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National Officers

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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

STRUCTURE AND SEQUENCES

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Throughout an introductory calculus course of a single real variable, the concept of an open interval of real numbers is prevalent, and the definition is encountered early

Let a and b be real numbers with $a < b$.

Set $(a,b) = \{x \in \mathbb{R} \mid a < x < b\}$, then (a,b) is

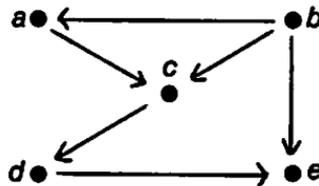
called an open interval with end points a and b [1, p.32].

Given this definition, the student continues the study of calculus, and soon it is realized that the open interval plays a key role in the sequence of development of basic ideas. A small sample of these fundamental concepts that depend of the notion of open interval is continuity of functions, the intermediate value theorem, the definition of the derivative of a function, the mean value theorem for derivatives, and the definition of convergence of sequences [1]. Actually, these concepts help to form the groundwork of calculus and since they all rely on the open interval to convey their meanings accurately, an inquisitive student might wonder what changes would result if the usual notion of open interval was somehow altered.

To examine these changes, the usual open interval must first be altered in an acceptable manner, and such a task requires distinguishing key properties of open sets from superfluous ones. Taking a closer look at the open interval of real numbers (a,b) , it is seen that, since the end points a and b are not included in the set, if $x \in (a,b)$ then there exists an $\epsilon > 0$ such that $(x-\epsilon, x+\epsilon) \subset (a,b)$. That is, given any x in the open interval (a,b) , x is surrounded by a buffer set which is itself a subset of (a,b) . In an intuitive light, it can be seen that the usual definition of open interval has structured the set of real numbers in such a way that every element of an open interval (a,b) is uninfluenced by all elements of the complement of (a,b) . With these ideas in mind, a method of determining the open subsets of an arbitrary point set is described.

Instead of letting the universal set be the set of real numbers \mathbb{R} , the finite point set $U = \{a,b,c,d,e\}$ will be taken as the universal set. The individual elements of U do not display any relationship to one another; that is, it cannot be determined how a is related to c or how d is related to e . U is said to be unstruc-

tured, and any statement regarding the open subsets of U is meaningless. However, it is a simple matter to define relationships between the elements of U , thereby, giving U structure [2, p.72].



Read " $x \rightarrow y$ " as " x influences y ", and define a subset O of U to be open if every element of O is uninfluenced by all elements of the complement of O . For example, $\{b\}$, $\{a,b,c,d\}$, and U are open, however, $\{a,b,c,e\}$ is not open because $d \rightarrow e$, and d is not an element of $\{a,b,c,e\}$. Note that the similarity between an open subset of U and an open interval or real numbers is that, in both cases, every element of an open set is uninfluenced by all elements of the complement of the open set.

Returning to the set of real numbers, the usual concept of open interval will be altered in a manner consistent with the similarities between the usual notion of open interval and the open subsets of U . Let x and y be real numbers with $x > y$, we can say that " x is uninfluenced by y ." Recall that an interval O of real numbers is open if every element of O is uninfluenced by all elements of the complement of O . The above criterion for open interval translates into "an interval O of real numbers is open if every element of O is greater than all elements of the complement of O ." With this definition, it is seen that the only open intervals of R are of the form

$$[a, \infty) = \{x \in R \mid x \geq a\}$$

$$(a, \infty) = \{x \in R \mid x > a\} \quad [2, p.83].$$

To arrive at this new characterization of open intervals, an intuitive and seemingly arbitrary development has been presented. It is offered in conjunction with the following three axioms, which were taken from the study of topology where open sets are studied in the most general light possible to convince the reader that intervals of the form (a, ∞) and $[a, \infty)$ are indeed, in a general sense, open [2, p.100].

1. The universal set and the empty set are both open.
2. Finite intersections of open sets are open.
3. Any union of open sets is open.

A few moments should be taken to verify that intervals of the form $[a, \infty)$ and (a, ∞) , where R is the universal set, really do satisfy the three open set axioms. It's enlightening to note that the open subsets of U satisfy the axioms. Returning to the usual definition of open interval, it is seen that intervals of the form $(-\infty, a)$, (a, ∞) , and (a, b) also satisfy the axioms.

With the task of sorting key properties of open sets from superfluous ones accomplished, a brief summary of what has been said so far and the introduction of some convenient notation will help to clarify what is to follow. It has been seen that the set of real numbers R can be paired with a structuring system in which all open intervals are of the form $(-\infty, a)$, (a, ∞) , and (a, b) . This pairing is denoted (R, S_U) . S_U denotes "the usual structure." Also, it has been demonstrated that R can be paired with a structuring system in which all open intervals are of the form $[a, \infty)$ and (a, ∞) ; this pairing is denoted (R, S_r) . (S_r denotes the "right structure" for obvious reasons.)

The set of real numbers has been restructured in an acceptable manner, and the fundamental concepts of calculus may now be examined in the two contrasting lights of (R, S_U) and (R, S_r) . The convergence of sequences provides several interesting examples of diverse behavior with regard to these two structuring systems, and with this end in mind, the following two definitions are offered.

1. A sequence s of real numbers is a function from the set N of natural numbers into R ; it is denoted $s: N \rightarrow R$. $s(n) \in R$ for $n \in N$ [1, p.37].
2. A sequence $s: N \rightarrow R$ is said to converge to a point $p \in R$ if, given any open interval O containing p , there corresponds a positive integer M such that $s(n) \in O$ whenever $n > M$. Convergence is denoted $s(n) \rightarrow p$ [2, p.121].

The ground work has been laid, and the behavior of some specific sequences may now be examined in close detail.

EXAMPLE 1

Claim: In (R, S_U) the sequence $s(n) = n$ does not converge to any real number.

Most students of calculus would feel confident in readily accepting the previous proposition. However, at first glance, the following contrasting statement may appear quite absurd.

Claim: In (R, S_r) the sequence $s(n) = n$ converges to every real number.

Justification: Recall that in (R, S_r) open intervals are of the form $[a, \infty)$ and (a, ∞) . Let p be any real number; the smallest open interval containing p is $[p, \infty)$. If $p \leq 0$, let M be any positive integer, and if $p > 0$, the Archimedean property guarantees the existence of a positive integer M greater than p . In either case, $s(n) \in [p, \infty)$ whenever $n > M$. Since p is arbitrary, and since $[p, \infty)$ is a subset of any open interval containing p , the sequence $s(n) = n$, in (R, S_r) , converges to every real number.

EXAMPLE 2

Claim : In (R, S_U) the constant sequence, $s(n) = k$, converges to k and only k .

Again the following statement exemplifies the diverse behavior of sequences when the set of real numbers is restructured.

Claim: In (R, S_r) the constant sequence, $s(n) = k$, converges to every real number less than or equal to k . In symbols, $p \leq k$ implies $s(n) \rightarrow p$.

Justification: Let p be any real number less than or equal to k . The smallest open interval containing p is $[p, \infty)$, and since $p \leq k$, $k \in [p, \infty)$. But $s(n) = k$, so $s(n) \in [p, \infty)$ for all n ; therefore, $s(n) \rightarrow p$. The only restriction on p is that $p \leq k$; therefore the sequence $s(n) = k$ converges to every real number less than or equal to k .

EXAMPLE 3

Claim: In (R, S_U) the sequences $s(n) = 1/n$ and $s(n) = -1/n$ converge to 0.

Once more the right structure provides a surprising example.

Claim: In (R, S_r) the sequence $s(n) = 1/n$ converges to 0, however the sequence $s(n) = -1/n$ does not converge to 0.

Justification: The range values of the sequence $s(n) = 1/n$ are always greater than zero. But any open interval containing zero also contains all values greater than zero. It is seen, then, that the entire range of s lies in any open interval containing zero. Therefore, the sequence $s(n) = 1/n$ converges to 0.

Now consider the sequence $s(n) = -1/n$. The range values of this sequence are less than zero for all n so that they never lie in the open interval $[0, \infty)$. It is seen that there is an open interval containing zero but none of the range values of s . Therefore $s(n) = -1/n$ does not converge to 0.

The intuitive development of (R, S_r) and the three open set axioms demonstrate that the notion of open set is much more general than the usual calculus definition would lead us to believe. Here, then, are three more acceptable definitions of open sets.

1. Let O be a subset of R . O is open if and only if the complement of O is finite.
2. Every subset of R is open.
3. R and the empty set are the only open subsets of R .

It is seen that the open sets characterized by each of these definitions satisfy the axioms [2, p.83]. The interested reader should look at convergence of sequences and continuity of functions in the light of these open set definitions. Many unusual and surprising examples will be found.

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THE LONG AND THE SHORT OF IT: When Is the Maximum the Minimum?

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Calculus teachers often wish to apply the concepts of maximum and minimum to real life situations. They also wish to find problems which provide practice involving trigonometric functions. The following is an example of a problem which meets both of the needs of the teacher: Given two corridors of widths a and b that meet at right angles, what is the length of the longest rod that can be turned from the first corridor to the second corridor without bending or tilting?

This problem is difficult to solve directly as stated; therefore replace it with the following essentially equivalent problem: Given corridors I and II of respective widths a and b , let L be the set of all line segments containing P and having endpoints on l_1 and l_2 . Of all the line segments in L find the shortest one. Figure 1 depicts AB as this minimal segment.

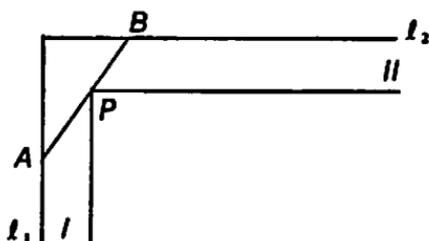


Figure 1

Why is this problem concerning minimal line segments essentially equivalent to the problem involving maximal rods?

Figure 2 is a duplicate of Figure 1 with AB as the minimal line segment in L with the additional inclusion of a rod CD which is being turned from corridor I into corridor II. The rod CD is being turned so that point C slides along l_1 , and the rod is always in contact with point P .

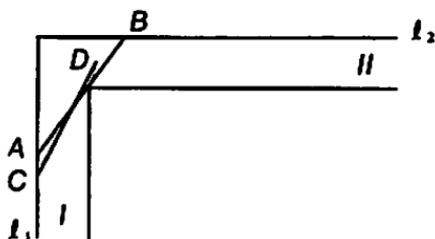


Figure 2

If rod CD is to be turned from corridor I into corridor II what is its maximum possible length? To analyze this problem consider these three cases:

1. Suppose CD is shorter than AB . In this case at no time can the end of the rod (point D) touch l_2 . If it does, then CD is a line segment of L but is shorter than AB which is the minimal line segment in L . A longer rod CD could then be used.

2. Suppose CD is longer than AB . In this case when endpoint C coincides with A , endpoint D will have to lie *behind* the wall l_2 . The rod CD is then too long to be turned from corridor I into corridor II.

3. Suppose CD is the same length as AB . Let C slide along l_1 , and consider these two subcases.

A. If C does not coincide with A , then D lies in corridor II and doesn't touch l_2 . Why? If D did lie on l_2 then CD would represent a second minimal line segment of L . The calculus development which we shall present later shows that AB is unique. If D lies behind the wall l_2 , then there is a segment shorter than CD (in fact a subset of CD) which is in set L . Since CD is the same length as AB , we again have a contradiction.

B. If C coincides with A , then D will also coincide with B since CD and AB have the same length.

Since D touches l_2 only in the event that C coincides with A , the rod AB can be turned from corridor I into corridor II with only one "close call" in transit. At the instant at which C and D lie on l_1 and l_2 , respectively, the rod appears stuck; however, point D immediately slides past l_2 and again lies in the interior of corridor II. Consequently, from cases 1 to 3 we conclude that the minimum length of segment AB (which passes through P and has endpoints on l_1 and l_2) is the same as the maximum length of a rod which can be turned from corridor I to corridor II. Figure 3 represents this minimal segment.

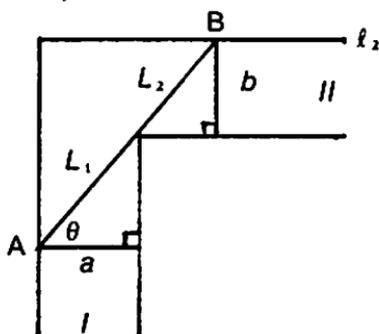


Figure 3

We have AB (to be minimized) $= L_1 + L_2 = a \sec \theta + b \csc \theta$, thus

$$\begin{aligned}\frac{d(AB)}{d\theta} &= a \sec \theta \tan \theta - b \csc \theta \cot \theta \\ &= \frac{a \sin \theta}{\cos^2 \theta} - \frac{b \cos \theta}{\sin^2 \theta} \\ &= \frac{a \sin^3 \theta - b \cos^3 \theta}{\cos^2 \theta \sin^2 \theta}\end{aligned}$$

To maximize AB , set $d(AB) / d\theta$ equal to 0 and then solve for θ . Since $0 < \theta < \pi/2$, $\cos^2 \theta \sin^2 \theta > 0$. The derivative is then equal to 0 if and only if its numerator is equal to 0. Thus we wish to solve

$$\begin{aligned}a \sin^3 \theta - b \cos^3 \theta &= 0 \\ \sin^3 \theta / \cos^3 \theta &= b/a \\ \tan^3 \theta &= b/a \\ \tan \theta &= \sqrt[3]{b/a} \\ \theta &= \arctan \sqrt[3]{b/a}\end{aligned}$$

Since the minimum value of AB is attained for a unique value of θ and since each value of θ gives rise to a unique point A on l_1 , this establishes that AB is unique. This verifies the assertion referred to in 3A above.

To illustrate this problem numerically let $a = 192$ cm and $b = 273$ cm. Then $\theta = \arctan \sqrt[3]{b/a} = \arctan \sqrt[3]{273/192}$
 $= \arctan \sqrt[3]{1.4219}$
 $= \arctan 1.1245$
 $\theta = .8439$ (or 48.35°)

Then AB (the length of the minimal line segment of L or the longest rod that can be turned from corridor I into corridor II) is $a \sec \theta + b \csc \theta$.

$$\begin{aligned}AB &= 192(\sec .8439) + 273(\csc .8439) \\ &= 192(1.5048) + 273(1.3382) \\ &= 288.92 + 365.34 \\ &= 654.26 \text{ cm}\end{aligned}$$

Challenges for the reader:

1. What happens if a rectangle instead of a rod is to be turned from corridor I into corridor II? This situation would arise if a cart or vehicle were involved — for example, a wheel chair leaving an elevator to turn into a corridor.
2. What would happen if the problem were considered in three dimensions so that the rod could be tilted?

THE MULTIPLICATION PRINCIPLE FOR COUNTING

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In counting courses two principles are usually listed early, the Addition Principle and the Multiplication Principle. Most books give a set theoretic version of the Addition Principle similar to: if (A_1, \dots, A_k) is a k -tuple of pairwise disjoint finite sets, then $|\cup\{A_i; i=1, \dots, k\}| = |A_1| + \dots + |A_k|$ (where $|S|$ denotes the number of elements in the finite set S). The following intuitive version of the Multiplication Principle seems to be most prevalent: if an event can be performed in n_1 ways and, after it is performed in any one of these ways, a second event can be performed in n_2 ways and, after the first two events have been performed, a third event can be performed in n_3 ways, and so on for k events, then the k events can be performed together in $n_1 \dots n_k$ ways. For a variety of statements of the Multiplication Principle see the references. It seems desirable to have a set theoretic version of the Multiplication Principle which is easily stated, easily proven, and easy to use when counting by use of set theoretic techniques. The most prevalent set theoretic version of the Multiplication Principle is usually stated as the following theorem; for a notable exception see [5, p.5].

Theorem 1. *If (A_1, \dots, A_k) is a k -tuple of sets with $|A_i| = n_i$, then $|A_1 \times \dots \times A_k| = n_1 \dots n_k$.*

Theorem 1 is easily proven by use of the Addition Principle and induction.

Following a few definitions and a lemma, two other set theoretic versions of the Multiplication Principle will be stated.

A relation R (i.e. a set of ordered pairs) is called an (S, n) -relation if the domain of R is S and if each s in S appears as the first coordinate of exactly n pairs in R . A set T can be viewed as an (S, n) -relation means T can be put into a one-to-one correspondence with an (S, n) -relation. Henceforth, in this article all sets will be finite.

Lemma. *T can be viewed as an (S, n) -relation if and only if $|T| = |S|n$.*

Proof. Suppose T can be viewed as an (S, n) -relation, then choose an (S, n) -relation R with $|T| = |R|$. For each s in S , let R_s denote the set of all elements of R having s as their first coordinate. Then $|R_s| = n$ and R is the union of the pairwise disjoint R_s 's. Hence $|R| = \sum_{s \in S} |R_s| = n |S|$.

For the converse, suppose $|T| = |S|n$. According to Theorem 1, $|S \times \{1, \dots, n\}| = |S|n$. Hence $|T| = |S \times \{1, \dots, n\}|$. Since $S \times \{1, \dots, n\}$ is an (S, n) -relation and $|T| = |S \times \{1, \dots, n\}|$, T can be viewed as an (S, n) -relation.

In view of the lemma, the following two theorems are different wordings of each other.

Theorem 2. *If (T_1, \dots, T_k) is a k -tuple of sets with $|T_i| = n_i$ and, for all $2 \leq i \leq k$, T_i can be viewed as a (T_{i-1}, n_i) -relation, then $|T_k| = n_1 \dots n_k$.*

Theorem 3. *If (T_1, \dots, T_k) is a k -tuple of sets with $|T_i| = n_i$ and, for all $2 \leq i \leq k$, $|T_i| = |T_{i-1}|n_i$ then $|T_k| = n_1 \dots n_k$.*

Theorem 2 is essentially the version of the Multiplication Principle given in [5]. Theorem 3 is interesting because its proof is extremely easy. When Theorem 3 is stated for $k = 2$, it becomes amazingly apparent that it is an extremely basic theorem.

In order to demonstrate how Theorem 2 or Theorem 3 can be used, the number of k -permutations on a set with n elements will be counted. Recall that a k -permutation on S is a k -tuple (x_1, \dots, x_k) where the x_i 's are distinct elements of S (here k satisfies $1 \leq k \leq n$). For all $1 \leq i \leq k$, let $P_i(S)$ denote the set of all i -permutations on S . By the range of a k -tuple $t = (x_1, \dots, x_k)$, denoted by $Rng\ t$, is meant $\{x_1, \dots, x_k\}$. For $2 \leq i \leq k$, let $R_i = \{(t, x) : t \in P_{i-1}(S), x \in S - Rng\ t\}$ and notice that R_i is a $(P_{i-1}(S), n - (i-1))$ -relation. Also, for $2 \leq i \leq k$, observe that $f_i: P_i(S) \rightarrow R_i: (x_1, \dots, x_i) \rightarrow ((x_1, \dots, x_{i-1}), x_i)$ is a one-to-one correspondence. Thus $|P_i(S)| = |R_i| = |P_{i-1}(S)| [n - (i-1)]$ for all $2 \leq i \leq k$. So according to Theorem 3, $P_k(S) = n(n-1) \dots [n - (k-1)]$. Use of Theorem 1 to count the number of elements of $P_k(S)$ seems more tedious than the process just described.

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HOW TO DERIVE YOUR OWN MEAN VALUE THEOREM

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As is well known, the mean value theorem states, if $f(x)$ is continuous on $a < x < b$ and is differentiable on $a < x < b$, then there is an X , with $a < X < b$, such that

$$\frac{f(b) - f(a)}{b - a} = f'(X)$$

Since $(f(b) - f(a))/(b - a)$ is the slope of the line thru $P(a, f(a))$ and $Q(b, f(b))$, this theorem essentially equates the slope of an arbitrary (differentiable) function at some interior point with the slope of the line \overline{PQ} . It is not necessary to use the slope of \overline{PQ} . In fact, the theorem is just a special case of the following:

Let $f(x)$ and $g(x)$ be continuous on $a \leq x \leq b$, be differentiable on $a < x < b$, and $g(a) = f(a)$, $g(b) = f(b)$. Then there is an X , with $a < X < b$, such that

$$(1) \quad f'(x) = g'(x)$$

The hypotheses allow Rolle's theorem to apply to $f(x) - g(x)$, and the result follows.

As an illustration, suppose we let $f(x)$ be a function continuous on $a < x < b$ and differentiable on $a \leq x \leq b$. Then there exists X , with $a < X < b$, such that

$$(2) \quad f'(X) = 2 \frac{f(b) - f(a)}{b^2 - a^2} \quad b + a \neq 0$$

This follows from (1) where $g(x) = rx^2 + s$, where r and s are determined from $g(a) = f(a)$ and $g(b) = f(b)$.

Applied to $f(x) = x/(1 + x^2)$ with $a = 1$, $b = 2$, we have $f(a) = 1/2$ and $f(b) = 2/5$; $f'(x) = (1 - x^2)/(1 + x^2)^2$. This gives us (from (2))

$$\frac{1 - X^2}{(1 + X^2)^2} = \frac{2(2/5 - 1/2)}{4 - 1} X$$

$$X^5 + 2X^3 - 15X^2 + X + 15 = 0$$

$$\text{and} \quad X = 1.23479865 +$$

As a second illustration let $f(x)$ be a function continuous on $a \leq x \leq b$ and differentiable on $a < x < b$. Then there exists X , with $a < X < b$, such that

$$(3) \quad f'(X) = 2 \frac{af(b) - bf(a)}{ab(b - a)} X - \frac{a^2f(b) - b^2f(a)}{ab(b - a)}, \quad ab \neq 0$$

Here $g(x) = rx^2 + sx$ with r and s determined as above. Applied to $f(x) = x/(1 + x^2)$ with $a = 1$, $b = 2$ as before, we have

$$\frac{1 - X^2}{(1 + X^2)^2} = \frac{2(2/5 - 1)}{2}X - \frac{2/5 - 2}{2} = -3/5X + 4/5$$

Then $3X^5 - 4X^4 + 6X^3 - 13X^2 + 3X + 1 = 0$
and $X = 1.53390574 +$

The Mean Value Theorem is obtained by setting $g(x) = rx + s$ and determining r, s as above.

Using $g(x) = r/x^n + s$ or $g(x) = r \ln x + s$ or whatever, you can make up your own Mean Value Theorem.

MULTIPLICATION AND DIVISION OF VECTORS IN A PLANE

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Since points in a plane, complex numbers, and vectors are interrelated with one another, it is surprising that vector multiplication has not been defined in the following obvious way.

Let A and B be two vectors with inclinations θ_1 and θ_2 , respectively. Define AB to be the vector with inclination $\theta_1 + \theta_2$ such that $\|AB\| = \|A\| \|B\|$ where as usual the symbol $\|$ means the norm of a vector. Likewise, define A/B to be the vector with inclination $\theta_1 - \theta_2$ such that $\|A/B\| = \|A\|/\|B\|$

We can now derive the following five results from the above definition.

(1) If $A = \langle a, b \rangle$ and $B = \langle c, d \rangle$, then,

$$\begin{aligned} AB &= \langle \|AB\| \cos(\theta_1 + \theta_2), \|AB\| \sin(\theta_1 + \theta_2) \rangle \\ &= \langle \|A\| \|B\| (\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2), \|A\| \|B\| (\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2) \rangle \\ &= \langle \|A\| \cos\theta_1 \|B\| \cos\theta_2 - \|A\| \sin\theta_1 \|B\| \sin\theta_2, \|A\| \sin\theta_1 \|B\| \cos\theta_2 + \|A\| \cos\theta_1 \|B\| \sin\theta_2 \rangle \\ &= \langle ac - bd, bc + ad \rangle \end{aligned}$$

$$\begin{aligned} A/B &= \langle \|A/B\| \cos(\theta_1 - \theta_2), \|A/B\| \sin(\theta_1 - \theta_2) \rangle \\ &= \left\langle \frac{\|A\|}{\|B\|} (\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2), \frac{\|A\|}{\|B\|} (\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2) \right\rangle \\ &= \left\langle \frac{\|A\| \|B\| (\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2)}{\|B^2\|}, \frac{\|A\| \|B\| (\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2)}{\|B^2\|} \right\rangle \\ &= \left\langle \frac{\|A\| \cos\theta_1 \|B\| \cos\theta_2 + \|A\| \sin\theta_1 \|B\| \sin\theta_2}{\|B^2\|}, \frac{\|A\| \sin\theta_1 \|B\| \cos\theta_2 - \|A\| \cos\theta_1 \|B\| \sin\theta_2}{\|B^2\|} \right\rangle \\ &= \left\langle \frac{ac + bd}{c^2 + d^2}, \frac{bc - ad}{c^2 + d^2} \right\rangle \end{aligned}$$

(2) The so-called dot or scalar product of two vectors A and B can now be written as

$$\begin{aligned} A \cdot B &= \|A\| \|B\| \cos(\theta_1 - \theta_2) = \|AB\| \cos(\theta_1 - \theta_2) \\ &= \|A/B\| \cos(\theta_1 - \theta_2) \|B\|^2 \end{aligned}$$

$$= \frac{(ac + bd)}{c^2 + d^2} (c^2 + d^2) = ac + bd$$

(3) Let us define another type of scalar product $A \times B$ to be equal to $\|A\| \|B\| \sin(\theta_1 - \theta_2)$

By an argument similar to (2), we finally obtain

$$A \times B = bc - ad$$

From this "star" product as defined above, we deduce that

$$(bc - ad)^2 \leq (a^2 + b^2)(c^2 + d^2)$$

(4) We can use the vector product and vector division as defined in this article to prove some trigonometric formulae. For example, since $AB = \langle ac - bd, bc + ad \rangle$, it follows that

$$\tan(\theta_1 + \theta_2) = \frac{bc + ad}{ac - bd} = \frac{(bc + ad)/ac}{(ac - bd)/ac}$$

$$= \frac{(b/a) + (d/c)}{1 - (b/a)(d/c)} = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$$

We can similarly derive the formula for $\tan(\theta_1 - \theta_2)$ from the quotient vector A/B .

Let us derive another formula. Suppose we have a vector $V = \langle \cos\theta, \sin\theta \rangle$. It is clear that V is a unit vector. Thus the vector $V^2 = VV$ is also a unit vector that makes angle 2θ with the x -axis. Therefore, we can write

$$V^2 = \langle \cos 2\theta, \sin 2\theta \rangle$$

On the other hand,

$$\begin{aligned} V^2 &= VV = \langle \cos\theta, \sin\theta \rangle \langle \cos\theta, \sin\theta \rangle \\ &= \langle \cos^2\theta - \sin^2\theta, 2 \sin\theta \cos\theta \rangle \end{aligned}$$

Thus, we conclude,

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

Similarly, from V^n , $n \geq 3$, we can derive the formulae for $\cos n\theta$ and $\sin n\theta$.

(5) It is clear from the quotient vector A/B that should A and B be perpendicular to each other, A/B must be of the form $\langle 0, k \rangle$ and should they happen to be parallel to each other, A/B must be of the form $\langle k, 0 \rangle$. The converse is also true. Stated differently, it means that $A = \langle a, b \rangle$ and $B = \langle c, d \rangle$ are perpendicular to each other iff $ac + bd = 0$ and are parallel to each other iff $bc - ad = 0$.

As usual, these conclusions can be applied to prove several geometrical results which are left to the reader. As examples, he can delight himself proving that the diagonals of a rhombus are perpendicular to each other, that the line joining the middle

points of two sides of a triangle is parallel to the third side, or that the angle in a semicircle is always a right angle.

We can also use the concept of vector multiplication and vector division for obtaining equations to different loci. For example, say, we want to obtain the equation to the line passing through two points, namely (a,b) and (c,d) . We take any point (x,y) on this line and then we consider the following vector

$$\frac{\langle x,y \rangle - \langle a,b \rangle}{\langle c,d \rangle - \langle a,b \rangle}$$

Since the vectors $\langle x,y \rangle - \langle a,b \rangle$ and $\langle c,d \rangle - \langle a,b \rangle$ are parallel to each other, being based along the same line, we have from the above discussion

$$\frac{\langle x,y \rangle - \langle a,b \rangle}{\langle c,d \rangle - \langle a,b \rangle} = \langle k,0 \rangle$$

for some real number k which is equivalent to the statement that

$$\frac{\langle x-a,y-b \rangle}{\langle c-a,d-b \rangle} = \langle k,0 \rangle$$

Thus

$$\begin{aligned} \langle x-a,y-b \rangle &= \langle c-a,d-b \rangle \langle k,0 \rangle \\ &= \langle k(c-a), k(d-b) \rangle \end{aligned}$$

which is the vector equation for our line. From the definition of the equality of two vectors and upon eliminating k , we obtain the usual cartesian form of the equation of this line. The reader may now try to obtain the equation of a circle with center (a,b) and radius equal to r .

Let me finally remark that the above discussion runs parallel to the discussion of complex numbers and the results of that system can easily be translated to this system.

THE MATHEMATICAL SCRAPBOOK

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgement will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

"Someone asked Euclid, 'But what shall I get by learning these things?' Euclid called his slave and said, 'Give him three coins, for he must make gain out of what he learns.'" —Stobaeus

A collection of interesting mathematical problems are those in which some or all the numerical digits are replaced by letters of the alphabet. Unfortunately, there is not a general rule which leads to the solutions of problems of this type. Hence, one must resort to careful examination of the problem and the various properties of integers.

An example of a problem of this type is the following: Find the digits represented by the letters where

$$(AA)^2 = BCDE,$$

and

$$(EE)^2 = EDCB.$$

Specifically, we are asked to determine the numerical value of the five letters indicated, where each letter must consist of one of the integers 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9.

From number theory we may recall the following twelve basic facts.

1. An even integer squared is even and the square of an odd integer is odd.
2. The square root of an odd positive integer is odd and that of an even is even.
3. The product of four consecutive integers cannot be a square.
4. Every square is of the form $3K$ or $3K + 1$, where K is a non-negative integer.
5. The square of an integer does not end in a single 0, in any two odd numbers, or in the digits 2, 3, 7, or 8.
6. The square of any integer ending with 5 always ends in 25 and the square of any integer ending in 25 always ends in 625.
7. The square of any number ending in 24, 26, 74, or 76 ends with 76.

8. Only five 4-digit squares begin with the same digit as the original integer. They are:

$$9025 = 95^2$$

$$9216 = 96^2$$

$$9409 = 97^2$$

$$9604 = 98^2$$

$$9801 = 99^2$$

9. The pairs of digits 25 and 76 are the only two pairs that occur at the end of a number which also reappear at the end of its square, cube, or a higher power of that number. That is, note:

$$25^2 = 625$$

$$76^2 = 5776$$

$$25^3 = 15625$$

$$76^3 = 438976$$

$$25^4 = 390625$$

$$76^4 = 33362176$$

$$25^5 = 9765625$$

$$76^5 = 2535525376$$

$$\vdots$$

$$\vdots$$

10. If the sum of the digits of a number is 15, it is neither a square nor a cube.
11. Every cube has one of the three forms $7K$, $7K+1$, or $7K-1$, where K is a non-negative integer.
12. If a number ending in the digits AB is multiplied by a number ending in CD and we get a product ending in AB or CD , then either $AB = 51$ or $CD = 51$.

Let us now return to our original problem. We first note that the two squares have reverse digits. Also, since $(EE)^2 = EDCB$ we note that our 4-digit square begins with the same digit as the original number. Hence, by 8 above, our number EE must be 99; that is, F is 9. Hence, $(EE)^2 = 99^2 = 9801 = EDCB$, and so $BCDE = 1089 = 33^2 = (AA)^2$. We thus conclude $A = 3$, $B = 1$, $C = 0$, $D = 8$, and $E = 9$.

Now consider the problem:

$$(AAB)^2 = BCDAB.$$

Examining a table of squares, one finds AAB must be less than 317 since $317^2 = 100489$ while $316^2 = 99856$, and the problem requires a 5-digit square. We also note that the square ends in AB as does the original number. Hence, fact 9 implies AB is either 25 or 76. Thus AAB must be 225 or 776. But 776 is impossible here since it is above 316. Hence, AAB is 225, so $(AAB)^2 = (225)^2 = 50625 = BCDAB$.

Another slightly more difficult problem is the following one, requiring the simultaneous solution of the three equations:

$$\begin{aligned}(DC)^2 &= BACF, \\ (FG)^2 &= CFAB, \text{ and} \\ C^2 &= F.\end{aligned}$$

Assuming each letter represents a distinct digit and that the digits are unequal, our third equation implies C is either 2 or 3 and F is either 4 or 9. But here $F \neq 9$ since the second equation would then imply $C = 8$ (since $90^2 = 8100$, $91^2 = 8281$, $92^2 = 8464$, $93^2 = 8649$, $94^2 = 8836$, and the squares of 95 to 98 are between 9025 and 9604). Hence, $F = 4$, so $C = 2$.

Our first two equations can now be written as:

$$\begin{aligned}(D2)^2 &= BA24, \text{ and} \\ (4G)^2 &= 24AB.\end{aligned}$$

Again examining our table of squares, we find the only 4-digit square which begins with 24 is 2401, which is the square of 49. Hence $(4G)^2 = (49)^2 = 2401 = 24AB$, and $BA24 = 1024 = (32)^2 = (D2)^2$.

Still another problem is the following: Find A, B, C, D , such that $ABCDC \times 9 = CDCBA$.

We first note that both the original number $ABCDC$ and the product have exactly 5 digits, so A must be 1, which implies B must be 0. Hence, we have:

$$10CDC \times 9 = CDC01.$$

But $C \times 9$ ends in 1, so C must equal 9. Hence,

$$109D9 \times 9 = 9D901,$$

which implies D equals 8. Can you show 10989 is the only 5-digit number to have its digits reversed when multiplied by 9?

A similar problem is the following:

$$ABCDE \times 4 = EDCBA.$$

Can you show that $ABCDE$ is 21978?

A partial test of interesting problems of this type would include the following:

- (1) $(ABA)^2 = ACDC A$, and $C = B^2$
- (2) $(CBA)^2 = DEFBA$
- (3) $AB \times CB = DDD$
- (4) $AB \times CDB = BBBBB$
- (5) $(AB)^2 - (BA)^2 = ACD$
- (6) $(AB)^2 - (BA)^2 = EDC$

Then finally for the story problem buff, the following problem may be interesting:

- (7) The numbers AB, BC, DC, EB , and FB are all primes. We

are given $(ABC)^2 = FBEBDC$. Find the 6 distinct digits.

Solutions to the above seven problems are as follows: (1) $ABA = 121$, (2) $CBA = 176$, (3) 37×27 with $D = 9$, (4) 41×271 with $B = 1$, (5) $21^2 - 12^2 = 297$, (6) $31^2 - 13^2 = 792$, and (7) $731^2 = 534,361$.

Additional problems involving sums are as follows, where each letter represents a distinct digit.

$$\begin{array}{r} EAT \\ + MORE \\ \hline CAKES \end{array}$$

$$\begin{array}{r} PAY \\ + THE \\ \hline RENT \end{array}$$

$$\begin{array}{r} FOUR \\ + FOUR \\ \hline EIGHT \end{array}$$

Can you find their solutions?

THE PROBLEM CORNER

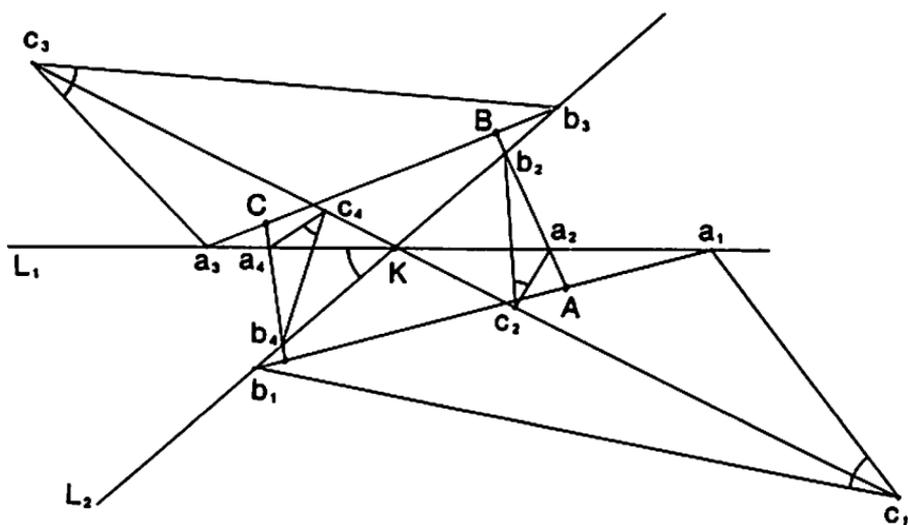
EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 February 1980. The solutions will be published in the Spring 1980 issue of *The Pentagon*, with credit being given for other solutions received. Preference will be given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

312. *Proposed by John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico.*

Let L_1 and L_2 be the axes of a plane coordinate system which cut off line segments $a_i b_i$ ($i = 1, 2, 3, 4$) of the sides (extended if necessary) of a quadrilateral $ABCD$ in such a manner that each point a_i lies on L_1 and each point b_i lies on L_2 . Let K denote the intersection of L_1 and L_2 . Now if similar triangles $a_i b_i c_i$ are drawn on each line segment $a_i b_i$ such that each angle with its vertex at c_i is equal to the angle formed by L_1 and L_2 , then show that the vertices c_i and the intersection K of the axes are collinear.



313. *Proposed by Michael W. Ecker, Scranton, Pennsylvania.*
 Joe and Moe plan to meet for lunch at the pizza parlor between noon and 1:00 P.M. but they can't decide what time to meet. Joe suggested that whoever arrives first should wait 10 minutes for the other before leaving. Moe likes Joe's suggestion but he wonders if a 10 minute wait will guarantee that they will have at least an even chance of meeting for lunch. Assuming that each of Joe's and Moe's times of arrival is random, what is the minimum time the first to arrive must wait to guarantee that their probability of having lunch together is at least $\frac{1}{2}$?
314. *Proposed by H. Laurence Ridge, University of Toronto, Toronto, Ontario.*
 One day John met Bill on the street.
 "Hi Bill."
 "Hi John. How's your family?"
 "Fine," replied John.
 "Tell me," asked Bill. "How old are your three children?"
 "Well," said John, "The product of their ages is 36 and the sum is one less than the address of the white building across the street."
 Bill noted the number on the building and went home. The next day Bill called John and complained that he wasn't given enough information.
 John said "My eldest child is a girl," whereupon Bill immediately gave the correct ages.
 What were the ages?
315. *Proposed by H. Laurence Ridge, University of Toronto, Toronto, Ontario*
 4 married couples meet for dinner.
 There is some shaking of hands. No one shakes hands more than once with the same person. Spouses do not shake hands.
 When the hand shaking is finished one husband asks all of the other people how many times they shook hands. Everyone gives a different answer.
 How many times did the questioner's wife shake hands?
316. *Proposed by Randall J. Covill, Mansfield, Massachusetts.*
 A repunit is an integer in the decimal system whose representation consists of a finite string of ones; e.g., the numbers 11, 11111, 1111111 are all repunits. A Fer-

mat number has the form $2^{2^k} + 1$ for any integer $k > 0$. Are any Fermat numbers also repunits?

SOLUTIONS

297. *Proposed by Charles W. Trigg, San Diego, California.*
R E T I R E is a square number in the decimal system with $- R E + T I = R E$.
 Each letter represents a different digit and the sum of three digits equals the fourth. What is this square number?
Solution by Leo Souve, Algonquin College, Ottawa Ontario, Canada.

If we set $R E = x$ and $T I = y$, then $y = 2x$ and

$$\begin{aligned} R E T I R E &= 10001 R E + 100 T I = 10001x + 100y \\ &= 10201x = 101^2x. \end{aligned}$$

Since **R E T I R E** is square, x is a two digit square such that its double consists of two digits distinct from the digits of x . The only possibilities are $(x,y) = (16,32)$ or $(36,72)$. Only the first satisfies the requirement that the sum of three digits equals the fourth. Thus the unique solution is $x = R E = 16$, $y = T I = 32$ and

$$R E T I R E = 163216 = 404^2.$$

Also solved by the proposer.

302. *Proposed by Randall J. Covill, Newburyport, Massachusetts.*

Consider the following digital display problem. A character is a set of parallel and/or perpendicular non-intersecting line segments of constant length. If a character has height, the height is equal to a constant whole number of line segments. If a character has width, the width is equal to a different constant whole number of line segments. If any segment or subset of segments can be either displayed or not displayed, what is the minimum number of segments necessary to represent all ten digits 0 to 9?

Solution by Michael W. Ecker, Scranton, Pennsylvania.

Clearly, the digits must have a height of at least two units, in view of the non-intersection requirement applied, for example, to the digit 2. Accordingly a minimal display of characters of height two units and width one unit is easily constructed; e.g.,

0123456789 utilizing 49 line segments of unit length. By superimposing all the characters it is easy to see that each of the characters can be constructed by removing appropriate line segments from the seven unit arrangement  which is minimal.

Also solved by the proposer.

303. *Proposed by Charles W. Trigg, San Diego, California.*
Show that the ratio of the volume of a sphere to the volume of its inscribed regular octahedron is π .

Composite solution from the solutions of Fred A. Miller, Elkins, West Virginia, and the proposer.

The regular octahedron consists of two square pyramids each with an altitude of R and a base with diagonals of $2R$, where R is the circumradius of the octahedron. Thus the volume of the octahedron is given by

$$V_o = 2(1/3) (R \sqrt{2})^2 R = (4/3) R^3.$$

The volume of the circumsphere is given by $V_s = (4/3)\pi R^3$.

Hence $V_s/V_o = \pi$.

304. *Proposed by Charles W. Trigg, San Diego, California.*
Does any three-digit number, N , equal 11 times the sum of the squares of its digits?

Solution by Fred A. Miller, Elkins, West Virginia.

Let $N = 100a + 10b + c = 11(a^2 + b^2 + c^2)$ where $a \neq 0$.

Then $a(100 - 11a) = b(11b - 10) + c(11c - 1)$ where $a, b, c < 10$, can be rewritten as

$$a(1 + 11(9 - a)) = b(11(b - 1) + 1) + c(11c - 1) \text{ or}$$

$$a - b + c = 11b(b - 1) + c^2 - a(9 - a) \tag{1}$$

Then since $a, b, c < 10$, $a - b + c = 11$ or 0.

Case I, $a + c = b$.

Equation (1) becomes $0 = 2(a^2 + ac + c^2) - 10a - c$.

Hence c is even. Testing the possible values of c , only $c = 0$ yields a solution which is $N = 550$.

Case II, $a + c - 11 = b$.

Equation (1) becomes $1 = (a + c - 11)(a + c - 12) + c^2 + a^2 - 9a$ or

$$23c - 131 = 2(a^2 + ac + c^2) - 32a.$$

Hence c is odd. Testing the possible values of c , only $c = 3$ yields a solution which is $N = 803$.

Hence the only solutions to the problem are $N = 803$ or $N = 550$.

Also solved by Michael Ecker, Scranton, Pennsylvania, John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico, Kenneth M.

Gustin, LaSalle College, Philadelphia, Pennsylvania, and the proposer.

305. *Proposed by John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico.*

If $(x-h)^2 + (y-g)^2 = r^2$ represents a circle tangent to three given circles, then (h, g, r) is called an Apollonian triple. Given the three circles

$$(x + 3)^2 + (y - 3)^2 = 6^2$$

$$(x - 1)^2 + (y + 5)^2 = 2^2$$

$$(x - 2)^2 + (y + 2)^2 = 1^2$$

find all Apollonian triples (h, g, r) for the given circles such that $h, g,$ and r are rational and such that $r > 0$.

Solution by Fred A. Miller, Elkins, West Virginia, and the proposer.

Let (h, g, r) be an Apollonian triple for the given three circles where $r > 0$ and $h, g,$ and r are all rational. These may be found by simultaneously solving the permutations by threes of the equations

$$(h + 3)^2 + (g - 3)^2 = (r \pm 6)^2 \quad (1)$$

$$(h - 1)^2 + (g + 5)^2 = (r \pm 2)^2 \quad (2)$$

$$(h - 2)^2 + (g + 2)^2 = (r \pm 1)^2 \quad (3)$$

Here (h, g) is the center of the Apollonian or tangent circle and r is its radius, hence $r > 0$. Since there are eight distinct ways of selecting the choice of signs in three's, eight systems of equations need to be solved for rational $h, g,$ and r such that $r > 0$. These systems generate the five following Apollonian triples: $(h, g, r) = (13/24, -5/2, 13/24), (0, -23/4, 13/4), (9/4, -2, 5/4), (39/20, -31/15, 13/12),$ and $(13/3, -23/4, 65/12)$.

Also solved by the proposer.

Editor's comment: The proposer's solution essentially parallels the solution given, but the proposer's solution yields in addition the equations of the common tangents and the equations of the conic sections upon which lie the centers of the Apollonian circles. For a discussion of the geometry underlying the solution given the reader should refer to Wanda L. Garner's article entitled "The Problem of Apollonius" which appeared in *The Pentagon* in the Spring 1974 issue (Vol. XXXIII, No. 2, pp 81-86).

306. *Proposed by the editor.*

Let $F(n) = \frac{(2n+1)(3n^2+3n-1)}{15}$ be a function whose

domain is the positive integers. If n is a positive integer selected at random, what is the probability that $F(n)$ is an integer?

Solution by Bob Prielipp, The University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

We begin by constructing the table exhibited below where each entry in the body of the table is its least non-negative residue modulo 15.

n	1	2	3	4	5	6	7	8	9	10
$2n+1$	3	5	7	9	11	13	0	2	4	6
$3n^2+3n-1$	5	2	5	14	14	5	2	5	14	14
$(2n+1)(3n^2+3n-1)$	0	10	5	6	4	5	0	10	11	9

n	11	12	13	14	15
$2n+1$	8	10	12	14	1
$3n^2+3n-1$	5	2	5	14	14
$(2n+1)(3n^2+3n-1)$	10	5	0	1	14

Observe that if the table were extended to include the next fifteen values of n the entries in the table under $n = 16$ would be the same as those under $n = 1$, the entries under $n = 17$ would be the same as those under $n = 2$, etc...

Hence given any 15 consecutive positive integers, $F(n)$ is an integer for exactly 3 of them (when $n = 15k + 1$, $15k + 7$, or $15k + 13$ for some non-negative integer k - check the 0's in the bottom line of our table).

Therefore if n is a positive integer selected at random, the probability that $F(n)$ is an integer is $3/15 = 1/5$.

Also solved by Michael Ecker, Scranton, Pennsylvania, John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico, Richard A. Gibbs, Fort Lewis College, Durango, Colorado, and the proposer.

Kappa Mu Epsilon News

Edited by Sister Jo Ann Fellin, *Historian*

News of chapter activities and other noteworthy KME events should be sent to Sister Jo Ann Fellin, Historian, Kappa Mu Epsilon, Benedictine College, North Campus Box 43, Atchison, Kansas 66002

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence

Chapter President - Judy Muse Thompson

57 actives

Chapter members offer tutoring services to mathematics students of the university and area high schools. Twenty new members were added to the chapter roll at the fall initiation banquet which featured Dr. John L. Locker, head of the mathematics department, as speaker. Vice-president Wade Auten gave a program of mathematical games, puzzles, and novelties at a chapter meeting. AL Beta was saddened by the death of their beloved and faithful corresponding secretary, Mrs. Jean T. Parker. The position of corresponding secretary has been assumed by Elizabeth T. Wooldridge.

Alabama Gamma, University of Montevallo, Montevallo

Chapter President - Debbie Kelly

10 actives

Two new members were initiated on 16 November. Other officers not published previously: Susan Cupp, vice-president; Naomi Hamilton Flanagan, secretary; Susan Mays, treasurer; Angela Hernandez, corresponding secretary; William Foreman, faculty sponsor.

California Gamma, California State Polytechnic University, San Luis Obispo

Chapter President - Jeff Jones

58 actives, 19 pledges

Social activities included a fall faculty-student picnic and a Christmas social and pledge ceremony. The dean of students spoke at the latter. Other talks during the semester were given by representatives of the Navy and the Air Force, both members of the faculty. The chapter sponsored a seminar on how to handle job interviews. Workshops were held for the preparation of the testing materials for use in the spring county-wide Junior High Math Field Day and a newsletter was mailed out to chapter alumni.

**California Delta, California State Polytechnic University,
Pomona**

Chapter President - Karen D. Britt
15 actives, 6 pledges

Other officers remain as published in the fall issue.

Colorado Beta, Colorado School of Mines, Golden

Chapter President - Shelby Switzer
10 actives, 30 pledges

Two faculty presentations were given at chapter meetings. Dr. Robert Underwood talked on "The Golden Mean and Searching for Oil" at the September meeting and Dr. Thomas Kelley spoke on Pascal's triangle at the October meeting. CO Beta members met in November to display and solve mathematical tricks and puzzles. Other officers not published previously: Jim Wallace, vice-president; John Henderson, secretary; Russ Kemp, treasurer; Ardel J. Boes, corresponding secretary; Robert G. Underwood, faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton

Chapter President - Wade Gibbs
20 actives, 15 pledges

The chapter remains active on a local and national level and continues to attract new members.

Illinois Zeta, Rosary College, River Forest

Chapter President - George Beranek
8 actives

Other officers elected on 3 October: Kathleen Tracy, vice-president; Betty Morrell Kurz, secretary; Laura Lenzi, treasurer. Sister Nona Mary Allard serves as corresponding secretary and faculty sponsor. At the October meeting fund raising plans were initiated to aid members to attend the spring national convention. Secretary Betty Kurz gave a presentation at the November meeting and Petra Horst was initiated into IL Zeta.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President - Steven Hild
36 actives

At the September meeting member Kyle Stravers presented a paper on "The Compass By Itself." Following an October pizza supper Reuben Collins and Walter Wise discussed their summer co-op jobs. The annual IA Alpha alumni homecoming

breakfast, held at the home of Professor and Mrs. Carl Wehner on 21 October, was well attended by faculty, current chapter members, and fourteen alumni. At the November initiation banquet new initiate Anita Doehrmann talked on "The Golden Mean." The semester activities concluded with the Christmas party hosted by Professor and Mrs. Greg Dotseth.

Iowa Delta, Wartburg College, Waverly

Chapter President - Mark Behle
22 actives, 3 pledges

Programs included an introduction to the new computer science program at Wartburg and talks on actuarial science and computer science data systems. The annual Christmas party, preparations for the spring math field day, and playing of mathematical games were some of the activities sponsored by the chapter.

Kansas Alpha, Pittsburg State University, Pittsburg

Chapter President - Howard Thompson
50 actives

The chapter hosted a fall picnic for all mathematics and physics majors. Professor J. Bryan Sperry, faculty sponsor of KS Alpha, discussed ruled surfaces at the October meeting. Eleven new members were initiated in November at which time Mary Huning presented a talk on "Proving the Equality of Flowcharts." Members gathered at the home of Dr. Helen Kriegsman, mathematics department chairperson, for the special Christmas meeting in December. At that time Howard Thompson reviewed a book which focused on the life of Gauss and Tom Pope discussed his experiences as a student teacher.

Kansas Beta, Emporia State University, Emporia

Chapter President - Greg Hayward
30 actives

Business at the four fall meetings centered around preparations for the nation convention -- financial arrangements for the trip and the preparation of papers for submission. The new vice-president of the university spoke to the chapter group at the initiation banquet. KS Beta held a Halloween party and helped with Emporia State University's annual Math Day.

Kansas Gamma, Benedictine College, Atchison

Chapter President - Daniel Kaiser
17 actives, 10 pledges

Monthly meetings followed the annual fall picnic in late August for members and prospective members. At one of these meetings Dr. Scheweppe of the University of Kansas talked on the history of computer hardware. During the fall semester the chapter prepared for the March Mathematics Tournament for area high school students. Chapter members for the second year organized a computer dance for the student body. The dance was held 10 November. The annual Wassail party took place on 4 December at the home of Richard Farrell, chairperson of the mathematics department, and his wife Carol.

Kansas Delta, Washburn University, Topeka

Chapter President - Susan Glotzbach

17 actives, 9 pledges

Guest speakers at the monthly meetings have included persons who use mathematics in their careers. The March Math Day is sponsored by KS Delta for students from those high schools within a sixty mile radius of Topeka. The day includes testing, a speaker, and an awards ceremony. Chapter members had a Christmas party in the mathematics department and decorated it for the holidays.

Kansas Epsilon, Fort Hays State University, Hays

Chapter President - Ken Eichman

20 actives

Members Terri Hooper and Debora Cate reported at a chapter meeting on their experiences and knowledge gained while attending the Kansas City fall meeting of NCTM. Members of KS Epsilon enjoyed a Halloween party at the home of Dr. and Mrs. Jeffery Barnett with games and homemade pizza. Some came thoroughly disguised in full costume. Debora Cate has assumed the position of chapter historian.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President - Laura Nesbitt

7 actives, 4 pledges

MD Alpha members toured the Science Center of the Maryland Academy of Sciences during the fall semester.

Maryland Beta, Western Maryland Collège, Westminster

Chapter President - Jeffrey Gates

17 actives

New faculty member Dr. Jack Clark spoke on networks and

their applications at the fall induction meeting of five new members. MD Beta co-sponsored with the Career Counseling Center a meeting on careers in mathematics. Returning to campus to speak on this occasion about computer applications and actuarial work were charter member Diane Briggs Martin ('65) and alumnus Robert Read ('72). MD Beta is proud of charter member Sherry Fischer Manning ('65) who was inaugurated last October as President of Colorado Women's College. Mrs. Manning received her M.S. from College of William and Mary and her D.B.A. from Colorado State University.

Maryland Delta, Frostburg State College, Frostburg

Chapter President - C. Michael Dicken

15 actives, 7 pledges

Following the installation of the chapter by National President James E. Lightner on 17 September, MD Delta began holding biweekly meetings. In addition to the meetings the chapter held a calculator raffle to raise funds for activities, initiated a free weekly tutoring service for students in lower level mathematics courses, and assumed responsibility for displays in the departmental showcase. Seven pledges were inducted in February. Spring plans include three guest speakers and programs featuring student presentations. Chapter members will also work with the departmental annual symposium on mathematics education. Other 1978-79 officers: Debra Boyd, vice-president; Janet Jessup, secretary; Reinaldo Machado, treasurer; Roberta L. White, corresponding secretary; Walter J. Rissler, faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg

Chapter President - Andrew Green

24 actives

Twelve members were initiated at the fall cookout on 6 October. Other officers not published previously: Debra Lopez, vice-president; Martha Holloway, secretary.

Missouri Alpha, Southwest Missouri State University, Springfield

Chapter President - Charles Armstrong

27 actives

Dr. Michael Awad, mathematics department faculty member, spoke on "Straight Lines" at the 14 September

meeting. The talk "Rainy Day Math Problems" was given by graduate student Carol Wickstrom on 19 October. The third speaker for the semester was Mrs. Vonna Vahldick, retired faculty member. Her presentation on 16 November was titled "Infinite Sums in the History of Mathematics."

Missouri Beta, Central Missouri State University, Warrensburg

Chapter President - Kimberly Owen
27 actives, 5 pledges

In addition to three fall meetings the chapter held initiation in November and a Christmas party in December. Chapter members conducted a fall fund raising project and made plans for a similar project for the spring semester.

Missouri Epsilon, Central College, Fayette

Chapter President - Janet Doll
5 actives

Other officers remain as published in the fall issue.

Missouri Zeta, University of Missouri-Rolla, Rolla

Chapter President - Karen Avery
12 actives, 10 pledges

The chapter and the mathematics department cosponsored an October outing. Other officers not published previously: John Reed, vice-president; Debbie Carleton, secretary; Jeff Jost, treasurer; Steven Sallwasser, historian; Farroll Wright, corresponding secretary; James Joiner, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne

Chapter President - Lois Bright
21 actives

The chapter holds monthly meetings and provides free tutoring service for calculus and pre-calculus students. The tutors help individual students and conduct review sessions for groups of students. The chapter has been involved in preparations for spring events. Included among these were the selection of two teams of members to compete in the annual Wayne State College Bowl, the selection of the "Ugly Professor," the annual spring banquet, and the annual high school mathematics contest. Other officers not published previously: Karen Reestman, vice-president; Kay Nickelson, secretary and treasurer; Rod Bubke, historian; Fred Webber, corresponding secretary; Jim Paige and Hilbert Johs, faculty sponsors.

Nebraska Beta, Kearney State College, Kearney

Chapter President - Doug Demmel

23 actives

Cecil Bykerk, chairperson of the department of actuarial science at the University of Nebraska, spoke to the entire student body about the actuarial profession. His talk was sponsored by NE Beta. Chapter members gave presentations on mathematical topics at the regular meetings. Eleven new members were admitted to the chapter in the initiation ceremony which preceded the semi-annual initiation banquet.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President - Tina Sibbitt

40 actives

Two customs have been established for chapter initiations. Refreshments consist of punch and a cake decorated with the Kappa Mu Epsilon crest. The cake, beautiful in its artistry, is provided by a local high school class in cake decoration. Each initiate is challenged with a quiz problem, in the form of a puzzle or recreational problem, to be completed mentally. Other officers not published previously: Beverly Lawson, vice-president; Chuck Paine, secretary; Mike Wester, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara University

Chapter President - Brian Covney

19 actives, 9 pledges

The chapter sponsored a reception for mathematics students and faculty in September, a seminar on career planning given by Mr. John Parker in November, and a Christmas party in December. Four senior members presented papers for senior seminar class. The students and their topics were: Brian Covney on "Gaussian Integers," Maureen Swiercznski on "Galois Theory," Nancy Heier on "Lattice Theory," and Brenda Weaver on "Inversion Theory and Hyperbolic Geometry." Sister John Frances Gilman serves as faculty sponsor for the chapter.

New York Kappa, Pace University, New York

Chapter President - Christine Szalay

29 actives, 7 pledges

The following seven members were inducted at the fifth annual initiation dinner on 3 May, 1978: Roman Bobak, Rosemary

Essig, Doris Generale, Jo Ann Kealy, Barbara Taylor, Ralph Trombetta, and John Xouris. Professor John Ogle spoke to the group at that time on "A Computer Technique for Public Encryption and Private Decryption on Electronic Mail." Other 1978-79 officers: Susan Fogli, vice-president; Joann Cristoforo, secretary; Joan Hoffman, treasurer; Lorraine Fu, corresponding secretary; Martin Kotler, faculty sponsor.

Ohio Gamma, Baldwin-Wallace College, Berea

Chapter President - Kevin Cullen
25 actives

Lynn Jones has undertaken the office of secretary for OH Gamma. Other officers remain as published in the fall issue.

Ohio Epsilon, Marietta College, Marietta

Chapter President - Brett Gant
19 actives

New initiates received into OH Epsilon on 8 November were: Catherine S. Arnold, Milton B. Wallace, Mark A. Zoller, Jeffery D. Myers, Brian P. Reid, William M. Greenlees, and James A. Dech. The members enjoyed a post-initiation party at the home of faculty sponsor John Michel. Other officers not published previously: Don Southard, vice-president; Mary Boyle, secretary; Kathy Daly, treasurer; Neil Berstein, corresponding secretary.

Ohio Zeta, Muskingum College, New Concord

Chapter President - Barbara Bauer
37 actives

Dr. Larry Zettel of Muskingum College talked about the Graphics Terminal at the September meeting. The ten new members inducted at the October meeting made short presentations at that time. Two speakers from Miami University made presentations in November. Dr. Richard Laatsch talked on n-dimensional solids and Dr. Vasant Walkar spoke on the relationship between birthdate and deathdate. The chapter Christmas party was held at the home of faculty sponsor James L. Smith.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford

Chapter President - Maryetta Unruh
40 actives, 12 pledges

In addition to the fall picnic, chapter activities included a bake sale held in conjunction with the annual Math Day sponsored by the mathematics department. A short program on careers in business and industrial firms was presented at the October meeting by Karen Anderson, a representative for Southwestern Bell Telephone Company.

Pennsylvania Alpha, Westminster College, New Wilmington

Chapter President - John Stafford

38 actives, 7 pledges

PA Alpha began fall activities with a party for all mathematics majors. New associate members were recognized at that time. The chapter updated its bylaws and assisted with the annual Westminster High School Mathematics Competition. Members joined the local chapter of the Association of Computing Machinery in a tour of the local computer facilities. Speakers and a campus-wide backgammon tournament are being planned.

Pennsylvania Gamma, Waynesburg College, Waynesburg

Chapter President - David Dell

14 actives

Other officers not published previously: Terry Murray, vice-president; Chau Vo, secretary and treasurer; Rosalie B. Jackson, corresponding secretary; David S. Tucker, faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President - Steven E. Fox

14 actives, 1 pledge

Ten new members were inducted into PA Epsilon last fall. At chapter meetings Maxine Cranage and Dr. Edward Evans spoke on mathematical fallacies and game theory, respectively. The tenth biennial Mathematics Conference, held on 10 November, was highlighted by Dr. Isaac Asimov's presentation on "Man, Computers, and Technology." Edward W. Evans is faculty sponsor for the chapter.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President - Luann Murtiff

24 actives

Nine new members were initiated at the October meeting. The initiation program consisted of a slide presentation given

by corresponding secretary Ida Z. Arms on her tour of the Soviet Union. Mathematics faculty member Dr. Arlo Davis gave a talk on "Tensors" at the November meeting. For the December meeting student members Terry Gillis, Mary L. Luciani, and Brad Strine explained their internships in mathematics and computer science with industries and the state government. Other officers not published previously: Cynthia Roman, vice-president; Terri Caligiuri, secretary; Amy Sitzler, treasurer.

Pennsylvania Theta, Susquehanna University, Selinsgrove

Chapter President - Jeff Towne

21 actives, 15 pledges

Other officers not published previously: Mark Kramm, vice-president; Karl Reuther II, secretary and treasurer; Carol Harrison, corresponding secretary and faculty sponsor.

Pennsylvania Iota, Shippensburg State College, Shippensburg

Chapter President - Jim Strayer

51 actives, 4 pledges

The chapter sponsored a get-acquainted gathering early in the semester for the Mathematics and Sciences faculty, their families, students and their guests. The semester activities closed with the initiation of four new members on 17 November. Refreshments were served following the ceremony. Other officers not published previously: Gail Hess, vice-president; Patricia Estock, secretary; Howard Bell, treasurer; Jack Mowbray, corresponding secretary; Winston Crawley, faculty sponsor.

Tennessee Beta, East Tennessee State University, Johnson City

Chapter President - Mike Bell

17 actives

The chapter sponsored a trip for its members to the University of Tennessee at Knoxville to learn about opportunities in the field of mathematics. A group picture was taken for the university annual. Other officers not published previously: Regina Wice, vice-president; Karen Dowdy, secretary; Lyndell Kerley, corresponding secretary and faculty sponsor.

Texas Alpha, Texas Tech University, Lubbock

Chapter President - Margaret Street

40 actives, 7 pledges

Seven new members were inducted into TX Alpha at the initiation banquet on 14 December. Jack Brown will replace mid-year graduate Donna Terral as secretary for the chapter.

Virginia Beta, Radford College, Radford

Chapter President - Patty Goodson

13 actives

The fall semester's program included a speaker on the topic of career opportunities for the non-teaching mathematics major, a Christmas party, and a film festival. "Limit," "Theorem of the Mean," "The Dot and the Line," "Golden Section," and "Computer Perspective" were some of the films included in the showing. Other officers not published previously: Carlotta Browning, vice-president; Kathy Sellars, secretary; Martha Ellen Peake, treasurer; Janet S. Milton, corresponding secretary; J. D. Hansard, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President - Jane Simeth

8 actives, 2 pledges

October actives included a doughnut sale to raise funds to support WI Alpha members who attend the spring national meeting and a session to make the table favors used during the NCTM Milwaukee meeting. At a November meeting short reports were presented on interesting features of the lives of some mathematicians. Laurie Woodruff has assumed the position of secretary.

Wisconsin Beta, University of Wisconsin - River Falls, River Falls

Chapter President - Sylvia Behm

12 actives

The chapter held monthly meetings with speaker and film presentations and sponsored a mathematics night on 1 November. Chapter members offered afternoon and evening help sessions for mathematics students. Other officers not published previously: Mark Drangsveit, vice-president; Kerry Kading secretary; Mark Hoffman, treasurer; Lyle Oleson, corresponding secretary; Lillian Gough, faculty sponsor.

