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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

# A Note On Extrema

CHI-MING CHOW

Highland, Michigan

There are many mathematics problems in which we are concerned with extrema values. The usual procedures for finding the extrema of a continuous and differentiable function in certain intervals are: 1. Find all zeros of its first derivative. 2. Use second derivative test to find extrema. 3. If the second derivative test fails, use first derivative test. 4. Find and eliminate all possible inflectional points which also have zero first derivatives. All these procedures are somewhat tedious because of the lengthy calculations involved.

This note will present a procedure which omits some of these calculations. Using the three theorems proved in this article, the possible inflectional points with zero first derivatives can be determined. After that all extrema can be found directly without using first and second derivative tests. Six examples are given at the end of this note. For convenience, a real polynomial function will serve as the model of the discussion that follows.

LEMMA 1. Let  $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1} + a_n$ ,  $n \geq 2$ , be a real polynomial function:

1. If  $a_0 > 0$  and  $n$  is odd, then the graph of  $f(x)$  is concave downward on the extreme left and concave upward on the extreme right.
2. If  $a_0 > 0$  and  $n$  is even, then the graph of  $f(x)$  is concave upward on both the extreme left and right.
3. If  $a_0 < 0$  and  $n$  is odd, then the graph of  $f(x)$  is concave upward on the extreme left and concave downward on the extreme right.
4. If  $a_0 < 0$  and  $n$  is even, then the graph of  $f(x)$  is concave downward on both the extreme left and right.

*Proof:* (Case 1) We know that a real polynomial function is continuous and differentiable for all  $x$ , Thus

$$f'(x) = a_0nx^{n-1} + a_1(n-1)x^{n-2} + \dots + a_{n-1},$$

and

$$f''(x) = a_0 n(n-1)x^{n-2} + a_1(n-1)(n-2)x^{n-3} + \dots + a_{n-2}.$$

Since  $n$  is odd and  $a_0 > 0$ , we have

$$\lim_{x \rightarrow \pm \infty} f''(x) = x^{n-2} \left[ a_0 n(n-1) + \frac{a_1(n-1)(n-2)}{x} + \dots + \frac{a_{n-2}}{x^{n-2}} \right]$$

This result shows that the graph of  $f(x)$  on the extreme left is always concave downward and the graph of  $f(x)$  on the extreme right is always concave upward.

The other cases can be similarly proved.

The four cases can be easily remembered by the following graphs:



**THEOREM 1.** Let  $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$ ,  $n \geq 2$ , be a real polynomial function and  $E$  be the number of extrema:

1. If  $a_0 > 0$  and  $n$  is odd, then  $E$  is even. If  $E \neq 0$ , the first left extrema is a maximum, then the minima and the maxima alternate. There are equal number of maxima and minima.
2. If  $a_0 > 0$  and  $n$  is even, then  $E$  is odd. The first left extrema is a minimum, then the maxima and minima alternate. The number of minima is one more than the number of maxima.
3. If  $a_0 < 0$  and  $n$  is odd, then  $E$  is even. If  $E \neq 0$ , the first left extrema is a minimum, then the maxima and minima alternate in order. There are equal number of maxima and minima.
4. If  $a_0 < 0$  and  $n$  is even, then  $E$  is odd. The first left extrema is a maximum, then the minima and the maxima alternate. The number of maxima is one more than the number of minima.

*Proof:* From Lemma 1, we know that the extreme left side of  $f(x)$  is always concave downward and increasing, and the extreme right side of  $f(x)$  is always concave upward and increasing. If it is continuously increasing from left to right, then there will be no turning point ( $E = 0$ ). Otherwise it must reach a maximum from the left first. Then it becomes concave downward and decreasing, thus it must pass through another turning point (minimum) before it becomes concave upward and increasing. This change can be repeated. Therefore the total number of extrema must be even, and the maxima and minima must be equal in number and alternate in order. Thus the proof is completed.

The other cases are similarly proven.

**THEOREM 2.** If  $f(x)$  is a real polynomial of degree  $n$ , then the number of different zeros of  $f'(x)$ ,  $F'_0$ , is the sum of the number of extrema of  $f(x)$ ,  $E$ , and the number of inflectional points with zero first derivatives of  $f(x)$ ,  $I_0$ . That is,  $F'_0 = E + I_0$ .

*Proof:* Let  $P$  be a point of  $f(x)$  with zero first derivative. If the first derivatives of both the closest left and right neighborhoods have different signs, then we know that  $P$  is an extremum. If they have the same sign, then  $P$  is known as an inflectional point. These facts show that if  $f(x)$  has  $F'_0$  points with zero first derivatives, then they must

be either extrema ( $E$ ) or inflectional points ( $I_0$ ). Therefore  $F'_0 = E + I_0$ . Thus the proof is completed.

**LEMMA 2.** *If  $f(x)$  is a real polynomial function of degree  $n \geq 2$ , then it can have  $n - 2$  or less inflectional points.*

*Proof:* If  $(x_1, f(x_1))$  is an inflectional point of  $f(x)$ , then  $x_1$  is a zero of  $f''(x)$ . The degree of  $f''(x)$  is  $n - 2$ , therefore there are  $n - 2$  or less possible inflectional points.

**LEMMA 3.** *Let  $f(x)$  be a real polynomial function of degree  $n \geq 2$ . If  $n$  is even, then the total number of possible inflectional points must be even but no more than  $n - 2$ . If  $n$  is odd, then the total number of possible inflectional points must be odd and no more than  $n - 2$ .*

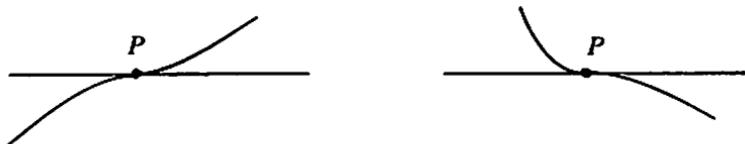
*Proof:* If  $n$  is even, by Theorem 1 we know that both ends of  $f(x)$  must be either concave upward or concave downward. But each time an inflectional point occurs, it changes the concavity direction once. Therefore it must occur an even number of times to make both ends have the same directed concavities. Also by Lemma 2, inflectional points can only occur  $n - 2$  or less times. Thus the total number of possible inflectional points must be even and no more than  $n - 2$ . Similarly, we can prove the second part of the lemma.

**LEMMA 4.** *There is only an odd number of inflectional points between two consecutive extrema.*

*Proof:* A maximum point is contained in a concave downward curve and a minimum point is contained in a concave upward curve. Thus by the definition of a inflectional point, we know that there is at least one inflectional point between two consecutive extrema. If there is more than one inflectional point between two consecutive extrema, then an even number of inflectional points must be added in between so that the directed concavity of the second extremum will not be changed.

**LEMMA 5.** *A necessary condition for the first derivative of  $f(x)$  at an inflectional point to be zero is that the curve on its left is either concave downward and increasing or concave upward and decreasing.*

*Proof:* The following two figures represent the two cases of this Lemma. The first derivative at inflectional point  $P$  can be zero only if the first derivatives of both sides of point  $P$  approach zero. From the two figures we can see that in order to have the first derivative of the left side of point  $P$  approach zero, it is necessary to have the left side of point  $P$  be either concave downward and increasing or concave upward and decreasing.

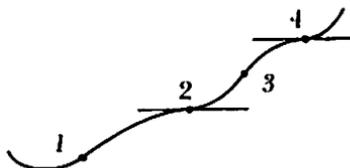


**LEMMA 6.** *A real polynomial function can not have two consecutive inflectional points with zero first derivatives.*

*Proof:* In both cases of Lemma 5, the curves on the right side of the inflectional points do not have the necessary condition of Lemma 5. Therefore the following inflectional point of each mentioned inflectional point can not have a zero first derivative.

**LEMMA 7.** *If there are  $I$  possible inflectional points between two consecutive extrema, then at most there can be  $I_{\text{..}} = \frac{I-1}{2}$  inflectional points with zero first derivatives.*

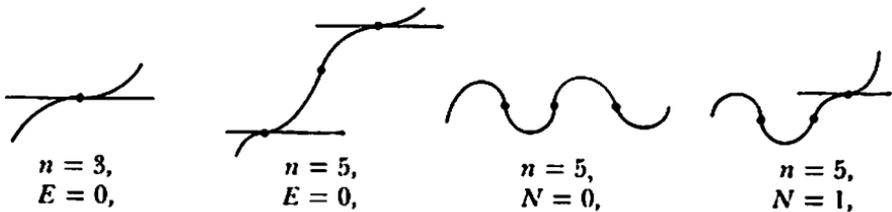
*Proof:* By Lemma 6 and also from the following graph, we can see clearly that only the even numbered inflectional points can possibly have zero first derivatives because they alone satisfy the



requirements of Lemma 5. But by Lemma 4, we know that  $I$  is always an odd number. Thus we can conclude that at most  $I_{\text{..}} = \frac{I-1}{2}$

LEMMA 8. Let  $f(x)$  be a real polynomial of degree  $n \geq 2$ , then there are at most  $I_o = \frac{n-E-1}{2}$  inflectional points with zero first derivatives.

*Proof:* Case 1. If  $n$  is odd, then by Theorem 1 we know that  $E$  must be an even number. If  $E = 0$  and  $n = 3$ , then at most  $f(x)$  may have one inflectional point with zero first derivative. Thus  $I_o = \frac{3-1}{2} = 1$ . If  $E = 0$  and  $n = 5$ , then at most  $f(x)$  may have two inflectional points with zero first derivatives. Thus  $I_o = \frac{5-1}{2} = 2$ . (Refer to figures below.) In general, if  $E = 0$  and  $n$  is odd, then at most  $f(x)$  can have  $I_o = \frac{n-1}{2}$  inflectional points with zero first derivatives.



If  $E$  is a non-zero even natural number, then  $E = (n-1) - 2N$  where  $N$  is some whole number. The curve of  $f(x)$  will need  $E-1$  inflectional points with non-zero first derivatives to maintain the concavity directions. If  $n = 5$  and  $N = 0$ , then  $E = (5-1) - 2(0) = 4$  and  $I_o = \frac{5-1}{2} - \frac{4}{2} = 0$ . If  $n = 5$  and  $N = 1$ , then  $E = (5-1) - 2(1) = 2$  and  $I_o = \frac{5-1}{2} - \frac{2}{2} = 1$ . (Refer to figures above.) Thus each time two extrema are added to the curve, two more inflectional points with non-zero first derivatives are needed. Consequently one possible inflectional point with zero first derivative will be eliminated. Therefore we conclude that when  $n$  is odd and  $E$  is even, at most  $I_o = \frac{n-1}{2} - \frac{E}{2} = \frac{n-E-1}{2}$ .

Case 2. If  $n$  is even, then  $E$  must be odd. We can prove, as in Case 1, that at most  $I_o = \frac{(n-2)}{2} - \frac{E-1}{2} = \frac{n-E-1}{2}$ .

**THEOREM 3.** If  $f(x)$  is a real polynomial function of degree  $n \geq 2$ ,  $F'_o$  is the number of different zeros of  $f'(x)$ , and  $I_o$  is the possible number of inflectional points with zero first derivatives of  $f(x)$ , then at most

$$I_o = n - (F'_o + 1)$$

or is less than this number by a multiple of even natural number.

*Proof:* From Theorem 2 and Lemma 8, we know that at most

$$I_o = \frac{n - E - 1}{2} \quad (1)$$

and

$$F'_o = E + I_o \quad (2)$$

From (2) we substitute  $E = F'_o - I_o$  into (1), we have at most

$$\begin{aligned} I_o &= \frac{n - (F'_o - I_o) - 1}{2}, \\ 2I_o &= n - F'_o + I_o - 1, \\ I_o &= n - (F'_o + 1). \end{aligned}$$

From this result and Lemma 6, we know that at most  $I_o = n - (F'_o + 1)$  or is less than this number by a multiple of even natural number.

The following examples illustrate the use of the three Theorems proved above.

**Example 1:** Find the extrema of  $f(x) = x^3$ .

*Solution:* Here  $n = 3$  and  $F'_o = 1$ . By Theorem 1, we know that  $E$  must be even. Also from Theorem 2,  $F'_o = E + I_o$ . It is clear that  $E$  must be 0. Therefore we conclude that there is no extremum.

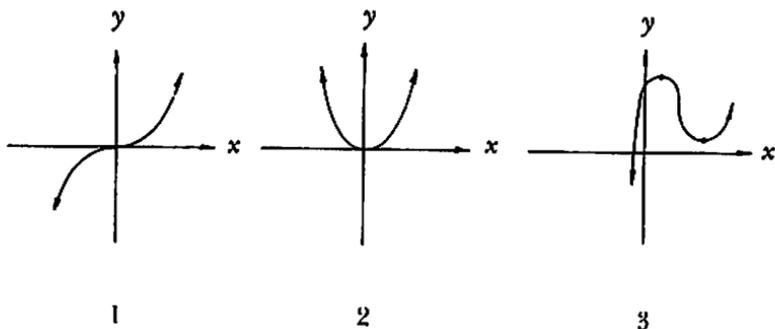
**Example 2:** Find the extrema of  $f(x) = x^4$ .

*Solution:* Here  $n = 4$  and  $F'_o = 1$ . By Theorem 3, at most  $I_o = 4 - 1 - 1 = 2$ , or  $2 - 2 = 0$ . But  $I_o = 2$  is impossible since by Theorem 2,  $F'_o \geq I_o$ . Thus by Theorem 1, we know that  $(0, f(0))$  is the minimum point.

Example 3. Find the extrema of  $f(x) = x^3 - 6x^2 + 9x + 1$ .

Solution:  $f'(x) = 3x^2 - 12x + 9$ . The zeroes are  $x = 1$  or  $3$ .

Here  $n = 3$ ,  $F'_0 = 2$ . By Theorem 3, we know that at most  $I_0 = 3 - 2 - 1 = 0$ . Thus by Theorem 1, we know that  $(1, f(1))$  is a maximum point and  $(3, f(3))$  is a minimum point.



Example 4. Find all extrema of  $f(x) = x^4 + \frac{4x^3}{3} - 4x^2$ .

Solution:  $f'(x) = 4x^3 + 4x^2 - 8x$ . The zeroes are  $x = -2, 0$ , or  $1$ . Here  $n = 4$ ,  $F'_0 = 3$ . By Theorem 3, at most  $I_0 = 4 - 3 - 1 = 0$ . Thus by Theorem 1, we know that  $(-2, f(-2))$  is a minimum point,  $(0, f(0))$  is a maximum point, and  $(1, f(1))$  is a minimum point.

Example 5. Find all extrema of  $f(x) = \frac{x^5}{5} - \frac{7x^3}{3} + 3x^2 + 1$ .

Solution:  $f'(x) = x^4 - 7x^2 + 6x$ . The zeroes are  $x = -3, 0$ , or  $2$ .

Here  $n = 5$ ,  $F'_0 = 4$ . By Theorem 3, at most  $I_0 = 5 - 1 - 4 = 0$ . By Theorem 1, we know that  $(-3, f(-3))$  is a maximum point,  $(0, f(0))$  is a minimum point,  $(1, f(1))$  is a maximum point, and  $(2, f(2))$  is a minimum point.

Example 6. Find the extrema of  $f(x) = \frac{x^5}{5} - \frac{3x^4}{4} + \frac{2x^3}{3} + 1$ .

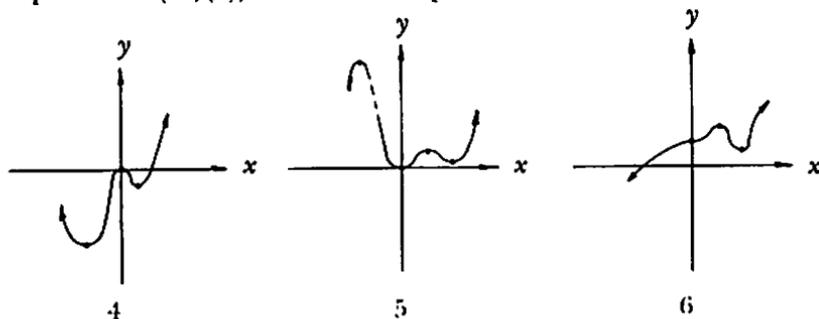
Solution:  $f'(x) = x^4 - 3x^3 + 2x^2$ . The zeroes are  $x = 0, 1$  or  $2$ .

Here  $n = 5$ ,  $F'_0 = 3$ . By Theorem 1, we know that  $E$  must be even. Also by Theorem 3, at most  $I_0 = 5 - 3 - 1 = 1$ . Thus there must be one inflectional point with zero first derivative. Now we have to find it:

$$f''(x) = 4x^3 - 9x^2 + 4x = x(4x^2 - 9x + 4) = 0.$$

$$f''(0) = 0, f''(0^-) < 0 \text{ and } f''(0^+) > 0.$$

Thus  $(0, f(0))$  is the only inflectional point with zero first derivative. Hence by Theorem 1, we know that  $(1, f(1))$  is a maximum point and  $(2, f(2))$  is a minimum point.



From the above examples, we can see that we have indeed eliminated many unnecessary calculations and tests for extrema.

# Antifactorials

MARK ATKINS

*Student, Monte Vista High School, Spring Valley, California*

The factorial function is a familiar and frequently used function when defined over the positive integers. It is usually not defined over a larger domain. In particular, the factorial function is not usually defined so as to be continuous and to have an inverse. The purpose of this article is to discuss such an extension of definition.

Over the positive integers, the factorial function of  $x$ , written  $x!$ , is defined as follows

$$\begin{aligned}1! &= 1 \\2! &= 2 \cdot 1 \\3! &= 3 \cdot 2 \cdot 1 \\4! &= 4 \cdot 3 \cdot 2 \cdot 1 \\&\vdots \\x! &= x \cdot (x-1) \cdot (x-2) \dots 3 \cdot 2 \cdot 1 \\&\text{or} \\x! &= \sum_{i=1}^x i\end{aligned}$$

Although  $x!$  is easily computed for any positive integer value of  $x$ , it is clear that the computation is dependant on the size of  $x$ . There exists no fixed equation in which the value of  $x$  can be "plugged in."

As long as one works with the positive integers, it is relatively easy to solve for  $x$  in equations such as  $x = 6!$  or  $6 = x!$ . But is there a solution for something like  $x! = 10$ ? Within the positive integers there is no solution since  $3! < 10$  and  $10 < 4!$

When the values of  $x$  are positive integers, the graph of  $y = x!$  is a set of isolated points. Suppose that the points are connected to form a continuous curve instead, such as shown in Figure 1. From such a curve, a value of  $x$  such that  $x! = 10$  could be estimated by finding where the line  $y = 10$  intersects the curve and dropping

a vertical line from that point. Where that line intersected the  $x$ -axis would be the value of  $x$ , about 3.6 in this case.

But the resulting estimation is useless unless a suitable definition or meaning of a non-integer factorial can be found. The definition will have to be extended so as to still yield the same values for positive integers as before.

One way  $3.5!$  might be interpreted is to form factors by subtracting one from each previous factor, stopping short of zero and then finding the product. Thus we would have for any positive number  $x$ ,

$$x! = (x)(x-1)(x-2)\dots(x-n), \\ 0 \leq x-n < 1$$

Then, for example,

$$3.5! = (3.5)(2.5)(1.5)(0.5) = 6.5625.$$

Unfortunately, this function is discontinuous at each integer, since the limit from the left is clearly the usual factorial value while the limit from the right is zero. The desired continuous curve is clearly absent and consequently this formula for finding factorials over the positive reals must be abandoned.

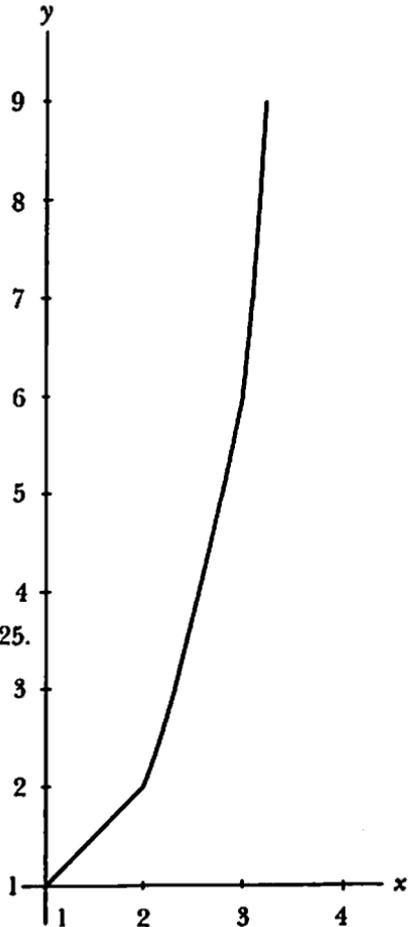


FIGURE 1

There is only one minor point wrong in the preceding procedure. Note that the partial products of  $3.5!$  are constantly increasing until the very last factor is used, when the answer is suddenly decreased:

$$\begin{aligned}
 (3.5) &= 3.5 \\
 (3.5) (2.5) &= 8.75 \\
 (3.5) (2.5) (1.5) &= 13.125 \\
 (3.5) (2.5) (1.5) (0.5) &= 6.5625.
 \end{aligned}$$

The assumption was that the last factor should be greater or equal to zero. Clearly it would be better if it is greater than or equal to one. Thus we can define  $x!$  for the positive reals greater than 1 as follows:

$$x! = (x)(x-1)(x-2)\dots(x-n) \text{ where } 1 \leq x-n < 2$$

Using this new definition we have

$$(3.5)! = (3.5)(2.5)(1.5) = 13.125.$$

Using this new definition a continuous factorial curve as in Figure 1 is produced, beginning with  $x = 1$ .

We can now begin to study the inverse of  $x!$  as defined above. The name "antifactorial" is appropriate to name this inverse, just as antilogarithm names the inverse function of logarithms. We need a useful symbol to mean "antifactorial". The factorial symbol "!" reversed looks like "!", but this new symbol looks much like the complex unit  $i$ . There is another symbol used for factorial  $x$ ,  $\lfloor x$ ; its reverse,  $\overline{x}$ , is simple to write, and has the advantage of covering a large expression. Thus, we will adopt  $\overline{x}$  to read "antifactorial  $x$ " and thus  $\overline{x!} = x$  and  $\overline{x}! = x$  for  $x \geq 1$ .

Now if  $1 \leq x < 2$ , then  $x! = x = \overline{x}$ . If  $2 \leq x < 3$ , then  $x! = x(x-1)$  from our definition above. Furthermore,  $2! \leq x! < 3!$  or  $2 \leq x! < 6$  for this interval. Let  $y = x!$ , so we have

$$\begin{aligned}
 y &= x! = x(x-1) \\
 y &= x^2 - x \\
 0 &= x^2 - x - y
 \end{aligned}$$

Applying the quadratic formula and taking the positive square root (the negative of the square root is useless here) yields  $x = \frac{1 + \sqrt{1+4y}}{2}$ . But  $\overline{y} = \overline{x!} = x$ , so  $\overline{y} = \frac{1 + \sqrt{1+4y}}{2}$  whenever  $2 \leq y < 6$ .

If  $3 \leq x < 4$ , then  $3! \leq x! < 4!$  and if  $y = x!$ ,  $6 \leq y < 24$ . In this interval, we have the equation

$$y = x! = x(x-1)(x-2) \\ = x^3 - 3x^2 + 2x$$

If, for example,  $y = 10$ , then we have the equation  $x^3 - 3x^2 + 2x - 10 = 0$  to solve. This cubic equation can be solved using the classical solution

$$\sqrt[3]{y} = 1 + \sqrt[3]{\frac{y}{2} + \sqrt{\frac{y^2}{4} - \frac{1}{27}}} + \sqrt[3]{\frac{y}{2} - \sqrt{\frac{y^2}{4} - \frac{1}{27}}}$$

The calculations in this solution are very tedious and difficult, so iteration techniques are recommended for finding  $\sqrt[3]{y}$  for  $6 \leq y < 24$ . If a computer is available, the above formula is the easier method of solution.

Finding antifactorials for the interval  $24 \leq y < 120$ , involves solving the fourth degree equation:

$$y = x! = x(x-1)(x-2)(x-3) \\ = x^4 - 6x^3 + 11x^2 - 6x$$

This would appear to be even more difficult than the previous cubic. However, solving the resolvent cubic equation always yields the answer of 2. This fact about the biquadratic solution simplifies the general formula for  $24 \leq y < 120$  to

$$\sqrt[3]{y} = \frac{3 + \sqrt{5 + 4\sqrt{y+1}}}{2}$$

There is no general solution for  $\sqrt[3]{y}$  if  $y > 120$ , since the resulting equations are of fifth degree or higher and, as is well known, no general solution exists.

Although presently there appears to be no need for the general factorial definition and its inverse as presented here, they are an excellent example of how a strictly integer function can be extended to a continuous function with an inverse, much as square roots and logarithms were extended. Factorials are becoming increasingly more useful and antifactorials could suddenly become a vital part of physics or some other field.

Here is a brief table of some factorials and antifactorials as calculated using a computer:

$x$	$x!$	$\sqrt{x}$
1	1	1.00000
2	2	2.00000
3	6	2.30278
$\pi$	7.68069	2.34163
4	24	2.56155
5	120	2.79129
6	720	3.00000
7	5040	3.08674
8	40320	3.16631
9	362880	3.24004
10	3628800	3.30891
11	39916800	3.37364
12	479001600	3.43484
13	6227020800	3.49294
14	87178291200	3.54831
15	1307674368000	3.60125
16	20922789888000	3.65202
17	355687428096000	3.70080
18	6402373705728000	3.74784
19	121645100408832000	3.79323
20	2432902008176640000	3.83714

# One Possible Algebraic Structure For Ordered Pairs of Real Numbers

EARL W. KING AND F. MAX STEIN

*Student and Faculty, Colorado State University\**

1. *Introduction.* When one studies ordered pairs of real numbers, he is often concerned with functions. In other cases ordered pairs may be thought of as vectors, and certain binary operations are defined between these ordered pairs. In this paper we will consider ordered pairs with an alternate definition of addition as one binary operation and will examine the resulting algebraic structure.

We will also examine the multiplication of ordered pairs as a second binary operation and see some of the geometrical consequences.

2. *Addition of ordered pairs.* Let  $S = \{x_1, x_2, \dots\}$  be a set of ordered pairs of real numbers, and let the ordered pair  $x$  be denoted by

$$x = (x, y).$$

We shall refer to  $x$  as the *first coordinate* and  $y$  as the *second coordinate*. The ordered pairs  $x_1 = (x_1, y_1)$  and  $x_2 = (x_2, y_2)$  are defined to be equal if and only if  $x_1 = x_2$  and  $y_1 = y_2$ , and we write

$$x_1 = x_2.$$

The *addition* of two ordered pairs  $x_1$  and  $x_2$  is defined by

$$x_1 + x_2 = x_3 \quad \text{where} \quad x_3 = (x_1 y_2 + x_2 y_1, y_1 y_2).$$

**THEOREM 1.** *If  $S_1$  is the set of elements of  $S$  with second coordinate zero, then the set  $S_1 = \{x_1, x_2, \dots\}$  forms an abelian group under addition.*

*Proof:* We need to check the requirements for  $S_1$  to form a group under addition.

\*Prepared in an Undergraduate Research Participation Program at Colorado State University by Mr. King under the direction of Professor Stein.

(a) The set  $S_1$  is *closed* under addition since for  $x_1, x_2 \in S_1$ , then

$$x_1 + x_2 = x_3$$

is an ordered pair of real numbers with the second coordinate not zero; i.e.,  $x_3 \in S_1$ .

(b) Addition of the elements of  $S_1$  is *associative* since

$$\begin{aligned}(x_1 + x_2) + x_3 &= (x_1y_2 + x_2y_1, y_1y_2) + (x_3, y_3) \\ &= (x_1y_2y_3 + x_2y_1y_3 + x_3y_1y_2, y_1y_2y_3)\end{aligned}$$

and

$$\begin{aligned}x_1 + (x_2 + x_3) &= (x_1, y_1) + (x_2y_3 + x_3y_2, y_2y_3) \\ &= (x_1y_2y_3 + x_2y_1y_3 + x_3y_1y_2, y_1y_2y_3),\end{aligned}$$

the same result.

(c) The left and right additive *identity* is  $0 = (0,1)$ , since for any  $x$  in  $S_1$ ,

$$x + 0 = 0 + x = x,$$

a result which follows from the definition of addition.

(d) The left and right additive *inverse* of  $x = (x,y)$  is  $-x = (-x/y^2, 1/y)$  such that

$$x + (-x) = (-x) + x = 0$$

a result easy to check from the definition of addition. (Note that elements of  $S$  with second coordinate zero do not have additive inverses.)

(e) The elements of  $S_1$  *commute* under addition since

$$\begin{aligned}x_1 + x_2 &= (x_1y_2 + x_2y_1, y_1y_2) \\ &= (y_2x_1 + y_1x_2, y_2y_1) = x_2 + x_1.\end{aligned}$$

This completes the proof of the theorem.

**3. Multiplication of Ordered Pairs.** The *multiplication* of two ordered pairs of  $S$ , say  $x_1$  and  $x_2$  is defined by

$$x_1 \cdot x_2 = (x_1x_2, y_1y_2).$$

Let the set  $S_2$  consist of the elements of the set  $S$  except for those elements of  $S$  that have at least one zero as a coordinate.

**THEOREM 2.** *The set  $S_2$  forms an abelian group under multiplication.*

*Proof:* We need to check the requirements for  $S_2$  to form a group under multiplication.

(a) If  $x_1$  and  $x_2$  are elements of  $S_2$ , then  $S_2$  is closed under multiplication since

$$x_1 \cdot x_2 = (x_1x_2, y_1y_2)$$

is again an ordered pair of real numbers in  $S_2$ .

(b) If  $x_1$  and  $x_2$  are elements of  $S_2$ , then

$$\begin{aligned} (x_1 \cdot x_2) \cdot x_3 &= (x_1x_2, y_1y_2) \cdot (x_3, y_3) \\ &= (x_1x_2x_3, y_1y_2y_3) \\ &= (x_1, y_1) \cdot (x_2x_3, y_2y_3) \\ &= x_1 \cdot (x_2 \cdot x_3). \end{aligned}$$

Thus the *associative* law holds.

(c) The left and right multiplicative *identity* for any element  $x$  in  $S_2$  is  $I = (1,1)$  and is such that

$$x \cdot I = I \cdot x = x.$$

(d) The left and right multiplicative *inverse* for  $x = (x,y)$  is  $x^{-1} = (1/x, 1/y)$ , an element of  $S_2$ . For any  $x$  in  $S_2$ ,

$$x \cdot x^{-1} = x^{-1} \cdot x = I.$$

(e) The elements of  $S_2$  *commute* under multiplication since

$$x_1 \cdot x_2 = (x_1x_2, y_1y_2) = (x_2x_1, y_2y_1) = x_2 \cdot x_1.$$

This completes the proof of the theorem.

4. *Preliminary Conclusions.* We have shown that the set  $S_1$  forms an abelian group under addition and the set  $S_2$  forms an abelian group under multiplication. However, the set  $S_2$  together with the additive identity for  $S_1$  does not form a field with respect to the two operations defined. First, the distributive law for elements of  $S_2$  does not hold for addition since

$$\begin{aligned} x_1 \cdot (x_2 + x_3) &= (x_1, y_1) \cdot (x_2 y_3 + x_3 y_2, y_2 y_3) \\ &= (x_1 x_2 y_3 + x_1 x_3 y_2, y_1 y_2 y_3) \\ x_1 \cdot x_2 + x_1 \cdot x_3 &= (x_1 x_2, y_1 y_2) + (x_1 x_3, y_1 y_3) \\ &= (x_1 x_2 y_1 y_3 + x_1 x_3 y_1 y_2, y_1^2 y_2 y_3), \end{aligned}$$

which is not the same result. Secondly, the set  $S_2$  is not closed under addition since there exist elements  $x_1$  and  $x_2$  in  $S_2$  such that  $x_1 + x_2$  is not in  $S_2$ . For example, let  $x_1 = (-2, 3)$  and  $x_2 = (4, 6)$ , then

$$x_1 + x_2 = (-2, 3) + (4, 6) = (-12 + 12, 18) = (0, 18)$$

which is not an element of  $S_2$ .

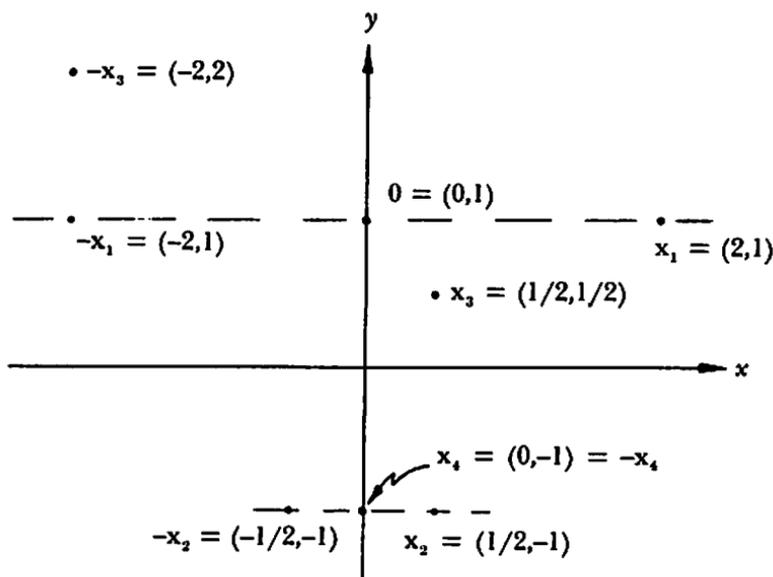


FIGURE 1

5. *The Geometry Associated With Elements of S.* From Figure 1 we are able to see the relationship between an ordered pair  $x$  and its additive inverse (if it has one). Under addition, points in the first and fourth quadrant have additive inverses in the second and third quadrants respectively. It is interesting to observe that for points which have 1 or  $-1$  as a second coordinate, the additive inverses are found by reflecting these points in the  $y$ -axis. Also note that the only points which are their own additive inverses are  $(0,1)$  and  $(0,-1)$ . Recall that all points on the  $y$ -axis have additive inverses while no points on the  $x$ -axis do.

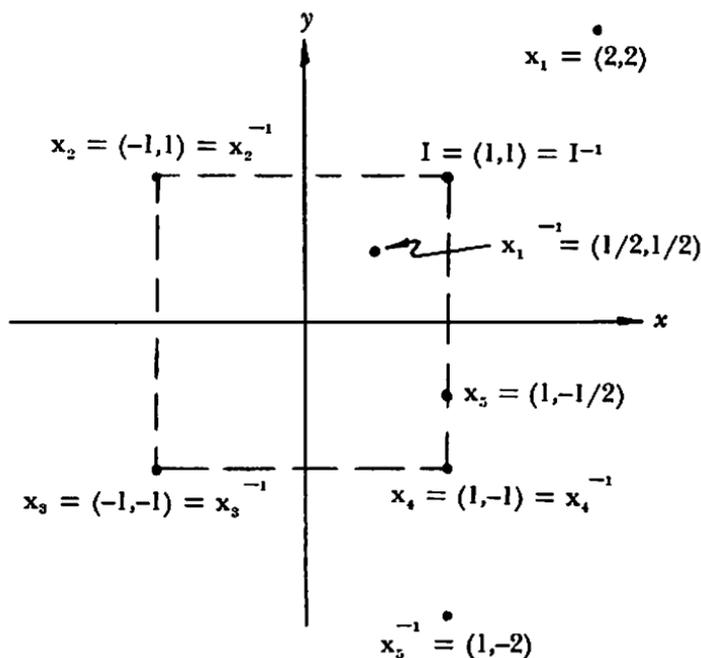


FIGURE 2

The multiplicative inverse of a point (having an inverse) which lies within the square of side 2 is outside this square, see Figure 2. In particular, a point  $(x,y)$  lying in the first quadrant has an inverse  $(u,v)$  such that if  $0 < x < 1$ ,  $0 < y < 1$ , then  $1 < u$ ,  $1 < v$ . If  $0 < x < 1$ ,  $1 < y$ , then  $1 < u$ ,  $0 < v < 1$ . If  $1 < x$ ,  $0 < y < 1$ , then  $0 < u < 1$ ,  $1 < v$ . Finally, if  $1 < x$ ,  $1 < y$ , then  $0 < u < 1$ ,  $0 < v < 1$ . Inverses of points in the other quadrants

can be determined similarly. Points having inverses and which lie on a side of the square have inverses on the same side of the square extended and in the same quadrant, and vice versa. The points  $(1,1)$ ,  $(1,-1)$ ,  $(-1,1)$ , and  $(-1,-1)$  are their own inverses.

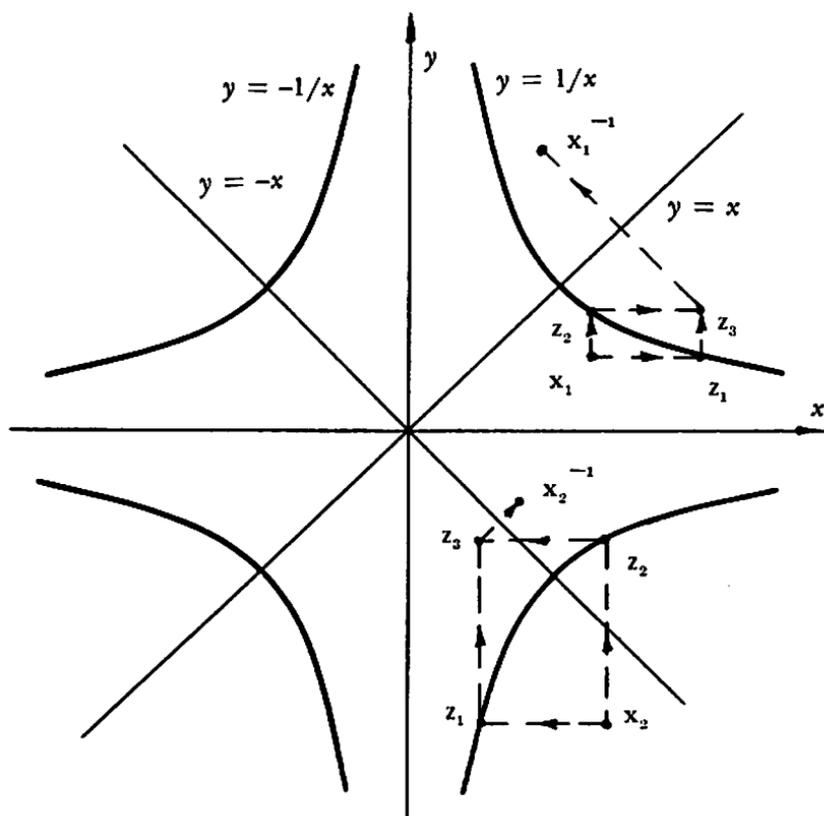


FIGURE 3

Figure 3 illustrates geometrically how  $x^{-1}$  can be obtained for a point  $x$  in the first quadrant. For any point  $x_1 = (x_1, y_1)$  in the first quadrant, draw a horizontal line from  $x_1$  until it intersects the curve  $y = 1/x$  in the point  $z_1 = (1/y_1, y_1)$ . Now draw a vertical line from  $z_1$ . From  $x_1$  draw a vertical line until it intersects the curve  $y = 1/x$  in the point  $z_2 = (x_1, 1/x_1)$ . Draw a horizontal line from  $z_2$  until it

intersects the vertical line drawn from  $z_1$ . Call the point of this intersection  $z_2$ , where  $z_2 = (1/y_1, 1/x_1)$ . Finally, reflect  $z_2$  in the line  $y = x$  to obtain the point  $x_1^{-1} = (1/x_1, 1/y_1)$ . For a point in another quadrant the inverse can be obtained using the other branch of the hyperbola  $y = 1/x$  or the hyperbola  $y = -1/x$  and the line  $y = -x$ .

6. *Conclusion.* The reader may have already noticed that the operations of addition and multiplication that we have used for ordered pairs are similar to those used for adding and multiplying fractions. For example, adding the two fractions  $a/b = (a,b)$  and  $c/d = (c,d)$ , we get  $(a,b) + (c,d) = a/b + c/d = (ad + bc)/bd = (ad + bc, bd)$ . For fractions we customarily reduce to lowest terms, and thus, for example,  $2/4 = 1/2$ ; in our case  $(2,4) \neq (1,2)$ .

For fractions if  $x_1 = x_2$ , then  $x_1 = a x_2$  and  $y_1 = a y_2$ ,  $a \neq 0$ . Here we have a set of *equivalence classes* which geometrically are straight lines through the origin. The additive identity  $(0,1)$  is then equivalent to any point on the  $y$ -axis, and the additive inverse of  $x = (x,y)$  is  $-x = (-x/y^2, 1/y) = (-x,y)$ , after the fraction is reduced to lowest terms. The additive inverse of  $x$  is thus the reflection of  $x$  in the  $y$ -axis.

The multiplicative identity is  $(1,1) = 1/1$ ; and the equivalence class for the identity is thus any point on the line  $y = x$ , except the origin, of course.

For fractions the multiplicative inverse of  $x = (x,y)$  would apparently be  $x^{-1} = (1/x, 1/y) = (y,x)$ , the reciprocal. A look at the following simple example shows that something more is needed for a geometrical interpretation in this case. If  $x = (2,3)$ , the inverse would apparently be  $(3,2)$ . But both points  $(2,3)$  and  $(3,2)$  lie outside the unit circle with center at the origin — and both points lie at the same distance from the origin. The discussion is terminated with this problem unresolved. A simple explanation can be made, however, if the ordered pairs are considered as fractions.

#### REFERENCES

1. Curtis, Charles W. *Linear Algebra*. Boston: Allyn and Bacon Inc., 1968.
2. Olmsted, John M. H. *Advanced Calculus*. New York: Appleton-Century-Crofts, 1961.

# Logarithms

JANET MAUCK

*Student, Anderson College*

College Algebra was a breeze  
Until logarithms.  
Math was easy as the ABC's  
Until logarithms.  
Until logarithms  
I had no problems in my life  
Until logarithms  
My mind was never torn by strife  
Until logarithms  
I liked mathematics (well, sort of).

Logarithms are  
Base systems in reverse  
But really they are  
The exponential inverse.  
Got that all O.K.?  
You didn't  
Too bad, I'll have to go on anyway.

We were puzzled enough  
Learning logs in base ten  
(Where everything is backwards, you see)  
Then our minds became confused again  
When we're told a better base is e.

Then I discovered antilogs  
Which filled my heart with gladness  
At last! A term that described  
How I felt about all this madness.  
But my newfound joy turned to sadness  
When I learned antilogs aren't against logs  
Instead they reverse the inverse  
Or inverse the reverse  
Or something like that.

Until logarithms  
I thought polynomial functions were tough  
Now they seem as easy as two plus two  
Oh, logarithms, of you I've had enough!

# A Lattice Point Problem

DANA S. NAU

*Student, University of Missouri at Rolla*

One of the problems on the 32nd annual William Putnam Mathematical Competition was the following: "Let there be given nine lattice points (points with integral coordinates) in three dimensional Euclidian space. Show that there is a lattice point on the interior of one of the line segments joining two of these points." After finishing the test, the author became interested in generalizing the problem to  $n$  dimensions for arbitrary  $n$ ; i.e., how many lattice points can be given in  $n$  dimensional Euclidian space without there being a lattice point on the interior of at least one of the lines joining any two of the points? As it turns out, the answer is  $2^n$ , and the proof of this theorem is the subject of this paper.

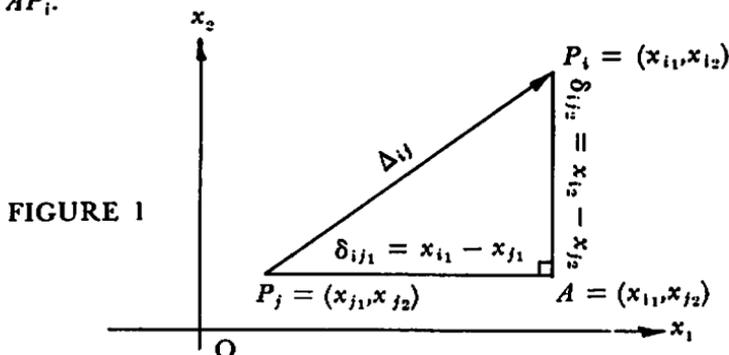
The author is not familiar with any conventional symbols (if there are any) used to characterize lattice points and related concepts. Therefore, it is necessary to define notation for a number of the concepts used below. The use of so much notation may be confusing to the reader; however, without the use of notation, the proofs presented would be even more confusing, as well as much longer. The following listing presents the various notations, with explanatory notes where necessary.

$$1. \quad \left( \begin{array}{l} P_1 = (x_{11}, x_{12}, \dots, x_{1n}) \\ P_2 = (x_{21}, x_{22}, \dots, x_{2n}) \\ \vdots \\ P_m = (x_{m1}, x_{m2}, \dots, x_{mn}) \end{array} \right)$$

denotes a set of  $m$  lattice points in  $n$  dimensional Euclidian space. Since a lattice point is defined to be a point with integral coordinates, it follows that each one of the  $x_{ij}$  is an integer.

2.  $L(P_i, P_j)$  denotes the set of all points on the interior of the line segment between  $P_i$  and  $P_j$ , where  $P_i$  and  $P_j$  are lattice points. In other words, this is the set of all points on the line segment between  $P_i$  and  $P_j$  except for the endpoints  $P_i$  and  $P_j$  themselves.

3.  $\delta_{ijk} = x_{ik} - x_{jk}$ , where  $x_{ik}$  and  $x_{jk}$  are the  $k$ 'th coordinates of the lattice points  $P_i$  and  $P_j$ . This is equivalent to taking the  $x_k$ 'th component of the vector starting at  $P_j$  and ending at  $P_i$ . For an illustration in two dimensions, see Figure 1, where  $\delta_{ij_1}$  is the length of the line segment  $\overline{P_jA}$  and  $\delta_{ij_2}$  is the length of the line segment  $\overline{AP_i}$ .



Since  $P_i$  and  $P_j$  both have integral coordinates, it follows that the  $\delta_{ijk}$  are all integers.

4.  $\Delta_{ij}$  is defined to be the vector  $(\delta_{ij_1}, \delta_{ij_2}, \dots, \delta_{ij_n})$ , i.e., the vector having initial point  $P_j$  and terminal point  $P_i$ . (See Figure 1)

5.  $M(P_j) = ((-1)^{\delta_{1j_1}}, (-1)^{\delta_{1j_2}}, \dots, (-1)^{\delta_{1j_n}})$ . This expression can be thought of as a point, a vector, or a row matrix; it really doesn't matter which one, since the only property needed here is that any two of the  $M$ 's are equal if and only if their corresponding components are equal. Given a lattice point  $P_j$ ,  $M(P_j)$  takes the vector  $\Delta_{ij}$ , looks at the value of each component of the vector, and assigns to it a 1 or a -1, depending on whether the component is even or odd. Thus  $M(P_j)$  is a collection of  $n$  1's and -1's, separated by commas.

Now that the reader has been introduced to the notation, we will proceed with the derivation of the answer to the problem, by means of two lemmas and the theorem.

**LEMMA 1.** *Let  $P_i$  and  $P_j$  be lattice points in  $n$  dimensional Euclidean space. Then  $L(P_i, P_j)$  contains no lattice points if and only if the greatest common divisor  $g = \text{GCD}(\delta_{ij_1}, \delta_{ij_2}, \dots, \delta_{ij_n})$  is 1.*

*Proof:* ( $\rightarrow$ ) Suppose  $g \neq 1$ . Then  $g \geq 2$ , since the GCD is a positive integer. Furthermore, by the definition of the GCD,  $\delta_{ijk}/g \in I$ ,  $k = 1, 2, \dots, n$ , where  $I$  is the set of integers. The vector equation of the line through  $P_i$  and  $P_j$  is

$$P = P_j + t \Delta_{ij}, \quad (1)$$

where  $P$  is any point on the line,  $\Delta_{ij}$  is the vector between  $P_j$  and  $P_i$  as defined above, and  $t$  is a parameter. For  $0 < t < 1$ ,  $P$  will be in  $L(P_i, P_j)$ . Certainly,  $0 < 1/g < 1$ . Therefore,  $P_{1/g} = P_j + (1/g)\Delta_{ij}$  is in  $L(P_i, P_j)$ . But then,

$$\begin{aligned} P_{1/g} &= (x_{j_1}, x_{j_2}, \dots, x_{j_n}) + (\delta_{ij_1}/g, \delta_{ij_2}/g, \dots, \delta_{ij_n}/g) \\ &= (x_{j_1} + \delta_{ij_1}/g, x_{j_2} + \delta_{ij_2}/g, \dots, x_{j_n} + \delta_{ij_n}/g). \end{aligned}$$

Therefore, since  $x_{jk}$  and  $\delta_{ijk}/g$  are both integers,  $k = 1, 2, \dots, n$ , it follows that  $P_{1/g}$  is a lattice point.

( $\leftarrow$ ) Suppose  $L(P_i, P_j)$  contains a lattice point  $P_{1/g}$ . Then from (1),

$$P_{1/g} = P_j + t \Delta_{ij} \text{ for some } t \in (0, 1), \quad (2)$$

and

$$(t) (\delta_{ijk}) \in I, \quad k = 1, 2, \dots, n. \quad (3)$$

But since  $\delta_{ijk}$  is an integer, (3) can be true if and only if  $t$  is rational. Therefore, there exist integers  $a$  and  $b$  such that  $b > 0$ ,  $t = a/b$ , and  $\text{GCD}(a, b) = 1$ . Since  $(a) (\delta_{ijk})/b$  is an integer,  $b$  divides  $(a) (\delta_{ijk})$ . Since  $\text{GCD}(a, b) = 1$ ,  $b$  divides  $\delta_{ijk}$ ,  $k = 1, 2, \dots, n$ . Therefore,  $g \geq b$ . Suppose  $b = 1$ . Then  $t = a/b = a \in I$ , so  $t \notin (0, 1)$ , contradicting (2). Therefore,  $b \neq 1$ , so  $g \geq b \geq 2$ .

**LEMMA 2.** *Let  $P_i$  and  $P_j$  be lattice points in  $n$  dimensional Euclidian space such that  $M(P_i) = M(P_j)$ . Then  $L(P_i, P_j)$  contains a lattice point.*

*Proof:* If  $M(P_i) = M(P_j)$ , then from the definition of  $M$  we have

$$(-1)^{\delta_{1j_1}} = (-1)^{\delta_{1i_1}}, \text{ so } \delta_{1j_1} \text{ and } \delta_{1i_1} \text{ are of like parity,}$$

$$(-1)^{\delta_{1j_2}} = (-1)^{\delta_{1i_2}}, \text{ so } \delta_{1j_2} \text{ and } \delta_{1i_2} \text{ are of like parity,}$$

$$\vdots$$

$$(-1)^{\delta_{1j_n}} = (-1)^{\delta_{1i_n}}, \text{ so } \delta_{1j_n} \text{ and } \delta_{1i_n} \text{ are of like parity,}$$

Therefore,

$$\delta_{1j_1} - \delta_{1i_1} = (x_{11} - x_{j_1}) - (x_{11} - x_{i_1}) = x_{11} - x_{j_1} = \delta_{1j_1} \text{ is even.}$$

$$\delta_{1j_2} - \delta_{1i_2} = (x_{12} - x_{j_2}) - (x_{12} - x_{i_2}) = x_{i_2} - x_{j_2} = \delta_{1j_2} \text{ is even.}$$

$$\vdots$$

$$\delta_{1j_n} - \delta_{1i_n} = (x_{1n} - x_{j_n}) - (x_{1n} - x_{i_n}) = x_{i_n} - x_{j_n} = \delta_{1j_n} \text{ is even.}$$

Therefore,  $\text{GCD}(\delta_{1j_1}, \delta_{1j_2}, \dots, \delta_{1j_n}) \geq 2 > 1$ , so by Lemma 1,  $L(P_i, P_j)$  contains a lattice point.

**THEOREM.** *If  $S = \{P_1, P_2, \dots, P_m\}$  is a set of  $m$  lattice points in  $n$  dimensional Euclidian space, then the maximum value of  $m$  such that  $L(P_i, P_j)$  contains no lattice points,  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, m$ , is  $m = 2^n$ .*

*Proof:* We must show (1) that there can be values assigned to  $P_1, P_2, \dots, P_{2^n}$  such that  $L(P_i, P_j)$  contains no lattice points,  $i = 1, 2, \dots, 2^n$  and  $j = 1, 2, \dots, 2^n$ , and (2) that for any values of  $P_1, P_2, \dots, P_m$  where  $m > 2^n$ , there exist  $i$  and  $j$  such that  $L(P_i, P_j)$  contains at least one lattice point.

(1) There exist exactly  $2^n$  points in  $n$  dimensional Euclidian space whose coordinates consist solely of ones and zeros. Let  $P_1, P_2, \dots, P_{2^n}$  be these points. Then  $\delta_{ijk} = x_{ik} - x_{jk} = 1$  or  $0$  or  $-1$ ,  $i = 1, 2, \dots, 2^n$ ,  $j = 1, 2, \dots, 2^n$ ,  $k = 1, 2, \dots, n$ . Therefore,  $\text{GCD}(\delta_{1j_1}, \delta_{1j_2}, \dots, \delta_{1j_n}) = 1$ , so by Lemma 1,  $L(P_i, P_j)$  contains no lattice points,  $i = 1, 2, \dots, 2^n$ ,  $j = 1, 2, \dots, 2^n$ .

(2) Suppose  $m > 2^n$ . There exist only  $2^n$  different ways of arranging 1's and -1's. Since each  $M(P_i)$  is an arrangement of  $n$  1's and -1's, there are only  $2^n$  distinct  $M(P_i)$ . Therefore, since  $m > 2^n$ , there must exist  $P_i$  and  $P_j$  in  $S$  such that  $M(P_i) = M(P_j)$ . Therefore, by Lemma 2, there is at least one lattice point in  $L(P_i, P_j)$ .

The author would like to thank those who compiled the problems for the 32nd annual William Lowell Putnam Mathematical Competition for including on the examination the problem that gave him the idea for this paper.

# Our Intuition Can Sometimes Fail Us

FRANCIS A. FUSSENEGGER

*Student, Washburn University*

Most students of calculus are familiar with the limiting process used in defining arc length. Since an arc in the plane bears a slight resemblance to a surface in space, it would seem that surface area could be defined by using a method similar to that used for arc length. It so happens that the concept of surface area is more complicated than it might first appear. Even though it intuitively seems possible, an attempt to define surface area using a similar limiting method will fail.

The definition of arc length is very straight forward. Suppose we wish to define the length of an arc  $C$  from point  $A$  to point  $B$  in the plane. First we choose a number of points on the arc and label them  $P_1, P_2, P_3, \dots, P_{n-1}$  as in Figure 1. Setting  $A = P_0$  and  $B = P_n$  we draw a straight line segment from point  $P_i$  to the next point  $P_{i+1}$ , as in Figure 2.

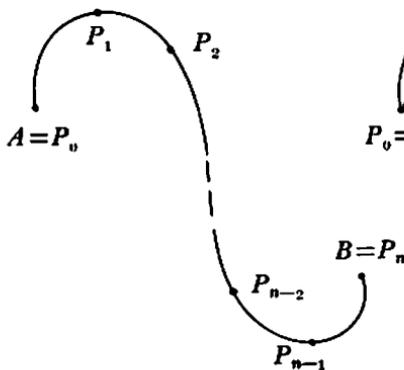


FIGURE 1

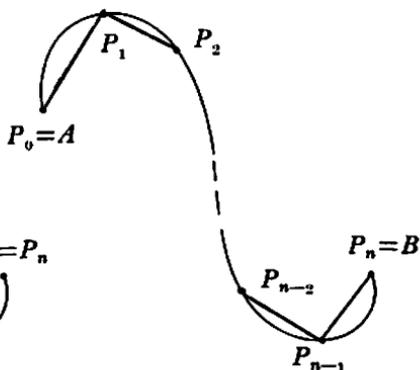


FIGURE 2

We then compute the length of these line segments

$$\overline{P_0P_1}, \overline{P_1P_2}, \overline{P_2P_3}, \dots, \overline{P_{n-1}P_n}$$

and add them to get

$$\sum_{i=1}^n \overline{P_{i-1}P_i}$$

If we let these points  $P_i$  get closer together, the sum of the lengths will approach what is called the arc length. Therefore, we denote the length of the longest line segment as  $\|\Delta\|$ , called the norm of the subdivision, and allow it to approach zero. Using this method we can define arc length.

**DEFINITION.** An arc from  $A$  to  $B$  has length  $L$  if for each  $\epsilon > 0$  there is a  $\delta > 0$  such that

$$\left| \sum_{i=1}^n \overline{P_{i-1}P_i} - L \right| < \epsilon$$

for every subdivision  $A = P_0, P_1, \dots, P_n = B$  with  $\|\Delta\| < \delta$ .

In other words the length of the subdivisions approach  $L$  as  $\|\Delta\|$  approaches zero.

Let us apply this method to surface area. Given a surface  $S$ , we select a number of points on this surface and draw lines between them so that we define triangles in space as in Figure 3. We thereby

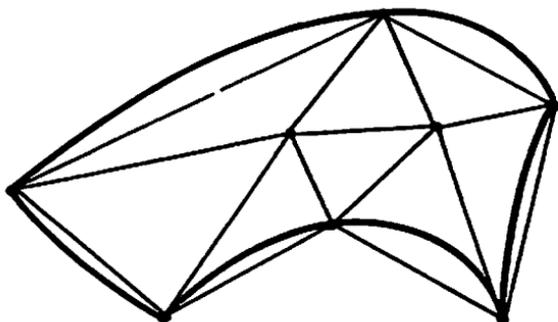


FIGURE 3

approximate the surface with a polyhedron. If we let the points get closer together so that the area of the largest triangle approaches

zero, just like we let  $||\Delta||$  approach zero, it would seem intuitive that the sum of the areas of the triangles would approach the area of the surface. We will show in an example due to H. A. Schwarz that, using this method to define surface area, the lateral surface of a right circular cylinder can be assigned an arbitrarily large finite area as well as an infinite area.

Let  $S$  be the right circular cylinder

$$S = \{(x,y,z) \mid x^2 + y^2 = 1, 0 \leq z \leq 1\}$$

and let  $m$  and  $n$  be positive integers. Let  $2m+1$  circles  $C_k$  be defined for  $k=0,1,2, \dots, 2m$  :

$$C_k = S \cap \{(x,y,z) \mid z = k/2m\}$$

On each of these  $2m+1$  circles let  $n$  equally spaced points  $P_{kj}$  be defined for  $j=0,1,2, \dots, n-1$  :

$$P_{kj} = \begin{cases} \left( \cos \frac{2j\pi}{n}, \sin \frac{2j\pi}{n}, \frac{k}{2m} \right), & \text{if } k \text{ is even} \\ \left( \cos \frac{(2j+1)\pi}{n}, \sin \frac{(2j+1)\pi}{n}, \frac{k}{2m} \right), & \text{if } k \text{ is odd} \end{cases}$$

For each circle  $C_k$  the points  $P_{kj}, j=0,1, \dots, n-1$  are the vertices of a regular polygon of  $n$  sides. If  $0 < k \leq 2m$  each side of the polygon with vertices lying on the circle  $C_k$  lies above a vertex of the polygon in  $C_{k-1}$  and thus determines a triangle in space. Similarly, if  $0 \leq k < 2m$ , each side of the polygon in  $C_k$  lies below a vertex of the polygon in  $C_{k+1}$  and thus determines a triangle.

Figure 4 shows a top view of  $C_k$  and  $C_{k-1}$  with inscribed polygons. Figure 5 shows a side view of a small section of the cylinder.

To compute the area of the triangles we must find the length of the base  $L$  and the height  $H$ . Since the cylinder has a radius of one, from Figure 4 we have

$$\begin{aligned} L &= \sqrt{1 + 1 - 2\cos(2\pi/n)} \\ &= \sqrt{2 - 2\cos(2\pi/n)} \end{aligned}$$

## The Pentagon

$n=6$

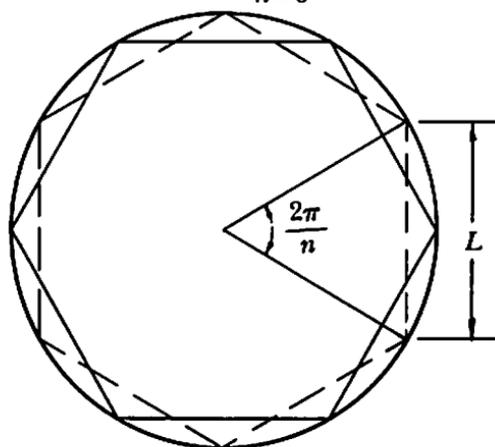


FIGURE 4

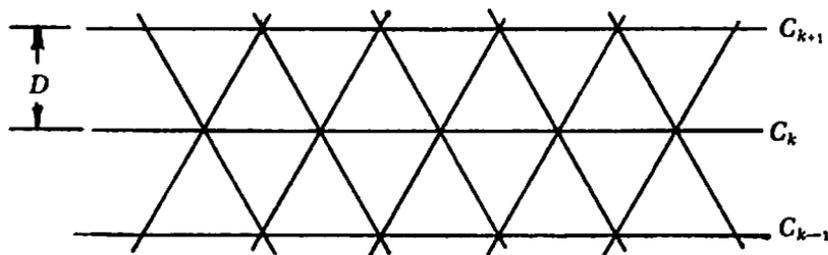


FIGURE 5

$$\begin{aligned}
 &= \sqrt{4\sin^2(\pi/n)} \\
 &= 2\sin(\pi/n)
 \end{aligned}$$

From Figure 6 we can see that the height  $H$  can be expressed as the hypotenuse of a right triangle with the distance between the circles  $D$  as one side and the distance from the center of the base to the side of the cylinder  $A$  as the other side.

From Figures 4 and 6 we have

$$\begin{aligned}
 1 - A &= \sqrt{1^2 - (L/2)^2} \\
 &= \sqrt{1 - \sin^2(\pi/n)} \\
 &= \sqrt{\cos^2(\pi/n)} \\
 &= \cos(\pi/n)
 \end{aligned}$$

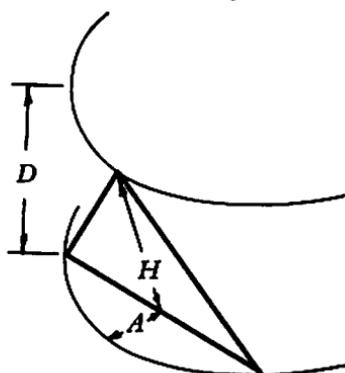


FIGURE 6

Since the height of the cylinder is one, the distance  $D = 1/2m$ , and we have

$$\begin{aligned} H &= \sqrt{D^2 + A^2} \\ &= \sqrt{1/4m^2 + (1 - \cos \pi/n)^2} \end{aligned}$$

It is not difficult to see that there are  $4mn$  triangles, therefore the total area of the inscribed polyhedron is

$$\begin{aligned} A(S_{mn}) &= 4mn \frac{1}{2} (2 \sin \pi/n) \sqrt{\frac{1}{4m^2} + (1 - \cos \pi/n)^2} \\ &= 2\pi \frac{\sin \pi/n}{\pi/n} \sqrt{1 + 4m^2(1 - \cos \pi/n)^2} \end{aligned}$$

Now if we let  $m$  and  $n$  approach infinity the area of each triangle approaches zero, and we would expect the total area to approach  $2\pi$ . Let  $m, n \rightarrow \infty$ , then

$$\begin{aligned} A(S) &= \lim_{m, n \rightarrow \infty} A(S_{mn}) \\ &= \lim_{m, n \rightarrow \infty} 2\pi \frac{\sin \pi/n}{\pi/n} \sqrt{1 + 4m^2(1 - \cos \pi/n)^2} \\ &= \lim_{m, n \rightarrow \infty} 2\pi \sqrt{1 + 4m^2(1 - \cos \pi/n)^2} \end{aligned}$$

We can now concentrate on the quantity within the radical, or more specifically on the function

$$f(m, n) = 2m(1 - \cos \pi/n) = \frac{\pi^2 m}{n^2} - \frac{2\pi^4 m}{4!n^4} + \frac{2\pi^6 m}{6!n^6} - \dots$$

We shall consider two cases:

(i) Let  $m = [\alpha n^2]$ , where  $0 < \alpha < \infty$ , and where  $[\alpha n^2]$  is the integer such that  $\alpha n^2 - 1 < [\alpha n^2] \leq \alpha n^2$ , then

$$f([\alpha n^2], n) = \frac{\pi^2 [\alpha n^2]}{n^2} - \frac{2\pi^4 [\alpha n^2]}{4!n^4} + \dots$$

thus  $\lim_{n \rightarrow \infty} f([\alpha n^2], n) = \alpha \pi^2$  and  $\lim_{n \rightarrow \infty} A(S_{[\alpha n^2], n}) = 2\pi \sqrt{1 + \alpha^2 \pi^4}$ .

(ii) Let  $m = n^3$ , then

$$f(n^3, n) = \pi^2 n - \frac{2\pi^4}{4!n} + \dots$$

thus  $\lim_{n \rightarrow \infty} f(n^3, n) = \infty$ , and  $\lim_{n \rightarrow \infty} A(S_{n^3, n}) = \infty$ .

We have thus assigned an arbitrarily large finite area and an infinite area to the lateral surface of a right circular cylinder with height one and radius one. The fact that the method fails for one example shows that surface area cannot be defined in the way we usually define arc length. We can see now that our intuition will sometimes fail us.

# Some Famous Mathematicians

ROBERT W. PRIELIPP

*Faculty, Wisconsin State University, Oshkosh*

How many of the famous mathematicians described by the clues given below can you recognize? Why not take some time right now to check on your knowledge of the history of mathematics. Just enter each individual's name, one letter per cell, in the space provided.

1.				A					
2.				L					
3.				G					
4.				E					
5.				B					
6.				R					
7.				A					

1. This French mathematician discovered that an irreducible algebraic equation of degree  $n$  is solvable by radicals if and only if the symmetric group on its roots is solvable; he died as the result of a duel at the age of 20.

2. This British mathematician was one of the first men to study matrices; his name is generally associated with the theorem which states that every finite group of order  $n$  is isomorphic to a subgroup of  $S_n$ , the symmetric group on  $n$  elements.

3. This French mathematician introduced the commonly used notation for derivatives of various orders,  $f'(x)$ ,  $f''(x)$ ,  $\dots$ ,  $f^{(m)}(x)$ ,  $\dots$ ; his name is generally attached to the theorem which states that if  $G$  is a finite group of order  $n$  and if  $H$  is a subgroup of  $G$  of order  $k$  then  $k$  divides  $n$ .

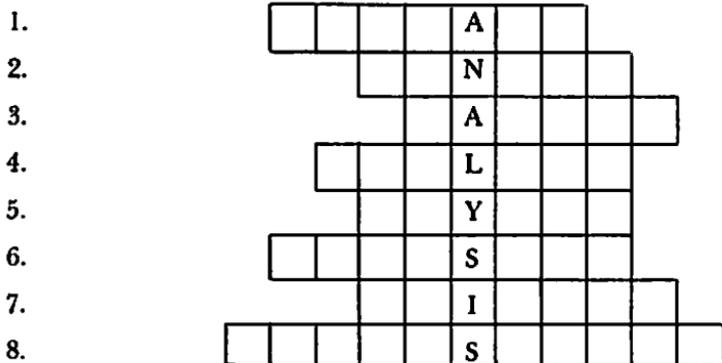
4. This German mathematician seems to have been the first to give an explicit definition of a number field; he also formulated a

method for constructing the real numbers using "cuts."

5. This Norwegian mathematician proved that the general quintic equation is not solvable by radicals; another name for a commutative group commemorates his work.

6. This German mathematician discovered the theory of ideals while attempting to prove Fermat's Last Theorem.

7. This German mathematician is generally regarded as one of the greatest mathematicians of all time; no other mathematician of the nineteenth century exerted so profound an influence on the development of science as he did; he proved the fundamental theorem of algebra, which may be stated: every algebraic equation of degree  $n$  has exactly  $n$  roots; the concept of congruence modulo  $m$  was introduced by him.



1. This Czechoslovakian mathematician probably constructed the first continuous but nowhere differentiable function; he disclosed some important properties of infinite sets in a posthumous work *Paradoxen des Unendlichen* (Paradoxes of the Infinite).

2. This Polish mathematician introduced a normed linear space (a more general form of a vector space) which is complete in the metric determined by the norm; together with Fréchet and Riesz he is considered to be one of the founders of modern functional analysis.

3. This German mathematician along with Abel laid the foundation of the theory of elliptic functions; his name is attached to a particular type of functional determinant but he was not among the first to use this kind of determinant.

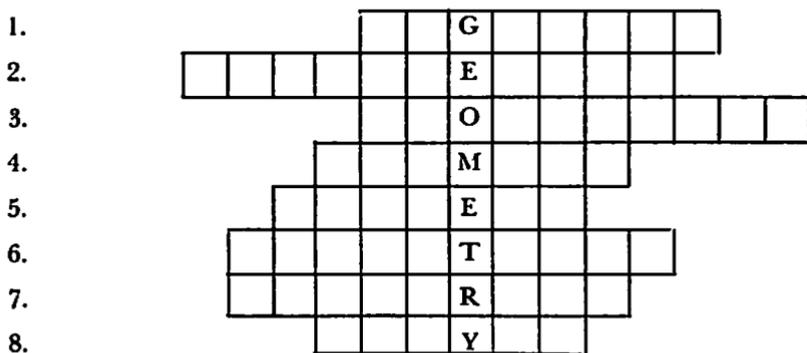
4. This French mathematician's book, *Théorie analytique*, contains a very useful transform; the theory of probability perhaps owes more to him than to any other person.

5. The name of this British mathematician is attached to a series which is a generalization of the so-called Maclaurin series, actually this series had been known long before to James Gregory and, in essence, to Jean Bernoulli.

6. This French mathematician introduced an integral more general than the Riemann integral and a type of measure for a set; he is sometimes called the father of the modern theory of integration.

7. This German mathematician discovered the calculus independently of Newton; although his discovery was after that of Newton, he is entitled to priority of publication.

8. This German mathematician, along with Cauchy and Riemann, is generally thought of as being one of the founders of the theory of analytic functions; his name, along with that of Bolzano, is attached to the theorem which states that every bounded infinite set has at least one limit point.



1. This French mathematician's book, *Eléments de Géométrie*, played a very significant role in the teaching of geometry in the United States during much of the nineteenth century; he made significant advances in many fields including nongeometrical-differential equations, calculus, theory of functions, theory of numbers, and applied mathematics.

2. This Russian mathematician was the first person to take the revolutionary step of publishing a geometry specifically built on an assumption in direct conflict with Euclid's parallel postulate.

3. This Greek mathematician was born at Perga, his chief work is the *Conics*; his name is also associated with the following problem: given three things, each of which may be a point, a line, or a circle, draw a circle that is tangent to each of the three given things (where tangency to a point is understood to mean that the circle passes through the point).

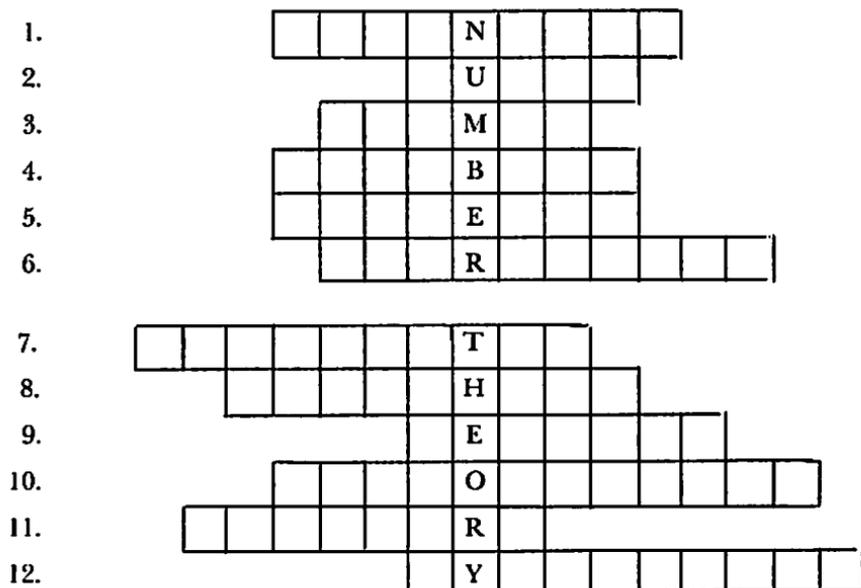
4. This German mathematician suggested that geometry be viewed as a study of manifolds of any number of dimensions in any kind of space; this view ultimately made the theory of general relativity possible; he is considered the father of elliptic geometry.

5. This German mathematician attempted to give geometry a strict axiomatic basis in his *Grundlagen der Geometrie* (Foundations of Geometry); his proposal of twenty-three problems in 1900, which he believed would be or should be among those occupying the attention of mathematicians in the twentieth century, has played an important role in research during the last seventy years.

6. This Greek mathematician made one of the most extensive early studies of the five regular solids; the theorem that there are precisely five regular polyhedra is probably due to him.

7. This French mathematician's *Discours de la méthode* contained an appendix entitled *La géométrie*, which contained some of the antecedents of present-day analytic geometry; his name is frequently attached to a "rule of signs".

8. This Hungarian mathematician discovered a non-Euclidean geometry; his "Absolute Science of Space" was an appendix to a treatise written by his father; upset when Gauss failed to praise his work and deeply hurt when Gauss indicated that he had had similar ideas years ago, he published nothing more.



1. This Italian's book, *Liber abaci*, strongly advocated the use of the Hindu-Arabic numerals; his name is generally associated with a famous sequence which comes from a celebrated problem involving rabbits.

2. The mathematical research of this Swiss mathematician averaged about 800 pages a year during his lifetime; he is generally recognized as the most successful notation-builder of all time; for example, the definitive use of the Greek letter  $\pi$  for the ratio of circumference to diameter in a circle is largely due to him and the symbol  $i$  for  $\sqrt{-1}$  was first used by him.

3. This French mathematician is sometimes called the founder of the modern theory of numbers; he was among the first to use a process that he called "infinite descent", a sort of inverted mathematical induction, to prove some of his theorems; since his time many mathematicians have wished that the margin of his copy of Bachet's *Diophantus* had not been quite so narrow.

4. In 1742 this Russian mathematician suggested that every even integer greater than 2 can be expressed as a sum of two (not neces-

sarily distinct) prime numbers; his name is generally associated with this conjecture which is still unsolved.

5. This French mathematician served as a clearing house for mathematical information; prime numbers of the form  $2^p-1$  are named in his honor.

6. This Polish mathematician helped to found *Fundamenta Mathematicae*, one of the world's most distinguished mathematical journals, in 1920; he made contributions to the theory of sets, to topology, and to the theory of numbers.

7. This Greek mathematician is often called the father of algebra, although such a designation should not be taken too literally; his *Arithmetica* is almost entirely devoted to the exact solution of equations, both determinate and indeterminate.

8. This German mathematician proved that if  $a$  and  $b$  are natural numbers such that  $a$  and  $b$  are relatively prime, then there exist infinitely many primes of the form  $ak+b$ , where  $k$  is a natural number; his name is also associated with a test for uniform convergence of a series, a series of the form  $\sum_{n=1}^{\infty} \frac{a_n}{n^s}$  and a principle which states that if you put  $n+1$  objects in  $n$  boxes at least one box will contain two or more objects.

9. This Russian mathematician established that if  $\alpha$  and  $\beta$  are algebraic numbers,  $\beta$  is not a rational number, and  $\alpha$  is neither 0 nor 1, then any value of  $\alpha\beta$  is transcendental; this theorem establishes the transcendence of such numbers as  $e^{\pi}$ ,  $5^i$ , and  $2^{\sqrt{2}}$

10. This Greek mathematician from Cyrene determined a very accurate estimate for the circumference of the earth even before the birth of Christ; he is well known in number theory for his sieve, which is a systematic procedure for determining which positive integers are prime numbers.

11. This French mathematician proved the prime number theorem in 1896; his *Essai sur l'étude des fonctions données par leur développement de Taylor* gave impetus to much of the later research on power series and their singularities; he also contributed important papers on the theory of entire functions with applications to the Riemann zeta function.

12. This Greek mathematician remains a very obscure figure due in part to the loss of documents which covered the period of his life; although a theorem involving a right triangle which bears his name does not appear in any form in surviving documents from Egypt, tablets from the Old Babylonian period show that in Mesopotamia the theorem was widely used; it is quite likely that this theorem was derived from the Babylonians.

#### REFERENCES

1. Boyer, Carl B. *A History of Mathematics*. New York: John Wiley and Sons, Inc., 1968.
2. Eves, Howard. *An Introduction to the History of Mathematics* (third edition). New York: Holt, Rinehart, and Winston, 1969.
3. Feller, William. *An Introduction to Probability Theory and Its Applications*, Volume I. New York: John Wiley and Sons, Inc., 1957.
4. Hille, Einar. *Analytic Function Theory*, Volumes I and II. New York: Blaisdell Publishing Company, 1959 and 1962.
5. Sanford, Vera. *A Short History of Mathematics*. Boston: Houghton Mifflin Company, 1930.
6. Smith, David Eugene. *A Source Book in Mathematics*, Volumes I and II. New York: Dover Publications, Inc., 1959.

(Answers to this article are located on p. 49.)

# Installation of New Chapters

EDITED BY LORETTA K. SMITH

KENTUCKY ALPHA CHAPTER

*Eastern Kentucky University, Richmond, Kentucky*

The installation of the Kentucky Alpha Chapter of Kappa Mu Epsilon was held on 27 March 1971, at the Student Union Building of the Eastern Kentucky University. Professor William R. Smith, the National Vice-President of Kappa Mu Epsilon, was the installing officer.

The officers of the newly-formed Chapter are:

President .....	Brenda J. Speagle
Vice-President .....	Robert W. Slone
Secretary .....	Paula S. Kinker
Treasurer .....	Ann Mackin
Reporter .....	Michael Allan Kettler
Corresponding Secretary .....	Bennie R. Lane
Sponsors .....	Glynn Creamer Sydney Stephens, Jr.

The Chapter members are:

Joyce A. Allsmiller	Alvin McGlasson
Jean Katherine Bertrand	Bobby L. Nayle
Patricia Ann Goins	David K. W. Ng
George W. Halsey	Charles D. Robinson
Linda K. Himes	Francesco Scorsone
Aughtum Howard	Betty L. Stephens
Larry Byron Hurt	Carol Teague
Pamela Marks	Jerry Michael Wesley
Richard Marks	Gary Dale Whitaker

After the banquet meal, Professor Smith presented a very interesting and informative talk entitled "How to Choose a Wife."

# **The Problem Corner**

EDITED BY ROBERT L. POE

Due to the unavoidable late release of the Spring 1972 issue of THE PENTAGON it has been decided that The Problem Corner should not appear in the Fall 1972 issue of THE PENTAGON. The editor regrets the necessity of this decision but feels it will best suit the interests of participating contributors. Your indulgence is respectfully requested.

Solutions to problems posed in the Spring 1972 issue will appear in the Spring 1973 issue. At that time a new list of problems for solution will be posed. Solutions to the problems appearing in the Spring 1972 issue should be submitted on separate sheets before 1 February 1973. The best solutions submitted by students will be published in the Spring 1973 issue of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Also The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Address all communications to Professor Robert L. Poe, Department of Mathematics, Berry College, Mount Berry, Georgia 30149.

# The Book Shelf

EDITED BY ELIZABETH T. WOOLDRIDGE

This department of THE PENTAGON brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. Elizabeth T. Wooldridge, Department of Mathematics, Florence State University, Florence, Alabama 35630.

*Introduction to Linear Algebra*, Franz E. Hohn, Macmillan Company, New York, 1972, 335 pp., \$9.95.

This text is intended for students at the freshman-sophomore level. No calculus background is required. This reviewer found the text to be very sound but not very exciting. Most of the material is presented along traditional lines with a strong emphasis on geometric interpretation whenever possible.

The author begins his study by developing the sweep out process to solve linear systems. Matrices are then introduced as the coefficient array of such systems and matrix algebra is derived.

Chapters Three and Four give a detailed presentation of two and three dimensional Euclidean space and the extension to  $n$ -dimensional space. It is in Chapter Five that one first finds the general definition of a vector space. The standard material such as invariance of number of vectors in a basis, dimensions of subspaces, and isomorphic spaces is presented.

Chapter Six covers the ideas of rank of a matrix, invertible matrices, and solutions of linear systems by matrix techniques. The theory of determinants is the subject of Chapter Seven. The product rule is derived by first establishing its validity for elementary matrices. Linear transformations and their matrix representatives are given in Chapter Eight. The final chapter of the book defines the characteristic value problem and presents much of the relevant theory. Also quadratic, bilinear, and definite forms are examined.

The exercises are numerous and varied. No answers are given in the text. I would recommend Professor Hohn's book to anyone seeking a complete, sound linear algebra text at the elementary level.

Eddy Joe Brackin  
Florence State University

*Essentials of Mathematics*, Max A. Sobel, Evan M. Maletsky, Thomas J. Hill, Ginn and Company, Boston, 1970; Book 1, 436 pp., \$5.88; Book 2, 406 pp., \$5.88; Book 3, 472 pp., \$6.28.

These books are designed for low-achieving students of grades 7, 8, and 9. Many exercises could be used for students in lower grades.

In each book, each chapter has a chapter test with chapters five and ten as review chapters.

Matt E. Matix is a little character who takes the student through many adventures in mathematics. Many of the problems are shown as they would be done with a computer.

The topics in Book 1 include whole numbers, prime numbers, fractions, decimal fractions, and geometric activities. The topics presented in Book 2 are integers, positive and negative numbers, fractions, per cent, formulas, and geometric activities. Book 3 presents integers, decimals, per cent, equations and inequalities, graphs, and geometric experiments.

In all the books there is a variety of activities. The illustrations are timely and the material presented shows the need for mathematics and that mathematics is fun.

Vaulda Welke  
Superior, Nebraska

*The Power of Calculus*, K. L. Whipkey and M. N. Whipkey, John Wiley & Sons, Inc., New York, 1972, 297 pp., \$9.95.

The Whipkeys have written a textbook which presents some concepts of elementary calculus for undergraduates in management, social, and behavioral sciences. CUPM recommendations for teaching a one-term calculus course to students in nontechnical fields are generally practiced throughout the content of the book. The text represents an acceptable addition to the present list of titles in this area.

Assuming students have previously obtained adequate training in high school algebra, the authors develop mechanical techniques for differentiation and integration of real single-variable rational func-

tions after mentioning sets and intuitively (but briefly) investigating the notions of relations, functions, and limits. This essentially describes the material of the first six chapters except for a smattering of low level applications concerning the derivatives and integrals of functions of one real variable. In fact, the applications of the calculus for business, sociology, and biology students found in this book are rather disappointing. Mostly they are the usual applications found in standard calculus texts published during this century.

Chapter Seven and Chapter Eight lightly cover exponential and logarithmic functions, differentiation and integration of logs and exponentials, touch on variables separable differential equations, hastily examine functions of two variables, barely introduce partial derivatives of functions of two variables, and conclude the body of the textbook with two sections on extrema of functions in Euclidean three space. An appendix, which has sets of exercises, follows Chapter Eight. It contains differentiation and integration of the basic trigonometric functions, some discussion of infinite sequences, and an extremely scanty look at Taylor's and Maclaurin series.

The entire book is well written, presents topics simply, and appears free of errors. It cannot be said that the book is especially attractive either artistically or academically. There are ample exercises and a large number of lucid examples. Answers to some exercises in each set of exercises are printed at the back of the textbook. It has many fine illustrations. What proofs are attempted are detailed and easy to follow. Considering its intended audience the book does well in that theory is absent, trigonometry is practically ignored, and any complicated calculus is avoided.

Anyone who is ancient enough to have studied or taught from one of the Granville, Smith, and Longley calculus books will become nostalgic when examining the format of the Whipkeys' textbook. However, infinitesimals do not appear.

Robert L. Poe  
Berry College

*Computer Hardware Theory*, W. J. Poppelbaum, Macmillan, New York, 1972, 749 pp., \$16.95.

From the preface of this book we discover that "it is assumed that the reader has taken an introductory computer science course, that he is reasonably familiar with the elements of calculus, and that he has had, at one time or another, a high school physics sequence." The reviewer strongly recommends that a student with *only* the above background avoid this book. In the same preface we find that "the audience we aim at is, of course, principally the junior or senior electrical engineer, computer scientist, and mathematician." That should give you some idea of the level of sophistication of this book. The reviewer believes that this text is indeed most suitable for the junior or senior electrical engineer.

A mathematician or computer science student whose background did not include severe doses of topics in applied mathematics and physics would find this text very difficult, but not impossible, and quite worth the effort demanded by its study. Under the guidance of a skilled lecturer students of the two latter-mentioned categories would benefit greatly from this text, though only with a considerable expenditure of time and effort. These statements are made not to frighten you away from any encounters with this text but to inform you of its demands so that you may make an intelligent decision regarding its suitability for your individual needs.

For those who are well prepared in the above-mentioned areas, as would be, presumably, the junior or senior electrical engineer, you will find this text complete, comprehensive, and well written. The reviewer can recommend it highly, either as a textbook or for self-study by those at the junior/senior level in electrical engineering.

As a final note I will comment on the statement from the preface which reads "Some elementary notions of vector calculus, complex numbers, tensors, and Fourier and Laplace transforms are developed in the text, so as to permit its use for liberal arts students of relatively restricted mathematical background." My comment is that the student described in this sentence will need a skilled, patient and understanding instructor in order to gain any more than the most cursory benefits from this book.

James E. McKenna  
State University College at  
Fredonia, New York

## MINIREVIEWS

*Finite Mathematics*, 2nd Edition, John G. Kemeny, Arthur Schleifer, Jr., J. Laurie Snell, Gerald L. Thompson, Prentice-Hall, Englewood Cliffs, N.J., 1972, 542 pp., \$12.95.

This book was written to meet the needs of behavioral and social science students in a course which would provide a sophisticated introduction for the non-mathematician to topics in modern mathematical analysis. The nine chapters develop the foundation of logic, set theory, probability, and linear algebra. They also illustrate the most important applications of these mathematical methods to modern business problems. The second edition provides new chapters on decision theory and analysis, Markov decision processes, and greatly expanded coverage of linear programming and its interpretations.

*Exploration in Elementary Mathematics*, 2nd Edition, Seaton E. Smith, Jr., Prentice-Hall, Inc., 1971, 443 pp., \$10.50.

The purpose of this book is to present the basic concepts of elementary mathematics in such a manner that "the reader will be able to understand these ideas and see their relevance to teaching." In addition to chapters on mathematical ideas, sets, systems of numeration, and operations with whole numbers and with rational numbers, the author has included chapters on elementary number theory and informal geometry. At the end of each chapter, there is a test for self-evaluation. Most of the recently developed devices for introducing various topics have been included. The book is designed as a text for either an in-service program or a college course for prospective teachers or others interested in extending their knowledge of some basic mathematical concepts.

*Introductory Mathematics for Technicians*, Alvin B. Averbach and Vivian S. Groza, Macmillan Company, New York, 1972, 813 pp., \$12.50.

This book is designed for a foundation technical mathematics course; it attempts to develop the basic skills that are needed by technicians in all classifications. Mastery of the topics presented in this text will give proficiency in performing directed calculations of some complexity by use of the slide rule and tables of functions of numbers. In addition to calculations, the student should have developed

the ability to use formulas and perform algebraic manipulations pertaining to problems met in industrial practice and to solve problems by applications of algebra, geometry, and trigonometry of the right triangle.

*Computer Appreciation*, T. F. Fry, Philosophical Library, New York, 1971, 245 pp., \$15.00.

The fourteen chapters of this book cover a wide range of topics from a short account of the historical development of calculating devices, through computer programming, to the organization of a modern data-processing department. It concludes with a brief consideration of the applications of computers and a discussion on the effects of computers upon management matters. The book is designed primarily for students, but should be useful to all who feel the need to become familiar with this field.

*Answers to SOME FAMOUS MATHEMATICIANS*

I. Algebra

1. Galois
2. Cayley
3. Lagrange
4. Dedekind
5. Abel
6. Kummer
7. Gauss

II. Analysis

1. Bolzano
2. Banach
3. Jacobi
4. Laplace
5. Taylor
6. Lebesgue
7. Leibniz
8. Weierstrass

III. Geometry

1. Legendre
2. Lobachevsky
3. Apollonius
4. Riemann
5. Hilbert
6. Theaetetus
7. Descartes
8. Bolyai

IV. Number Theory

1. Fibonacci
2. Euler
3. Fermat
4. Goldbach
5. Mersenne
6. Sierpinski
7. Diophantus
8. Dirichlet
9. Gelfond
10. Eratosthenes
11. Hadamard
12. Pythagoras

# The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgment will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

Geometric constructions of irrational numbers have interested many a student of mathematics from ancient times to the present. One of the more fascinating irrational numbers to construct is  $\pi$ . It is impossible to construct  $\pi$  with only a straight edge and compass. In fact, not even a curve of higher order defined by an integral algebraic equation, for which  $\pi$  is the ordinate corresponding to a rational value of the abscissa, has been found to exist. The geometric construction of  $\pi$  requires the use of a transcendental curve which can be constructed using a transcendental apparatus such as the integraph which traces the curve by continuous motion.

The integraph was invented by the Russian engineer Abdank — Abakanowicz and constructed by Coradi of Zürich. It enables one to trace the curve of the integral  $Y = F(x) = \int f(x)dx$  given the differential curve  $y = f(x)$ . The integraph is so constructed that, when the guiding point of the linkwork of the integraph follows the differential curve, the tracing point will trace the integral curve.

Consider any point  $(x, y)$  of the differential curve  $y = f(x)$  and construct the auxiliary triangle having for its vertices the points  $(x, y)$ ,  $(x, 0)$  and  $(x-1, 0)$  as shown in Figure 1.

Note that the resulting triangle is a right triangle whose hypotenuse forms angle  $\theta$  with respect to the  $x$ -axis and that  $\tan \theta = y$ . Therefore, the hypotenuse of the triangle is parallel to the tangent to the integral curve  $Y = F(x)$  at the point  $(X, Y)$  corresponding to the point  $(x, y)$  on  $y = f(x)$ . The integraph is thus constructed so that the tracing point shall move parallel to the variable direction of the hypotenuse of the triangle, while the guiding point follows the differential curve  $y = f(x)$ . This is accomplished by connecting the tracing point of the integral curve with a sharp-edged roller whose plane is vertical and which moves so as to be always parallel to the hypote-

nuse of the auxiliary triangle. A weight is used to press the roller firmly upon the paper so that its point of contact can advance only in the plane of the roller. The integraph can be used to approximate definite integrals which will allow us to construct  $\pi$  as follows.

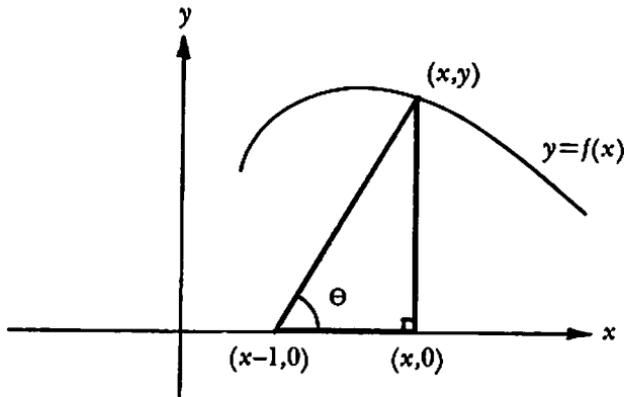
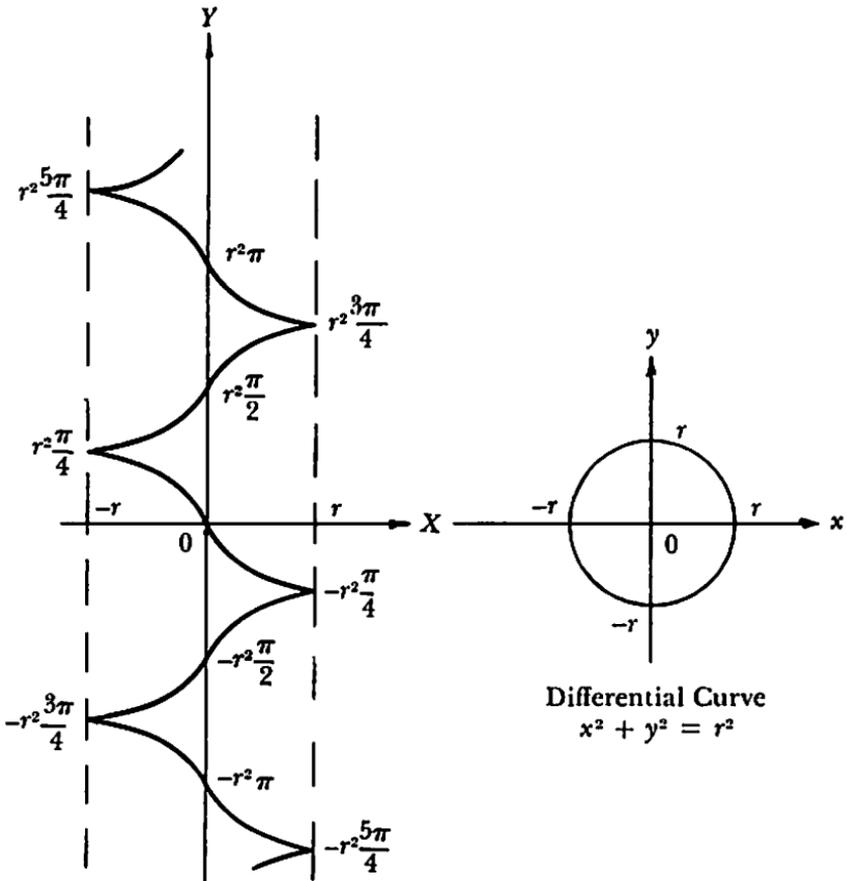


FIGURE 1.

Let the differential curve  $y = f(x)$  be the circle  $x^2 + y^2 = r^2$ . Hence  $y = \sqrt{r^2 - x^2} = f(x)$ . The integral curve  $Y = F(x) = \int \sqrt{r^2 - x^2} dx$ , by the use of the trigonometric substitution  $x = r \sin \phi$ , becomes  $Y = \frac{r^2}{2} \sin^{-1} \frac{x}{r} + \frac{x}{2} \sqrt{r^2 - x^2}$ .

The integral curve thus consists of a series of congruent branches, the  $Y$ -intercepts of which have ordinates  $0, \pm \frac{r^2\pi}{2}, \pm r^2\pi, \pm \frac{r^23\pi}{2}, \dots$ . The lines  $x = \pm r$  intersect the curve  $Y = F(x)$  at ordinates  $\pm r^2 \frac{\pi}{4}, \pm r^2 \frac{3\pi}{4}, \pm r^2 \frac{5\pi}{4}, \dots$ , as shown in Figure 2.



Integral Curve

Differential Curve  
 $x^2 + y^2 = r^2$ 

FIGURE 2

By letting  $r = 1$ , the ordinates of these intersections will construct the irrational number  $\pi$  and its multiples. The integraph thus allows us to trace the curve efficiently and with unusual sharpness. Using an integraph, can you construct  $3\pi$ ?

The current interest in chess has also renewed interest in other games of strategy. One such game is the mathematical game of Hex, which mathematicians have been playing the past 25 years or so. Its origin has been attributed to Piet Hein, a Danish mathematician, and to John Nash, an American mathematician. Hein is given the credit for the invention of the game and Nash devised a proof that the normal outcome of the game is a white win.

The game of Hex is played on a board comprised of  $n$  by  $n$  adjacent hexagons arranged in a rhombus, where  $n$  is some integer, as shown in Figure 3.

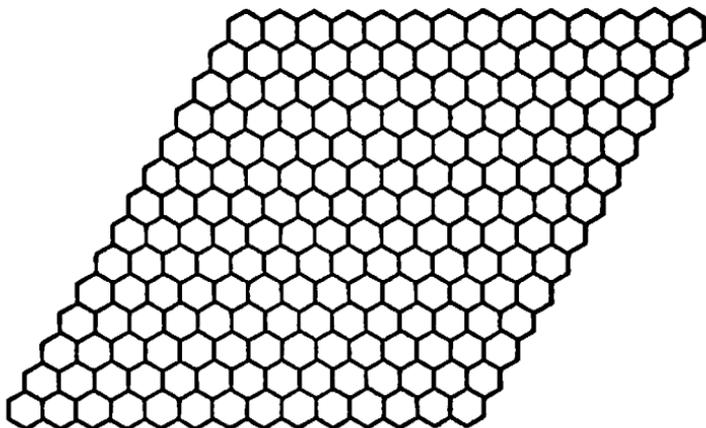


FIGURE 3.

If  $n$  is odd, it appears advantageous for the starting player to start in the center. If  $n$  is even (preferred by most players), the game becomes more interesting and has varied strategies. For our example, we shall take  $n = 14$  as shown in Figure 3. This is the form of the game of Hex played in the Yale Mathematics Common Room in 1952.

The game is played with 196 round markers, usually Black and White. Each color has 98 markers which can be placed in any of the 196 hexagons. Each hexagon is large enough to accommodate only

one marker. White plays first and places one of his white markers in any one of the 196 hexagons. Once placed on the board, it remains for the duration of the game. Next, Black places a black marker in any one of the 195 remaining hexagons. The game continues, alternating plays between White and Black. White attempts to join the top edge of the board to the bottom edge with an unbroken string of white markers adjacent to one another. Black wishes to join the two sides in a similar fashion. The following conclusions can be proven:

1. The game of Hex never ends in a draw.
2. If the Hex board is completely covered with white and black markers, then either there is a white chain joining the top to the bottom or a black chain joining the two sides.
3. Either White can force a win or Black can force a win.
4. If Black can force a win, then White can force a win.

The unusual aspect concerning the game of Hex is that no one knows a winning strategy. The person starting the game (white) has an advantage, and so any player should want the first move. But having the first move does not assure victory.

During the past few years, a number of mathematicians have attempted to devise an explicit strategy for Hex. So far, however, no one has been successful. Can you prove the four conclusions stated above?

# Kappa Mu Epsilon News

EDITED BY ELSIE MULLER, *Historian*

News of Chapter activities and other noteworthy **KME** events should be sent to Elsie Muller, Historian, Kappa Mu Epsilon, Department of Mathematics, Morningside College, Sioux City, Iowa 51106.

## CHAPTER NEWS

### Alabama Beta, Florence State University, Florence

Chapter President — Robert O'Conner  
52 actives

At the initiation banquet 25 new members were added which makes a total of 522 members for Alabama Beta. The chapter sponsors a tutoring program. At one of the chapter meetings Dr. Bill Bryant, Vanderbilt University, was the featured speaker about job opportunities. Larry Smith received a graduate assistantship to the University of Alabama and Euel Cutshall received one to the University of Florida. Other officers: Jane Bickel, vice-president; Teresa Guthrie, secretary and treasurer; Jean Foster, historian; Dr. Elizabeth T. Wooldridge, corresponding secretary; Lionel Isbell, reporter; and Dr. E. J. Brackin, faculty sponsor.

### California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President — Jim Pearce  
61 Actives

With the assistance of the chapter the department of mathematics plans and executes an annual mathematics contest which involves over 500 high school students. **KME** members conduct a regularly scheduled mathematics laboratory (tutorial service) for the use of all students on campus. Other officers: Nancy Miller, vice-president; Gayle Simon, secretary; Susan Genung, treasurer; Dr. George R. Mach, corresponding secretary; and Dr. Ralph M. Warten, faculty sponsor.

**Colorado Alpha, Colorado State University, Fort Collins**

Chapter President — Kate Legge

34 Actives, 21 pledges

As an activity Colorado Alpha sponsored an alumni seminar on employment opportunities in the mathematical sciences. Bi-weekly meetings are held on such topics as *The Role of Corporation in Society*, *Combinatorial Mathematics*, and *Japanese Education*. Other officers: Marilyn Hull, vice-president; Margita Stauers, secretary; Michael Colby, treasurer; Dr. Howard Frisinger, corresponding secretary; Dr. Kenneth Klopfenstein, faculty sponsor.

**Illinois Alpha, Illinois State University, Normal**

Chapter President — Frank Hirsch

28 actives, 6 pledges

In addition to providing tutoring service, the chapter ordered and distributed mathematical tables to all mathematics and science students. Eight meetings, featuring both students and faculty, were held during the year. Other officers: Thomas Horsley, vice-president; Linda Phillips, secretary; Gary Abramson, treasurer; Dr. Robert K. Ritt, corresponding secretary; Dr. Orlyn Edge, faculty sponsor.

**Illinois Delta, College of St. Francis, Joliet**

Chapter President — Linda Smith

11 actives, 5 pledges

Following the collapse of an attempted merger between the College of St. Francis and Lewis College, Illinois Delta has returned to its affiliation with the College of St. Francis. Other officers: Julie Hardy, secretary; Arnold Good, faculty sponsor.

**Illinois Eta, Western Illinois University, Macomb**

Chapter President — Michael Griswold

10 actives, 7 pledges

Other officers: Charles Smith, vice-president; Victoria Lange, secretary; Charles Smith, treasurer; Dr. Larry Morley, corresponding secretary; Dr. James Calhoun and Mr. Lucian Wernick, faculty sponsors.

**Indiana Alpha, Manchester College, North Manchester**

Chapter President – Helen Taylor

Other officers: Jeanette Klotz, vice-president; Doug Warrick, secretary; Hank Nietert, treasurer; Dr. Ralph McBride, corresponding secretary.

**Indiana Delta, University of Evansville, Evansville**

Chapter President – Mark Newlin

90 actives

In an attempt to generate wider participation in KME, two meetings for the coming year will be devoted to reports on mathematics where each member has from 5 to 20 minutes to present one of a variety of things – a favorite calculus problem, an application of mathematics, an interesting theorem, a book review, a project report, etc. *The Straight Line* is the official newsletter of Indiana Delta KME which is free to members. Others may subscribe at the rate of \$1 per volume. Interesting programs during the year were *Higher Plane Curves from an Elementary View* and *Königsburg Bridge Problem in Three Dimensions*. The first KME opinion questionnaire showed little dissatisfaction of the members with the organization, but there was only a 50% response. Other officers: Sandra Spillman, vice-president; Debra Austin, secretary; Dr. Gene Bennett, treasurer and corresponding secretary; Mr. Kenneth Stofflet, faculty sponsor.

**Iowa Alpha, University of Northern Iowa, Cedar Falls**

Chapter President – Brian Hogue

35 actives

Monthly meetings are held in the homes of faculty members with students presenting papers. The annual homecoming breakfast was held Saturday, 7 October at the home of the past national president, Dr. Fred W. Lott. Other officers: Don Trebil, vice-president; Mary Kay, secretary; John S. Cross, corresponding secretary and faculty sponsor.

**Iowa Beta, Drake University, Des Moines**

Chapter President – Denise Cindrich  
10 actives, 3 pledges

Three new members were initiated on 23 April: Mark Jones, John Diehl, and Luann Goodrich. Other officers: John Diehl, vice-president; Luann Goodrich, secretary; Rod Luther, treasurer; Dr. Wayne Woodworth, corresponding secretary; and Alex Kleiner, faculty sponsor.

**Iowa Gamma, Morningside College, Sioux City**

Chapter President – Stephen Bolks  
25 actives

The chapter sponsored a mathematics seminar for the consortium, Colleges of Mid-America. Six colleges were represented. The guest lecturer, Professor Hans Weinberger of University of Minnesota, gave two talks, *How Can Mathematics be Applied* and *Does Symmetry Beget Symmetry*. The luncheon featured mathematics in the liberal arts. Harlan Hullinger has a graduate assistantship in mathematics at the University of Iowa. Other officers: Dave Frevert, vice-president; Cheryl Cornwell, secretary; Paul Franken, treasurer; Elsie Muller, corresponding secretary and faculty sponsor.

**Kansas Beta, Kansas State Teachers College, Emporia**

Chapter President – Ron Stair  
75 actives

Other officers: Debbie Atkins, vice-president; Mary Bender, secretary; Randy Robertson, treasurer; Charles Tucker, corresponding secretary; Dr. Thomas Bonner, faculty sponsor.

**Kansas Gamma, Benedictine College, Atchison**

Chapter President – Robert Croll  
11 actives, 1 pledge

Student programs have included the following topics: *Factoring Functions*, *Cantor's Work with Transfinite Numbers*, and *Simplex Algorithm*. The chapter hosted its 5th biennial invitational mathematics tournament for high school students of the surrounding area on 29 April 1972. Several Kansas Gamma students attended the

regional convention where one of the members presented a paper on *Anti-isomorphisms*. Other officers: Julia Croghan, secretary; Anita Botzen, treasurer; Sister Jo Ann Fellin, corresponding secretary and faculty sponsor.

**Maryland Beta, Western Maryland College, Westminster**

Chapter President — Ronald R. Jemmerson  
12 actives

Guest speakers have been Dr. Mario Borelli of Notre Dame University and Sister John Frances Gilman of St. Joseph College. Senior members attending graduate school are Bonnie Green at University of Kansas and Robert Chapman at University of Indiana. Other officers: Michael Foster, vice-president; Linda Swift, secretary; Donald Dulaney, treasurer; Dr. James Lightner, corresponding secretary; and Dr. Robert Boner, faculty sponsor.

**Maryland Gamma, Saint Joseph College, Emmitsburg**

Chapter President — Maureen Hinke  
8 actives

Sr. Marie Augustine Dowling of the Maryland Alpha chapter presented a lecture, *The Super Egg and the Works of Piet Hein*. Another program was *How to Turn a School Bus Around in a Closet* by Dr. Robert Boner of the Maryland Beta chapter. In April there was an evening of mathematics films which was a part of the college campus cultural series. A joint meeting of the Maryland Beta and Gamma chapters was held where Sr. John Frances Gilman lectured on *One of the Last Set of Wednesdays*. Other officers: Joyce Reichert, vice-president; Mary McFadden, secretary; Patricia Thornton, treasurer; Sr. John Frances Gilman, corresponding secretary and faculty sponsor.

**Massachusetts Alpha, Assumption College, Worcester**

Chapter President — Thomas Curran  
14 actives

Five new members were initiated in February. Following a dinner honoring the new members, Dr. Sumner Cotzin spoke on the topic, *Puzzles and Paradoxes in Mathematics*. Other officers: Donald

Parker, vice-president; Kerrith Chapman, secretary; Charles Brusard, corresponding secretary; Rev. Richard P. Brunelle, faculty sponsor.

#### **Missouri Alpha, Southwest Missouri State University, Springfield**

Chapter President — Alan Washburn  
40 actives

Meetings are held monthly with a picnic in May. The chapter KME merit awards were given to Nelda Burton and Peggy Struckmeyer. Other officers: Carol Letterman, vice-president; Barbara Leamer, secretary; Denise Wray, treasurer; Eddie W. Robinson, corresponding secretary; L. T. Shiflett, faculty sponsor.

#### **Missouri Beta, Central Missouri State College, Warrensburg**

Chapter President — Shana McCann  
31 actives, 25 pledges

There were seven meetings during the year which included two initiations, a Christmas party, and a spring banquet. Other officers: Dwain Schreimann, vice-president; June Hlavacek, secretary; Deborah Distler, treasurer; Velma Birkhead, corresponding secretary; and Homer Hampton, faculty sponsor.

#### **Missouri Gamma, William Jewell College, Liberty**

The convention for Region 4 was held at William Jewell College on 15 April 1972 with the Missouri Gamma chapter acting as hosts and Harold L. Thomas as director. Papers were presented by eight students representing Washburn University, Kansas State Teachers College, Morningside College, Benedictine College, University of Missouri at Rolla, Kansas State College, and University of Northern Iowa. The guest lecturer at the luncheon was Dr. Glen Haddock of the University of Missouri at Rolla.

#### **Missouri Zeta, University of Missouri at Rolla**

Chapter President — Dana Nau  
25 actives

During the spring semester the chapter put up a new bulletin board with a pictorial directory of UMR's mathematics faculty.

Weekly help sessions were held for the three calculus courses and integral tables were given to all the differential equations students. At the pledge smoker Dr. Charles Hatfield spoke on *How to Write a Mathematics Paper*. At the regional convention in Liberty, Missouri on 15 April, with nine members in attendance, Dana Nau received first place for his paper, *A Lattice Point Problem*. Other officers: Robert Holliday, vice-president; Curt Killinger, treasurer; Debbie Fugitt, recording secretary; Peggy Shackles, Historian; Dr. Roy Rahestraw, corresponding secretary; and Dr. Jim Joiner, advisor.

### Missouri Theta, Evangel College, Springfield

Chapter President – Keith Sorbo  
9 actives, 3 pledges

Program topics have included *Non-Euclidean Geometry*, *Mathematics in Hydrology*, *Mathematical Puzzles*, and *Mathematics and Theology*. Five members attended the Region 4 convention. Other officers: Dave Earle, vice-president; Sandy Butler, secretary and treasurer; Glenn H. Bernt, corresponding secretary and faculty sponsor.

### Nebraska Alpha, Wayne State College, Wayne

Chapter President – Dale Ruehling  
22 actives, 16 pledges

Other officers: Wayne Breyfogle, vice-president; Ellen Hummel, secretary; Charles Wendt, treasurer; Fred Webber, corresponding secretary; Frank Prather and Jim Paige, faculty sponsors.

### Nebraska Gamma, Chadron State College, Chadron

Chapter President – Terry Welke  
25 actives

Meetings are held the first and third Thursdays of every month. Each senior gives a program on some aspect of mathematics at one of these meetings. A book market will be held on the first two days of the Spring semester. Other officers: Larry Ruzicka, vice-president; Cheri Landrey, secretary; Cheryl Sheldon, treasurer; James Kaus, faculty sponsor.

**New Mexico Alpha, University of New Mexico, Albuquerque**

Chapter President – Pat Russell  
50 actives

Other officers: Declan Rieb, vice-president; John Gilbert, secretary; Tim Burns, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

**New York Gamma, State University of New York, College at Oswego**

Chapter President – Michael Murray  
25 actives, 11 pledges

Other officers: Deborah Perry, vice-president; Jacquelyn Schaefer, secretary; Dawn Rickard, treasurer; Steven Reyner, corresponding secretary; J. Burling and J. Walcott, faculty sponsors.

**New York Zeta, Colgate University, Hamilton**

24 actives

A talk, *Why are there only Five Regular Solids*, by Professor John Baum of Oberlin College was given in March. L. D. Shatoff is the corresponding secretary and the faculty sponsor.

**New York Theta, St. Francis College, Brooklyn**

Chapter President – Timothy Marco

Students have been the speakers at the chapter meetings. Members have also sponsored intercollegiate mathematics contests as well as field trips. Other officers: James Tuthill, vice-president; Dee Lucarelli, secretary; Edward Krygowski, treasurer; Donald Coscia, corresponding secretary and faculty sponsor.

**New York Iota, Wagner College, Staten Island**

Chapter President – Brian Manske  
14 actives

During the Spring semester the chapter held two faculty-student mathematics bowls. The first annual award from the chapter (a \$25 savings bond) was presented to Karen Dybing (class of 1972) for her excellence in and dedication to mathematics. Four new initiates were

taken in on 1 February. Other officers: Linda Chacon, vice-president; Beverly Fraser, secretary; Lois Bredholt, treasurer; Mrs. Mary Petras, corresponding secretary; Raymond Traub, faculty sponsor.

**Pennsylvania Gamma, Waynesburg College, Waynesburg**

Chapter President – Larry Fordyce  
9 actives, 6 pledges

A variety of meetings made up the eight held during the year: two of them had outside speakers, two had student speakers, an initiation, a banquet, a panel discussion on the curriculum, and a planning and development meeting. As for activities the group sponsors a freshman mathematics tutoring program and publishes a local mathematics newsletter each semester. Other officers: David Wildman, vice-president; Stephanie Milinovich, secretary and treasurer; Gabriel J. Basil, corresponding secretary; and Lee Hagg-lund, faculty sponsor.

**Pennsylvania Delta, Marywood College, Scranton**

Chapter President – Maureen O'Malley

Seven meetings are held during the year. The first of these is a tea for the incoming freshmen to acquaint them with the Honor Society and the mathematics club, *Semi-Group*. One of the fall gatherings features a guest speaker. The final dinner is an installation of the new officers and a welcome to the new members. Other officers: Ann Marie Cascio, vice-president; Mary Ann Dorofee, secretary and treasurer; Miss Marie Loftus, faculty sponsor.

**Pennsylvania Epsilon, Kutztown State College, Kutztown**

Chapter President – Perry Leshner  
13 actives

The highlight for the year was a trip to Franklin Institute. Meetings were held biweekly of which two of them were initiation banquets. Other officers: Curtis Shappell, vice-president; Lila Taylor, secretary; Irving Hollingshead, corresponding secretary; Edward Evans, faculty sponsor.

**Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana**

Chapter President — John Nelson  
50 actives, 7 pledges

In February Dr. Charles Bertness spoke on *Graph Theory*; in March Mr. Wallace Morrell gave a lecture on *The Snow Plow Problem*; in April John Smith, a student, presented a paper, *A Proof of the Limit of  $e$* . In May the annual banquet was held at which time Dr. Edwin Smith talked on *Fun With Polyhedra*. Other officers: John Schutte, vice-president; Peggy Barkman, secretary; Germaine Fotto, treasurer; Professor Ida Z. Arms, corresponding secretary; Professor William R. Smith, faculty sponsor.

**Pennsylvania Eta, Grove City College, Grove City**

Chapter President — David Darkee

Other officers: Mary Lou McCracken, vice-president; Paige Mosser, secretary; Marty Houser, treasurer; Marvin Henry, corresponding secretary; Cameron Barr, faculty sponsor.

**Pennsylvania Iota, Shippensburg State College, Shippensburg**

Chapter President — Brenda Csencsits  
31 actives, 12 pledges

Region I held its first regional convention on 3-4 November with Pennsylvania Iota serving as hosts. Preliminary plans called for a session opening the conference on Friday evening, some social events, and several sessions devoted to presentation of papers by students. Professor William Smith, national Vice-President of KME, will be the keynote speaker.

At the annual banquet of the chapter a talk, *How to Catch Lions in the Sahara Desert*, was given by Dr. Howard Bell and Dr. William McArthur. A constitution revision committee presented a new constitution which was approved. Kathy Fischer headed the *Newsletter Committee*. Other officers: Mary Ibberson, vice-president; Gloria Brady, secretary; Dr. Howard Bell, treasurer; Dr. John Mowbray, corresponding secretary; Dr. James Sieber, faculty sponsor.

**Pennsylvania Kappa, Holy Family College, Philadelphia**

Chapter President — Mario Herczeg  
2 actives

Five meetings were held this past year at which the members solved problems from THE PENTAGON and from old Putnam examinations. Dr. Robert Beck from Villanova University was a guest speaker on *How to Stuff a Large Group into a Small Computer*. Other officers: Brenda Nadijka, vice-president and treasurer; Mario Herczeg, secretary; Allan Becker, corresponding secretary; Sr. Mary Grace Kuzawa, faculty sponsor.

**Tennessee Beta, East Tennessee State University, Johnson City**

Chapter President — Barney Taylor  
16 actives

Ten new members were initiated at a lovely spring banquet on 12 May. Dr. Eduardo Zayas-Bazan, associate professor of languages and one of those for whom our government paid \$100,000, spoke on the *Bay of Pigs*. The versatility of the members was shown by the awards in the spring: James Cloyd — KME senior mathematics award, chemistry award, dean's award; Nellie Woolsey — art award; Kenneth Oster — political science award and dean's award; Beverly Taylor, Nancy Forrester, Shannon Whitehead — dean's award. Other officers: Nellie Woolsey, vice-president; Hilda Street, secretary; Gregory Heuberger, treasurer; Lora McCormick, corresponding secretary; Sallie Carson and T. H. Jablonski, faculty sponsors.

**Texas Alpha, Texas Tech University, Lubbock**

Chapter President — Katie Updike  
18 actives

The chapter meets monthly and also holds special meetings and pledge meetings. Other officers: Marcus Rasco, vice-president; Marilyn Baker, secretary; James Bain, treasurer; Marilyn Baker, corresponding secretary; Dr. Bob Moreland, faculty sponsor.

**Wisconsin Alpha, Mount Mary College, Milwaukee**

Chapter President — Geri-lynn O'Boyle  
9 actives

Wisconsin Alpha celebrated its twenty-fifth anniversary by hosting the regional convention for Region 2 on 24-25 March. David R. Johnson of Nicolet High School in Milwaukee used as his topic, *The Magical Aftermath*, at the banquet. In addition to student papers, there were some short movies and a reading from *Dialogues on Mathematics* by A. Renyi.

The chapter sponsored its usual annual mathematics contest for high school students on 12 April. Four new members were initiated on 25 April, Christine Amrhein, Denise Diorio, Mary Doyle, and Cindy Sporleder. The speaker, Jim Morgeneau, talked on *Piaget and His Theory*. Other officers: Betty Witt, vice-president; Catherine Starck, secretary; Mary Kathleen Doyle, treasurer; Sister Mary Petronia, corresponding secretary and faculty sponsor.

**Wisconsin Beta, Wisconsin State University, River Falls**

Chapter President — Steven Hesperich  
18 actives, 22 pledges

The chapter meets once a month and holds some special meetings. The 1972 winner of the plaque for the outstanding mathematics student was Stephen Hesperich. Other officers: Terry Desjarlais; vice-president; Chris Goldsmith, secretary; Dick Ruhsam, treasurer; Lyle Oleson, corresponding secretary; Dr. Ed Mealy, faculty sponsor.



