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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Rotation of Convex Curves In Regular Polygons*

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A curve and a polygon! In the light of modern mathematics even a primary school student could establish a relationship between the two—for instance, that the circle is the limiting value of a polygon. But more advanced study of mathematics can produce more intricate and complex relationships on this topic of curves and polygons. Recently, mathematicians have investigated relationships stemming from the rotation of curves in regular polygons and have advanced many interesting properties and theorems. One particular theory produced by this study concerns the characteristics of convex curves rotated in regular polygons such that the curves always maintain contact with the sides of the polygons. It is the purpose of this paper to discuss this theory by investigating two particular types of convex curves—those rotated in regular polygons of three sides, or the equilateral triangle, and those rotated in regular polygons of four sides, or the square. The relationship is so well established between the curves and regular polygons in these two cases that the curves have come to be named and defined by the type of polygon in which they rotate. Furthermore, the perimeter of each of these two curves proves to be typical of convex curves inscribed in all regular polygons. Thus, by discussing these two types of convex curves and paying special attention to their perimeters, this paper will introduce a significant relationship involving curves and polygons—a relationship mathematically noted in Barbier's Theorem.

For purposes of brevity, discussion will be limited to the plane, although many analogues can be found in spaces of higher dimension.

Prior to the actual discussion, however, certain basic definitions must be considered. Recall that a plane figure is called *convex* if it wholly contains the line segment that joins any two points of the figure. A *convex curve* is the boundary of such a plane convex figure. An equally vital concept to the study of convex curves is that of lines of support. A *line of support* is defined to be a line l

*A paper presented at the 1969 National Convention of KME and awarded first place by the Awards Committee.

drawn such that it passes through at least one boundary point of a planar convex figure S and such that all the points of the figure not on l lie on one side of this line l . The corresponding diagrams clarify this idea and illustrate some of the relative possible positions of lines of support.

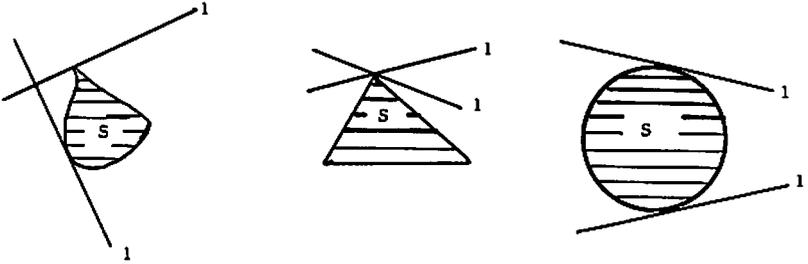


Figure 1

It is intuitively obvious that in any given direction, there can be exactly two parallel lines of support to a bounded convex figure. In the light of this intuitive judgment, the *width* of a convex figure can be defined as follows: Given a line l in the plane of a bounded convex figure, the distance between the two parallel lines of support perpendicular to the direction of l is called the *width* of the convex figure in a given direction. In other words, the width of a convex figure in a given direction is the smallest distance between parallel supporting lines of a bounded convex figure.

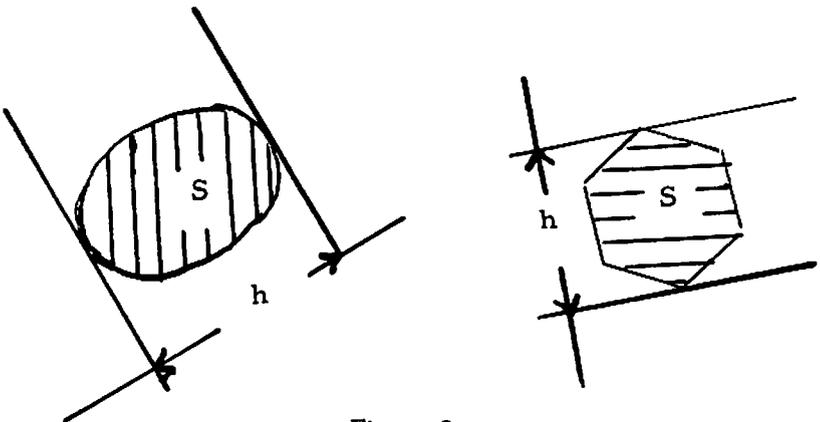


Figure 2

With this introduction, the discussion of convex curves that rotate freely within a regular polygon can begin. Making an exception to the logical, numerical order, curves that rotate freely in regular polygons with four sides will be discussed prior to those curves in three-sided regular polygons.

Recalling the definition of width of a convex curve in a given direction, a curve with the property that its width is the same in *all* directions is designated a *curve of constant width*. An excellent example of this type of curve is the circle, in which width equals diameter. There are infinitely many other examples of curves of constant width, however. The reason this type of curve (the curve of constant width) has been introduced here is that it completely defines the category of convex curves that can be rotated in regular polygons of four sides. It is an accepted property of the theory of convex bodies that a curve of constant width can be rotated inside a square and always maintain contact with the sides. Conversely, it has been established that a curve rotated inside a square such that it always maintains contact with the sides is necessarily a curve of constant width. An intuitive approach will help in understanding the validity of these two statements.

If two pairs of perpendicular supporting lines are drawn to a curve of constant width (the circle in the illustration below), the parallelogram which they form will be a square of width h . Therefore, it follows that all squares circumscribed about a curve of constant width are congruent. Thus, the square may be rotated in any direction so as to always remain circumscribed about the given curve. Also, since the square has sides equal to the width of the curve, the curve may be rotated in any direction so as always to remain in contact with the sides of the square.

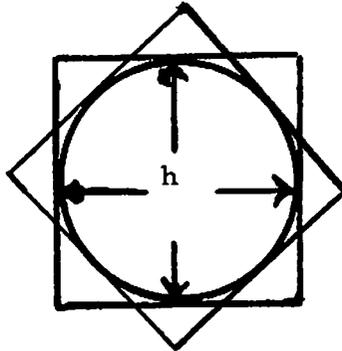


Figure 3

It is this movement property that completely characterizes curves of constant width. It is important to note here that this circle inscribed in the square of side h has the perimeter of πh . This fact will be important in the discussion of the construction and perimeter of other curves of constant width which now follows.

The simplest example of a curve of constant width, besides the circle, is the Reuleaux Triangle, named after the French technologist who in 1875 discovered the curve as having this property. The basic construction of the Reuleaux Triangle, indicated below, begins with an equilateral triangle of side h . With each vertex as center, circular arcs of radius h are drawn connecting the other two vertices. The composite of these three arcs is the curve of constant width.

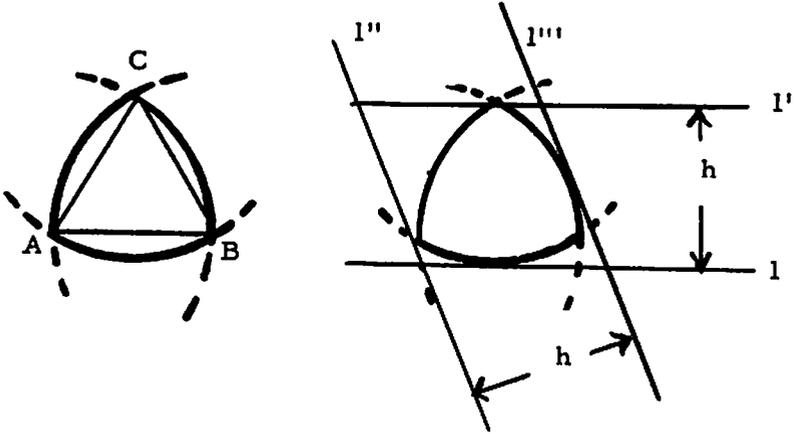


Figure 4

Notice that given any two parallel supporting lines of the Reuleaux Triangle, one of them passes through some vertex of the triangle ABC (which serve as the corner points of the curve) while the other is tangent to the opposite circular arc. Hence, the distance between any two parallel supporting lines, (that is, the width), of a Reuleaux Triangle is the radius of the arc—or h . Notice also, then, that each of the three arcs has a central angle of 60° and each arc has a length of $\frac{2\pi h}{6}$. This arc length is determined,

for example, by considering circular sector AB with radius h as $\frac{1}{6}$

of a circle with perimeter $2\pi h$. The perimeter of the entire curve then, is $3 \cdot \frac{2\pi h}{6} = \pi h$. Observe that the perimeter of the Reuleaux Triangle is equal to the perimeter of a circle of the same width h . This is not a coincidence. In fact, it is an excellent indication of the relationship to be established concerning curves rotated in regular polygons. But more will be said of this later.

An entire group of curves of constant width, sometimes referred to as Reuleaux polygons, are constructed according to the same principle as the Reuleaux Triangle. All that is required is that the polygon be regular with an odd number of sides. This stipulation of an odd number of sides is necessary, since for each vertex there must be two opposite vertices connected by an arc of radius h . Here is an example of the curve developed from the pentagon.

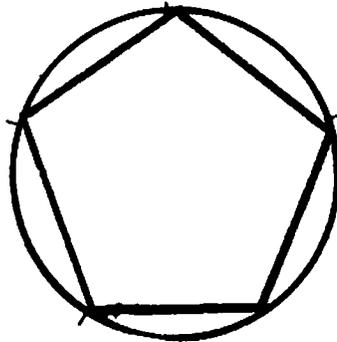


Figure 5

As in the case of the Reuleaux Triangle, the central angle of the arc, the length of the arc, and the perimeter of the entire curve can be discovered in the Reuleaux polygon. In general, if a regular polygon has $2n - 1$ sides, then from Euclidean geometry, the central angle of each arc has a measure of $\frac{2\pi}{2(2n - 1)}$, the length of each arc is $\frac{2\pi h}{2(2n - 1)}$, and the total perimeter of the curve is $2n - 1 \cdot \frac{2\pi h}{2(2n - 1)} = \pi h$. For example, let the initial figure be a regular pentagon. Then $2n - 1 = 5$. The central angle of each arc is $\frac{2\pi}{2(5)}$ or $\frac{\pi}{5}$ or 36° . The length of each arc is $\frac{2\pi h}{2(5)}$

or $\frac{\pi h}{5}$ since each circular sector is $\frac{1}{10}$ of an entire curve of radius

h . Following this, then, the total perimeter of the curve is $5 \cdot \frac{2\pi h}{2(5)}$

$= \pi h$. Once again, the perimeter of any of the Reuleaux polygons of width h is equal to the perimeter of the circle of the same width—the circle inscribed in the square of side h .

Consider one more type of curve of constant width. The non-circle curves of constant width examined thus far have all been obtained from regular polygons with an odd number of edges, possessing corners, and consisting of a number of equal circular arcs of the same radius. There are other types of curves of constant width and the one next to be examined is without corners, but is again obtained from regular polygons with an odd number of sides. The construction is indicated in the following figure for an equilateral triangle of side s . But keep in mind that the construction can be used on any regular n -gon.

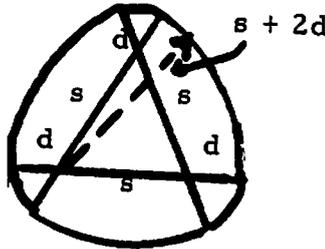


Figure 6

With each vertex of the triangle as center, two circular arcs—one of radius d and one of radius $s + d$ —can be drawn. The figure resulting will have three arcs of radius d about the vertices of the triangle, and three arcs of radius $s + d$ inside each angle of the triangle. If $s + 2d$ is referred to as h , the curve will be a curve of constant width $h = s + 2d$. This result follows from the fact that of any two parallel supporting lines of the curve, one is tangent to an arc of the larger circle and one is tangent to an arc of the smaller circle. (Fig. 6) Since both arcs always have the same center, the width is constant— $s + 2d$. Keeping in mind that this is an equilateral triangle, the length of each arc and the perimeter of the entire curve can be determined. For the smaller arcs, the length

of each is $\frac{2\pi d}{6}$, and for the larger arcs, the length of each is $\frac{2\pi(s+d)}{6}$. Thus, the total perimeter of the curve is $3 \cdot \frac{2\pi d}{6} + 3 \cdot \frac{2\pi(s+d)}{6} = \pi(s+2d) = \pi h$ which again is equal to the perimeter of a circle of width $h = s + 2d$.

The regularity of the curves of constant width which have thus far been discussed—that of equal arcs of the same radius or two sets of equal arcs of the same radius—is not necessary for this type of curve. The accompanying diagram illustrates another method of constructing curves of constant width. To save time, it will merely be stated that upon calculation of the perimeter, the result once again is πh where h is the width of the curve.

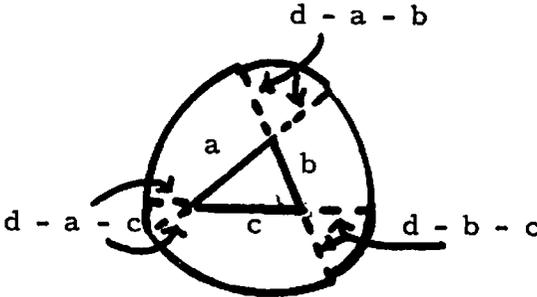


Figure 7

Thus, there is the possibility that all curves of constant width h have perimeter πh —the perimeter of a circle of the same width. The question arises—is this always the case? By first pursuing another question, the answer to this one will be apparent. Is this property of the perimeter being equal to πh characteristic of the curve of constant width only, or is there some deeper relationship between this value of the perimeter and any convex curve which rotates freely in a regular polygon?

For an answer to this question, discussion will now concern the second case of convex curves mentioned earlier. This is the case of the polygon where $n = 3$, or the equilateral triangle. Convex curves which rotate freely in equilateral triangles are called Δ -curves. As before, the curve derives its definition from the regular polygon in which it rotates. And, again as before, the simplest example is the circle.

Corresponding to the property of the width of the curve of constant width is the property of height of the Δ -curves. Since the triangles are equilateral, the altitudes from each vertex are equal. Hence, Δ -curves may also be described as curves whose height or altitude in every direction is the same, or as curves of constant height. Many other properties of Δ -curves are similar to those of curves of constant width. By considering a few examples of Δ -curves, the perimeter of the curve rotated in an equilateral triangle will be compared to the perimeter of the circle inscribed in the same triangle.

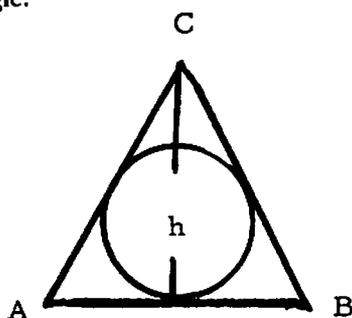


Figure 8

As mentioned earlier, the circle is an example of a Δ -curve since it can rotate freely within an equilateral triangle and always maintain contact with the sides. From Euclidean geometry, it is known that the radius of the circle inscribed in an equilateral triangle is $\frac{1}{3}$ the altitude of the triangle. Therefore, if the triangle has an altitude of h , the perimeter of the inscribed circle or Δ -curve is $\frac{2\pi h}{3}$.

Another Δ -curve can be found in the following manner. Let a circle whose radius equals the altitude of an equilateral triangle roll along one side of such a triangle. (Fig. 9) The triangle will cut an arc out of the circle which when reflected in the corresponding chord forms the shape of a "lens." The boundary of this lens is called a Δ -biangle, and the Δ -biangle is a Δ -curve. The fact that the specified circle rolls along the side of the triangle is important here. As it does, at each instance, the Δ -biangle is the same shape and size, and in a sense, is rotating in the equilateral triangle. For the sake of simplicity, this will be accepted as an intuitive proof that the Δ -biangle is a Δ -curve.

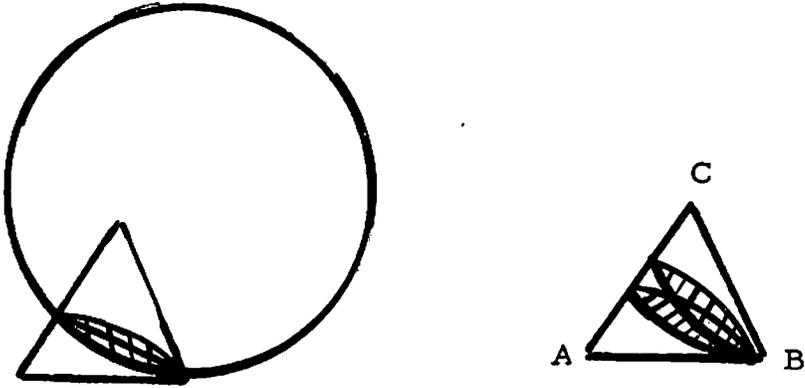


Figure 9

Since the triangle is equilateral, each arc of the Δ -biangle is 60° . Thus, the perimeter of the curve is $\frac{2}{6}$ or $\frac{1}{3}$ the perimeter of a circle of radius h , that is $\frac{2\pi h}{3}$. Notice that this result is the same perimeter as that of a circle rotating in an equilateral triangle of altitude h , as found above.

A square of side h becomes the basis for the construction of the next Δ -curve to be considered. About the vertices of the square, four circles of radius h are drawn. The boundary curve of the figure resulting from the intersection of these circles has been found to be a Δ -curve of height h . Thus it rotates freely in an equilateral triangle of altitude h . To keep the discussion simplified, no proof will be given of the fact.

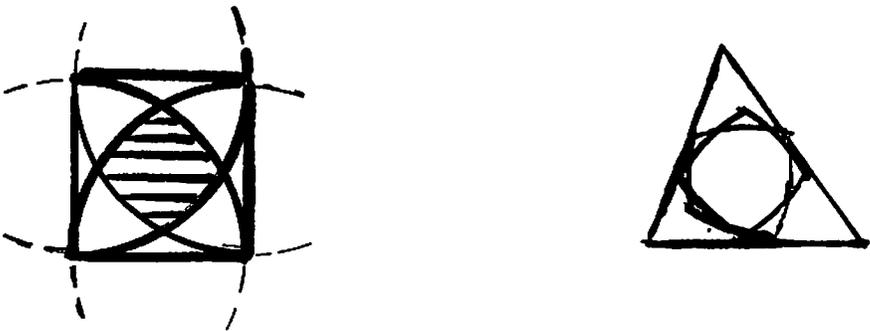


Figure 10

Notice that when an equilateral triangle is circumscribed about this Δ -curve, two sides of the triangle pass through adjacent corners of the curve while the third is tangent to the opposite arc. The altitude of any such circumscribed equilateral triangle equals the side of the square, that is, h . Through a detailed proof based on this construction, the curve is found to be defined by four circular arcs of 30° and radius h . As such, the arc length is $\frac{2\pi h}{12}$ or $\frac{\pi h}{6}$

and the perimeter of the total curve is $4 \cdot \frac{2\pi h}{12}$ or $\frac{2\pi h}{3}$ —once again the same as the perimeter of a circle inscribed in an equilateral triangle of altitude h .

The investigation could continue, yet the results would remain the same. For the two specific cases discussed here, a possibility has been suggested that curves rotated in regular polygons of n sides have perimeters very characteristic. (See table below.) In the case of curves of constant width, the perimeter of each curve investigated proved to be πh , the perimeter of a circle inscribed in a square of side h . The perimeter of each Δ -curve, $\frac{2\pi h}{3}$, is the same as the perimeter of a circle inscribed in an equilateral triangle of altitude h . Could some generalization be made? A French mathematician, Barbier, found the answer to be yes when he stated and proved the following theorem: The length of any curve K which rotates inside a regular polygon of n sides equals the length of the circle inscribed in this n -sided polygon. The examples constructed

Type of curve	Polygon in which rotated	Perimeter of curve	Perimeter of circle inscribed in same polygon
Curve of constant width	Square	πh	πh
Δ -curve	Equilateral Triangle	$\frac{2\pi h}{3}$	$\frac{2\pi h}{3}$

Figure 11

(continued on p. 42)

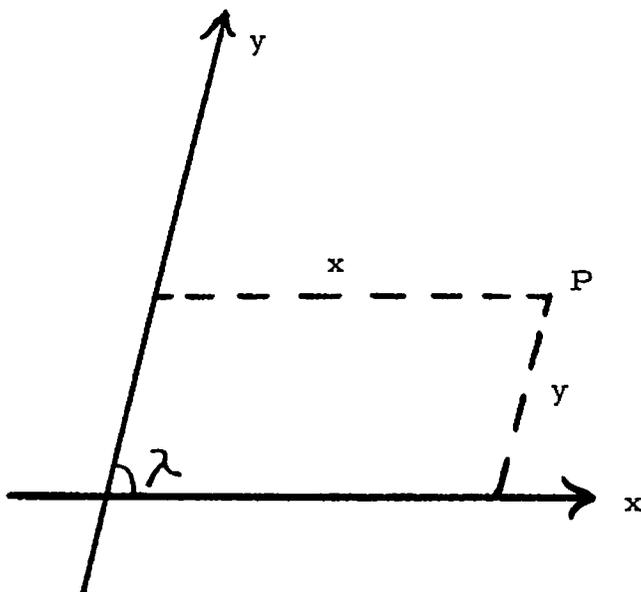
An Introduction to Quasi-Trigonometries*

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Speaking of trigonometry in the plural form may come as a surprise to many students of mathematics since one is well acquainted with the conventional trigonometric functions defined on the unit circle in the XY -plane. The existence of other functions which behave in much the same way as the familiar sine, cosine, and tangent should not, however, be excluded.

To begin the study, let us consider the quasi-plane which is simply two-space on which has been imposed a set of oblique axes.



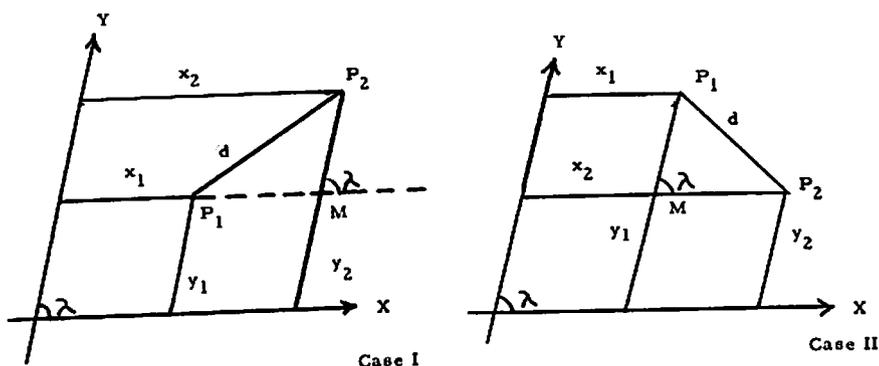
(Figure 1)

These axes meet at an angle λ where $0 \leq \lambda < 2\pi$. Given any point, P , its x coordinate is its distance to the Y -axis along a line drawn

*A paper presented at the 1969 National Convention of KME and awarded second place by the Awards Committee.

parallel to the X -axis, and similarly its y coordinate is the distance to the X -axis along a line drawn parallel to the Y -axis.

The first step in defining quasi-trigonometric functions is to define the distance between two arbitrary points, P_1 and P_2 in the quasi-plane. Consider two points with quasi coordinates (x_1, y_1) and (x_2, y_2) . Let d be the distance between these two points. To find d extend the lines through P_1 and P_2 parallel to the X - and Y -axes such that triangle P_1MP_2 is formed. Since P_1M and P_2M are parallel to the X - and Y -axes, the interior angle at M will either equal λ or $180 - \lambda$.



(Figure 2)

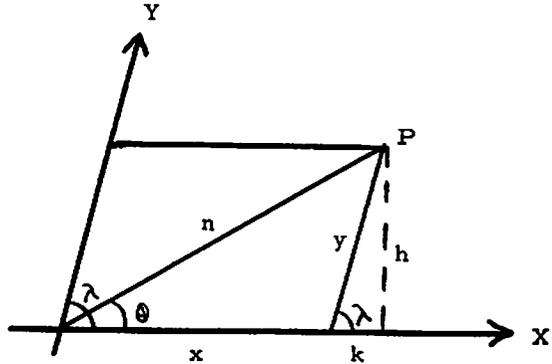
Now recalling the law of cosines, $c^2 = a^2 + b^2 - 2ab \cos C$, one can determine d . In Case I, $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 - 2 |x_2 - x_1| |y_2 - y_1| (-\cos \lambda)$. In Case II, $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 - 2 |x_2 - x_1| |y_2 - y_1| \cos \lambda$. In Case I, $(x_2 - x_1)$ and $(y_2 - y_1)$ are of the same sign while in Case II $(x_2 - x_1)$ and $(y_2 - y_1)$ differ in sign. Thus, by removing the absolute value signs, one can have one general formula for distance in the quasi-plane:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + 2(x_2 - x_1)(y_2 - y_1) \cos \lambda.$$

While the diagrams exhibit only the situation when $\lambda \leq 90^\circ$, this formula can be shown to be valid for all $\lambda \in (0, 2\pi)$. Note that when $\lambda = \pi/2$, $\cos \lambda = 0$ and it reduces to the familiar formula for distance in an orthogonal coordinate system.

Given these relations in the quasi-plane, quasi-trigonometric functions may now be defined. Let $P(x, y)$ be any point such that \overline{OP} determines an angle θ with the X -axis. If n is the length of \overline{OP} , or the distance from P to the origin, the quasi-trigonometric functions for any given λ are defined as:

- nis $\theta = y/n$
- nat $\theta = y/x$
- ces $\theta = n/x$
- conis $\theta = x/n$
- conat $\theta = x/y$
- coces $\theta = n/y$.



(Figure 3)

The analogy between these functions and the familiar trigonometric functions is readily seen. One can easily prove such relations as: $\text{nat } \theta = \text{nis } \theta / \text{conis } \theta$, $\text{ces } \theta = 1 / \text{conis } \theta$, etc.

It is also possible to express these functions in terms of the regular trigonometric functions of λ and θ . For instance, referring to Figure 3, one can see that, for $h \neq 0$, $\text{nis } \theta = y/n = y/n \cdot h/n = \csc \lambda \sin \theta = \sin \theta / \sin \lambda$. This relationship is also true for $h = 0$ in which case $\sin \theta = \text{nis } \theta = 0$. It is interesting to observe that since $\sin \lambda \leq 1$ for $0 \leq \lambda \leq \pi/2$, $\text{nis } \theta \geq \sin \theta$ when λ is in the interval $[0, \pi/2]$. This conclusion is justified in Figure 3 since $y \geq h$ when λ is less than 90° .

Examining $\text{conis } \theta$, one sees that, for $y \neq 0$, $\text{conis } \theta = x/n = x/n + k/n - k/n = (x+k)/n - k/n \cdot y/n = \cos \theta - \cos \lambda \text{nis } \theta$. Since $\text{nis } \theta = \sin \theta / \sin \lambda$, $\text{conis } \theta = \cos \theta - \cos \lambda \sin \theta / \sin \lambda = \cos \theta - \cot \lambda \sin \theta$. This result leads one to conclude that, for $0 \leq \lambda \leq \pi/2$, $\text{conis } \theta \leq \cos \theta$. Again this result is easily seen in Figure 3 since $x \leq x+k$.

These derived relations are valid when $\lambda = \pi/2$, in which case $\sin \lambda = 1$ so that $\text{nis } \theta = \sin \theta$ and $\cot \lambda = 0$ making $\text{conis } \theta = \cos \theta$.

One of the most important relations in regular trigonometry is: $\sin^2 \theta + \cos^2 \theta = 1$. This relation is seen quite naturally when the functions are defined on the unit circle where $x = \cos \theta$ and $y = \sin \theta$. Is there something analogous to this relation in quasi-trigonometry?

The relations discovered above can be rewritten, solving for $\sin \theta$ and $\cos \theta$: $\sin \theta = \text{nis } \theta \sin \lambda$ and $\cos \theta = \text{conis } \theta + \cos \lambda \text{nis } \theta$. Now one can write: $1 = \sin^2 \theta + \cos^2 \theta = \text{nis}^2 \theta \sin^2 \lambda + \text{conis}^2 \theta + 2 \cos \lambda \text{conis } \theta \text{nis } \theta + \cos^2 \lambda \text{nis}^2 \theta$. Notice that

$\sin^2 \theta$ appears in two terms. Factoring this term and remembering that $\sin^2 \theta + \cos^2 \theta = 1$, our equations simplify to: $1 = \sin^2 \theta + 2 \cos \lambda \cos \theta \sin \theta + \cos^2 \theta$.

Now tabulate some of these relations. In the "ordinary" coordinate system:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$1 = \sin^2 \theta + \cos^2 \theta.$$

In the quasi-coordinate system, one sees that both of these formulas are similarly altered and the factor $2(\cos \lambda)$ plays an important role:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + 2 \cos \lambda (x_2 - x_1)(y_2 - y_1) \text{ and}$$

$$1 = \sin^2 \theta + \cos^2 \theta + 2 \cos \lambda \cos \theta \sin \theta.$$

In the last equation, making the substitutions, $x = \cos \theta$, and $y = \sin \theta$ yields: $1 = x^2 + (2 \cos \lambda)xy + y^2$. This equation can now enable one to classify different types of quasi-trigonometries. It is a general quadratic equation and represents a conic section. The particular conic can be determined quite easily by examining the discriminant, which in this case is $d = 4 \cos \lambda - 4$. If $d = 0$, the equation is a parabola and quasi-parabolic trigonometry results. This happens when $\cos^2 \lambda = 1$, i.e. when $\lambda = 0$ or $\lambda = \pi$. The X and Y axes are then concurrent so quasi-parabolic is a type of degenerate trigonometry.

If $d < 0$ or $\cos^2 \lambda < 1$, the equation is an ellipse and the result is quasi-elliptic trigonometry when $\lambda \in (0, \pi)$ or $(\pi, 2\pi)$. Just as a circle is a special type of ellipse, regular circular trigonometry is a special type of quasi-elliptic trigonometry.

Finally, if $d > 0$, a hyperbola is obtained, but this implies $\cos^2 \lambda > 1$ in which case λ would have to be imaginary.

As an illustration consider one particular quasi-elliptic trigonometry—that for $\lambda = \pi/3$ or 60° . Draw a unit circle with center at the origin and superimpose the X' and Y' axes at an angle of 60° . Examine the coordinates of some points on this curve with respect to the two coordinate systems (Figure 4). Point A which has coordinates (1, 0) in rectangular coordinates has these same coordinates in quasi-coordinates, but the point B which is (0, 1) in rectangular coordinates is $(-1/\sqrt{3}, 2/\sqrt{3})$ in the specific quasi-coordinate system. These coordinates, as well as the others

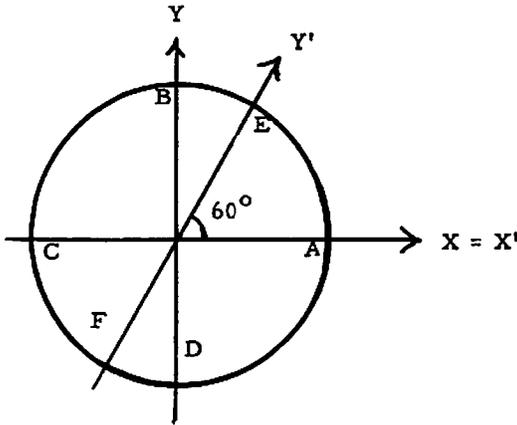


Figure 4

in the table, are found by using the properties of right triangles having angles of 30° and 60° .

Point	Rectangular Coordinates	λ -Coordinates
A	(1, 0)	(1, 0)
B	(0, 1)	$(-1/\sqrt{3}, 2/\sqrt{3})$
C	(-1, 0)	(-1, 0)
D	(0, -1)	$(1/\sqrt{3}, -2/\sqrt{3})$
E	$(1/2, \sqrt{3}/2)$	(0, 1)
F	$(-1/2, -\sqrt{3}/2)$	(0, -1)

Now it is easy to verify that the rectangular coordinates all lie on the "circle" $x^2 + y^2 = 1$. Can one find an algebraic equation for this curve in this quasi-coordinate system? Observe the equations relating quasi and regular trigonometries. In these equations make the substitutions $x = \cos \theta$, $y = \sin \theta$, $x' = \cos \lambda$, and $y' = \sin \lambda$, and these two coordinate systems are related such that $x' = x - y \cot \lambda$ and $y' = y/\sin \lambda$. These are the equations of transformation between the two coordinate systems. If one were to solve these

equations of transformation for x and y and substitute those values into $x^2 + y^2 = 1$, it would yield the equation: $x'^2 + 2x'y' \cos \lambda + y'^2 = 1$. In this specific case, $\cos \lambda = 1/2$, so that $x'^2 + 2x'y' + y'^2 = 1$, but this equation is the general equation describing the quasi-elliptic trigonometry. Examination of its discriminant shows that the equation is that of an ellipse. It is a simple matter to show that the points given in quasi-coordinates all satisfy this equation.

Using the relations between circular trigonometric functions and quasi-trigonometric functions, one can also derive formulas for the quasi-trigonometric functions of the sum and difference of two angles. For $\lambda = 60^\circ$, some of these are given here:

$$\begin{aligned} \text{nis}(\theta + \phi) &= \text{nis } \theta \text{ conis } \phi + \text{nis } \phi \text{ conis } \theta + \text{nis } \theta \text{ nis } \phi \\ \text{nis}(\theta - \phi) &= \text{nis } \theta \text{ conis } \phi - \text{nis } \theta \text{ nis } \phi \\ \text{conis}(\theta + \phi) &= \text{conis } \theta \text{ conis } \phi - \text{nis } \theta \text{ nis } \phi \\ \text{conis}(\theta - \phi) &= \text{conis } \theta \text{ conis } \phi - \text{nis } \theta \text{ nis } \phi + \text{conis } \theta \text{ nis } \phi \\ \text{nis}(2\theta) &= 2 \text{nis } \theta \text{ conis } \theta + \text{nis}^2 \theta \\ \text{conis}(2\theta) &= \text{conis}^2 \theta - \text{nis}^2 \theta. \end{aligned}$$

The formula for $\text{nis}(\theta + \phi)$ differs from that of $\sin(\theta + \phi)$ by the addition of the term $\text{nis } \theta \text{ nis } \phi$. The formulas for the nis of the difference of two angles and the conis of their sum are directly analogous to the formulas of regular trigonometry. The last two equations are simply special cases of the first and third.

Quasi-trigonometric functions are more than interesting exercises in mathematics. The quasi-hyperbolic functions are related to conventional hyperbolic functions; quasi-trigonometric functions have been found to be the solutions to certain non-linear differential equations and also they can be defined geometrically in three-space. They show that the ordinary trigonometric functions with which one is familiar are only a very specialized subset of a much larger set of functions.

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An Introduction to Probability Over Infinite Sample Spaces

JAMES WALSH

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When an experiment of some kind is undertaken, associated with the experiment is a set of results. For instance when a coin is tossed it will show either "heads" or "tails". These are the only possible results. In this experiment the set {heads, tails} is called the sample space. The set of all possible results of an experiment is called a sample space for the experiment. Each element of a sample space is called an outcome. In the experiment above "heads" and "tails" are the outcomes; if the coin is unbiased, they are equally likely outcomes.

Let us now consider another experiment. Suppose a regular unbiased six-sided die is rolled. The sample space, of course, is the set {1, 2, 3, 4, 5, 6}. The possible (equally likely) outcomes would be 1, 2, 3, 4, 5, and 6. Suppose we were interested in the outcomes which are greater than 4. Of course these outcomes would form the set {5, 6}, a subset of the sample space. Any subset of a sample space is called an event. Thus the subset {5, 6} represents the event defined by all possible outcomes greater than 4. For the sake of convenience we will denote an event by the letter E .

If, in an experiment, the number of outcomes is finite, then the probability of an event is defined by the equation:

$$P(E) = \frac{n(E)}{n(S)}$$

Where $n(E)$ is the number of elements in the event,
 $n(S)$ is the number of elements in the sample space, and
 $P(E)$ is read "the probability of E ".

A few elementary problems might be useful at this point for illustrative purposes.

1.) In the coin problem above the probability of heads is $1/2$. In the die problem above the probability of a number greater than 4 is $2/6$.

2.) Given the set of integers from 12 to 24 inclusively, what is the probability of randomly selecting a specified number?

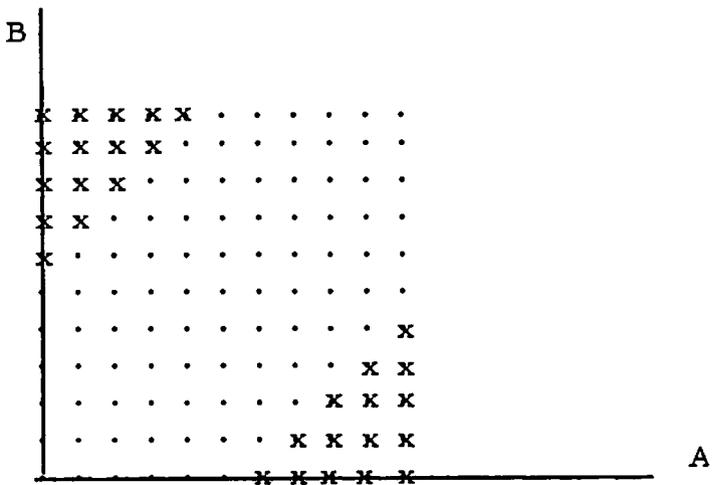
Since $n(E) = 1$ and $n(S) = 13$, $P(E) = 1/13$.

3.) Given the set of points on the coordinate plane (ordered

pairs of real numbers) which fall into the four standard quadrants, what is the probability of randomly selecting the quadrant in which a specified point lies?

Here $n(E) = 1$, $n(S) = 4$ and $P(E) = 1/4$.

4.) Somewhat different is the problem: Given the line segment $[0, 10]$ and two random points A and B , (A and B integers) what is the probability that the distance between them is greater than 5? We can graphically represent the sample space and the event defined by the distance being greater than 5.



x denotes the ordered pairs such that $|A - B| > 5$.

From the drawing it can be seen that $n(S) = 121$

and $n(E) = 30$

thus $P(E) = \frac{30}{121}$

Each of these examples follows the given definition of probability in as much as neither the size of the event nor the size of the sample space is infinite.

However, by using the same type of reasoning similar problems involving infinite sets may be worked. However, the equation

$P(E) = \frac{n(E)}{n(S)}$ must be altered slightly.

Consider the problem of finding the probability of selecting at random a specified number from the set of all real numbers from 12 to 24. Here $n(E) = 1$, but S is infinite. It seems reasonable to think of $\frac{n(E)}{n(S)}$ as a fraction whose numerator is constant but

whose denominator is larger than any given number and whose value therefore approaches zero. In other words, if $n(E)$ is finite and $n(S)$ is infinite we can define $P(E)$ to be zero.

But what of the case where both $n(E)$ and $n(S)$ are infinite? Special cases can be attacked by special methods. For example what is the probability that a randomly selected element from the set of all integers will be an even integer? Consider any finite set S_i of consecutive integers and let E_i be its subset of even integers. Then

$\frac{n(E_i)}{n(S_i)}$ is either $\frac{k}{2k+1}$, $\frac{k}{2k}$, or $\frac{k}{2k-1}$ (k an integer). Since

$\frac{n(E_i)}{n(S_i)}$ can be made as close to $1/2$ as we please by using large

enough sets E_i , it seems reasonable to accept $P(E) = 1/2$ as the answer to the given problem.

Another special method of attack may be applied to the following:

Two concentric circles of radius 1 and 2 respectively are given. What is the probability that a specified point inside the large circle will be outside the smaller? Clearly there are infinitely many elements in E and in S but instead of using notions of cardinality to define probability we can use notions of area. It is reasonable to replace $n(S)$ by the area of the larger circle rather than the cardinality of its interior points. Similarly $n(E)$ can be replaced by the area of the region between the circles. Thus

$$P(E) = \frac{A(E)}{A(S)} = \frac{3\pi}{4\pi} = \frac{3}{4}$$

where $A(E)$ is the size of E graphically represented and

$A(S)$ is the size of S graphically represented.

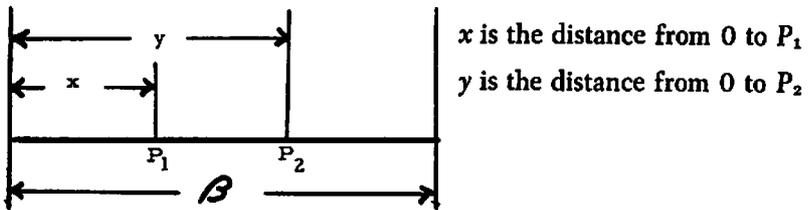
This last method of attack, suggested by example 3 above, leads

directly to the problem which was originally put to the author and which led to the writing of this paper. Namely:

“Let α and β be given positive real numbers, with $\alpha < \beta$. If two points are selected at random from a straight line segment of length β , what is the probability that the distance between them is at least α ?”

We shall use this problem as vehicle for the introduction of the notion of probability over infinite sample spaces.

Drawing the straight line segment of length β and the two points on it, we introduce two variables thusly:



But both x and y are bounded:

$$0 \leq x \leq \beta,$$

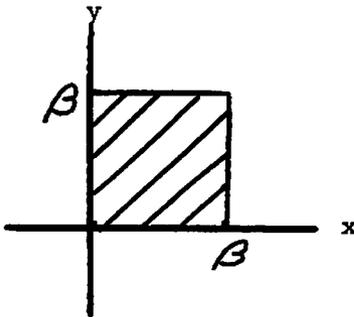
$$0 \leq y \leq \beta.$$

Thus the region of possible values of x and y can be graphed on the Cartesian plane: $\{(x, y) \mid 0 \leq x \leq \beta \text{ and } 0 \leq y \leq \beta\}$

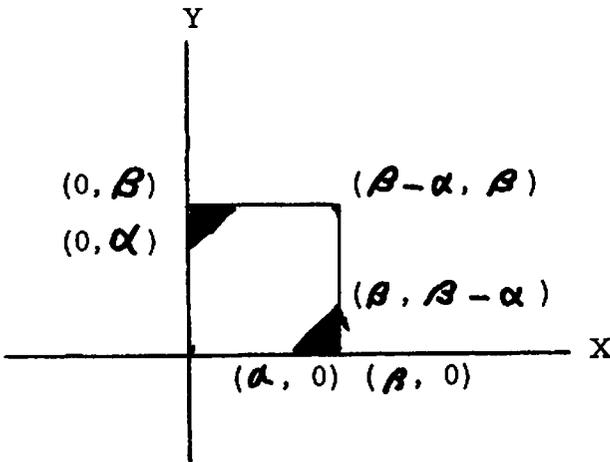
Given: β is a positive real number,

$$0 \leq x \leq \beta,$$

$$0 \leq y \leq \beta.$$



Now we wish to find the portion of this region which satisfies the inequality $|x - y| \geq \alpha$:



The shaded portions satisfy the inequality. The area of the shaded portion is: $\frac{1}{2}(\beta - \alpha)(\beta - \alpha) + \frac{1}{2}(\beta - \alpha)(\beta - \alpha) = (\beta - \alpha)^2$ so the probability that $|x - y| \geq \alpha$ with given information is the area of these portions divided by the area of the possible values which is $(\beta)^2$, the area of the square region which contains all possible values of x and y .

So the probability is $\frac{(\beta - \alpha)^2}{\beta^2}$.



I do not know what I may appear to the world, but to myself I seem to have been only a boy playing on the seashore, and diverting myself in now-and-then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

—I. NEWTON

Linear Diophantine Equations and Congruences^{*}

JANET LOVE

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Sun-Tsu in the first century found a least positive integer having remainders 2, 3, and 2 when divided by 3, 5, and 7 respectively.

$$X = 3 \cdot w + 2$$

$$X = 5 \cdot y + 3$$

$$X = 7 \cdot z + 2.$$

These equations are in the form of the division algorithm, that is
dividend = divisor \cdot quotient + remainder.

Letting $-w = W$, $-y = Y$, and $-z = Z$, the system of equations takes on this form:

$$X + 3 \cdot W = 2$$

$$X + 5 \cdot Y = 3$$

$$X + 7 \cdot Z = 2.$$

The problem is to solve such a system for an integral solution for X . Brahmagupta in the seventh century found a least positive integer, 59, having remainders 2, 3, 4, and 5 when divided by 3, 4, 5, and 6 respectively.

$$59 = 3 \cdot 19 + 2$$

$$59 = 4 \cdot 14 + 3$$

$$59 = 5 \cdot 11 + 4$$

$$59 = 6 \cdot 9 + 5$$

Problems of a similar nature include integral solutions to equations of the form $ax + by = n$ where a , b , and n are given integers. Because the solutions are restricted to integers, the equations are called Diophantine equations in honor of Diophantus of the third century; however, this is only a modern conformance because Diophantus would have sought all fractional solutions as

^{*}A paper presented at the 1969 National Convention of KME and awarded third place by the Awards Committee.

well. While some problems involving Diophantine equations are merely a curiosity, there exist many important Diophantine equations. This paper will first consider a curious problem and then the important linear Diophantine equation.

Here is the problem. On a desert island, five men and a monkey gathered coconuts all day, then went to sleep. The first man awoke and decided to take his share. He divided the coconuts into five equal shares with one coconut left over. He gave the extra one to the monkey, hid his share, and then went back to sleep. Later the second man awoke, took his fifth from the remaining pile. He too had one extra and gave it to the monkey. He hid his pile, then went back to sleep. Each of the remaining men did likewise in turn. In the morning, the five men divided the remaining coconuts into five equal shares, and again one was left for the monkey. The problem is to find the minimum number of coconuts present originally. This problem is probably the most often worked on and least often solved of all Diophantine brain teasers.

Let K = total number of coconuts. F is the number each man received on the final division. A , B , C , D , and E are the number of coconuts the first, second, third, fourth, and fifth man, respectively, took when he awoke and divided the coconuts. The total number of coconuts equals five equal piles which the first man had, + 1 left over for the monkey.

$$K = 5A + 1.$$

If the first man hid his fifth, then there were $4A$ coconuts left.

$$4A = 5B + 1.$$

Continuing in the same manner,

$$4B = 5C + 1$$

$$4C = 5D + 1$$

$$4D = 5E + 1$$

$$4E = 5F + 1.$$

By substitution, the system of equations simplifies to become $-1024K + 15625F = -11529$, a Diophantine equation. What is the smallest value for K which will satisfy this equation?

To begin, it is necessary to state a few fundamental definitions and theorems which are necessary in order to solve Diophantine equations and congruences. (See [4] for proof of theorems)

Definition 1: Let a and b be integers. a divided b , written $a \mid b$, if and only if $ak = b$ where k is an integer. If a does not divide b , then it is written $a \nmid b$.

Definition 2: a is congruent to b modulo m , written $a \equiv b \pmod{m}$, if and only if $a - b = km$ where k is an integer. By definition 1, $a - b = km$ implies that $m \mid a - b$.

Definition 3: Let d be a common divisor of a and b where a , b , and d are integers. If every common divisor of a and b is a divisor of d , then d is called the greatest common divisor of a and b and is denoted by $d = (a, b)$. If $d = (a, b)$, then $dA = a$, $dB = b$, where A and B are integers, and $(A, B) = 1$. If $(A, B) = 1$, A and B are said to be relatively prime.

Theorem 1: If $ab \equiv ac \pmod{m}$, $(a, m) = d$, $m = m_1 d$, then $b \equiv c \pmod{m_1}$.

Example: $2 \cdot 3 \equiv 2 \cdot 8 \pmod{10}$. Let $a = 2$, $b = 3$, $c = 8$, $m = 10$. Since $(a, m) = 2$ and $m = 5 \cdot 2$, $3 \equiv 8 \pmod{5}$.

Theorem 2: If $ab \equiv ac \pmod{m}$ and a and m are relatively prime, then $b \equiv c \pmod{m}$.

Example: $3 \cdot 7 \equiv 3 \cdot (-3) \pmod{10}$. Let $a = 3$, $b = 7$, $c = -3$, $m = 10$. Since $(a, m) = 1$, $7 \equiv -3 \pmod{10}$.

Theorem 3: If $(a, m) = 1$, then $ax \equiv b \pmod{m}$ has a unique solution.

Theorem 4: The congruence $ax \equiv b \pmod{m}$ has a solution if and only if $d = (a, m) \mid b$.

These definitions and theorems will be basic to the solution of linear Diophantine equations and congruences.

Consider a linear equation in two unknowns.

Theorem 5: The equation $ax + by = n$ where a , b , and n are given integers has a solution in integers if and only if $(a, b) \mid n$.

Proof:

Part A

Suppose X and Y are solutions to $ax + by = n$.

$$aX + bY = n; \quad aX - n = -bY$$

$$aX \equiv n \pmod{b}$$

$aX \equiv n \pmod{b}$ has a solution if and only if $(a, b) \mid n$.

Part B

Suppose $(a, b) \mid n$

Then $ax \equiv n \pmod{b}$ has a solution, call it X .

$$aX \equiv n \pmod{b}$$

$$aX - n = kb; aX - kb = n.$$

Let $-k = Y$

$$aX + bY = n.$$

The solution to $ax + by = n$ may be found by solving the congruence $ax \equiv n \pmod{b}$.

Example: Find the complete solution to $2x + 3y = 11$. Now since $(2, 3) = 1$ and $1 \mid 11$, there exists a solution in integers.

$$2x \equiv 11 \pmod{3}$$

$$2x \equiv 2 \pmod{3}$$

$$x \equiv 1 \pmod{3}$$

$$x = 1 + 3t \text{ where } t \text{ is an integer.}$$

$$\text{By substitution, } 2(1 + 3t) + 3y = 11; y = 3 - 2t.$$

Example: Find the complete solution to $4x + 6y = 11$. Since $(4, 6) = 2$, but $2 \nmid 11$, there does not exist a solution in integers.

Consider once again the Diophantine equation of the coconut problem.

$$-1024K + 15625F = -11529$$

Since $(-1024, 15625) = 1 \mid 11529$, there exists a solution in integers to the congruence $-1024K \equiv -11529 \pmod{15625}$ which can also be written as $-1024K \equiv -11529 \pmod{5^6}$. Since the mod is so large, the congruence is not easily solved by trial and error. Stewart, in his *Theory of Numbers*, develops a method for solving such a congruence where the mod is a power of a prime number. Following Stewart's discussion, the general solution $K = 15621 + 15625T$, where T is an integer, is derived. If $T = 0$, then $K = 15621$, the minimum number of coconuts present originally.

Consider now a general linear equation in k unknowns.

Theorem 6: $a_1x_1 + a_2x_2 + \dots + a_kx_k = n_2$ where a_1, a_2, \dots, a_k are given integers has a solution in integers if and only if $d_2 \mid n_2$ where $d_2 = (a_1, a_2, \dots, a_k)$.

Part A. Prove: If $a_1x_1 + a_2x_2 + \dots + a_kx_k = n_2$ has a solution in integers, then $d_2 = (a_1, a_2, \dots, a_k) \mid n_2$.

Proof by induction:

Let $k = 2$. $a_1x_1 + a_2x_2 = n_1$. If $a_1x_1 + a_2x_2 = n_1$ has a solution in integers, $(a_1, a_2) \mid n_1$.

Induction hypothesis: If $a_1x_1 + a_2x_2 + \dots + a_kx_k = n_2$ has a solution in integers, then $d_2 = (a_1, a_2, \dots, a_k) \mid n_2$.

Show by induction that if $a_1x_1 + a_2x_2 + \dots + a_kx_k + a_{k+1}x_{k+1} = n_3$ has a solution in integers, then $(a_1, a_2, \dots, a_k, a_{k+1}) \mid n_3$.

Let $a_1x_1 + a_2x_2 + \dots + a_kx_k + a_{k+1}x_{k+1} = n_3$ have a solution $B_1, B_2, \dots, B_k, B_{k+1}$.

Then by substitution, $a_1B_1 + a_2B_2 + \dots + a_kB_k = n_3 - a_{k+1}B_{k+1}$.

By induction hypothesis, $d_2 = (a_1, a_2, \dots, a_k) \mid n_3 - a_{k+1}B_{k+1}$.

Thus $cd_2 = n_3 - a_{k+1}B_{k+1}$ where c is an integer or $a_{k+1}B_{k+1} - n_3 = -cd_2$.

$a_{k+1}B_{k+1} \equiv n_3 \pmod{d_2}$.

If $a_{k+1}B_{k+1} \equiv n_3 \pmod{d_2}$ has a solution, $(a_{k+1}, d_2) \mid n_3$.

But $(a_{k+1}, d_2) = (a_{k+1}, (a_1, a_2, \dots, a_k)) = (a_1, a_2, \dots, a_k, a_{k+1})$.

Thus by induction if $a_1x_1 + a_2x_2 + \dots + a_kx_k = n_2$ has a solution, then $(a_1, a_2, \dots, a_k) \mid n_2$.

Part B. Prove: If $d_2 = (a_1, a_2, \dots, a_k) \mid n_2$, then $a_1x_1 + a_2x_2 + \dots + a_kx_k = n_2$ has a solution in integers.

Proof by induction:

Let $k = 2$. Thus $d_1 = (a_1, a_2) \mid n_1$.

If $d_1 \mid n_1$, then $a_1x_1 + a_2x_2 = n_1$ has a solution in integers.

Induction hypothesis: If $d_2 = (a_1, a_2, \dots, a_k) \mid n_2$, then $a_1x_1 + a_2x_2 + \dots + a_kx_k = n_2$ has a solution in integers.

Show by induction that if $d_3 = (a_1, a_2, \dots, a_k, a_{k+1}) \mid n_3$, then $a_1x_1 + a_2x_2 + \dots + a_kx_k + a_{k+1}x_{k+1} = n_3$ has a solution in integers.

Suppose $d_3 = (a_1, a_2, \dots, a_k, a_{k+1}) \mid n_3$.

Consider $a_1x_1 + a_2x_2 + \dots + a_kx_k + a_{k+1}x_{k+1} = n_3$.

$a_1x_1 + a_2x_2 + \dots + a_kx_k = n_3 - a_{k+1}x_{k+1}$.

$a_1x_1 + a_2x_2 + \dots + a_kx_k \equiv n_3 \pmod{a_{k+1}}$ has a unique solution in integers since $d_3 \mid n_3$.

Call it B_1, B_2, \dots, B_k .

By substitution, $a_1B_1 + a_2B_2 + \dots + a_kB_k \equiv n_3 \pmod{a_{k+1}}$

$a_1B_1 + a_2B_2 + \dots + a_kB_k - n_3 = fa_{k+1}$ where f is an integer.

$a_1B_1 + a_2B_2 + \dots + a_kB_k - fa_{k+1} = n_3$.

Let $-f = B_{k+1}$.

Thus by induction if $d_2 = (a_1, a_2, \dots, a_k) \mid n_2$, then $a_1x_1 + a_2x_2 + \dots + a_kx_k = n_2$ has a solution in integers.

Consider once again the problem of Sun-Tsu. If such a system of Diophantine equations is to be solved, one must first solve one of the equations and then substitute this solution into the second equation.

$$X + 3 \cdot W = 2 \qquad X \equiv 2 \pmod{3}$$

$$X + 5 \cdot Y = 3 \qquad X \equiv 3 \pmod{5}$$

$$X + 7 \cdot Z = 2 \qquad X \equiv 2 \pmod{7}$$

$$X \equiv 2 \pmod{3}$$

$$X = 2 + 3t_1$$

$$2 + 3t_1 \equiv 3 \pmod{5}$$

$$3t_1 \equiv 1 \pmod{5}$$

$$3t_1 \equiv 6 \pmod{5}$$

$$t_1 \equiv 2 \pmod{5}$$

$$t_1 = 2 + 5t_2$$

$$X = 2 + 3(2 + 5t_2); \quad X = 8 + 15t_2$$

$$8 + 15t_2 \equiv 2 \pmod{7}$$

$$15t_2 \equiv -6 \pmod{7}$$

$$15t_2 \equiv 1 \pmod{7}$$

$$t_2 \equiv 1 \pmod{7}$$

$$t_2 = 1 + 7t_3$$

$$X = 8 + 15(1 + 7t_3); \quad X = 23 + 105t_3.$$

Let $t_3 = 0$. Thus $X = 23$ is the least positive integer having remainders 2, 3, and 2 when divided by 3, 5, and 7 respectively.

There is an endless variety of Diophantine equations. While only the linear Diophantine equation has been considered here, the study includes finding an integer, n , such that n is the sum of two squares or finding an integer, m , such that m is the sum of four squares. Whether Diophantus drew much or little from the work of his predecessors is not known. But it is certain that his work has had a profound influence on the development of number theory of which Diophantine equations and congruences are an important part.

(Continued on p. 48)

The Problem Corner

EDITED BY ROBERT L. POE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before March 1, 1970. The best solutions submitted by students will be published in the Spring 1970 issue of *The Pentagon*, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Professor Robert L. Poe, Department of Mathematics, Berry College, Mount Berry, Georgia 30149.

PROPOSED PROBLEMS

226. *Proposed by John Caffrey, Washington, D.C.*

A, B, C, and D are arranged in a Greek square. The sum of the four columns is given using letters of the same digit value. Obviously, the simple sum would be the same if the square were rotated. What are the four digits?

$$\begin{array}{r} BCDA \\ CABD \\ DBAC \\ ADCB \\ \hline ABBBA \end{array}$$

227. *Proposed by Leigh Jones, Pleasantville, New Jersey.*

The exact value of $N!$, N a positive integer greater than or equal to 5, is an even positive integer which terminates in one or more zeros; that is, the exact value of $N!$, $N \geq 5$, has one or more trailing zeros. Are there any factorials with exactly 5 trailing zeros? If we say that $N!$ has k trailing zeros is it possible to determine which values of k have no corresponding values of N ?

228. *Proposed by Leigh Jones, Pleasantville, New Jersey.*

If a polynomial over the field of complex numbers containing k distinct terms is raised to the n th power what is the least possible number of distinct terms in the result and what is the greatest possible number of terms in the result?

229. *Proposed by the Editor.*

Today students who either take drugs, or belong to student activist organizations which oppose the Establishment, or are

in opposition to the Vietnam War give college and university administrators many uneasy moments. It is not unreasonable to believe that many of these administrators would like to enroll in their schools as few students as possible who may be classified as belonging to one or more of the above categories. Certainly then, such schools must attempt to enroll the minimum number of students which may be identified with all three of these groups. However, the government and other organizations that provide institutions with lucrative research and student aid grants insist that such colleges and universities maintain fair and unbiased admission policies. Recently a study conducted by Trivia Researchers, Inc. and sponsored by the Senate revealed that 5% or less of the student body in the average university was composed of students who could be characterized as drug takers who oppose the Establishment and the Vietnam War. Now Dr. Grant Ghettar, the president of BIG University (Bountiful Institutional Grants University), through subjective questions asked on admission applications has been able to determine that a fraction, p , of the total applicants for admission to BIGU next fall are potential drug users, a fraction, q , of them will belong to activist organizations which oppose the Establishment, and a fraction, r , of them oppose the Vietnam War. Since Dr. Grant Ghettar is more of a politician than an academician, will you determine for him the least number of students who may be members of the drug-taking opposition to the establishment and the Vietnam War that should be admitted to BIG University next fall? Keep in mind that some must be admitted in order to obtain grants.

230. *Proposed by the Editor.*

In the long division problem below each of the odd digits has been replaced by the letter O and each of the even digits has been replaced by the letter E. Can you state the problem in digits?

$$\begin{array}{r}
 \text{OOE} \\
 \text{OOE} \overline{) \text{EEOOE}} \\
 \underline{\text{EOE}} \\
 \text{OOO} \\
 \underline{\text{OEE}} \\
 \text{EOE} \\
 \underline{\text{EOE}} \\
 \text{EOE}
 \end{array}$$

SOLUTIONS

221. *Proposed by R. S. Luthar, University of Wisconsin, Waukesha, Wisconsin.*

Show that for any positive integer n ,

$2^n + 3^n + 4^n \equiv (-1) \pmod{10}$ iff 4 does not divide n .

Solution by S. Ron Oliver, Morningside College, Sioux City, Iowa.

$2^i \equiv 6 \pmod{10}$ iff $i \equiv 0 \pmod{4}$,

$2^i \equiv 2 \pmod{10}$ iff $i \equiv 1 \pmod{4}$,

$2^i \equiv 4 \pmod{10}$ iff $i \equiv 2 \pmod{4}$,

and $2^i \equiv 8 \pmod{10}$ iff $i \equiv 3 \pmod{4}$.

$3^i \equiv 1 \pmod{10}$ iff $i \equiv 0 \pmod{4}$,

$3^i \equiv 3 \pmod{10}$ iff $i \equiv 1 \pmod{4}$,

$3^i \equiv 9 \pmod{10}$ iff $i \equiv 2 \pmod{4}$,

and $3^i \equiv 7 \pmod{10}$ iff $i \equiv 3 \pmod{4}$.

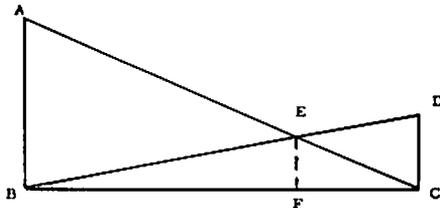
$4^i \equiv 6 \pmod{10}$ iff $i \equiv 0 \pmod{2}$

and $4^i \equiv 4 \pmod{10}$ iff $i \equiv 1 \pmod{2}$.

Therefore $i \equiv 0 \pmod{4}$ iff $2^i + 3^i + 4^i \equiv (6 + 1 + 6) \pmod{10}$ or $2^i + 3^i + 4^i \equiv 3 \pmod{10}$ and $2^i + 3^i + 4^i \equiv (-1) \pmod{10}$; $i \equiv 1 \pmod{4}$ iff $2^i + 3^i + 4^i \equiv (2 + 3 + 4) \pmod{10}$ or $2^i + 3^i + 4^i \equiv 9 \pmod{10}$ or $2^i + 3^i + 4^i \equiv (-1) \pmod{10}$; $i \equiv 2 \pmod{4}$ iff $2^i + 3^i + 4^i \equiv (4 + 9 + 6) \pmod{10}$ or $2^i + 3^i + 4^i \equiv 9 \pmod{10}$ or $2^i + 3^i + 4^i \equiv (-1) \pmod{10}$; $i \equiv 3 \pmod{4}$ iff $2^i + 3^i + 4^i \equiv (8 + 7 + 4) \pmod{10}$ or $2^i + 3^i + 4^i \equiv 9 \pmod{10}$ or $2^i + 3^i + 4^i \equiv (-1) \pmod{10}$.

Also solved by Jim Polkey, Texas Technological College, Lubbock, Texas; Kenneth M. Wilke, Topeka, Kansas; Charles E. Wilson, Florence State University, Florence, Alabama; David P. Zerbe, Kutztown State College, Kutztown, Pennsylvania.

222. *Proposed by Rosser J. Smith, III, Sun Oil Company, Dallas, Texas.*



Given the two right triangles, $\triangle ABC$ and $\triangle DCB$, with $AC = 70$, $DB = 60$, and $EF = 10$, find BC .

Solution by C. N. Mills, Illinois State University, Normal, Illinois.

Let angle $ACB = \phi$, angle $DCB = \theta$, and $X = BC$. Also, $\tan \theta = DC/BC$, $\tan \phi = AB/BC$, $(DC)^2 = (BD)^2 - (BC)^2$, and $(AB)^2 = (AC)^2 - (BC)^2$. Substituting for BF and FC in the equation $BC = BF + FC$ and rearranging one arrives at

$$10(4900 - X^2)^{\frac{1}{2}} + 10(3600 - X^2)^{\frac{1}{2}} = [(4900 - X^2)(3600 - X^2)]^{\frac{1}{2}}.$$

Rationalizing the above equation one will get an eighth degree equation. Using Horner's Method, and a computer one finds that BC is approximately 58.46.

Note. Professor C. N. Mills, who is now Professor Emeritus of Mathematics at ISU, was the National Treasurer of KME during 1934-35.

Also solved by Kenneth M. Wilke, Topeka, Kansas; David P. Zerbe, Kutztown State College, Kutztown, Pennsylvania.

223. *Proposed by Charles W. Trigg, San Diego, California.*

Show that no pentagonal number can be partitioned into three three-digit primes which together contain the nine positive digits.

Solution by David P. Zerbe, Kutztown State College, Kutztown, Pennsylvania.

The smallest number possible which can be formed by adding three odd three-digit numbers, utilizing all nine digits is 801. And the largest such number obtainable is 2529. Therefore we eliminate all pentagonal numbers less than 801 and greater than 2529. Now the sum of three three-digit primes is odd, hence we can eliminate all even pentagonal numbers between 801 and 2529. Also, a three-digit prime must end in 1, 3, 7, or 9 and if the three three-digit primes have distinct digits their sum must end in 1, 3, 7, or 9. We thus eliminate all pentagonals ending in 5 from within the indicated range. This leaves only 1001, 1247, 1617, 1717, 2147, and 2501 as possible candidates. (From here on we do not consider numbers ending in 5.) The largest number with a one in the units position which can be formed by adding three odd three-digit numbers, utilizing all nine digits is 2421. Thus, eliminate 2501. The smallest

such number is 1341. Eliminate 1001. The only number which ends in 17 and is formed by adding three odd three-digit numbers, utilizing all nine digits is 1917. 1617 and 1717 will not work. The only such number which ends in 47 is 1647 and therefore 1247 and 2147 are eliminated.

224. Proposed by R. S. Luthar, University of Wisconsin, Wankesha, Wisconsin.

$$\text{Evaluate limit } \left(\frac{x + \log \sec x}{x} \right)^{\frac{x}{\log \cos x}}.$$

Solution by Jim Polskey, Texas Technological College, Lubbock, Texas.

$$y = \lim_{x \rightarrow 0} \left(\frac{x + \log \sec x}{x} \right)^{\frac{x}{\log \cos x}} = \lim_{x \rightarrow 0} \left(\frac{1 - \log \cos x}{x} \right)^{\frac{x}{\log \cos x}}.$$

Let $a = \frac{x}{\log \cos x}$ and $b = 1/a$. Then

$$y = \lim_{x \rightarrow 0} (1 - b)^a. \text{ Using the binomial expansion,}$$

$$\begin{aligned} y &= \lim_{x \rightarrow 0} \left(1 - ab + \frac{a(a-1)b^2}{2!} - \frac{a(a-1)(a-2)b^3}{3!} + \dots \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{1-b}{2!} - \frac{(1-b)(1-2b)}{3!} + \dots + \frac{(-1)^n(1-b)(1-2b)\dots(1-(n-1)b)}{n!} + \dots \right) \\ &= \lim_{x \rightarrow 0} \left\{ \sum_{n=2}^{\infty} \left[\frac{(-1)^n}{n!} \prod_{m=1}^{n-1} (1 - mb) \right] \right\} \\ &= \sum_{n=2}^{\infty} \left\{ \frac{(-1)^n}{n!} \prod_{m=1}^{n-1} \left[\lim_{x \rightarrow 0} (1 - mb) \right] \right\}. \text{ But } b \text{ is of the inde-} \end{aligned}$$

terminate form $0/0$ as $x \rightarrow 0$. Using l'Hopital's rule, one has

$$\begin{aligned}
 y &= \sum_{n=2}^{\infty} \left\{ \frac{(-1)^n}{n!} \prod_{m=1}^{n-1} \left[\lim_{x \rightarrow 0} (1 + m \tan x) \right] \right\} \\
 &= \sum_{n=2}^{\infty} \left\{ \frac{(-1)^n}{n!} \prod_{m=1}^{n-1} (1) \right\} = \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} .
 \end{aligned}$$

Therefore $y = 1/e$.

225. *Proposed by Thomas P. Dence, University of Colorado, Boulder, Colorado.*

Consider six different points in the plane. Draw all line segments connecting one point with another. Now color each line segment with one of two different colors. Prove that there will always exist a triangle of the resulting colored drawing whose sides have the same color.

Solution by Thomas P. Dence (proposer of the problem), University of Colorado, Boulder, Colorado.

Let black and red be the two colors. Let a_1, a_2, \dots, a_6 be the six points. Consider the five lines connecting a_1 to a_2, a_3, \dots, a_6 . Three of these lines must have the same color, say it is black. Without loss of generality say these three lines are from a_1 to $a_2, a_3,$ and a_4 . Consider the triangle formed by these three points $a_2, a_3,$ and a_4 . If all sides are red then we are done. Otherwise one side is colored black. Then that side together with its two adjacent line segments drawn to a_1 form an all black triangle.



Editor's note: The article submitted by Gerald Esch in the spring 1969 issue of *The Pentagon* had an error. For $F(X) = (1 - X)^{-n}$ with $n = 3$, the Maclaurin series for $F(X)$ should have been $1 + 3x + 6x^2 + 10x^3 + \dots$.(See p. 90)

The Book Shelf

EDITED BY JAMES BIDWELL

This department of *The Pentagon* brings to the attention of its readers published books (both old and new) which are of a common nature to all students of mathematics. Preference will be given to those books written in English or to English translations. Books to be reviewed should be sent to Dr. James Bidwell, Central Michigan University, Mount Pleasant, Michigan 48858.

Modern Principles of Mathematics, Robert T. Craig, Prentice-Hall, Englewood Cliffs, N.J., 1969, 400 pp., \$8.95.

After having examined this survey-type book, intended (according to the author) primarily for college non-mathematics majors, the reviewer finds that her feelings are similar to those she has experienced at the end of a smorgasbord dinner. She has had a taste or two of numerous items, some of which really whetted her appetite for more, and others of which she could have done without altogether. At the conclusion of the meal, she really didn't know what she had eaten, and had the uneasy feeling that the combination was chosen too indiscriminately and unwisely to be a truly satisfying repast.

Craig's smorgasbord book consists of two parts, ten chapters, and ninety sections. To describe what is in the book, one would have to list all the section headings, since a given chapter will contain a weird assortment of topics. The student is exposed, among other things, to set theory, logic, number theory, combinatorial mathematics, probability, the number system, trigonometry, analytic geometry, differential and integral calculus, function theory, measure theory, matrix theory, group theory, projective geometry, non-euclidean geometries, design of experiments, and topology. The abundant exercises range all the way from the trivial one of deciding whether or not $\sqrt{169}$ is irrational to the sophisticated one of proving that if F is a finite field, then the intersection of all subfields of F has a prime number of elements. Many of the proofs requested in the exercises would challenge an upper division honors student who is majoring in mathematics.

The author has attempted to cover too many topics, and in so doing he has treated some of them in too sketchy a fashion. For example, he devotes one section, a little over two pages in length, to point set topology. In this section he defines topological space, open and closed sets, open covering, and compact set. The only

theorem that he states and proves is the one to the effect that if a subset A of the real numbers is compact, then A is closed and bounded; he then asks the student, in an exercise, to prove the difficult converse!

It is perhaps Craig's brief treatment of topics that has resulted in several inaccurate or misleading statements. For instance, he asserts that three linear equations in three unknowns have a solution if and only if the determinant of the matrix of the system is not zero. (The proof is left as an exercise.) In a section in which he gives the epsilon definition of a Cauchy sequence of rational numbers, he elaborates on his definition by making the confusing statement: "This says that the sequence gets close to itself but not necessarily close to some fixed element in Q ." [Q is the set of rationals.]

Modern Principles of Mathematics is attractive in format, uses the most widely accepted notation, and contains much good material, including some interesting historical notes at the beginning of each chapter. It could conceivably be used as a textbook by an experienced instructor who is adept at organizing and supplementing topics in a course. However, the book promises much more than it can realistically deliver to the audience for whom it was supposedly written.

Violet Hachmeister Larney
State University of New York at Albany

An Intuitive Approach to Elementary Geometry, Beauregard Stubblefield. Brooks/Cole Publishing Company, Belmont, California, 1969, 254 pp., \$7.95.

It is encouraging indeed to find a book in geometry by a noted scholar written particularly for the student who plans to be an elementary teacher. We may continue to hope for better and better mathematics teaching on all levels so long as reputable writers are implementing the recommendations of the CUPM with textbooks such as this one.

The author carefully integrates the various branches of geometry in such a manner as to point up the unitary character of all mathematics and to emphasize its interrelatedness with other areas. This feature alone is a strong selling point for the book.

The reader moves easily into the comparative study of formal deductive geometry which characterized his high school course with a more informal or experimental geometry.

The axiomatic approach in studying parallelism and the construction of models enables the student to grasp easily that the addition of axioms prepares the way for a whole new type of geometry; the important role played by models in testing consistency and independence of axioms is well presented. Such basic concepts as congruence, measurement of segments and angles, constructions and the study of similarities are treated with clarity and backed up with numerous examples and exercises.

The principle of integration is manifest in the author's treatment of space figures where he moves quickly into fundamental topological considerations and Euler's formula. This same principle is operative in the establishment of geometric proofs for algebraic identities.

Cartesian analytic geometry is discussed simultaneously with the vector representation of space; conic sections are given brief mention.

The axiomatic note struck by the author in the early pages of the book is repeated in the two final chapters dealing with Non-Euclidean Geometry and the Postulational Method.

The problems are original and appropriate both as to content and timing i.e., they underscore with freshness a topic that is being presented at a given time.

The book is a must for all preparing to teach mathematics in the elementary school.

Sister Helen Sullivan
Mount St. Scholastica College

Manual for the Slide Rule (second edition), Irving Drooyan and William Wooton, Wadsworth Publishing Company, Inc., Belmont, California, 1969. Paperback (Spiral Bound), 134 pp.

This text could be classified as a handbook to serve the same purpose in the area of slide rule work as professional materials do for engineers. High school and technical college students, engineers, and technical personnel employed in industry should find this manual a very useful text.

The preface states that a student pursuing the study of this text should have a knowledge of simple algebra and numerical trigonometry. I would suggest that a knowledge of logarithms be included, particularly with regard to the final sections on logarithms.

The first eight lessons dealing with the fundamental operations

of multiplication, division, squares, square roots, cubes, and cube roots along with ratio, proportion, percentage and the basic work of logarithms to base ten can be used with most slide rules from the simple Mannheim type to the more advanced log log duplex slide rule. The material on trigonometric scales (sines, cosines, and tangents), the solution of right and oblique triangles, and the more advanced work on logarithms was written for the modern log log duplex deci-trig slide rule.

Each scale is introduced by describing the scale and explaining its purpose at the time it is to be used in performing an operation. Scales are shown in diagrams and examples that correspond to the diagrams are utilized to illustrate the concepts under discussion. Every student should profit from the large number of worked-out examples.

The approximation method is used to determine the decimal point in the various computations considered. This method is helpful both as a check for the student and also as an aid in basic understanding. Ample problems are given to be solved with the answers to odd-numbered exercises included at the end of the book. The book is short, appealing in format, and contains a wealth of stimulating and varied problems in each unit.

Ronald D. Dettmers
Wisconsin State University—Whitewater

An Introduction To Mathematical Logic by Gerson B. Robison, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1969, xiii + 212 pp., \$5.95.

This book is truly an introductory text designed for undergraduate students. It could be safely used in sophomore or junior level classes. It requires no unusual sophistication in mathematical language. Its length is excellent for a three-semester hour course.

The first four chapters are devoted to introduction and an analysis of statements. Standard elementary symbols are used throughout these chapters. Tautologies are shown by means of truth tables. Quantifiers and bound and free variables are explained as simply as possible. The student can easily understand this introductory material with a minimum of confusion. This part of the book introduces only twelve special symbols, a thankfully small number.

Chapters 5, 6, 7 and 8 introduce theorem and proof. The

author uses standard rule for proof and has several sample demonstrations in Chapter 5. The next chapters introduce a general deduction theorem, more standard inference rules which do not stress classical latin names, and some "universal theorems." Universal theorem is the author's term for any one of sixteen metatheorems of first order theory. (The author does not use the term "first order.") Chapter 8 also includes some standard group theorem proofs, carefully done with a clear explanation of reasons in the demonstration.

The last two chapters develop two different systems, carefully mentioning undefined symbols, rules for statements and axioms. The first system is basic set theory and most standard theorems are stated. Some theorems are proved. The second system develops Boolean algebra. A more relaxed view on proof is allowed at this point.

Complete answers to odd-numbered exercises are included. There is a list of symbols facing page one. The book contains a minimum of symbolism throughout; many of these are used only in the problem sections. The print is relatively large and the format of the book is excellent for reading.

In summary, the text is indeed an elementary introduction to mathematical logic, which is clear, full of explanation and yet is concise. It avoids a large amount of symbolism which frequently complicates "introductory" texts. However, it goes beyond the mathematics of "logical thinking" which is the content of many liberal arts oriented mathematics texts. The book is quite teachable and readable. I recommend it as a true undergraduate introduction to logic.

James K. Bidwell
Central Michigan University

Modern Trigonometry, Timothy D. Cavanagh, Wadsworth Publishing Company, Inc., Belmont, California, 1969.

The text is very "standard" in terms of the topics that are covered but "modern" in the manner that the material is covered. The author does a fine job of laying a good mathematical foundation in a separate chapter on sets, relations, and functions. These concepts, once introduced, are then used throughout the book. For example, the trigonometric functions are introduced in terms of "wrapping" the real number line around the unit circle.

There are plenty of exercises which would challenge all stu-

dents. One drawback is the size of type used for illustrations; it seems rather small in places.

The book would fit nicely into a high school college preparatory program in mathematics. It would also serve the needs of junior colleges and universities offering a separate course in trigonometry.

Ramon L. Avila
Ball State University

MINIREVIEWS

Analytic Geometry and the Calculus, Second Edition, A. W. Goodman, The Macmillan Company, New York, 1969, 819 pp., \$12.95.

This text covers the standard material of an introductory integrated course in analysis. It also contains chapters on differential equations and linear algebra. It is designed for a twelve semester hour sequence. The author claims to have gained clarity for the student by reducing the rigor of theorem statements and definitions.

Introduction to Mathematics, Second Edition, Bruce Meserve and Max Sobel, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1969, 420 pp., \$7.95.

This is a "liberal arts" text for the undergraduate. It covers a wide range of topics: numeration, systems geometry, equations and graphing, probability, logic. Excellent answer keys are included. It is designed to be intellectually stimulating without being restrictively rigorous and to therefore remove "your fears of mathematics."

Elementary Algebra and Intermediate Algebra, S. J. Bryant, L. Nower, D. Saltz; Glencoe Press, Beverly Hills, 1968, 304 pp. and 343 pp.

Both texts cover standard secondary school mathematics. Modern concepts and definitions are used. The second book includes fifty pages devoted to matrices, set theory, probability, and linear programming. Rational and transcendental functions and sequences are also treated extensively. Answers keys are provided.

Beginning Algebra, J. H. Mininck and R. C. Strauss, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1969, 378 pp., \$7.95.

This book is designed for college level elementary algebra students. General principles are drawn from examples. Color is utilized and answers are provided for half of the exercises. Set notions

are used with equalities and inequalities and graphing in one and two variables. An instructor's manual is available.

Elementary College Arithmetic, David Ledbetter, Goodyear Publishing Company, Inc., Pacific Palisades, California, 1969, 266 pp., \$7.95.

This text is designed to provide background for courses in algebra, business, computer programming and technology. Modern structure ideas are utilized, but number systems are not formally developed. The book includes a chapter on measurement and geometric formulas. An answer key is provided. The text appears to be genuinely practical for college students with very weak mathematical background. It is *not* a text for elementary teachers.



(Continued from p. 12)

in this paper suggested and verified this fact—however, the proof for general curves requires ideas and methods which are beyond the scope of this paper. This paper has attempted to discuss curves of constant width and Δ -curves in order to provide merely an introduction to Barbier's Theorem.

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Installation of Chapters

EDITED BY SISTER HELEN SULLIVAN

MASSACHUSETTS ALPHA CHAPTER

Assumption College, Worcester, Massachusetts

The installation of Assumption College of Massachusetts Alpha marks the first chapter of Kappa Mu Epsilon in Massachusetts.

The ceremony was conducted by Professor Emmet C. Stopher of State University College at Oswego, New York, on November 19, 1968. After brief introductory remarks concerning the goals and aims of the organization, Professor Stopher installed the new chapter and the following officers:

President	-	-	-	-	Roger Liesegang
Vice President	-	-	-	-	Paul Proulx
Secretary	-	-	-	-	David LaFratta
Corresponding Secretary	-	-	-	-	Richard Houde
Faculty Adviser	-	-	-	-	Sumner Cotzin

OHIO ZETA CHAPTER

Muskingum College, New Concord, Ohio

The installation of Ohio Zeta Chapter of Kappa Mu Epsilon was held on Saturday, May 17, 1969, at 3:45 p.m. in the lounge of Brown Chapel. Professor Harry Mathias from Bowling Green State University was the installing officer.

Members of the Muskingum College Mathematics Honorary who became charter members of the chapter were Ralph Beattie, Bruce Beidient, Dennis Berkey, Carol Butler, Richard Butler, Thomas Clifford, Linda Cooper, Meryl Hestwood, Professor Dorothy Knight, Dr. Coleman Knight, Cheryl Knox, James Kohl, Earl Linser, Linda Maximuke, Lynn Myers, Sue Roshong, Harold Rouster, Dr. Allen Strand, Ardith Strathern, Richard Thompson, Patricia Zettle, and David Zuro. The following officers were installed:

President	-	-	-	-	-	David Zuro
Vice President	-	-	-	-	-	Susan Roshong
Secretary-Treasurer	-	-	-	-	-	Patricia Zettle
Corresponding Secretary	-	-	-	-	-	Dr. James Smith.

Three members of the Mathematics Honorary who were already **KME** members were Miss Lottie Brown, Professor Stephen Kublank, and Dr. Smith.

After the installation, Miss Brown presented portions of the research that she had done on lattice theory, which she had carried out for her senior honors paper.

Following a banquet which was held in the Patton Hall Private Dining Room, Professor Mathias presented an interesting and informative account of the history and purposes of Kappa Mu Epsilon.



PENNSYLVANIA THETA CHAPTER

Susquehanna University, Selinsgrove, Pennsylvania

The Pennsylvania Theta Chapter of Kappa Mu Epsilon was installed on May 26, 1969. Professor William R. Smith, National Vice-President, presided over the installation ceremonies. Assisting him was Miss Arms, corresponding secretary of the **KME** chapter at Indiana University of Pennsylvania.

There were fourteen Susquehanna students and five faculty members installed at the ceremonies. The following are the charter members of Pennsylvania Theta Chapter: Timothy Byrnes Robert Clyde, William Cooke, Alan Cooper, Linda Garber, Peggy Harris, Jill Heffelfinger, Stephen Herrold, Frederick Mayer, Carol Riley, Elizabeth Sautter, Elinor Thompson, Elizabeth Varner, Dennis Zimmerman, Mr. James Handlan, Miss Carol Jensen, Mr. Barry Peiffer, Mr. John Reade, and Mrs. Margaret Rogers.

The following officers were installed:

President	-	-	-	-	Peggy Harris
Vice President	-	-	-	-	Elizabeth Varner
Recording Secretary	-	-	-	-	Elinor Thompson
Treasurer	-	-	-	-	Elizabeth Sautter
Corresponding Secretary	-	-	-	-	Miss Carol Jensen
Adviser	-	-	-	-	Mr. James Handlan

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the editor. Material will be used where possible and acknowledgment will be made in THE PENTAGON.

In the usual calculus course, much time is spent calculating derivatives given the equation of a function. If a function is given only graphically (such as the results from an experiment), a derivative can also be found by using a method of graphical differentiation.

Consider the function in figure 1. If we wish to find the value of the derivative at point P , we first draw, as accurately as possible, the tangent line to the graph at point P .

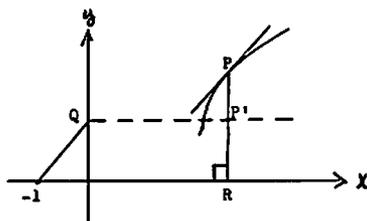


Figure 1.

Next draw a line parallel to this tangent line through the point $(-1, 0)$. Let Q be the point of intersection of this line and the y -axis. Through Q draw a line parallel to the x -axis which intersects the vertical line PR at a point P' . Then, RP' gives the value of the derivative for the function at point P . Can you prove this? By careful measurement of RP' , the value of the derivative is found.

To find the approximate value of the derivative for the points in the domain of a function, take the domain and divide it into subintervals. For a point in each subinterval, repeat this method, thus finding the derivative for each of these chosen points. By joining these points P' , an approximate graph of the derivative function will result.

For example, consider the function graphed in figure 2. This function S represents the observed distance traveled by a particle as a function of time.

The Pentagon

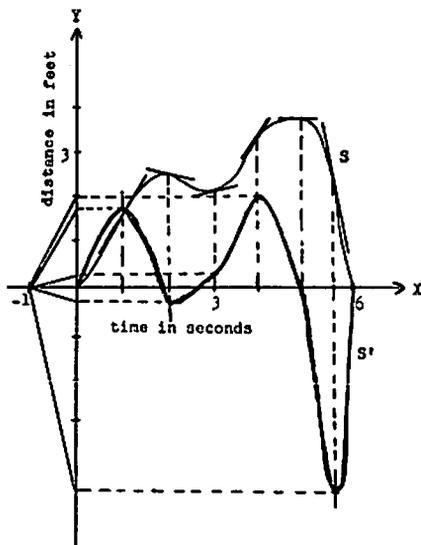


Figure 2.

Take the domain $[0; 6]$ and divide it into six equal subintervals. By choosing a point (one possible case is shown) in each subinterval and following the procedure described above, the velocity (derivative of S) will be the curve S' sketched. Hence, if one wanted the velocity when $T = 3$ seconds, it would be approximately $\frac{1}{4}$ ft/sec, at 2 second, approximately $-\frac{1}{4}$ ft/sec. Can you find the acceleration of this particle over this interval?

- Δ -

Many stories have been told concerning the apparent absent-mindedness of mathematicians. One excellent one appeared in *THE AMERICAN MATHEMATICAL MONTHLY*, Vol. 76, Number 7, in an article 'Some Mathematicians I Have Known', by George Polya. This widely told story concerns Norbert Wiener, but is hardly true.

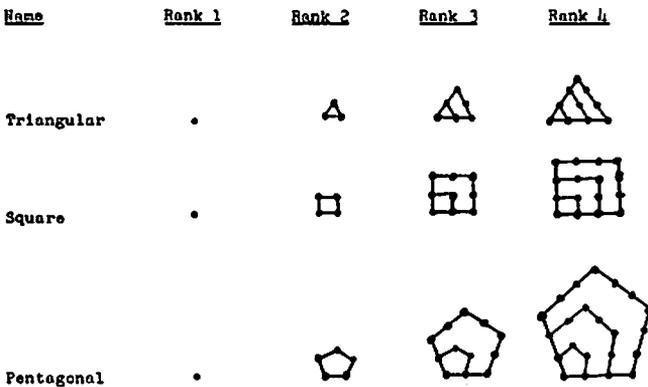
"It is about a student who had a great admiration for Wiener, but never had an opportunity to talk to him. The student walked into a post office one morning. There was Wiener, and in front of Wiener a sheet of paper on the desk at which he looked with tremendous concentration. Suddenly Wiener ran away from, and then back to, the paper, facing it again with tremendous concentration.

The student was deeply impressed by the prodigious mental effort mirrored in Wiener's face. He had just one doubt: Should he speak to Wiener or not? Then suddenly there was no doubt, because Wiener, running away from the paper, ran directly into the student who then had to say, 'Good morning, Professor Wiener.' Wiener stopped, stared, slapped his forehead and said: 'Wiener—that's the word.'



The early Greeks were interested in numbers associated with geometric figures. These numbers were called polygonal numbers and have geometric as well as algebraic foundations.

Consider the following regular polygons.



The points are equally spaced along the sides of each figure. Counting the points in each case, the following table of polygonal numbers results.

Name	Rank							
	1	2	3	4	5	6	7	8
Triangular	1	3	6	10	15	21	28	36
Square	1	4	9	16	25	36	49	64
Pentagonal	1	5	12	22	35	51	70	92
Hexagonal	1	6	15	28	45	66	91	120
Heptagonal	1	7	18	34	55	81	112	148

Examining the arithmetic progressions

$$1, 2, 3, 4, 5, 6, 7, 8, \dots, n;$$

$$1, 3, 5, 7, 9, 11, 13, 15, \dots, 2n - 1;$$

$$1, 4, 7, 10, 13, 16, 19, 21, \dots, 3n - 2;$$

$$1, 5, 9, 13, 17, 21, 25, 29, \dots, 4n - 3;$$

$$1, 6, 11, 16, 21, 26, 31, 36, \dots, 5n - 4;$$

one would note that the sum of the first k terms give the k th rank of the triangular, square, pentagonal, hexagonal, and heptagonal numbers. In general, the k th n -agonal number is equal to the sum of the first k terms of the arithmetic progression beginning with 1 and having a common difference between terms of $n - 2$. Can you develop a formula for the k th rank of a n -agonal?

— \triangle —

Editor's note: The following was submitted by Robert E. Koernke. I checked this formula with a computer and found it to be accurate.

For years mathematicians have searched for new formulas which yield prime numbers. A formula which yields sixty primes is $(16x^2 - 328x + 1722) \pm (4x - 41)$; where x is an integer, $1 \leq x \leq 30$.



(Continued from p. 29)

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Kappa Mu Epsilon News

EDITED BY EDDIE W. ROBINSON, *Historian*

The Seventeenth Biennial Convention of Kappa Mu Epsilon was held May 2-3, 1969, with Iowa Alpha at the University of Northern Iowa as host chapter.

THURSDAY, MAY 1, 1969

Following the registration, a mixer was held in University Hall. The National Council met in the Colombia Room of the Student Union.

FRIDAY, MAY 2, 1969

The meetings were held in University Hall with National President, Fred W. Lott of Iowa Alpha, presiding. Dean Clifford G. McCollum, College of Natural Sciences, University of Northern Iowa gave the address of welcome and National Vice-President, George R. Mach of California Gamma, responded for the society. The roll call of the chapters was made by Laura Z. Greene, National Secretary. The following chapters, approved for membership since the last national convention, were welcomed:

South Carolina Beta, South Carolina State College, Orangeburg,
Pennsylvania Eta, Grove City College, Grove City,
Texas Zeta, Tarleton State College, Stephenville,
Connecticut Alpha, Southern Connecticut State College,
New Haven,

New York Eta, Niagara University, Niagara,
Massachusetts Alpha, Assumption College, Worcester,
Missouri Eta, Northeast Missouri State College, Kirksville.

Petitions for new chapters at Muskingum College, New Concord, Ohio, Susquehanna University, Selinsgrove, Pennsylvania, St. Francis College, Brooklyn, New York, and Shippensburg State College, Shippensburg, Pennsylvania, were presented and approved.

Dr. George R. Mach presided during the presentation of the following papers:

1. *Rotation of Convex Curves in Regular Polygons*, Norma Henkenius, Kansas Gamma, Mount St. Scholastica College.
2. *A Discussion of a Theorem of Pfister*, Craig Bainbridge, Iowa Gamma, Morningside College.
3. *How Children Form Mathematical Concepts*, Ann Riess, New York Epsilon, Ladycliff College.

4. *A New Approach to Conic Sections*, Charlotte Cleis, California Gamma, California State Polytechnic College.

After lunch and the taking of the group picture, the faculty members and students met separately in two "Let's Exchange Ideas" discussion sections. Dr. R. G. Smith, Kansas Alpha, presided at the Faculty Section, and Jerry Jurschak, Iowa Alpha, was in charge of the Student Section.

The convention reconvened at 2:30 p.m., and after reports from the recorders of the two sections, Sister Malachy Kennedy, Kansas Gamma, and Katherine Peterson, Kansas Alpha, the following student papers were presented:

5. *JCAIP, UO!!!!.K*, Barbara Elder, Kansas Delta, Washburn University.
6. *An Introduction to Quasi-Trigonometries*, Elizabeth Struckhoff, Kansas Gamma, Mount St. Scholastica College.
7. *Finite Metric Space*, Charles Breindel, Pennsylvania Zeta, Indiana University of Pennsylvania.
8. *Coordinatization of the Euclidean Line*, Martha Thompson, Kansas Beta, Kansas State Teachers College.
9. *A Commentary on 'Spaceland: As Viewed Informally from the Fourth Dimension,'* Ron Oliver, Iowa Gamma, Morningside College.

A banquet was served in the Commons Dining Center. The guest speaker, Professor Robert V. Hogg, University of Iowa, spoke on "A Problem in Maximum Likelihood Estimation."

SATURDAY, MAY 3, 1969

The program began at 8:30 a.m. with the following student papers:

10. *Linear Diophantine Equations and Congruences*, Janet Love, Kansas Delta, Washburn University.
11. *An Introduction to Probability over Infinite Sample Spaces*, James Walsh, Kansas Beta, Kansas State Teachers College.
12. *Equilateral Triangles and the Parallelogram*, Susan O'Connor, Wisconsin Alpha, Mount Mary College.
13. *A Study of One-Operation Algebraic Structures*, Judy Graney, Kansas Gamma, Mount St. Scholastica College.

The following papers were listed by title:

1. *Influence of Religion on Hindu Mathematics*, Kathleen Cue, Iowa Alpha, University of Northern Iowa.

- 2. *A New Universe*, Nora Garton, Nebraska Beta, Kearney State College.
- 3. *The Theory of Positional Numeration*, John Flaig, California Delta, California State Polytechnic College.
- 4. *Computer-Assisted Instruction*, Lynann Brinkworth, New York Epsilon, Ladycliff College.

Professor Carl V. Fronabarger, Missouri Alpha, reported for the nominating committee. The following list of national officers was elected for 1969-1971:

- President ----- Dr. George R. Mach
California State Polytechnic College
- Vice-President ----- Professor William R. Smith
Indiana University of Pennsylvania
- Secretary ----- Professor Laura Z. Greene
Washburn University of Topeka
- Treasurer ----- Professor Walter C. Butler
Colorado State University
- Historian ----- Professor Eddie W. Robinson
Southwest Missouri State College.

The Awards Committee announced the following awards to the students listed below for papers presented during the convention:

- First Place ----- Norma Henkenius, Kansas Gamma
- Second Place ---- Elizabeth Struckhoff, Kansas Gamma
- Third Place ----- Janet Love, Kansas Delta

Professor Eugene Etter, Kansas Epsilon, reported for the Resolutions Committee. The following resolutions were adopted:

Whereas the Seventeenth Biennial Convention on this beautiful campus of the University of Northern Iowa has been a very enjoyable and profitable conference, be it resolved that we express our appreciation and gratitude to:

- 1. The host chapter, Iowa Alpha, its moderator and corresponding secretary, John S. Cross, and his colleague, Calvin Irons, and the University of Northern Iowa of Cedar Falls, Iowa, for their hospitality and efficient organization of all major and minor details that contributed so well to the success of the convention.
- 2. Each National officer: President Fred W. Lott, Vice-President George R. Mach, Secretary Laura Z. Greene, Treasurer Walter C. Butler, Historian Eddie W. Robinson for their gracious and efficient assistance.
- 3. Helen Kriegsman, the editor of THE PENTAGON, and to Wilbur J. Waggoner, the Business Manager of THE PENTAGON,

who have satisfactorily maintained the high quality of our magazine.

4. Professor Robert V. Hogg of the University of Iowa, who provided the challenging and entertaining program at the convention banquet.

5. The thirteen students who have prepared and presented the excellent papers which formed an integral part of the convention program; and also the four students who prepared alternate papers.

REPORT OF THE NATIONAL PRESIDENT

Kappa Mu Epsilon has continued its growth during the past biennium. In the two years since the last convention in 1967, seven new chapters have joined our ranks and an eighth, approved by the chapters last winter, will be installed next week. These chapters are located at: South Carolina State College; Grove City College in Pennsylvania; Tarleton College in Texas; Southern Connecticut State College; Niagara University in New York; Assumption College in Massachusetts; Northeast Missouri State College; and on May 9, 1969, a new chapter will be installed at Western Illinois University. We will thus have eighty-four active chapters and by the vote of this convention four more chapters will soon be added to make a total of eighty-eight.

There were two regional conventions during the biennium. Rosary College in Illinois was the host chapter for the North Central Regional Convention attended by members from eight chapters. The Middle West Regional Convention was hosted by Northeastern State College in Oklahoma with an attendance of 111 from fifteen chapters. We should continue to support these regional conventions in the even-numbered years and encourage other regions to plan such meetings. For this purpose the national organization provides up to \$100 to help defray the expenses of the host chapter of each such regional conference.

The membership of Kappa Mu Epsilon in the Association of College Honor Societies which began in February of 1968 was an important step forward and will have significant implications for the future development of Kappa Mu Epsilon.

The action of this convention in adjusting initiation fees and delegate travel allowances will enable the organization to proceed on a sound financial basis and make it possible to encourage increased participation by the individual chapters in national affairs.

As any observer of the present scene is well aware, it is evident that colleges and universities in this country are presently facing

troubled days. It is no exaggeration to say that the traumatic events in some institutions during the past year have many of the elements of a crisis and there is no question in my mind that these events will bring about substantial changes in higher education. It is not clear at this time what some of these changes will be but I am optimistic enough to believe that the good sense of students, faculty, and general public that will ultimately be developed will enable us to emerge strengthened by the ordeal. The honor society movement with its strong commitment to honest intellectual endeavor has a responsibility to help preserve this most basic element of education and I hope that the local chapter on each of your campuses as well as the national organization may serve in every way possible to sustain and re-emphasize our faith in the community-of-scholars concept.

This is my last report to you as National President. I would like to pay tribute to all those whose conscientious efforts make an organization like Kappa Mu Epsilon possible. The cooperation of national officers, THE PENTAGON officials, and members of chapters throughout the country have served to lighten my tasks and it has been an honor and privilege to serve as president of Kappa Mu Epsilon.

Fred W. Lott

REPORT OF THE NATIONAL VICE-PRESIDENT

I have enjoyed serving Kappa Mu Epsilon as its national Vice-President and as a member of the National Council during the past biennium. I have had opportunities to confer personally with the National President and the National Secretary and by mail with the other officers. Working with the National Council under President Lott's direction has been very gratifying.

One of my major efforts this past year has been the arranging and conducting of the student presentations for this convention. Announcements were placed in THE PENTAGON and mailed to all chapters. I appointed and acted as chairman of the Student Paper Selection Committee which included as members Sister Mary Philip, Illinois Zeta; Claris Marie Armstrong, Mississippi Beta; and William R. Smith, Pennsylvania Zeta. Seventeen papers were submitted from eleven chapters. Thirteen were selected for presentation and four as alternates. All correspondence with the students was done by me. I have enjoyed meeting these students and introducing them to you.

An extensive project which I initiated was the constitution and by-laws revision presented to this convention. With the cooperation

of the whole National Council, we hope to have found all of the errors, discrepancies, and internal inconsistencies in those documents. We feel that the name change is most appropriate.

In the past two years I have represented Kappa Mu Epsilon on several occasions. The first of these was on October 24, 1967, at the inauguration of Robert C. Kramer as president of California State Polytechnic College, Kellogg-Voorhis. The second was on April 3, 1968, at the inauguration of Robert E. Kennedy as president of my own college, California State Polytechnic College, San Luis Obispo. From February 27 to March 1, 1969, I represented Kappa Mu Epsilon at the 1969 meeting of the Council of the Association of College Honor Societies.

George R. Mach

REPORT OF THE NATIONAL SECRETARY

Kappa Mu Epsilon now initiates approximately three thousand members each biennium. The present membership is more than twenty-six thousand.

The secretary has the responsibility of maintaining the permanent records of the organization. Much of the information for these records is taken from the chapter reports. Prompt, accurate reports from the corresponding secretaries are most important. All orders for supplies and jewelry are sent to the secretary. The jewelry orders are sent to the L. G. Balfour Company for shipment to you. This process usually takes four to six weeks.

I appreciate the cooperation of the corresponding secretaries and the excellent support of the National Officers. It has been a privilege to work with Dr. Lott during the past several years. He has given generously of his time and energy both when he served as Editor of *THE PENTAGON* and more recently as Vice-President and President. His devotion and contribution to Kappa Mu Epsilon could not have been greater.

Laura Z. Greene

REPORT OF THE NATIONAL HISTORIAN

Two years ago I succeeded J. D. Haggard as national historian and the historical files of each chapter and *THE PENTAGON* files were transferred to my office. The files were in excellent condition, so I have had only to maintain the files of the chapters which were in existence at the time of the transfer and to create new folders for the chapters installed in the present biennium. Each folder for the new chapters contains the original petition and as much of the installation news as I could gather.

I have continued the practice of past historians of soliciting news items semi-annually from each of the chapters for the **KME** News section of THE PENTAGON, but I changed the format of the form letter which has been used in the past. Each semester, I have requested a list of officers and news items covering the previous semester. The response has been very gratifying, causing me to need to severely edit the material for the fall issue of THE PENTAGON. Since members like to have their activities recorded and to see their names in print, and, particularly, since the publication of an item is a method of keeping records, I have endeavored to print all the news submitted to me for the Spring issue.

Using the philosophy that history is past news, I have begun each news section report with anecdotes and happenings of past conventions and chapter activities which were recorded in the previous issues of THE PENTAGON. This was done so that long-time members could recall events of the past and that new members could develop appreciation of the heritage of the society. This past news is possible only because of the complete files of THE PENTAGON.

Numerous requests for information have come to my office regarding past conventions, past constitutions, and past national officers. These requests could not have been answered if it were not for the marvelous files maintained by past historians, and to them I give my thanks.

Eddie W. Robinson

REPORT OF THE EDITOR OF THE PENTAGON

Numerous changes have occurred in the personnel of the editorial staff of THE PENTAGON during the past biennium. At the last convention Eddie Robinson, Southwest Missouri State College, was elected National Historian and, therefore, became responsible for editing the Kappa Mu Epsilon News. H. Howard Frisinger, Colorado State University, served as Problem Corner Editor until the Fall 1968 issue when he was replaced by Robert L. Poe, Texas Technological College. During the entire period George R. Mach, California State Polytechnic College, has edited the Mathematical Scrapbook. He initiated the idea of different **KME** chapters being responsible for the contributions of each issue and the members of California Gamma, Iowa Gamma, and Nebraska Beta have assisted him in this project. James Bidwell, Central Michigan University, replaced John C. Biddle, who is on a two-year assignment in Saudi Arabia, as Book Shelf Editor. One familiar name continues to appear

in the list of associate editors, that of Sister Helen Sullivan, who is responsible for reporting the section on the Installation of Chapters. Each of these individuals has made his unique contribution to our publication, and I appreciate the support given.

Recognition is also due the business manager, Wilbur J. Waggoner, who has most effectively contributed to the publication of THE PENTAGON. The new manager of the University Press, Bruce Eldred, has continued the cooperative service of the former manager, Irwin Campbell. The assistance of these three members of the Central Michigan University staff has made a valuable contribution.

In addition to these individuals the editorial staff is well aware of the untiring efforts of the sponsors, corresponding secretaries, and other faculty members of the chapters who have supplied information and who have stimulated students to submit papers. As the official journal of Kappa Mu Epsilon THE PENTAGON attempts to reflect the objectives of the organization and to stress the role of the undergraduate student of mathematics. It is through the active support of the faculty that students are encouraged to prepare manuscripts and to propose problems or submit solutions to ones already posed and, thereby, derive the benefits of such an experience.

The major portion of each issue has been derived from the materials submitted by these interested students. Three issues of THE PENTAGON have been published since the last convention and the fourth issue is now in the process of being printed. In addition to the regularly published sections there have been twenty-one articles published, involving fourteen students and nine faculty members.

The staff invites each of you to offer suggestions and comments about THE PENTAGON and encourages you to contribute your articles, problems, solutions, and other appropriate items.

Helen Kriegsman

REPORT OF THE BUSINESS MANAGER OF THE PENTAGON

This is the sixth report of the activities and duties of the Business Manager of THE PENTAGON that I have presented to a biennial convention of Kappa Mu Epsilon. My primary responsibility as Business Manager of THE PENTAGON is to see that subscriptions are properly recorded and that each subscriber is mailed a copy of our journal as it is published. Many PENTAGONS, although mailed to the address that is filed, are not received by the

subscriber. The class of mail under which THE PENTAGON is mailed is not forwardable. The subscription cards of subscribers whose journals are returned because of incorrect addresses are pulled from the files and no more PENTAGONS are mailed to this subscriber unless a change of address is sent. I would ask each delegate and corresponding secretary to impress upon the members of your respective chapters who have subscriptions to THE PENTAGON the necessity of having on file with the Business Manager a correct address.

Since THE PENTAGON is published semi-annually in late May and late December, there are many subscription cards received in the periods between publication dates. So that new subscribers do not have to wait as long as six months for their first journal to arrive a stock of PENTAGONS large enough to fill new subscriptions for approximately the next three months after each publication date is kept. Some subscribers for example will receive a Fall PENTAGON in April, but no new subscriber waits a half year to get his first issue.

Let me briefly give you some statistics on the circulation of our journal over the last biennium. Just under thirteen thousand PENTAGONS have been printed during the biennium. All of these have been mailed to subscribers except for a limited number which have been kept to fill requests for back issues. In the mailing of the past four issues, our journal has gone to every state except Hawaii and Idaho as well as to several foreign countries. THE PENTAGON is found in the library of many high schools, colleges, and universities throughout the United States and Canada. More than one-half of all PENTAGONS have gone to nine states. These states in order of number of PENTAGONS mailed to addresses within the state are Illinois, Pennsylvania, New York, Kansas, Missouri, Texas, California, Tennessee, and Nebraska.

The library of each college or university which has an active chapter of Kappa Mu Epsilon receives a complimentary copy of THE PENTAGON. Authors of articles published in THE PENTAGON receive five copies of the issue in which their article is published. Each student presenting a paper to this convention will have his subscription extended for two years.

The work of my office is made easier by the cooperation that is given by chapter corresponding secretaries, by our National Secretary, Miss Greene, and by our editor, Dr. Kriegsman. My thanks to these individuals.

Wilbur Waggoner

KAPPA MU EPSILON

FINANCIAL REPORT OF THE NATIONAL TREASURER

For the period April 1, 1967 to April 22, 1969

1. CASH ON HAND April 1, 1967		\$ 8,391.42
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RECEIPTS

2. RECEIPTS FROM CHAPTERS		
Initiates (2,944 at \$5.00)	\$14,720.00	
Miscellaneous (Supplies, Jewelry, Installations, etc.)	<u>4,314.01</u>	
Total Receipts from Chapters		\$19,034.01
3. MISCELLANEOUS RECEIPTS		
Interest on Bonds and Savings	860.03	
Pentagon (Surplus)	175.12	
Treasury Bond Refund	56.00	
Over Payments	53.67	
Short Checks	216.95	
From Savings to Checking Account	<u>3,580.00</u>	
Total Miscellaneous Receipts		<u>4,941.77</u>
4. TOTAL RECEIPTS		<u>23,975.78</u>
5. TOTAL RECEIPTS PLUS CASH ON HAND		<u>\$32,367.20</u>

EXPENDITURES

6. NATIONAL CONVENTION, 1967		
Paid to Chapter Delegates	2,156.88	
Officers Expenses	606.79	
Miscellaneous (Speaker, Prizes)	87.00	
Host Chapter	<u>75.00</u>	
Total National Convention		2,925.67
7. BALFOUR COMPANY (Memberships, Certificates, Stationery, etc.)		7,205.78
8. PENTAGON (Printing, Mailing of 4 Issues)		6,702.11
9. INSTALLATION EXPENSE		282.78
10. NATIONAL OFFICE EXPENSE		349.70
Two Regional Conventions		<u>177.73</u>
11. MISCELLANEOUS EXPENDITURES		
Association of College Honor Societies	495.19	
Refunds	53.67	
Treasurer's Bond	126.00	
Short Checks	216.95	
Secretarial Expense and Postage	964.05	
Deposit to Savings	<u>7,500.00</u>	
12. TOTAL EXPENSE		26,999.63
13. CASH BALANCE ON HAND April 22, 1969		<u>5,367.57</u>
14. TOTAL EXPENDITURES PLUS CASH ON HAND		<u>\$32,367.20</u>
15. BONDS ON HAND April 22, 1969	\$ 1,000.00	
16. SAVINGS ACCOUNT	<u>9,532.60</u>	
		\$10,532.60
17. TOTAL ASSETS AS OF April 22, 1969		\$15,900.17
18. TOTAL ASSETS 1967		<u>14,904.27</u>
19. NET GAIN FOR PERIOD		\$ 995.90

CHAPTER NEWS

Editor's Note:

At the 1969 National Convention, an inquiry was made concerning a past connection between Pi Mu Epsilon and Kappa Mu Epsilon. The following is an excerpt from the report of National President and Founder, Kathryn Wyant, at the 1935 National Convention at Kansas State Teachers College of Pittsburg, Kansas:

"Some of you know that I was active in local state and national affairs of ΠΔΝ, ΠΜΕ, and ΜΑΑ when I taught at the University of Missouri. In 1930 I was asked by Northeastern Teachers College of Tahlequah, Oklahoma, to leave my Alma Mater and to teach with them. That fall I was initiated into their Mathematics Club and worked with the local group. At Thanksgiving time I attended a sectional meeting of the American Mathematical Society at Columbia, Missouri, and talked with people from various schools about a mathematics fraternity for the four year college. At Christmas time I went to Cleveland, Ohio to an A.S.A.S. meeting. At the business meeting of ΠΜΕ there, I asked what they would do to us if we petitioned for a chapter. Since I had made several talks at Pi Mu Epsilon conventions asking them to keep their University rank and to not let the small University nor the four year college enter, several members looked at me curiously. I was told by representatives of three chapters that they would "black ball" my group. Then one said, "Form a new fraternity." I told them that I would if they would back me up. They said they would and they have. In January, 1931, I began—with the help of the local mathematics group—to form a National Fraternity. Some of the reasons for the various symbols are stated in the Book of the Ritual. It, however, may not be amiss to restate them and to state others here. I wrote to Dr. Walter Miller, Professor of Greek at the University of Missouri. He sent several mottoes to us. Most of them were on the idea, "Mathematics disciplines the mind." These we did not like, so we sent the English of several of them to him and he translated them for us. From these translations the Greek for "Develop an appreciation for the beauty in Mathematics" was chosen for our motto and from the initial letters our name, Kappa Mu Epsilon, resulted. Our name is accidentally very similar to Pi Mu Epsilon, but the more I think about it the more pleased I am that this accident happened."

Alabama Beta, Florence State University, Florence

There were thirty-seven active members in 1968 and the chapter had the following officers:

President — Ronald J. Williams
 Vice-President — Sherry B. Timbulake
 Secretary-Treasurer — Frances Wallace
 Historian — Linda Schutz
 Corresponding Secretary — Dr. Elizabeth T. Wooldridge
 Faculty Sponsor — Mrs. Mary R. Hudson
 Reporter — Judy Elkins

Nineteen new members were initiated at a banquet in April, 1968. The speaker was Carl Prince, who was initiated into Alabama Beta in 1948. He is now Deputy Chief of the Computations Lab at NASA in Huntsville. Other programs included a student report by Janie Montgomery, a lecture by Professor Smith of the University of Mississippi, a film, "The Teaching Methods of R. L. Moore," and a show at the University Planetarium.

At the 1968 homecoming, a coffee hour was attended by fifty-seven alumni from twelve different years of initiations. Members of Alabama Beta provide a tutoring service for elementary and secondary students in the local area as well as for college students.

James Weatherly, 1962 Chapter President, received his Ph.D. in Mathematics from the University of Kentucky in January, 1969. Barbara Wright received a scholarship to the University of Alabama. Mary Virginia Darby, a 1968 graduate received a scholarship to the University of Georgia.

Arkansas Alpha, Arkansas State University, State College

Meetings at Arkansas Alpha were held on the second and fourth Wednesday of each month. The chapter had twenty-six active members. The following officers served last year:

President — Gene Boerner
 Vice-President — Suzanne Truxton
 Secretary — Mrs. Jim Albright
 Treasurer — Jim Albright
 Corresponding Secretary — Dr. John L. Kent
 Faculty Sponsor — Mr. John Bennett

Each program consisted of a business meeting, a 15-25 minute talk by a member or a guest speaker and a refreshment period. A Christmas Party was held in December at which members wrapped gifts in support of a local drive to help under-privileged families furnish gifts for the children.

Illinois Alpha, Illinois State University, Normal

The 1969-70 officers are:

President — Carmen Couceyro
Vice-President — Douglas Malinsky
Secretary — Marilyn McAtee
Treasurer — Steven Bell
Social Chairman — Susan McGovern
Historian — Mary May Tan
Newsletter Editor — Cheryl Leibel

Clifford Newton Mills, Professor Emeritus of Mathematics at Illinois State University is now living at 1611 South Summit Avenue, Sioux Falls, South Dakota 57105. Professor Mills was National Treasurer of Kappa Mu Epsilon in 1934 and 1935.

Iowa Beta, Drake University, Des Moines

Each of the five pledges was required to present a paper to the chapter. A picnic was held in October with prospective pledges. The chapter officers were:

President — Gary Bagly
Vice-President — Dennis Hult
Secretary — Hugh Meried
Treasurer — Randolph Bulger
Corresponding Secretary — Joseph Hoffert
Faculty Sponsor — Basil Gillan

Massachusetts Alpha, Assumption College, Worcester

The first set of officers for this newly installed chapter are:

President — Roger Liesegang
Vice-President — Paul Praulx
Secretary — David La Fratta
Corresponding Secretary — Richard Haude
Faculty Adviser — Sumner Catzin

Michigan Beta, Central Michigan University, Mount Pleasant

At the spring 1969 initiation, Michigan Beta honored Dr. Cleon Richtmeyer. Dr. Richtmeyer retired on December 1, 1968, after forty-four years of service to Central Michigan University. He was initiated into Michigan Alpha, Albion College and then was a charter member of Michigan Beta when it was installed in April, 1942. He and Dr. Judson Foust were sponsors of Michigan Beta for many years. In 1947 Dr. Richtmeyer was elected National Historian and served four years. In 1951, he was elected Vice-President and served for four years and he was elected National President in 1955, again serving four years. Since 1924, Dr. Richtmeyer has been an instructor of mathematics, the chairman of the

Mathematics Department, the Director of Instruction and the Dean of the School of Arts and Sciences.

The following is an excerpt from a letter from National President, Fred W. Lott, to Michigan Beta Chapter.

"It is a privilege for Kappa Mu Epsilon to honor Dr. Cleon Richtmeyer on the occasion of his retirement at Central Michigan University.

"Dr. Richtmeyer's distinguished service to Kappa Mu Epsilon has extended over many years—first as the Michigan Beta chapter sponsor, then sixteen years as a national officer; Historian 1947-51, Vice-President 1951-55, President 1955-1959, followed by four years on the National Council as Past President 1959-1963.

"It is impossible to measure the full significance and nationwide impact on the cause of mathematics education made by his capable leadership and devoted efforts to Kappa Mu Epsilon over the past quarter of a century, but we can be sure that it has been substantial. An organization like Kappa Mu Epsilon can exist only because some people are willing to devote their services, and the mathematics community is richer because of Dr. Richtmeyer's unstinting contributions over a period of many years.

"Dr. Richtmeyer, our best wishes for many pleasant years of retirement and we hope you will be able to maintain your ties with Kappa Mu Epsilon in the years to come."

Missouri Zeta, University of Missouri at Rolla, Rolla

Joseph Meuser, chapter historian from Spring 1968 to Spring 1969, is a graduate assistant in physics at Kansas State College of Pittsburg, Kansas.

New York Beta, S.U.N.Y. at Albany, Albany

The chapter has sixty active members and ten pledges. The following officers served last year:

President — Sally Malik

Vice-President — Cheryl Schaibe

Secretary — Emmy Chemnitz

Treasurer — James Nickerson

Corresponding Secretary — John Therriou

Business meetings were held in October, November, and December with induction of new members in November and a Christmas Party in December.

Nebraska Alpha, Wayne State College, Wayne

Maurice D. Anderson will serve as both Faculty Sponsor and

Corresponding Secretary during the year 1969-1970. Fred Webber, who has served as corresponding secretary for the last couple of years, is on leave.

Nebraska Gamma, Chadron State College, Chadron

The chapter has twenty-three active members and eight pledges. Chapter meetings were held on the first and third Thursdays of each month. Included in the programs were films pertaining to mathematics. A picnic was held in the fall with the biology and physical science fraternities.

New Mexico Alpha, University of New Mexico, Albuquerque

The chapter has fifty active members with Tim Burns serving as President and Merle Mitchell serving as Faculty Sponsor.

New York Eta, Niagara University, Niagara

The chapter has twenty-one active members and fourteen pledges. The chapter has joint meetings with the mathematics club on campus. These meetings feature a lecture by a student or a faculty member. Fourteen new members were inducted in March.

This fall, the chapter was host for a joint meeting of the mathematics clubs of Western New York colleges and universities. It was attended by eighty-five people from five different schools. Professor H. S. M. Coxeter from Toronto lectured.



Someone asked Euclid "But what shall I get by learning these things?" Euclid called his slave and said "Give him three coins, for he must make gain out of what he learns."

—STOBAEUS



Kappa Mu Epsilon Convention, May 2-3, 1969.
University of Northern Iowa, Cedar Falls, Iowa.