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National Officers

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Kappa Mu Epsilon, national honorary mathematics society, was founded in 1931. The object of the fraternity is fourfold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievements in the study of mathematics at the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Progress in Mathematics and Its Implications for the Schools*

G. BALEY PRICE

*Executive Secretary, Conference Board of the Mathematical Sciences,
Washington, D. C.*

The changes in mathematics in progress at the present time are so extensive, so far reaching in their implications, and so profound that they can be described only as a revolution.

Let us examine the causes of this revolution. It has been caused in the first place by the tremendous advances made by mathematical research. Many members of the general public are surprised to learn that mathematics is a live, active, and growing subject. They seem to feel that mathematics was completed by Newton, and that undergraduate, and even graduate, courses in the subject never change—indeed, that there is no opportunity, need, nor occasion for them to change. It is true that if a theorem is once true, it is always true. But theorems, like airplanes, become obsolete because new and better theorems are discovered.

The twentieth century has been the golden age of mathematics, since more mathematics, and more profound mathematics, has been created in this period than during all the rest of history. *Mathematical Reviews* is an international abstracting journal that publishes brief reviews of research papers and books; the typical review is about two or three inches long in one column. In spite of the brevity of the reviews, the volume of *Mathematical Reviews* for 1960 will contain 1652 large, double-column pages; and it is estimated that the volume for 1961 will contain 2400 such pages. The present century has seen the introduction and extensive development of subjects in pure mathematics such as abstract algebra, topology, measure theory, general theories of integration, and functional analysis, including the theory of Hilbert space. These subjects were not extensively taught in even the best graduate departments of mathematics until after 1930; as a result, many members of the older generation of mathematicians in the United States did not have courses in these new subjects when

*An address delivered at the Regional Orientation Conferences in Mathematics during the Fall of 1960. The Conferences were sponsored by The National Council of Teachers of Mathematics with the financial support of the National Science Foundation.

they were in graduate school. Since it is impossible to be a mathematician today without a knowledge of these new subjects and their continuing developments, the university mathematician has been forced to continue his "in-service training" throughout his entire career. It would be out of place here to enter into a discussion of the details of the new subjects I have named; it is sufficient to say that the changes made in mathematics by modern research are equally as profound as those in chemistry, physics, and biology.

There has been rapid development in certain other fields of mathematics which are more closely related to important applications than those already named. Probability and statistics are studied not only for their own sake, but also because of their extensive and important applications in the physical and engineering sciences, in the biological sciences, and the social sciences. The recent development of this field is indicated by the fact that the Institute of Mathematical Statistics was not organized until 1935. The theory of games is a mathematical theory of games of strategy; the history of the subject dates essentially from 1944, when John Von Neumann and Oscar Morgenstern published their *Theory of Games and Economic Behavior*. As the title of the book indicates, the theory of games was developed not only for its mathematical interest, but also as a mathematical model in terms of which economic forces and behavior could be explained and understood. Linear programming is usually dated from 1948; it has provided an important tool for the more efficient management of large-scale industrial and governmental operations. Operations research was introduced by England and by the United States to support their war efforts during World War II; after the war, many industrial firms employed operations research methods in an effort to make their operations more efficient and more productive. Operations research employs many mathematical and statistical techniques. The Operations Research Society of America was organized after World War II, and it holds several large national meetings each year. Quality control is concerned with techniques for the efficient control of quality in large-scale manufacturing processes. For example, millions of light bulbs are made by automatic machines; what steps can the manufacturer take to insure that the quality remains at specified levels? There are so many lamp bulbs (and similarly for many other items) that it is not economically feasible to test all of them. Furthermore, a complete test of a lamp bulb (and of many other items also) is a destructive test, and only those bulbs that are tested can be sold. Quality control employs a variety of sta-

tistical techniques. The history of the subject dates from 1929 when Shewhart, of the Bell Telephone Laboratories, published a book entitled *Economic Control of Quality of Manufactured Product*, but quality control methods and techniques were not widely employed until they were demanded by the necessities of World War II. The field now has its own professional organization, the American Society for Quality Control, which was organized soon after World War II, and now has more than 12,000 members.

Let us return to our original question, namely, what caused the revolution in mathematics? I have said that this revolution was caused first of all by the advances resulting from mathematical research. In the second place, the revolution in mathematics was caused by the automation revolution.

The automation revolution consists of the introduction of machines that control machines, and of the consequences of the use of such machines. Examples of automation abound everywhere. Long distance telephone dialing is a simple but impressive example of automation that is so commonplace that it often passes unappreciated. The automatic pilot that flies our jet airplane is another example of automation. Guided missiles provide still another example. Complicated computers and control mechanisms are required to lift a missile from its launching pad and to place it in orbit. A final example is provided by the computing machines that are programmed to control milling machines for cutting complicated three-dimensional shapes from wood or metal.

The automation revolution has influenced the revolution in mathematics in two ways. First, it has made possible the construction and operation of machines of enormous size, complexity, and cost; and it has thereby created the necessity for the design and development of such machines. Until fairly recent years, most of the design and development problems could be solved by simple experimental procedures. I once heard the late Charles F. Kettering, Director of Research for the General Motors Corporation, explain how to design a better piston ring. He prescribed a simple experimental procedure as follows: "Make several hundred piston rings," he said, "using different combinations of design, metal, finish, and heat treatment. Then put them all in engines and try them out. The piston ring that gives the best performance has the design you want."

I am sure that Mr. Kettering was fully aware of the importance of mathematical and other analytical procedures, but the simple experimental procedures he suggested would undoubtedly be quite

successful in the design and development of an item as small and simple as a piston ring. But the typical problem of today does not concern the piston ring, but rather something of the size, complexity, and cost of the B-70 airplane. This plane will be made of stainless steel, and it will be somewhat less than 200 feet long. Its range will be 7,000 miles, and it will fly at 2,000 miles per hour at an altitude of 70,000 feet. It will be capable of carrying in its thirty-foot bays enough nuclear bombs to blow a small nation off the map. Originally, the B-70 program called for 162 planes at a cost of \$10,000,000,000, a figure the Senate Preparedness-sub-committee said was unrealistically low. (Added later: An article entitled "Program for B-70 at Mock-Up Stage" in the *New York Times* for February 12, 1961, contains the following statement: "Wonder or blunder, more than \$797,300,000 has been spent on it so far and the only tangible product is a wood-metal-plastic contrivance that looks like a cross between a plane and a spaceship.") The experimental approach outlined by Mr. Kettering would suggest that we build a hundred of the proposed planes, using different combinations of wing types, fuselage designs, engine types and mounts, and control systems. This approach to the problem would be absurd. No test pilot would fly a plane that had been built in this fashion. Everyone knows the design and development of a plane of this type requires that the analysis—much of it mathematical—be carried to such a point that the first one built flies and performs essentially according to specification.

But the automation revolution has influenced the revolution in mathematics in another way. Not only has it created the necessity for solving complicated design and development problems, but it has contributed an important tool for their solution. This tool is so important that I would list it as the third cause of the revolution in mathematics.

The introduction of the large-scale, high-speed, automatic digital computing machine is the third cause of the revolution in mathematics. This computer has made it possible for mathematical theory to be teamed with the computing machine to produce answers that are required by physicists, engineers, and others.

One example will illustrate the change in our ability to compute. About one hundred years ago an Englishman named Shanks computed π to 707 decimal places. Working with pencil and paper, he devoted 20 years to this undertaking. In 1949, however, the computing machine known as the ENIAC computed π to more than

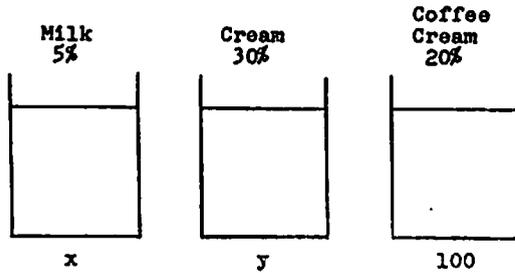
2,000 decimal places in 70 hours. Furthermore, the modern calculations of π have shown that Shanks made a mistake in the 528th decimal place. Some time after 1949, another machine computed π to more than 3,000 decimal places in 13 minutes. Still later, a smaller machine computed π to 10,000 decimal places; after the result was published in 1957, it was discovered that the machine had made a mistake in the 7,480th decimal place. By 1960, π has been computed correctly to 10,000 decimal places so many times that history does not record all of them.

The importance of the electronic digital computing machine arises not from the fact that certain calculations can be carried out more quickly than heretofore, but rather from the fact that computations which were formerly completely impossible can now be made quickly and efficiently. Consider again the launching of a guided missile. The computing machine remains on the ground, but radar supplies information to it about the flight of the missile. The computing machine makes the necessary calculations and, through a radar connection, sets the controls in the missile. The flight of the missile can be influenced only during the period the engine is in operation, a period which is usually not more than two or three minutes. No group of human computers could possibly receive the data, make the necessary calculations, and transmit the results back to the missile in so few seconds. The electronic digital computer handles the problem with ease.

Several examples will illustrate the changes in the nation's need for mathematics and in the nature of the mathematics courses taught in our schools. In 1850 almost no one was engaged in research. The members of the general public needed to know how to keep simple accounts and how to solve simple problems in measurement. Bookkeeping requires a knowledge of the four operations of arithmetic: addition, subtraction, multiplication, and division. The problems of measurement encountered in 1850 included the determination of the number of acres of land in a field, of the number of cords in a stack of wood, and of the number of bushels of grain in a bin. The public school courses in arithmetic included a treatment of all of these topics.

By the early years of the twentieth century, new needs for mathematics had arisen. I shall now describe one which resulted from two advances in dairy technology. The first advance was the invention and widespread introduction of the cream separator, a machine which separates milk into cream and skim milk; and the second was

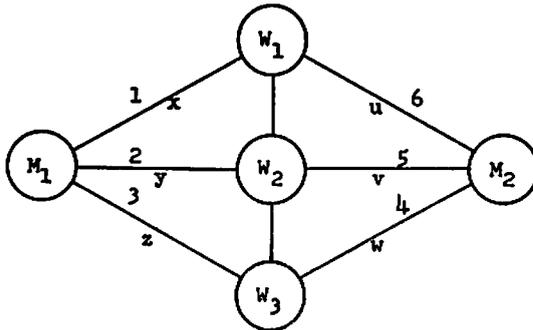
the development of a simple test for determining the percentage of butterfat in a sample of milk or cream. Given the cream separator and the butterfat test, a common problem for the dairyman is illustrated by the following: how many pounds of milk, testing 5 percent butterfat, and how many pounds of cream, testing 30 percent butterfat, must be mixed to give 100 pounds of coffee cream, which tests 20



percent butterfat. Let x and y denote respectively the number of pounds of milk and cream required. Then

$$\begin{aligned}x + y &= 100, \\ .05x + .30y &= 20.\end{aligned}$$

The solution of the problem has led to the solution of two linear equations in two unknowns. For many years high school algebra has included the treatment of the solution of systems of this type.



Next, consider a simple problem in linear programming. A certain manufacturer has warehouses W_1 , W_2 , and W_3 which contain

100, 200, and 100 tons, respectively, of his product. The manufacturer receives an order for 125 tons of his product from market M_1 and an order for 225 tons from market M_2 . The freight rates from the warehouses W_1 , W_2 , and W_3 to market M_1 are respectively 1, 2, and 3 dollars per ton; and the freight rates from W_1 , W_2 , and W_3 to M_2 are respectively 6, 5, and 4 dollars per ton. How many tons should the manufacturer ship from each warehouse to each market to fill the two orders?

Let x , y , and z denote the number of tons to be shipped from W_1 , W_2 , and W_3 respectively to M_1 ; and let u , v , w denote the number of tons to be shipped from W_1 , W_2 , and W_3 respectively to M_2 . Then from the statement of the problem we obtain the following equations and inequalities:

$$\begin{aligned}x + y + z &= 125, \\u + v + w &= 225, \\x + u &\leq 100, \\y + v &\leq 200, \\z + w &\leq 100.\end{aligned}$$

Finally, if C denotes the total freight charges for making the shipments, then

$$C = x + 2y + 3z + 6u + 5v + 4w.$$

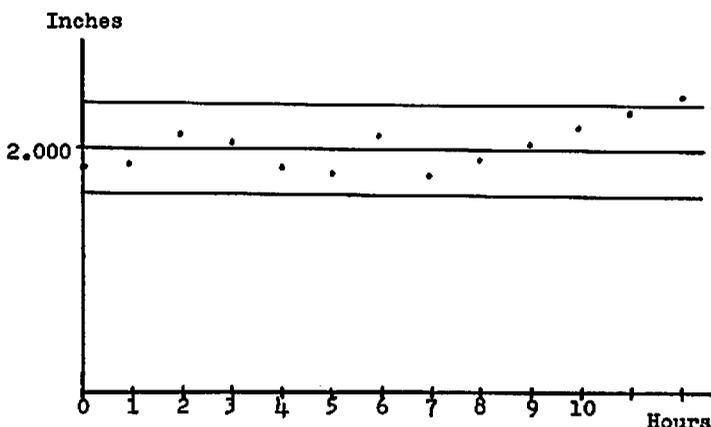
The solution of the problem is obtained by finding the values of x , y , z , u , v , w which satisfy the five equations and inequalities, and which give C its minimum value.

Problems of this type are of great practical importance to business, industry, and government. Many examples arise in the oil industry. A given oil company will usually have several sources of crude oil, several refineries, many storage facilities, and widely scattered markets. The problems encountered involve many unknowns, and methods must be devised for solving them on larger computing machines.

This problem in linear programming involves considerations which have not been taught in our high school mathematics courses heretofore. These courses have treated linear equations but not linear inequalities. A study of inequalities of all kinds is one of the new topics included in the new mathematics programs for high schools.

Consider the following problem. A certain manufacturer receives an order for 100,000 rods of a certain kind, each of which is

to have a diameter of two inches. The buyer knows, however, that it is not economically feasible to produce rods whose diameters are exactly 2.000 inches; accordingly, his order states that rods whose diameters lie between 1.995 inches and 2.005 inches are acceptable. The manufacturer finds that although he cannot manufacture rods whose diameters are exactly 2.000 inches on an automatic lathe, he can successfully make rods whose diameters lie between 1.995 and 2.005 inches on this lathe. When the cutting tool is dull or when the lathe is out of adjustment, however, the lathe produces rods whose diameters fall outside the specified tolerances. The manufacturer finds that he must institute a quality control procedure to assist



him. In a typical procedure a random sample of five rods will be drawn each hour. The diameters of the five rods will be measured and their average will be computed and plotted on a quality control chart. If the average falls within certain limits that have been established for the lathe, the manufacturing process is continued; if it falls outside these limits, the lathe is stopped and put back in proper working order.

The operation of the quality control procedure described is extremely simple, but the mathematics involved in establishing the limits and justifying the procedure includes very deep results in the theory of probability. In the past, probability and statistical inference have not been included in our high school mathematics courses. The Commission on Mathematics, however, considered the subject so important that it wrote a textbook on probability and statistical in-

ference for a course in the second semester of the twelfth grade. Furthermore, the second semester of Contemporary Mathematics, the mathematics course on Continental Classroom, will be devoted to probability and statistical inference, partly because of the importance of this subject for high school teachers, but even more because of its importance for many members of the general public.

Another example will illustrate further the importance of probability and statistics in the everyday affairs of the nation. As mentioned earlier, accounting in the past employed the four fundamental operations of arithmetic; there are strong indications, however, that accounting in the future will involve important applications of probability and statistics.

Consider the telephone companies. A long distance telephone call from New York to San Francisco will use the lines of several different companies, and each of them must receive its share of the revenue. There are undoubtedly millions of such calls each month. The determination of the exact amount of revenue due each telephone company requires only the four operations of arithmetic, but the amount of work involved is enormous. The telephone companies are now investigating the possibility of employing sampling theory in the solution of this problem; the Ohio Public Utilities Commission heard testimony in September 1960 regarding sampling to split revenues between Ohio Bell and the General Telephone Company, which operates in many areas of Ohio. Clearly the total amount of work involved in the accounting will be greatly reduced if the total revenue is divided in the same ratio as that in a small sample of the calls. Important mathematical and legal questions are involved, however. How large should the sample be to insure that the total revenue is divided fairly within certain limits? Will the stockholders accept dividends based on revenues divided by sampling theory methods; will the Bureau of Internal Revenue accept taxes based on income obtained by such methods?

It is freely predicted that sampling theory methods, based on probability and statistical inference, will be widely introduced into accounting procedures in the near future. These developments emphasize once more the importance of probability and statistics for the general public, and the importance of introducing a course on these subjects into the high school curriculum.

Thus far I have emphasized the importance of new developments in mathematics. It would be a mistake to believe, however, that the only important parts of mathematics are those which have

been discovered and developed recently. Many old subjects are still highly important and we must continue to teach them. Frequently, however, the emphasis must be placed on a different aspect of the subject, and an effort must be made to teach the subject so that the student gains a deeper understanding of it. The teaching of trigonometry and logarithms provides two examples. Trigonometry became a part of the college curriculum in mathematics about 300 years ago when the American colonies were located on the Atlantic seaboard. In the large majority of cases a college graduate became a sea captain, a surveyor, or a minister. A sea captain needed trigonometry for navigation; a surveyor needed it to lay out the farms and cities of the new continent; and the minister needed trigonometry for astronomy and the calculation of the date of Easter. Trigonometry was the all-important applied mathematics of this early period, and the solution of triangles was its important aspect.

Today, the important part of trigonometry is the study of the properties of the trigonometric functions rather than the solution of triangles. Radio beams and radar aids have made navigation easy; the new country has been staked out, and only a few, even among the engineers, study surveying; and our observatories now compute the date of Easter. The trigonometric functions, however, have many important applications, for example, in electrical engineering; and trigonometry is still an important subject in applied mathematics if the emphasis is placed on analytic trigonometry rather than on the solution of triangles.

Logarithms were introduced about 300 years ago, and they have been widely taught as an important tool for calculation. But logarithms are no longer important for calculations; small calculations are performed on desk calculators, and large calculations are performed on electronic digital computers. Shall we stop teaching logarithms? Not at all, but the emphasis should be shifted from logarithms as a tool for calculation to a study of the properties of the logarithm function.

Consider a final example. The study of the flow of heat and the distribution of temperatures in a solid body is a problem of great importance at the present time; it was first studied extensively by the French mathematician Fourier early in the nineteenth century. His discoveries had few practical applications at the time, but they have many applications of the highest importance today. Many problems related to the flow of heat occur in the design of every steam power plant, of every air conditioning system, and of every nuclear power

plant. The study of the flow of heat, begun by Fourier, and of the mathematical problems which have arisen from this original problem, have had a profound influence on the development of modern mathematics. Some of the changes that are being made in high school mathematics are designed to provide a better foundation for the study of some of the old problems in mathematics and their modern developments. The study of the flow of heat is an advanced problem which cannot be studied in high school. Nevertheless, it is important to develop the points of view and to lay the foundation that will permit the student to understand the old problems and the new methods which have been developed to solve these old problems.

As a result of the revolution in mathematics, there is an unprecedented demand for mathematicians and mathematics teachers; it is impossible to foresee a time when there will be an adequate supply. This demand for mathematicians is part of a larger demand for highly trained personnel in all fields. This demand represents a long-term development in our civilization—a civilization which is increasingly dependent on scientific and technological advances. This long-term increase in the demand for highly trained personnel was obscured first by the depression of the 1930's and second by the dislocations caused by World War II. The realization of the true situation burst upon the nation with startling suddenness in the 1950's, long after efforts should have been initiated to deal with it.

The Rockefeller Report on Education, entitled *The Pursuit of Excellence*, contains an account of the automation revolution, the accompanying long-term increase in the demand for highly educated personnel, and the crisis that confronts the nation. *The Pursuit of Excellence* contains the following table (see page 7), which shows

OCCUPATIONAL DISTRIBUTION OF LABOR FORCE

(Selected skills and occupations as percent of labor force)

	<u>1910</u>	<u>1957</u>
Professional and technical workers	4.4%	9.9%
Proprietors, managers and officials, excluding farm	6.5	10.3
Clerical workers	10.2	14.1
Skilled workers and foremen	11.7	13.3
Total selected skills and occupations	<u>32.8%</u>	<u>47.6%</u>

that the percentage of the labor force in selected skills and occupations increased from 32.8% in 1910 to 47.6% in 1957.

The Rockefeller Report on Education stresses the crisis in science and mathematics education in the following paragraphs (see pages 27-28):

“Though we cannot discuss in detail each of the fields of study, it is worthwhile to say a few words about education in science and mathematics. The public reactions to this subject have been so intense and so diverse that it has not been easy for the informed citizen to appraise the issues. The simplest way to avoid confusion is to keep a few basic ideas firmly in mind.

“First, the crisis in our science education is not an invention of the newspapers, or scientists, or the Pentagon. It is a real crisis.

“Second, the USSR is not the ‘cause’ of the crisis. The cause of the crisis is our breath-taking movement into a new technological era. The USSR has served as a rude stimulus to awaken us to that reality.

“The heart of the matter is that we are moving with headlong speed into a new phase of man’s long struggle to control his environment, a phase beside which the industrial revolution may appear a modest alteration of human affairs”

How well is our educational system meeting the demands placed upon us The Rockefeller Report on Education answers as follows (see page 22):

“The fateful question is not whether we have done well, or whether we are doing better than we have done in the past, but whether we are meeting the demands and unparalleled opportunities of the times. And the answer is that we are *not*.”

The implications of this crisis for our schools are clear. We must put forth whatever effort may be required to insure that the education provided by our schools—and, in particular, the mathematics education provided by our schools—is adequate for the needs of our times. I shall now indicate some of the components of the mathematics education that is adequate for our times.

The first component consists of mathematics courses with the

proper mathematical content. Many of the topics in such courses are old mathematics, but they are presented in such a way that the student gains greatly increased understanding and insight into the subject. Algebra, an old subject, is one of the central topics in the new courses. Algebra has usually been presented as a collection of rules, which if followed, produce the answer; proofs were reserved entirely for geometry. Algebra in the new courses will now be taught so that its structure—its deductive character—is apparent. Many of the topics in courses with the proper mathematical content concern subjects which are entirely new in the high school curriculum. For example, a chapter on vectors is now available in a mathematics course for the eleventh grade. Vectors form a proper subject for study not only because they form an interesting new mathematical structure, but also because they have important applications in physics and engineering. Another new topic in the high school curriculum is probability and statistics. I have already mentioned the textbook for a semester course on this subject in the twelfth grade. The course has already been taught in a number of high schools and it has been an immediate success everywhere. The engineers, among others, are demanding that their students know more and more about probability, statistics, and their applications. The theory of matrices is a final example of a new mathematical topic in the high school curriculum. Matrices are relatively new in mathematics, being only about a century old; they provide an example of an important new type of algebraic structure, and their study yields a tool of great significance and power in many fields in which mathematics is applied.

The second component in mathematics education adequate for our times consists of well-qualified teachers. A teacher must know a great deal of mathematics in order to be a satisfactory teacher of school mathematics. Superintendents and principals should now realize that the day has passed when any teacher who happens to have an otherwise free period can be assigned to teach mathematics. Many of our high school students must now reach a level of attainment expected of college sophomores only 15 or 20 years ago.

The well-qualified teacher must know mathematics, and in addition he must teach the subject with interest and enthusiasm. Your first reaction may be that in speaking of the importance of the school mathematics teacher, I am considering the wrong group. You may agree that mathematics is important, but you may feel that the elementary school teacher, the junior high school teacher, and the senior high school teacher present only the elementary parts of the

subject. But I must insist that their role is an important one, for they provide the foundation for the later study of more advanced mathematics. The high school mathematics teacher must present the elements of algebra, geometry, and trigonometry, but in addition he must preserve and strengthen the student's native interest in and enthusiasm for mathematics. He must make the subject interesting and appealing, so that his pupils will continue to study it with enthusiasm. A teacher who fears and dislikes mathematics will not teach very much mathematics to his students, but he will readily convey his fear and dislike of the subject to many of them. Such a teacher will often build up in his students a permanent fear and dislike of mathematics, and they will abandon the study of the subject at the first opportunity. One of the best ways to attract students to the study of mathematics is to know and like mathematics, and then to teach good, significant courses.

The third component in mathematics education adequate for our times consists of counselors who will make certain that those students who have mathematical interests and abilities take at least four years of good mathematics in high school, and that all of those who have the ability to do college work take at least three years of high school mathematics. A student who does not take at least three years of high school mathematics is so handicapped that many fields of study are permanently closed to him when he reaches college. The best courses and the finest teachers are to no avail if students do not take the courses. For this reason, there must be counselors to help students plan their high school programs.

Several comments are now in order. First, a small high school cannot provide the mathematics courses and the teachers I have described above as necessary; Conant has suggested that a high school with a graduating class of 100 is the minimum size. Students in a smaller school almost certainly are denied proper mathematics courses. The nation cannot waste its limited supply of good mathematics teachers by placing them in schools where they teach their specialty to less than full capacity. The nation cannot afford the waste of talent that results from sending gifted students (they occur also in small schools!) to schools with poor mathematics programs and poor teachers.

A second comment is that many high school mathematics teachers already teaching must undertake retraining immediately. Many excellent opportunities exist. The mathematics course on Continental Classroom provides an opportunity for in-service training

which is available to most of the teachers in the United States. In addition, there are in-service training programs organized by the high school itself or by a nearby college or university; and finally there are summer and academic year institutes.

A third comment concerns the training of new teachers. Schools must encourage and help colleges and universities that train teachers to revise their teacher-training programs so that their graduates are adequately prepared to teach the new courses. A vigorous program to modernize teacher-training programs has been launched by the Panel on Teacher Training of the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics.

Finally, high school mathematics teachers should re-examine their teaching techniques. Some highly effective new techniques have been developed by some of those who have developed the new courses. The introduction of new mathematics into the courses and the development of new teaching techniques have proceeded hand-in-hand.

In conclusion, I must emphasize that the elementary school, the junior high school, and the senior high school lay the foundation. I must emphasize that the elementary school teacher, the junior high school teacher, and the senior high school teacher are absolutely essential to the success of our program to provide better mathematics; for these teachers must teach mathematics, and these teachers must teach with enthusiasm so their their students continue the study of mathematics. Finally, I must emphasize that the National Council of Teachers of Mathematics is right in arranging this Conference to provide information and help to schools that wish to improve their mathematics programs.

I wish you a profitable two days in the Conference and much success during the coming year in your efforts to improve mathematics in the schools of the United States.



What science can there be more noble, more excellent, more useful for men, more admirably high and demonstrative, than this of the mathematics?

—BENJAMIN FRANKLIN

An Adventure with Spirals

CAROL BAUGHER

Student, Montclair State College, New Jersey

Designs and shapes have interested man from the beginning of time. As he looked around him into his environment, it would be impossible for him to miss the variety and the different types in the structure of things. Many forms that are dealt with in plane and solid geometry have found their way into the man-made products of today. One of these forms that is often encountered is the spiral. Many varieties of this occur, and they form the basis for many interesting phenomena as well as for mathematical and biological work.

Let us look therefore into the mathematical background of the spiral, and the basic types of properties that are shown. There are at least three types of curves which fulfill the requirements of a spiral. The first type that we will discuss is called the Spiral of Archimedes. It is defined to be a curve such that the radius vector at any point is directly proportional to the vectorial angle. The equation in polar coordinates is $r = a \theta$.

The following problem, adapted from [1]*, will illustrate the Spiral of Archimedes. A long straight rod rotates in a plane about a fixed point at the rate of 60 revolutions per minute. An object slides along the rod, moving outward from the center at the rate of 3 cm per minute. Using polar coordinates, with the pole at the center of rotation and the polar axis in the direction of the rod when the object is at the center, let (r, θ) be the position of the object at time t . If we start measuring time when $r = 0$, then in t minutes the object will have moved $3t$ cm away from the pole and will have turned through $60(2\pi)t$ radians. Hence

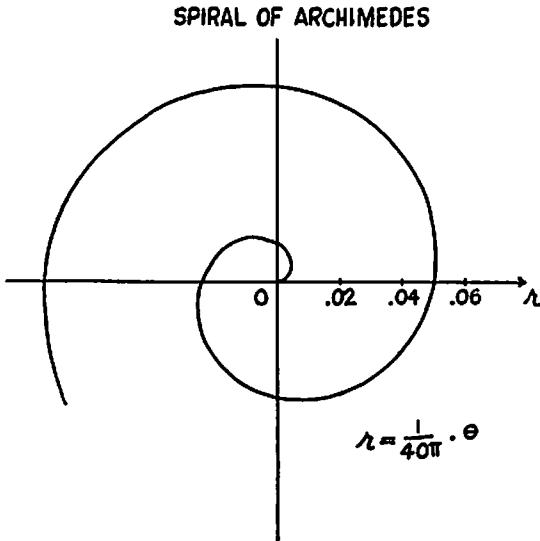
$$\begin{aligned}r &= 3t \\ \theta &= 120\pi\end{aligned}$$

are parametric equations for its path. Eliminating the parameter yields

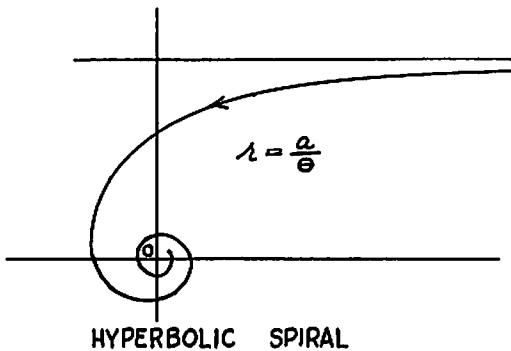
$$r = \frac{1}{40\pi} \theta,$$

*The numbers in the brackets refer to the references at the end of this article.

and the locus is a Spiral of Archimedes. This is the simplest type of spiral curve found.

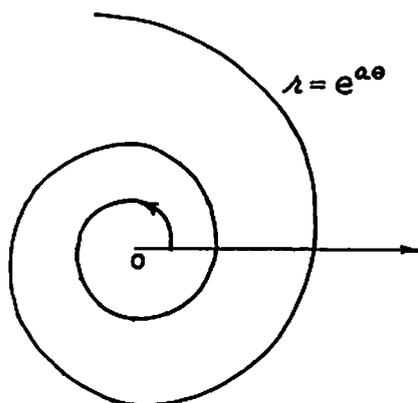


Another spiral associated with this type is the hyperbolic or reciprocal spiral. In this the radius vector at any point is inversely proportional to the vectorial angle, thus giving the equation $r\theta = a$. The resulting graph would be as follows [4]:



The third type of spiral is one that occurs with great frequency in nature. This is the logarithmic or equiangular spiral which is de-

defined as a curve for which the natural logarithm of the radius vector at any point is directly proportional to the vectoral angle, giving $\ln r = a\theta$ or $r = e^{a\theta}$. It can also be said to be a plane curve that cuts all its radius vectors at the same angle [5, p. 88]. Its graph is shown in the figure.



LOGARITHMIC SPIRAL

Another curious method of finding this last spiral makes use of other properties which are relevant to forms found in nature. It is necessary therefore to discuss the golden ratio or divine proportion which is frequently found in nature. The value of this can be determined by working with squares and triangles. First a 2×2 square is drawn (see figure), then connect the mid-point of one side, E , with the opposite vertex, C . Of course the length of this line, EC , may be found by the use of the Pythagorean Theorem.

$$x^2 = 4 + 1$$

$$x = \sqrt{5}$$

Then from B extend side AB so that EF is equal to $\sqrt{5}$. The resultant rectangle is one with 'divine proportions' and has been found to be favored when working in art. The golden ratio can be obtained from this by setting up a ratio from the dimensions. Thus $2 : 1 + \sqrt{5}$ or $2 : 3.236 = .6180$ (approximately). As was stated before, this ratio is found in many forms in nature.

Another variation of the spiral must be considered if we are to give it the three dimension effect it sometimes possesses in nature. A helix is such a variation. This can be shown by taking a stick or any other object and attaching a piece of string to the bottom. Then wind the string up the stick. This will produce a more or less spread out spiral [2, p. 20]. Also try to take a watch spring and lift it up from the center. This also gives you the effect of a different variation. A point of interest here is the direction in which you wind the string up the stick. If you go to the left you form what is called a sinister spiral, if to the right you form a dextral spiral. These become of interest when one considers forms in nature. It is found that the left one is rather infrequent. Left handed people usually make left handed spirals. The Japanese however, possibly because they write to the left, also make left handed spirals. Rather interesting to the police, I imagine!

Now that we have considered the spiral in terms of mathematics, let us consider it in terms of occurrence. First, let us begin with Nature. We discussed previously the golden ratio. Fibonacci, a thirteenth century mathematician, developed a sequence of numbers for which the ratios of successive numbers approach the golden ratio, and which have been related to growth in plants. The series begins with 1 and each new term is found by adding the previous two numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, etc.

Combining this work with that of the botanist, Church, one can see the relationship with the way that disc flowers occur, for example, on the head of a sunflower. He found that they were arranged on two sets of log spirals, one set going in one direction, the other set in the opposite direction. The number of spirals however, is fixed by the Fibonacci sequence and the law of the golden section. There is, thus, a system set up for this type of growth. This occurs in pine cones as well. If there are five spirals in one set there will be eight in the other, or eight and thirteen, or thirteen and twenty-one. Regular patterns of growth are established [3, p. 29].

The Fibonacci sequence again is found in plants when one considers the placement of leaves on the stem. Leaves occur in the following sequence: $1/2$, $1/3$, $2/5$, $3/8$, $5/13$, etc. again making use of the golden ratio. The numerator denotes the number of turns a string makes around the stem through the leaf axils while the denominator shows the number of leaves in the interval. This presents quite an interesting relationship between a spiral and alternate leaf development.

Leaves are often arranged spirally because it is the best adaptation for maximum light in a minimum area. One interesting plant, Lady's Tresses, even has its buds so orientated. They grow spirally on their stalk in a sinistral curve. Thus all the buds are well spaced for light. The curve of the stalk is similiar to the everyday corkscrew which can easily be pictured for this example [2, p. 55].

One of the best ways that plants have used the spiral comes to us from the climbing plants. Tendrils upon striking an object will curl and hence give support to the plant. It has been found that almost all of these so called "curls" are dextral spirals, and have twisted contrary to the sun.

Many of the algae, primitive forms of plants, also show many spirals in their make-up. Spirogyra, a truly classical specimen, displays a perfect spiral in its type of chloroplast. Actually this is where it gets its name. Spiral thickenings for support in plant vascular systems are also easy to find. Many primitive relationships have thus been found with spiral connections.

Let us leave the field of botany and enter the animal kingdom. Here as well, we shall see the influence of the spiral on forms and structures.

The classic example of the spiral is to be found at the seashore in the various types of shells. Each shell from the smallest to the largest displays some type of spiral. Perfect log spirals have even been projected from some shells; some are dextral, others are sinister, but they all show to the seeker great grace and beauty in design. Shells had the spiral staircase long before man thought to design it.

The possible reason for this type of shell development is easy to comprehend. The outer shell surface is harder and contains more muscles than the inner, which is softer and much more ductile. Thus when growth occurs, the outer edge pulls farther out rather than straight. Continuing over long periods of time, you can see how the spiral form is obtained [2, p. 88].

The horns of various animals also show spiral curves. Here we must pause and make a distinction between a spiral curve and a spiral twist. Curves are those which are primarily in a flat plane. A twist is similiar to a baker's cruller, and is usually vertically orientated. Sheep, for example, have the former type of horns. They are homonymously arranged on its head, that is a dextral curve on the right and a sinister curve on the left. It is necessarily this way, otherwise they would grow through the head and it would be rather un-

comfortable. The twist occurs on goats and antelopes, and usually goes back off the head. Spirals have a definite place here, for if the bony structure were straight instead of curved it would be rather inconvenient [2, p. 79].

Other animals also reflect the spiral in their forms. The norwhal's front tooth is usually spiraled. It is felt that since the norwhal usually favors one side as he goes through the water, that this causes the spiral twisting. Rabbit's teeth and elephant's tusks will also spiral when there is some abnormality at the base when they are forming [2, p. 40]. Also, in the antennae of many insects spirals are found.

Man's body itself is not exempt from spiral forms. An intricate part of the ear, the cochlea, also is built in this form. There is no escape, spirals are an important design in the animal world.

The physical world is not excused either. The ocean waves actually roll over on their way to shore along the curve of a log spiral. The whirlpool, and the tornado, one wet and one dry, are also depicted by an inverted, pulled out watch spring. In the heavens also, one may have the chance of seeing a spiral nebula, which follows the curve again of a log spiral [5, p. 92].

The world around man is full of spiral forms, thus let's see what use man has made of them. Engineers today are making use of them in road construction. Have you ever ridden a Spiral of Archimedes off your favorite highway? Most roads exits are thus built because they conserve space and force the driver to slow down. When engineers work with open mines, a log spiral type of roadway is employed because it permits slow but deep grading of a hillside. A form so common in nature has a big place on drawing boards of today.

The helix is a form used often by man. The traditional candy cane and the red and white barber pole show this. Also very few women today are content with straight hair. The more log spirals they can force their hair into, the happier they are.

Man's classical use for the spiral has come, however, from architecture and the spiral staircase. In Europe, especially, your castle wasn't complete without such a stairway. The most beautiful and enduring types of these stairs have their counterpart in nature. One of the most unique of these is the staircase in the Château of Chambou in France. It is a double spiral, one going up and the other down. A man walking on one spiral could not see anyone walking on the other. One stairway was supposed to be for the women and the other for the men.

We in America also have our share of spiral stairs. The traditional lighthouses, many monuments, and even some of our modern structures make use of them. All one needs to visit is the new Guggenheim Museum designed by Frank L. Wright to see a perfect example of the inverted spiral. Styles change but the spiral is constant.

Artists have also made broad use down through the ages of the spiral. They have stimulated people through the uplifting influence of this form.

The spiral has affected different people in varied ways. For example, the story is told of the Swiss mathematician Jacob Bernoulli, who was intrigued by the famous log spiral of nature. To him its swirls showed many fascinating properties. Because he saw a sort of eternal recurrence in the log spiral he ordered his tombstone to be engraved with it, as well as the inscription: "Though changed I shall arise the same." [3, p. 183].

While none of us shall probably be so affected by the influence of the spiral, we nevertheless are affected somewhat. What nature has had all along, man has made use of, in his mathematics, his art, and his architecture, and the spiral with its graceful lines will exist forever.

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Quadrispace*

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A question repeatedly asked in mathematics circles is, "If there are one, two, and three dimensions, why don't we have four, five, or more dimensions?" It was in search of an answer to this question that the contemplation of n dimensions began about the middle of the last century. Since then a complete geometry has been built on the basis of n dimensions, and there has been much heated argument as to the possible existence of these higher dimensions. It may be well to point out here that the question of its physical existence is not a mathematical one. Pure mathematics is simply symbolic or formal logic, and it is on this basis only that the subject has been explored.

We are three dimensional beings and our senses are limited to three dimensional objects. Man can make no movement which cannot be resolved into three mutually perpendicular directions. It is therefore next to impossible for us to imagine what a fourth dimension would be. The object of this paper then is to give some idea, something as near a picture as possible, of the fourth dimension. Generally speaking we will discuss only the fourth dimension, and for simplicity's sake we will discuss only those figures composed of straight lines of unit length, that is squares, cubes, etc.

The geometries of higher dimensions extend from three separate categories in mathematics. The first of these consists of a natural extension from the geometries of two and three dimensions. The second considers higher dimensions as an offshoot of the so-called "non-Euclidean" geometries. And the third is concerned with the important parallelism between algebra and geometry.

In plane and solid geometry, we have corresponding figures for x , x^2 , and x^3 , but what about x^4 , x^5 , . . . x^n ? Coordinate geometry can plot figures for equations in one, two, and three unknowns, but algebra continues to equations in four, five, and more unknowns. It is impossible for geometry to make models of these figures of higher dimensions. But through a study of the progressions of our known figures, and by examining the fourth dimension from different points of view, we can arrive at some of the properties of these higher spaces.

*A paper presented at a Regional Convention of KME at Washburn University, Topeka, Kansas on March 26, 1960.

In a progression from the simplest to the more complex, we have a point, a line, a square, a cube, and a corresponding fourth dimensional figure which we will call a tesseract. Let us examine these figures and record our results on a chart in hopes of a suggestion of some of the properties of the tesseract.

A point can be said to contain only one point (itself). But a line contains an infinite number of points. Let our line of unit length contain P equally spaced points. Then the square would contain P^2 points, the cube P^3 points, and the tesseract P^4 points. Correspondingly a line contains one line, but a square contains P lines, a line being generated by each of the points on the original line. A cube then would contain P^2 lines, and a tesseract would contain P^3 lines. This logic is carried further in Table I.

CONTENTS

Dimensions	Name	Points	Lines	Squares	Cubes
0	Point	1	0	0	0
1	Line	P	1	0	0
2	Square	P^2	P	1	0
3	Cube	P^3	P^2	P	1
4	Tesseract	P^4	P^3	P^2	P

Table I.

We can get a better idea of the properties of the fourth dimension if we study these figures as objects of generation. This can best be done by filling out successive generation charts such as those on page 88. In the first chart we study the generation of a line by a point. The original figure being a point we recall that a point contains one point, a point generates a line which also ends in a point. Thus by adding from left to right we get the characteristics of the generated figure, a line. The second chart shows the generation of a square by a line, giving us the characteristic boundaries of the square: four points, four lines, and one square. And in chart four we find that a tesseract is bounded by sixteen points or vertices, thirty two lines or edges, twenty four squares or surfaces, and eight cubes. By studying these charts we see that every space is generated by a lower space, that is the same numbers are used in the "initial"

"GENERATION CHARTS"**GENERATION OF A LINE BY A POINT**

	In initial figure	Generated	In final figure	In generated figure
points	1	-	1	2
lines	0	1	0	1

Chart 1.**GENERATION OF A SQUARE BY A LINE**

	In initial figure	Generated	In final figure	In generated figure
points	2	-	2	4
lines	1	2	1	4
squares	0	1	0	1

Chart 2.**GENERATION OF A CUBE BY A SQUARE**

	In initial figure	Generated	In final figure	In generated figure
points	4	-	4	8
lines	4	4	4	12
squares	1	4	1	6
cubes	0	1	0	1

Chart 3.**GENERATION OF A TESSERACTION BY A CUBE**

	In initial figure	Generated	In final figure	In generated figure
points	8	-	8	16
lines	12	8	12	32
squares	6	12	6	24
cubes	1	6	1	8
tesseracts	0	1	0	1

Chart 4.

and "final" columns but these same numbers are simply dropped one space in the "generated" column. It follows immediately that the properties of any space may be obtained by further successions of these charts. A generalization of the results obtained by these charts is presented in Table II.

BOUNDARIES

Dimensions	Name	Points	Lines	Squares	Cubes
0	Point	1	0	0	0
1	Line	2	1	0	0
2	Square	4	4	1	0
3	Cube	8	12	6	1
4	Tesseract	16	32	24	8

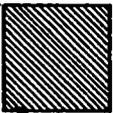
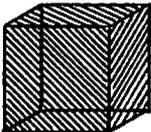
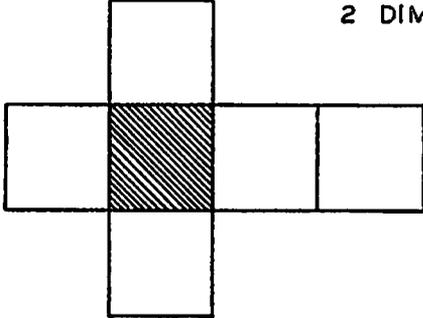
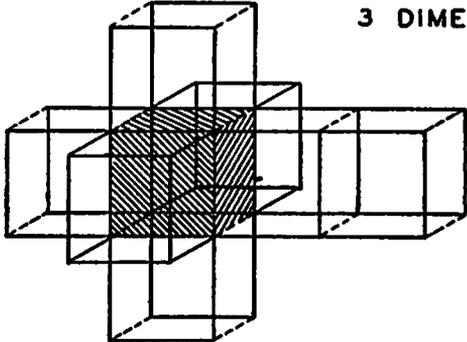
Table II.

Our chief difficulty now lies in the fact that it is extremely difficult to think of a cube, a solid in our space, as a mere boundary to the corresponding figure of a fourth dimensional solid. A somewhat better picture may be obtained by a study of the projections of our figures. If a two dimensional square is cut at one of its vertices, it can be laid out into a one dimensional line with a unit length to the left of the base and two unit lengths to the right of the base. A cube also, if cut in the proper places, can be laid out into a two dimensional surface with four squares enclosing the base and another square to the right of these. This is called an orthographic projection of the cube. Let us now study the relation of these two projections. If the line of four unit lengths was moved in a direction perpendicular to its one dimension through a distance of one unit length, it would generate four squares. Adding two more squares to enclose the base, we have the orthographic projection of the cube.

Hence, by analogy, an orthographic projection of a tesseract would be the generation of the projection of a cube through a unit length plus two additional cubes completely enclosing the cubic base (See figures page 90).

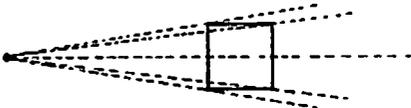
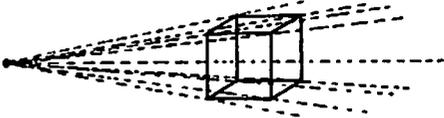
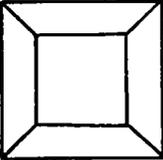
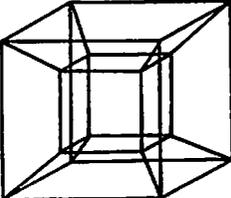
Perhaps a still better projection is the projection from a point source of light at a small distance. Here the projection of a one dimensional line would be a zero dimensional point. The projection of a two dimensional square would be a one dimensional line, and the projection of a three dimensional cube would be two dimensional. From this last projection we can study the properties of a cube without having a three dimensional model. We see a square inside of a square with lines joining the corresponding vertices of the two squares. We can also count the number of vertices, surfaces, and edges. Thus it follows that the projection of a tesseract should be a

ORTHOGRAPHIC PROJECTIONS

<p>SQUARE</p>  <p>2 DIMEN.</p>	<p>1 DIMEN.</p> 
<p>CUBE</p>  <p>3 DIMEN.</p>	<p>2 DIMEN.</p> 
<p>TESSERACT</p>  <p>4 DIMEN.</p>	<p>3 DIMEN.</p> 

three dimensional cube inside another with each of the vertices of the smaller cube connected with the corresponding vertices of the larger cube. If we compare the properties of this projection with those calculated in our charts of generation, we will find that they coincide exactly (See figures page 91).

PROJECTIONS
FROM A POINT SOURCE

OBJECT	PROJECTION
	
	
	
<p data-bbox="172 1194 583 1322">• TESSERACTION ?</p>	

Leaving the progressions of geometrical forms, we now consider the problem of symmetry. In a plane the only kind of rotation possible is that about a point, in the third dimension rotation can take place about an axis line, and it logically follows that rotation in the

fourth dimension should be possible about an axis plane! If given a line labeled from left to right B, A, C, A', B' , segments AB and $A'B'$ being equal and symmetrical with respect to point C ; we cannot possibly in the first dimension make these segments coincide. However, if we temporarily remove one of these segments from the first dimension and rotate it with respect to point C through a two dimensional plane, these two segments can be made to coincide exactly. Similarly, two symmetrical triangles cannot be made to coincide by any movements in their plane, but by rotating one of them 180° in the third dimension it may be replaced into the plane exactly coincident to the other triangle. Correspondingly, if one of two symmetrical solids from our space was removed into the fourth dimension and rotated 180° , it could then be replaced in our space occupying the same position as the other one. If, for example, we could put our left hand into fourth dimensional space and rotate it 180° , it would return into the third dimension as a right hand. The mirror image of such a solid represents the solid after such a rotation. This possibility suggests some of the more fascinating aspects of the fourth dimension. If such a space existed, a manufacturer of gloves or shoes would be able to simplify his production greatly by making only left handed articles and sending half of them on a round trip through the fourth dimension. Moreover, if the pilot of the space ship was right-handed at take-off, he would be left-handed upon his return.

The inability of man to perceive the fourth dimension, even if it be close at hand, is further comparable to the inability of a hypothetical two dimensional man to detect the existence of our space, even though he is a part of it. Thus in order to throw a little more light upon our relations to the proposed fourth dimension, let us examine these two dimensional beings.

Since they exist in a plane, they will have length and breadth, but no thickness. They can move freely in the confines of their two dimensional space, but they cannot rise above nor sink below this surface. If we were to draw a circle around a two dimensional man, he would be completely imprisoned. And though it would be a simple matter for us to pick him up into the third dimension and replace him in his plane outside the circle, he would be thoroughly perplexed and would not know what had happened. If his house had more than one window and two of them were left open at one time, his house would be divided into two separate parts. A circle, triangle, square, or any closed plane figure would be completely impenetrable

to him, just as a closed sphere, pyramid, or box are impenetrable to us. However, we find it quite easy to enter and leave the inside of his forbidden circle without ever touching its circumference. This implies that a fourth dimensional being should be able to enter and leave the confines of our solids without ever disturbing a molecule of its surface. Nothing in the world of the two dimensional man's space can be enclosed so that a three dimensional man cannot enter. Thus we can imagine a fourth dimensional robber stealing our most precious jewels from our strongest vaults without ever opening a door or touching its walls, or a fourth dimensional physician performing a delicate operation on our heart without ever piercing our skin.

The image formed on the retina of our eye is two dimensional, since the retina is itself a two dimensional surface. We have been able to gain the perception of depth only through the assimilation of past experience. In the same manner, objects in a plane would appear to be simply lines of varying lengths to a two dimensional man, since the retina of his eye would be a curved one dimensional line.

If a two dimensional being were exposed to the third dimension, that is, if a three dimensional solid were passed in front of his eye, he would simply see lines of varying lengths. Even if he knew what the properties of the third dimension were, he would not be able to see it. Correspondingly if we were, "or are," exposed to the fourth dimension, we would be only able to see three dimensional sections of it at one time, and consequently would not recognize it as the fourth dimension.

Putting this in a slightly different form, a point can be considered as an infinitely small slice of a line, a line as an infinitely thin slice of a plane, a plane as an infinitely thin slice of a solid, and a solid merely as an infinitely thin slice of a fourth dimensional solid! Taking this a little further, we see that our universe may be thought of as composed of an infinite number of planes, and if the fourth dimension existed, it would contain an infinite number of universes such as ours.

If a fourth dimension did exist, what would its properties be? We know that, like our three dimensions taken one at a time, it would have to be mutually perpendicular to our present three dimensions, it would have to extend to infinity in a linear direction, and it would be invisible to our sense of sight. Einstein and a great number of others have suggested time as such a space.

Time is certainly linear, since it extends in one dimension from the birth of Christ to infinity in both directions, and, to be

sure, we cannot see the passing of time. Moreover, if we consider a finite section of time, such as the maturing of a watermelon, we can see some more of its implications. From its birth in the blossom until it is full-grown, we know that it grows a little larger every day. Further, if an infinitely thin slice of the fourth dimension is a three dimensional solid, and if time is considered as the fourth dimension, then our perception of the watermelon at any instant is an infinitely small portion of the fourth dimension. If a cone were passed slowly in front of a two dimensional man's eye, tip first, he would see successive lines gradually growing in length until it disappeared at the base, and so we are able to see successive stages in the development of a watermelon until it dies, rots, and returns to the earth. Our inability to perceive the growth of a watermelon as an entity, but only successive stages, is comparable to the inability of our two dimensional friend to envision the three dimensional cone as a whole.

Thus we see that although the fourth dimension is significant only in the fields of mathematical contemplation and logic, its implications in other fields are at least disturbingly interesting. If it were possible to prove the existence of the fourth dimension, its implications in the fields of philosophy, theology, biology, the physical sciences and many others would be almost without bounds.

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Isotropic Elements of an N-dimensional Space

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The two lines $y = \pm ix$, $i = \sqrt{-1}$, have been called isotropic lines [1]*. These lines have the following properties:

- I. $y = \pm ix$ is self orthogonal.
- II. The distance between any two points of any of these lines is zero.

In this note we study first the curves in the plane xOy , having similar properties; then we generalize the idea to an n -dimensional space. Note that the plane xOy is not the so called complex plane. This note explains one of the reasons that in complex spaces the inner product of the vectors (x_1, \dots, x_n) and (y_1, \dots, y_n) is defined to be

$$\sum_{j=1}^n x_j \bar{y}_j,$$

where \bar{y} is the complex conjugate of y .

1. Isotropic curves of the plane: Let $f(x,y) = 0$ be the equation of a curve in the plane. Suppose that $f(x,y)$ is analytic, i.e., $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist. Since the inner product of two vectors $\mathbf{u} = (x_1, y_1)$ and $\mathbf{v} = (x_2, y_2)$ is $\mathbf{u} \cdot \mathbf{v} = x_1 x_2 + y_1 y_2$, the necessary and sufficient condition for \mathbf{u} and \mathbf{v} to be orthogonal is

$$(1) \quad x_1 x_2 + y_1 y_2 = 0.$$

Thus the relation

$$\frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = 0$$

implies that the vector $\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$ is normal to the curve. If we impose the condition that this vector is self orthogonal, by (1) we get

$$(2) \quad \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = 0, \quad \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \text{ not both zero.}$$

*The numbers in brackets refer to references at the end of this article.

We see that $y = \pm ix + c$ is a solution of (2). These lines do not have envelopes. Thus isotropic curves in the plane are isotropic lines.

2. Isotropic elements of an n -dimensional space: Let $F(x_1, \dots, x_n)$ be analytic in n variables x_1, \dots, x_n . Again

$$\frac{\partial F}{\partial x_1} dx_1 + \dots + \frac{\partial F}{\partial x_n} dx_n = 0$$

implies that $\left(\frac{\partial F}{\partial x_1}, \dots, \frac{\partial F}{\partial x_n}\right)$ is a vector normal to the hypersurface

$F(x_1, \dots, x_n) = c$; since for two vectors $\mathbf{u} = (x_1, \dots, x_n)$ and $\mathbf{v} = (y_1, \dots, y_n)$ the inner product is

$$\mathbf{u} \cdot \mathbf{v} = \sum_{j=1}^n x_j y_j,$$

and the equality

$$\sum_{j=1}^n x_j y_j = 0$$

implies that \mathbf{u} and \mathbf{v} are orthogonal. Now for $F = c$ to be self orthogonal we must have

$$(3) \quad \sum_{j=1}^n \left(\frac{\partial F}{\partial x_j}\right)^2 = 0.$$

We observe that

$$(4) \quad \sum_{j=1}^n a_j x_j + b = 0, \quad \text{with} \quad \sum_{j=1}^n a_j^2 = 0$$

is a solution of (3). These solutions are also linear.

There are also non-linear solutions of (3). We shall give a proof for the three-dimensional space and leave the generalization to the reader. In the three-dimensional space

$$(5) \quad z = ax + by + c, \quad \text{with}$$

$$(6) \quad a^2 + b^2 + 1 = 0$$

is a solution of

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1 = 0$$

which is a special case of (3). Here we actually suppose the equation of the surface is of the form $z = z(x,y)$. In this case

$\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$ is normal to the curve.

Now to prove the theorem. Consider the locus

$$(7) \quad \begin{cases} x = 0 \\ z = g(y) \end{cases}, g \text{ analytic.}$$

We would like to find a condition for (7) to be tangent to (5) at a point $(0, t, g(t))$. Thus we have $b = g'(t)$. This is proved by comparing the two loci

$$\begin{cases} x = 0 \\ z = g(y) \end{cases} \quad \text{and} \quad \begin{cases} x = 0 \\ z = ax + by + c. \end{cases}$$

We observe that direction numbers of the tangents are respectively $(0, 1, g'(t))$ and $(0, 1, b)$.

Now from (7) and (5) at the point $(0, t, g(t))$ we have

$$c = z - ax - by = g(t) - tg'(t).$$

Thus

$$(8) \quad z = ax + by + c = ax + g'y + g - tg'.$$

But (6) can be written as

$$a^2 + b^2 + 1 = a^2 + (g')^2 + 1 = 0.$$

Therefore we get

$$a = \pm \sqrt{-1 - (g')^2}.$$

Substituting in (8) and taking all terms to one side of the equation we get a function of x, y, z , and t , namely

$$(9) \quad F(x,y,z,t) = -z \pm x\sqrt{-1 - (g')^2} + g'y + g - tg' = 0.$$

It is clear that F is analytic. Now the one parameter family of surfaces $F(x,y,z,t) = 0$ has an envelope in the neighborhood of a point

if the point satisfies the differential equation and $\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}$ are

not all zero, and $\frac{\partial^2 F}{\partial t^2} \neq 0$ [2]. Applying these conditions to (9) in

the particular case $g(t) = t^2$, and choosing the plus sign for the radical, we have

$$\frac{\partial F}{\partial z} \neq 0, \text{ and}$$

$$\frac{\partial^2 F}{\partial t^2} = \frac{4x}{(-1-4t^2)^{3/2}} - 2 \neq 0, \text{ if } t \neq \pm \frac{i}{2}.$$

Therefore there is an imaginary envelope for (9) passing through the curve (7) in the neighborhood of (0,0,0).

3. Measure of a portion of an $(n-1)$ -dimensional element of an n -dimensional space: Let $n \neq 1$. Then the formula

$$A_k = \int \int \cdots \int_S \left[1 + \sum_{j=1}^k \left(\frac{\partial z}{\partial x_j} \right)^2 \right]^{1/2} \prod_{j=1}^k dx_j,$$

where S is a region of the real n -space, gives the k -measure of a portion of the hypersurface $z = f(x_1, \dots, x_k)$, [2]. Here the integral is of order k , in the case z is real. Note that the dimension of the space is $n = k + 1$. It is clear that A_k for any isotropic element of the space is zero.

4. Inner product in a unitary space: It is desirable to have a commutative inner product. If for $\mathbf{u} = (x_1, \dots, x_n)$ and $\mathbf{v} = (y_1, \dots, y_n)$ we define the inner product to be

$$\mathbf{u} \cdot \mathbf{v} = x_1 y_1 + \cdots + x_n y_n,$$

it is obvious that $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$. But what was said in sections 1, 2, and 3, shows that we get all sorts of lines and surfaces with zero measure. This is interesting, but many geometric definitions can not be carried to complex spaces such as, "Distance between two distinct points is positive." Thus we sacrifice commutativity for positive distance, and we define the inner product to be

$$\mathbf{u} \cdot \mathbf{v} = x_1 \bar{y}_1 + \cdots + x_n \bar{y}_n.$$

In this case $\mathbf{v} \cdot \mathbf{u} = \overline{(\mathbf{u} \cdot \mathbf{v})}$.

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- [1] Amir-Moez, A. R., Wonders of "i", *The Pentagon*, Spring 1958, p. 82, 88.
 [2] Taylor, A. E. *Advanced Calculus*, Ginn and Co., N.Y., 1955, pp. 399-401.

The Problem Corner

EDITED BY J. D. HAGGARD

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before October 1, 1961. The best solutions submitted by students will be published in the Fall, 1961, issue of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to J. D. Haggard, Department of Mathematics, Kansas State College, Pittsburg, Kansas.

PROPOSED PROBLEMS

141. *Proposed by Tom Wood, University of Missouri, Columbia.*

Find: The natural numbers $a, b, c, d, e, g, h, i, j$.

Given:	$a \ b \ c$	and	$b = e - i$
	$- \ c \ b \ a$		$j = a + c$
	<hr style="width: 100%; border: 0.5px solid black;"/>		$f = 7$
	$d \ e \ f$		$a > c$
	$+ \ f \ e \ d$		
	<hr style="width: 100%; border: 0.5px solid black;"/>		
	$g \ h \ i \ j$		

142. *Proposed by R. C. Weger, William Jewell College, Liberty, Missouri.*

m and n are digits in a number system whose base is b , $m < b$ and $n < b$. Find the largest $k < b$ such that

$$mn \equiv nm \pmod{k}$$

143. *Proposed by Ronald L. Hammitt, San Jose State College, San Jose, California.*

With compasses only, find the vertices of a square inscribed in a given circle.

144. *Proposed by Paul R. Chernoff, Harvard University, Cambridge, Massachusetts.*

Let $a_1(x) = x$ ($x \geq 0$) and $a_{n+1}(x) = \sqrt{a_n(x) + 1}$ for all $n \geq 1$. Show that $\lim_{n \rightarrow \infty} a_n(x)$ exists for all $x \geq 0$, and compute its value.

145. *Proposed by C. W. Trigg, Los Angeles City College.*

If three alternate primes are in arithmetic progression, show that the difference between any two of them is greater than 5.

SOLUTIONS

136. *Proposed by Jimmy M. Rice, Fort Hays Kansas State College, Hays, Kansas.*

A schoolboy on Neptune solves the quadratic equation $x^2 - 10x + 31 = 0$ and finds the roots to be 5 and 8. What is the base of the number system he is using?

Solution by Thomas Sharpe, Evansville College, Evansville, Indiana.

Let the base of the Neptune system be b . Then

$$10 = 1 \cdot b + 0$$

$$31 = 3 \cdot b + 1$$

and the equation can now be written $x^2 - bx + 3b + 1 = 0$. Since 5 is a root, and using base 10, we may obtain $25 - 5b + 3b + 1 = 0$ or $b = 13$. Substituting 8 into the equation also yields $b = 13$.

Solution by David Forslund, James Lick High School, San Jose, California.

Since 5 and 8 are roots, $5 + 8 = 10_b$.

Thus $13 = 10_b = 1 \cdot b + 0$, so the base b is 13.

Also $5 \cdot 8 = 31_b = 3 \cdot b + 1$, which gives $b = 13$.

Also solved by Mark Bridger, Columbia University, New York City; James Brooking, State University of New York, Albany; Paul R. Chernoff, Harvard University, Cambridge, Massachusetts; Donald Dittner, Central Missouri College, Warrensburg; Richard Lewis, Kansas State College of Pittsburg, Pittsburg, Kansas; Ralph Milano, Montclair State College, Upper Montclair, New Jersey; Elizabeth Ann O'Connell, San Jose State College, San Jose, California; Arie Poldervaart, University of New Mexico, Albuquerque; R. C. Weger, William Jewell College, Liberty, Missouri; Tom Wood, University of Missouri, Columbia; Nelson Zinsmeister, Manchester College, North Manchester, Indiana.

137. *Proposed by Robert Myers, Chicago Teachers College. (From Introduction to Theory of Equations, Conkwright)*

Determine p and q so that 5 will be a double root of $x^4 - 9x^3 + px^2 + qx + 25 = 0$.

Solution by Arie Poldervaart, University of New Mexico, Albuquerque.

Since 5 is a double root $x^2 - 10x + 25$ must be a factor of $x^4 - 9x^3 + px^2 + qx + 25$. Dividing the latter by the former we get $(q + 10p - 175)x + (400 - 25p)$ for a remainder, which must be zero for all x . Thus

$$\begin{aligned}q + 10p - 175 &= 0 \\400 - 25p &= 0\end{aligned}$$

which yields

$$p = 16, q = 15.$$

Solution by Paul R. Chernoff, Harvard University, Cambridge, Massachusetts.

Since 5 is a double root of the equation, the equation formed by differentiating the left side,

$$4x^3 - 27x^2 + 2px + q = 0,$$

has 5 as a root. Thus

$$5^4 - 9 \cdot 5^3 + 25p + 5q + 25 = 0$$

and

$$4 \cdot 5^3 - 27 \cdot 5^2 + 10p + q = 0$$

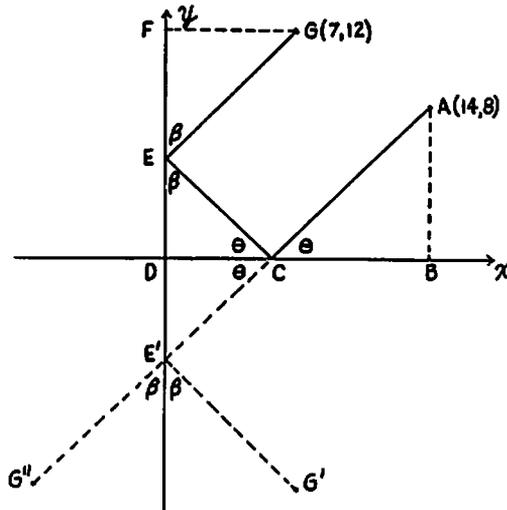
which yield

$$p = 16 \text{ and } q = 15.$$

Also solved by Patrick J. Boyle, San Jose State College, San Jose, California; Mark Bridger, Columbia University, New York City; James Brooking, State University of New York, Albany; David Farlund, James Lick High School, San Jose, California; Ronald Hammett, San Jose State College, San Jose, California; Richard Hannes, Upsala College, East Orange, New Jersey; Ronald W. Harris, University of Southwestern Louisiana, Lafayette; Mauriel P. Harris, James Lick High School, San Jose, California; John Hunt, James Lick High School, San Jose, California; Ralph Milano, Montclair State College, Upper Montclair New Jersey; Kathleen Ott, Nebraska State Teachers College, Wayne; James R. Paris, East Tennessee State College, Johnson City; S. M. Steele, Jr., Kansas State College of Pittsburg, Pittsburg; Mary Sworske, Mount Mary College, Milwaukee, Wisconsin; James Wasserman, St. Meinrad College, St. Meinrad, Indiana; Tom Wood, University of Missouri, Columbia; Nelson Zinsmeister, Manchester College, North Manchester, Indiana.

130. *Proposed by Mark Bridger, Columbia University, New York, New York.*

Professor Umbugia has had his pool table inscribed in the first quadrant of a cartesian coordinate system, with a corner pocket located at the origin and a coordinate axis along two of the sides. In a game with one of his students the Professor finds his ball on the spot $(14, 8)$. Taking careful (though inaccurate) aim he hits the ball in such a way that it misses every other ball, bouncing off the x and y axes in that order, and finally coming to rest at the spot $(7, 12)$. To cover up his "scratch", the Professor asks, "How far has the ball traveled?"



Solution by Tom Wood, University of Missouri, Columbia.

Let L be the total distance traveled by the ball and θ the angle the path of the ball makes with the x axis. Then

$$L = AC + CE + EG,$$

where

$$AC = 8 \csc \theta$$

$$CE = CD \sec \theta = DE \csc \theta$$

$$EG = 7 \sec \theta$$

and

$$CD = BD - BC = 14 - 8 \cot \theta$$

$$DE = DF - EF = 12 - 7 \tan \theta.$$

Therefore

$$L = 8 \csc \theta + (12 - 7 \tan \theta) \csc \theta + 7 \sec \theta = 20 \csc \theta$$

or

$$L = 8 \csc \theta + (14 - 8 \cot \theta) \sec \theta + 7 \sec \theta = 21 \sec \theta.$$

Thus

$$20 \csc \theta = 21 \sec \theta$$

$$\tan \theta = 20/21$$

$$\theta = 43.60^\circ$$

giving

$$\begin{aligned} L &= 20 \csc \theta = 21 \sec \theta \\ &= 20(1.450) = 21(1.381) \\ &= 29. \end{aligned}$$

Solution by R. C. Weger, William Jewell College, Liberty, Missouri.

Reflect the broken line GEC about the x axis, obtaining the broken line $CE'G'$. Now reflect $E'G'$ about the y axis obtaining $E'G''$. Thus $ACE'G''$ is a straight line segment and

$$AC + CE + EG = AG'' = \sqrt{(14 + 7)^2 + (12 + 8)^2} = 29$$

Also solved by Patrick J. Boyle, San Jose State College, San Jose, California; James A. Brooking, State University of New York, Albany; David Forslund, James Lick High School, San Jose, California; Ronald L. Hammett, San Jose State College, San Jose, California; Ronald W. Harris, University of Southwest Louisiana, Lafayette; Ralph Milano, Montclair State College, Upper Montclair, New Jersey; Keith F. Purcell, Central College, Fayette, Missouri; S. M. Steele, Jr., Kansas State College of Pittsburg, Pittsburg; Dean Stringer, Nebraska State Teachers College, Wayne; Mary Sworske, Mount Mary College, Milwaukee, Wisconsin.

139. *Proposed by Paul R. Chernoff, Harvard University, Cambridge, Massachusetts.*

Evaluate $\sum_{p=0}^n \binom{2n+1}{2p}$, where $\binom{m}{n} = \frac{m!}{n!(m-n)!}$.

Solution by Mark Bridger, Columbia University, New York, New York.

From the definition of $\binom{m}{n} = \frac{m!}{n!(m-n)!}$ we note that:

$$\binom{m}{n} = \binom{m-1}{n-1} + \binom{m-1}{n} \text{ for } n \geq 1$$

and

$$\binom{m}{0} = \binom{m-1}{0} = 1.$$

For $n < 0$, $\binom{m}{n} = 0$.

$$\begin{aligned} \sum_{p=0}^n \binom{2n+1}{2p} &= \sum_{p=0}^n \left[\binom{2n}{2p-1} + \binom{2n}{2p} \right] \\ &= \binom{2n}{-1} + \binom{2n}{0} + \sum_{p=1}^{2n} \binom{2n}{p} \\ &= 1 + \sum_{p=1}^{2n} \binom{2n}{p} = \sum_{p=0}^{2n} \binom{2n}{p} \end{aligned}$$

But this is the sum of the binomial coefficients of $(a+b)^{2n}$. Letting $a = b = 1$,

$$\sum_{p=0}^{2n} \binom{2n}{p} = (1+1)^{2n} = 2^{2n}$$

Also solved by David Farslund, James Lick High School, San Jose, California.

140. *Proposed by the Editor. (From The American Mathematical Monthly).*

Evaluate the infinite product

$$\prod_{n=1}^{\infty} \left(1 + \frac{1}{a_n} \right) \text{ where } a_1 = 1, a_n = n(a_{n-1} + 1).$$

Solution by Paul Chernoff, Harvard University, Cambridge, Massachusetts.

First we show that

$$\begin{aligned} \prod_{n=1}^m \left(1 + \frac{1}{a_n} \right) &= \sum_{n=0}^m \frac{1}{n!}. \\ \prod_{n=1}^m \left(1 + \frac{1}{a_n} \right) &= \prod_{n=1}^m \frac{a_n + 1}{a_n} = \prod_{n=1}^m \frac{a_{n+1}}{(n+1)a_n} \\ &= \frac{a_2}{2a_1} \cdot \frac{a_3}{3a_2} \cdots \frac{a_{m+1}}{(m+1)a_m} = \frac{a_{m+1}}{(m+1)!} = \frac{a_m + 1}{m!} \\ &= \frac{a_m}{m!} + \frac{1}{m!} = \frac{a_{m-1}}{(m-1)!} + \frac{1}{(m-1)!} + \frac{1}{m!} \\ &= \frac{a_{m-2}}{(m-2)!} + \frac{1}{(m-2)!} + \frac{1}{(m-1)!} + \frac{1}{m!} \end{aligned}$$

Continuing in this manner we obtain $\sum_{n=0}^m \frac{1}{n!}$,

whence

$$\prod_{n=1}^{\infty} \left(1 + \frac{1}{a_n} \right) = \sum_{n=0}^{\infty} \frac{1}{n!} = e$$

Also solved by Mark Bridger, Columbia University, New York, New York.

EDITOR'S NOTE: The Problem Corner is in need of problems for future issues. Both faculty and students may submit what appears to them to be interesting problems that will challenge the reader.

The Mathematical Scrapbook

EDITED BY J. M. SACHS

In mathematics, as in any scientific research, we find two tendencies present. On the one hand, the tendency towards abstraction seeks to crystallize the logical relations inherent in the maze of material that is being studied, and to correlate the material in a systematic and orderly manner. On the other hand, the tendency toward intuitive understanding fosters a more immediate grasp of the objects one studies, a live rapport with them, so to speak, which stresses the concrete meaning of their relations.

—D. HILBERT

= Δ =

In most texts in analytical geometry the subjects of translation and rotation are presented and discussed, usually, from the point of view of changing the coordinate system. This same material can be presented as the mapping of a line onto a line, a plane onto a plane or 3-dimensional space onto itself with a fixed coordinate system. This approach would be more satisfactory, it seems to me, as a preparation for further work in mathematics in which mappings and transformations play a large role. What differences would it involve? In the plane a translation would be thought of as a sliding along a line, coordinate system fixed, with point (x, y) mapped into (x', y') where $x' = x + h$, $y' = y + k$. This could be viewed as a single slide in the direction indicated by the slope k/h a distance $\sqrt{h^2 + k^2}$ or as a slide in the x -direction a distance h followed by a slide in the y -direction a distance k .

How does this differ from a translation of coordinate system? The latter, moving the origin to the point whose coordinates were formerly (h, k) is done by means of the equations $x' = x - h$, $y' = y - k$. Our approach leaves the coordinate system unchanged and maps the origin into the point (h, k) . A similar comparison can be made in 3-space.

What would this mean for a rotation? In analytical geometry we usually considered rotations in the plane leaving the origin fixed and rotating the axes through an angle θ maintaining their perpendicularity. Suppose we consider leaving the axes fixed and perform a mapping in which each point goes into an image point by rotating the ray joining the point and the origin through the angle θ . What would be the differences between the equations so obtained and the

traditional equations for the rotation of the coordinate system? How could you accomplish this in 3-space?

Of even greater interest to me is the almost complete absence in analytical geometry of material on reflections. Once one adopts the mapping idea instead of the change of coordinate system idea, the concept of reflection is a simple, useful, and fundamental one. Reflection in a point P maps any point Q into a point Q' which is on the line determined by P and Q , the same distance from P as Q and on the opposite half-line to Q , boundary P . Reflection in a line is done by perpendiculars to the line with the image Q' on the perpendicular through Q , the same distance from the line as Q and in the opposite half-plane to Q , boundary the line of reflection. A similar thing can be done for reflections in a plane. I would like to leave with you two questions.

1. Can you show that any translation can be done by means of successive reflections?
2. Can you show that any rotation can be accomplished by successive reflections?

= Δ =

Mathematicians attach great importance to the elegance of their methods and their results. This is not pure dilettantism. What is it indeed that gives us the feeling of elegance in a solution, in a demonstration? It is the harmony of the diverse parts, their symmetry, their happy balance; in a word it is all that introduces order, all that gives unity, that permits us to see clearly and to comprehend at once both the ensemble and the details. But this is exactly what yields great results, in fact the more we see this aggregate clearly and at a single glance, the better we perceive its analogies with other neighboring objects, consequently the more chances we have of divining the possible generalizations. Elegance may produce the feeling of the unforeseen by the unexpected meeting of objects we are not accustomed to bring together; . . . this esthetic satisfaction is bound up with the economy of thought.

—H. POINCARÉ

= Δ =

Suppose a friend picks a positive integer less than a given limit. Can you devise a series of questions which can be answered by YES or NO so that you can determine the integer your friend has chosen? What relation is there between the magnitude of the limit and the minimum number of questions you need to ask? Note that the same

questions must be asked in all cases, that is you cannot change question 2 depending upon what answer you got to question 1. For example suppose the limit is 16. The following four questions will always give you enough information to determine the number:

1. Is the chosen integer 8 or larger?
2. Is the remainder upon division by 8 four or larger?
3. When this remainder is divided by 4 is the next remainder 2 or larger?
4. When this last remainder is divided by 2 is the remainder 1?

These questions amount to asking whether the digits in the binary notation of a four place binary number are zeros or ones. The largest number we could get this way would be 1111 which as a binary represents $8 + 4 + 2 + 1 = 15$ which is the largest integer we could have. How many questions would you need if the limit were 32? Suppose the limit were not a power of 2, what would you do? For example, how many questions would you need for limit 100?

Suppose the rules of the game were changed to permit the questioner to produce three possible answers from which the person who knows the number must choose one if the correct answer is there. How would you word your questions if the limit number is 27? 81? 243?

$$=\triangle=$$

There are certain focal points in history toward which the lines of past progress converge, and from which radiate the advances of the future. Such was the age of Newton and Leibniz in the history of mathematics. During fifty years preceding this era several of the brightest and acutest mathematicians bent the force of their genius in a direction which finally led to the discovery of the infinitesimal calculus by Newton and Leibniz. Cavalieri, Roberval, Fermat, Descartes, Wallis and others had each contributed to the new geometry. So great was the advance made, and so near was their approach toward the invention of the infinitesimal analysis, that both Lagrange and Laplace pronounced their countryman, Fermat, to be the true inventor of it. The differential calculus, therefore, was not so much an individual discovery as the grand result of a succession of discoveries by different minds. Indeed, no great discovery ever flashed upon the mind at once, and though those of Newton will influence mankind to the end of the world, yet it must be admitted that Pope's lines are only a "poetic fancy":

"Nature and Nature's laws lay hid in night;
 God said, 'Let Newton be,' and all was light."

—F. CAJORI

=Δ=

Consider the second differences of products of consecutive primes beginning with 5.

	First Difference	Second Difference
5 · 7 = 35		
	42	
7 · 11 = 77		24
	66	
11 · 13 = 143		12
	78	
13 · 17 = 221		24
	102	
17 · 19 = 323		12
	114	
19 · 23 = 437		116
	230	
23 · 29 = 667		102
	232	
29 · 31 = 899		16
	248	
31 · 37 = 1147		

Notice the apparent patterns in the units digit for integers in the column for the second differences. Does a pattern of such patterns persist. Is there a discernible relationship as the pattern shifts?

=Δ=

The appreciation of the structure of ideas is that side of a cultured mind which can only grow under the influence of a special study. I mean that eye for the whole chessboard, for the bearing of one set of ideas on another. Nothing but a special study can give any appreciation for the exact formulation of general ideas, for their relations when formulated, for their service in the comprehension of life. A mind so disciplined should be both more abstract and more concrete. It has been trained in the comprehension of abstract thought and in the analysis of facts. . . .

—A. N. WHITEHEAD

The Book Shelf

EDITED BY H. E. TINNAPPEL

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Harold E. Tinnappel, Bowling Green State University, Bowling Green, Ohio.

Analytic Function Theory, Volume I, Einar Hille, Ginn and Company, Boston, 17, 1959, 308 pp., \$6.50.

This book is the first to appear in a series of texts under the heading "Introduction to Higher Mathematics" designed for use in upper undergraduate courses and in the early years of graduate study.

The foreward emphasizes that a modern treatise on function theory must take account of the abstract and postulational approach to mathematics. The author integrates the theory of analytic functions with modern analysis as a whole, stressing the structural and algebraic aspects of the theory as well as the topological concepts involved.

There are nine chapters in Volume 1 with material presented as follows: After a preliminary study of number systems, the geometry of the complex plane is developed, and simple functions such as linear fractions, powers, and roots are studied. The main theory begins in Chapter 4 with the definition of holomorphic functions, the Cauchy-Riemann equations, inverse functions, and the elements of conformal mapping. This is followed by a chapter on power series and one on the elementary transcendental functions. The systematic study of holomorphic functions occupies the last three chapters, devoted to complex integration, representation theorems, and the calculus of residues. Supplementary material on point sets, polygons, and Riemann and Riemann-Stieltjes integration is to be found in three Appendixes. There is a brief Bibliography at the end of the volume, and suggestions for collateral reading are appended to the various chapters.

While the only prerequisite suggested is a good course in advanced calculus and familiarity with abstract mathematical reasoning, the text is clearly designed for the student who will continue in mathematics as opposed to one "taking a course in complex vari-

ables." But it is essentially a student text with instructive exercises following each section—some being very straight-forward, others extending the theory.

This volume is preparatory for a second, which is due to follow fairly soon, to provide additional material at a higher level—for example, the excellent chapter on power series concludes with a mention of analytic continuation and examples of series with a natural boundary, leaving closer study to Volume II.

Hille's presentation is precise and elegant and his book certainly suggests that this series of texts will offer a real contribution to the teaching of modern mathematics.

—T. ROBERTSON
Occidental College

Rings of Continuous Functions, Leonard Gillman and Meyer Jerison, D. Van Nostrand Company, Inc., Princeton, New Jersey, 1960, 300 pp., \$8.75.

This is one of the University Series in Higher Mathematics, and has the same general format as the earlier books in the series. The authors have avoided the twin perils of a rigid theorem-proof structure, and a loose discussion structure in which the reader never knows what has been proven. While theorems are stated formally, set off in italics, and followed by formal proofs, the theorems and proofs are interspersed with many examples and much discussion material, telling the reader what the authors are doing, and what it all means. Care is taken to distinguish between theorems and proofs, and the less formal discussion, so that the reader is in no danger of becoming confused as to what has been proved and what has been discussed.

The book is a study of the ring $C(X)$ of all continuous real-valued functions on a topological space X , and the ring $C^*(X)$ of all bounded continuous real-valued functions on X . In particular, the relationships between the algebraic properties of $C(X)$ and $C^*(X)$ on the one hand, and the topological properties of X , on the other hand, are of major concern.

The book falls roughly into three parts. The first, Chapters 1 through 5, covers the fundamentals of the theory. The second, chapters 6 through 11, introduces the Stone-Cech compactification and its applications to the theory. The third part consists of somewhat independent chapters on Discrete Spaces and Nonmeasurable Car-

dinals, Hyper-Real Residue Class Fields, Prime Ideals, Uniform Spaces, and Dimension.

At the end of each chapter are numerous problems, grouped by subject matter, which serve to further illuminate the material. At the end of the book there is a section on the historical development of the subject, with references.

—E. R. DEAL

Colorado State University

Ordinary Differential Equations and Their Solutions, George M. Murphy, D. Van Nostrand Company, Inc., Princeton, New Jersey 1960, 451 pp., \$8.50.

This book is, as the author indicates in the preface and introduction, a source book for any person who has occasion to solve ordinary differential equations. The bibliography of more than one hundred fifty different books is adequate indication of considerable research by the author as he was preparing the manuscript.

The first part of the book gives a complete, clear and detailed method of solution for each type of ordinary differential equation. As a reader can discover by examining the table of contents, each general classification of equation is broken down into subdivisions; e.g., the equation of first order and first degree has thirteen subtopics, the equation of first order and higher degree has eleven, the linear equation of second order has twelve, nonlinear equation of second order has eleven, the linear equation of order greater than two has nine, and the nonlinear equation of order greater than two has five.

The second part of the book is a classified list of ordinary differential equations and their solutions or "some directions as to a method". Usually the method so indicated is the shortest or simplest of several that could be used. The list of more than two thousand equations is systematically selected to include the types and methods of solution described in Part I. Each equation is carefully cross referenced to the proper paragraph or paragraphs of part I, so that a reader can quickly find the necessary explanation for a solution. The equations are kept as general as possible by use of literal coefficients.

An index of equations lists the equations of part II by number with reference to their type as explained in part I. The index of terms and topics lists pages on which such items are found.

The book is one which should be found in technical libraries, Chemistry, Physics, Engineering, and Mathematics departments.

With regard to misprints my sympathy is always with the au-

thor and publisher of a book. This fact was especially true as I read the preface and found, "No one knows better than I the annoyance *than* can result from a misprint".

—O. E. ETTER

Fort Hays Kansas State College

Basic Concepts of Elementary Mathematics, William L. Schaaf, John Wiley & Sons, Inc., (440 Fourth Avenue) New York 16, 1960, 386 pp., \$5.50.

The announced purpose of this book is to provide a content course in the mathematics that an elementary or junior high school teacher of arithmetic should understand for an adequate background and is designed to replace the traditional arithmetic methods course usually taken by elementary teachers. It stresses some important aspects of modern mathematics such as the logical structure of mathematical systems, the nature of number, basic set theory, and the nature of proof in algebra and geometry which should be understood by elementary teachers. Some notion of the content of the book is suggested by the topics discussed. These include: the nature of mathematics and the axiomatic point of view, sets and relations, elementary logic and deductive reasoning, demonstrative, modern, and analytical geometry, the number concept, axiomatic development of number systems from the natural numbers to the complex number system, scales of notation, logarithms, measurement and mensuration, trigonometry of the right triangle, functions and their graphs, simple and compound interest, and simple probability and insurance. A thorough grasp of this material will impart a more substantial knowledge of mathematics than that possessed by the typical elementary teacher of arithmetic.

The range of the material demands that the topics be covered somewhat superficially with emphasis on the broad aspects. The author strives diligently to explain clearly but in his attempt to achieve lucidity sometimes falls into the trap of mathematical looseness, lack of rigorous statement, and inaccuracies. For example, in treating fractional exponents, $a^{1/n}$ is defined with no warning concerning the case with a negative and n even. Again, finite sets are said to be always ordered when "can be ordered" is meant. The $\sqrt{+9}$ is given as $+3$ and -3 contrary to the meaning usually assigned this symbol. Mathematical induction is said to mean "if whatever holds true for any n also holds true for $n + 1$, then it holds for all n " without pointing out the importance of verification for $n = 1$.

Some will find objections to the book because of these lapses

but the reviewer feels that considering the audience to which the text is directed, it makes a very useful contribution to the effort to up-grade the teaching of mathematics at the elementary level. There is a need to strengthen the background of the elementary teacher so as to include some understanding of the modern concepts of mathematics. This text makes a worthy contribution toward this end.

In format, readability, diagrams, and quality of paper, the text is excellent. I have no doubt that we will see more texts in the same vein.

—LESTER R. VANDEVENTER
Eastern Illinois University

Intermediate Algebra, Roy Dubisch, Vernon E. Howes and Steven J. Bryant, John Wiley and Sons, Inc., (440 Fourth Avenue) New York 16, 1960, 286 pp., \$4.50.

This book is an intermediate algebra (secondary level) presenting the traditional topics in a traditional manner. However, an effort has been made to stress the axiomatic nature of algebra at opportune places throughout the book and more so in the latter chapters.

In discussing the traditional rules, care has been taken to distinguish between those that are definitions and those that are theorems. Although all the theorems are not proven, the basic principles are usually indicated. The presentation in the chapter on exponents is much better than the usual traditional treatment. The symbols are carefully defined and absolute value notation used. Here again the role of the definition is stressed.

Judging by the aim of the authors as set forth in the preface that they plan to emphasize the "how to do it" aspect of the subject, to "tie it in as closely as possible with the rest of mathematics", and not to be completely rigorous in their approach to the subject but make it clear to the student that "proofs are just as much a feature of algebra as they are of geometry", I would say that they have been successful.

—RUTH ERCKMANN
Iowa State Teachers College

Modern Trigonometry, Dick Wick Hall and Louis O. Kattsoff, John Wiley & Sons, Inc., (440 Fourth Avenue) New York 16, 1961, x + 236 pp., \$4.95.

This text has a good analytical approach to precise definitions and the study of trigonometric functions. The rectangular coordinate

system is introduced on page 10 and the polar coordinate system on page 34. After work on functional ideas and notation, the trigonometric functions are introduced on page 47. Since radians are introduced on page 29, the convenient tools of radian measure, rectangular coordinates, and polar coordinates are not only available, but nicely and consistently used along with the traditional sexagesimal measure of angles for the trigonometry and the study of trigonometric functions throughout the remainder of the book.

The solution of triangles is not ignored, but subordinated and used largely for the application of trigonometric methods. Chapter VIII develops the common logarithms for computational purposes. A four-place common logarithm table is included and all problems involve computations to four figures or less.

It is pleasing to note that the book contains a treatment of significant digits, rounding off numbers, and calculations using approximation. A short chapter is devoted to "Linear Interpolation." The graphs of the trigonometric functions are treated neatly and extensively early in the book, starting on page 78.

Trigonometric identities and equations are naturally introduced quite late in the textbook. These are very adequately covered and solutions of equations are requested to be in both radians and degrees. The last chapter deals with complex numbers, laying a good foundation for the formulas and use of these.

The textbook has answers for the odd-numbered problems, and contains tables of powers and roots, common logarithms, four-place values for the six trigonometric functions in 10' intervals, four-place logarithms for the same trigonometric functions, degrees to radians to three places, and radians to two decimal places to degrees and minutes.

As the authors state in their Preface, this book is both elementary and modern, analytic in its approach, and usable at both senior high school and college levels.

—CHARLES B. TUCKER

Kansas State Teachers College, Emporia

Classics in Science, (A course of selected reading by Authorities), "Introduction" by E. N. Da C. Andrade, Philosophical Library (15 East 40th Street) New York, 1960, 322 pp., \$6.00.

A "Classic" is a writing of the first rank, or of the first class. It is a writing of acknowledged excellence or which has authoritative standing among the representatives of a given body of knowledge.

This volume is a collection of selected readings drawn from authorities in the various sciences, and is devoted to the major questions concerning the origin and meaning of science, the universe, matter and energy, and the pressing questions of science and everyday life. Many of the selections included occupy the esteemed position of the "classic" in the body of scientific literature.

In the "Introductory Reading Guide" Dr. Andrade states that science "is the body of observed facts of Nature, and. . . the test of the truth of a deduction is its agreement with experiment or observation." This general theme is discussed carefully in relation to the special quests of the biological and the physical sciences and their special relationships to the concept of human welfare. This discussion ends on a proper note of admonition to the serious student who would understand the classic writings in his field in his quest for proficiency: "Let us see to it that those who stand outside both the scientific and moral discipline do not direct the high pursuit of knowledge, which, apart from its loftier aspects, is fraught with untold material benefit for mankind, to baleful and pernicious ends."

The "Origin and Meaning of Science" is discussed in writings contributed to the scientific search by a glittering array of ancient and modern representatives: Aristotle, Kant, Geikie, Pearson, William Peddie, and Andrade. The nature of the universe in which man lives, moves, and has his being is the theme of selections representing Pythagoras (as interpreted by Ovid), Aristotle, Ptolemy, Copernicus, Galileo, Newton, Lyell, von Helmholtz, Ball, and Eddington. Representatives of the great tradition of experimentation on the nature of Matter and Energy include: Lavoisier, Farrady, Clerk Maxwell, Dalton, J. J. Thomas, Rutherford, F. W. Aston, and Schrodinger. Science and everyday life is discussed in the selections drawn from the writings of Marconi, John Russell, E. F. Armstrong, Andrade, and Edward Appleton.

A specific criterion for the selections of readings was their success in speaking to certain leading questions marking the progress of the sciences, e.g., "How and where did science begin?" "What gives science its power in the modern world?" "How did relativity modify Newton's system?" "What mental rewards does the study of science provide?"

The first of these questions is discussed in a selection drawn from the writings of Sir John Myers, who stated that science began, as poetry began, "from the need of explanation." This whole process of "explaining" the meanings of facts and their relationships has

been expressed through the use of hypotheses, or through the exposition of the "underpinning" of these facts. This explanation has always involved "filling in, beneath and behind the facts, whatever is conceived in imagination as really going on, and presenting these appearances to the observer."

This book of readings is commended to the reading of the students of mathematics in particular. So much of the classic and crucial experimentation in the various histories of the sciences has involved the use of the exact science of mathematics that it would be paradoxical for the mathematics student not to be very familiar with these historically-significant applications of his subject-matter. A book of carefully selected readings, particularly the "classic" readings, always presents a most stimulating and formidable encounter with past moments of greatness. This point might be true most of all with respect to the scientific classics. Whether this be true or not, a student undeniably will grow toward greater proficiency in his field by encountering those high-water marks of past intellectual achievements contributed to his field of study by renowned predecessors.

—SHERMAN M. STANAGE
Bowling Green State University

A Structure of Science, Joseph H. Simons, Philosophical Library
(15 East 40th Street) New York, 1960, 269 pages, \$4.75.

This little volume is a student's introduction to the meaning, practices, and applications of science, a discussion of some of the fundamental concepts of science (e.g., matter, orderliness, and change), and of special ideas (e.g., the collisions of the primary particles of matter) which have occupied the author's attention over a period of years. It is particularly interesting to the student as a kind of autobiographical logbook of the experimentalist who has also been intrigued by the philosophical aspects of his work and who has considerable experience in transmitting these interests to students in general education courses in the sciences.

As an introductory discussion the book has some features to commend itself to an audience of undergraduate mathematics students. The style is readable and the explanations and expositions are lucid. Perhaps its strongest offering is an acceptable definition of science which could serve as a tentative, working definition for a more careful study of the sciences. The author states that "science concerns itself with relationships between facts, the dependence of one fact upon other facts, the systematic arrangement, reorganiza-

tion of facts, and the underlying principles governing the relationships between facts." A "fact" is defined as "a respectable observation," or an observation which has been determined by "a significant number of observers competent to make the observation."

The importance of the term "respectable," and its obvious implications for the definitions of science, fact, and observation, clearly stipulates that the knowledge acquired by science makes no claim to certainty. The definition of science is also ably used by the author as a means of cutting through some of the common—but fallacious—hasty and ill-conceived identifications of "science" with something else, e.g., "intelligence," "skillful," "complicated," even "logic." Sciences are special ways of specifying the interrelationships of certain kinds of facts, and the sciences cannot be reduced to mere use of logic, or a mere attempt to achieve a complex precision in miscellaneous studies.

The attempt to simplify for the introductory student has led the author to make certain awkward, ambiguous, and unnecessary statements: "Art is concerned with doing and science with knowing" (p. 15). "Science is . . . passive while art is active" (p. 15). "Science has little respect for tradition as such" (p. 17). "There can be no serious conflict between science and religion, as there are no places of overlapping of the two areas" (p. 19). Serious students would find it necessary to test these statements carefully, for in the context of the book they promise far more than they perform as serious interpretations.

Part II of the volume discusses some of the fundamental concepts upon which all science seems to be based. This discussion considerably expands an earlier discussion of the "creed" of science. The author maintains that these concepts are more or less intuitive and seldom questioned. The list is impressive, but embarrassedly incomplete: Matter, Force, Inertia, Potential, Orderliness, Conservation, Equilibrium, Chance, Discreteness, Symmetry, and Change. The precise criteria dictating the choice of just these concepts are not disclosed, and this omission undoubtedly marks a weakness in an introductory volume on this subject. An additional weakness would be the extreme brevity of the discussion of each concept.

Part III of the book is a discussion of ideas which "have been simmering in the mind of the author for many years," e.g., the collisions of primary particles of matter. This theme is the subject of most of the eight chapters in this part of the book.

Finally, it must be pointed out that this volume is weakened in that it contains neither an index of subjects treated nor a bibliography of works related to the study, and the reasons offered for this omission are surprisingly inept for a mature scientist writing for a student audience, or for any audience, for that matter. A student, above all other readers, once interested in a serious subject, should be assisted to read further in a manner accelerated by a carefully prepared index and by a bibliography pointing in the direction of more exacting readings.

—SHERMAN M. STANAGE
Bowling Green State University

Aerospace Dictionary, Frank Gaynor, Philosophical Library Inc., (15 East 40th Street), New York 16, 1960, 260 pp., \$6.00.

This is a well written dictionary, quite up-to-date in the terms presented. Its division into twenty-six chapters: A, B, C, etc. is a desirable feature. An especially good feature is the large clear type with each term set out in heavy dark type. This makes each term easily located.

The book seems to be remarkably free of errors although furfuryl alcohol is misspelled on page 97. The formula for this compound is slightly incorrect too. A typographical error was also noticed in the paragraph describing hyperbolic guidance on page 115.

The term JP4 does not appear in the book although it would seem to belong there.

—JOSEPH E. WEBER
Bowling Green State University

Dictionary of Aeronautical Engineering, J. L. Nayler, Philosophical Library Inc., (15 East 40th Street), New York 16, 1959, 318 pp., \$10.00.

The book seems to be quite complete and yet is of such a size as to fit into ones pocket quite easily. It is up-to-date including such current discoveries as the Van Allen Radiation Belts, etc. However, the term (q.v.), which is used irregularly along with "see", might send a student to another dictionary unnecessarily.

—JOSEPH E. WEBER
Bowling Green State University

Kappa Mu Epsilon News

EDITED BY FRANK C. GENTRY, HISTORIAN

Colorado Alpha, Colorado State University, Fort Collins.

We have initiated 42 new members this spring which brings our total membership to 395. Mr. Walter Butler, the National Treasurer, and 3 student members plan to attend the National Convention this spring.

Illinois Gamma, Chicago Teachers College, Chicago.

Six of our members attended the Regional Conference at Illinois State Normal University last spring. Several papers were presented by our delegates.

Indiana Gamma, Anderson College, Anderson.

Our chapter initiated eleven new members this year. Dr. C. W. Curtis, M. A. A. Visiting Lecturer from the University of Wisconsin was on our campus for lectures on "What is Mathematics?" and "Group Theory and the Concept of Symmetry." Officers for the year are: Evelyn Ward Bowen, President; Ruben Schwieger, Vice-President; Owen Kardatzke, Recorder-Treasurer; and Professor Gloria Olive, Corresponding Secretary.

Iowa Alpha, Iowa State Teachers College, Cedar Falls.

We initiated 6 new members last year. In addition to programs given by members of our own chapter, we had addresses by Dr. W. T. Reid, Professor of Mathematics, State University of Iowa, Dr. David Blackwell, Visiting Lecturer for the Mathematical Association of America, and Dr. H. S. M. Coxeter of the University of Toronto. We expect a visit from Dr. W. Maak of Gottingen University this spring.

Kansas Alpha, Kansas State College of Pittsburg, Pittsburg.

Among the lecture topics we have had this year are "Laplace's Expansion Theorem in Determinant Theory", "Applications of Mathematics in the Field of Astronomy", and "Mathematics Taught in the Public Schools and Colleges of China".

Kansas Beta, Kansas State Teachers College, Emporia.

We now have a total membership of 566 of whom 69 are active and on campus this year. We have initiated 40 new members this year. We have had Dr. George Springer of Kansas University as

guest speaker, His topic was "Axiom of Choice." Most of the energies of our chapter have been directed this year toward plans for the National Convention for which we will be hosts in April. We are looking forward to a most pleasant week-end with many guests from other chapters.

Kansas Gamma, Mount St. Scholastica College, Atchison.

Approximately 30 of our members and pledges are planning to attend the National Convention at Emporia this spring. St. Benedicts and Mount St. Scholastica sponsored a 3-day conference dealing with space and related problems in January. Special guest was Dr. Arthur L. Quirk, Chairman of the Department of Physics at the University of Rhode Island. He spoke on the "Conquest of Space" and the "Nuclear Reactor—an Aid to Research." Sister Helen Sullivan, Chairman of our Department of Mathematics, was a visiting lecturer at St. Xaviers College in Chicago in December. She spoke on "An Orientation to the Various Types of Geometry" and "The Use of Involution to Effect the Geometric Number Systems." She also appeared on the St. Benedicts College lecture series in February when her topic was "Some Liberal Aspects of Modern Mathematics."

Kansas Epsilon, Fort Hays Kansas State College, Hays.

Since our chapter was organized in 1952, we have had representation at each National Convention and at all except one regional convention. We plan to keep our record by sending a delegation to Emporia, Kansas, in April.

Michigan Alpha, Albion College, Albion.

Dr. C. F. Brumfiel of the University of Michigan will speak to a combined meeting of Kappa Mu Epsilon and the Albion chapter of Mu Alpha Theta on his work in algebra and geometry while at Ball State Teachers College this spring. One of our members is submitting a paper for possible presentation at the National Convention. We expect to have several of our members attend.

Mississippi Gamma, Mississippi Southern College, Hattiesburg.

Dr. Charles Curtis of the University of Wisconsin was guest speaker at our annual banquet in February.

Missouri Alpha, Southwest Missouri State College, Springfield.

In addition to papers prepared by local members, we had as a special feature this year a program entitled "Voices Across Time and Space" presented by the Bell Telephone Laboratories.

New Mexico Alpha, University of New Mexico, Albuquerque.

In March our chapter sponsored a visit to Sandia Corporation for all interested students on the campus. After viewing a film on research and development in the field of nuclear weapons, the group browsed around the exhibits in the Sphere of Science. We are planning a session in the future on problem solving in which the group will submit their problems for solving.

Ohio Alpha, Bowling Green State University, Bowling Green.

Our members assist students in freshman and sophomore classes in mathematics by conducting regularly scheduled help sessions. Our chapter mailed out an Alumni Directory of the Kappa Mu Epsilon members this year. The interest in this project seemed quite good and the response was gratifying. Our President of last year, Douglas Cornell, received a National Science Foundation fellowship at Yale University this year and Elizabeth Moorhead, last year's Vice President is in Graduate School at Columbia University. David Weisgerber and Marilyn Pile are at the University of Illinois and Indiana University respectively.

Ohio Gamma, Baldwin-Wallace College, Berea.

The month of February was set aside this year as "Science Emphasis Month" and our chapter sponsored several assemblies and lectures. Among them was one on "Linear Programming" and another on "Mathematical Foundations of Measurement Theory" by Prof. Robert M. Thrall of the University of Michigan. One of our programs this year was a talk by Dave Kaiser, a Baldwin-Wallace graduate who is now teaching mathematics at Parma Senior High School. We also had Dr. Jason Nassau of Case Institute of Technology as a guest speaker. His subject was "The Structure of our Galaxy."

Ohio Epsilon, Marietta College, Marietta.

As the youngest chapter of Kappa Mu Epsilon our big news is the story of the installation of our chapter which appeared in the Fall 1960 issue of THE PENTAGON.

Oklahoma Alpha, Northeastern State College, Tahlequah.

We have had 10 meetings so far this year, most of the programs having been presented by student members.

(Continued on page 124)

Installation of New Chapters

THE PENTAGON is pleased to report the installation of a new chapter.

ALABAMA DELTA CHAPTER

Howard College, Birmingham, Alabama

Alabama Delta Chapter was installed on February 10, 1961 by Alabama Beta Chapter from Florence State College. Miss Orpha Ann Culmer, former National Historian, was the installing officer. Mrs. Mary Hudson, Mr. Henry Johnson, and Mr. James W. Hooper from Alabama Beta assisted with the ritual. Miss Christine Kinnear, Miss Martha Kimbrell, Mrs. Bonnie Sturgis, and Mr. Larry Montgomery also represented Alabama Beta at the installation. Mr. Sanders Bishop from Oklahoma Alpha, now on the faculty at Howard was the conductor. The installation was held after a banquet at the Hitching Post.

Charter members are: Jane Brummett, Dexter H. Burdeshaw, Linda Cospers, Delila Daniel, Howell Glenn, Peggy Guffin, James Hart, Fred Massey, Shirley McGuff, Henry Minshew, Barbara Money, Jerry Mooney, Azalia Osborn, Jean Peacock, Mae Lynn Todd, Jo Ann Turner, Laura Weaver; and from the faculty: Mr. Ben Chastain, Mrs. Mary Hudson, Mr. Richard Morris, Dr. W. D. Peebles, Jr., Dr. R. E. Wheeler, and Mrs. Lettia Yeager.

The new chapter's officers are: James Hart, president; Mae Lynn Todd, vice-president; Shirley McGuff, secretary; Laura Weaver, treasurer; Dr. Ruric Wheeler, corresponding secretary and Dr. W. D. Peebles, Jr., faculty advisor.

The Howard College Mathematics Club from which charter members came was organized in 1954. Since that time it has maintained an average membership of approximately 25 students. The purpose of the Mathematics Club is to foster an interest in mathematics by acquainting students with the different fields in which mathematics is used; to provide meetings whereby the needs and wants of students interested in mathematics can best be met; and to unite mathematics students so as to develop a spirit of mutual helpfulness.

Howard College was founded at Marion, Alabama in 1842. It was moved from Marion to Birmingham in 1887 and has only recently moved to a new campus in Birmingham. Eleven buildings are

completed and construction is still continuing. The college grants four degrees: the Bachelor of Arts, the Bachelor of Science, the Bachelor of Music, and the Bachelor of Science in Pharmacy. Of the 2000 students enrolled at Howard College, approximately 60 are majoring in mathematics.

We are happy to welcome this new chapter and to wish them well.

(Continued from page 122)

Pennsylvania Beta, La Salle College, Philadelphia.

Among the lecture topics for 1960-61 are the following: "Abstract Vector Spaces and Linear Operators," "LaPlace Transforms," "Elementary Ideal Theory," "Difference Equations," "Inverse Laplace Transforms of Rational Functions," "Eigenvalues," and "Foundations of Projective Geometries." Our chapter visited the Philco Computer Center at Willow Grove and listened to the lecture on the Philco 2000 and a problem in linear programming.

Pennsylvania Gamma, Waynesburg College, Waynesburg.

Professor Clyde Martz of the Physics Department spoke to the chapter on "Nuclear Reactors." We made a trip to the Rockwell Meter Plant in Uniontown, Pennsylvania.

Tennessee Beta, East Tennessee State College, Johnson City.

We initiated 21 new members in January to make a total of 67 since our installation in May 1959. We sponsored a lecture by Dr. Avron Douglass of the University of Maryland who spoke on "Cultural Aspects of Mathematics." He visited us under the sponsorship of the Society for Industrial and Applied Mathematics. We plan to assist the Mathematics Department in administering the State Mathematics Contest for High School Students. We regularly present an award to the outstanding graduating senior who is a mathematics major on Honors Day.

Virginia Alpha, Virginia State College, Petersburg.

We expect Dr. Begle of Yale University to visit our chapter in March. Dean John M. Hunter, Dr. Reuben R. McDaniel, Dr. T. N. Baker, Mr. Joseph H. Trotter, Mrs. Emma B. Smith, and Louise S. Hunter have worked with the Virginia State sponsored Mathematics and Science Institutes and the National Science Foundation Summer Institutes during the past year.