger

2. George R. Mach
   National Vice President, 1966-1969
   President, 1969-1973
   Now serving as National Secretary, since 1977

3. Sister Jo Ann Fellin
   National Historian, 1975-1979

4. Helen Kriegsman
   Editor, The Pentagon, 1965-1971

5. Dana Sudborough
   Business Manager, The Pentagon, 1945-1957,and Mrs. Sudborough

6. Wilbur Waggoner
   Business Manager, The Pentagon, 1957-1977
   Now serving as National Treasurer, since 1977

We are all delighted you could join us on this happy occasion and we express our thanks to you for all you did to bring us to this time in our Society's history.

Kappa Mu Epsilon has as one of its prime objectives the honoring of superior undergraduate academic performance in mathematics. Every year we induct national ly over 1100 new student members, an average of about 10 per chapter across the country. In almost every instance, those whom we have initiated as undergraduates have gone on to do fine things with their lives, professionally and personally, following their graduation. As a special feature of this golden anniversary celebration, the National Council invited the local chapters to nominate individual members who, after their graduation, had distinguished themselves professionally and who continue to bring credit to their alma mater and to Kappa Mu Epsilon. It is now my privilege to present to these fifty distinguished individuals the certificates naming them the Golden Anniversary Distinguished Members. I am asking Dr. Troy Hicks, our speaker of the evening, to represent them, all of whom are listed in your program along with their accomplishments; he will receive for all of them, collectively, this honor. The individual certificates will be mailed to the recipients following the convention, since many of them could not be with us tonight. Dr. Fronabarger, Sister Jo Ann Fellin, Dr. Kriegsman, and Professor Sudborough are also recipients, and I will ask them to stand at their places as I read the citation:

CITATION

Our very best wishes to all these Distinguished Members. It is a special privilege to honor them on this occasion, and we share with the nominating chapters the pride that they are members of Kappa Mu Epsilon. Their successes will serve as an inspiration to all of our present student members, providing high goals to strive for and hard work to emulate. Heartiest Congratulations!

IF YOUR SUBSCRIPTION HAS EXPIRED

We hope you have found THE PENTAGON both interesting and helpful. Your suggestions are always welcome and may be written on this form. They will be forwarded to the Editor.

If you wish to renew your subscription for two years, please send $5* to THE PENTAGON, Department of Mathematics, Central Michigan University, Mt. Pleasant, Michigan 48859.

Please give your name below in the manner in which you wish your mail addressed (Miss, Mrs., Prof., etc.). If your name has changed, please indicated also the name to which THE PENTAGON was sent.

Name ____

Address ____

City ____________________ State ____________ Zip __________

*Foreign subscriptions require an additional $1 per year for postage.

KME members are reminded that pins, keys, and tie-tacs are available and may be ordered through corresponding secretaries.