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On The Evaluation Of $\int \csc x \, dx$ And $\int \sec x \, dx$

HARSH LUTHAR

Student, Beloit College

$$\begin{aligned} 1. \quad \int \csc x \, dx &= \int \frac{1}{\sin x} \, dx \\ &= \int \frac{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \, dx \\ &= \int \frac{\frac{1}{2} \cos \frac{x}{2}}{\sin \frac{x}{2}} \, dx + \int \frac{\frac{1}{2} \sin \frac{x}{2}}{\cos \frac{x}{2}} \, dx \\ &= \ln \sin \frac{x}{2} - \ln \cos \frac{x}{2} + C \\ &= \ln \tan \frac{x}{2} + C \quad (A) \end{aligned}$$

$$\begin{aligned} 2. \quad \int \sec x \, dx &= \int \csc \left(\frac{\pi}{2} - x \right) dx \\ &= - \ln \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) + C \text{ by virtue of (A)} \\ &= \ln \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) + C \end{aligned}$$

A Graphical Representation of Roots of Quadratic Equations or The Derivation of An Unusual Curve

MARK A. HORNEY AND F. MAX STEIN

*Student and Faculty, Colorado State University**

1. *Introduction.* The graph of the *unusual curve* we derive is shown in Figure 1. It is tangent to the parabola $y = x^2/4$ at $(-8,16)$ and at the origin, and it lies below and to the left of the parabola otherwise. It passes through the points $(1,-2)$ and $(0,-16)$, and it has no real points to the right of the point $(1,-2)$.

This unusual curve arises from the consideration of the roots of the quadratic equation

$$x^2 + Ax + B = 0, \quad (1)$$

where A and B are real constants. In this equation A and B can be interpreted as the coordinates of a point in the plane. The roots of (1), say x_1 and x_2 , can then be interpreted as the coordinates of two more points, say (x_1, x_2) and (x_2, x_1) . These points will be called the *images* of the point (A, B) . We consider the locus of images for certain points and the mapping of certain curves and the possibility of invariant points under the transformation defined below.

2. *The Transformation.* If A and B are real, the roots of (1) are

$$x_1 = \frac{-A + \sqrt{A^2 - 4B}}{2} \quad \text{and} \quad x_2 = \frac{-A - \sqrt{A^2 - 4B}}{2} \quad (2)$$

These roots will be real if $A^2 - 4B \geq 0$ and imaginary if $A^2 - 4B < 0$. Thus to each point in the A, B -plane there corresponds an equation of the form (1) which has roots x_1, x_2 under the *transformation*

$$x_1, x_2 = \frac{-A \pm \sqrt{A^2 - 4B}}{2} \quad (3)$$

*Prepared in an Undergraduate Research Participation Program at Colorado State University by Mr. Horney under the direction of Professor Stein.

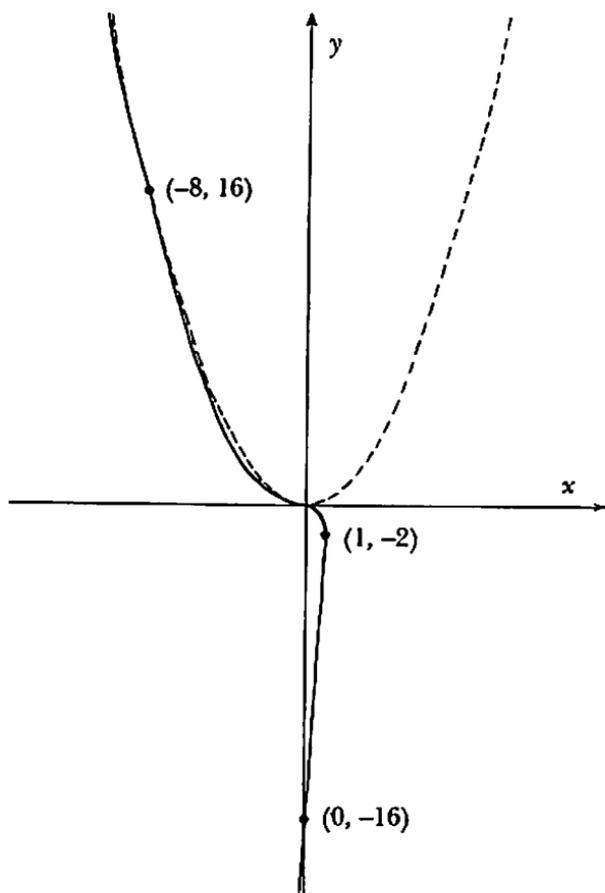


FIGURE 1

The transformation (3) leads to the first question: Which part of the A, B -plane corresponds to equations such that $A^2 - 4B \geq 0$? That is, which equations (1) have real roots?

Since the graph of the equation $A^2 - 4B = 0$, or $B = A^2/4$, is a parabola with vertex at the origin, points on or below the parabola satisfy the inequality $A^2 - 4B \geq 0$, while points above the parabola do not. This result is stated as a lemma.

LEMMA. *Points on or below the parabola $B = A^2/4$ in the A, B -plane map into real points, while points above the parabola map into imaginary points.*

Our subsequent discussion is concerned with the real images of points; i.e., in this paper we do not examine the images of points lying inside the parabola.

3. *The Images.* When x_1 and x_2 in (2) are real numbers we can use them to define two new points

$$(x_1, x_2) \text{ and } (x_2, x_1) \quad (4)$$

which we call the *images* of the point (A, B) . The line defined by the two images is called a *root line*. For future use the vertical ray extending downward, with origin of the ray on the parabola $B = A^2/4$, and which passes through the point $(A_0, 0)$ will be called the *half-line* $A = A_0$.

We now ask: More precisely where are the real images of points in the A, B -plane under the transformation (3)? The question is answered by the following theorem.

THEOREM 1. *For the point (A_0, B_0) in the A, B -plane:*

(i) *If $A_0^2 - 4B_0 > 0$, the images (x_1, x_2) and (x_2, x_1) of (A_0, B_0) define the root line*

$$B = -A - A_0 \quad (5)$$

(ii) *If $A_0^2 - 4B_0 = 0$, the point (A_0, B_0) lies on the parabola $B = A^2/4$ and has images $(-A_0/2, -A_0/2)$.*

(iii) *The locus of images of points (A_0, B_0) which lie on the half-line $A = A_0$ is the line $B = -A - A_0$.*

Proof: For part (i) the equation of the line defined by (x_1, x_2) and (x_2, x_1) is

$$B - x_2 = \frac{x_2 - x_1}{x_1 - x_2} (A - x_1) = -A + x_1,$$

or

$$B = -A + x_1 + x_2 = -A + \frac{-A_0 + \sqrt{A_0^2 - 4B_0}}{2} + \frac{-A_0 - \sqrt{A_0^2 - 4B_0}}{2},$$

or simply

$$B = -A - A_0$$

For part (ii), since $A_0^2 - 4B_0 = 0$, then (4) is simply the single point $(-A_0/2, -A_0/2)$.

For part (iii) we observe that since B_0 does not occur in (5), the root line of the point (A_0, B_0) , all points having the same abscissa have their images on the same root line.

We now make some observations regarding the images of points (A_0, B_0) , see Figure 2.

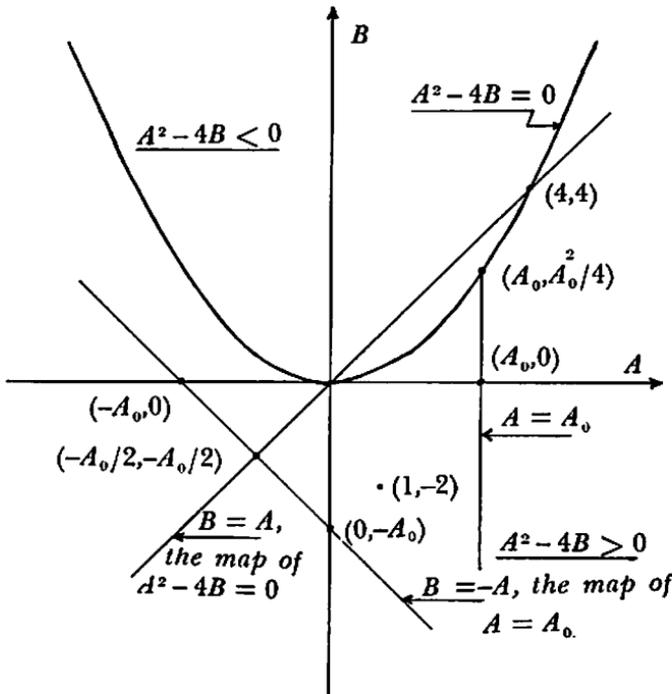


FIGURE 2

(a) The images of points (A_0, B_0) which lie on the parabola $B = A^2/4$ lie on the line $B = A$, which we call the *symmetric line*. This follows since both coordinates of the image of (A_0, B_0) are the same by Theorem 1 (i).

(b) All root lines have slope -1 , see (5), and are thus perpendicular to the symmetric line in (a).

(c) The images of a point on the A -axis, say $(A_0, 0)$, are $(-A_0, 0)$ and $(0, -A_0)$. These points are obtained from the intercepts of the root line (5) or from the map of $x^2 + A_0x = 0$, an equation with roots 0 and $-A_0$.

(d) As a point varies downward down a half-line, the images of the point diverge along the root line $B = -A - A_0$ from the symmetric point $(-A_0/2, -A_0/2)$.

Using the preceding observations it is not difficult to determine that points in the vertical crosshatched regions in Figure 3 map into points in the corresponding diagonal crosshatched regions.

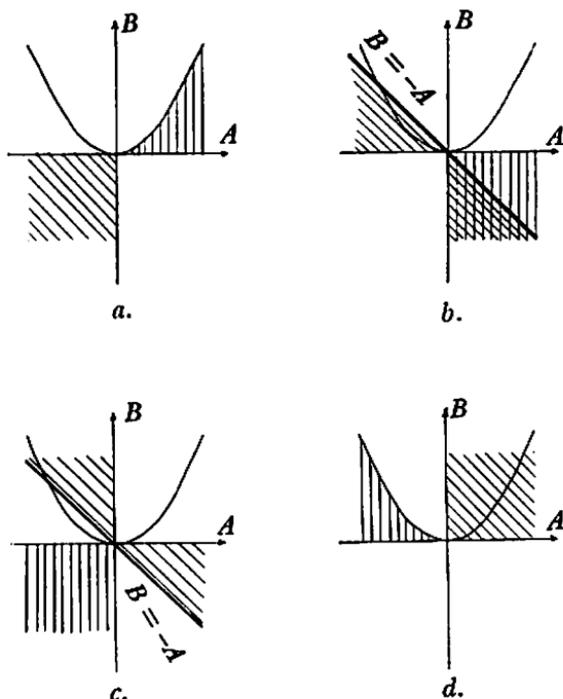


FIGURE 3

We repeat that points inside the parabola have imaginary images. Also we observe that the only points which are candidates to be their own images lie in the doubly crosshatched region in Figure 3 (b).

4. *Invariant Points.* Are there any points which are their own images?

For a point to be its own image requires that

$$A = \frac{-A + \sqrt{A^2 - 4B}}{2} \quad \text{and} \quad B = \frac{-A - \sqrt{A^2 - 4B}}{2},$$

or vice versa. If we add these equations we get

$$A + B = -A, \text{ or } 2A = -B. \quad (6)$$

By isolating the radical in the first equation and squaring we get

$$9A^2 = A^2 - 4B, \text{ or } 2A^2 = -B. \quad (7)$$

From (6) and (7) we get that (0,0) and (1,-2) are candidates for *invariant points*. Since $x^2 = 0$ has only the one image (0,0), the origin is an invariant point. However,

$$x^2 + x - 2 = 0$$

has (1,-2) and (-2,1) as images. Thus we call (1,-2) a *semi-invariant point*. Incidentally the point (-2,1) is on the parabola $B = A^2/4$, and the root line through these two points is tangent to the parabola at (-2,1).

5. *Predecessor Points.* Since there are *two* images for each point in general, complications arise when we examine sequences of points under the transformation (3). Thus we choose to go backwards and find the quadratic equation which has the given point as a root. From the coefficients in the resulting quadratic equation we get the *predecessor point* of the given point (A,B).

THEOREM 2. *The predecessor point of the point (A,B) is the point*

$$(-A - B, AB). \quad (8)$$

Proof: If (A,B) is the given point, it came from the equation

$$(x - A)(x - B) = 0 \text{ or } x^2 - (A + B)x + AB = 0.$$

The predecessor point is thus $(-A - B, AB)$, and it is, of course, unique.

Are there any *cyclic points*? That is, which points are their own predecessors? For a cyclic point we need

$$A = -A - B \text{ and } B = AB.$$

Again we get that $B = 0$, and hence $A = 0$, and also $A = 1$, and hence $B = -2$, as expected from the discussion of invariant points.

Are there any *bi-cycle points*? That is, which points are such that the predecessor of the predecessor of a point is the point itself? For a bi-cycle (or bicycle) point we need

$$A = A + B - AB \text{ and } B = -(A + B)AB.$$

If $B = 0$, then A is arbitrary. If $B \neq 0$, then $A = 1$ and $B = -2$, again as expected. Note that any cyclic point is a bicycle point. Also note that any point on the A -axis is a bicycle point.

The case of tricycle points or multicycle points can be examined in a similar manner.

6. *The Unusual Curve.* Finally we get to the *unusual curve*. We ask: What are the predecessor points for points on the parabola $B = A^2/4$ or $A^2 = 4B$?

The roots of (1) yield points (4). For the first point to lie on the parabola we must have

$$\left(\frac{-A + \sqrt{A^2 - 4B}}{2} \right)^2 = 4 \left(\frac{-A + \sqrt{A^2 - 4B}}{2} \right). \quad (9)$$

By expanding, gathering terms, and simplifying this equation reduces to

$$(A - 4)\sqrt{A^2 - 4B} = 2B - 4A - A^2.$$

Upon squaring both sides and simplifying we get

$$B^2 + B(16 - 12A) + 4A^3 = 0$$

or

$$B = 6A - 8 \pm 2\sqrt{16 - 24A + 9A^2 - A^3} \quad (10)$$

The graph of this equation is the *unusual curve* given in Figure 1 and Figure 4. The same result is obtained if the second point is used in (9).

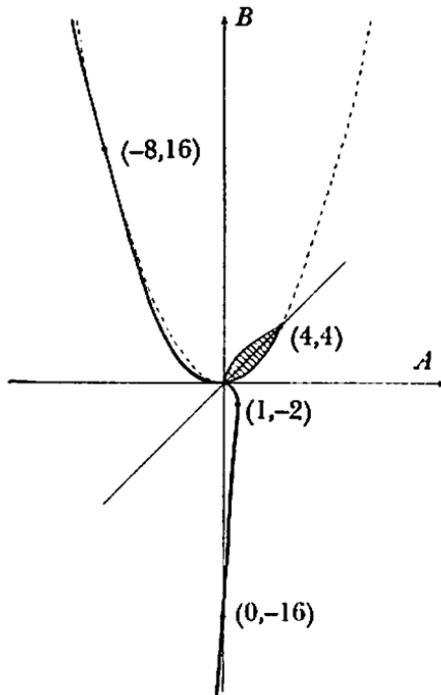


FIGURE 4

Observations:

- (a) The radicand in (10) is zero for $A = 1$ and is negative for $A > 1$. Thus there are no real points to the right of $A = 1$.
- (b) Both the parabola and the unusual curve have equal slopes at $(-8,16)$ and the origin.
- (c) Between the two points given in (b) the curve (10) lies below the parabola (but not far below).

7. *Double Images.* The images of images, called *double images*, of points are of interest. Recall that points inside the parabola have imaginary images. Thus:

- (a) The double images of points below the right half of the parabola and to the right of the unusual curve (10) are all real.

(b) Points to the left of the unusual curve (10) have one image point inside the parabola and one image point outside. Thus half the double image points are real and half imaginary.

(c) Points in the second quadrant bounded above by the parabola and below by the unusual curve and between the points $(-8,16)$ and $(0,0)$ have double image points that are all imaginary. The images lie in the crosshatched region shown in Figure 4, and thus the double images are imaginary.

(d) Points above $(-8,16)$ and between the parabola and the unusual curve fall in category (a) above; they thus all have real double image points.

8. Under the transformation (3) other interesting curves can be obtained. E.g., points on the line $A = B$ transformed by (3) have a real locus which is a hyperbola. The possibility of the convergence of a sequence of predecessor points is a topic which remains to be considered.

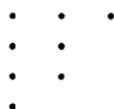
On Self-Dual Partitions

HUGO S. SUN

Faculty, California State University, Fresno

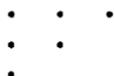
Let n be a positive integer. $n = a_1 + a_2 + \dots + a_k$ is called a partition of n if each a_i is a positive integer, $0 < a_i \leq n$, and $1 \leq k \leq n$. For instance, the partitions for 3 are 3, $1 + 2$, and $1 + 1 + 1$. Let $P(n)$ denote the number of partitions of the number n . It is easy to see that $P(1) = 1$, $P(2) = 2$, $P(3) = 3$, $P(4) = 5$, $P(5) = 7$, etc.

Let $n = a_1 + a_2 + \dots + a_k$ be a partition. The diagram with a_1 points on the first row, a_2 points on the second row, \dots , a_k points on the k th row is called the *diagram* of the partition. For example, the diagram of $8 = 3 + 2 + 2 + 1$ is



The partition $n = a_1 + \dots + a_k$ is called the *dual* of $n = b_1 + \dots + b_m$ if the rows of one diagram equal to the columns of the other. For example, $8 = 4 + 3 + 1$ is the dual of $8 = 3 + 2 + 2 + 1$.

A partition is called *self-dual* if its dual is itself, i.e., its diagram is symmetric about the diagonal. For example, $6 = 3 + 2 + 1$ is self-dual; its diagram is



THEOREM 1. *Any positive integer except 2 has a self-dual partition.*

Proof: $1 = 1$, $3 = 2 + 1$, $4 = 2 + 2$, $5 = 3 + 1 + 1$, $6 = 3 + 2 + 1$, $7 = 4 + 1 + 1 + 1$, $8 = 4 + 2 + 1 + 1$ are the self-dual partitions for positive integers less than 9.

For $n \geq 9$. If n is even, let $k = \frac{n-4}{2}$. Then $n = (k+2) + 2 +$

$\underbrace{(1 + \dots + 1)}_{k \text{ times}}$ is self-dual.

If n is odd, then $n-9$ is even. Let $\frac{n-9}{2} = k$. $n = (k+3) + 3 + 3 + \underbrace{(1 + \dots + 1)}_{k \text{ times}}$ is self-dual.

Denote by $P_s(m)$ the number of partitions of m with each summand $\leq s$. For example, $P_2(4) = 3$, $P_2(5) = 3$.

THEOREM 2. *If n is even, the number of self-dual partitions is*

$\sum_{i=1}^k P_{2^i} \left(\frac{n - (2i)^2}{2} \right)$, where k is the largest integer for which

$(2k)^2 \leq n$. *If n is odd, the number of self-dual partitions is*

$\sum_{i=1}^k P_{2^i-1} \left(\frac{n - (2i-1)^2}{2} \right)$, where k is the largest integer for

which $(2k-1)^2 \leq n$.

Proof: We first observe that a square of size 1 or 2^2 or 3^2 , etc., may be extracted from a corner of the diagram of a partition. For each self-dual partition of an even number, only a square of even size may be extracted. The number of self-dual partitions containing the largest square of size $(2i)^2$ is $P_{2^i} \left(\frac{n - (2i)^2}{2} \right)$, for example, for $n = 10$, the self-dual partitions are $5 + 2 + 1 + 1 + 1$ and $4 + 3 + 2 + 1$, the number is precisely $P_2(3)$. Summing up the number of self-dual partitions containing the maximal squares of different sizes, we have $\sum_{i=1}^k P_{2^i} \left(\frac{n - (2i)^2}{2} \right)$.

If n is odd, the diagram cannot contain a square of even size, and the number of self-dual partitions containing a maximal square of size $2i-1$ is $P_{2^i-1} \left(\frac{n - (2i-1)^2}{2} \right)$. Summing up according to squares

of different sizes we have $\sum_{i=1}^k P_{2^i-1} \left(\frac{n - (2i-1)^2}{2} \right)$.

In Theorem 2 we count the number of self-dual partitions with the help of partitions with each summand less than a certain fixed number. We can also use the number of self-dual partitions to count the number of partitions. For this end, we denote by $P(n)$ the number of self-dual partitions of n . With the same counting technique, we can prove

$$\text{THEOREM 3. } P(n) = \sum_{i=1}^k \sum_{s=0}^{n-i^2} P_i(s) P_i(n-i^2-s) - P(n).$$

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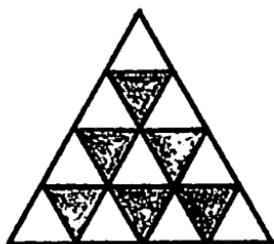
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Triangles and Triangular Numbers: A Geometric Counting Problem

DAVID R. DUNCAN AND BONNIE H. LITWILLER

Faculty, University of Northern Iowa

Consider the following geometric counting problem: How many triangles appear in Figure 1? This triangle is a 4-unit equilateral triangle since it is 4 units per side.



4-unit equilateral triangle

FIGURE 1

A common immediate reaction is to count the small equilateral triangles of side 1-unit to achieve a total of 16 small triangles. The next common reaction is to count the large 4-unit equilateral triangle to obtain 17 triangles as the total; however, more careful inspection reveals that there are still more triangles.

Figure 2 depicts examples of various equilateral triangles. Clearly this problem is more complex than one might think at first glance. Therefore, a counting strategy needs to be developed which would apply to any n - unit equilateral triangle. To aid the counting process, two types of triangles need to be distinguished, those whose vertices point upward (type U) and those whose vertices point downward (type D). Here and henceforth "triangle" will mean equilateral triangle.

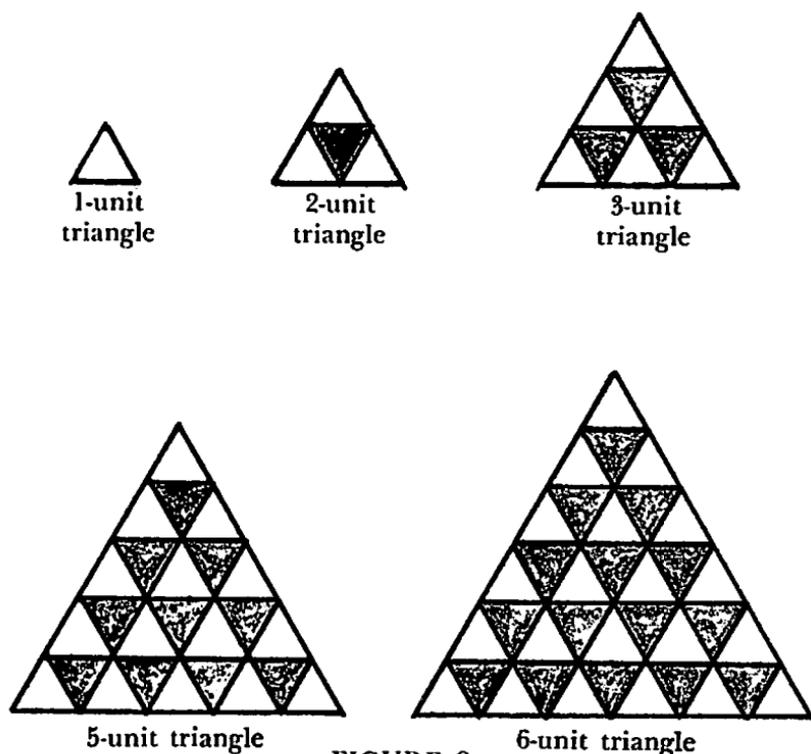


FIGURE 2

First consider the 1-unit triangle: clearly there is but one triangle formed and it is type *U*. By carefully studying the triangles in Figure 1 and Figure 2, Table 1 can be constructed.

TABLE 1.

No. of units per side in the original triangle	Number of triangles of side <i>n</i>						Total number of triangles
	<i>n</i> = 1 <i>U</i> <i>D</i>	<i>n</i> = 2 <i>U</i> <i>D</i>	<i>n</i> = 3 <i>U</i> <i>D</i>	<i>n</i> = 4 <i>U</i> <i>D</i>	<i>n</i> = 5 <i>U</i> <i>D</i>	<i>n</i> = 6 <i>U</i> <i>D</i>	
1	1 0						1
2	3 1	1 0					5
3	6 3	3 0	1 0				13
4	10 6	6 1	3 0	1 0			27
5	15 10	10 3	6 0	3 0	1 0		48
6	21 15	15 6	10 1	6 0	3 0	1 0	78

The informed reader will recognize the numbers 1, 3, 6, 10, 15, and 21 as triangular numbers. These are numbers which can be arranged in the form of an equilateral triangle. Figure 3 illustrates a geometric pattern for the first five triangular numbers. The relation between these triangular patterns and the geometric figure, the triangle, can be seen.

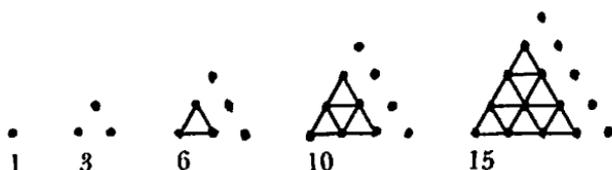


FIGURE 3

Let S_i be the total number of triangles in an i -unit triangle and

let T_i be the i th triangular number ($T_i = \sum_{j=1}^i j = \frac{i(i+1)}{2}$). Using

Table 1, we can write

$$S_1 = 1 = T_1$$

$$S_2 = 3 + 1 + 1 = T_2 + 2T_1$$

$$S_3 = 6 + 3 + 3 + 1 = 6 + 2(3) + 1 = T_3 + 2T_2 + T_1$$

$$\begin{aligned} S_4 &= 10 + 6 + 6 + 1 + 3 + 1 = 10 + 2(6) + 3 + 2(1) \\ &= T_4 + 2T_3 + T_2 + 2T_1 \end{aligned}$$

$$\begin{aligned} S_5 &= 15 + 10 + 10 + 3 + 6 + 3 + 1 = 15 + 2(10) + 6 + 2(3) + 1 \\ &= T_5 + 2T_4 + T_3 + 2T_2 + T_1 \end{aligned}$$

$$\begin{aligned} S_6 &= 21 + 15 + 15 + 6 + 10 + 1 + 6 + 3 + 1 \\ &= 21 + 2(15) + 10 + 2(6) + 3 + 2(1) \\ &= T_6 + 2T_5 + T_4 + 2T_3 + T_2 + 2T_1 \end{aligned}$$

Observe that the number of terms is the same as the number of units per side of the original triangle. We make these two conjectures:

$$\text{If } n \text{ is odd, } S_n = \sum_{i=1}^n c_i T_i \text{ where } c_i = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 2 & \text{if } i \text{ is even} \end{cases}$$

$$\text{If } n \text{ is even, } S_n = \sum_{i=1}^n d_i T_i \text{ where } d_i = \begin{cases} 1 & \text{if } i \text{ is even} \\ 2 & \text{if } i \text{ is odd} \end{cases}$$

The proof of these two conjectures is by mathematical induction on n .

$$\text{If } n = 1 \text{ by inspection, } S_1 = T_1 = \sum_{i=1}^1 c_i T_i = c_1 T_1.$$

Assume that the conjecture is true for $n = k$, then prove the conjecture is true for $n = k + 1$.

Case I. k is odd.

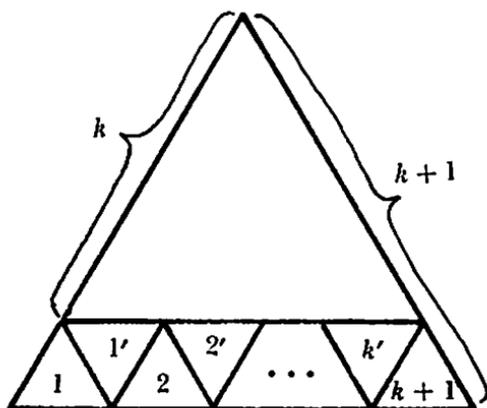


FIGURE 4

First, count the number of triangles in the top k -unit triangle; by the induction hypothesis this sum is

$$S_k = \sum_{i=1}^k c_i T_i \text{ where } c_i = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 2 & \text{if } i \text{ is even} \end{cases}$$

Next count the number of triangles contained all or in part in the "strip" which was added to the base of the original k -unit triangle (Figure 4). These may be organized in the following way:

First count the triangles of type U using triangles in the bottom "strip". By inspection, the number of 1-unit triangles is $k + 1$ and the number of 2-unit triangles is k . In general, the number of i -unit triangles is $(k+2)-i$.

Thus: The number of 1-unit triangles is $k+1$.

The number of 2-unit triangles is k .

The number of 3-unit triangles is $k-1$.

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The number of $k+1$ -unit triangles is 1.

There are $\sum_{i=1}^{k+1} i = T_{k+1}$ triangles of type U using triangles in the bottom "strip."

Next, count the triangles of type D using triangles in the bottom "strip". To accomplish this, notice that any such triangle of type D contains exactly one of triangles $1', 2', \dots, k'$ (Figure 4). Exactly one such triangle contains triangle $1'$; similarly one such triangle contains triangle k' . Two triangles contain triangle $2'$ (of, respectively 1-unit and 2-units); similarly two triangles contain triangle $(k-1)'$.

In general, if $0 < i \leq \frac{k-1}{2}$, exactly i triangles contain triangle i' (of respectively, 1-unit, 2-units, . . . , i -units). By symmetry, ex-

actly i triangles contain triangle $(k-i+1)'$. If $i = \frac{k+1}{2}$, exactly

$\frac{k+1}{2}$ triangles contain triangle $\left(\frac{k+1}{2}\right)'$.

Note that this is the "middle" triangle in the bottom "strip". Thus the number of triangles of type D using triangles in the bottom "strip" is :

$$1 + 1 + 2 + 2 + 3 + 3 + \dots + \left(\frac{k-1}{2}\right) + \left(\frac{k-1}{2}\right) + \left(\frac{k+1}{2}\right)$$

$$= 2\left(1 + 2 + 3 + \dots + \left(\frac{k-1}{2}\right)\right) + \left(\frac{k+1}{2}\right) = 2 T_{\frac{k-1}{2}} + \left(\frac{k+1}{2}\right)$$

Recall that we wish to show that

$$S_{k+1} = \sum_{i=1}^{k+1} d_i T_i \text{ where } d_i = \begin{cases} 1 & \text{if } i \text{ is even} \\ 2 & \text{if } i \text{ is odd} \end{cases}$$

(since $k+1$ is even)

We know:

$\underbrace{S_{k+1}}$	=	$\underbrace{S_k}$	+	$\underbrace{T_{k+1}}$	+	$\underbrace{\frac{2T_{\frac{k-1}{2}} + \frac{k+1}{2}}}$.
Number of triangles in $k+1$ unit triangle		number of triangles in original k -unit triangle		number of triangles of type U contained all or in part in the bottom strip		number of triangles of type D contained all or in part in the bottom strip	

We also know that

$$c_i = d_{i+1} \text{ and } T_i = T_{i+1} - (i+1).$$

$$\text{Furthermore } 2T_{\frac{k-1}{2}} + \frac{k+1}{2} = 2 \left(\frac{(k-1)}{2} \right) \left(\frac{k+1}{2} \right) + \frac{k+1}{2}$$

$$= \left(\frac{k+1}{2} \right)^2 = 1+3+5 + \dots + k$$

Using summation notation we must show:

$$\sum_{i=1}^{k+1} d_i T_i = \sum_{i=1}^k c_i T_i + T_{k+1} + 2T_{\frac{k-1}{2}} + \frac{k+1}{2}$$

$$= \sum_{i=1}^k c_i T_i + \sum_{i=1}^{k+1} i + 1+3+5 + \dots + k$$

$$\begin{aligned}
&= \sum_{i=1}^k c_i T_i + \sum_{i=1}^{k+1} d_i(i) \\
&= \sum_{i=1}^k d_{i+1} \left(T_{i+1} - (i+1) \right) + \sum_{i=1}^{k+1} d_i(i) \\
&= d_1(T_1-1) + \sum_{i=1}^k d_{i+1} \left(T_{i+1} - (i+1) \right) + \sum_{i=1}^{k+1} d_i(i) \\
&= \sum_{i=1}^{k+1} d_i(T_i - i) + \sum_{i=1}^{k+1} d_i(i) \\
&= \sum_{i=1}^{k+1} d_i T_i .
\end{aligned}$$

Case I of the inductive step is thus complete.

Case II. k is even

Again use Figure 4 as a reference. We can conclude by the inductive hypothesis that there are S_k triangles in the top k -unit triangle where, in this case, $S_k = \sum_{i=1}^k d_i T_i$ where $d_i = \begin{cases} 1 & \text{if } i \text{ is even.} \\ 2 & \text{if } i \text{ is odd} \end{cases}$

Again, we must count the number of triangles using triangles in the "strip" which was added to the base of the original k -unit triangle. These may be organized, as in Case I, in the following way: First count the triangles of type U . The procedure used in Case I applies here since the parity of k was not an issue in this counting process. Thus there are $\sum_{i=1}^{k+1} i = T_{k+1}$ triangles of type U using triangles in the bottom "strip".

Next count the triangles of type D using triangles in the bottom "strip". To accomplish this, note that any such triangle of type D contains exactly one of the triangles $1', 2', \dots, k'$. Exactly one such triangle contains triangle $1'$; similarly, exactly one such triangle contains triangle k' . Two triangles contain triangle $2'$ (of respectively, 1-unit and 2-units); similarly, two triangles contain triangle $(k-1)'$. In general, the same pattern of counting holds as in Case I, except that there is no "middle" triangle in the bottom "strip". Thus the number of such triangles of type D is $(1 + 1) + (2 + 2) +$

$$\dots + \left(\frac{k}{2} + \frac{k}{2} \right) = 2(1 + 2 + 3 + \dots + \frac{k}{2}) = 2T_{\frac{k}{2}}.$$

We wish to show: $S_{k+1} = \sum_{i=1}^{k+1} c_i T_i$ where $c_i = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 2 & \text{if } i \text{ is even} \end{cases}$

We know: $S_{k+1} = \underbrace{S_k}_{\substack{\text{number of} \\ \text{triangles in} \\ k+1\text{-unit} \\ \text{triangle}}} + \underbrace{T_{k+1}}_{\substack{\text{number of} \\ \text{triangles} \\ \text{of type } U \\ \text{contained all} \\ \text{or in part} \\ \text{in the bottom} \\ \text{strip}}} + \underbrace{2T_{\frac{k}{2}}}_{\substack{\text{number of} \\ \text{triangles} \\ \text{of type } D \\ \text{contained} \\ \text{all or in} \\ \text{part in the} \\ \text{bottom strip}}}$

$$\begin{aligned} \text{Also we know } T_{k+1} + 2T_{\frac{k}{2}} &= \sum_{i=1}^{k+1} i + 2(1 + 2 + 3 + \dots + \frac{k}{2}) \\ &= \sum_{i=1}^{k+1} i + 2 + 4 + 6 + \dots + k \\ &= \sum_{i=1}^{k+1} c_i(i) \end{aligned}$$

Using summation notation we must show:

$$\begin{aligned} \sum_{i=1}^k c_i T_i &= \sum_{i=1}^k d_i T_i + T_{k+1} + 2T_{\frac{k}{2}} \\ &= \sum_{i=1}^k d_i T_i + \sum_{i=1}^{k+1} c_i(i) \\ &= c_1(T_1-1) + \sum_{i=1}^k c_{i-1} (T_{i+1} - (i+1)) + \sum_{i=1}^{k+1} c_i(i) \\ &= \sum_{i=1}^{k+1} c_i(T_{i-i}) + \sum_{i=1}^{k+1} c_i(i) \\ &= \sum_{i=1}^{k+1} c_i T_i. \end{aligned}$$

Case II of the inductive step is thus complete.

The induction is thus completed; we have shown our two conjectures are true. If S_n is the number of triangles contained in an n -unit equilateral triangle:

If n is odd, $S_n = \sum_{i=1}^n c_i T_i$ where $c_i = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 2 & \text{if } i \text{ is even} \end{cases}$.

If n is even, $S_n = \sum_{i=1}^n d_i T_i$ where $d_i = \begin{cases} 1 & \text{if } i \text{ is even} \\ 2 & \text{if } i \text{ is odd} \end{cases}$.

The reader may consider the analogous problem in 3-dimensions for an n -unit tetrahedron. How many tetrahedrons can be located in such a figure?

Nature Of The Roots of $x^3 + 3Hx + G = 0$

R. S. LUTHAR

Faculty, University of Wisconsin, Janesville

I would like to share the following method for determining the nature of the roots of $x^3 + 3Hx + G = 0$ with the readers of THE PENTAGON. I believe this method is not widely known in this country, if known at all.

Consider the two functions $y = x^3 + 3Hx$ and $y = -G$. It is clear that the x -coordinates of the points of intersection of the graphs of these two functions determine the roots of the given equation. Whereas the graph of $y = -G$ is a straight line parallel to the x -axis, the graph of $x^3 + 3Hx$ is a cubic curve that resembles either Figure 1 (when $H > 0$) with no bend or Figure 2 (when $H < 0$) with two bends, one at $x = (-H)^{1/2}$ and the other at $x = -(-H)^{1/2}$, the absolute value of the function at both of these two points being equal to $2(-H)^{3/2}$. Let us now make the following observations:

(1) When $H > 0$. Clearly, the graph of $y = -G$ will intersect the graph in Figure 1 only in one point. Thus, the equation $y = x^3 + 3Hx + G = 0$, in this case, will have one real and two imaginary roots.

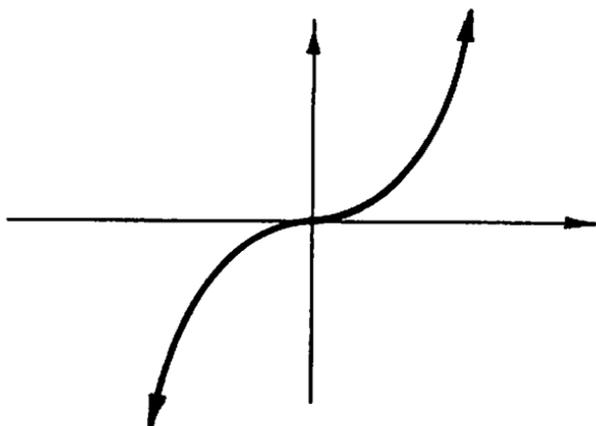


FIGURE 1

(2) When $H < 0$. In this case the graph of $y = -G$ will intersect the graph in Figure 2 in one point, two points (one of them being a double point), or three points according as $2(-H)^{3/2} \begin{cases} \equiv \\ > \\ < \end{cases} |-G|$, that is, $4H^3 + G^2 \begin{cases} \equiv \\ > \\ < \end{cases} 0$. Thus the equation $x^3 + 3Hx + G = 0$ has one real and two imaginary roots, two real roots (one of them being a double root), or three distinct real roots according as $4H^3 + G^2 \begin{cases} \equiv \\ > \\ < \end{cases} 0$.

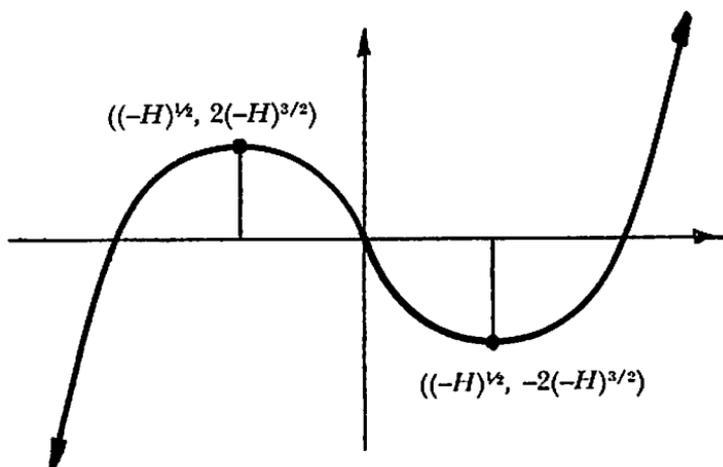


FIGURE 2

It may be noticed that (1) is included in (2), for, $H > 0$ implies $4H^3 + G^2 > 0$. Therefore, we may finally conclude that the equation

$$x^3 + 3Hx + G = 0$$

(a) Has one real and two imaginary roots if

$$4H^3 + G^2 > 0$$

(b) Has two real roots (one of them being a double root) if

$$4H^3 + G^2 = 0$$

(c) Has three distinct real roots if

$$4H^3 + G^2 < 0.$$

It may be remarked that the above graphical way of discussing the roots of a reduced cubic equation is applicable, in principle, to every equation. In practice, however, it will not help much where equations of higher order are involved. The reader though may wish to test it against the quadratic equation $x^2 + bx + c = 0$ by considering the functions $y = x^2 + bx$ and $y = -c$.

The Book Shelf

EDITED BY O. OSCAR BECK

This department of THE PENTAGON brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. O. Oscar Beck, Department of Mathematics, University of North Alabama, Florence, Alabama 35630.

Topics in Pre-Calculus Mathematics and *Topics in College Algebra*, Donald R. Burluson, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1974, 484 pp. and 338 pp., \$10.50 and \$8.95.

As stated in the preface, a student with two successful years of high school algebra should find these books quite readable. The author has an intuitive and common-sense approach in his development of the topics. He gives his readers the history behind many of the concepts being presented, and occasionally dots his prose with bits of humor.

The reader should pay particular attention to the books' numerous examples, for the author uses them as a vehicle for developing the theory. Complete details are given in all of the examples and proofs. In addition, the author points out commonly-made errors which the student should avoid.

In summary, I highly recommend these books as pre-calculus (college algebra) texts. Their readability and clarity of detail should make them accessible to most students — even those with a weak high school background.

The following typographical errors were noted: p. 49, line 25 replace " $(a+b)$ " by " $(a+b)'$ "; and p. 87, line 12 replace "0,]" by "[0.2]".

Jack C. Sharp
Western Carolina University

Arithmetics, Robert L. Johnson and Charles R. McNerney, Macmillan Publishing Co., New York, 1974, 490pp., \$9.95.

In the words of the authors this book was written with three major areas of application in mind. First, teachers of mathematics courses for elementary education students could use the book as a resource book. Second, it was intended that elementary teachers use it as an "on the teacher's desk" resource book. Third, the book was intended to be a resource for in-service workshops in mathematics for teachers.

The authors admit that there are a large number of such books already in existence, but, they make the claim that their goals in writing the books are different, essentially, "by using a rhetoric throughout . . . which helps the teacher to identify with mathematics, to realize that children continually recreate mathematics concepts, and to see relationships between what he should know and what he will teach."

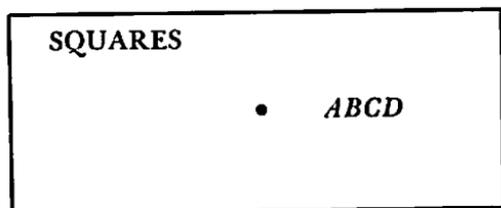
The task for this reviewer was to assess how well the authors succeeded in attaining these goals. The reviewer has no quarrel with the goals as stated. Their interpretation and implementation is another matter, however.

To this reviewer the strength of this book lies in the exercises. Certainly, a good course for elementary school teachers could be devised by treating the exercises as a source of problematic material. The course would then be directed toward the solution of these exercises as problems. This direction seems desirable for two reasons: (1) interpreting exercise sets as mere practice in applying the topics from the previous expository section seems to do harm to the mathematical objectives, and (2) at times, the exercise sets or parts of them seem divorced from the expository material (e.g., Exercises (Section 10.8), in which basic geometric constructions were related to geometric reflections with no previous preparation). Statement (1) above refers to the opportunity afforded the student to do some good mathematical investigating by starting with the Exercise sets interpreted as problems. A concentrated effort on the Exercise sets would define a better course for elementary school teachers than would be possible with many other books of the same type.

In implementing the stated goals for the book, the reviewer differs with the authors on a number of points. These differences will be summarized chapter by chapter.

In Chapter One the use of truth tables to explain the logical connectives for statements is not particularly enlightening for a beginner. These definitions seem more reasonable when explained with Venn Diagrams. A truth table is essentially a computational device and is not rich pedagogically. The use and interpretation of variables was another problem in Chapter One. In the sentence $17 - \Delta = 9$, is " Δ " a variable? In one Exercise set, instructions were given to replace Δ with name of a number implying the use of " Δ " as a variable. Then, what purpose is served by the use of " Δ ", " \square ", etc? Another confusion arises when the elements of a set are represented with small letters. In the set $C = \{x,y\}$, are " x " and " y " distinct letter objects or are they variables? The interpretation as variables is more defensible mathematically. Interpreting the letters as unique objects leads to nothing interesting or useful mathematically and can only confuse a beginner.

Chapter Two is devoted to mathematical thinking. Primarily, it is an exposition of logical inference patterns. This can be done with Venn Diagrams of the right kind, i.e., those that illustrate class relationships. Depicting the statement " $ABCD$ is a square" with the following Venn Diagram contributes nothing to understanding. (p. 76)



On the other hand, the use of Venn Diagrams in the section on negation of quantifiers is most enlightening.

Chapter Three concerns numbers and numeration. The treatment is standard using correspondences and equivalence classes. It ignores the counting aspects which seems unfortunate for the preparation of elementary school teachers. The statement in the section "How Chil-

dren Learn to Count," to the effect that children beginning school have a rote level ability to count, ignores too many teaching possibilities, (*italics are the authors'*). There are a couple of printing errors in this chapter: {&, *, #} is matched with "4" in Figure 3-3 on page 101, and on page 117 we find $7^2 = 491$. Finally, on page 120, in working with different number bases, though it is true that, 'five times six is 36 in the base eight system', it is not true that, 'five times six is thirty-six in the base eight system', because the term *thirty-six* is appropriate only in a *decimal* system.

Chapter Four is directed toward the system of whole numbers. The development is standard and well done. However, this reviewer objects to the implicit idea that this is the only right and proper pedagogical development, e.g., multiplication defined in terms of the Cartesian Product of two sets. This criticism applies to other books of the same type as well.

In Chapter Five the system of integers is developed. This chapter seems adequate.

Chapter Six is devoted to number theory. This is generally a strong chapter with one notable exception. Without going into details, the argument as given as a rationale for the Euclidean Algorithm is only half complete. The proof should demonstrate that the last non-zero remainder is a divisor of each of the two original numbers and that no larger divisor is possible.

In Chapter Seven the set of rational numbers is developed. It is done formally for the most part. In the words of the authors, showing that the set of rational numbers with its two operations is a field was one of their principal objectives. Why this should concern elementary school teachers was not explained.

Beyond that, the reviewer objected to the cavalier treatment of "percent" as just another name for rational numbers. This obscures the relationship between percent and ratio. Finally, drawing a solid line and then stating that there are holes in it, because the line was to represent the set of rational numbers and not the real numbers, is not defensible mathematically, or pedagogically.

Chapter Eight is devoted to the real and complex numbers. The intuitive development of the irrational numbers with square root approximations is good but confusing because the authors' flow-

charted algorithm is not applied in their own examples. Also, the very formal development of the complex numbers seems inappropriate for the stated objectives of the book.

Chapter Nine is a short but adequate introduction to probability and statistics.

Chapter Ten, the last chapter in the book, is devoted to geometry. Geometric transformations deserve a more thorough treatment than they receive in this chapter. Lacking this, the chapter simply doesn't reflect modern thinking in geometric instruction.

With the reservations noted, the reviewer would recommend this text as a good resource for a mathematics course aimed at elementary school teachers.

Alton T. Olson
University of Alberta

MINIREVIEWS

Problem Solvers, edited by L. Marder, Crane, Russak and Company, New York, 1973-1974, approx. 100 pp., \$4.95-\$5.25.

This series of sixteen paper bound books is intended "to provide an inexpensive source of fully solved problems in a wide range of mathematical topics." The early volumes in the series are devoted to topics usually encountered by students at the freshman-sophomore level in mathematics, engineering, and physics. Later volumes, including those provided this reviewer, cover more advanced material. Explanatory materials and theory is kept at a minimum so as to allow more space for problem-solving. The books may be used in place of, or in addition to conventional textbooks. The individual volumes differ in authors, length, and price; however the issues received by the reviewer fall within the bounds listed above. The topics of the sixteen volumes are as follows: (1) Ordinary differential equations, (2) Calculus of several variables, (3) Vector algebra, (4) Analytical mechanics, (5) Calculus of one variable, (6) Complex numbers, (7) Vector fields, (8) Matrices and vector spaces, (9) Calculus of variations, (10) Laplace transforms, (11) Statistics, (12) Fourier series, (13) Electromagnetism, (14) Stochastic process, (15) Fluid Mechanics, (16) Groups.

The Power of Calculus (2nd Edition), Kenneth L. Whipkey and Mary Nell Whipkey, John Wiley and Sons, New York, 1975, 381 pp., \$10.95.

Mr. and Mrs. Whipkey have written this textbook from their experiences in teaching calculus to students in business and the social sciences. It is intended as a readable text for a one-term course. This new edition contains the topics recommended for students in the biological, management, and social sciences by the Committee on the Undergraduate Program in Mathematics. These include a review of algebraic topics, limits, continuity, differentiation techniques and applications, integration, logarithmic and exponential functions, and partial derivatives and their applications. Abstract theory and symbolism is minimized, but the authors stress correct and precise definitions of terms and statements of theorems. Changes from the 1972 edition include the adding, at the students' request, to the appendix a review of basic algebra, the inclusion of answers to more problems, and the use of more graphs to illustrate concepts. Also, the formal definition of limit has been moved to the appendix, the definite integral is defined in a less formal way, and sections on inequalities, limits, logarithms, and exponentials have been rewritten. The examples have been expanded, and more exercises are given which relate to business, economics, and biology. Each chapter contains a set of optional review problems. The appendix contains a self-contained review of trigonometry, although the text itself requires no knowledge of trigonometry. For the applied problems the authors provide the needed definitions of terms, so that an extensive background in the social sciences or business is not required.

Patterns in the Sand (2nd Edition), Maurice Bosstick and John L. Cable, Glencoe Press, Beverly Hills, 1975, 400 pp., \$10.95.

This new edition of a work first published in 1971 has the same objectives and spirit as its predecessor. It is geared to an understanding of the language and ideas of mathematics useful in making decisions. Written with a light-hearted spirit, it is aimed primarily at those persons who are not mathematics majors. The academic prerequisites are "a working knowledge of arithmetic computation, a speaking acquaintance with algebra . . ." The authors have re-

tained the chapters on numbers, algebra, probability, statistics, mathematical systems, and trigonometry and calculus; three sections have been added to the chapter on numbers, and the discussions of probability have been "mellowed." Perhaps the most noticeable change is the inclusion of sections on sets and their relationship to logic in the first chapter. Throughout the book more solved examples are presented, additional exercises are given, and some rearrangement of the order of the exercises has been made. The text contains answers to the odd-numbered exercises and an instructor's answer booklet is available for the others.

+ + +

(Continued from page 132)

Virginia Beta, Radford College, Radford

Chapter President — Jo Ann Winkler
9 actives

The chapter has set up a tutoring program in which students can get help in any mathematics class in which they are having trouble. Other officers: Patricia Moore, vice-president; Jo-Ann Wright, secretary; Glenda Wright, treasurer; Roxie Novak, corresponding secretary; Dr. J. D. Hansard, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President — Barbara Junghans

At one of the meetings there was a talk *Instant Insanity and Graph Theory* by Sister Adrienne Eickman. The chapter is very busy making preparations for the national KME convention. Other officers: Mary Lou Meyers, vice-president; Linda Starr, secretary; Karen Loesl, treasurer; Sister Mary Petronia, corresponding secretary and faculty sponsor.

The Problem Corner

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before March 15, 1975. The best solutions submitted by students will be published in the Spring 1975 issue of *The Pentagon*, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

272. *Proposed by Charles Trigg, San Diego, California.*

A partition of a positive integer n is a representation of n as a sum of positive integers,

$$n = a_1 + a_2 + \dots + a_k.$$

If the order of the parts is considered insignificant, the partition is called unordered. Thus $5 = 3 + 2$ and $5 = 2 + 3$ are the same unordered partition of 5.

Let $P_k(n)$ be the number of partitions of n into k unordered parts.

Show that $[P_2(2n)][P_2(2n + 1)]$ is a perfect square.

273. *Proposed by Gary Schmidt, Washburn University, Topeka, Kansas.*

Show that $\frac{d^2y}{dx^2} = \frac{-d^2x/dy^2}{(dx/dy)^3}$ is an identity.

274. *Proposed by R. S. Luthar, University of Wisconsin, Janesville, Wisconsin.*

Show that the greatest integer contained in $(2 + \sqrt{2})^n$ is odd, where n is any positive integer.

275. *Proposed by the editor.*

In Professor Hy Potenuse's Geometry class he proved that any two triangles are similar if all their corresponding angles are equal and two triangles are congruent if they have two sides and their included angle respectively equal. One bright student observed, to the professor's amazement, that two similar triangles can be drawn which are not congruent even though two sides of one triangle are equal to two sides of the second triangle. How did he do it and what relationship is necessary for this to occur?

276. *Proposed by the editor.*

A class of school children were to run an unusual race. In the school yard there were two flagpoles, one located 60 feet due south of the wall of the building and the other located 90 feet due southeast from the first pole. Each child starts at the first pole, runs to any point in the wall, makes a chalk mark on the wall, and then runs to the other pole. One child's time was much better than any other's. Assuming that all the children are equally fast, what path did the winner take?

SOLUTIONS

262. *Proposed by the editor.*

A rope hangs over a pulley. On one end of the rope hangs a weight. On the other end of the rope hangs a monkey equal in weight to the weight. The combined ages of the monkey and its mother are four years, and the rope weighs four ounces per foot. The monkey's weight in pounds equals the mother's age in years. The mother is twice as old as the monkey was (A) when the mother was one-half as old as the monkey will be (B) when the monkey is three times as old as the mother was (C) when the mother was three times as old as the monkey. The weight of the rope plus the weight of the weight is one-half again as much as the difference between [the weight of the weight and the weight of the weight] and the monkey. How long is the rope?

Solution by Frank Longo, University of Michigan, Ann Arbor, Michigan.

Let w = weight of the weight, m = weight of the monkey, r = weight of the rope, ma = age of the monkey, and MA = age of the mother. Ignoring the sentence about the age relationships between the mother and the monkey, the following equations may be set up:

$$(1) w = m$$

$$(3) m = MA$$

$$(2) ma + MA = 4$$

$$(4) r + w = \frac{3}{2} [(w + w) - m]$$

$$\text{or } r = \frac{1}{2} m = \frac{1}{2} MA$$

So the weight of the rope (which completely determines its length) can be found entirely from the present age of the monkey's mother.

Next, start from the end of the mother-monkey sentence and work backwards:

"... when the mother was three times as old as the monkey"

Assume this happened k years ago $(MA - k) = 3(ma - k)$

"... when the monkey is three times as old as the mother was when ..."

The monkey's age at that time will be $3(MA - k)$ or $9(ma - k)$

"when the mother was one-half as old as the monkey will be when ..."

The mother's age at that time was $4\frac{1}{2}(ma - k)$

"as the monkey was when ..."

Since at one time $MA - k = 3(ma - k)$, the difference between the Mother's and the monkey's ages at that time (and at any time) is $2(ma - k)$. So at the time in question, the monkey's age was

$$4\frac{1}{2}(ma - k) - 2(ma - k) = 2\frac{1}{2}(ma - k)$$

"The mother is twice as old as . . ."

Therefore $MA - 2[2\frac{1}{2}(ma - k)] = 5(ma - k)$. But by the first equation in this section $(MA - k) = 3(ma - k)$. Eliminating k we have $\frac{3}{5}MA = ma$. Substituting this value of ma into equation (2) above

$$\frac{3}{5}MA + MA = 4 \text{ or } MA = 2\frac{1}{2}.$$

Hence the mother is $2\frac{1}{2}$ years old; the monkey is $1\frac{1}{2}$ years old; and the rope weight $1\frac{1}{4}$ pounds. At 4 ounces per foot, the rope must be 5 feet long.

Also solved by Debra Evans and Mark McAndrew, Washburn University, Topeka, Kansas.

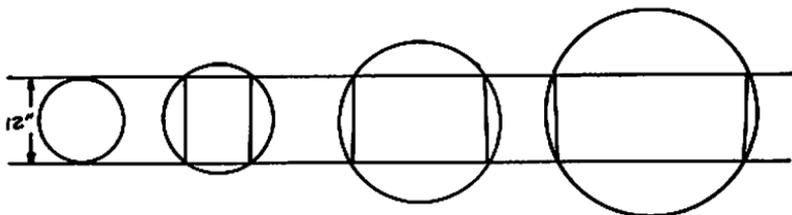
Editor's Comment: This problem was propounded by America's greatest puzzlist Sam Loyd and may be found in Sam Loyd's *Cyclopedia of Puzzles*.

263. *Proposed by the editor.*

Suppose a hole is drilled through the center of a sphere of radius r such that all sides of the hole are 12 inches long. How much volume remains? How do you interpret your answer?

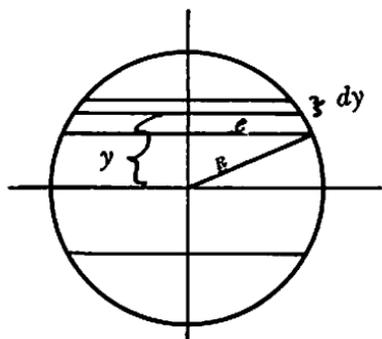
Solution by Leigh James, Rocky Hill, Ct.

Since neither the radius of the sphere nor the radius of the hole are mentioned, one suspects that these dimensions are irrelevant provided that the radius of the sphere is at least 6 inches. Then the symmetry of the sphere suggests that the volume of the remaining portion of the sphere is independent of the radius of the sphere as the following diagram shows:



Hence, once the hole is drilled, the spherical caps above and below the lines denoting a vertical distance of 12" and the cylinder between the two vertical lines in each sphere disappear leaving a small "doughnut-like" shell which has the same volume as the sphere of radius 6" when the radius of the hole is zero. Hence the desired volume is $\frac{4}{3}\pi(6)^3 = 288\pi$ cubic inches.

Editorial Comment: This problem is easily handled by the use of calculus. In the diagram let R be the radius of sphere and $2l$ be the length of the hole. By the "shell method" the volume is



$$V = 2\pi \int_{-\sqrt{R^2 - l^2}}^{\sqrt{R^2 - l^2}} \frac{(2x)y}{\sqrt{R^2 - l^2}} dy$$

$$= 2\pi \int_{-\sqrt{R^2 - l^2}}^{\sqrt{R^2 - l^2}} 2y \sqrt{R^2 - y^2} dy$$

since $x^2 + y^2 = R^2$.

$$\text{Then } V = -\frac{4\pi}{3} (R^2 - y^2)^{3/2} \Big|_{-\sqrt{R^2 - l^2}}^{\sqrt{R^2 - l^2}} = \frac{4\pi}{3} l^3$$

Then since $l = 6''$ in our problem, $V = 288\pi$ cubic inches. Hence, the volume remaining equals the volume of a sphere whose radius equals one half the length of the hole. Thus, if a hole 12 inches long were drilled through the center of the earth only a volume of 288π cubic inches would remain.

264. *Proposed by the editor.*

Professor Hy Potenuse noticed that an algebra student solved the equation $(4 - x)(x - 6) = -3$ as follows: either $4 - x = -3$ and $x = 7$ or $x - 6 = -3$ and $x = 3$. The answers are correct. The professor would like to find all quadratic equations which can be "solved" by the same method. Can you help him?

Solution by Janet Guyer and Debra Evans, Washburn University, Topeka, Kansas.

Given an equation in the form $(c - x)(x - d) = e$, where $e \neq 0$, $x = c - e$ and $x = d + e$ will be the solutions if and only if $e = c - d - 1$.

Proof: Let $x = c - e$ and $x = d + e$ be the solutions to $(c - x)(x - d) = e$. If $x = c - e$, then $(c - (c - e))(c - e - d) = e$. Then since $e \neq 0$, $c - e - d = 1$ or $e = c - d - 1$. Similarly, $x = d + e$ implies $e = c - d - 1$. Therefore, $x = c - e$ and $x = d + e$ are the solutions to $(c - x)(x - d) = e$ when $e = c - d - 1$ and $e \neq 0$. Next let $e = c - d - 1$. Substituting for e in the original equation $(c - x)(x - d) = c - d - 1$ or $x^2 - (c + d)x + (cd + c - d - 1) = 0$. Factoring, we obtain $(x - d - 1)(x - c + 1) = 0$. Consider the following two cases:

$$\text{Case I. } x - d - 1 = 0 = x - d - (c - d - e) \text{ or } x = c - e$$

$$\text{Case II. } x - c + 1 = 0 = x - c + (c - d - e) \text{ or } x = d + e.$$

Therefore, when $e = c - d - 1$, then $x = c - e$ and $x = d + e$ are the solutions to $(c - x)(x - d) = e$. Thus, $x = c - e$ and $x = d + e$ are the solutions to $(c - x)(x - d) = e$ where $e \neq 0$ if and only if $e = c - d - 1$. If $e = 0$, then $(c - x)(x - d) = e$ is a quadratic equation in factored form.

Also solved by Leigh James, Rocky Hill, Ct.

Editor's Comment: This solution shows that all quadratic equations can be put into the desired form. Suppose that the quadratic equation has roots a, b . Then by choosing $c = a + 1$ and $d = b - 1$ and $e = a - b + 1$. For a complete discussion of this problem, see H. A. J. Allen's article entitled "Making

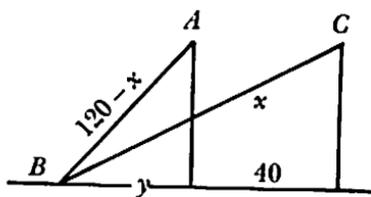
Sense Out Of Nonsense" which appears at page 270 of the March 1974 issue of *Mathematics Teacher*.

265. *Proposed by the editor.*

Two flagpoles, each 40 feet tall, are 40 feet apart. One piece of rope 120 feet long is fastened to the top of each flagpole and pulled taut by an iron ring in the ground as shown below. Assuming that the rope loses no length in being fastened to the flagpoles and assuming the rope remains in the same plane as the flagpoles, how far in front of the first flagpole is the iron ring?

Solution by Jean Herfordt, Wayne State College, Wayne, Nebraska.

Since the rope is 120 feet long, which is the distance down one flagpole, across the ground, and up the second flagpole the ring cannot lie between the flagpoles (within the same plane) so that the rope is pulled taut. Therefore, the ring (*B*) must lie to the side of one flagpole as in the diagram:



If we let $BC = x$, then $AB = 120 - x$. Let y equal the distance from the ring to the nearest flagpole.

Using the Pythagorean Theorem, we can obtain two equations involving x and y :

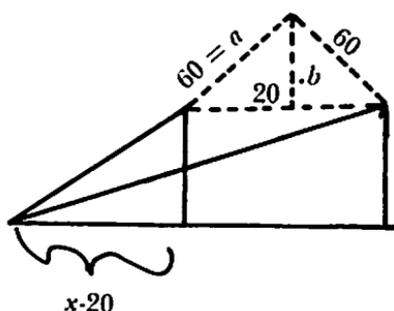
$$40^2 + y^2 = (120 - x)^2$$

$$40^2 + (40 + y)^2 = x^2.$$

Combining these equations we find $y = 3x - 200$. Substituting this value into the first equation we have $40^2 + (3x - 200)^2 = (120 - x)^2$ or $x^2 - 120x + 3400$ and $x = 60 \pm 10\sqrt{2}$ where only the plus sign applies. Hence $y = 30\sqrt{2} - 20$ feet.

Also solved by Sebastian Maurice, Eastern Illinois University, Charleston, Illinois, and Leigh James, Rocky Hill, Ct.

Editor's Comment: Leigh James points out that since the length of the rope is constant, the ring lies on an ellipse whose foci are located at the tops of the flagpoles and whose center is located at the midpoint of the line joining the foci as shown in the diagram below.



$$a = 60 \quad b = 40\sqrt{2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ where } y = 40.$$

$$\text{Hence } \frac{x^2}{60^2} + \frac{40^2}{(40\sqrt{2})^2} = 1 \text{ or}$$

$x = 30\sqrt{2}$ but we need to deduct 20 feet as the horizontal distance from the center of the ellipse to the pole. Hence, the ring is $30\sqrt{2} - 20$ feet from the pole.

266. Proposed by the editor.

Let $abcd$ denote a four digit square in the decimal system. If $abcd$ is broken up into two two-digit numbers ab and cd , then our square has the property that $abcd = (ab + cd)^2$. Find all squares $abcd$ in the decimal system having this property provided that $a > 0$.

(No solution was submitted. The following solutions are supplied by the editor).

I. $abcd = (ab + cd)^2$. There is an x , $0 \leq x < 50$, such that $abcd = (50 \pm x)^2$, $ab + cd = 50 \pm x$
 $abcd = 100(ab) + cd$

$$abcd = (50 \pm x)^2 = 100(25 \pm x + p) + q \text{ where } x^2 = 100p + q$$

$$\text{so } ab = 25 \pm x + p$$

$$cd = q$$

$$ab + cd = 25 \pm x + p + q$$

$$50 \pm x = 25 \pm x + p + q$$

$$25 = p + q$$

Since q must be a two digit ending of a square and $q < 25$, $q = 01, 04, 09, 16, 21, 24$, or 25 . Of these only $q = 01$ and $q = 25$ make $100p + q$ perfect squares. Hence $p = 24$, $q = 1$ and $x = 49$; or $p = 0$, $q = 25$, and $x = 5$. Thus, the solutions sought are $99^2 = 9801$, $55^2 = 3025$, and $45^2 = 2025$. The solution $01^2 = 0001$ is excluded since $a = 0$.

II. Let $abcd = 100p + q$

Then $(p + q)^2 = 100p + q$ or $(p + q)(p + q - 1) = 99p$ where $31 < p + q < 100$ to insure four-digit squares. Now, $99 \mid p + q$ implies that $p + q = 99$ and $abcd = 9801$. $11 \mid p + q$ implies $9 \mid p + q - 1$ so that $p + q = 55$ and $abcd = 3025$. $9 \mid p + q$ implies $11 \mid p + q - 1$ so that $p + q = 45$ and $abcd = 2025$.

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgment will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

Random numbers have been used extensively for many years by statisticians to determine random samples chosen from a large population. Tables of random numbers are readily available and with the increased use of computers and electronic calculators which can generate random numbers, new applications for random numbers have drawn the attention of mathematicians. One such application of interest is approximate integration using random numbers.

Suppose the continuous function $f(x) > 0$ is bounded by the line $y = c$, c a constant, for all $x \in [a, b]$ as shown in Figure 1.

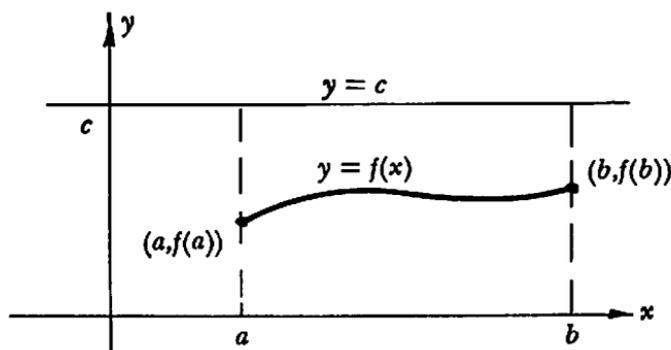


FIGURE 1.

If we were to randomly select points in the interval $[a,b]$ for x , we could calculate an approximate value for the area under the curve $y = f(x)$ for $x \in [a,b]$ as follows. Using a random number generator (computer, calculator, or table), we generate an X -value in the interval $[a,b]$. We then calculate the value of $y = f(x)$ for the X -value generated and record its value. We repeat this process many times, each time recording the functional value for the value of $x \in [a,b]$ generated. Hence, an *average* functional value $f(x)$ for $f(x)$ on the interval $[a,b]$ is

$$\hat{f}(x) = \frac{\sum_{i=1}^n f(x_i)}{n},$$

where n is the number of random values of $x \in [a,b]$ which we generated and calculated the value of $f(x)$. Hence the total area under the curve $y = f(x)$ for $x \in [a,b]$ is

$$\int_a^b f(x) dx \approx \hat{f}(x) \cdot (b-a).$$

For example, if we wish to calculate an approximate value of the area under the curve $y = \sin x$ for $x \in [0,1]$ using a random number generator on the IBM 360, where each of the five experimental trials listed consists of generating 1000 random numbers (hence $n = 1000$); the author obtained the following results; where

$$\int_0^1 \sin x dx \approx \hat{f}(x) \cdot (1-0) = f(x).$$

<u>Trial Number</u>	<u>Average Functional Value $\hat{f}(x)$</u>	<u>Area Approximation</u>
1	0.4522319	0.4522319
2	0.4694775	0.4694775
3	0.4590085	0.4590085
4	0.4593489	0.4593489
5	0.4563996	0.4563996

The actual value (to five decimal places) for $\int_0^1 \sin x \, dx = 0.45970$ by exact integration. One will note that for the five trials listed above that the smallest approximation was .4522319 in trial 1 and the largest was .4694775 in trial 2. If we were to take the average value of all five trials (and hence equivalent to a single trial of 5000) we would have the value of 0.4592929 which is very close to the exact value of 0.45970.

This same approach may be used to calculate the approximate value of a double integral of the form

$$\int_a^b \int_{c(x)}^{d(x)} g(x,y) \, dy \, dx$$

Here we wish to find an average functional value $\hat{g}(x,y)$ for $x \in [a,b]$ and $y \in [c(x),d(x)]$ where $g(x,y)$ is assumed to be a positive continuous function which is bounded above for all values of (x,y) in this region.

To calculate the value of $g(x,y)$, we this time need to generate pairs of random numbers, one for $x \in [a,b]$ and another for $y \in [c(x),d(x)]$. Then we calculate the value of $z = g(x,y)$ for the values of x and y generated and record the functional value. We repeat this process n times as before. Then the average functional value of $z = g(x,y)$ for $x \in [a,b]$ and $y \in [c(x),d(x)]$ is

$$\hat{g}(x,y) = \frac{\sum_{i=1}^n g(x_i,y_i)}{n}$$

and hence an appropriate value of the total volume under $z = g(x,y) > 0$ for $x \in [a,b]$ and $y \in [c(x),d(x)]$ is

$$\int_a^b \int_{c(x)}^{d(x)} g(x,y) \, dy \, dx \approx \hat{g}(x,y) \cdot (\text{Base area in } xy\text{-plane}).$$

Suppose that we wish to calculate the approximate value of $\int_0^1 \int_0^x \sin xy \, dy \, dx$. Here we are actually finding the volume as shown below in Figure 2.

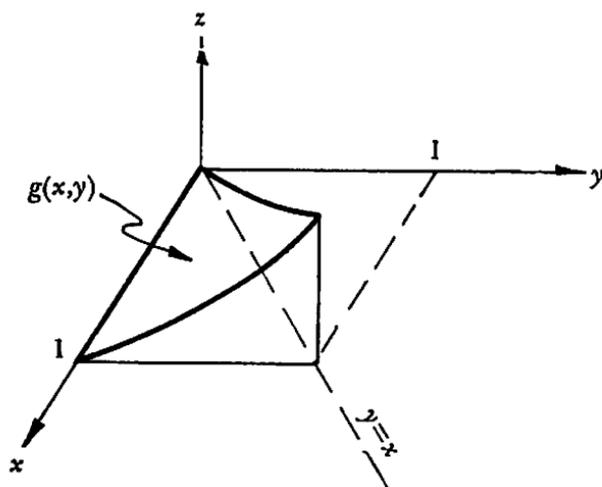


FIGURE 2.

To find our desired value, we must generate pairs of random numbers in each case, $x \in [0,1]$ and $y \in [0,x']$ where x' is the value of x generated. Again using the IBM 360, the author obtained the following results where each trial consisted of 1000 random number pairs (x,y) satisfying the above requirements.

<u>Trial Number</u>	<u>Approximate Value of Integral</u>
1	0.1206040
2	0.1219669
3	0.1229281
4	0.1190198
5	0.1204925
6	0.1175579
7	0.1212409
8	0.1229811
9	0.1190462
10	0.1204315

One will note that the smallest value obtained was 0.1175579 in trial 6 and the largest was 0.1229811 in trial number 8. The average of all ten trials is 0.1206268 which would be equivalent to one trial of size 10,000. Can you evaluate this integral by exact integration?

Another approach to the two problems above using random numbers for approximate integration is to use probability. Suppose we wish to approximate $\int_a^b f(x) dx$, where $f(x) > 0$ and bounded above by $y = c$, c a constant, as in Figure 1. We could generate random numbers in pairs (x,y) . Such that $x \in [a,b]$ and $y \in [0,c]$. Then we will calculate the value of $y = f(x)$ for our value of x randomly selected calling this y -value y' . We then compare the values of y' and y . If $y > y'$ then we will call this case a failure as the point (x,y) does not lie below or on the curve in the desired area. But if $y \leq y'$, we will call this case success since the point (x,y) lies in the desired region.

We record the number of successes and number of trials and keep a tally. Hence the desired area under $y = f(x)$ for $x \in [a,b]$ is the following:

$$\int_a^b f(x) dx \approx \frac{\text{Number of successes}}{\text{Number of trials}} \cdot c(b-a)$$

Using the same example as before, suppose we wish to calculate the area under the curve $y = \sin x$ for $x \in [0,1]$ using a random number generating function on the IBM 360. Each of the ten trials below consists of generating 1000 pairs of random numbers where $x \in [0,1]$ and $y \in [0,1]$ since $y = \sin x$ is bounded above by $y = 1$ as shown in Figure 3.

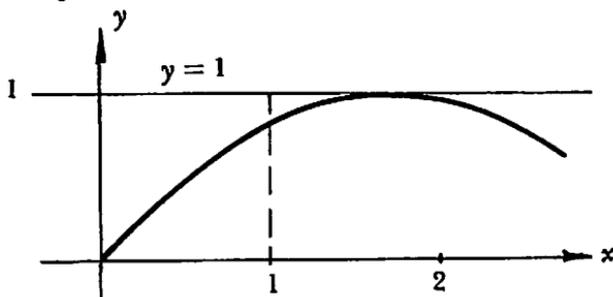


FIGURE 3.

The author obtained the following results:

Trial No.	No. Successes	Area = $\frac{\text{No. Successes}}{1000} \cdot 1$
1	458	.458
2	456	.456
3	470	.470
4	488	.488
5	446	.446
6	456	.456
7	469	.469
8	443	.443
9	448	.448
10	474	.474

The actual area was previously found to be $\int_0^1 \sin x \, dx \approx .45970$ and the above ten trials are fairly close to this value. One will note that in the ten trials the largest value was .488 in trial 4 and the smallest was .443 in trial 8. The average for all ten trials (or for 10,000 random numbers) is .461 which is extremely close to the actual value of .460. One would also note that in comparing the two methods; that is, the average functional value method vs the probability case, that the probability model case was slightly less accurate for the integral $\int_0^1 \sin x \, dx$.

The probability case can be extended to the three-dimensional or double integral case. Can you extend this case to the approximate evaluation of the integral $\int_0^1 \int_0^x \sin xy \, dy \, dx$?

Kappa Mu Epsilon News

EDITED BY ELSIE MULLER, *Historian*

News of Chapter activities and other noteworthy KME events should be sent to Elsie Muller, Historian, Kappa Mu Epsilon, Department of Mathematics, Morningside College, Sioux City, Iowa 51106.

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence

Chapter President – Vann Bush
30 actives

One of the most profitable meetings included the award winning John von Neumann film. The members provide a tutoring service for elementary, high school, and university students in mathematics. KME initiate of 1964, Dr. Marjory Johnson, gave a paper at the national meeting of AMS and MAA in January. Other officers: Dan Copeland, vice-president; Carolyn Malone, secretary-treasurer; Jean T. Parker, corresponding secretary; and Dr. Eddy J. Brackin, faculty sponsor.

Alabama Gamma, University of Montevallo, Montevallo

Chapter President – Cheryl Mays
9 actives, 5 pledges

The chapter sponsors an annual spring cookout to which they invite all mathematics majors and minors as well as the mathematics faculty for a picnic and mathematically oriented games. In the fall, they sponsored the sale of the CRC Mathematical Handbook. Other Officers: David Combs, vice-president; Joan Cherry, secretary-treasurer; Dr. Angela Hernandez, corresponding secretary; and Dr. William Foreman, faculty sponsor.

**California Gamma, California Polytechnic State University,
San Luis Obispo**

Chapter President – Terry L. Shell
17 student members, 18 faculty members

Monthly meetings include chapter business and faculty speakers or special visiting speakers. During the year, there will be one initiation and one banquet. In the mathematics laboratory, the chapter provides a tutorial service available for all students on campus. Members also assist the mathematics faculty with the annual mathematics contest on campus for over 500 high school students. Other officers: Doug Morgan, vice-president; Sandra McKaig, secretary; Karen Barks, treasurer; Dr. Mach, corresponding secretary; and Dr. Warten, faculty sponsor.

California Delta, California Polytechnic University, Pomona

Chapter President – Paul Devine
15 actives, 12 pledges

All of the science majors are invited to a **KME** picnic. Meetings are held once a quarter. As for activities, the members do tutoring and also organize an exhibit during the weekend of *Poly Vue*, an open house conducted by the university. In the spring the chapter awards a book scholarship. Other officers: Nick Mansur, vice-president; Dennis Neal, secretary and treasurer; Samuel Gendelman, corresponding secretary; Joseph Kachun, faculty sponsor.

Colorado Alpha, Colorado State University, Fort Collins

Chapter President – Jacquie Ostrom
6 actives, 10 pledges

Monthly meetings are held in the homes of professors. Guest speakers have included an astronomer, a logician, a solar energy research scientist, and professors from the mathematics department. Activities include a pot-luck supper at the end of each quarter, a snow-shoeing trip in the winter, and a picnic in the spring. Plans are being made for an Alumni Seminar on Employment Opportunities in the Mathematical Sciences. Other officers: Debbie Mea-

cham, vice-president; Danelle Lencz, secretary; Cindy Druva, treasurer; Duane Clow, corresponding secretary; Bennett Manuel, faculty sponsor.

Colorado Beta, Colorado School of Mines, Golden

Chapter President – Lloyd Scheidt
10 actives

The following lectures have been given at the programs, *A Mathematical Model of an Oil Shale Retort* by Dr. Don Iausett, assistant professor of mathematics, and *Survival in the Real World* by R. E. D. Woolsey, Professor of Mineral Economics, CSM. A departmental display is being prepared for Engineers' Day in April. Other officers: John Turner, vice-president; Russ Hoffman, secretary; Carol Payne, treasurer; A. J. Boes, corresponding secretary; D. W. Iausett, faculty sponsor.

Illinois Alpha, Illinois State University, Normal

Chapter President – Bruce Iverson
30 actives, 10 pledges

The chapter conducts study sessions for those who wish to prepare for Parts 1 and 2 of the Actuarial Examination. Guest speakers provide the programs for the monthly meeting. Activities include the sponsoring of a volleyball team this semester and a mathematics major tournament. Other officers: Rose Danelczuk, vice-president; Rita King, secretary; Carol Born, treasurer; Marcella Travernicht, corresponding secretary; D. Orlyn Edge, faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston

Chapter President – Bill Taber
44 actives

Money in the chapter treasury is to be used to send some students to the national convention. Other officers: Kevin Settle, vice-president; Connie Kutosky, secretary and treasurer; Mrs. Ruth Wheeler, corresponding secretary; Mr. Larry Williams, faculty sponsor.

Illinois Zeta, Rosary College, River Forest

Chapter President—Christine Stephens
16 actives

At the monthly meetings junior members present problems and senior members present research papers. Last fall the chapter sponsored a picnic to acquaint students, particularly the underclassmen, with the mathematics department. The chapter also provides an on-campus tutoring service for mathematics and mathematically-oriented subjects. Other officers: Georgina Lunardon, vice-president; Anita Koziol, secretary; Colette Zurkowski, treasurer; Mr. Mordechai Goodman, corresponding secretary and faculty sponsor.

Illinois Eta, Western Illinois University, Macomb

Chapter President — Ron Jordan
17 actives

The bi-monthly meetings feature a presentation of a talk by students. Topics have included *Simulation* and *Relativity*. Each meeting also includes *Meet the Prof* when one professor is asked to talk about his accomplishments and interests. Every Monday night the members tutor students in the lower level mathematics courses. Other officers: Mark Adler, vice-president; Nancy Boettner, secretary; Sandy Kammerman, treasurer; Dr. Kent Harris, corresponding secretary; Dr. James Calhoun, faculty sponsor.

Indiana Delta, University of Evansville, Evansville

Chapter President — Michael Lachance
65 actives

Topics on the programs at the monthly meetings have been as follows: *Fundamentally Speaking*, *What if Euclid was Wrong*, *My Bead is Quicker than your Bead*, *Using the Computer with Children*, and *A Stroll Through the Queen's Garden*. At the high school science fair the chapter is presenting an award of a \$25 U.S. Savings Bond for the best mathematics project. Members are offering a free tutoring service at a designated time throughout the week in a

designated room. Other officers: Thomas Becker, vice-president; Betty Conditt, secretary; Dr. Gene Bennett, treasurer and corresponding secretary; Mr. Kenneth Stofflet, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President – Patricia Johnson Fox
25 actives

At the monthly meetings, held in the homes of faculty members, a student presents a mathematical paper. Other officers: Diane J. Schmitt, vice-president; Jo Ann F. Wenthold, secretary and treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines

Chapter President – George Carr
4 actives – 3 pledges

At the monthly meetings, one or two student presentations are made. The initiation banquet is being held in April. Other officers: Jeanette Becker, vice-president; Carol Behrens, secretary; Mary Bauer, treasurer; Wayne Woodworth, corresponding secretary; Alex Kleiner, faculty sponsor.

Iowa Gamma, Morningside College, Sioux City

Chapter President – Marc Burkhart
23 actives

Professor Walter Mientka of the University of Nebraska was the guest speaker at the initiation dinner. The number of people in attendance at the **KME** Homecoming Breakfast increases each year. The names of all **KME** members with telephone numbers are given to each mathematics student so that he may contact anyone of them for free tutoring. Other officers: Mark Fegan, vice-president; Rae-jean Pacholke, secretary; Dale Lenderts, treasurer; Michael Sorn, secretary; Elsie Muller, corresponding secretary and faculty sponsor.

Iowa Delta, Wartburg, College, Waverly

Chapter President – Robert Basham
25 actives

On 21 January, the chapter co-sponsored with the Wartburg mathematics department two public lectures by Stuart Klugman, instructor of statistics and actuarial science at the University of Iowa. His lectures were entitled *How to Gamble if you Must – Strategies for Red-and-Black* and *A Statistical Analysis of the 1970 Draft Lottery*. At the monthly meetings programs are provided by its members. Other officers: David Neve, vice-president; Pamela Snyder Egts, secretary; Laurel Kunta, treasurer; Dr. August W. Waltmann, corresponding secretary and faculty sponsor.

Kansas Alpha, Kansas State College of Pittsburg, Pittsburg

Chapter President – Gary Morella
40 actives

Programs at the monthly meetings were given by Randy Timi, *The Inversion Transformation*; Cathy Baird, *An Analysis of the Difference Components and Behavior of the Equations in a Given Macroeconomic Model*; Gary Morella, *A BASIC Program that Composes Music*; Roy Bryant, *A Runge-Kutta Solution to Differential Equations*. Eleven new members were initiated at the November meeting. Other officers: Laura Spain, vice-president; Judy Wilson, secretary; Roy Gryant, treasurer; Dr. Harold Thomas, corresponding secretary; Prof. J. Bryan Sperry, faculty sponsor.

Kansas Beta, Emporia Kansas State College, Emporia

Chapter President – Pam Shirley
20 actives

In honor of Charles Tucker, former KME national officer and a founding member of Kansas Beta Chapter, the chapter is collecting funds for a scholarship to be given to KME members. Professor Tucker had been the corresponding secretary for many years. Other officers: Beth Ann Ridinous, vice-president; Jane Niefeld, secretary;

Gregg Stair, treasurer; Donald L. Bruyr, corresponding secretary; Thomas Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison

Chapter President — Mary Kay Stewart
9 actives

The film, *Mauritit Escher: Painter of Fantasies*, was sponsored by the chapter and well attended by both mathematics and art students. Comments from faculty members of both departments followed the showing of the film. A very profitable invited speaker night was also held during the fall semester on the topic of finite fields. Social activities included a picnic for members and invited guests in the fall and the annual Christmas Wassail Party. Other officers: Gary Burton, vice-president; Diane Beckman, secretary; Sister Jo Ann Fellin, corresponding secretary and faculty sponsor.

Kansas Delta, Washburn University, Topeka

Chapter President — Janet Gayer
21 actives, 4 pledges

Guest speakers for programs have been Jahn Howe of the Security Benefit Life Insurance and Mr. Kenneth Wilke, the problem editor of THE PENTAGON. Activities included a buffet supper and initiation service at the International House and a *Math-o-Rama* for high school seniors in the surrounding area. Other officers: Lonnie Stouffer, vice-president; Sandra Peer, secretary; A. Allen Riveland and Emmanuel Colys, faculty sponsors.

Kansas Epsilon, Fort Hays Kansas State College, Hays

Chapter President — Jean Ingersoll
15 actives

The annual initiation banquet was held on 24 March. Other officers: Brena Mauch, vice-president; Camellia Tuttle, secretary and treasurer; Eugene Etter, corresponding secretary; Dr. Charles Votaw, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President – M. Claire Wagner

80 actives, 3 pledges

The following students have given talks at the programs: Jean Sewell – *Card Shuffling* and Patricia Creed – *Polyhedra – Beauty and Truth*. Ms. Jean Ciconte, an IBM Staff Programmer, was featured on the career program in September. In October there was a joint meeting with Maryland Beta. Other officers: Cathy Brown, vice-president; Colleen Baum, secretary; Cathy Brown, treasurer; Sister Marie Augustine Dowling, corresponding secretary and faculty sponsor.

Maryland Beta, Western Maryland College, Westminster

Chapter President – Mark Miller

16 actives

Joint meetings have been held with Maryland Alpha on 16 October and 9 April. As for activities the members had a May Carnival booth during the May Day celebration and had a picnic. Other officers: Bette Gemma, vice-president; Carol Zynel, secretary; Douglas Bitz, treasurer; James Lightner, corresponding secretary; Robert Boner, faculty sponsor; Virginia Bevans, historian.

Michigan Beta, Central Michigan University, Mt. Pleasant

Chapter President – Don Palmer

15 actives – 7 pledges

The monthly meetings have included speeches on mathematics problems by members and the film, *Donald Duck in Mathemagic Land*. The chapter is very active in promoting the help sessions every Tuesday and it sponsors the *Freshman Mathematics Examination*. Other officers: Elaine Lachapelle, vice-president; Pat Carls, secretary; Vicki Hagley, treasurer; Dean Hinshaw, corresponding secretary and faculty sponsor.

**Mississippi Gamma, University of Southern Mississippi,
Hattiesburg**

Chapter President – Harry Dole
20 actives

The April meeting will be a combination of the initiation and a cook-out. Other officers: Sherry Fortenberry, vice-president; Frances Weber, secretary; Jack D. Munn, corresponding secretary; Alice Essary, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield

Chapter President – Dan Wilson
40 actives

One of the programs was a talk, *Nonstandard Analysis*, by Dr. Alex Cramer. Dan Wilson was awarded the KME Merit Award. A tutoring program is carried on in connection with the mathematics laboratory. Other officers: Esther Key, vice-president; Sherry Collier, secretary; Greg Darnaby, treasurer; Eddie W. Robinson, corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg

Chapter President – Stephen Lacey
27 actives – 7 pledges

The chapter conducts 6 formal meetings during the year. A Christmas party, a spring honors banquet, and an annual field trip to sites of interest in the Kansas City area represents the major activities. Other officers: Robert Stuckmeyer, vice-president; Deborah West, secretary; Sandra Robertson, treasurer; Dr. Homer F. Hampton, corresponding secretary; Dr. Velma Birkhead, faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville

Chapter President – Leila Barge
32 actives

Monthly meetings have featured senior presentations and problem solving sessions. Activities include the conducting of the an-

nual high school mathematics contests and a treasure hunt with mathematical clues. Other officers: Ken Eccher, vice-president; Pam Keller, secretary; Janet Sundstrom, treasurer; Sam Lessig, corresponding secretary; Mary Sue Beersman, faculty sponsor.

Missouri Zeta, University of Missouri at Rolla, Rolla

Chapter President — Joe Tull
28 actives — 20 pledges

Programs have included Dr. Charles Hatfield, *How to Stack the Deck or the Mathematics of the Perfect Shuffle*, and Dr. Lyle Pursell, *How to Keep Your Professors from Getting Bored*. Help sessions are conducted for calculus, trigonometry, algebra, and differential equations. The chapter gives a prize for the best mathematical exhibit at the area high school science fair. Other officers: Jim Fricke, vice-president; Kim Coleman, secretary; Larry Harris, treasurer; Peter Sawtelle, corresponding secretary; James Joiner, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne

Chapter President — Jean Herfordt
25 actives

The monthly meetings are followed by entertainment in the form of mathematical puzzles, problems, or novelties. Presentations are usually made by two members who challenge the remaining members for solutions. Each spring the club administers a mathematics examination to three freshmen mathematics majors. On this basis David Lashier of Odebolt, Iowa, was selected outstanding freshman mathematics major for 1973-74. On 21 April the members assisted the mathematics department in administering the annual high school mathematics contest for approximately three hundred secondary students. Other officers: Deborah Dubs, vice-president; Craig Hellwege, secretary and treasurer; Deanna Fey, reporter and historian; Fred Webber, corresponding secretary; Jim Paige and Frank Prather, faculty sponsors.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President — David Billingsly

54 actives

The initiation was held at the club room of a local apartment complex. The club provided a roast beef and members brought potluck. Other officers: Michael Huffman, vice-president; Beverly Riese, secretary; John Starner, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Alpha, Hofstra University, Hempstead

Chapter President — Diane Goldman

30 actives

In October there was a faculty-student get-together picnic and in November Donald McKinnon spoke on actuarial science. Other officers: James Wilson, vice-president; Lillian Nilsen, secretary; Alan Blayne, treasurer; Prof. Stanley Kertzner, corresponding secretary and faculty sponsor.

New York Iota, Wagner College, Staten Island

Chapter President — Diane Morse

During the fall semester the chapter held meetings every two weeks. At the initiation meeting Dr. Raymond Traub spoke on probability in the talk, *Your Influence in Committee Voting*. The chapter also sponsored the film, *Nim and Other Oriented Graph Games*, for the entire community. Anita Giobfie was selected as one of six American College students to spend 16 weeks studying at the Atomic Energy Commission, Argonne National Laboratory in Illinois. She will concentrate on computer science. Other officers: Ellen Lepowsky, vice-president; Joan Sangivanni, secretary; Anthony Tropeano, treasurer; William Horn, corresponding secretary; Raymond Traub, faculty sponsor.

New York Kappa, Pace University, New York

Chapter President – Richard Girard
30 actives – 30 pledges

The second annual induction dinner is being held in April. Other officers: Thomas DeLuca, vice-president; Lawrence Jermyn, secretary; Salvatore Vittorio, treasurer; Mrs. Sandra Pulver, corresponding secretary and faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green

Chapter President – Craig Cooper
50 actives

The meetings have featured an initiation banquet, a *Meet the Profs* night, and a career night. **KME** has engaged in a volleyball match with the professors and graduate students. Professor Dean Neumann was the recipient of the **KME** Excellence in Teaching Award. Other officers: Renee Rebera, vice-president; Julie Osmon, secretary; Alan Schleimer, treasurer; Waldemar Weber, corresponding secretary; L. D. Sabbagh and Thomas Hern, faculty sponsors.

Ohio Gamma, Baldwin-Wallace College, Berea

Chapter President – Barb Walker
20 actives

Other officers: Barb Galla, vice-president; Sherry Sieman, secretary; Lee Thomas, treasurer; Prof. R. E. Schlea, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord

Chapter President – Daniel Ledsome
35 actives

At the initiation banquet each new initiate presented a talk about the mathematician of his choice. Other officers: Mary Galani, vice-president; Barbara Beerman, secretary and treasurer; Dr. James L. Smith, corresponding secretary and faculty sponsor.

**Oklahoma Gamma, Southwestern Oklahoma State University,
Weatherford**

Chapter President – Blaine Greenhagen
13 actives

Dr. William Jay Conover from Texas Tech of Lubbock, Texas, spoke to the chapter on *Careers in Statistics*. The chapter also took a field trip to Western Electric in Oklahoma City. Other officers: Joel Newcombe, vice-president; Jane Holmstrom, secretary; Michelle Klaassen, treasurer; Dr. Wayne Hayes, corresponding secretary; Dr. Don Prock, faculty sponsor.

Pennsylvania Beta, LaSalle College, Philadelphia

Chapter President – Mathew Coleman
21 actives – 6 pledges

The chapter offers free tutoring to those who desire it. The colloquium included Mr. Stephen Leonard who spoke on *Balanced Incomplete Blocks* and a representative from Reading Railroad who talked about *Computer Applications in Railroading*. Three mathematics graduates have returned to speak about their careers as actuaries and computer programmers. To raise money the chapter sponsored a pinochle and bridge tournament. Other officers: Susan Szczepanski, vice-president; Linda Pantano, secretary; Debbie Wissman, treasurer; Brother Damian Conelly, corresponding secretary and faculty sponsor.

Pennsylvania Gamma, Waynesburg College, Waynesburg

Chapter President – Paul Christian
18 actives

The programs have included lectures by the faculty on astronomy, electrostatics, and number theory; lectures by students on Gauss, mathematical games, and computer applications; MAA films on probability theory and unsolved problems. The chapter also sponsors and develops a departmental newsletter each semester, sponsors freshman tutoring sessions, sponsors a mathematical game

booth at the Homecoming Carnival, and takes a trip to the Ohio State Open House. Other officers: Geri Bezek, vice-president; Cecilia Dusha, secretary and treasurer; Gabriel J. Basil, corresponding secretary; Lee O. Hagglund, faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President – Maxine Cranage
20 actives

In addition to the regular programs the chapter is going to sponsor an evening of mathematical games in the mathematics laboratory for elementary school teachers. Other officers: Kathi Bauer, vice-president; Barbara Diehl, secretary; Diane Green, treasurer; Irving Hollingshead, corresponding secretary; Edward Evans, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President – David Elko
52 actives

At the October meeting Dr. Charles Bertness gave a talk on *Graph Theory*. At the December meeting Mr. Joseph Peters gave a talk on *Linear Programming*. Other officers: Kristine Mangone, vice-president; Sharon Evans, secretary; Teresa Pavlekovsky, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City

Chapter President – Daniel McWhertor
28 actives – 17 pledges

Other officers: Karen Williams, vice-president; Carolyn Lape, secretary; Betsy Guffey, treasurer; Marvin Henry, corresponding secretary; Cameron Barr, faculty sponsor.

Pennsylvania Iota, Shippensburg State College, Shippensburg

Chapter President — Belinda Bowersox
32 actives — 15 pledges

At the monthly meetings mathematical talks were given by pledges, professors and visiting lecturers. Social activities included a banquet at which the fall pledges were installed, a roller skating party, a basketball game against the mathematics faculty, and a summer reunion picnic. Trips are planned to visit the Mack Plant in Chambersburg and the Mechanicsburg Naval Depot to see the computer facilities. Other officers: Billie Belles, vice-president; Lynn Dotter, secretary; Howard Gell, treasurer; John Mowbray, corresponding secretary; James Sieber, faculty sponsor; Peggy Benfer, historian.

Tennessee Delta, Carson-Newman College, Jefferson City

Chapter President — Candace Hamner
23 actives

At the meetings Dr. Howard Chitwood gave a talk on *The Fibonacci Sequence and Related Topics in Geometry* and Dr. Sherman Vanaman gave one on *Mathematics for Enjoyment*. Included in activities was a hike in the Great Smoky Mountains National Park. Other officers: Steven Hansen, vice-president; Jane Rohrer, secretary; John Fraley, treasurer; Denver Childress, corresponding secretary; Howard Chitwood, faculty sponsor.

Texas Gamma, Texas Women's University, Denton

Chapter President — Ellen Durrance
14 actives

Six new members were initiated into the chapter in November. Other officers: Mrs. Debby Guthrie, vice-president; Margaret Petey, secretary and treasurer; Dr. D. T. Hogan, corresponding secretary and faculty sponsor.

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