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CONTENTS

National Officers	2
Notice	3
A Definition of Number	4
Continued Fractions On February 29	12
The "Matching Game": A Mathematical Analysis	19
Proofs of the Infinitude of Primes	25
The Bookshelf	28
The Problem Corner	33
The Mathematical Scrapbook	39
Kappa Mu Epsilon News	43
Convention Photograph	64

National Officers

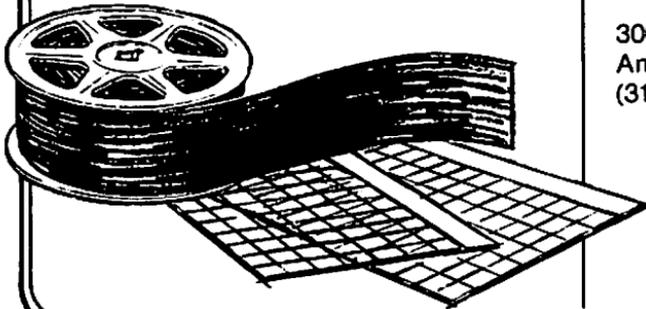
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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Notice

Due to increased costs of printing and mailing, the National Council of Kappa Mu Epsilon, publisher of THE PENTAGON, have authorized a rise in the price of the journal. Effective with this issue the price is \$1.00 per copy, \$2.00 per year, and \$4.00 for two years. The last change in price was in the Fall 1969 issue. In light of the inflation rate of the last few years, the increase at this time does not seem to be excessive.

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A Definition of Number*

SANDRA K. PEER

Student, Washburn University

Bertrand Russell's book, *Principia Mathematica*, written with Alfred North Whitehead attempts the momentous task of providing a reduction of mathematics to a system of logic. Central to this task is Russell's definition of natural number. This definition follows a suggestion offered by Gottlob Frege in his book, *Grundlagen der Arithmetik*, written in 1884. Frege's book received almost no attention; consequently, his definition of number remained practically unknown until it was rediscovered by Russell in 1901.

Why was Russell interested in the definition of a number? In 1889, Peano discovered that the entire theory of natural numbers could be derived from three primitive terms and five axioms. One of Peano's primitive terms is number, specifically, natural number. The other two primitive terms are zero and successor. Using today's terminology and symbolism, we shall give meaning to number as well as to Peano's other primitive terms, describe how these terms satisfy Peano's five axioms, and briefly examine the consequences. Keep in mind that no attempt is made to make this discussion a formal axiomatic treatment of the problem.

According to Russell, each number is a certain collection of sets. We illustrate these collections by considering the number 2. Two will be the collection of all sets whose elements can be put in a one-to-one correspondence with the set $\{a, b\}$, for example, where a and b are distinct elements. This one-to-one correspondence between sets is the same as a one-to-one, onto function between two sets. The set $\{a, b\}$ is an element of the collection which we call 2, since obviously, $\{a, b\}$ can be put in a one-to-one correspondence with itself. There are, of course, many other sets which can be put in a one-to-one correspondence with the set $\{a, b\}$. The number 2, then, is a set of sets. For the sake of clarity, we will call 2 a *barrel* of sets to avoid confusion as to which set we are speaking of when we use the word "set." Two is

$$\{ \{a, b\} , \{0, 1\} , \{ _ , _ \} , \dots \} .$$

*A paper presented at the 1975 National Convention of KME and awarded first place by the Awards Committee.

Similarly, we can describe 1 as the barrel of sets whose elements can be put in one-to-one correspondence with the set $\{a\}$, for example. Zero can be described as the barrel of sets which can be put in a one-to-one correspondence with the set of all square circles, for example. This is, of course, the empty set and the zero barrel contains only that one set.

We will use the symbolism $[A]$ to denote the barrel which contains the set A .

Using today's mathematics, we make the number concept precise by defining on sets A and B the following relation: $A \sim B$ if and only if there exists a one-to-one correspondence between A and B . It can be proven that \sim is an equivalence relation. Using the symbolism introduced earlier, we now define barrel $[A]$ as follows. $[A] = \{X \mid X \sim A\}$. The barrel $[A]$ is the equivalence class containing A . A set X gets to be a member of barrel $[A]$ if and only if there exists a one-to-one correspondence between X and A . $[A] = [B]$ if and only if $A \sim B$. This result will be used frequently.

$$2 = [\{a, b\}] = [\{0, 1\}] = [\{_, _ \}] = \dots$$

At first glance, it might appear that we could simply take the set of all barrels to be the set of natural numbers; however, this is not the case. Some barrels contain sets that are too large and do not deserve recognition as natural numbers. If we are allowed to use our previous knowledge of numbers for a moment, we can illustrate this. Intuitively, we will have

$$\begin{aligned} 7 &= [\{0, 1, 2, \dots, 6\}] \text{ and} \\ 832 &= [\{0, 1, 2, \dots, 831\}]. \quad \text{In general,} \\ n &= [\{0, 1, 2, \dots, n-1\}]. \end{aligned}$$

But what about the barrel which contains as a set *all* of the natural numbers, the barrel $[\{0, 1, 2, \dots\}]$? What natural number name should we give to this barrel? Indeed, we do not want this barrel to be a natural number. We do want to give it a name, however. The traditional symbolism for this barrel is \aleph_0 (Aleph null). Unfortunately, there are barrels which contain even larger sets than \aleph_0 . Consider the set of all subsets of \aleph_0 , symbolized by $P(\aleph_0)$. We call $P(\aleph_0)$ the power set of \aleph_0 . It can be shown that there is no one-to-one correspondence between a set in \aleph_0 and a set in $P(\aleph_0)$ ([6], p. 24). In other words, \aleph_0 and $P(\aleph_0)$ are not equal. $P(\aleph_0)$ contains sets "larger" than those in \aleph_0 . $P(P(\aleph_0))$ is a barrel which contains even "larger" sets.

We do not want \aleph_0 or anything larger to be a natural number. Therefore, we must define natural number in such a way so that \aleph_0 and everything larger than \aleph_0 is excluded from being a member.

In order that we restrict our natural numbers to include only those numbers less than \aleph_0 , we introduce a few definitions.

Given a set A , we define the successor of a barrel $[A]$, symbolized by $s([A])$, to be the barrel $s([A]) = [A \cup \{x\}]$ where $x \notin A$. To justify the use of function notation, we argue that s is a function between barrels. We need to show that $[A] = [B]$ implies that $s([A]) = s([B])$. $[A] = [B]$ implies that there exists a one-to-one correspondence between sets A and B . $s([A]) = [A \cup \{x\}]$, where $x \notin A$, and $s([B]) = [B \cup \{y\}]$, where $y \notin B$. We already have a one-to-one correspondence between sets A and B . Therefore, we just need to extend this one-to-one correspondence to include x corresponding to y . We now have the desired one-to-one correspondence between $A \cup \{x\}$ and $B \cup \{y\}$, which implies that $s([A]) = s([B])$. Thus, s is a function between barrels.

To illustrate,

$$s(1) = s[\{0\}] = [\{0\} \cup \{x\}] = [\{0, x\}] = 2.$$

Similarly, $s(842) = 843$. But what about the successor of \aleph_0 ?

$s(\aleph_0) = [\{0, 1, 2, \dots\} \cup \{x\}]$ where $x \in \{0, 1, 2, \dots\}$.
 $s(\aleph_0) = [\{x, 0, 1, 2, \dots\}]$. Note that the members of the set $\{x, 0, 1, 2, \dots\}$ can easily be put in a one-to-one correspondence with the members of the set $\{0, 1, 2, \dots\}$ in the following manner.

$$\begin{array}{cccc} \{0, 1, 2, 3, \dots\} & \notin & \aleph_0 & \\ \uparrow & \uparrow & \uparrow & \uparrow \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \{x, 0, 1, 2, \dots\} & \notin & s(\aleph_0) & \end{array}$$

Since we have an element of $s(\aleph_0)$ being in one-to-one correspondence with an element of \aleph_0 , the theorem which we mentioned earlier tells us that

$$\aleph_0 = s(\aleph_0).$$

In other words, \aleph_0 and $s(\aleph_0)$ are just different names for the

same barrel. We emphasize that nowhere have we defined \aleph_0 . All of our discussion concerning \aleph_0 so far has been totally intuitive. We have introduced this concept only to help motivate definitions which will follow.

We will define what is meant by a hereditary class. A class H of barrels is defined to be a hereditary class if and only if $s([A]) \in H$ whenever $[A] \in H$.

A comment about the word "class" is in order. We now have sets in barrels, and barrels in classes. As was the case with the word "barrel," the word "class" is used here instead of "set" simply to avoid confusion and to help indicate the level of set structures with which we are working.

We give some intuitive examples of hereditary classes.

$\{0, 1, 2, \dots\}$ is a hereditary class.

$\{1, 2, 3, \dots\}$ is a hereditary class.

\aleph_0 is a hereditary class since the successor of \aleph_0 is \aleph_0 itself, as we discussed earlier.

$\{0, 1, 2, \dots, \aleph_0\}$ is a hereditary class.

$\{0, 1, 2, \dots, \aleph_0, P(\aleph_0)\}$ is a hereditary class since $s(P(\aleph_0)) = P(\aleph_0)$ just as $s(\aleph_0) = \aleph_0$.

$\{0, 5, 6, 7, \dots\}$ is not a hereditary class since the successor of 0 is not a member of the class.

We are now prepared to give meaning to Peano's three primitive terms.

ZERO: $0 = [\phi]$

NATURAL NUMBERS: The set of natural numbers is the intersection of all hereditary classes containing zero.

(Note that if we intersect hereditary classes such as

$\{0, 1, 2, \dots\}$, $\{0, 1, 2, \dots, \aleph_0\}$,

$\{0, 1, 2, \dots, \aleph_0, P(\aleph_0)\}$, this intersection would simplify to our desired set of natural numbers. By taking the intersection of classes such as these, we see that we do indeed eliminate those numbers which are too large to be natural numbers. The intersection of all hereditary classes containing zero would certainly contain exactly the desired set of natural numbers.)

SUCCESSOR: The successor of a number $[A]$ is defined as before, with the restriction that the domain of s be N . $s([A]) = [A \cup \{x\}]$, where $x \notin A$.

(Note that our previous definition of successor was more general than desired here; it applied to any barrel we might choose. However, we are concerned only with those barrels which are natural numbers, we need only restrict the original definition of successor so that its domain is only the natural numbers.)

We have now given meaning to Peano's three primitive terms and will next argue briefly that Peano's five axioms are indeed satisfied by our definitions.

Peano's axioms may be translated into modern-day terminology in the following way.

Let N be the set of natural numbers.

1. $0 \in N$.
2. $n \in N$ implies that $s(n)$ is uniquely determined in N . (This says that s is a function from N to N .)
3. For all n and $m \in N$, $n \neq m$ implies that $s(n) \neq s(m)$. (This says that s is a one-to-one function.)
4. For all $n \in N$, $s(n) \neq 0$. (This says that 0 is not the successor of any natural number.)
5. Let $P \subseteq N$. Then $N = P$ if
 - a) $0 \in P$ and
 - b) $n \in P$ implies that $s(n) \in P$.

(This is the traditional math induction principle.)

We demonstrate that the axioms are satisfied as follows:

1. N , by definition, is the intersection of all hereditary classes of barrels which contain 0, so 0 is clearly in N .
2. We argued earlier that s was a function when defined on *all* barrels. When we restrict the successor function so that its domain is the set of natural numbers, the uniqueness property of the function is preserved. What remains to be shown is that the successor function takes N into N . To show that $s(n) \in N$ whenever $n \in N$, first assume that $n \in N$. It follows that n is an element of every hereditary class of barrels containing 0. Therefore, by definition of hereditary class, $s(n)$ is an element

of every hereditary class of barrels containing 0. $s(n)$ is an element of the intersection of all hereditary classes which contain 0. Thus, $s(n) \in N$, whenever $n \in N$.

3. Using the contrapositive of the axiom, we want to show that $s(n) = s(m)$ implies that $n = m$. Assume $s(n) = s(m)$. Let $n = [A]$ and $m = [B]$. Then $s(n) = [A \cup \{t\}]$ where $t \notin A$ and $s(m) = [B \cup \{r\}]$ where $r \notin B$. We want to show that $[A] = [B]$. We can show this by showing that $A \sim B$. Since $s(n) = s(m)$, we know that $[A \cup \{t\}] = [B \cup \{r\}]$; i.e., there exists a one-to-one correspondence between $A \cup \{t\}$ and $B \cup \{r\}$. If t is associated with r , then just restrict the correspondence between A and B and delete the correspondence between r and t . If, on the other hand, t is associated with some $b \in B$ and r is associated with some $a \in A$, then leave all other members of A and B in the same correspondence as they were originally and associate a with b . Thus, either way, there exists a one-to-one correspondence between A and B , which says that $n = m$.
4. Let $n = [A]$. Then $s(n) = [A \cup \{x\}]$, where $x \notin A$. If $s(n) = 0$, then $s(n) = [A \cup \{x\}] = 0 = [\phi]$. This implies that there is a one-to-one correspondence between $A \cup \{x\}$ and ϕ . Since $A \cup \{x\}$ contains at least one member, and the empty set, of course, contains no members, we know there cannot exist a one-to-one correspondence between the two sets. Thus, we have reached a contradiction. By indirect proof, we have shown that 0 is not the successor of any natural number.
5. Let $P \subseteq N$, and let $n \in N$ arbitrarily. This, by definition, implies that n is an element of every hereditary class containing zero. In particular, n is an element of P , since, by conditions a) and b) in the axiom, P is a hereditary class. We now have $N \subseteq P$. Since we assumed that $P \subseteq N$, we can combine our two results to arrive at the conclusion that $N = P$.

We have thus given meaning to Peano's primitive terms and have described how these terms satisfy Peano's five axioms.

It may seem that we started with some basic terms and developed our meanings for Peano's primitive terms without using any axioms. Of course, we cannot get something from nothing. We used an intuitive approach. In *Principia Mathematica*, Russell and Whitehead do indeed have to begin with some primitive terms and basic

axioms. Our discussion does not give evidence of the complications involved in the formal justification of Peano's axioms from these basic axioms.

Before concluding, we should briefly investigate the significance of the Russell-Whitehead work by putting it in the perspective of seventy years of mathematics. Why did they undertake this task? Their intent, as was Frege's, was, simply stated, to show that mathematics is an extension of logic and, in so doing, to show that the truth of all mathematics is dependent on the truths of some universally accepted logical principles.

Before the writing of *Principia Mathematica*, it was known that the arithmetization of mathematics, beginning with the natural numbers, had been completed. Using today's mathematics, we can outline this arithmetization process. It is possible to define the negative numbers by introducing a set disjoint from the natural numbers which has a one-to-one correspondence with the nonzero natural numbers ([4], p. 244). It is then possible to extend the integers and define the rational numbers by using a certain equivalence relation and the associated equivalence classes ([2], p. 213). One can then proceed to extend the rational numbers to define the real numbers, using a method developed by Dedekind ([7], p. 9). Finally, the real numbers can be extended by considering a set of ordered pairs, which are called complex numbers.

Was the goal of providing the link from logic to natural numbers accomplished? Not totally. Recall that in the definition of the successor function, we used, without mention, what is called the "axiom of infinity." In this definition we said that $s([A]) = [A \cup \{x\}]$ where $x \notin A$. How do we know that there is always an $x \notin A$? In order to say this, we would have to know that there are an infinite number of distinct objects in the world. This is not a universal truth; i.e., it cannot be said for certain that there are an infinite number of distinct objects in the world. But, as Russell himself said, neither is there conclusive logical evidence that there are only a finite number of objects in the world. Because the axiom of infinity is not a universally accepted truth, mathematics was not successfully reduced to basic logical truths.

An ultimate difficulty was brought to light by Kurt Gödel. In 1931, he showed that there are serious difficulties encountered if one bases the number system on any given set of axioms. If the axioms are consistent with each other; i.e., if they are not contradictory, then

the system is not complete, which means that there are mathematical statements which can neither be proved nor disproved. In other words, there exists a statement such that neither the statement nor its negation can be deduced from the axioms of the system. Therefore, not all mathematics can be derived from Russell's basic axioms nor, for that matter, from any other set of axioms.

Since *Principia Mathematica*, other set theoretic foundations have been developed which are more intuitive and therefore more readily accepted by practicing mathematicians ([3], p.317).

Stephen Barker writes, "They (Russell and Frege) visualized themselves as explorers of a hitherto unknown level of abstract reality, explorers who were able to discover that the vast region of mathematical reality is really only a peninsula of the larger continent of logical reality. It was a bracing and exhilarating way of picturing one's own activity. But like many bright and fresh morning visions, it had begun to fade even before it came clearly into focus" ([1], p. 81).

Nevertheless, Russell's contribution to mathematics has influenced the evolution of mathematical thought in this century. Had it not been for his work, mathematics may have taken a different path.

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Continued Fractions on February 29*

MARC J. BURKHART

Student, Morningside College

The calendar of today has been debated upon, altered, and reformed in order to try to make it better. It's complexities are hidden by its simpleness. The calendar is designed to mark the progress of man and chart his position through time, yet it is one of the most unprogressive things that man has designed. It must hang somewhere in everybody's house in order for a person to keep track of dates and days for months in advance because it seems to have no order.

Many of the difficulties of a satisfactory calendar are inherent. Foremost of these perhaps is the fact that the three most natural units of time: the periods of the rotation of the earth on its axis, the revolution of the moon about the earth, and the earth about the sun are incommensurable. That is, neither the year nor the lunar month contains an integral number of days, nor is the fractional residue of a day a rational number such as $\frac{1}{4}$ or $\frac{1}{2}$.

When the modern form of the calendar was first introduced on January 1, in the year 45 B.C., one of the innovations was the shifting of the first day of the year from March to January which previously was the eleventh month. The calendar was called the Julian calendar after Julius Caesar. More than sixteen centuries later, in 1582, inaccuracies in the Julian calendar had thrown the year off ten days. That year, Pope Gregory XIII recommended that two major revisions take place. One was that ten days dropped from the calendar so the seasons would line up again. The second was that the leap year of the Julian calendar which had provided for a leap year every fourth year, was modified so that there would be 97 instead of 100 leap years every four centuries ([2], p. 42).

Pope Gregory had the advice of the astronomer Christopher Clavius in this correction of the ten minutes per year error in the Julian calendar, but even his calendar is not perfect. The average length of the year according to the Gregorian calendar is about 24 seconds longer than it should be. This remaining error is so small that it will not be until about the year 4600 that our calendar

*A paper presented at the 1975 National Convention of KME and awarded second place by the Awards Committee.

will be as much as a day in error. The problem is that while the earth makes a trip around the sun, it turns almost $365\frac{1}{4}$ times. More accurately the number can be expressed as 365.24219, but even that is not exact. The two periods of time are incommensurable—one cannot be expressed precisely in terms of the other, no matter how many figures are used after the decimal point. Therefore, the best calendar is the one that most closely approximates this relation.

In the Gregorian calendar, there is an error of .373 minutes per year or one day in 3,861 years, and one day and fifty-two minutes in 4,000 years. This trifling error in the Gregorian calendar may be corrected by suppressing the intercalations in the year 4,000 and its multiples so that it will not amount to a day in 100,000 years. On the other hand, there are 48,699 Julian years for every 48,700 Gregorian years giving a much larger error ([3], p. 37).

This error brings us to a part of modern mathematics, the application of continued fractions. It is known that a year contains slightly more than 365 days but the 365 days are regarded as a year. The year of 365 days must be corrected in a practical and convenient way so that over a long period of time such as several thousand years, the calendar will be as accurate as possible. The addition of an extra day every four years is an example of an attempt to keep the calendar accurate.

To solve the problem of finding a practical and convenient way to correct the calendar, you have to know the precise number of days. *The Handbook of Mathematical Functions* states that one second is fixed as $1/31,556,925.9747$ of the tropical year 1900 at 12h ephemeris time ([4], p. 24). By definition, there are 86,400 seconds in one day giving us 365.2422 days in one year. The problem is how to determine a fraction that closely approximates the decimal portion of the year (.2422 days) and also has a practical and convenient interpretation. This is the kind of problem where continued fractions are very useful. .2422 must now be expanded as follows:

$$0.2422 = 0 + 2,422/10,000 = 0 + 1/(10,000/2,422), \text{ or}$$

$$0.2422 = 0 + \frac{1}{4 + \frac{312}{2422}}$$

The fraction $312/2422$ also can be written as $1/(2422/312)$, so that $312/2422 = 1/(7 + (238/312))$: then

$$0.2422 = 0 + \frac{1}{4 + \frac{1}{7 + \frac{238}{312}}}$$

Continuing in this way, there results the complete continued fraction expansion of 0.2422 ([4], p. 24).

$$0.2422 = 0 + \frac{1}{4 + \frac{1}{7 + \frac{1}{1 + \frac{1}{3 + \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}}}}}}}$$

This continued fraction terminates because 0.2422 is a number with a finite number of decimal places.

The "partial quotients" or "convergents" of 0.2422 are:

$$0, \frac{1}{4}, \frac{1}{4 + \frac{1}{7}} = \frac{7}{29}, \frac{1}{4 + \frac{1}{7 + \frac{1}{1}}} =$$

$$\frac{8}{33}, \frac{31}{128}, \frac{132}{545}, \frac{163}{673}, \frac{458}{1890}, \frac{1211}{5000} \cdot$$

An expression of the form

$$a_1 + \frac{b_1}{a_2 + \frac{b_2}{a_3 + \frac{b_3}{a_4 + \frac{b_4}{a_5 + \dots}}}}$$

is called a continued fraction. The numbers $a_1, a_2, a_3, \dots, b_1, b_2, b_3, \dots$, may be real or complex and the number of terms may be finite or infinite ([1], p. 12). Fractions of the above form in which

$b_1 = b_2 = b_n = 1$, a_1 is a positive integer or zero, and a_2, a_3, a_4, \dots are either positive or negative integers are called simple continued fractions.

In order to conserve space in writing continued fractions, it is permissible to use the form:

$$a_1 + \frac{b_1}{a_2 + \frac{b_2}{a_3 + \dots \frac{b_{n-1}}{a_n}}},$$

where the lowered plus sign indicates that the following fraction is part of the denominator of the given fraction. Every rational number may be expressed as a simple continued fraction.

Let p/q , $q > 0$, be any rational fraction. Then $\frac{p}{q} = a_1 + \frac{r_1}{q}$,

where $0 \leq r_1 < q$. If $r_1 = 0$, the process terminates. If $r_1 \neq 0$,

$$\frac{p}{q} = a_1 + \frac{1}{\frac{q}{r_1}}, \quad 0 < r_1 < q. \quad \text{But } \frac{q}{r_1} = a_2 + \frac{r_2}{r_1}, \quad 0 \leq r_2 < r_1.$$

If $r_2 = 0$, the process terminates. If $r_2 \neq 0$, $\frac{q}{r_1} = a_2 + \frac{1}{\frac{r_1}{r_2}}$,

$$\frac{r_1}{r_2} = a_3 + \frac{r_3}{r_2}, \quad 0 \leq r_3 < r_2. \quad \text{Substituting: } \frac{p}{q} = a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \dots \frac{1}{a_n}}}$$

which may terminate at any point. The partial quotients (convergents), c_i , of the simple continued fraction $a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{a_4 + \dots \frac{1}{a_n}}}}$ are defined as follows:

$$c_1 = \frac{p_1}{q_1} = a_1.$$

$$c_2 = \frac{p_2}{q_2} = a_1 + \frac{1}{a_2} = \frac{a_1 a_2 + 1}{a_2}.$$

$$c_3 = \frac{p_3}{q_3} = a_1 + \frac{1}{a_2 + \frac{1}{a_3}} = \frac{(a_1 a_2 + 1) a_3 + a_1}{a_2 a_3 + 1}.$$

$$c_n = \frac{p_n}{q_n} = a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \dots \frac{1}{a_n}}}.$$

The odd convergents, c_{2n-1} , of an infinite simple continued fraction form an increasing sequence; the even convergents form a decreasing sequence. Every odd convergent is less than any even convergent. Moreover, each convergent c_n , $n > 3$, lies between the two preceding convergents. Algebraically, this theorem states $c_1 < c_3 < c_5 < \dots < c_{2n+1} < \dots < c_{2n} < \dots < c_6 < c_4 < c_2$ ([1], p. 16). This same statement is true for a finite continued fraction. Thus going back to our calendar problem, the odd convergents of 0.2422 are $0, \frac{7}{29} = 0.2413793, \frac{31}{128} = 0.2418750, \frac{163}{673} = 0.2419911$, and $\frac{1211}{5000} = 0.2422$, which all clearly increase toward 0.2422, the last convergents actually taking on the desired value 0.2422. The even convergents are $\frac{1}{4} = 0.25, \frac{8}{33} = 0.2424242, \frac{132}{545} = 0.2422018, \frac{458}{1891} = 0.2421999$, which clearly decrease to 0.2422.

The odd convergents, which are fractions smaller than the number represented by the continued fraction, are called "approximations in defect," and the even convergents are called "approximations in excess." These even convergents are fractions greater than the number represented by the continued fraction.

Now that we have all the convergents, it is possible to obtain a precise and practical way to determine a leap year rule. Since the true year is 365.2422 days, but 365 days is given as a normal year, we look for a simple way of adjusting the calendar periodically so that it is correct.

The first nonzero convergent is $\frac{1}{4}$, which would mean add one day every four years which is the normal leap year sequence. Since this is an even convergent, it will be an overcorrection of the calendar. If this is not corrected, it will result in a large error. The next convergent is $\frac{7}{29}$ which would mean adding a full week every twenty-nine years but this would not be practical. It is an odd convergent and so it would be an undercorrection. That is, it would not correct the calendar enough. If it were not corrected, the calendar would lag behind real time.

The third nonzero fraction is $\frac{8}{33}$ which needs to be multiplied times $\frac{3}{3}$ to make a significant correction factor of $\frac{24}{99}$. This

can be interpreted to mean add 24 days every 99 years. This is what we have now with our Gregorian calendar. Since this is an even convergent, it is an over-correction.

The convergents $31/128$, $132/545$, $163/673$, and $458/1891$ are not good correction factors in that they are impractical and not simple to work with. The next convergent, $1211/5000$ is a perfect correction in that it is both practical and convenient. $1211/5000$ means there must be a correction of 1211 days out of every 5000 years. By making every fourth year a leap year, in 5,000 years, there would be 1250 days correction. By subtracting the ones that occur at the end of the century, we have 1200 additional days every 5000 years. If every fifth century was to be counted as a leap year, we would have 1210 additional days. If every 5000th year was a double leap year, in other words, a February 29th and a 30th, our calendar would be perfect ([4], p. 25).

Intermediate approximations of 0.2422 can be determined so that you don't have to wait 5,000 years for a perfect year. If p_m/q_m and p_{m+2k}/q_{m+2k} , where m is a specific positive integer and $k = 1, 2, 3, \dots$, are two convergents in the continued-fraction expansion of a given number, and if a and b are positive integers, then the fraction $ap_m + bp_{m+2k}$ divided by $aq_m + bq_{m+2k}$ lies between p_m/q_m and p_{m+2k}/q_{m+2k} . The fraction $(ap_m + bp_{m+2k})/(aq_m + bq_{m+2k})$ is not a convergent but it will be a close approximation because it lies between two of them ([4], pp. 25-26).

When p_m/q_m and p_{m+2k}/q_{m+2k} are odd convergents, both are approximations in defect so this means that $(ap_m + bp_{m+2k})/(aq_m + bq_{m+2k})$ will also be in defect. The odd convergents increase, so

$$\frac{p_m}{q_m} < \frac{ap_m + bp_{m+2k}}{aq_m + bq_{m+2k}} < \frac{p_{m+2k}}{q_{m+2k}}$$

In the same way, the even convergents decrease and so

$$\frac{p_{2m}}{q_{2m}} > \frac{ap_{2m} + bp_{2m+2k}}{aq_{2m} + bq_{2m+2k}} > \frac{p_{2m+2k}}{q_{2m+2k}}$$

Taking this theorem and the first two even convergents gives us

$$\frac{1}{4} > \frac{1a + 8b}{4a + 33b} > \frac{8}{33}$$

No values of a and b give us a denominator of 100, if a and b are integers. If $a = 17$ and $b = 4$, $\frac{1}{4} > \frac{49}{200} > \frac{8}{33}$, which means

a correction of 49 days in 200 years. This could be accomplished by every fourth year being a leap year except at the end of every other century. Since these are even convergents, this is an approximation in the excess. If $a = 9$ and $b = 8$, $\frac{1}{4} > \frac{73}{300} > \frac{8}{33}$ which would mean a correction of 73 days in every 300 years which could be accomplished by every fourth year being a leap year except when it occurs at the end of the century, but every 300th year will be a leap year. Once again, this is an approximation of an excess. This process for finding practical approximations can be carried on indefinitely.

Two odd convergents, $7/29$ and $31/128$, will give an under-correction. Using $\frac{7}{29} < \frac{7a + 31b}{29a + 128b} < \frac{31}{128}$ with $a = 4$ and $b = 3$, we obtain $\frac{7}{29} < \frac{121}{500} < \frac{31}{128}$. Multiplying the middle term by $\frac{10}{10}$, we obtain $\frac{1210}{5000}$, which is one day too few from the perfect $\frac{1211}{5000}$, so it is an approximation in defect due to the fact that it does not call for the double leap year.

In all of these ways our calendar might be improved so that it better keeps track of mankind's passage through time.

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The "Matching Game": A Mathematical Analysis*

JAMES HIMMELREICH

Student, Indiana University of Pennsylvania

One of the more popular versions of solitaire is known as the "matching game." In this game, eight cards are drawn from a well-shuffled deck and laid face up on a table. The cards are then examined for a pair with matching denomination, without regard to suit. If a match is found, the next two cards from the deck are also placed face up on top of each match. The game continues until either no match is found or the deck is exhausted. At this point, the cards laying face up are counted and the number is recorded. Using some of the techniques of mathematical analysis, the following questions about the game will be answered: What is the probability of getting all of the cards out? Which number occurs more frequently than any other? If a number less than 21 costs \$5 and a number greater than 21 pays \$10, what is the expected gain (or loss) after 100 games? Obviously 21 can never occur, since cards are drawn two at a time. As a matter of fact, only even numbers between 8 (if there are no matches) and 52 (if the deck is exhausted) can ever occur.

The basic tool of analysis is probability. Since most of the desired information about the matching game concerns expectation, this is a logical place to begin. The easiest probability to find is that of no matches, i.e., the resulting number is 8. Recall from elementary probability that if A and B are independent events, and $P(A)$ is the probability of event A , then $P(A \text{ and } B) = P(A) \cdot P(B)$. For the first card, no match will occur in any event, so of 52 cards from which to choose, 52 will not match. For the second card not to match the first, of the 51 cards remaining, all but 3 (the three with denomination identical to the first card) will not match. Therefore, probability that the first two do not match = $52/52 \times 48/51 = 48/51 = .941176$. Similarly, for the third card to not match either of the first two, of the 50 remaining cards, 6 will match either one

A paper presented at the 1975 National Convention of KME and awarded third place by the Awards Committee.

or the other, leaving 44 which will not match. Thus it can be shown that if $P(X = 8)$ is the probability of obtaining 8 cards for a final result of the game,

$$\begin{array}{cccccccc}
 \text{card} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 P(X = 8) & = \frac{52}{52} \times \frac{48}{51} \times \frac{44}{50} \times \frac{40}{49} \times \frac{36}{48} \times \frac{32}{47} \times \frac{28}{46} \times \frac{24}{45} \\
 & = .11268.
 \end{array}$$

$P(X = 10)$ can be found by finding: (1) the probability that the first seven cards do not match times (2) the probability that the eighth card matches only one of the preceding seven times (3) the number of different ways that the match can have occurred (i.e., 1st and 3rd cards matched, 2nd and 7th cards matched, and so on) times (4) the probability that the two cards which cover the matched pair do not themselves match any of the other cards or each other. (1) This is merely the first seven terms of $P(X = 8)$. (2) There are now 45 cards left, only three of which match only one of the preceding cards; therefore, $3/45$ is the probability that the eighth card matches just one of the preceding seven. (3) The number of ways a matched pair can occur from eight cards is ${}_8C_2$, the combination of 8 things taken two at a time ([1], p. 23), which is $8!/2!6! = 28$. (4) For the first covering card, there are 44 cards left with the number of non-matches the same as for the seventh card drawn (28), since only a matching card has been drawn since. For the second covering card, there are three less non-matching cards (25) out of 43 cards remaining. The probability of both events is $28/44 \times 24/43$. Thus (1) \cdot (2) \cdot (3) \cdot (4) yields

$$\begin{aligned}
 P(X = 10) &= \frac{52}{52} \cdot \frac{48}{51} \cdot \frac{44}{50} \cdot \frac{40}{49} \cdot \frac{36}{48} \cdot \frac{32}{47} \cdot \frac{28}{46} \\
 &\quad \cdot \frac{3}{45} \cdot 28 \cdot \frac{28}{44} \cdot \frac{24}{43} \\
 &= .1393304
 \end{aligned}$$

Obviously, the exact probabilities of higher numbers are much more difficult to find, and for the purposes of this analysis, are not that important. It would seem that close approximations (estimates) of the theoretical probabilities would serve just as well here. This calls on another tool of analysis, sampling techniques.

The principle of sampling is very basic to statistics: The theoreti-

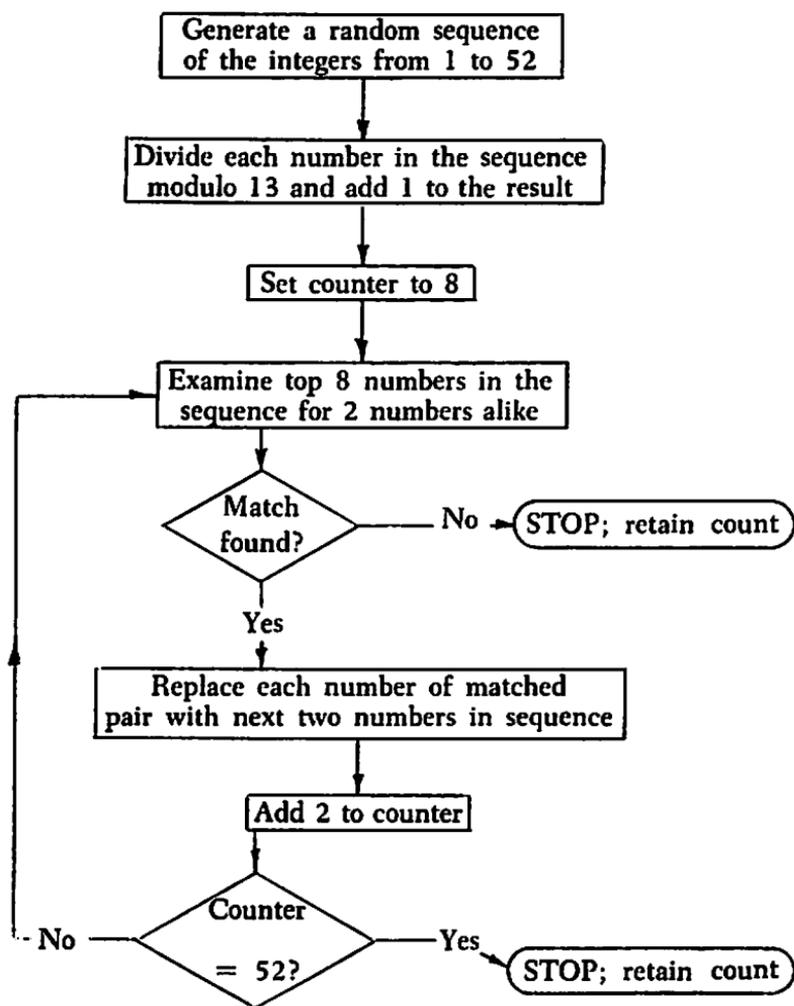


Figure 1—Mathematical Model of the “Matching Game”

TABLE 1

THE MATCHING GAME

RESULTS OF 15 SAMPLES OF SIZE 1000

X	ΣX	ΣX^2	\bar{x}	s^2	s	\hat{p}_i
8	1707	195,445	113.8	79.246	8.902	.1138
10	2076	291,008	138.4	245.983	15.684	.1384
12	1875	236,031	125.0	110.400	10.507	.1250
14	1579	167,165	105.3	63.263	7.954	.1053
16	1321	118,079	88.1	116.199	10.780	.0881
18	1018	69,670	67.9	38.784	6.228	.0679
20	850	48,890	56.7	48.224	6.944	.0567
22	765	39,847	51.0	55.467	7.448	.0510
24	594	23,726	39.6	13.571	3.684	.0396
26	519	18,347	34.6	25.973	5.096	.0346
28	421	12,151	28.1	22.330	4.725	.0281
30	331	7709	22.1	26.996	5.196	.0221
32	313	6761	20.9	15.316	3.914	.0209
34	254	4498	16.9	19.796	4.449	.0169
36	206	2950	13.7	8.062	2.839	.0137
38	184	2502	12.3	16.329	4.041	.0123
40	95	741	6.3	9.289	3.048	.0063
42	52	234	3.47	3.582	1.893	.00347
44	26	66	1.73	1.396	1.181	.00173
46	0	0	0	0	0	0
48	0	0	0	0	0	0
50	0	0	0	0	0	0
52	814	44,806	54.3	42.198	6.496	.0543

$$\bar{x} = \Sigma X / 15$$

$$s^2 = \Sigma X^2 / 15 - \bar{x}^2$$

$$\hat{p}_i = \bar{x} / 1000$$

cal probability of an event can be estimated by performing the experiment many times and recording the number of occurrences of the event. If an event p_i occurs 5 times out of 100 tries, then its estimate \hat{p}_i is $5/100 = .05$. There are two ways to improve upon this estimate: enlarge the sample size and increase the number of samples taken. For this particular problem, 15 samples of size 1000 were taken and probability estimates were found (see Table 1). Compare the sample estimates for $X = 8$ and $X = 10$ with their respective theoretical probabilities. Notice also that the standard deviation of the sample (s), the measure of distribution about the mean of the sample, is much smaller for the larger values of X , indicating a better approximation of p_i ([1], pp. 55, 103). The game was played 15,000 times in all, a gargantuan task had it not been for yet another analytical tool, the mathematical model.

The mathematical model is an abstraction of a physical process which is much more convenient to observe than its physical counterpart. In this case, the model served as an algorithm for a computer program which simulated the matching game (see Figure 1). This made it possible to obtain the fifteen 1000-size samples rather quickly in comparison to the time it would have taken to play the game by hand.

The samples having been taken, a histogram, (see Figure 2) was constructed from the sample data. One question can be answered immediately from the graph. The number 10 (one match) occurs more frequently than any other. From the table, it can also be found directly that $P(X = 52) = .0543$. Notice that the probabilities of $X = 46$, $X = 48$, $X = 50$ are all zero. If 44 cards are out and there is a match, the two cards used to cover the match bring the total cards left in the deck to six, each of which has a matching card lying face up on the table. Therefore, the entire deck will be used, which explains the sudden upward jump the histogram takes at $X = 52$.

The final problem is one of mathematical expectation. If $M.E.$ is the mathematical expectation of one game, then $M.E. = P(\text{loss})(\text{amount of loss}) + P(\text{gain})(\text{amount of gain})$.

$$P(\text{loss}) = P(X < 21) = P(X = 8) + P(X = 10) + \dots + P(X = 20) = .6951$$

$$P(\text{gain}) = P(X > 21) = P(X = 22) + P(X = 24) + \dots + P(X = 52) = .3049$$

amount of loss = $-\$5$

amount of gain = $\$10$

Therefore, $M.E. = (.6951)(-5) + (.3049)(10) = -.4265$

The expectation over 100 games = $100 \times M.E. = -42.65$

This last result represents a net loss of over 42 dollars, which is definitely an unsound financial investment.

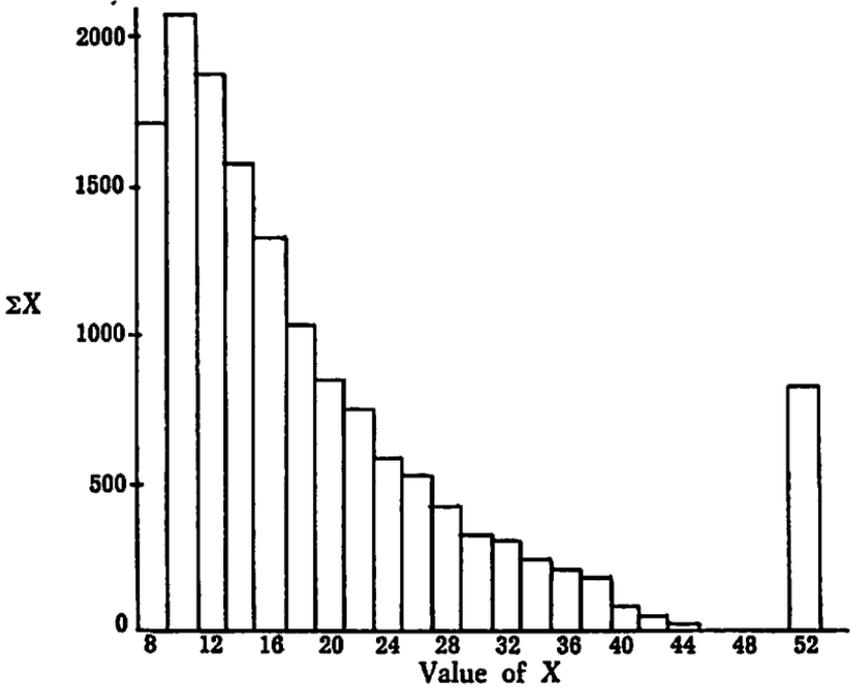


Figure 2—Histogram of Table 1

The information gleaned from the matching game was obtained with a modicum of time and effort, mostly through the tools of mathematical analysis and simple problem-solving techniques. This approach can, and is, being used every day to solve a wide variety of games, simulations, and problems with extensive physical applications.

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Proofs of the Infinitude of Primes

ROBERT E. KENNEDY

Faculty, Central Missouri State University

Bagnato has pointed out that productive mathematicians not only discover; they use the results of others [1]. This article will show how it is possible to introduce the student to the use of results which may be easy to understand, but whose proofs are beyond his immediate capabilities. In what follows, the area of number theory is considered.

Euclid proved that the set of primes is infinite over 2000 years ago by a proof which is often characterized as one of the most elegant examples of an indirect proof. In fact when one first comes into contact with the concept of the infinitude of primes, it is very likely that the proof is verbatim that of Euclid.

Of course there are many proofs that there are infinitely many primes. In what follows, some of these proofs will be pointed out. In particular, proofs are mentioned that use slightly more advanced knowledge of number theory. In this way, the proof of the infinitude of primes may be used to develop one's competency in techniques of proof. Since Euclid's proof is well known, it will not be reproduced here except to mention that it involves a consideration of the expression

$$p_1 p_2 p_3 \cdots p_n + 1$$

where p_i represents the i^{th} prime.

It is not unusual that the first time one encounters Euclid's proof, he suspects that larger primes are actually being generated. This suspicion is encouraged by the fact that

$$2 + 1, (2)(3) + 1, (2)(3)(5) + 1, \\ (2)(3)(5)(7) + 1, (2)(3)(5)(7)(11) + 1$$

are indeed primes. However, it is found that

$$(2)(3)(5)(7)(11)(13) + 1$$

is not a prime, and one is tempted to dismiss his original suspicion. But in a sense, larger primes are being generated, since any prime which divides a number in the above form will be larger than the primes used to obtain that number.

Another elementary proof for the infinitude of primes is found by considering a result of C. A. Grimm [2] which states that for any integer n , each member of the finite sequence

$$n! + 2, n! + 3, n! + 4, \dots, n! + k, \dots, n! + n$$

is divisible by a prime that does not divide any other member. Thus if it is assumed that there exist a finite number of primes say m , then one considers

$$(m + 2)! + 2, (m + 2)! + 3, \dots, (m + 2)! + k, \\ \dots, (m + 2)! + (m + 2)$$

which, as implied by the above theorem, represents $m + 1$ distinct primes. This contradicts the assumption that there were only m primes to begin with, and thus there must be infinitely many primes.

Both of the above proofs are examples of indirect proofs. One might ask if there is a direct proof for the infinitude of primes. To answer this question, suppose that there existed an infinite sequence of distinct integers which were relatively prime in pairs. Then each would have a prime divisor shared with no other member of the sequence. Since the number of terms of the sequence is infinite, we have an infinite set of primes, and it follows that the set of primes is infinite.

Of course one might ask, and rightly so, about the existence of such a sequence. A typical exercise found in number theory textbooks, is to show that any two Fermat numbers are relatively prime. In other words, show that any two terms in the infinite sequence

$$2^{2^1} + 1, 2^{2^2} + 1, 2^{2^3} + 1, \dots, 2^{2^n} + 1, \dots$$

are relatively prime. Each term of this sequence is clearly distinct. Thus, we have demonstrated a sequence with the desired properties, and we have a direct proof for the infinitude of primes.

The above three proofs are all within the grasp of the beginning student of number theory. It goes without saying that there are proofs which use more advanced techniques. In practice, however, these proofs are usually "spin-offs" of other theorems.

In 1837, G. L. Dirichlet proved that if a and b are relatively prime, then there are infinitely many primes of the form $a + bk$. His proof involved certain analytic methods of complex function theory and thus not considered an "elementary" proof. However,

certain special cases are simple to prove by a technique similar to Euclid's method. For example, it can be shown that there exist infinitely many primes of the form $4n + 3$. For suppose that there are only finitely many primes of this form, say p_1, p_2, \dots, p_n . Now consider the number

$$N = 4p_1p_2 \cdots p_n - 1 = 4(p_1p_2 \cdots p_n - 1) + 3.$$

Since N is of the form $4n + 3$ and N is greater than p_i for each i , it follows that N is composite and must have prime factors of the form $4M + 1$ or $4M + 3$. But since the product of any two numbers of the form $4M + 1$ is again of that form, it follows that N has at least one prime divisor of the form $4M + 3$. This implies, however, that this prime also divides 1 which is impossible. Therefore, there must be an infinite number of primes of the form $4n + 3$, and the set of primes is infinite.

Bertrand's Postulate states that if N is any integer greater than 1, then there exists a prime p such that $N < p < 2N$. From this we may obtain perhaps the shortest proof of the infinitude of primes. For suppose that there is a last prime q . But q is an integer greater than 1, and by Bertrand's Postulate, there exists a prime p such that $q < p < 2q$, which contradicts the assumption that q is the largest prime. Thus the number of primes is infinite.

It should be pointed out, that as short as the above proof seems, the proof of Bertrand's Postulate uses inequalities and estimates of the size of various expressions and thus is not so short.

In the above, the outlines of the theorems were given but their complete proofs may be found in most number theory textbooks. They are, in terms of the background material needed, given in order. Thus Euclid's proof is the most accessible to the novice. Even though his proof seems easy to us, we must realize that since Euclid had little of the background material available to modern mathematicians, the above proof makes his genius all the more apparent.

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The Book Shelf

EDITED BY O. OSCAR BECK

This department of THE PENTAGON brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. O. Oscar Beck, Department of Mathematics, University of North Alabama, Florence, Alabama 35630.

An Introduction to Complex Analysis, Peter L. Walker, Halsted Press, New York, 1975, 149 pp., \$11.95.

This physically attractive little book, with six short chapters and three appendices, at first glance appears to be a brief and easy introduction to complex analysis. But a second glance reveals a different picture. The author indicates that he has covered the first five chapters (12 sections, 93 pages) in 25 lectures given to a group of honors students. It is this reviewer's opinion that the book would not be a good choice as a text for the usual undergraduate course; it is too compact, plunges ahead too rapidly, assumes too much previous knowledge, and possesses almost no examples.

While most texts begin with elementary properties of the complex numbers, the first *nine pages* of the Walker book cover, among other things: the Heine-Borel theorem [he proves that if a set K in C is compact, then it is closed and bounded, and he refers the reader to Rudin, *Principles of Mathematical Analysis* for the proof of the converse]; proof of the Bolzano-Weierstrass theorem [a bounded infinite subset of C has a limit point in C]; proof that if f is a continuous map of a compact set into C , then f is uniformly continuous, and the range of f is compact; and some properties of the exponential function e^z . This is enough to discourage anyone who has not had a course in either topology, real analysis, or both.

The rapid pace of this book is best illustrated by noting the location of some of the "name" theorems in Walker's book, as compared with their location in two well-known texts, which we shall call Book C and Book F. The basic material for a one-semester undergraduate course is covered in the following number of pages in these three books:

Walker: 93	Book C: 188	Book F: 239
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Pages on which some of the important theorems are found are given in the following table:

<i>Theorem</i>	<i>Page Location in</i>		
	<i>Walker</i>	<i>Book C</i>	<i>Book F</i>
Cauchy-Riemann Equations	11	35	27
Cauchy-Goursat Theorem	30	106	136
Cauchy Integral Formula	31	118	142
Morera's Theorem	33	123	159
Liouville's Theorem	35	125	166
Fundamental Theorem of Algebra	35	125	166
Laurent Series	45	134	182
Residue Theorem	69	155	197

In addition to giving very few examples, Walker includes only a small number of exercises, there being but 58 as compared with 340 in Book C over comparable material. So in a typical one-semester course the instructor could assign no more than one or two exercises each class period.

This book might make interesting reading for someone who has already had a course in complex analysis, but who wants to obtain a new, concise point of view. It would be excellent for airplane travel, since it is very lightweight!

Violet H. Larney

State University of New York at Albany

Analysis in Euclidean Space, Kenneth Hoffman, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1975, 445 pp., \$15.95.

This book is a substantial introduction to real and complex analysis, designed for junior, senior, or graduate level students having (at least) one year of serious calculus. A wide range of topics is presented. The first half of the book presents the ideas of completeness, convergence, compactness, and continuity in the context of (finite dimensional) Euclidean space; also a rapid review of elementary calculus is given, emphasizing power series methods. The second half begins with a review of the ideas in the first half in the more general context of a normed linear space. This transition

should not be too troublesome for the student, as the notation is almost identical. Furthermore, the author does a good job of emphasizing that the completeness of n -space and the Heine-Borel property depend heavily upon finite-dimensionality. The next chapter is a nicely motivated introduction to the Lebesgue integral for real or complex valued functions defined on n -space and the book concludes with a chapter on the theory of differentiation for functions on one Euclidean space to another. Vector Calculus, along with such topics as the Theorem of Stokes are not included in the book.

Overall, the book is up-to-date and well written; the author has taken care to include examples and motivation. Nevertheless, the student will find the reading of this text extremely difficult. Indeed, as the author comments in the preface, the regular lectures were always supplemented by tutorial sessions, designed to develop the student's mathematical maturity. In the opinion of the reviewer, such tutorial sessions or their equivalent would be essential in any course using this book as a text.

The exercises are adequate in number and they seem to be very well chosen. There are several interesting true-false questions. These questions are thought-provoking and yet many of them will be within the range of the student to answer. A small number of starred exercises are included; these questions are, in general, very difficult.

In the hands of a skilled instructor, this book could be made the basis of an excellent one-year course in analysis. However, the reviewer would not recommend this book to a student for the purpose of self-study.

Richard Dowds
State University College, Fredonia

A Bridge to Advanced Mathematics, Dennis Sentilles, Williams and Wilkins, Baltimore, 1974, 400 pp., \$14.95.

This book is intended as a transition course between the cookbook freshman calculus and the rigor of upper division courses in mathematics. The Library of Congress cataloging lists it under "Proof Theory," which is clearly the intent of the author. For material from which to draw proofs, he uses set theory, functions, and point set topology. His "strategic attack," as he calls it, is to ask two questions, and then develop material leading to the answers to these questions, including a clarification of the terms used, and to explore ideas arising from these questions. The questions are: I. Can one, in

sequence, pick out all the points of a "connected point set" of the real number line? and II. Given a connected object, to what extent can it be changed, distorted, or transformed in such a way that the resulting object will still be connected?

Although the author states and proves theorems, his basic approach is a heuristic one, designed to lead the student to an understanding of the material, rather than to present him with a polished model of rigor and concise reasoning. This leads more to verbosity than terseness. The author points out that one must constantly keep in front of the student the two questions, and the relationship of the material to those questions. This is certainly true, but even more is true. The author's style and teaching method come through very strongly in the text. If it matches your style, an excellent course can result. If it does not, I do not believe either you or the students will enjoy using this book.

I do not understand why the author uses \square for the empty set, nor why the "map for the wayfaring stranger" on page 152 has such an obscure relationship to the table of contents. Figure 2.1 shows a "straight line" on the surface of a sphere as distinct from a great circle. There seems to be no explanation of what that "straight line" is, and the discussion on page 80 about the "flattened surface of a perfectly spherical earth" doesn't help much.

The quality of paper is very poor. One can read both sides of a sheet at the same time, which I find very bothersome. The price seems high for the physical quality of the book.

I have taught a course whose intention was the same as that for which this book was written, and this book is better for this purpose than most of what I have used. I would be hard pressed to get through the material in a three semester-hour course, even if I took the author's suggestion and quit after section 6.5 and left out some of the material in the first two chapters. Since one cannot quit before finishing Question I, one must do some careful planning, but not much more than would be expected of a conscientious teacher. If you have a transition course, or have students who need such a course, you ought to consider this text.

E. R. Deal
Colorado State University

Technology Mathematics Handbook, Jan J. Tuma, McGraw Hill, New York, 1975, 384 pp., \$15.95.

This book is a summary of definitions, formulas, and tables of general use to engineers, scientists, teachers, and students of mathematics or science. An amazing amount of information is contained in the book. All of the usual formulas for areas, volumes, integrals, and various geometric quantities are presented plus many more obscure but useful ones. There are formulas for various sums (finite and infinite) and formulas for all the various moments of inertia for a variety of solids, for example. The tables included are conventional (square roots, logarithms, values of trigonometric functions, etc.).

The text seems relatively error free but it would take years to check every formula. Some errors were found in definitions. In the definition of a function, for example, the author includes so-called multiple-valued functions. In modern mathematical usage a function has a unique value at each point in its domain. There are a number of other points which mathematicians might find objectionable in regard to definitions but none are really serious for the user of mathematics in the sciences.

All factors considered, this book should be a valuable reference for students, engineers, and many other scientific workers.

Ben F. Plybon
Miami University (Ohio)

The Problem Corner

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 September 1976. The best solutions submitted by students will be published in the Fall 1976 issue of *The Pentagon*, with credit being for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

277. *Proposed by the editor.*

Consider the fraction $\frac{a^3 + 2a}{a^4 + 3a^2 + 1}$ where a is a positive integer. Show that the fraction is reduced to lowest terms for any permissible choice of a .

278. *Proposed by the editor.*

Dr. Knowitall noticed that his clock had stopped. So he wound it, noted the time to be 6:00 p.m. and went to a friend's house to play chess. He arrived at 8:30 according to the clock in the friend's house. Dr. Knowitall left at 11:00. When he arrived home, the time, according to his clock, was 12:30 a.m. He reset his clock to the correct time. Assuming that Dr. Knowitall walked at the same rate in both directions and assuming that his friend's clock kept perfect time, what was the correct time when Dr. Knowitall reset his clock?

279. *Proposed by the editor.*

In Moldavia, people pay an income tax equal to a percentage of the weekly wage equal to the number of ducats earned each week; e.g., on a weekly wage of 10 ducats, the rate is 10 percent. Assuming the maximum salary is 100 ducats per week, what is the optimum salary in Moldavia?

280. *Proposed by the editor.*

Solve the cryptarithm $THAT = (AH)(HA)$ where each letter represents a unique digit in the decimal system.

281. *Proposed by the editor.*

An algebra student encountered the following problem on an exam: Evaluate $\frac{\log A}{\log B}$. Being pressed for time, he cancelled common factors from both numerator and denominator (including the common factor \log) to obtain the correct answer. $\frac{\log A}{\log B} = \frac{A}{B} = \frac{3}{4}$. What are A and B ?

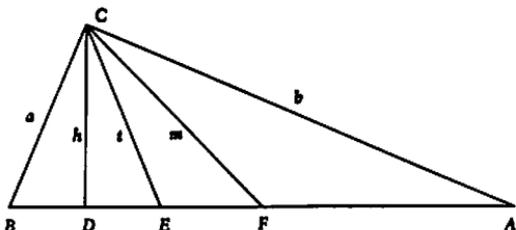
SOLUTIONS

267. *Proposed by Charles W. Trigg, San Diego, California.*

In what triangles do an altitude, median and angle bisector from one vertex of the triangle divide the angle at the vertex into four equal parts?

Solution by the proposer.

In the triangle BAC of the figure, let the altitude $CD = h$, the angle bisector $CE = t$, the median $CF = m$, and the angles $BCD = DCE = ECF = FCA = x$.



Then h bisects angle BCE so $t = a$ and $BE = 2a \sin x$.

Also, $a/b = (2a \sin x)/EA$, so $EA = 2b \sin x$, $BA = 2(a + b) \sin x$, $FA = (a + b) \sin x$, and $EF = (b - a) \sin x$.

Now m bisects angle ECA , so

$$a/b = [(b - a) \sin x]/[(a + b) \sin x]$$

whereupon

$$a^2 + ab = b^2 - ab$$

$$b^2 - 2ab - a^2 = 0$$

$b = a(1 \pm \sqrt{2})$, where only the positive sign applies, so $a/b = \sqrt{2} - 1$.

From the well-known formula for the bisector of an angle in terms of the sides:

$$t^2 = a^2 = ab - (2a \sin x)(2b \sin x)$$

$$a/b = 1 - 4 \sin^2 x = \sqrt{2} - 1.$$

$$\sin^2 x = (2 - \sqrt{2})/4.$$

$$2x = \arccos [1 - 2(2 - \sqrt{2})/4]$$

$$= \arccos (1/\sqrt{2}) = 45^\circ.$$

It follows that ABC is a right triangle with legs in the ratio $(1 + \sqrt{2}):1$.

Editor's comment: The formula for the angle bisector t in terms of the sides is given by

$$t_c^2 = \frac{ab(a+b+c)(a+b-c)}{(a+b)^2}$$

Here $c = \overline{BA} = \overline{BE} + \overline{EA} = 2a \sin x + 2b \sin x = 2(a+b) \sin x$. To avoid the use of this formula, note that the diagram shows $\triangle ABC = \triangle CBE + \triangle CEA$. Their respective areas imply $\frac{1}{2}a \sin 2x + \frac{1}{2}ab \sin 2x = \frac{1}{2}ab \sin 4x = ab \sin 2x$

$\cos 2x$ or $a + ab = 2ab \cos 2x$. Hence $\cos 2x = \frac{a+b}{2b} =$

$$\frac{1}{2} + \frac{1}{2} \cdot \frac{a}{b} = \frac{1}{2} + \frac{\sqrt{2}-1}{2} = \frac{\sqrt{2}}{2}$$

Thus $2x = \frac{\pi}{4}$ and angle c is a right angle.

268. Proposed by Charles W. Trigg, San Diego, California.

Find the unique four-digit integer in the decimal system which can be converted into its equivalent in the septenary system by moving its left hand digit to the far right.

Solution by Rosann F. Viel, Eastern Kentucky University, Richmond, Kentucky.

If the four digit integer, $abcd_{ten}$, is equal to the four digit integer, bcd_{seven} , then

$$a \cdot 10^3 + b \cdot 10^2 + c \cdot 10^1 + d \cdot 10^0 = b \cdot 7^3 + c \cdot 7^2 + d \cdot 7^1 + a \cdot 7^0$$

$$1000a + 100b + 10c + d = 343b + 49c + 7d + a$$

$$999a - 243b - 39c - 6d = 0$$

Since $bcda$ is in the septenary system, all digits must be less than seven. By use of a computer, all possible combinations of a , b , c , and d from zero to seven were calculated. The combination yielding zero in the above equation was $a = 1$, $b = 3$, $c = 6$, and $d = 6$.

Therefore, $1366_{ten} = 3661_{seven}$

Solution by the proposer.

All of the digits in the integers are less than seven. If $abcd_{10} = bdca_7$, then

$$10^3a + 10^2b + 10c + d = 7^3b + 7^2c + 7d + a.$$

$$999a - 243b - 39c - 6d = 0.$$

$$9(37c - 9b) = 13c + 2d.$$

Now since $13c + 2d$ is divisible by 9, then $(c, d) = (2, 5)$, $(3, 3)$, $(4, 1)$, or $(6, 6)$, to which correspond the values for $37a - 9b$ of 4, 5, 6, or 10. The last value is the only possible one for values of a and b less than 7, namely: $a = 1$ and $b = 3$.

Thus the unique solution is $1366_{10} = 3661_7$.

269. *Proposed by Leigh James, Rocky Hill, Connecticut.*

Find a formula for $\sum_{k=0}^{n-1} (-1)^k (n-k)^2$ where n and k are integers.

Solution by Michael Reis, Loyola College, Baltimore, Maryland.

Solution: $\sum_{k=0}^{n-1} (-1)^k (n-k) = \frac{n(n+1)}{2}$

Proof: $\sum_{k=0}^{n-1} (-1)^k (n-k)^2 = n^2 - (n-1)^2 + (n-2)^2 - (n-3)^2 + \dots + (-1)^{n-1} = [n^2 + (n-2)^2 + \dots] - [(n-1)^2 + (n-3)^2 + \dots]$

There are two cases to consider: n is even or n is odd. If $n = 2m$, the first bracketed expression becomes the sum of the first m even squares, and the second the sum of the first m odd squares. Since we know that

$$\sum_{j=1}^s (2j) = \frac{2}{3}s(s+1)(2s+1) \text{ and } \sum_{j=1}^s (2j-1) = \frac{s}{3}(2s-1)(2s+1),$$

we have for $n = 2m$

$$\begin{aligned} \sum_{k=0}^{n-1} (-1)^k (n-k)^2 &= \sum_{j=1}^m (2j)^2 - \sum_{j=1}^m (2j-1)^2 = \\ &= \frac{2}{3}m(m-1)(2m+1) - \frac{m}{3}(2m-1)(2m+1) = \\ &= \frac{m(2m+1)}{3} [2m+2 - (2m-1)] = m(2m+1) \\ &= \frac{n(n+1)}{2} \end{aligned}$$

The case $n = 2m + 1$ is treated similarly.

Also solved by Richard De Cesare, Southern Connecticut State College, Trumbull, Connecticut, by using mathematical induction.

Editor's comment: Another solution can be based upon the fact that $a^2 - b^2 = (a + b)(a - b)$. Suppose $n = 2m$.

$$\begin{aligned} \text{Then } \sum_{k=0}^{n-1} (-1)^k (n-k)^2 &= [n^2 - (n-1)^2] + \\ &+ [(n-2)^2 - (n-3)^2] + \cdots + 2^2 - 1^2 = \\ &= [n + (n-1)](1) + [(n-2) + (n-3)](1) + \cdots \\ &+ (2+1)(1) = \frac{n(n+1)}{2} \text{ and the case } n = \end{aligned}$$

$2m + 1$ can be treated similarly.

270. Proposed by Charles W. Trigg, San Diego, California.

The expression $abcd$ is capable of two interpretations: (1) the product of the four quantities a, b, c , and d ; (2) an integer with digits in position; i.e., the sum of the four digits a, b, c , and d each multiplied by an appropriate power of the base in which the integer is expressed. Does any integer exist in any system of notation for which the two interpretations lead to the same final value?

Solution by the proposer.

The solution $a = b = c = d = 0$ in every base is trivial. Otherwise, the answer is "NO!" in any base r , the digits are less than r , so

$$\begin{aligned} (a)(b) < (a)(r) < ar + b = \bar{a}\bar{b} \text{ for some integers } \bar{a} \text{ and } \bar{b}. \\ \text{In general, } (a_1)(a_2)(a_3) \cdots (a_n) < a_1 r^{n-1} < a_1 r^{n-1} + \\ a_2 r^{n-2} + a_3 r^{n-3} + \cdots + a_{n-1} r + a_n. \end{aligned}$$

271. *Proposed by the editor.*

A regular polygon is cut from a piece of cardboard. A pin is put through the center to serve as an axis about which the polygon can rotate. Find the least number of sides which the polygon can have in order that a rotation through an angle of $27\frac{1}{2}$ degrees will put the polygon in coincidence with its original position.

Solution by the proposer.

Let s be the number of sides in the polygon. Imagine that the required polygon is inscribed in a circle. Then each side corresponds to an angle of $360/s$ degrees. Next suppose that r of the sides are contained in an angle of $27\frac{1}{2}$ degrees.

$$\text{Hence, } r \cdot \frac{360}{s} = 27\frac{1}{2} \text{ or } \frac{r}{s} = \frac{11}{144} .$$

Hence $r = 11t$, $s = 144t$ for some integer t . Taking $t = 1$, the smallest number of sides in the polygon is 144.

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgment will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

The practical applications of group theory many times appear to be somewhat impractical and insignificant to the undergraduate mathematics student. The chemist, however, has found many important applications of group theory quantum mechanics and is handicapped without some general knowledge of groups and matrices. This area of quantum mechanics is called representation theory and is extensively studied by the chemist today.

We begin with the following definition.

DEFINITION: A homomorphism of a group G onto a group $D(G)$ of $n \times n$ matrices is called a representation of G of degree n . The representation is called faithful when the homomorphism is an isomorphism. The matrices in $D(G)$ represent linear transformations on a vector space V over C . The dimension of V is called the dimension of the representation.

It can be shown that for every finite group there is a group of matrices onto which the group elements are homomorphic. In quantum mechanics the symmetry operations which transform a molecule or an infinite crystal into itself form a group. The symmetric operations which leave molecules unchanged are called point groups. Each of these point groups will have a group representation; that is, they will be expressible as a set of matrices, each corresponding to a single operation in the group. It is sufficient to specify only a few molecular operations of the group as the closure property of a group yields the entire group.

A concise notational scheme has been devised to indicate the symmetry operations which generate each of the molecular point groups. A portion of that notation normally used is the following:

E —represents the identity element; that is, the symmetric operation which leaves the molecule alone.

C_n —represents an n -fold rotation axis whereby the (right-handed) rotation is through an angle of $2\pi/n$ radians about some indicated axis, usually the z -axis. Obviously $C_1 = E$,

which is usually not considered a rotation. If an element has more than one axis of rotation, they are denoted as C'_n , C''_n , etc., with one of the axes considered the major axis.

C_n^k —represents k rotations of $2\pi/n$ each around the axis C_n .

σ_r , σ_h , σ_d —represents mirror reflections about some plane where the subscript h is used when the reflection is a plane perpendicular to the element's major axis, v when the plane contains the major axis, and d for a diagonal case where the planes bisect the angles between C_2 's and contain the major axis. If there exist more than one mirror reflection of any one type, they are denoted using primes as in the C_n case.

Examples of molecules with C_n axes are water ($n = 2$), ammonia ($n = 3$), square-pyramidal B_5H_9 ($n = 4$), ferrocene $C_5H_5FeC_3H_5$ ($n = 5$), benzene ($n = 6$), and hydrogen fluoride ($n = \infty$). Having mirror reflections are NH_3 and BF_3 which each have three σ_v 's; ethylene, cyclopropane, $PtCl_4^{2-}$, benzene and H_2 which have a σ_h ; and benzene which has three σ_d 's.

Molecules of the elements are classified as to their symmetric properties, and molecular point groups are formed having the usual group properties. Special notation has been established. For example, C_n represents the groups whose only symmetric element is C_n ; C_{nh} represents the groups whose elements are E , C_n , σ_h , and all possible products; and C_{nr} represents the groups whose elements are E , C_n , σ_r , and all possible products.

Let us consider H_2O which is an example of the C_{2v} group. This group consists of the set $\{E, C_2, \sigma_r, \sigma'_v\}$ where C_2 coincides with the z -axis of the usual Cartesian coordinate system, σ_r the xz -plane, and σ'_v the yz -plane. Hence the matrices representing the transformations effected on a general point (x, y, z) are:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sigma_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \sigma'_v = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence the group multiplication table is

•	E	C ₂	σ _v	σ' _v
E	E	C ₂	σ _v	σ' _v
C ₂	C ₂	E	σ' _v	σ _v
σ _v	σ _v	σ' _v	E	C ₂
σ' _v	σ' _v	σ _v	C ₂	E

The above representation is not unique.

To further examine the molecular point group, we will need additional information. We define the character of a representation to be the set of traces of the matrices comprising the group representation. We will denote the order of the group by h . The dimension of the i^{th} representation, which is the order of each of the matrices which constitute it, will be denoted by l_i . The various operations in the group will be given the generic symbol R . The element in the m^{th} row and the n^{th} column of the matrix corresponding to an operation R in the i^{th} irreducible representation will be denoted $\Gamma_i(R)_{mn}$. Let $\chi_i(A)$ be the character of the representation of A in the i^{th} irreducible representation. Then

$$\sum_R [\Gamma_i(R)_{mn}] \cdot [\Gamma_j(R)_{m'n'}] = \frac{h}{\sqrt{l_i l_j}} \delta_{ij} \delta_{mm'} \delta_{nn'}$$

Some important results necessary to determine the characters are the following:

- (1) The sum of the squares of the dimensions of the irreducible representations of a group is equal to the order of the group; that is, $\sum l_i^2 = h$.
- (2) The sum of the squares of the characters in any irreducible representation equals h ; that is, $\sum [\chi_i(R)]^2 = h$.
- (3) The vectors whose components are the characters of two different irreducible representations are orthogonal; that is, $\sum_R \chi_i(R) \chi_j(R) = 0$ when $i \neq j$.
- (4) In a given representation (irreducible or reducible) the characters of all matrices belonging to operations in the same class are identical.

- (5) The number of irreducible representations of a group is equal to the number of classes in the group.

If we again return to the group C_{2r} of which H_2O is an example, we note that C_{2r} contained four elements and each is a separate class. Hence by (5) above, there are four irreducible representations for this group. But (1) states that the sum of the squares of the dimensions of these representations equal h . Hence we are looking for a set of four positive integers, say l_1, l_2, l_3 and l_4 , such that $l_1^2 + l_2^2 + l_3^2 + l_4^2 = 4$. One solution is $l_1 = l_2 = l_3 = l_4 = 1$. Therefore the group C_{2r} has four one dimensional irreducible representations. The characters of this first irreducible representation are:

	E	C_2	σ_v	σ'_v
Γ_1	1	1	1	1

in each case we have $\sum[\chi_i(R)]^2 = 4$ as required. So each $\chi_i(R) = \pm 1$. But (3) implies that for each of the other representations to the orthogonal to Γ_1 , we must have two pluses and two minuses. Hence we have

	E	C_2	σ_r	σ'_v
Γ_1	1	1	1	1
Γ_2	1	-1	-1	1
Γ_3	1	-1	1	-1
Γ_4	1	1	-1	-1

One will note that all these representations are orthogonal to each other. These four representations are the four irreducible representations of the group C_{2r} of which H_2O is a well-known example.

Can you find the group representations for the group $C_{2h} = \{E, C_2, i, \sigma_h\}$, where $i = C_2\sigma_h = \sigma_h C_2$?

Kappa Mu Epsilon News

EDITED BY SISTER JO ANN FELLIN, *Historian*

News of Chapter activities and other noteworthy **KME** events should be sent to Sister Jo Ann Fellin, Historian, Kappa Mu Epsilon, Benedictine College, North Campus Box 43, Atchison, Kansas 66002.

The Twentieth Biennial Convention of Kappa Mu Epsilon convened on Thursday, 17 April 1975, on the Mount Mary College campus, with Wisconsin Alpha as the host chapter.

Following the registration in Bergstrom Hall, there was a mixer consisting of a German Dance Group, a skit, a sing-a-long in the North Dining Room. At the same time the National Council met in the lounge.

The first general session met in Kostka Hall on Friday, 18 April 1975, with the National President, William R. Smith, presiding. Sister Mary Nora Barber, President, Mount Mary College, welcomed the group. James E. Lightner, National Vice-President, responded for the society. The roll call of the chapters was made by Elizabeth T. Wooldridge, National Secretary.

The National President, William R. Smith, greeted the new chapters which have been installed during the past two years. Resumes of four groups petitioning for membership in Kappa Mu Epsilon were distributed to the voting delegates.

Professor James E. Lightner presided during the presentation of the following student papers:

1. *A Characterization of Automorphisms of Cyclic Groups*, Gregg Stair, Kansas Beta, Emporia Kansas State College.
2. *An Alternative Approach to Elliptic Geometry*, Louell Snodgrass and Leila Barge, Missouri Eta, Northeast Missouri State University.
3. *Field Extensions and a Beginning of Galois Theory*, Rhoda L. Ring, Pennsylvania Zeta, Indiana University of Pennsylvania.
4. *The Leontief Input-Output Matrices*, Kathy L. Rhen, Pennsylvania Lambda, Bloomsburg State College.
5. *A Definition of Number*, Sandra K. Peer, Kansas Delta, Washburn University.

After lunch and the taking of the group picture, sectional meet-

ings were held. The faculty section met in the North Dining Room with William R. Smith presiding; the student section met in Kostka Hall with Barbara Junghans presiding.

The convention reconvened at 2:45 p.m. and the following papers were presented:

6. *Euclid Today*, Janice K. Neely, Iowa Gamma, Morningside College.
7. *A Study of Tessellations*, Barbara Junghans, Wisconsin Alpha, Mount Mary College.
8. *The Matching Game — A Mathematical Analysis*, James Himmelreich, Pennsylvania Zeta, Indiana University of Pennsylvania.
9. *A Discussion of the Division Algorithm in Integral Domains other than Z*, Janet Guyer, Kansas Delta, Washburn University.
10. *Group Theory in Plain Bob — A Mathematical Approach to Bell Ringing*, Janet Fiora, Pennsylvania Lambda, Bloomsburg State College.

Barbara Junghans, President of Wisconsin Alpha Chapter, was master of ceremonies for the traditional banquet Friday evening, 18 April, in the dining room of Bergstrom Hall. The jazz choir of Mount Mary College furnished a delightful program. The guest speaker, Professor David R. Johnson, gave an interesting and magical address, *A Magical Aftermath*.

The convention resumed on Saturday morning with the presentation of the remainder of the papers:

11. *The Cantor Mapping*, David Elko, Pennsylvania Zeta, Indiana University of Pennsylvania.
12. *Continued Fractions on February 29*, Marc J. Burkhart, Iowa Gamma, Morningside College.

The following papers were listed by title as alternates:

1. *The Orchard Problem*, Gary LeFan, Alabama Beta, University of North Alabama.
2. *The Nine-Point Circle — Amazing!*, Lawrence Moerke, Pennsylvania Zeta, Indiana University of Pennsylvania.
3. *An Uncountable Set with Measure Zero?*, Cheryl K. Graves, Missouri Theta, Evangel College.

4. *The Amazing Cycloid*, John Williams, Oklahoma Alpha, Northeastern State College.
5. *On the Area Tiles in Different Lattices*, Jacqueline M. Ostrom, Colorado Alpha, Colorado State University.
6. *A Study of Business Cycles and Forecasting*, Cathy Baird, Kansas Alpha, Kansas State College of Pittsburg.
7. *And Pythagoras Said*, Mary Beth Breindel, Pennsylvania Zeta, Indiana University of Pennsylvania.
8. *The Minimum Dstance*, Peter John Fallon, New York Eta, Niagara University.
9. *Pseudo Groups*, William L. Taber, Illinois Beta, Eastern Illinois University.

President Smith called the second business meeting to order. The following were approved for chapters of Kappa Mu Epsilon:

1. Bethany College, which will be West Virginia Alpha.
2. West Georgia College, which will be Georgia Alpha.
3. Missouri State College, which will be Missouri Iota.
4. Hardin Simmons University, which will be Texas Eta.

Reports of the sectional meetings were made by William R. Smith for the faculty and Louell Snodgrass for the student section.

The following reports were given by the national officers:
Business Manager of *The Pentagon* — Wilbur Waggoner
Editor of *The Pentagon* — Wilbur Waggoner
National Historian — Elsie Muller
National Treasurer — Eddie W. Robinson
National Secretary — Elizabeth T. Wooldridge
National Vice-President — James E. Lightner
National President — William R. Smith

Lyle Oleson reported for the auditing committee that the books were in order.

Invitations for the twenty-first biennial convention were issued by
Gary Morella, Kansas Alpha
David Moore, Ohio Zeta
Steve Lacy, Missouri Beta
Wai Ming Fan, New York Eta

George R. Mach, chairman of the nominating committee reported. The following officers were elected for the biennium, 1975-1977.

President	William R. Smith Indiana University of Pennsylvania
Vice-President	James E. Lightner Western Maryland College
Secretary	Elizabeth T. Wooldridge University of North Alabama
Treasurer	Eddie W. Robinson Southwest Missouri State University
Historian	Sister Jo Ann Fellin Benedictine College

The new officers were installed by the Past President, George R. Mach.

Dr. Harold Thomas, Kansas Alpha, chairman of the awards committee, gave the following criteria upon which papers were judged:

A. The Paper

1. Originality in the choice of the topic.
2. Appropriateness of the topic to the meeting and audience.
3. Organization of the material
4. Depth and significance of the content
5. Understanding of the material

B. The Presentation

1. Style of presentation
2. Maintenance of interest
3. Use of audio-visual materials (if applicable)
4. Enthusiasm for the topic
5. Overall effect

Awards were then presented to:

Sandra K. Peer Kansas Delta	First Place
Marc J. Burkhart Iowa Gamma	Second Place
James Himmelreich Pennsylvania Zeta	Third Place

Barbara Junghans Wisconsin Alpha	Honorable Mention
Janet Guyer Kansas Delta	Honorable Mention
David Elko Pennsylvania Zeta	Honorable Mention

Homer Hampton, Missouri Beta, reported for the resolutions committee. The following resolutions were adopted:

Resolved: That the Twentieth Biennial Convention of Kappa Mu Epsilon express its gratitude to Elsie Muller who has served as Historian of Kappa Mu Epsilon and who has given so generously of her time and talents. *services*

Resolved: That the Twentieth Biennial Convention of Kappa Mu Epsilon express its appreciation —

1) To Sister Mary Petronia and the members of Wisconsin Alpha for their work in the expeditious planning of this Convention. *Sponsor*

2) To Sister Mary Nora, President of Mount Mary College, for the gracious hospitality and the many services rendered the chapters and officers of the Convention. *President*

4) To David R. Johnson for his most entertaining talk. *entertainment*

✓ 4) To the Selection Committee and the Awards Committee who gave so unselfishly of their time to the primary activity of Kappa Mu Epsilon.

5) To the students who prepared and presented papers at this Convention.

6) To the National Officers of Kappa Mu Epsilon for their diligent service during and preceding the Biennial Convention.

7) To the German Dance Group who performed and entertained so admirably on Thursday evening.

Resolved: That the National Council of Kappa Mu Epsilon —

1) Consider revising the dues structures and reimbursement for travel.

2) Consider the inclusion of Kappa Mu Epsilon student representatives at National Council Meetings held during each Biennial Convention.

REPORT OF THE NATIONAL PRESIDENT

During the last biennium we have lost three friends and active members of **KME**: Charles B. Tucker, President 1951-55, C. C. Richtmeyer, President 1955-59, and E. M. Hove, Historian and Secretary for a total of 22 years. I would ask that we take a moment of silence to honor these three to whom we all owe a great deal.

Since my election as President at the 19th Biennial Convention, it has been my pleasure to share with you in seeing our Society grow with the addition of eight new chapters; Weatherford, Oklahoma, installed by Mike Regan; Bloomsburg, Pennsylvania, installed by James Lightner; Pace University in New York City, which I installed; and Western Carolina University, installed by Elizabeth Wooldridge; together with the four new chapters we have approved at this convention. Currently we have twelve inquiries and one set of petitions due shortly. It is gratifying to see this type of growth when we are told that generally the number of mathematics majors is decreasing across the country. With the excellent brochure prepared due to the joint efforts of Jim Lightner and Sister John Frances Gilman which tells the **KME** story clearly and concisely, we should be able to do an even better job of gaining new chapters in the coming biennium.

The National Council made a few changes in our regional structure which were reported upon in a letter to all corresponding secretaries in November, 1973. This regional structure is a part of our Society which we are continually examining. We are currently seeking Directors for Regions I and VI, which had been served so well in the past biennium by Sister John Frances and Professor Joyce Curry. There have been three very successful regional meetings held since our last National Convention—one at LaSalle College in Philadelphia; one in New Concord, Ohio, held jointly by Muskingum College and Marietta College; and one at the University of Missouri at Rolla.

For the first time to my knowledge, the President of the Society has been invited to speak at the National meeting of the National Council of Teachers of Mathematics to be held next week in Denver, Colorado. The Council has approved my accepting this invitation and I will be sharing the platform with the President of Pi Mu Epsilon and Mu Alpha Theta. This meeting is especially important since it will give me a chance to have some private conversations

with Houston Kearnes, the President of Pi Mu Epsilon, at which time we can discuss each Society's goals and objectives.

At the meeting of the National Council last evening it was decided to invite the Regional Directors to attend our non-convention year meeting in order to closely examine the regional structure.

We are also studying the possibility of an executive secretary, but this is in the preliminary stage of discussion and an amendment to the constitution would be necessary before any action is taken.

Beginning with the next issue, we find it necessary to increase the price of *The Pentagon* to \$1.00 per issue. Xerox University Microfilms will preserve and distribute all copies of *The Pentagon* at no cost to us. We also took action to have the minutes of the convention published in *The Pentagon*.

The dues and mileage rate for conventions will be re-examined for possible amendment before the next convention.

One of the important functions of the President is to appoint all of the convention committees, with the exception of the Paper Selection Committee, which is appointed by its chairman. I would like to report to you that there was excellent cooperation from everyone I asked to serve on these committees, and I would like to publicly thank them for their efforts.

Once again, let me say that these past six years on the National Council, four as Vice President, and two as President, have been rewarding ones. I see the future of the organization to be a productive one for all of us.

William R. Smith

REPORT OF THE NATIONAL VICE PRESIDENT

The past two years have been most interesting and memorable, as I have had the pleasure of serving Kappa Mu Epsilon as Vice President. My appreciation goes to the Society for this opportunity.

In October following the 19th Biennial Convention, the National Council met for a two-day session to deal with many items of business of the Society. At this meeting I was asked to act as coordinator for the regions, assisting the Regional Directors and attempting to provide some intercommunication whenever possible. During the biennium I have tried to do this, and I am pleased to report that three regional meetings were held. Region 1 (Sister John Frances Gilman, Director) held its convention on 6 April 1974 at LaSalle College (Pennsylvania Beta chapter); I had the pleasure of being

the afternoon speaker for this meeting which featured 7 student papers. Region 2 (Mr. Dean Hinshaw, Director) held its convention 1-2 March 1974 at Muskingum College (Ohio Zeta chapter) with 8 student papers being presented. Region 4 (Dr. Harold Thomas, Director) held its convention 19-20 April 1974 at the University of Missouri, Rolla (Missouri Zeta chapter) with student papers being the major part of the meeting. My special thanks to these directors, the host chapters, and faculty sponsors for their hard work in planning these successful regional meetings, and to all those members and chapters which attended and supported the regional program.

During this biennium I have also been involved in designing, writing, editing, and having published a brochure which tells the Kappa Mu Epsilon story to prospective members and prospective chapters. With a great deal of help from Sister John Frances Gilman and the printing shop of Niagara University, I am pleased to report that the brochure is ready for distribution at this convention. I hope you are pleased with the results of this "first effort" and will find it useful.

In October 1973, I had the pleasure of being the installing officer for the Pennsylvania Lambda Chapter at Bloomsburg State College.

Finally, it is the Vice President's responsibility to arrange for the student papers which are presented at the biennial convention. I am pleased to report that 22 students submitted papers for judging in January for this 20th convention. My special thanks to a most efficient and dedicated selection committee: Sister Jo Ann Fellin (Kansas Gamma), Dr. James Pomfret (Pennsylvania Lambda) and Dr. James Smith (Ohio Zeta), who read and ranked all these papers at a very busy time of year. Their job was made all the more difficult because of the exceptionally high quality of the papers submitted; decisions on presenters and alternates were made reluctantly. Kappa Mu Epsilon thanks and congratulates all 22 students who submitted papers for their fine work. It is these papers which make our biennial convention so successful.

James E. Lightner

REPORT OF THE NATIONAL SECRETARY

On 18 April 1931, the first chapter of Kappa Mu Epsilon, Oklahoma Alpha was established at Northeastern State College at

Tahlequah. A month later, 27 May 1931, the second chapter, Iowa Alpha was established at Iowa State Teachers College at Cedar Falls. Today, forty-four years later there are 95 active and 14 inactive chapters in twenty-nine states. The membership of the 95 active chapters is 31,591, and the total membership of all chapters is 34,592.

During the past two years four new chapters were installed; New York Kappa at Pace University, North Carolina Beta at Western Carolina University, Oklahoma Gamma at Southwestern State College, and Pennsylvania Lambda at Bloomsburg State College.

The permanent record card of each member is filed in the office of the secretary. Orders for membership certificates, for jewelry, for invitations to membership and other supplies, and for charters for new chapters are approved and copies filed in the secretary's office.

It has been a privilege to work with all the corresponding secretaries, whose efficient work makes work of the secretary more pleasant.

Elizabeth T. Wooldridge

REPORT OF THE NATIONAL HISTORIAN

The files of the National Historian continue to be a repositiorium of the valuables for the national society as well as the chapters. Correspondence with the chapters and national officers as well as folders for the newly installed chapters are also being preserved.

News items have been solicited semi-annually from each chapter for the **KME** News section of *The Pentagon*. During the past bien-nium 74 chapters have responded at one time or another. The following chapters replied to all four questionnaires: Alabama Beta, California Gamma, Colorado Beta, Illinois Beta, Iowa Alpha, Iowa Gamma, Iowa Delta, Kansas Alpha, Kansas Gamma, Kansas Epsilon, Maryland Beta, Michigan Beta, Missouri Beta, New York Iota, Ohio Zeta, Pennsylvania Zeta, Pennsylvania Eta, and Oklahoma Gamma. Some chapters have reported late and as a result the news appeared in a later issue.

I wish to thank the society for giving me the privilege of serving in this office. I also want to thank all the national officers, Professor Bidwell, and the corresponding secretaries of each chapter for all the cooperation which I have received. You made the work pleasant.

Elsie Muller

KAPPA MU EPSILON
FINANCIAL REPORT OF THE NATIONAL TREASURER
For the Biennium 4 April 1973 to 4 April 1975

RECEIPTS

1. Cash on hand 4 April 1973		\$ 4,983.47
2. Receipts from Chapters		
Initiates 2567	\$17,970.00	
Jewelry	1,011.54	
Supplies	301.35	
	\$19,282.89	
3. Miscellaneous		
Charters	\$ 36.00	
Interest	2,118.60	
From Savings to Checking	10,324.70	
	\$12,479.30	
4. Total Receipts		\$31,762.19
5. Total receipts plus cash on hand		\$36,745.66

EXPENDITURES

6. National Convention 1973		\$ 3,040.59
7. National Officer Expense		2,926.26
8. Regional Officer Expense		936.06
9. Balfour Company		159.32
10. Blake Printery		3,351.78
11. Pentagon		9,520.45
12. Miscellaneous		
ACHS	100.00	
Savings	12,012.90	
	12,112.90	
13. Total Expenditures		\$32,047.36
14. Cash on Hand 4 April 1975		\$ 4,698.30
15. Total expenditures plus cash on hand		\$36,745.66
16. Total assets 4 April 1975	\$20,789.08	
17. Total assets 4 April 1973	19,386.05	
18. Net Gain for biennium	\$ 1,403.03	

I shall not bore you by reading you these figures. You can all see that the books do balance. You will have the auditor's report later.

There are some comparisons which are of interest: The number of initiates was not as large as the last biennium. The amount 17,970 does not fit the divisibility criteria for division by 7. This is because New York Theta sent in 8 dollars for one initiate and I have not refunded the one dollar, but I shall do so.

The amount of jewelry and supplies purchased from the National Secretary during the biennium was \$600 less than the previous biennium.

I am not in office to play a numbers game, to sell jewelry, or to sell supplies, but the following information might reflect chapter activities:

20 chapters did not initiate anyone during the biennium

79 chapters did not order any jewelry

91 chapters did not order any supplies during the biennium.

I am not chiding any chapter because of this, but simply reminding you that your national officers exist to be of service to you and, perhaps, you are not aware of all the services and products that we can provide. For instance, if you are not using the beautifully engraved invitations and reply cards, then you are missing an opportunity to advertise your chapter and you are missing an opportunity to show some real class.

Virginia Alpha ordered more jewelry than any other chapter. They initiated 22 members and ordered \$161 worth of jewelry.

Tennessee Alpha initiated 83 members during the biennium. Wisconsin Beta initiated 70 members. Colorado Beta and Pennsylvania Zeta each initiated 62 members. New Jersey Beta initiated 56 members and Kansas Alpha initiated 53 members. Kansas Alpha has the distinction of having initiated more members than any other chapter. During their 43 years of existence, they have initiated 1216 members.

This completes my report and I do respectfully submit it.

Eddie W. Robinson

REPORT OF THE EDITOR OF THE PENTAGON

THE PENTAGON is the journal of **KME**. It should serve the needs of the members. As editor I have tried to follow the suggestions and

criticisms you have made. However, we need more direct participation of the membership. I urge each of you to contribute to the journal by submitting articles, problems and solutions, contributions to the Scrapbook, and **KME** News items.

This is my second biennium report. Since the last one three issues have been printed and a fourth one is in press. Articles by students continue to outnumber those by non-students. I want this to continue. I hope many of the papers presented at this biennium will be printed in **THE PENTAGON**.

Although we have tried to improve the delivery dates of the journal issues, there have been delays. These delays are chiefly the responsibility of the printer. We are attempting to work as closely as possible with the printer to insure as early a delivery date as possible.

During the biennium there have been two changes in **THE PENTAGON** staff. The death of Robert Poe, our Problem Corner editor, was unexpected. His post has been filled by Kenneth Wilke of Topcka, Kansas. Dr. Elizabeth Wooldridge, who was the Bookshelf editor, became secretary of **KME** at the last biennium. Her position has been filled by Oscar Beck of the University of North Alabama. The other editors are Loretta Smith, Elsie Muller, and Richard Barlow. Wilbur Waggoner, the Business Manager, handles many details that make my job easier. I sincerely thank each of them for their effective and efficient work.

I also wish to thank members of the Mathematics Department of Central Michigan University who have reviewed manuscripts and helped me on technical matters. I particularly thank Robert Chaffer, Robert DeBruin, Maurice Eggan, Edward Lamie, and Douglas Smith.

James K. Bidwell

REPORT OF THE BUSINESS MANAGER OF THE PENTAGON

This is the ninth biennial report on the activities and duties of the Business Manager of *The Pentagon*, official journal of Kappa Mu Epsilon, that I have made. Over that period of time the number of *Pentagons* printed for each issue has increased by more than fifty per cent while the cost of printing and mailing has increased by nearly two hundred per cent.

During the past biennium, over twelve thousand *Pentagons* were

mailed to addresses in most of the fifty states. We have no subscribers in Alaska, Idaho, Nevada, Utah, or Vermont. Our journal goes to many foreign lands, including such countries as Yugoslavia, Chile, Netherlands, Denmark, Thailand, Republic of China, Africa, Iran, and India. Libraries in many high schools and institutions of higher education are among our subscribers.

I find it interesting to observe over a biennium the states which have the most *Pentagons* mailed to addresses within the state. During the past biennium, the five states receiving the most *Pentagons* in descending numerical order were Pennsylvania, Missouri, Illinois, New York, and Kansas. These five states received over one-third of the *Pentagons* that were mailed.

Many *Pentagons* are returned to the office of the Business Manager by the postal service as undeliverable due to incorrect addresses. Please inform your chapter members that to receive their journal they must keep a current address on file with the Business Manager. If a subscriber has any problem with receiving his *Pentagon*, he should contact this office and I will certainly attempt to resolve the problem.

Complimentary copies are sent to the library of each college or university with an *active* chapter of Kappa Mu Epsilon. Also, complimentary copies are sent to authors of articles in *The Pentagon*. Speakers of this convention will automatically have their subscriptions extended for two years.

I have received much cooperation from our editor, Dr. Bidwell, from our national secretary, Dr. Wooldridge, and from the chapter corresponding secretaries. This cooperation is gratefully acknowledged. Again it has been my privilege and pleasure to serve Kappa Mu Epsilon as Business Manager of *The Pentagon* over the past biennium.

Wilbur Waggoner

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence

Chapter President—Anthony Eckl

30 actives

Regular meetings were held and the usual Christmas party and spring picnic were well attended. Alabama Beta member Gary

LeFan received an assistantship to Auburn University. Other officers: Nancy Putman, secretary and treasurer; Jean Parker, corresponding secretary; Eddy Brackin, faculty sponsor.

Alabama Gamma, University of Montevallo, Montevallo

Chapter President—David R. Combs

10 actives, 6 pledges

Programs at the monthly meetings included speakers from mathematically related fields. The chapter handles sales and distribution of books of mathematical tables to all interested students each year—one of these is awarded each spring by the chapter to the outstanding freshman mathematics student. Senior math graduates were honored at the spring banquet. A spring cookout encouraged communication between faculty and students. Other officers: Jimmy Mills, vice-president; Helen Hines, secretary; Sandra K. Hayes, treasurer; Angela Hernandez, corresponding secretary; William Foreman, faculty sponsor.

**California Gamma, California Polytechnic State University,
San Luis Obispo**

Chapter President—Sandra McKaig

60 actives

Monthly meetings featured student and faculty speakers. The Math Lab (tutorial service) was made available to students through the chapter. Dr. Raymond L. Wilder spoke at the initiation banquet on 31 May. Other officers: Robert Kernaghan, vice-president; Marguerite Liem, secretary; Robert Watanabe, treasurer; George R. Mach, corresponding secretary; Adelaide T. Harmon, faculty sponsor.

Illinois Alpha, Illinois State University, Normal

Chapter President—Jim Murdock

21 actives, 7 pledges

In addition to monthly meetings presented by faculty and student speakers, the chapter organized a tutoring program for undergraduate mathematics students. Social activities included the Christmas party and spring picnic for the members and their guests. Other officers: Rita King, vice-president; Marcia Heinz, secretary and corresponding secretary; Colleen Kirby, treasurer; Orlyn Edge, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President—Cheryl A. Ross

35 actives

The annual Homecoming Breakfast was held 25 October 1975 at the home of Dr. and Mrs. E. W. Hamilton with many alumni in attendance. Monthly meetings consisting of student presentations were held in the homes of faculty members. Other officers: Richard A. Kroeger, vice-president; Christy C. Vandeventer, secretary and treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines

Chapter President—Mary Bauer

11 actives, 2 pledges

Other officers: Carol Behrens, vice-president; Alan Carpenter, secretary; David Trautman, treasurer; Christina Bahl, corresponding secretary; Alex Kleiner, faculty sponsor.

Iowa Delta, Wartburg College, Waverly

Chapter President—Gary Wipperman

18 actives, 18 pledges

Notable among the monthly meeting activities was a Citizen's Workshop on Energy and the Environment on 18 February. Participants in the workshop manipulated an analog computer to simulate energy usage over the next few decades. Eighteen initiates were inducted on 18 March. Other officers: Jennifer Zelle, vice-president; Deb Ehlers, secretary; Paul Koch, treasurer; Glenn Fenneman, corresponding secretary and faculty sponsor.

Kansas Alpha, Kansas State College of Pittsburg, Pittsburg

Chapter President—Laura Spain

45 actives

Programs at the monthly meetings were given by Dr. Bruce Daniel, "Mathematics Applied to Astronomy"; Dean Otey, Jeanne Spigarelli, and Gary Morella, mathematical experiences in industry, teaching and in the military. Twenty new members were initiated in February. Dr. Helen Kriegsman, Dr. Harold Thomas, and Professor Bryan Sperry and fourteen students attended the 20th biennial convention forming the largest delegation at the convention at Mount Mary College. Recipients of the Robert M. Mendenhall Awards for scholastic achievement were Timothy Harries, Judith

Wilson, and Paul Zafuta. Dr. Thomas assisted at the installation of the Missouri Iota Chapter on 8 May. Dr. Kriegsmann and four students from Kansas Alpha were also in attendance at the installation. Other officers: Roy Bryant, vice-president; Deanne Anderson, secretary; Darla Hedrick, treasurer; Harold L. Thomas, corresponding secretary; J. Bryan Sperry, faculty sponsor.

Kansas Beta, Emporia Kansas State College, Emporia

Chapter President—Kathy Du Vall

30 actives

On 25 September new initiates were welcomed into the chapter at a fall banquet. Chapter members are participating in fund raising projects to offset expenses for a 1976 spring regional convention. Other officers: Audrey Dunlap, vice-president; Mary Bender, secretary; Debbie Irwin, treasurer; Donald Bruyr, corresponding secretary; Tom Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison

Chapter President—Le Ann Fischer

11 actives

Highlighting second semester meetings was guest speaker Dr. J. C. Kelly of the University of Missouri. Dr. Kelly, Benedictine alumnus, presented an intriguing talk with props relating to the cycloid. Six students were initiated 18 March. **KME** member Betty Nickel gave the valedictory address for the class of '75. Kansas Gamma gave a farewell steak cookout 6 May for all senior members. Le Ann Fischer was named the 11th recipient of the Sister Helen Sullivan Scholarship given by Kansas Gamma. Other officers: Bruce Fisher, vice-president; Cathy Molini, secretary and treasurer; Sister Jo Ann Fellin, corresponding secretary; Jim Ewbank, faculty sponsor.

Kansas Epsilon, Fort Hays Kansas State College, Hays

Chapter President—Carol Hilt

35 actives

The annual spring banquet took place on 24 March. The members assisted with the activities at the annual math day. Other officers: Larry Hornbaker, vice-president; Mike Moyers, secretary and treasurer; Eugene Etter, corresponding secretary; Charles Votaw, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President—Claire Wagner

13 actives, 3 pledges

At the March meeting Mary Lou Brozena spoke on "The Josephus Problem". Maryland Alpha and Maryland Beta joined for the April meeting—Claire Wagner talked on "The Golden Section". Three new members were initiated at the May meeting at which time the topic "Movement in Four Directions" was explored by Cathy Brown. Other officers: Cathy Brown, vice-president and treasurer; Colleen Baum, secretary; Sister Marie Augustine Dowling, corresponding secretary and faculty sponsor.

Maryland Beta, Western Maryland College, Westminster

Chapter President—Barry Watson

25 actives

Other officers: Michael Kline, vice-president; Deborah Simmons, secretary; Virginia Bevans, treasurer; Carol Rouzer, historian; James Lightner, corresponding secretary; Robert Boner, faculty sponsor.

Michigan Beta, Central Michigan University, Mt. Pleasant

Chapter President—Alice Winter

25 actives

Monthly meetings were continued. An alumni breakfast was held during the fall 1975 homecoming festivities for former **KME** members. The chapter offers a tutorial service to students requesting help in mathematics. Other officers: Hans Heikel, vice-president; Joann Ostrowski, secretary; Laurie Baird, treasurer; Edward Whitmore, corresponding secretary and faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville

Chapter President—Don Hackman

18 actives

Senior members presented talks on mathematical topics at the monthly meetings. Fall and spring picnics, initiation of new members, a mathematical treasure hunt, and participation in a campus bowl comprised the varied local activities of the year. Other officers: Amy Barrow, vice-president; Debbie Reinker, secretary; Doyle Taylor, treasurer; Sam Lesseig, corresponding secretary; John Erhart, faculty sponsor.

Missouri Iota, Missouri Southern State College, Joplin

Chapter President—Cindy Carter

17 actives

Missouri Iota Chapter was installed on 5 May with seventeen charter members initiated at that time. Eddie Robinson, National Treasurer of **KME**, Southwest Missouri State University, Springfield, and Harold Thomas, Regional Director, Kansas State College of Pittsburg, were installing officers. A special project planned for the 1975-76 year is an investigation of career opportunities in mathematics. Other officers: Bob Dampier, vice-president; Sam Miller, secretary and treasurer; Sharon McBride, historian; Mary Elick, corresponding secretary; Charles Allen, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne

Chapter President—Joan Herfordt

31 actives

Monthly meetings were held by the chapter. Bill Pieper of Lyons, Nebraska was selected as outstanding freshman for the 1974-75 year on the basis of an examination given to selected mathematics students. Announcement of the award was made at the Honors Convocation and his name was engraved on a plaque listing honorees of this award. Other officers: Deborah Dubs, vice-president; Craig Hellwege, secretary and treasurer; Deanna Fcy, historian and reporter; Fred Webber, corresponding secretary; James Paige, faculty sponsor.

Nebraska Beta, Kearney State College, Kearney

Chapter President—Kay Marrow

22 actives

Meetings were held twice each month and help sessions were offered for mathematics students. Socially the chapter members enjoyed Halloween and Christmas parties and a spring picnic. Other officers; Donna Chramosta, vice-president; Linda Williams, secretary; Julie Mackey, treasurer, Charles Pickins, corresponding secretary; Randall Heckman, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President—Beverly Riese

50 actives

Other officers: Turner Laquer, vice-president; Bonnie Zimmer-

man, secretary; Gerald McNerney, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green

Chapter President—Julie Osmon

60 actives, 21 pledges

At the initiation banquet on 13 May Professor Cliff Long spoke on "Predator-Prey and Vibrating Membranes". W. Charles Holland and V. Frederick Rickey received the **KME** Excellence in Teaching Mathematics Award. Selection was made by a student committee. The annual chapter picnic included a softball game 29 May. Other officers: Debra Bird, vice-president; Diane Sparks, secretary; Jane Kline, treasurer; Waldemar Weber, corresponding secretary; Thomas Hern and Charles Holland, faculty sponsors.

Ohio Zeta, Muskingum College, New Concord

Chapter President—Warren Brown

32 actives

The spring mathematics department and **KME** banquet was held in conjunction with a talk given by a 1969 Muskingum graduate and Ohio Zeta member—Lottie Brown. Now with the NASA Goddard Space Flight Center, Greenbelt, Md., Lottie Brown spoke on the computer and satellite work she is doing in connection with ERTS 1 and 2 systems. Other officers: Debra Gutridge, vice-president; Sandra Mumaw, secretary and treasurer; James L. Smith, corresponding secretary and faculty sponsor.

**Oklahoma Gamma, Southwestern Oklahoma State University,
Weatherford**

Chapter President—Blaine Greenhagen

23 actives, 8 pledges

The chapter hosted two invited speakers at their monthly meetings—Dr. Jobe from Oklahoma State University and Dr. Conover from Texas Tech. On the tour of Western Electric in Oklahoma City students observed the processes used in the construction of complicated communication systems. Boating was the highlight of the spring picnic. Other officers: Joel Newcomb, vice-president; Jane Holmstrom, secretary; Michelle Klaassen, treasurer; Wayne F. Hayes, corresponding secretary; Don Prock, faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President—James Risko

30 actives, 15 pledges

Regular meetings were held during the year. Activities included an initiation dinner and attendance at the PCTM meeting in Shipensburg. Other officers: Jeffrey Coffin, vice-president; Roseanne Eroh, secretary; Rita Borillo, treasurer; Irving Hollingshead, corresponding secretary; E. Evans, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President—David Elko

56 actives

Regular monthly meetings included a presentation by Dr. Jack Shepler on statistics and a talk by Professor Henry Gould from West Virginia University about applied mathematics. At the annual banquet Dr. John Hoyt, retiring member of the mathematics department at Indiana University of Pennsylvania, spoke about some of his many experiences during his teaching career including 25 years at the U. S. Naval Academy at Annapolis. Other officers: Kristine Mangone, vice-president; Sharon Evans, secretary; Teresa Pavlekovsky, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Tennessee Delta, Carson-Newman College, Jefferson City

Chapter President—Kathy Allen

27 actives

The meetings featured faculty speakers and an honors project discussion by a mathematics major. Nine new members were initiated at the annual banquet on 12 April. Activities included hikes, bicycle trips, ice, cream parties, and a student-prepared spaghetti supper. Other officers: Doug Westberry, vice-president; Joy Rish, secretary; David Ford, treasurer; Denver R. Childress, corresponding secretary; Howard Chitwood, faculty sponsor.

Texas Beta, Southern Methodist University, Dallas

Chapter President—Albert Axe

Other officers: Larry Hamner and Gary Hall, vice-presidents; Laura Sand, secretary and treasurer; C. J. Pipes, corresponding secretary; Robert C. Davis, faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene

Chapter President—Tim Zukas

19 actives

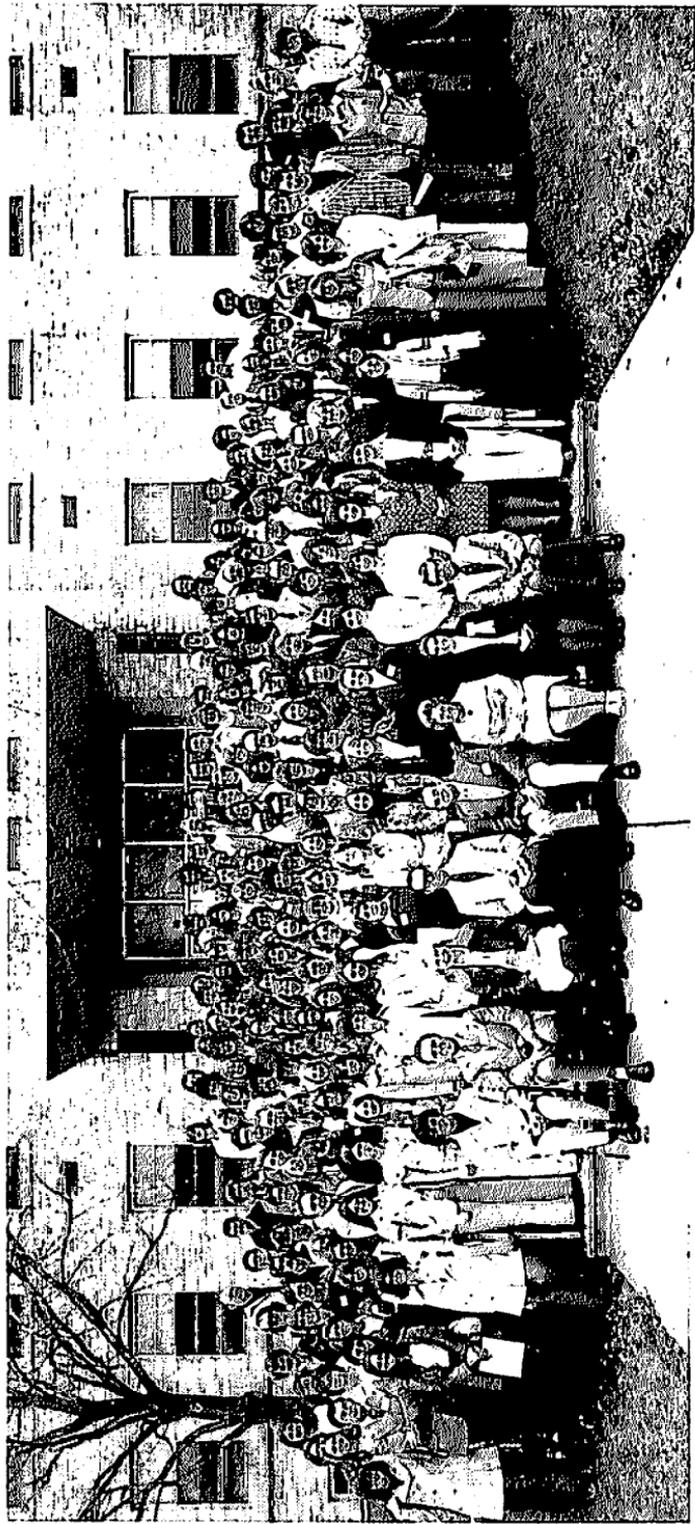
Professor Mike Reagan from Northeastern State College, Tahlequah, Oklahoma, installed the Texas Eta Chapter and initiated the charter members on 3 May. Other officers: Sharon Carver, vice-president; Susan Porter, secretary and treasurer; Anne Bentley, corresponding secretary; Edwin J. Hewett and Charles D. Robinson, faculty sponsors.

Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President—Jane Reinartz

7 actives, 4 pledges

Chapter activities during the spring were dominated by the preparations for the biennial convention held at Mount Mary. Three members were initiated 26 February. After the initiation ceremony and talks presented by the new initiates, Mary Bujack, Joy Rademacher, and Jane Reinartz, the chapter members had dinner with alumnae **KME** members in the area. Other officers: Joy Rademacher, vice-president; Cathy Borchert, secretary; Mary Bujack, treasurer; Sister Mary Petronia, corresponding secretary and faculty sponsor.



**Kappa Mu Epsilon Convention, 17-19 April 1975
Mount Mary College, Milwaukee, Wisconsin**