

# THE PENTAGON

Volume XV

FALL, 1955

Number 1

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## National Officers

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Kappa Mu Epsilon, national honorary mathematics society, was founded in 1931. The object of the fraternity is fourfold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievements in the study of mathematics at the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

## Editor's Page

This issue of THE PENTAGON is devoted largely to the publication of papers presented by students at the Tenth Biennial Convention of Kappa Mu Epsilon which was held May 6-7, 1955 on the campus of Nebraska State Teachers College, Wayne, Nebraska. A report on the convention is also included.

The Editor hopes that you will find the reading of these papers and reports both a pleasurable and profitable experience. You may be inspired to write a paper on some topic not ordinarily covered in mathematics courses for undergraduates yet within the bounds of understanding of undergraduate students. Perhaps you have already written such a paper and have not given consideration to the possibility of having it published. In either case, the Editor of THE PENTAGON would like for you to put your manuscript in proper form and present it for consideration for publication. For directions in preparing your paper see "Instructions for Preparation of Manuscripts", THE PENTAGON, 14:126, Spring, 1955.

A recommendation from the students attending the Tenth Biennial Convention was to the effect that THE PENTAGON should be made into a quarterly magazine. Until the number of contributors increases this will be impossible. The change will probably be made if there is a great increase in the number of worthwhile manuscripts presented for consideration.

=△=

Kappa Mu Epsilon was founded at Tahlequah, Oklahoma in 1931. Thus the organization will soon be celebrating it's twenty-fifth birthday.

It has been suggested that the Spring, 1956, number of THE PENTAGON might well be a silver anniversary issue. What do you think of this idea? What should be included in such an issue. Do you have a contribution for such an issue? The Editor would not only welcome suggestions but pleads for your assistance.

—C. V. F.

# Taxicab Geometry\*

MORRIS ROSEN  
*Student, Hofstra College*

In plane analytic geometry, much time is spent in studying the conic sections, the line, circle, ellipse, parabola, and hyperbola. We will define a new geometry called "Taxicab Geometry" and investigate what happens to the classical conic sections in this new geometry. We will find that they are vastly different.

Consider a city laid out in square blocks. A map of this city could be an ordinary sheet of coordinate paper. If we start at an arbitrary corner and travel by taxicab along these streets to a point three blocks over and four blocks up, we must ride a total of seven blocks and incidentally pay for that length of trip. We can imagine what the taxidriver would say if we tried to pay him for a five block trip since that was the Euclidean distance between the two points, *i.e.*, the hypotenuse of the 3-4-5 right triangle.

Specifically, we idealize the real streets as follows:

- (1) We assume that the set of streets is in a plane.
- (2) We assume that the streets have no width, *i.e.*, that they can be represented by straight lines.
- (3) We let the streets be represented by the grid of a Cartesian system where a point will have at least one integral coordinate if we take as unit the length of one block.

We define a taxi line between two points (on streets) as the shortest distance (along streets) between the two points.

It is immediately obvious that the straight line is not always unique. Thus the postulate that there is one and only one straight line between two points does not hold in taxicab geometry.

Let us investigate the number of straight lines between two points located at corners, say  $P_1$ ,  $P_2$ . Let us represent a horizontal move of one block by  $h$ , a vertical move of one block by  $v$ . Then if we write down a sequence of  $v$ 's and  $h$ 's in any order, a unique path between  $P_1$  and  $P_2$  will be determined. For example  $hhvv$  can only mean two horizontal moves followed by two vertical moves. If the two points are separated by  $n$  horizontal blocks,  $k$  vertical blocks, then the number of taxi lines between  $P_1$  and  $P_2$  will be given by the permutations of  $n + k$  things of which  $n$  are alike,  $k$  are alike.

$$N = (n + k)! / (n!k!)$$

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\* A paper presented at the 1955 National Convention of K.M.E. and awarded first place by the Awards Committee.

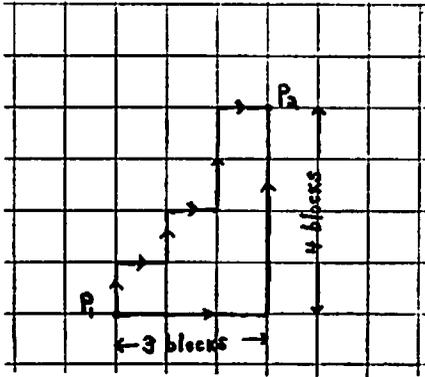


FIGURE 1. TAXI STRAIGHT LINES.

Let us now define our intuitive notion of a taxi distance analytically. Let the two points be given as  $P_1 (x_1, y_1)$ ,  $P_2 (x_2, y_2)$  where at least one coordinate will be an integer. Then, with two exceptions, the distance between  $P_1$  and  $P_2$  is given by the sum of the vertical and horizontal distances:

$$d(P_1, P_2) = |x_1 - x_2| + |y_1 - y_2|$$

To describe the exceptional cases we let  $z = [z] + z'$  where  $[z]$  denotes, for every real number  $z$ , the largest integer  $\leq z$ . If  $a$  and  $b$  are two unequal positive numbers, we define  $\min(a, b)$  to be  $a$  or  $b$ , whichever is the lesser. The exceptions are:

- (1) If  $x_1$  and  $x_2$  are different integers, and  $[y_1] = [y_2]$ ,  
 $d(P_1, P_2) = |x_1 - x_2| + \min(y_1' + y_2', 2 - y_1' - y_2')$
- (2) If  $y_1$  and  $y_2$  are different integers and  $[x_1] = [x_2]$ ,  
 $d(P_1, P_2) = |y_1 - y_2| + \min(x_1' + x_2', 2 - x_1' - x_2')$ <sup>1</sup>

We are now prepared to investigate some of the elementary loci mentioned in the beginning of this paper. If we choose an arbitrary point on a corner, it is quite easy to see by direct trial that the locus of points a distance of three blocks by taxi will lie on a square (Figure 2).

Analytically,

$$d(P, 0) = r = |x - 0| + |y - 0| \quad (x \text{ or } y \text{ integers})$$

For  $x > 0, y > 0, x + y = r$  ( $x$  or  $y$  integers)

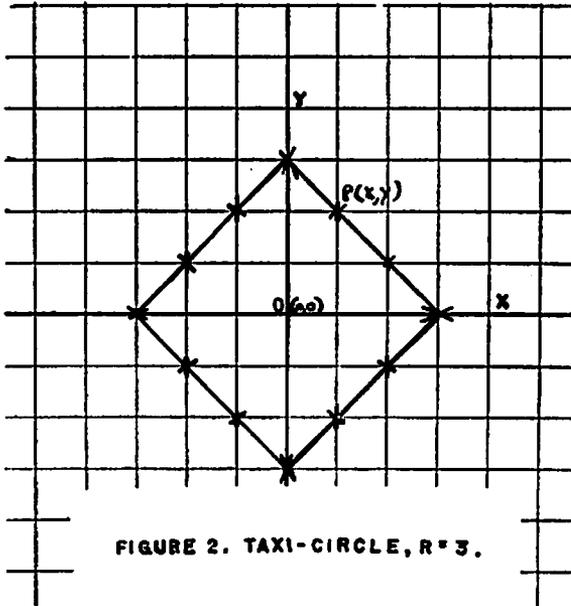
We can find the locus in the other quadrants in a similar manner. The points of the locus are the intersections of the Euclidean

<sup>1</sup> H. J. Curtis, "A Note on the Taxicab Geometry," *American Mathematical Monthly*, 60:416-417, June, 1953.

straight lines with the grid of streets.

If  $r$  is an integer, all points of the locus will be corners; if  $r$  is not an integer, no points of the locus will be corners.

It is intuitively obvious that the locus is the same if the center is taken at any arbitrary corner. In general integral translations are valid in Taxicab Geometry.



We now investigate the locus if the center is not a corner. We duplicate here parts of the proof given by H. J. Curtis in his note in the June 1953 issue of the *American Mathematical Monthly*.<sup>2</sup>

We wish to describe  $C(P,r)$ , the set of points  $r$  distant from point  $P$ , in the case  $P$  is not a corner. Then, referring to Figure 3, if  $d(P,P_1) = k$ , the set of points  $C(P,r)$  is contained in the sum of the sets  $C(P_1, r - k)$  and  $C(P_2, r + k - 1)$ . For any corner  $Q$ , we denote the interior of the square  $C(Q,s)$  as  $D(Q,s)$ . The taxi-circle of radius  $r$  about a point  $P$ , not a corner, is the intersection of the grid of streets with the octagon  $D(P_1, r - k) + D(P_2, r + k - 1)$ . This can also be shown analytically.

The ellipse can be defined as the locus of a point moving such that the sum of its distances from two fixed points, called foci, is a

<sup>2</sup> *Ibid.*, page 117.

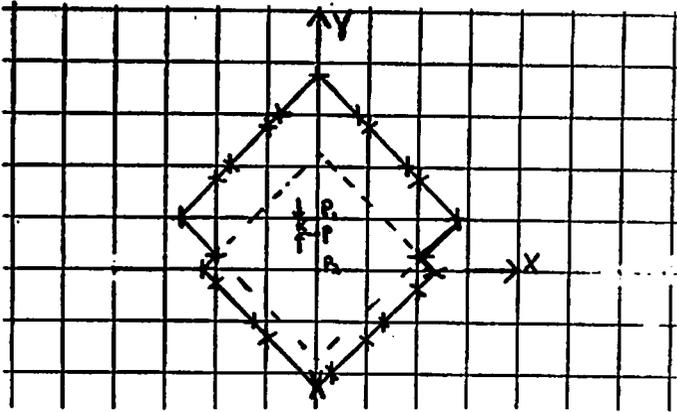


FIGURE 3. TAXI - CIRCLE,  $R=3$ , CENTER  $(0, 3/4)$

constant. The taxi-ellipse is defined similarly. Analytically if the foci are  $(a,0)$  and  $(-a, 0)$  where  $a$  is integral, the equation becomes  $|x - a| + |y| + |x + a| + |y| = c$  ( $x$  or  $y$  integers). See Figure 4.

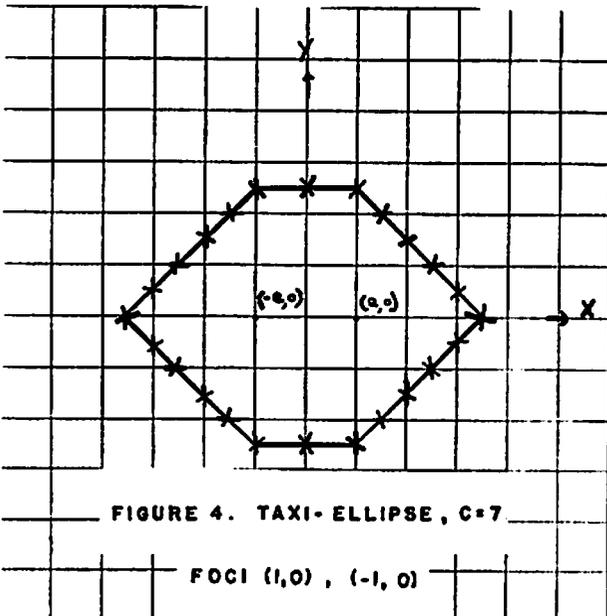


FIGURE 4. TAXI- ELLIPSE,  $C=7$

FOCI  $(1,0)$  ,  $(-1, 0)$

If we take the foci at two oblique corners, we obtain a locus similar to Figure 5. We see that the rotation of the foci leads to a different locus.

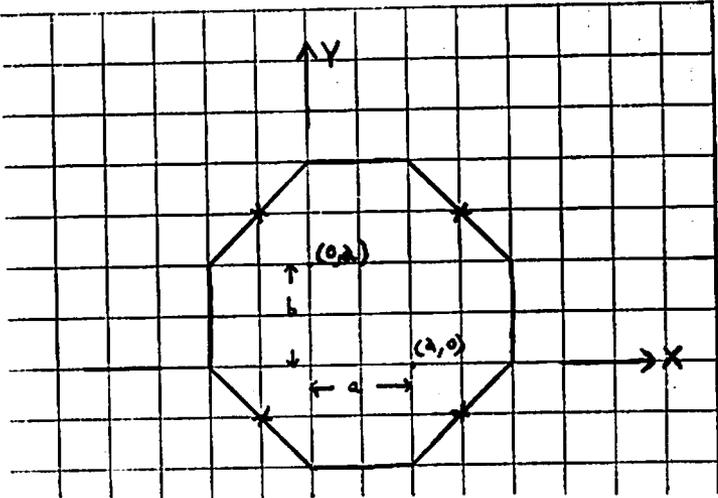


FIGURE 5. TAXI-ELLIPSE,  $C=8$ , FOCI  $(0,2)$ ,  $(2,0)$

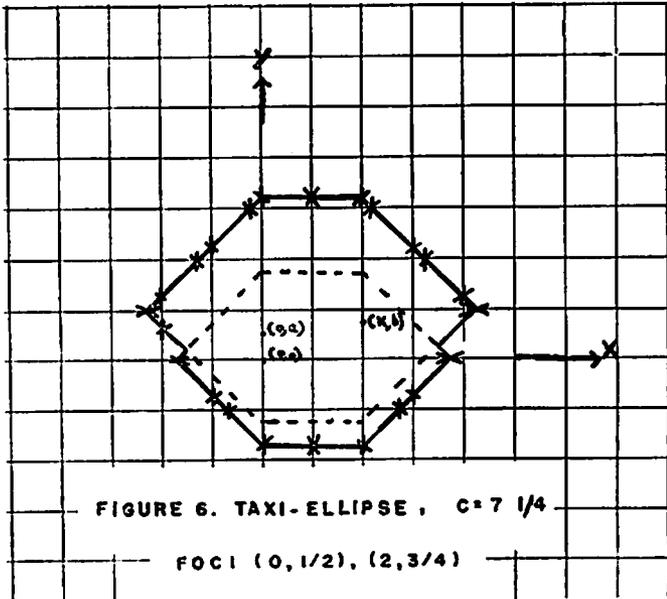
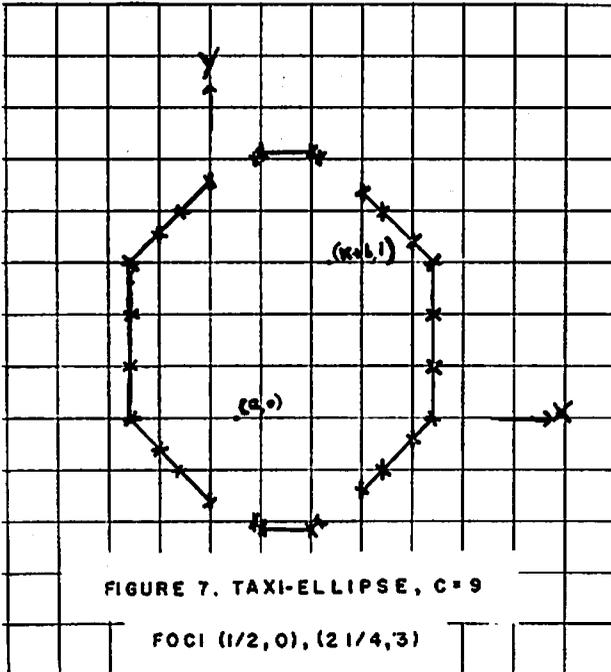


FIGURE 6. TAXI-ELLIPSE,  $C=7\frac{1}{4}$

FOCI  $(0, \frac{1}{2})$ ,  $(\frac{3}{4}, 1.5)$

As in the case of the taxi-circle, the taxi-ellipse becomes the sum of parts of two taxi-ellipses if a focus is not a corner (Figure 6). The locus in the next figure is the sum of parts of three different taxi-ellipses (Figure 7).



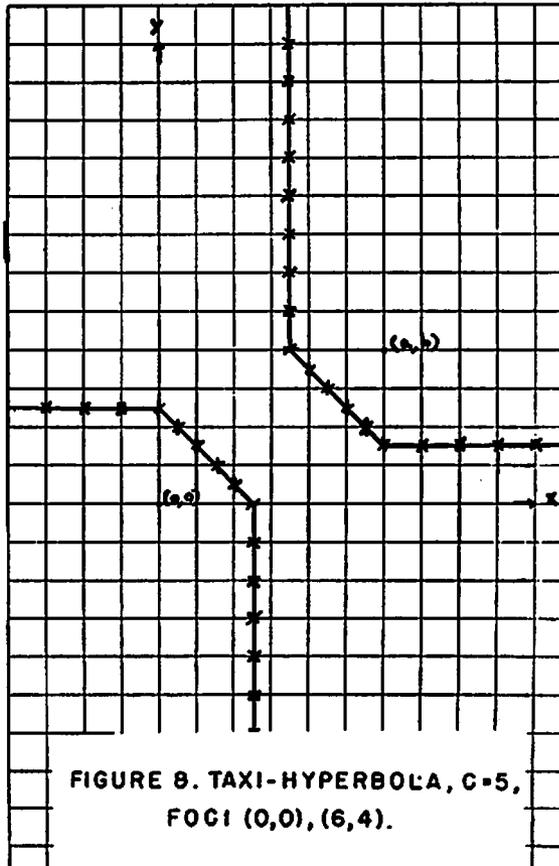
The hyperbola can be defined as the locus of a point moving such that the difference of its distances from two fixed points is a constant. The taxi-hyperbola is defined similarly. If the foci are corners on the same street, the locus becomes two vertical straight lines. If the foci lie on the corners of intersecting streets, *i.e.*, at  $(0,0)$ ,  $(a,b)$  where  $a$  and  $b$  are integers, then the locus resembles the Euclidean hyperbola more closely (Figure 8).

$$\pm c = |y| + |x| - |y - b| - |x - a| \quad (x \text{ or } y \text{ integers}).$$

If a focus is located at the point  $(a,b)$  where  $b$  is not an integer, then as in the case of the taxi-circle, we will have a combination of two or more taxi-hyperbolas.

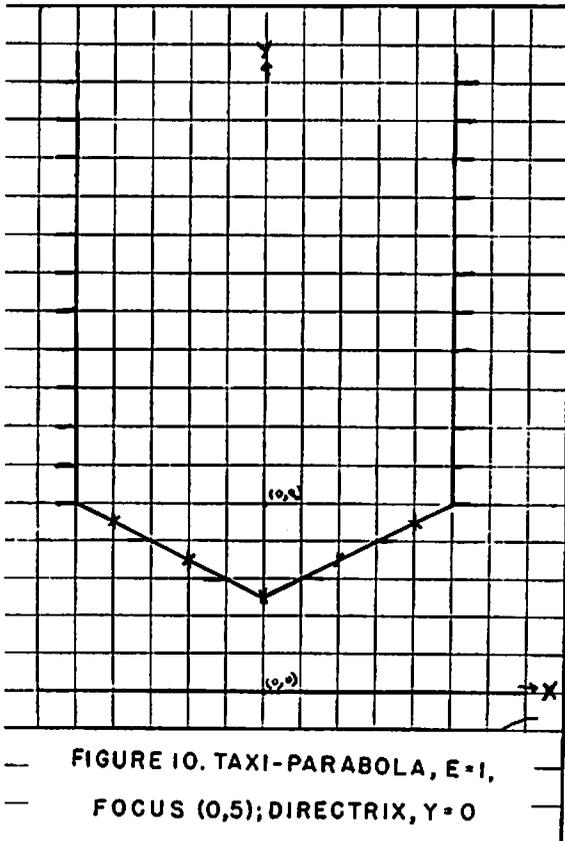
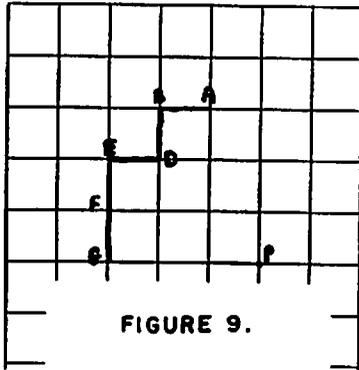
In ordinary geometry, an equivalent definition for the conic sections is as follows: The conic section is the locus of a point mov-

ing such that its distance from a fixed point, the focus, is in a constant ratio to its distance from a fixed line, the directrix. The ratio of the focal distance to the distance from the fixed line is called the eccentricity,  $e$ . If  $e < 1$ , the conic is an ellipse; if  $e = 1$ , the conic is parabola; if  $e > 1$ , the conic is a hyperbola.



If we should try to transfer the definition directly to taxicab geometry we would run into a great deal of trouble, since the taxi-line is not unique. Hence, we make the following definitions: A fixed straight line corresponds to a *specific* minimal path along streets between two points. We also define the distance between a

point and a fixed straight line as the minimal distance along streets between the point and the fixed line. In Figure 9, for example,  $PG = 3$ ,  $PF = PD = PA = 4$ ,  $PE = PB = 5$ . Hence the distance between  $P$  and fixed straight line  $ABDEFG$  is 3 blocks.

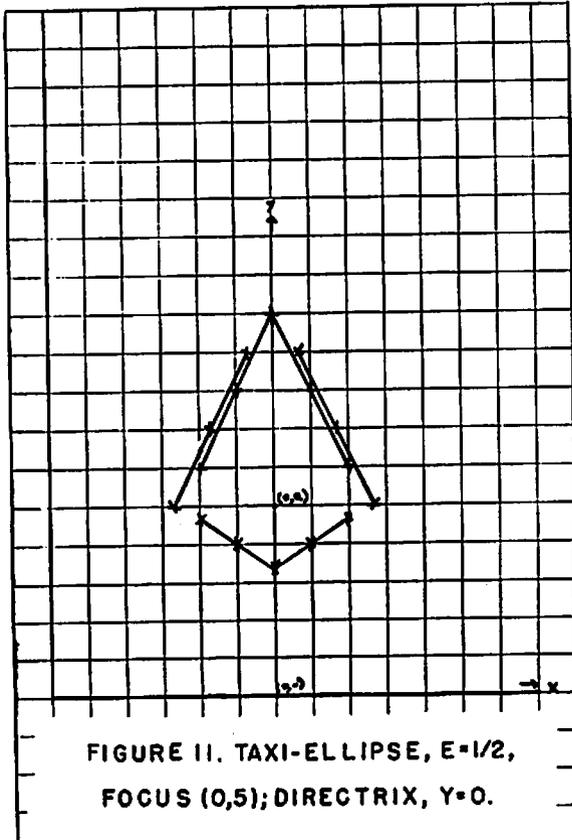


The simplest fixed line occurs when the two end points are on the same street, *i.e.*, the set of points  $(z,a)$  or  $(b,z)$  where  $z$  ranges over the set of real numbers, and  $a$  or  $b$  is any integer. The distance from a point  $P(x,y)$  to the fixed line,  $L$ , which consists of the set of points  $(z,a)$  is then:  $d(P,L) = |y - a| + \min(x', 1 - x')$  where  $x = [x] + x'$  as before.

If we take  $(0,a)$ , where  $a$  is an integer, as the focus and the fixed straightline  $y = 0$  as the directrix, the equation of the conic becomes:

$$|x| + |y - a| = e[|y| + \min(x', 1 - x')]$$

( $x$  or  $y$  integers).



This relation leads in general to much more complicated loci which do however resemble the Euclidean loci (Figures 10, 11, 12). I have found no case where the taxi-ellipse and taxi-hyperbola are equivalent to those obtained by the first definition. Variations of the loci may be found by taking the focus at positions other than a corner and taking more complicated directrices.

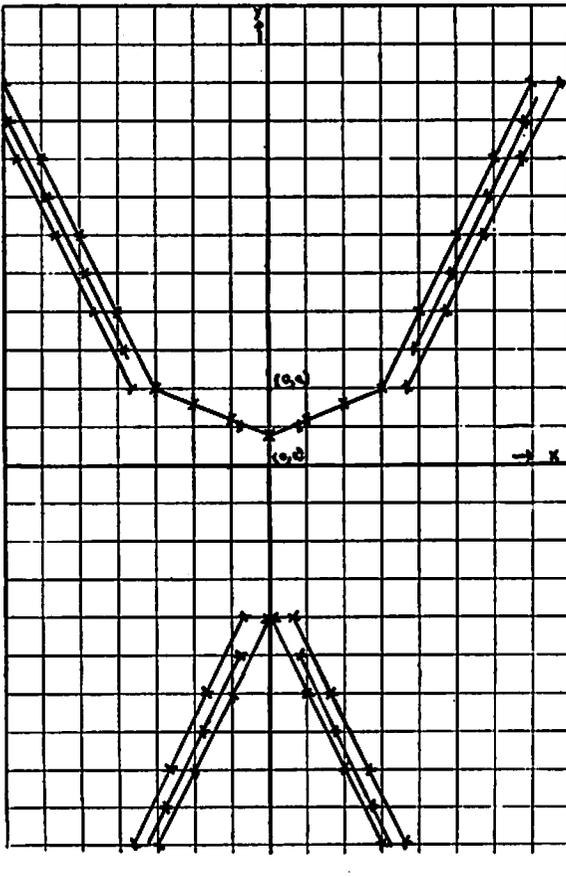


FIGURE 12. TAXI-HYPERBOLA,  $E=3/2$ ,  
 FOCUS  $(0,2)$ ; DIRECTRIX,  $Y=0$ .

This concludes a quite elementary survey of the conic sections as defined in taxicab geometry. Some of the more important aspects of this geometry are:

- 1) Integral translations are valid.
- 2) Rotations are not valid.
- 3) The straight line between two points is not unique.
- 4) Points of the taxi-circle lie on a square.
- 5) The two equivalent Euclidean definitions of the conics are not equivalent in taxi-geometry.

Some of the notions in this paper may be extended to:

- 1) Three dimensional taxi-geometry.
- 2) A system of circular streets and radial avenues.
- 3) Rectangular blocks.



“Excellence is evident in full and adequate solutions to problems; for whatsoever theorem solves the most complicated problem of the kind, does with a due reduction reach all the subordinate cases.”

—E. HALLEY

“The student who has thus far taken the system of real numbers for granted, and worked with them, may continue to do so to the end of his life without detriment to his mathematical thought. On the other hand, most mathematicians are curious, at one time or another in their lives, to see how the system of real numbers can be evolved from the natural numbers.”

—W. F. OSGOOD

# Mechanical Solution of Cubic Equations\*

JOHN COUCH

Student, Kansas State Teachers College, Emporia

It is my purpose to present a solution of cubic equations by means of a mechanical device.

I will assume that the cubic equation is in the form

$$(1) \quad x^3 - px^2 + qx - r = 0.$$

Any cubic equation can be put in this form by dividing by the coefficient of  $x^3$ . The quantities  $p$ ,  $q$ ,  $r$  may have any values, positive, negative or zero.

Let us consider the diagram below (Fig. 1) where the segment

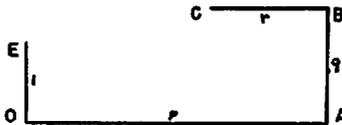


FIGURE 1.

$OE = 1$ ,  $OA = p$ ,  $AB = q$ ,  $BC = r$ . This diagram is the basis of the mechanical device. This arrangement of the segments holds if the quantities  $p$ ,  $q$ ,  $r$  are all positive. If any of the quantities  $p$ ,  $q$ ,  $r$  is negative, the corresponding segment is reversed in direction. For example if  $p > 0$ ,  $q < 0$ ,  $r < 0$ , the arrangement is as shown in Figure 2.

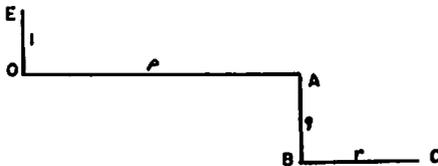


FIGURE 2.

Now let  $D$  be a point on  $OA$  and  $F$  be a point on  $AB$  such that the angles  $EDF$  and  $DFC$  are right angles.

\* A paper presented at the 1955 National Convention of K.M.E. and awarded second place by the Awards Committee.

Then the three triangles  $EOD$ ,  $DAF$ , and  $FBC$  are similar. If we denote  $OD$  by  $x$ , the remaining segments in the diagram can be easily determined by making use of the proportionality of corresponding sides of similar triangles.

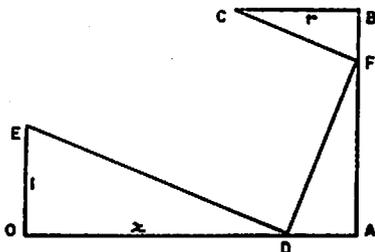


FIGURE 3.

$$DA = p - x.$$

$$AF/DA = x/1, \text{ or } AF = x(p - x).$$

$$FB = q - x(p - x) = q - px + x^2.$$

$$\text{Since } BC/FB = x/1, r/(q - px + x^2) = x/1,$$

$$r = x(q - px + x^2),$$

$$\text{and } x^3 - px^2 + qx - r = 0.$$

Therefore the value of  $x$  in the equation is the length of the line segment  $OD$ .

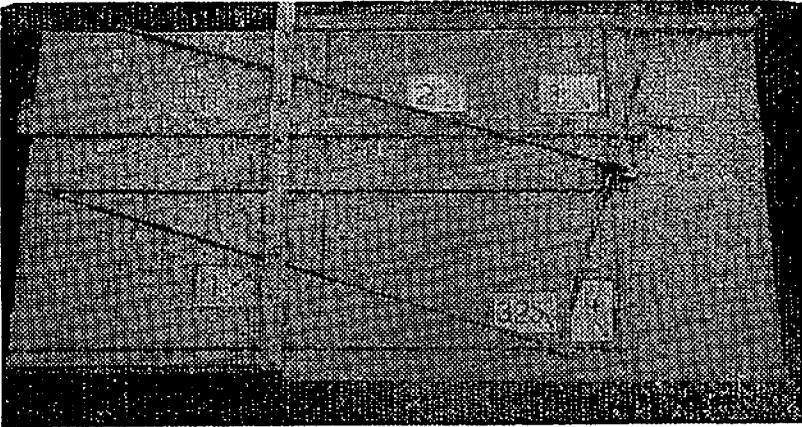
Figure 1 is used as a pattern for setting up the diagram for any equation. If a sign in a given equation differs from that of the corresponding term in Equation (1),  $p$ ,  $q$ ,  $r$  positive, the corresponding segments in Figure 1 are oppositely directed.

If, however, we have a cubic equation of the form  $x^3 + px^2 + qx + r = 0$ ,  $p, r, q > 0$ , then we may change the signs of the odd powers of  $x$  and solve in the manner described. The value of  $x$  thus obtained is the negative of the root of the given equation. That is, the equation  $x^3 + px^2 + qx + r = 0$  may be changed into the form  $x^3 - px^2 + qx - r = 0$ . The solution of the second equation will be the solution of the first except for the sign.

A mechanical device for solving cubic equations must have some way of setting up the diagrams for the various equations and also must have some sort of adjustable double square that may be fitted into the diagrams. My double square consists of two welding rods fastened permanently together so as to form a right angle and another rod that slides along one of these rods, always at a right angle to it.

In addition to the double square I have a slide which moves in a panel. Above the slide that moves back and forth, I have a screw eye mounted so that the slide moves under this screw eye. Below the screw eye at a distance of four inches there is a slot for the vertex  $D$  to slide in. The distance from the screw eye to the slot represents  $EO$ . By means of the slide,  $OA$  may be adjusted to any value. By means of a marker that moves up and down on the slide, the proper value of  $AB$  may be determined and the marker has a scale on it so that the value of  $r$  may be read. To solve an equation, I set up the required diagram, then insert one end of the welding rod in the screw eye, one vertex in the horizontal slot and a projection on the movable arm in a vertical slot and merely move the sliding arm until the proper value of  $r$  is intercepted on the marker. Now the value of  $x$  may be read directly from the scale along the horizontal slot.

The solution of the equation  $x^3 - 4x^2 + 3x^2 - 2 = 0$  by means of the device is shown in the picture below. A real root of this equation is approximately 3.25.



“These three notions, of the variable, of form, and of generality, comprise a sort of mathematical trinity which, preside over the whole subject.”

—A. N. WHITEHEAD

# Nomography\*

EDDIE DIXON  
*Student, William Jewell College*

Often in science courses a student is required to solve the same equation over and over again. Because of a similar situation occurring in industry, there developed a need for a rapid and simple method of solving these equations. The answer was nomography.

A nomogram is an arrangement of two or more scales in such a manner that the value of an unknown variable may be determined by the use of a straight edge. The scales of the nomogram may be either straight or curved, uniform or nonuniform. Also, the unknown variable may be dependent on not only one, but any number of independent variables.

Figure 1 is the nomogram for the equation  $C = \pi D$ , the well-known diameter-circumference relationship for a circle. This equation falls into the general class of  $f(u) = f(v)$ . The top scale is the diameter and the bottom scale is the circumference. Thus, if you have a wheel with a diameter of 8 inches, by finding 8 on the top scale, you see that the circumference is about 25.3 inches. A similar procedure is used to find the diameter if the circumference is given.

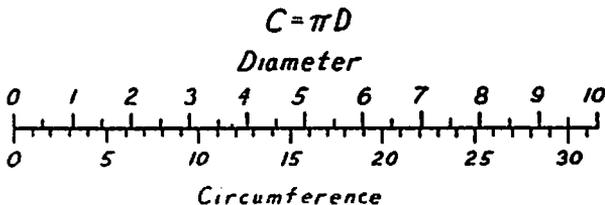


Figure 1

Now, let's take just one of these scales and examine it. Using the diameter scale,  $D$  is our variable and the function of  $D$  is  $\pi D$ . In order to plot this as a scale we will need to know three things: first, the different values of  $D$ ; second, the corresponding values of  $f(D)$ ; and third, at what distance  $X_D$  each value of  $D$  is to be placed from the point of reference.

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\* A paper presented at the 1955 Convention of K.M.E. and awarded third place by the Awards Committee.

Letting  $D$  vary from 0 to 10, we can determine the values for the first two lines of Table 1.

Table 1

|        |   |      |      |   |   |   |   |   |   |      |
|--------|---|------|------|---|---|---|---|---|---|------|
| D      | 0 | 1    | 2    | . | . | . | . | . | . | 10   |
| $f(D)$ | 0 | 3.14 | 6.28 | . | . | . | . | . | . | 31.4 |
| $X_D$  | 0 | 0.63 | 1.26 | . | . | . | . | . | . | 6.28 |

Now we must determine the value of  $X_D$ .  $X_D$  is directly proportional to  $f(D)$ . Therefore, if we let the inch be our unit of measure, and let  $X_D = \pi D$ , we will need a scale 31.4 inches long which is completely out of the question.

The solution of our problem is immediately apparent. In order to reduce the length of the scale to a more practical value, we introduce what is called the scale modulus or scale multiplier which we shall denote by  $m$ .

Our relation between  $X_D$  and the function of the variable now takes the form of  $X_D = m_D f(D)$ . Thus we see that the value of  $m$  depends on two things: (1) the range of values of the variable and (2) the length of the scale.

We have previously assigned a range of 0 to 10 to our variable so that  $f(D)$  has a range of 0 to 31.4. Now, if we let our scale be 6.28 inches long, the scale modulus,  $m_D = X_D/f(D) = 6.28/31.4 = .2$ . Thus  $X_D = .2\pi D$  and we can now add the third line to Table 1.

The general form,  $X_u = m_u f(u)$  of the scale equation is the fundamental equation of straight line nomography. Also, this method of forming a table of values is necessary in all work in nomography.

Now that we are able to construct a scale, let's make a nomogram of our equation  $C = \pi D$ .

Since we have two variables, we must have two scales and therefore two scale equations:  $X_C = m_C f(C)$  and  $X_D = m_D f(D)$ .

In using the adjacent scale method (Fig. 1) the length of the two scales must be equal, and since the two functions were given as equal, our scale moduli must be equal.

Therefore the equation for the C scale is  $X_C = .2C$  and it has the following table of values.

All that remains to be done now is to draw the line, measure out the distances along the scales, and letter the nomogram.

Now, let's review the steps in constructing this nomogram.

1. Decide which variable is to be independent.
2. Assign a range of values to the independent variable.
3. Decide how long a scale is desired.
4. Calculate the value of the scale modulus.
5. Make the table of values for  $D$ .
6. Make the table of values for  $C$ .
7. Draw the nomogram.

Table 2

|        |   |   |    |   |   |   |   |   |   |   |   |    |
|--------|---|---|----|---|---|---|---|---|---|---|---|----|
| $C$    | 0 | 5 | 10 | . | . | . | . | . | . | . | . | 30 |
| $f(C)$ | 0 | 5 | 10 | . | . | . | . | . | . | . | . | 30 |
| $X_C$  | 0 | 1 | 2  | . | . | . | . | . | . | . | . | 6  |

In constructing a nomogram, it is not necessary to use zero as the minimum value. As a matter of fact, it is quite often more practical to use some other value. Figure 2, the nomogram for the volume of a sphere, is an example of this. In this case the radius was varied from 5 to 10 with the volume assuming the corresponding values. The radius scale of this nomogram is an example of a nonuniform scale.

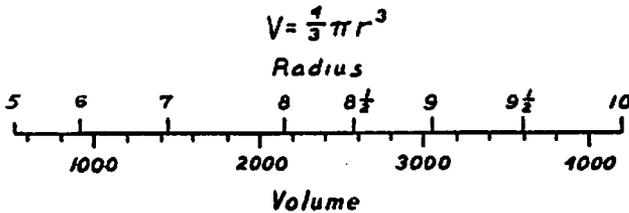


Figure 2

A more interesting and challenging type of nomogram is the one used for the addition of reciprocals. The general form of this equation is  $1/f(u) + 1/f(v) = 1/f(W)$ . This type of nomogram can be used in physics for the Gaussian form of the lens equation, the addition of resistance in parallel, or the addition of capacitance in series.

Figure 3 is an example of this type of nomogram. This nomogram consists of three scales intersecting at their common zero point

and with the middle scale representing the sum of the other two.

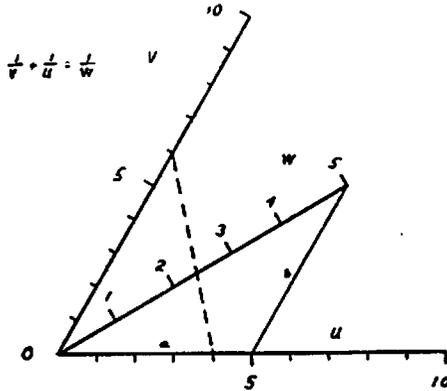


Figure 3

To use this nomogram, lay a straight edge across the drawing so that it intersects the given values of  $u$  and  $v$  and then read the value of the point of intersection on the  $W$  scale. For example, if  $v = 6$  and  $u = 4$ , then  $w = 2.4$ .

This type of nomogram is constructed in the following manner:

1. Select the range of values for  $u$  and the range of values for  $v$ .
2. Decide on the lengths  $X_u$  and  $X_v$ .
3. Now determine the values for  $m_u$  and  $m_v$  from the scale equations:  
 $X_u = m_u f(u)$  and  $X_v = m_v f(v)$ .
4. Select the value of  $\theta$ , the angle between the  $u$  and  $v$  scales.
5. Determine the value of  $m_w$  from the equation:<sup>1</sup>  
 $m_w = (m_u^2 + m_v^2 + 2m_u m_v \cos \theta)^{1/2}$ .
6. The  $w$  scale lies along the diagonal of a parallelogram having two of its sides on the  $u$  and  $v$  scales and with these sides in the ratio  $m_u/m_v$ .
7. Make the three tables of values.
8. Draw the nomogram.

Another type of nomogram is the alignment chart for equations of the form  $f(u) + f(v) = f(w)$ . Figure 4 shows an example of this type.

<sup>1</sup> A. S. Lovons, *Nomography*, (New York: John Wiley and Sons, Inc., 1948), p. 61.

To use this nomogram, lay a straight edge across it so that it intersects two of the scales at their given values, then the point of intersection on the third scale is the value of the unknown variable. For example, if  $u = 3$  and  $w = 15$ , then  $v = 6$ .

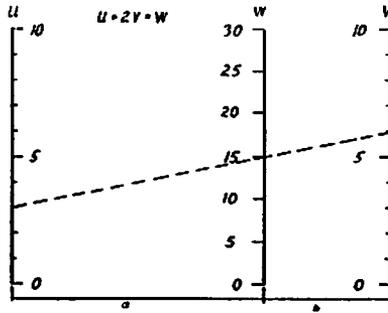


Figure 4

This nomogram has three unknowns and therefore three scale equations:  $X_u = m_u f(u)$ ,  $X_v = m_v f(v)$ , and  $X_w = m_w f(w)$ . The nomogram is constructed in the following manner:

1. Select the range of values for  $u$  and the range of values for  $v$ .
2. Decide on the lengths  $X_u$  and  $X_v$ .
3. Now determine the values for  $m_u$  and  $m_v$ .
4. Calculate the value of  $m_w$  from the equation:<sup>2</sup>  

$$m_w = m_u m_v / (m_u + m_v)$$
5. Make the three tables of values.
6. Draw the nomogram.
  - a. Construct the  $u$  and  $v$  scales parallel to each other and a convenient distance apart.
  - b. Locate the scale for  $w$  so that its distance from the  $u$  scale is to its distance from the  $v$  scale as

$$m_u / m_v = a / b.$$

This type of nomogram can also be used for an equation of the form  $f(u)f(v) = f(w)$  by merely taking the logarithm of each side. The equation then becomes  $\log[f(u)] + \log[f(v)] = \log[f(w)]$ .

This type of nomogram can also be drawn for an equation of

<sup>2</sup> Ibid., p. 28.

four or more variables of the form

$$f(u) + f(v) + f(w) + \dots = f(x).$$

This means that it can also be used for equations of the form

$$f(u)f(v)f(w) \dots = f(x)$$

after taking the logarithm of each side of the equation.

Figure 5 shows an example for an equation of this last type. To use the nomogram, connect the values of  $u$  and  $v$  by a straightedge and find its point of intersection with the blank scale  $Q$ . Now connect that point on  $Q$  with the value of  $w$  and read the answer on the  $x$  scale.

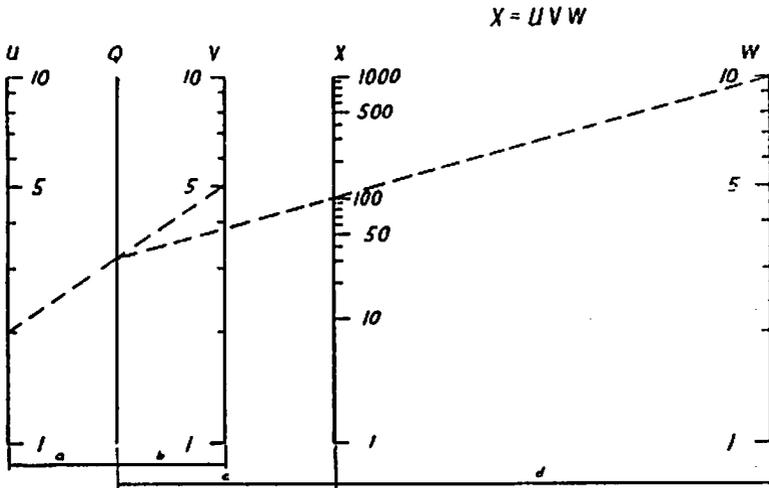


Figure 5

The construction of this type of nomogram is very similar to the previous one.

1. Break up the equation  $x = uvw$  into the two equations  $Q = uv$  and  $x = Qw$ .
2. Assign the ranges of values to  $u$  and  $v$  keeping in mind the fact that  $\log(1) = 0$ .
3. Decide on the lengths of the scales.
4. Determine  $m_u$  and  $m_v$ .
5. Now  $m_q = m_u m_v / (m_u + m_v)$ .
6.  $m_u / m_v = a/b$ .
7. Assign the range of values to  $w$  and determine  $m_w$ .

8. Now  $m_x = m_q m_w / (m_q + m_w)$ .
9.  $m_q / m_w = c/d$ .
10. Make the tables of values.
11. Draw the nomogram.
  - a. All scales are parallel.
  - b. Place the  $u$  and  $v$  scales at a convenient distance apart.



“Arithmetic has a very great and compelling effect, elevating the soul to reason about abstract number, and if visible or tangible objects are obtruding upon the argument, refusing to be satisfied.”

—PLATO

“Beyond the real world, though perceptibly connected with it, mathematicians have created intellectually an ideal world, which they attempt to develop into the most perfect of all worlds, and which is being explored in every direction.”

—A. PRINGSHEIM

“In the beginning everything is self-evident, and it is hard to see whether one self-evident proposition follows from another or not. Obviousness is always the enemy to correctness. Hence we must invent new and even difficult symbolism in which nothing is obvious.”

—BERTRAND RUSSELL

“The science of Pure Mathematics, in its modern developments, may claim to be the most original creation of the human spirit.”

—A. N. WHITEHEAD

# The Invention of Quaternions\*

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The invention of quaternions at the time of its discovery received much applause and many smiles as well. There were those who compared it with Newton's "Principia" while others called it an unmixed evil. Besides these, some interpreted the discovery solely in the light of its usefulness, and other men hailed it as an advance in pure mathematics—a step forward in mathematical thinking. (4)

From this last position, that of pure mathematics, we are considering the invention of quaternions. By this, I do not mean to say that this is the only laudable view to assume, but merely that we have not time to consider its many applications and that it is necessary to understand the principles upon which the quaternion theory rests before attempting to apply it or to judge it.

We will consider the invention of quaternions first from the geometrical interpretation of vectors, an essential concept of quaternion theory, and second from the algebraic interpretation given to it by Sir William Rowan Hamilton. Third, as a kind of a post script, I would like to indicate the usefulness of the quaternion theory.

Let me define a quaternion for you. Its explanation will become clear, I think, as the paper progresses. A quaternion is defined as "a number which alters a directed line segment in length and direction." (5) This, as will be shown, includes the notions of complex numbers and of vectors. The symbol representing a quaternion is a small  $q$ .

From what you know of complex numbers and from plane analytic geometry, you know that we may consider  $i$  to be an operator which causes a line to turn through an angle of 90 degrees. We may consider the complex number,  $x + iy$ , to represent a point, or we may consider it to be a displacement from the origin to a point. In elementary physics a portion of vector analysis is explained. It is sufficient to understand that the addition of complex numbers considered as displacements corresponds to the addition of vectors. This addition is accomplished by the method of the parallelogram of forces with which I am sure you are familiar. A vector is defined as a line having both magnitude, *i.e.*, length, and direction.

The vector is the first concept associated with the quaternion

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\* A paper presented at the 1955 National Convention of K.M.E. and awarded honorable mention.

theory. Briefly, let us review the fundamental notions connected with vectors. Addition, as I have said, is accomplished by the parallelogram. By the negative of a vector we mean a vector of the same magnitude as the given vector but with the opposite direction. Geometrical subtraction of two vectors is accomplished by using the parallelogram to add the first vector and the negative of the second one.

The multiplication of a vector by a second vector corresponds to the combination of a turning—changing the direction of the first vector—plus a stretching—changing the magnitude of this vector.

Division of vectors is defined as the inverse of multiplication. To find the quotient of vectors  $\alpha$  and  $\mathbf{b}$  we find the vector  $\mathbf{x}$  such that the product of  $\mathbf{b}$  and  $\mathbf{x}$  is  $\alpha$ .

If in the division of vectors, we limit ourselves to the division of one vector by a parallel vector, the quotient is called a scalar. A scalar is then defined as a positive or negative real number. We see that this is true since the quotient is obtained from two parallel vectors which, having the same direction, differ only in magnitude. Scalars are of the utmost importance in the algebra of quaternions since a quaternion may be defined as the sum or the combination of a scalar and a vector. This definition is a derivative of the one above: that a quaternion is a number which alters a directed line segment in length and direction.

Since the algebra of quaternions is based on the system of three dimensions, let us now turn to the geometry of space to consider a system similar to that of complex numbers in a plane.

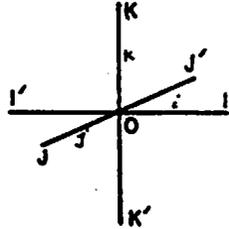
In the geometry of space, think for a moment upon how many different numbers the ratio of two vectors depends. To produce a vector by operation upon a given vector requires, in general:

- 1) a change in length caused by applying a stretching or a compressing factor, and
- 2) a change in direction which in three-dimensional geometry involves three distinct angles.

Therefore the ratio of two vectors depends upon four separate numbers, and from this comes the name "quaternion." We may think of a quaternion as a number which produces a vector by operation on another number or vector. The stretching factor is termed *tensor* and is symbolized by the  $T$  prefixed to a quaternion:  $Tq$ . The turning factor is termed *versor* and is symbolized by  $Uq$ . (5) It will be shown that a quaternion,  $q$ , is equal to the product of its tensor and its versor.

The geometrical theory of complex numbers—for quaternions are themselves complex numbers—concerned Sir William Hamilton for many years. It often happens that as geometry clarifies algebraic solutions, so also does algebra contribute to the interpretation of what has been considered geometrically. Hamilton, therefore, gave a purely algebraic treatment to his theory of quaternions. (The following is taken from *The Elements of Quaternions* by Hamilton with minor rearrangements.)

Suppose  $OI, OJ, OK$  are three co-initial, rectangular unit lines, the rotation around the first from the second to the third being positive. [If you would like to set up a little system for yourself, hold up your left hand and let your thumb, index, and second fingers form the axes. The thumb would be  $OJ$ , the index finger  $OK$ , and the big finger  $OI$ .]



Let  $OI', OJ'$ , and  $OK'$  be three unit vectors respectively opposite to these so that  $OI = -OI', OJ = -OJ', OK = -OK'$ . Let  $i, j, k$  denote a system of three right versors [remember that a versor is the turning factor] in three mutually rectangular planes with the three given lines for their axes so that Axis  $i = OI$ , Axis  $j = OJ$ , and Axis  $k = OK$ , and that

$$\begin{aligned} i &= OK/OJ = OJ'/OK = OK'/OJ' = OJ/OK' \\ j &= OI/OK = OK'/OI = OI'/OK' = OK/OI' \\ k &= OJ/OI = OI'/OJ = OJ'/OI' = OI/OJ' \end{aligned}$$

This symbolism means that

$i$  is an operator which turns  $OJ$  into  $OK$ ,  $OK$  into  $OJ'$ , etc.

$j$  is an operator which turns  $OK$  into  $OI$ ,  $OI$  into  $OK'$ , etc.

$k$  is an operator which turns  $OI$  into  $OJ$ ,  $OJ$  into  $OI'$ , etc.

The three respectively opposite versors then are:

$$-i = OJ/OK = OK'/OJ = OJ'/OK' = OK/OJ'$$

$$-j = OK/OI = OI'/OK = OK'/OI' = OI/OK'$$

$$-k = OI/OJ = OJ'/OI = OI'/OJ' = OJ/OI'$$

Consequently, we see that  $i^2 = (OJ'/OK) \cdot (OK/OJ) = OJ'/OJ = -1$ . Continuing, we find that  $j^2 = -1$  and  $k^2 = -1$ . This might have been inferred from the fact that the three radial quotients  $i, j, k$  are all right versors . . . . The squaring of  $i$  comes thus to be geometrically constructed by doubling an arc or an angle. This rotation is equivalent to an inversion of direction which is a passage from the radius  $OJ$  to the opposite radius  $OJ'$ .

Since  $i \cdot j = (OJ/OK') \cdot (OK'/OI) = OJ/OI$ , we have the values for the products when taken two by two in order of succession:  $ij = k$ ,  $jk = i$ ,  $ki = j$ . The multiplication of  $j$  by  $i$  which is  $ij$  equals  $k$  and is represented by a rotation round  $I$  from  $J$  to  $K$  being positive. The contrasted multiplication of  $i$  by  $j$  may be represented by a rotation round  $I$  from  $J$  to  $K'$  which we know is negative so that the new and opposite product  $ji = -k$ .

We see, that since we have  $ji = -ij$ , the laws of combination of the symbols  $i, j, k$  are not in all respects the same as in algebra. The Commutative Property of Multiplication does not hold here. In algebra we say that  $xy = yx$ , but in the quaternion theory, we see that this is impossible. This arises from the fact that the factors being combined are diplanar versors. These diplanar versors, however, do obey the Associative Property of multiplication as in algebra:  $ij \cdot k = i \cdot jk$ .

We have already found that  $i = jk$ . Substituting this value in the product of the three versors:  $i \cdot j \cdot k$ , we have  $i \cdot i = i^2 = -1$ . Therefore, we have  $i^2 = j^2 = k^2 = ijk = -1$ . In this is contained virtually the laws of the symbols  $i, j, k$  and is sufficient symbolical basis for the whole algebra of quaternions. This is true because every quaternion can be reduced to the quadrinomial form  $q = w + xi + yj + zk$  where  $w, x, y, z$  compose a system of four scalars while  $i, j, k$  are these same right versors. (1)

While Hamilton was considering the complex number,  $x + iy$ , in which he considered  $x$  and  $y$  as a "couple of real numbers," he saw no reason why algebra should correspond to operations confined to one plane. His attention was drawn to the study of three planes. Within three dimensions, he considered three real numbers:  $x, y, z$ . The puzzling factor was the determination of a complex number of this triple. He needed something like  $x + iy + jz$  where  $j$  had properties similar to those of  $i$ . To be more symmetrical he regarded  $xi + yj + zk$ . (4)

It was easy enough to conceive addition and therefore subtraction of such vectors, but the difficulty came when he attempted to multiply them. Hamilton surmounted this obstruction by positing the above laws:

$$i^2 = j^2 = k^2 = ijk = -1.$$

In any algebra one must be able to perform certain operations. The addition and subtraction of quaternions is essentially the addition of vectors. If we let two quaternions represented by two directed line segments, vectors, have a common origin their sum is the diagonal of the parallelogram, two adjacent sides of which are the two quaternions.

We turn to *The Encyclopedia Americana* for a fairly simple explanation of the multiplication of quaternions. This extract is taken almost directly.

Multiplication by a quaternion is equivalent to an orthogonal transformation in four variables plus an expansion or a contraction. The geometrical construction of a quaternion product needs the hypotheses of a four-dimensional space for its proper presentation . . . . It follows that the product of two quaternions is itself a quaternion.

Given the quaternions:  $q = w + xi + yj + zk$   
 $r = w' + x'i + y'j + z'k$ ,  
 the product is  $qr = W + Xi + Yj + Zk$ , where  
 $W = ww' - xx' - yy' - zz'$   
 $X = wx' + w'x + yz' - y'z$   
 $Y = wy' + w'y + zx' - z'x$   
 $Z = wz' + w'z + xy' - x'y$

Division is interpreted by introducing the reciprocal defined by the equation  $qr = 1$ , in which  $q$  and  $r$  are said to be reciprocal to each other. As in ordinary algebra, we write  $r = q^{-1}$  and  $q = r^{-1}$ . This product obviously obeys the commutative law of algebra. The reciprocal of a quaternion must have the form  $q^{-1} = (1/m) \cdot (w - xi - yj - zk)$  where  $m$  is a scalar. By forming the product  $q(q^{-1})$  and observing the laws  $i^2 = j^2 = k^2 = -1$  and  $ij = -ji = k$ , etc., it is easily shown that  $m = w^2 + x^2 + y^2 + z^2$ . This number is called the *norm* of the quaternion,  $q$ . The positive square root of the norm of a quaternion ( $+\sqrt{w^2 + x^2 + y^2 + z^2}$ ) is called its tensor, the stretching factor. The quotient of a quaternion by the tensor of that quaternion is called the versor, the turning factor. Thus, a quaternion is always the product of its tensor and versor. Symbolically this is written  $q = Tq \cdot Uq$  where  $T$  and  $U$  stand for tensor and versor respectively. (6)

In general, therefore, the application of the four fundamental processes of addition, subtraction, multiplication, and division to quaternions leads determinate quaternion results. When it was clear that these processes resulted in other quaternions and that a complete system could be built up, exploration began in order to find applications to physics, astronomy, and other sciences. These applications are mentioned by Prof. H. T. H. Piaggio in *The Significance and Development of Hamilton's Quaternions*. (4) Another not presented herein is given in *An Elementary Presentation of the Theory of Quaternions*, F. D. Murnaghan. (3)

One investigator applied physically the symbolic operator to the scalar which yields "the direction in which that scalar increases most and the magnitude of its rate of increase per unit distance."

The tensor calculus—though the term tensor has acquired a different meaning from the one given here—is a more recent development, and it has gained importance with the advent of the relativity theory. The tensor calculus appeals to mathematicians as it is applicable to any number of dimensions whereas the quaternions are essentially three-dimensional.

Some scientists, the physicists and vector analysts in particular, were willing to disregard what they termed a useless quaternion in favor of the scalar and vector parts which were most used. However, "many mathematicians think that the real value of the quaternion lies in its extension of the idea of number. Hamilton's theory of

couples was the first step in the creation of the abstract algebras which attract so much attention today." (4)

In this paper I have attempted to present a new algebra, one that involves other concepts and interpretations than the linear algebra familiar to us. This new algebra yields also to geometric interpretations, parts of which I have demonstrated. It is widely felt that, while the form in which Hamilton presented his invention has become only of interest historically, the concept will survive. It has been said (Prof. S. T. Whittaker, *Mathematical Gazette*, Vol. 25, 1941, p. 300) that "the development of relativity and quantum mechanics will sooner or later require quaternion methods for the more difficult problems and that when physicists have been forced to learn quaternions, they will use them for all purposes." (4)

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"Whether he drew much or little from the work of his predecessors, it is certain that the ARITHMETICA of Diophantus has exercised a profound influence on the development of number theory."

—R. D. CARMICHAEL

# A Substitution for Simplifying Powers of Sine and Cosine

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The process shown here is a trigonometric substitution for positive integral powers of the sine and the cosine and products of such powers of sine and cosine. The substitution expresses such powers or products in terms of sums of cosines of multiple angles. The process is applicable to both odd and even powers but the following examples will be restricted to the even powers as the substitution is more useful for these.

A practical application of this substitution is to the integration of even powers of sines and cosines and their products. This substitution is often found in English textbooks but virtually never appears in texts in this country.

(1) Let  $\cos \theta + i \sin \theta = z$ .

(2) Then  $1/z = 1/(\cos \theta + i \sin \theta)$ .  

$$= [1/(\cos \theta + i \sin \theta)] [(\cos \theta - i \sin \theta) / (\cos \theta - i \sin \theta)].$$
  

$$= \cos \theta - i \sin \theta.$$

(3)  $z + 1/z = 2\cos \theta$ .

(4)  $z - 1/z = 2i \sin \theta$ .

(5) By De Moivre's Theorem:

$$z^n = (\cos \theta + i \sin \theta)^n = (\cos n\theta + i \sin n\theta).$$

(6)  $1/z^n = z^{-n} = (\cos \theta + i \sin \theta)^{-n} = (\cos n\theta - i \sin n\theta)$ .

(7)  $z^n + 1/z^n = 2\cos n\theta$ .

(8)  $z^n - 1/z^n = 2i \sin n\theta$ .

**EXAMPLE A.** Express  $\cos^4 \theta$  in terms of sums of cosines of multiple angles.

By (3),  $(2\cos \theta)^4 = (z + 1/z)^4 = z^4 + 4z^2 + 6 + 4/z^2 + 1/z^4$   

$$= (z^4 + 1/z^4) + 4(z^2 + 1/z^2) + 6$$
  

$$= 2\cos 4\theta + 8\cos 2\theta + 6, \text{ by (7).}$$

Dividing through by  $2^4$  we get:

$$\cos^4 \theta = (\cos 4\theta)/8 + (\cos 2\theta)/2 + 3/8.$$

Using the method of the preceding example, a general formula can be derived for  $\cos^n \theta$ , where  $n$  is a positive even integer:

$$\begin{aligned} \cos^n \theta &= 1/2^{n-1} [\cos n\theta + n\cos(n-2)\theta \\ &+ [n(n-1)/(1 \cdot 2)] \cos(n-4)\theta \\ &+ [n(n-1)(n-2)/(1 \cdot 2 \cdot 3)] \cos(n-6)\theta + \dots \\ &+ n(n-1)(n-2) \dots (n/2 + 1)/(2 \cdot 1 \cdot 2 \cdot 3 \dots n/2)]. \end{aligned}$$

EXAMPLE B. Express  $\sin^4\theta$  in terms of sums of cosines of multiple angles.

$$\begin{aligned} \text{By (4), } (2i \sin \theta)^4 &= (z - 1/z)^4 = z^4 - 4z^2 + 6 - 4/z^2 + 1/z^4 \\ &= (z^4 + 1/z^4) - 4(z^2 + 1/z^2) + 6 \\ &= 2\cos 4\theta - 8\cos 2\theta + 6, \text{ by (8).} \end{aligned}$$

Dividing through by  $(2i)^4 = 16$ , we get:

$$\sin^4\theta = 1/8(\cos 4\theta - 4\cos 2\theta + 3).$$

In general, for  $n$  a positive even integer we find:

$$\begin{aligned} \sin^n\theta &= i^n/2^{n-1}[\cos n\theta - n\cos(n-2)\theta \\ &\quad + [n(n-1)/(1 \cdot 2)] \cos(n-4)\theta \\ &\quad - [n(n-1)(n-2)/(1 \cdot 2 \cdot 3)] \cos(n-6)\theta + \dots \\ &\quad + (-1)^{n/2} n(n-1)(n-2) \dots (n/2 + 1) \\ &\quad / (2 \cdot 1 \cdot 2 \cdot 3 \dots n/2)]. \end{aligned}$$

EXAMPLE C. Express  $\sin^4\theta\cos^6\theta$  in terms of sums of cosines of multiple angles.

$$\begin{aligned} (2i)^4\sin^4\theta(2)^6\cos^6\theta &= (2i \sin \theta)^4(2\cos \theta)^6 \\ &= (z - 1/z)^4(z + 1/z)^6 = (z^2 - 1/z^2)^4(z + 1/z)^2 \\ &= (z^8 - 4z^4 + 6 - 4/z^4 + 1/z^8)(z^2 + 2 + 1/z^2) \\ &= z^{10} + 2z^8 - 3z^6 - 8z^4 + 2z^2 + 2/z^2 - 8/z^4 \\ &\quad - 3/z^6 + 2/z^8 + 1/z^{10} + 12 \\ &= (z^{10} + 1/z^{10}) + 2(z^8 + 1/z^8) - 3(z^6 + 1/z^6) \\ &\quad - 8(z^4 + 1/z^4) + 2(z^2 + 1/z^2) + 12. \end{aligned}$$

By (7)  $\sin^4\theta\cos^6\theta$

$$\begin{aligned} &= 1/[(2i)^4(2)^6][2\cos 10\theta + 4\cos 8\theta - 6\cos 6\theta \\ &\quad - 16\cos 4\theta + 4\cos 2\theta + 12] \\ &= 1/512[\cos 10\theta + 2\cos 8\theta - 3\cos 6\theta - 8\cos 4\theta \\ &\quad + 2\cos 2\theta + 6]. \end{aligned}$$

EXAMPLE D. Using the results of Example C, we can easily get

$$\begin{aligned} \int \sin^4\theta\cos^6\theta d\theta &= 1/512[(\sin 10\theta)/10 + (\sin 8\theta)/4 \\ &\quad - (\sin 6\theta)/2 - 2\sin 4\theta + \sin 2\theta + 6\theta] + C. \end{aligned}$$

Reference: *Advanced Trigonometry*, C. V. Durell, M. A., and A. Robson, M. A.; (London: G. Bell and Sons, Ltd., 1930), pp. 169-171.

# **Job Opportunities in Mathematics\***

**ARNOLD LEE JANOUSEK AND CHARLES R. DEETER**  
*Students, Fort Hays Kansas State College*

This is a report on a paper entitled "A Survey of Job Opportunities for Mathematicians in Business and Industry." The survey was conducted by Arnold Lee Janousek and Charles R. Deeter at Fort Hays Kansas State College in October, 1954.

The primary objective of the survey is to aid persons who are seeking employment in the mathematical profession. It considers some of the available types of jobs and discusses the major factors which should be investigated by a prospective employee.

Questionnaires were sent to one hundred twenty-two firms that would be likely to employ mathematicians. Seventy-five percent of these firms replied with the requested information.

The survey did not by any means include all of the industries or businesses which might conceivably hire mathematicians. Neither did it consider the teaching profession. However, the number of mathematicians required by industry should indicate the need for teachers to train these persons.

With the trend toward specialization in the past decade the practice of using mathematicians in industry has become a profession. Prior to this time, the calculations which applied to physical problems were made by the engineers and physicists. But with the advent of high speed aircraft and nuclear energy, and the invention of complex electronic devices, these engineers and physicists found that they had less time to devote to mathematics. Therefore they began to call upon the students of applied mathematics.

Today the mathematician may expect to find employment in four major types of mathematics. These are computing, statistics, actuarial mathematics, and research mathematics.

The job of computing is simply the work of making the calculations which are involved in a problem of applied mathematics. In such a capacity the mathematician finds opportunities in the aircraft industry, in government research, and in the industries which manufacture electronic and mechanical computers.

Statisticians are employed by nearly all industrial and commercial firms. Many of the statisticians conduct surveys within the firm and apply statistical methods to the results of the survey. This information is used in improving manufacturing techniques and the

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\* A paper presented at the 1955 Convention of K.M.E.

quality of the product being manufactured. Some phases of research and testing also require the use of statistics in the interpretation of results.

An actuary is a mathematician whose field is a unique combination of pure mathematics and advanced statistics. He has opportunities almost exclusively in the insurance business where he calculates premiums, prepares tables of death and accident rates, and, in general, applies his mathematical knowledge to the problems of the insurance business.

The research field provides opportunities for the advanced mathematician and is probably the most diversified field mentioned in this report.

The jobs discussed above include opportunities for persons at all educational levels, but some firms have special requirements. The most notable of these requirements is found in the actuarial field where the employee must be a member of the Society of Actuaries. His professional rating within the Society is determined entirely by his successful completion of a set of examinations. Another irregularity occurs in the field of atomic energy where there are no openings especially suitable for a person at the Master's Degree level. This is due to the nature of the work which is either computing or advanced research.

Salaries vary with the qualifications of the person who is being employed, but the average starting salaries are approximately \$350, \$425, \$500 per month for persons at the Bachelor's, Master's, and Doctor's Degree levels respectively.

Advancement is rapid in all industries and a person could expect to be earning about \$10,000 to \$12,000 per year after about ten years of work. Government salaries are in accordance with the General-Schedule Wage Scale, where the grades range from GS-5 to GS-18 with salaries from \$3,100 to \$14,000 annually.

All employees in industry and business have the benefit of federal social security. In addition to this retirement security, most firms have retirement plans which enable the employee to provide for the security of his family.

Also available to most employees is a health and life insurance plan. Usually this is a group type such as the Blue Cross-Blue Shield insurance but occasionally it is company-sponsored insurance.

The opportunities mentioned here are not limited to men. Women have many of the same opportunities. Many corporations prefer women in their organizations due to the nature of the work.

In summary, the mathematician has many opportunities in business and industry. Not only can he adequately provide for his family's welfare and security but he can benefit by his association with other persons in his own profession.

# A Note on Conccyclic Points

L. T. SHIFLETT

*Faculty, Southwest Missouri State College*

The fairly well-known technique of writing the equation of the conic determined by five points suggests a procedure whereby one can determine if four points lie on a circle. The procedure is not offered as the best method for making such a determination, but as an interesting side light that captured the attention of this writer.

It is hoped that a review of the process of determining the equation of a conic through five points will not insult the intelligence of the reader. Suppose the five points are  $P_1, P_2, P_3, P_4, P_5$ . Let  $\alpha_{1j} = 0$  be the equation of the line through  $P_1$  and  $P_j$ . An equation of the system of conics through  $P_1, P_2, P_3, P_4$  is then given by forming the equation

$$(1) \quad \alpha_{12}\alpha_{34} + k\alpha_{13}\alpha_{24} = 0.$$

That (1) represents an equation of the second degree in  $x$  and  $y$  for any real value of  $k$  is evident, since its left member is the sum of products of two linear functions of  $x$  and  $y$ . That (1) is satisfied by the coordinates of  $P_1, P_2, P_3, P_4$  is also evident, since the substitution of the coordinates of any one of the four points causes one of the linear functions in each product to vanish.

If the coordinates of  $P_5$  are substituted in (1), a value for  $k$  will be determined which will single out the member of the system given by (1) which passes through  $P_5$ . Thus the equation of the conic through the five points is determined.

The procedure for determining if four points are conccyclic is based on a closer examination of (1). Suppose it is required to decide if the points  $P_1, P_2, P_3, P_4$  are on the same circle. In the stated manner, form the equation (1). Our procedure is to decide whether one of the members of this system can be a circle. It is well known that in the equation of a circle, the coefficients of  $x^2$  and  $y^2$  are equal and the coefficient of  $xy$  is zero. In (1) these coefficients are linear functions of  $k$ . Thus, if the value of  $k$  that makes the coefficients of  $x^2$  and  $y^2$  equal also makes the coefficients of  $xy$  vanish, the points are conccyclic; if not, the points are not conccyclic.

To illustrate the procedure, consider the points  $P_1(-2, 3)$ ;  $P_2(-2, -7)$ ;  $P_3(-7, -2)$ ;  $P_4(1, 2)$ . The equations of the lines are as follows:

$$\begin{aligned}\alpha_{12} &= x + 2 = 0 \\ \alpha_{34} &= x - 2y + 3 = 0 \\ \alpha_{13} &= x - y + 5 = 0 \\ \alpha_{24} &= 3x - y - 1 = 0.\end{aligned}$$

An equation of the system of conics through these points is  
 $(x + 2)(x - 2y + 3) + k(x - y + 5)(3x - y - 1) = 0$ , or  
 $(1 + 3k)x^2 + (-2 - 4k)xy + ky^2 + (5 + 14k)x$   
 $+ (-4 - 4k)y + 6 - 5k = 0$ .

The condition that the coefficients of  $x^2$  and  $y^2$  are equal is that  
 $1 + 3k = k$ , or  
 $k = -1/2$ .

This also makes  $-2 - 4k = 0$ ; thus the points are concyclic.  
 Consider also the points  $P_1(0, 0)$ ;  $P_2(1, 2)$ ;  $P_3(3, 3)$ ;  $P_4(0, 5)$ .  
 The equations of the lines are as follows:

$$\begin{aligned}\alpha_{12} &= 2x - y = 0 \\ \alpha_{34} &= 2x + 3y - 15 = 0 \\ \alpha_{13} &= x - y = 0 \\ \alpha_{24} &= 3x + y - 5 = 0.\end{aligned}$$

Equation (1) becomes

$$\begin{aligned}(2x - y)(2x + 3y - 15) + k(x - y)(3x + y - 5) &= 0, \text{ or} \\ (4 + 3k)x^2 + (4 - 2k)xy + (-3 - k)y^2 + (-30 - 5k)x \\ + (15 + 5k)y &= 0.\end{aligned}$$

The condition that the coefficients of  $x^2$  and  $y^2$  are equal is that  
 $4 + 3k = -3 - k$ , or  
 $k = -7/4$ .

This value of  $k$  makes the coefficient of  $xy$  equal to  $15/2$  rather than zero; thus it is seen that these four points are not concyclic.

# The Problem Corner

EDITED BY FRANK C. GENTRY

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond the calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before March 1, 1956. The best solutions submitted by students will be published in the Spring, 1956, issue of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Frank C. Gentry, Department of Mathematics and Astronomy, University of New Mexico, Albuquerque, New Mexico.

## PROBLEMS PROPOSED

82. *Proposed by Sam H. Sesskin, student, Hofstra College, Hempstead, New York.*

A box in the shape of a cube 4 feet on an edge is placed with one face against a wall. A ladder 12 feet long is leaned over the box and against the wall so as to touch one edge of the box. How high up the wall does the ladder reach, and how far from the wall is its foot?

83. *Proposed by Marie Smith, student, University of New Mexico, Albuquerque, New Mexico.*

Nine men: Brown, White, Adams, Miller, Green, Hunter, Knight, Jones, and Smith play the several positions on a baseball team. Determine from the following data the position played by each: 1) Smith and Brown each won \$10 playing poker with the pitcher; 2) Hunter is taller than Knight and shorter than White but each of these weighs more than the first baseman; 3) the third baseman lives across the corridor from Jones in the same apartment house; 4) Miller and the outfielders play bridge in their spare time; 5) White, Miller, Brown, the right fielder, and the center fielder are bachelors and the rest are married; 6) of Adams and Knight, one plays an outfield position; 7) the right fielder is shorter than the center fielder; 8) the third baseman is brother to the pitcher's wife; 9) Green is taller than the infielders and the battery, except for Jones, Smith, and Adams; 10) the second baseman beat Jones, Brown, Hunter, and the catcher at cards; 11) the third baseman,

the shortstop, and Hunter made \$150 each speculating in U. S. Steel; 12) the second baseman is engaged to Miller's sister; 13) Adams lives in the same house as his own sister but dislikes the catcher; 14) Adams, Brown, and the shortstop lost \$200 each speculating in copper; 15) the catcher has three daughters, the third baseman has two sons, but Green is being sued for divorce. (From Bennett and Baylis, *Formal Logic*.)

84. *Proposed by Leon Bankoff, D. D. S., 2410 Wilshire Boulevard, Los Angeles, California.*

A line through the incenter of a right triangle cuts the hypotenuse  $AB$  perpendicularly in  $F$  and the side  $BC$  in  $G$ , ( $BC > AC$ ). Show that the area of the triangle  $ABC$  is equal to  $CG \cdot GB + GF^2$ .

85. *Proposed by Carl V. Fronabarger, Southwest Missouri State College.*

In any quadrilateral circumscribed about a circle, the diagonals and the lines joining the points of tangency of opposite sides are concurrent.

86. *Proposed by Victor L. Osgood, 26 Willow Court, Oceanport, New Jersey.*

The number  $3\frac{5}{12}$  is such that if 5 be added to or subtracted from its square a perfect square results. Find another such number.

87. *Proposed by Alfred Moessner, Gunzenhausen, Germany.*

William has a smaller amount of money than his brother David. The sum of the squares of the numbers which represent the amounts of money that each has is equal to the sum of the amounts. What are the smallest possible amounts the two can have?

## SOLUTIONS

72. *Proposed by Frank Hawthorne, Hofstra College, Hempstead, New York.*

A rectangular piece of sheet metal with integral dimensions  $a$  and  $b$  has equal squares of side  $x$  cut from each corner. The sides are then bent up to form a rectangular box with open top. The value of  $x$  is chosen so as to make the volume of the box a maximum. If  $x$  is rational and if a triangle is formed with sides  $a$  and  $b$  the angle

C between them is  $60^\circ$ , show that the side  $c$  of this triangle is integral.

*Solution by Gabriel Elissa, Baldwin-Wallace College, Berea, Ohio.*

The volume of the box  $V = x(a - 2x)(b - 2x) = 4x^3 - 2ax^2 - 2bx^2 + abx$ .  $dV/dx = 12x^2 - 4ax - 4bx + ab$  and the condition for a maximum volume is that  $12x^2 - 4ax - 4bx + ab = 0$ . This equation will have rational roots in  $x$  if  $16(a + b)^2 - 48ab = 16(a^2 + b^2 - ab)$  is a perfect square. Now for the triangle  $c^2 = a^2 + b^2 - 2ab \cos 60^\circ = a^2 + b^2 - ab$ . Hence  $c$  is an integer.

Also solved by S. H. Sesskin, Hofstra College, Hempstead, New York.

73. *Proposed by Victor Thebault, Tennie, Sarthe, France.*

On the sides  $BC$ ,  $CA$ ,  $AB$  of the triangle  $ABC$ , determine points  $M$ ,  $N$ ,  $P$  such that  $BM = MN = NP = PA$ .

*Solution by Paul McKee, Baldwin-Wallace College, Berea, Ohio.*

Construct a circle with center  $M_1$ , any point on side  $BC$ , and with radius  $BM_1$ . Through any point  $P_2$  on  $BA$  construct line  $P_2N_2$  equal to  $BM_1$  so that angle  $BP_2N_2 = 2$  angle  $BAC$ . Through  $N_2$  draw  $N_2N_1$  parallel to  $AB$  meeting the circle inside the triangle at  $N_1$ . Join  $N_1$  to  $M_1$ . Draw  $N_1P_1$  parallel to  $N_2P_2$  meeting  $BA$  at  $P_1$ . Join  $B$  to  $N_1$ , extending it to meet  $AC$  at  $N$ . Draw  $NM$  parallel to  $N_1M_1$  meeting  $BC$  at  $M$ . Draw  $NP$  parallel to  $N_2P_2$  meeting  $BA$  in  $P$ . Then  $M$ ,  $N$ , and  $P$  are the required points. For, quadrilateral  $BMNP$  is similar to quadrilateral  $BM_1N_1P_1$  and  $M_1N_1 = M_1B = N_2P_2 = N_1P_1$ . Hence  $BM = MN = NP$ . Since angle  $BPN =$  angle  $BP_2N_2 = 2$  angle  $BAC$ , triangle  $PAN$  is isosceles and  $PN = PA$ .

74. *Proposed by H. D. Larsen, Albion College, Albion, Michigan.*

From a five place table of logarithmic sines, calculate the five place logarithms of 2, 3, and 7.

*Solution by S. H. Sesskin, Hofstra College, Hempstead, New York.*

Obviously  $\log 2$  and  $\log 3$  may be obtained successively from  $\log \sin 30^\circ$  and  $\log \sin 60^\circ$ . To develop a general method note that  $\sin 2\theta = 2 \sin \theta \sin (90^\circ - \theta)$ , so that  $\log 2 = \log \sin 2\theta - \log \sin \theta - \log \sin(90^\circ - \theta)$ . Now for very small values of  $\theta$   $\log$

$\sin(90^\circ - \theta)$  is very near zero. In fact  $\log 2 = \log \sin 2' - \log \sin 1' = 6.76476 - 10 - (6.46373 - 10) = 0.30103$ . For  $\log 3$ ,  $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$  or  $3 = (\sin 3\theta + 4 \sin^3 \theta) / \sin \theta$ . Setting  $\theta = 60^\circ$  leads to  $3 = 4 \sin^2 60^\circ$  or  $\log 3 = 2 \log 2 + 2 \log \sin 60^\circ = 0.47712$ . For  $\log 7$ ,  $\sin 7\theta = 7 \cos^6 \theta \sin \theta - 35 \cos^4 \theta \sin^3 \theta + 21 \cos^2 \theta \sin^5 \theta - \sin^7 \theta = 7 \sin \theta - 56 \sin^3 \theta + 112 \sin^5 \theta - 64 \sin^7 \theta$ . From this  $7 = \cos^2 \theta (\sin 7\theta + 64 \sin^7 \theta) / (\cos^2 3\theta \sin \theta)$  after substituting  $\cos 3\theta / (\cos \theta)$  for  $4 \cos^2 \theta - 3$ . Now set  $\theta = \pi/7$  and we find  $\log 7 = 2[\log \sin (5\pi/14) - \log \sin (\pi/14)] + 6[\log 2 + \log \sin (\pi/7)] = 0.84510$ .

77. *Proposed by Paul W. Healy, University of New Mexico, Albuquerque, New Mexico.*

Find all numbers less than 10,000 such that each may be divided by 2 by prefixing the 2 and erasing the last digit. What is the form of all such numbers?

*Solution by Gustavus J. Simmons, New Mexico Highlands University, Las Vegas, New Mexico.*

If  $a, b, c, \dots, m, N$  are the digits of the required number then  $(a \cdot 10^n + b \cdot 10^{n-1} + \dots + m \cdot 10 + N) / 2 = 2 \cdot 10^n + a \cdot 10^{n-1} + \dots + m$ , or  $8a \cdot 10^{n-1} + 8b \cdot 10^{n-2} + \dots + 8m + N = 4 \cdot 10^n$ . Since  $a/2 = 2$ ,  $a = 4$  or  $a = 5$ . If  $a = 5$  we have  $8b \cdot 10^{n-2} + \dots + 8m + N = 0$  which is only satisfied if  $b = c = d = \dots = m = N = 0$ . If  $a = 4$ , then  $8b \cdot 10^{n-2} + \dots + 8m + N = 8 \cdot 10^{n-1}$  is an identity. Hence successively  $N = 8$ ,  $m + 1 = 10$  so that  $m = 9$ , and in the same way  $b = c = \dots = 9$  for all the other digits. The general form of the number is then  $5 \cdot 10^n$  or  $5 \cdot 10^n - 2$  for  $n = 1, 2, 3, 4, \dots$ . In addition there is the trivial solution 4. The numbers less than 10,000 are 4, 48, 50, 498, 500, 4998, 5000.

Also solved by Virginia Castelluccio, Montclair State Teachers College; Gabriel D. Elissa, Baldwin-Wallace College; Sam Sesskin, Hofstra College; and Lois E. Sudborough, Central Michigan College.

78. *Proposed by Victor L. Osgood, Oceanport, New Jersey.*

From the Pythagorean Theorem  $2^2 + 3^2 = 13$ . Find 3 more pairs of rational numbers, the sums of whose squares is 13.

*Solution by Sam Sesskin, Hofstra College, Hempstead, New York.*

Since primes of the form  $4n + 1$  can be the sums of two integral squares in only one way, it follows; if we ignore the trivial

solutions  $(-2, -3)$ ,  $(-2, 3)$ ,  $(2, -3)$ ; that we seek integral solutions for the equation  $m^2 + n^2 = 13k^2$ . Now let  $k^2 = (u^2 + v^2)^2 = (2uv)^2 + (u^2 - v^2)^2$  or  $(2^2 + 3^2)[(2uv)^2 + (u^2 - v^2)^2] = 13(u^2 + v^2)^2$ . Substituting from this into the identity  $(a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 + (ad \mp bc)^2$  we obtain the general solution

$$[4uv \pm 3(u^2 - v^2)]/(u^2 + v^2)^2 + [2(u^2 - v^2) \mp 6uv]^2/(u^2 + v^2)^2 = 13.$$

For  $u = 1$ ,  $v = 2$  this gives  $(1/5)^2 + (18/5)^2 = 13 (17/5)^2 + (6/5)^2 = 13$ , for  $u = 2$ ,  $v = 3$ ,  $(9/13)^2 + (46/13)^2 = 13$  and  $(39/13)^2 + (26/13)^2 = 13$ , etc.

Also solved by Charles Thurston, M. I. T.; William King, Kansas State Teachers College at Emporia; Gustavus J. Simmons, New Mexico Highlands University; and Gabriel D. Elissa, Baldwin-Wallace College.

79. *Proposed by Frank Hawthorne, Hofstra College, Hempstead, New York.*

An Englishman had walked one-third of the way across a railway bridge 7920 feet long when he heard a train coming behind him at 45 miles per hour. He could just escape by running at uniform speed to either end of the bridge. What was his name?

*Solution by Charles Thornton, Massachusetts Institute of Technology, Cambridge, Massachusetts.*

The length of the bridge is  $1\frac{1}{2}$  miles. If the man runs toward the train, he will meet it after a  $\frac{1}{2}$  mile run. If he runs away from the train, he will have  $\frac{1}{2}$  mile to go when the train starts on the bridge. Since he reaches the end of the bridge at the same time as the train, he can run one third as fast or 15 miles per hour. This means that he ran the mile in 4 minutes. Now Roger Bannister is the only Englishman who has accomplished this feat.

Also solved by Gabriel D. Elissa, Baldwin-Wallace College; Gene W. Hiller and Joe F. Middleton, both of Mississippi Southern College; William King, Kansas State Teachers College at Emporia; Lois E. Sudborough, Central Michigan College; and Sam Sesskin, Hofstra College who pointed out that the above solution was unique until May 28, 1955, when two other Englishmen, Brian Hewson and Chris Chataway also ran a mile in four minutes.

80. *Proposed by the Editor* (From Rietz and Crathorne, *College Algebra*, Third Edition.)

An airplane flying at 75 miles per hour and following a long straight road, passed an automobile going in the opposite direction. One hour later it overtook a second automobile. The automobiles passed each other when the airplane was 100 miles away. If both automobiles travel at the same speed how far apart were they when the airplane passed the second one and what was their speed?

*Solution by Sam Sesskin, Hofstra College, Hempstead, New York.*

Let  $x$  be the number of miles from point where cars met to point where airplane met first car. Then  $(100 - x)/75$  is time for first car to meet airplane. When airplane met first car, second car was  $2x$  miles away. Hence in the next hour it traveled  $75 - 2x$  miles which is the speed of both cars. Consequently  $x/(75 - 2x)$  is time for first car to meet airplane. Then

$$(100 - x)/75 = x/(75 - 2x) \text{ or } x = 25 \text{ miles.}$$

Hence the speed of the two cars was 25 miles per hour and they were 100 miles apart when the airplane overtook the second car.

Also solved by Robert Diebold, University of New Mexico, Albuquerque, New Mexico.



"Now this establishment of correspondence between two aggregates and investigation of the propositions that are carried over by the correspondence may be called the central idea of modern mathematics."

—W. K. CLIFFORD

# The Mathematical Scrapbook

EDITED BY J. M. SACHS

*Many arts there are which beautify the mind of man; of all other none do more garnish and beautify it than those arts which are called mathematical.*

—H. BILLINGSLEY (1570)

= Δ =

"A mathematical problem should be difficult in order to entice us, yet not completely inaccessible lest it mock at our efforts. It should be to us a guide post on the mazy paths to hidden truths, and ultimately a reminder of our pleasure in the successful solution."

—D. HILBERT

= Δ =

Solve the equation  $x^2 - 68x + 999.9999 = 0$ , which has rational roots, by factoring.

SUBMITTED BY NORMAN ANNING

= Δ =

"There has not been any science so much esteemed and honored as that of mathematics, nor with so much industry and vigilance become the care of great men, and labored in by the potentates of the world, viz., emperors, kings, princes, etc."

—BENJAMIN FRANKLIN

= Δ =

A man who is at present (1955) serving in the Senate of The United States of America was  $x$  years old in the year  $x^2$ . Find  $x$ .

= Δ =

Do you have trouble making check stubs agree with the records of your bank? Take heart! Others, great mathematicians among them, have shared this problem.

"Sir Isaac Newton, though so deep in algebra and fluxions, could not readily make up a common account; and, when he was Master of the Mint, used to get somebody else to make up his accounts for him."

—REV. J. SPENCE

*Anecdotes, Observations,  
and Characters of Books  
and Men, 1858.*

A traveler is walking in a strange land. Each inhabitant of this land belongs to one of the two political parties. The members of the Fabulous Party always tell the truth and the members of the Colossal Party always lie. Our traveler comes to an unmarked fork in the road and, knowing that one of the two branches leads to the capital, decides to ask a native working nearby for directions. What single question can he ask to be sure which road will take him to the capital?

=Δ=

“As the sun eclipses the stars by his brilliancy, so the man of knowledge will eclipse the fame of others in assemblies of the people if he proposes algebraic problems, and still more if he solves them.”

—BRAHMAGUPTA

=Δ=

Here is an interesting variation on the, “What color was the bear?” problem. A hunter sighted a bear and hit the animal with his first shot. The wounded bear struggled a mile due east before expiring on the very spot where he had been standing when he was shot. Where must the bear have been? (The answer that the bear must have been on a circle of latitude of circumference one mile is not correct! Read the problem again!)

=Δ=

“What is exact about mathematics except exactness? And is this not a consequence of the inner sense of truth?”

—GOETHE

=Δ=

What’s wrong here? The following “pseudo-proof” that the line at infinity passes through the origin was sent in by Norman Anning who credits it to *The Mathematical Gazette* about twelve years back. Can you find any flaws?

$$\text{Consider } D = \begin{vmatrix} x + 2y & x + 2y - 10 \\ 3x + y & 3x + y - 10 \end{vmatrix} = 0.$$

This is the equation of a line.

The two lines

$$x + 2y = 0$$

$$x + 2y - 10 = 0$$

intersect in a point at infinity and the two lines

$$3x + y = 0$$

$$3x + y - 10 = 0$$

intersect at a different point at infinity. Thus the line  $D = 0$  contains two distinct points at infinity and must be the line at infinity. The two lines

$$\begin{aligned} x + 2y &= 0 \\ 3x + y &= 0 \end{aligned}$$

intersect in the origin, hence  $D = 0$  passes through the origin!  
 $=\Delta=$

“Discovery of postulates is often followed by the discovery that some of them are false. What of it? The discovery that a postulate is false is a discovery—it is a step in advance, understanding is increased, we are disillusioned, knowledge has grown. When a postulate is found to be false, it has to be abandoned. The history of postulational thinking, especially the wake of postulate detection is thickly strewn with abandoned postulates. In political economy the postulate of ‘the economic man’; in ethics the postulate of a universal law for the guidance of conduct; in law the centuries-old postulate of *lex naturae* or *jus naturae*, a kind of ‘brooding omnipresence in the sky’ as Justice Holmes wittily characterized it; in physics and cosmology the postulates of cosmic absolutes—absolute time, absolute matter, absolute natural law; these and many others have been cast overboard in our day. Indeed a very interesting and edifying work might be written on ‘Abandoned Postulates.’ It would be the history of the progress of knowledge.”

—C. J. KEYSER

$=\Delta=$

An artificial satellite is circling the earth, pole to pole. It is at a height of 300 miles and one complete circle takes 90 minutes. You see the reflection of sunlight from it just as it comes up over your horizon due north of you. You keep your eye on the satellite and call to a friend who is standing a few feet away. Do you have enough time to draw it to his attention before it disappears below the horizon?

SUGGESTED BY HENRY PATIN

$=\Delta=$

Her Radiancy, the Empress of Kgovjnia insisted that her two foreign visitors find a just way to judge the finals of The Imperial Scarf Making Competition. She urged them to consider speed, lightness, and warmth. Though she housed them in her best dungeon and fed them abundantly on bread and water, they were nevertheless anxious to resume their journey. The three finalists in the competition were Lolo, Mimi, and Zuzu. Lolo made five scarves while Mimi

was making two; but Zuzu made four while Lolo was making three! Again so fairy-like is Zuzu's handiwork, five of her scarves weigh no more than one of Lolo's; yet Mimi's is lighter still, five of hers will but balance three of Zuzu's! and for warmth one of Mimi's is equal to four of Zuzu's; yet one of Lolo's is as warm as three of Mimi's! How would you have judged the competition?

—LEWIS CARROL  
From "A Tangled Tale."

=△=

A man climbs a moving escalator two steps at a time. He covers a total of 32 steps to the top. A second man climbs the same moving escalator taking paces at the same rate as the first man but only one step at a time. He covers 48 steps to the top. If the escalator were not moving, how many steps would the second man have climbed to reach the top?

—FROM THE WILSON JUNIOR COLLEGE  
MATH. TOURNAMENT, 1954.

=△=

Two of Euler's pupils having computed to the 17th term, a complicated converging series, their results differed one unit in the fiftieth cipher; and an appeal being made to Euler, he went over the calculation in his mind, and his decision was found correct.

—LETTERS OF EULER  
DAVID BREWSTER, NEW YORK, 1872

=△=

"It has come to pass, I know not how, that Mathematics and Logic, which ought to be but the handmaids of Physics, nevertheless presume on the strength of the certainty which they possess to exercise dominion over it."

—FRANCIS BACON

=△=

"Every common mechanic has something to say in his craft about good and evil, useful and useless, but these practical considerations never enter into the purview of the mathematician."

—ARISTIPPUS, THE CYRENAIC

# The Book Shelf

EDITED BY REX D. DEPEW

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of **THE PENTAGON**. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Rex D. Depew, State Teachers College, Florence, Alabama.

*An Introduction to Deductive Logic*, Hugues Leblanc, John Wiley and Sons, Inc., (440 Fourth Avenue), New York, 1955, 12 + 244 pp., \$4.75.

Introductory logic texts today fall into three classes. First there are those texts that call themselves "applied" or "practical" logic, and pretend to teach correct thinking. Then there are introductions that try to present material from both classical and modern formal logic together with treatments of induction and scientific method in such a way as to be palatable to fine arts majors as well as students of the sciences. Finally there are those texts that offer a rigorous exposition of modern formal or symbolic logic in strict deductive form without apologies to the student untrained in mathematics. It is to this last class that Hugues Leblanc's excellent book belongs.

Leblanc, who is associate professor at Bryn Mawr, is one of the most accomplished disciples of Harvard's renowned symbolic logician W. V. Quine. Because of its relative completeness and its clarity of presentation, Leblanc's book will probably prove more useful as an introduction to deductive logic than either Quine's or Tarski's short texts.

The book falls into five sections. The first presents the familiar two-valued sentential logic. A section on quantificational logic follows in which classical as well as modern analysis is covered. A third chapter presents the sentential and quantificational logic in strict deductive form. The logic of identity, classes, and relations is treated in the fourth section. The final chapter includes analysis of four syntactical concepts: consistency, completeness, decidability, and independence. The treatment throughout is remarkably clear and rigorous. A number of new and economical modes of demonstration are included.

Students of mathematics will welcome Leblanc's book as a use-

ful introduction to contemporary logic as well as a concise demonstration of the twentieth century's challenging theorem that mathematics is contained in logic as the particular is comprehended by the universal.

—J. G. BRENNAN

*Integers and the Theory of Numbers*, Abraham A. Fraenkel, Scripta Mathematica, (Yeshiva University) New York, 1955, 102 pp., \$2.75.

The development of the topic, "Integers and the Theory of Numbers," is simple and comprehensible. Through his terse style the author, Abraham Fraenkel, conveys the information in a very direct manner. The absence of an involved formalism enables the reader to grasp the subject matter with relative ease of comprehension.

An effort has been made to lead the reader to a real understanding of the material considered rather than to an acquaintance with mere manipulative procedures. In this respect, Professor Fraenkel bridges the gap between the more theoretical approaches and the methods of secondary mathematics or beginning undergraduate instruction. Consequently, precision in statement has been achieved without sacrificing clarity or without becoming too deeply involved with questions of rigor.

The singular treatment of the author elicits the attention of the beginning student and sustains his interest throughout the unfoldment of the subject. The discussions are sufficiently complete to meet adequately the needs of any beginner in the study of the number theory.

The reader is fascinated by the succinct way the author reveals the simplicity of statement, characteristic of many problems in number theory. At the same time, he points out the difficulty involved in arriving at general solutions in some seemingly simple cases.

After seriously reading *Integers and the Theory of Numbers*, the student is likely to feel that some very definite information has been promulgated concerning the nature of numbers. The treatment is enriched by the introduction of interesting bits of history and the content is made more intelligible by relegating the more formal aspects of the subject to the appendix. This procedure seems to have considerable merit for neophyte readers.

Although the reviewer believes that *Integers and the Theory of Numbers* by Professor Fraenkel may be read to advantage by anyone, the book is especially addressed to laymen and beginning students.

—HARRY M. LINNETE

*Stochastic Models for Learning*, by Robert R. Bush and Frederick Mosteller, John Wiley and Sons, Inc., (440 Fourth Ave.) New York, 1955, 16 + 365 pp., \$9.00.

Few fields of mathematics have seen more creative activity during recent years than that of mathematical statistics. Problems have arisen in this field which are of such depth and interest as to have attracted some of the best mathematical minds. No less striking have been the many and varied applications of statistics to business, to technology, and to scientific research. The book under review here is concerned with an application of mathematical statistics to a problem in psychology.

The authors have undertaken to set up a statistical theory for the phenomenon of learning. The aspects of the learning process selected for treatment in this work are of a rather primitive nature. This is, of course, as it should be, not merely because this is a pioneer work in the field but also because in theory-construction one always seeks to proceed from the simple to the complex.

One of the simplest types of the learning situation will make clear the general procedure adopted in this book. Let us consider a maze used for studying the learning process in rats. This maze has the shape of a T, the vertical part of which is a passageway. At each end of the horizontal part of the T there is a box. The box on the right contains food; the box on the left is empty. A hungry rat is permitted to run through the passageway, and upon reaching the horizontal part of the T (at the "choice point") he must proceed either to the right or left. If he runs to the right he finds food—if to the left, he finds only the empty box. The maze is so constructed that once he makes his choice, he cannot retrace. Now, if this experiment is repeated with this rat (properly starved on each occasion, of course) there will result a sequence of right and left turns. Observation shows that repetition of the experiment will result in an ever-increasing proportion of right turns and that eventually the rat will make a right turn every time. That is, the rat is "learning" to find the food.

The rapidity with which the rat is learning may be measured by the probability of his making a right turn at each run. More precisely, if  $p_n$  is the probability of a right turn on the  $n$ th run, then we may expect that  $p_1 < p_2 < \dots < p_n$  and that eventually there will be an  $n$  for which  $p_n = p_{n+1} = \dots = 1$ .

It is at least intuitively plausible that each  $p_1$  will depend on the preceding one. Thus, if a rat has achieved a probability of

$p_s = 0.6$  he will be more likely to achieve  $p_s = 0.7$  on the next turn than if he had achieved only, say,  $p_s = 0.4$ . The main problem with which the authors are concerned is: given a set of observed  $p$ 's (for a given rat or group of rats), find a mathematical expression which will enable one to predict  $p_{i+1}$  from  $p_i$ . This will provide the "stochastic" model for the learning process.

In this simple case, it is shown that the problem is solved by the linear form

$$Q_1 p = p + a_1(1 - p) - b_1 p,$$

where  $Q_1$  is the linear operator giving rise to the result on the right;  $p$  is the proportion of successes achieved at the previous turn; and  $a_1$  and  $b_1$  are non-negative with  $0 \leq a_1 + b_1 \leq 1$ .

In the mathematical portion of this book the reader will find a remarkably clear exposition of such basic statistical tools and ideas as matrix operators, Markov chains, length of runs, the Monte Carlo method and maximum-likelihood estimators. It is the second part of this book which is supposed to be of primary interest to the psychologist. I feel compelled to say that the psychologist had better be a pretty good mathematician to get the full value of even this part. The mathematician does not, however, have to be a psychologist at all to gain considerable statistical knowledge and insight from this very readable and refreshingly original book.

—HARRY SILLER

#### BOOKS RECEIVED BY THE BOOK SHELF EDITOR

- Advanced Calculus*, Louis Brand, John Wiley and Sons, Inc., (440 Fourth Ave.) New York, 12 + 574 pp., \$8.50.  
*College Algebra*, Paul R. Rider, Macmillan, (60 Fifth Ave.) New York, 14 + 397 pp., \$4.00.  
*Fundamental Concepts of Mathematics*, R. H. Moorman, Burgess, (426 S. Sixth St.) Minneapolis, Minn., 92 pp., \$2.75.  
*Mathematics for Higher National Certificate*, Vol. 1, S. W. Bell and H. Matley, Cambridge University Press, 12 + 293 pp., \$2.75.  
*The Real Projective Plane*, H. S. M. Coxeter, Cambridge University Press, 11 + 226 pp., \$4.75.  
*The Teaching of Mathematics*, David R. Davis, Addison-Wesley Publishing Co., Cambridge, Massachusetts, 15 + 415 pp., \$5.00.  
*Theory of Functions of a Real Variable*, Leo F. Boron and Edwin Hewitt, Fredrick Ungar Publishing Co., New York, 277 pp., \$6.50.

## The Tenth Biennial Convention

The Tenth Biennial Convention of Kappa Mu Epsilon was held at Wayne State Teachers College, Wayne, Nebraska, on May 5, 6, 7, 1955, with Nebraska Alpha as host.

One hundred sixty official delegates, members, and guests were registered as follows:

| Chapter          | Students | Faculty | Chapter            | Students | Faculty |
|------------------|----------|---------|--------------------|----------|---------|
| Alabama Beta     | 1        | 2       | Michigan Gamma     | 2        |         |
| California Alpha | 1        |         | Mississippi Gamma  | 1        |         |
| California Beta  | 1        | 1       | Missouri Alpha     | 4        | 1       |
| Colorado Alpha   | 1        | 2       | Missouri Beta      | 4        | 1       |
| Illinois Alpha   | 4        | 1       | Missouri Gamma     | 2        | 1       |
| Illinois Gamma   | 7        | 1       | Missouri Epsilon   | 5        | 1       |
| Iowa Alpha       | 5        | 2       | Nebraska Alpha     | 29       | 5       |
| Iowa Beta        | 1        | 1       | New Mexico Alpha   |          | 1       |
| Kansas Alpha     | 1        | 3       | New York Alpha     | 1        | 1       |
| Kansas Beta      | 6        | 3       | Ohio Alpha         | 1        | 1       |
| Kansas Gamma     | 4        | 1       | Oklahoma Alpha     | 2        | 1       |
| Kansas Delta     | 3        | 2       | Pennsylvania Alpha | 1        |         |
| Kansas Epsilon   | 3        | 2       | Texas Delta        | 4        | 1       |
| Louisiana Beta   | 1        |         | Virginia Alpha     |          | 1       |
| Michigan Alpha   | 1        |         | Wisconsin Alpha    | 6        |         |
| Michigan Beta    | 9        | 2       |                    |          |         |

Nine student papers were read at the convention. The following is a list of these papers in the order they were read:

1. *Mathematics in Biology*, William Herrin, Missouri Epsilon, Central College.
2. *Integrating Even Powers of Sines and Cosines*, Jerome Donohue, Illinois Gamma, Chicago Teachers College.
3. *Mechanical Solution of Cubic Equations*, John Couch, Kansas Beta, Kansas State Teachers College, Emporia.
4. *Binary Systems and Digital Computers*, Carolyn Cusac, Missouri Alpha, Southwest Missouri State College.
5. *Invention of Quaternions*, Bernadine Law, Kansas Gamma, Mount St. Scholastica College.
6. *Nomography*, Eddie Dixon, Missouri Gamma, William Jewell College.
7. *Are Coordinate Systems Always Right?* Jean Crecelius, Missouri Beta, Central Missouri State College.
8. *Taxicab Geometry*, Morris Rosen, New York Alpha, Hofstra, College.

9. *Survey of Job Opportunities for Mathematicians*, Charles Deeter and Arnold Janousek, Kansas Epsilon, Fort Hays Kansas State College.

The Awards Committee, consisting of Rex Depew, Alabama Beta; Bernie Thelien, Iowa Alpha; Mabel Barnes and David Ford, California Beta; Emmet Stopher and Arnold Janousek, Kansas Epsilon; Phares O'Daffer and C. T. McCormick, Illinois Alpha, awarded the following prizes for excellenc of papers presented:

First place to Morris Rosen, New York Alpha.

Second place to John Couch, Kansas Beta.

Third place to Eddie Dixon, Missouri Gamma.

Honorable mention to Bernadine Law, Kansas Gamma.

Each of the first three place winners was presented with a mathematics dictionary.

Dr. R. H. Bing of the University of Wisconsin gave an invited address on *The Inventive Side of Mathematics* at the general banquet on Friday evening.

National officers were elected to serve for the biennium 1955-57. The results of the election were:

|                |                  |                |
|----------------|------------------|----------------|
| President      | C. C. Richtmeyer | Michigan Beta  |
| Vice-President | J. M. Sachs      | Illinois Gamma |
| Secretary      | Laura Z. Greene  | Kansas Delta   |
| Treasurer      | M. L. Madison    | Colorado Alpha |
| Historian      | Frank Hawthorne  | New York Alpha |

Invitations for the 1957 convention were extended by the following chapters: Louisiana Beta, Kansas Beta, Illinois Alpha, Kansas Alpha, Texas Delta.

The National Council voted life subscriptions to THE PENTAGON to C. V. Newsom, E. R. Sleight, and H. D. Larsen.

For her long and faithful service as national secretary, President Tucker presented Miss E. Marie Hove of New York Alpha with a secretary's symbol charm, the quill, mounted with the Kappa Mu Epsilon crest.

At the close of the convention, three resolutions were adopted, as follows:

1. Gratitude to Miss Hove who has served as national secretary since 1937.
2. Appreciation to (a) the host chapter, (b) the national offi-

- cers, (c) Dr. R. H. Bing of the University of Wisconsin, (d) the staff of THE PENTAGON, (e) the students who presented papers.
3. Suggestion to the President that he appoint committees to study (a) the possibility of regional meetings, (b) the desirability of a constitutional amendment to include the editor of THE PENTAGON as a member of the National Council, (c) to study travel expenses paid to delegates attending the convention.

### **Report of the President**

With the convention my term of office expires after four years as your national president. I wish to thank all of you for the opportunity to serve you. It has given me a tremendous amount of experience and contact with many of my contemporaries which I greatly appreciate.

It is difficult to know just what details to develop into needed changes of policies; the essential point being that I have no crystal ball to see what problems of my administration were minor and temporary, and which ones will develop into needed changes.

Last fall nearly all colleges had a good increase in their freshman enrollment. At Emporia we had an overall increase of about 50 per cent in enrollment. All colleges are anticipating an ever-increasing enrollment, reaching a peak between 1960 and 1964. Your national officers will see the effects of these increased enrollments as the years pass. I wish to mention two or three of the effects and their meanings to Kappa Mu Epsilon.

The freshmen of last fall will, during their sophomore and junior years, become eligible for membership in K.M.E. Our reports show that in the period 1949-1951 there were 1311 initiates; 1951-1953, 1133 initiates; 1953-1955, 1168 initiates. There is an increase of only 55 initiates in this biennium over the preceding biennium. My guess is that the next biennium will have well over 1300 initiates.

A second effect will be less obvious. Many chapters with only a small group from which to select their members may have found it necessary to use minimum standards in order to keep the size of the chapter large enough to be operative. This will become less necessary and the admission standards to K.M.E. chapters should improve while the size of the chapters will still increase.

The new officers should keep in touch with our inactive chapters. There is a possibility that the next few years will see some of the four inactive chapters become functional again. They should be encouraged to do so.

Last spring the chapters in Missouri and Kansas held a sectional meeting at Warrensburg, Missouri. The meeting was complete with student papers and a banquet with a speaker. I wish to recommend this idea of sectional meetings for the years between our biennial conventions. Roughly, I suggest that our groups may be divided into the following six sections:

- (1) Eastern Section: New York, New Jersey, Eastern Pennsylvania
- (2) Southeastern Section: Virginia, North Carolina, and South Carolina.
- (3) East Central Section: Illinois, Indiana, Michigan, Ohio, Wisconsin, and Western Pennsylvania
- (4) West Central Section: Colorado, Iowa, Kansas, Missouri, Oklahoma, and Nebraska
- (5) Southern Section: Alabama, Louisiana, Mississippi, New Mexico, Tennessee, and Texas
- (6) California Section: California.

To continue the ball rolling I should like to extend, on behalf of Kansas Beta, to the chapters of the West Central Section an invitation to hold a sectional K.M.E. meeting at Emporia next spring. Next fall invitations and information will be sent to all chapters concerned in order to make plans for this.

I advise the formation of a board of directors for K.M.E. This recommendation was suggested by our Past President, Dr. Henry Van Engen. I suggest that this be studied and, if feasible, that it be recommended as a constitutional change at the next convention. It seems to me this board should be composed of the national officers, the editor of *THE PENTAGON*, and business manager of *THE PENTAGON*, and a representative from each of the sections previously suggested which is not represented among the officers. Their duties and responsibilities should be carefully defined.

With 49 active chapters and prospects of continued or even more rapid growth, the problems of the secretary will become quite extensive. I appeal to each chapter to ease and simplify her work. Further, the national officers in the not-too-distant future will find need to arrange for a paid "executive secretary" to carry on the work of K.M.E. There is too much work to be handled in any other man-

ner. This executive secretary might well be a retired or semi-retired faculty member.

Again I would like to acknowledge the work of the various national officers, the committee members, the sponsors, and the corresponding secretaries over the past biennium. To all of these, I would like to express my appreciation for their support. It is their efforts that make K.M.E. what it is—on the basis of their efforts our organization will rise or fall.

Especially would I like to extend appreciation to Dr. Fronbarger for his work editing THE PENTAGON. He and his staff have done a splendid job, and we owe him a big debt of gratitude and high commendation. Also our Pentagon business manager, Professor Dana Sudborough, has done a splendid job.

In closing, I say again, thank all of you for the privilege of serving you for the past four years. I am sure that the officers you elect today will find it both a pleasure and an education to carry on this strong and ever-growing organization. They will serve you well and do their best, and may each and every one of you, as individuals and as chapters, give your new officers your full and unqualified support.

—CHARLES B. TUCKER

#### **Report of the Vice-President**

It has been a tradition that the length of the vice-president's report should be proportional to the amount of work carried on by that office. Since President Tucker has been most efficient, there has been little for me to do. Therefore this report will be brief.

I have enjoyed working with the members of the National Council for the past four years as vice-president. During that time I have seen the fraternity grow in size and strength. I am sure the future holds great promise of further growth and service by Kappa Mu Epsilon.

—CLEON C. RICHTMEYER

#### **Report of the National Secretary**

As secretary it is again my privilege to report to you upon the activity and the inactivity of the chapters of Kappa Mu Epsilon. Such a report must necessarily contain some statistics, but statistics which tell an interesting story.

Since our last convention, three new chapters have been installed, namely, Pennsylvania Beta, LaSalle College, Philadelphia, May 19, 1953, installed by Professor L. F. Ollmann, Hofstra College—Past National Treasurer; California Beta, Occidental College, Los Angeles, May 28, 1954, installed by Professor Chester G. Jaeger of California Alpha and Professor Mabel S. Barnes formerly of Nebraska Alpha; Virginia Alpha, Virginia State College, Petersburg, Virginia, January 29, 1955, installed by E. Marie Hove, Hofstra College, National Secretary.

It was my privilege to install the Virginia Alpha Chapter at Virginia State College in Petersburg, and a real privilege it was. Virginia State College is the first Negro college to be admitted to Kappa Mu Epsilon, and we have in them a chapter of which we can be extremely proud. I was much impressed with their faculty, their college, and their students. They are setting standards of which we can well be proud. It was with joy that I received the news that Dr. Hunter was coming to this convention. President McDaniel had told me when in Petersburg that he did not think that it would be possible to send a delegate this year, and Dr. Hunter had written that they would not be represented.

During the past biennium one chapter, Ohio Beta at the College of Wooster, has been placed on the inactive list. Kappa Mu Epsilon now has a total of 53 chapters located in 24 states—49 active chapters and 4 inactive chapters, with a total membership of 10,990.

During the past biennium 1142 persons have been initiated into Kappa Mu Epsilon in a total of 141 initiations. One chapter has held no initiation services and one chapter has held five initiations, but the more common practice is to hold one initiation per semester.

It has been my privilege to attend all ten of the national conventions, to have worked actively in four chapters of Kappa Mu Epsilon, and to have visited 13 other chapters by way of national conventions, installations, or speaking engagements. Kappa Mu Epsilon has come a long way since that first national convention held in Tahlequah, Oklahoma in 1933 where five of the then seven active chapters were represented.

I would not want to close this report without paying tribute to the many fine people with whom I have worked in Kappa Mu Epsilon. The correspondence in my files tells a most interesting story of

the work which has been done by the national officers during these many years. I must mention two persons not present in particular—both past presidents of Kappa Mu Epsilon, Professor E. R. Sleight, now retired, and Dr. C. V. Newsom, Associate Commissioner of Higher Education in New York and newly-elected vice-chancellor of New York University. To the past and present members of the National Council of K.M.E. who have given generously of their time and effort in carrying out the work of Kappa Mu Epsilon and to those chapters and officers who have given their fine support to my office, my sincere thanks. It has been a distinct pleasure to have served you these many years.

—E. MARIE HOVE

#### **Report of the National Historian**

During the past biennium THE PENTAGON has printed "Program Topics and News Items" in two of THE PENTAGONS. The Fall, 1953, issue, the convention issue, substituted biographical sketches of the National Council members and the minutes of the Convention for this item.

Several chapters had requested a history of Kappa Mu Epsilon so, in the fall of 1954, I submitted some information from the historian's files to Dr. Fronabarger who combined this data with material from the earlier PENTAGONS and minutes from some of the national conventions and wrote the article "Brief History of Kappa Mu Epsilon," which again replaced the usual items from the chapters. We hope that it has been of interest to your group.

At this time, I want to thank you for the privilege of serving the past four years. It has been a real pleasure.

—LAURA GREENE

#### **FINANCIAL REPORT OF THE NATIONAL TREASURER**

April 3, 1953 — April 4, 1955

|                            |           |            |
|----------------------------|-----------|------------|
| Cash on hand April 4, 1955 |           | \$ 3388.21 |
| Receipts from chapters     |           |            |
| Initiates 1168 at \$5.00   | \$5840.00 |            |
| Miscellaneous              | 141.08    |            |
|                            |           | \$5981.08  |

|                                                                     |                   |
|---------------------------------------------------------------------|-------------------|
| <b>Miscellaneous receipts</b>                                       |                   |
| Interest on bonds                                                   | 105.00            |
| Balfour Company (Commissions)                                       | 145.50            |
| THE PENTAGON (Surplus)                                              | <u>800.00</u>     |
|                                                                     | <u>1050.50</u>    |
| <b>Total receipts</b>                                               | <u>7031.58</u>    |
| <b>Total receipts plus cash on hand</b>                             | <b>\$10419.79</b> |
| <b>Expenditures (1953-1955)</b>                                     |                   |
| National Convention 1953                                            |                   |
| Paid to chapter delegates                                           | \$1472.42         |
| Officers expenses                                                   | 524.93            |
| Miscellaneous (Speaker, mimeo, entertainment, host chapter expense) | <u>95.10</u>      |
|                                                                     | 2092.45           |
| Balfour Company (membership certificates, charters, etc.)           | 556.22            |
| Installation expense                                                | 36.35             |
| <b>PENTAGON</b>                                                     |                   |
| (Printing, mailing 4 issues)                                        | 2582.93           |
| Prizes for 1955 Convention                                          | 25.94             |
| Bond for treasurer                                                  | 62.50             |
| National offices expense                                            | 137.25            |
| Refund Poudre Valley National Bank (Returned check)                 | <u>140.00</u>     |
| <b>Total expenditure</b>                                            | <u>5633.64</u>    |
| Cash balance on hand April 4, 1955                                  | \$4786.15         |
| Bonds on hand April 4, 1955 (Face value)                            | <u>2900.00</u>    |
| <b>Total assets as of April 4, 1955</b>                             | <b>\$7686.25</b>  |
| <b>Net gain for the period</b>                                      | <b>\$1397.94</b>  |

—M. LESLIE MADISON

**Report of the Editor of THE PENTAGON**

The first issue of THE PENTAGON appeared in 1941 with Dr. C. V. Newson as editor. He truly is the "father" of THE PENTAGON. Dr. H. D. Larsen served as editor from the fall of 1943 to the fall of 1952. Both of these men rendered a notable service to Kappa Mu Epsilon, giving unstintedly of their time and effort. They served as both editor and business manager of the magazine. The press of other duties made it necessary for Dr. Larsen to resign in the fall of 1952.

At the time of Dr. Larsen's resignation the National Council decided to separate the offices of editor and business manager. I was appointed editor and Professor Dana Sudborough of Michigan Beta was appointed business manager and has ably served in that capacity, thus relieving me of part of the responsibilities carried by the previous editors.

I accepted this appointment with some hesitancy, suspecting that the office would require a great deal of time and painstaking effort. My suspicions have been confirmed. However, there have been compensations for the effort in terms of my own personal development, and I hope that the readers have found the magazine to be both interesting and informative.

The constitution provides that the National Historian shall be the assistant editor of the official magazine, THE PENTAGON. Miss Laura Z. Greene, National Historian, has edited the Kappa Mu Epsilon News Department. Dr. Edward H. Matthews of the mathematics staff at Missouri Alpha has ably assisted with proof-reading of manuscripts, galley proofs, and page proofs. Associate editors: Book Shelf, Frank Hawthorne; Installation of New Chapters, Jerome M. Sachs; Mathematical Scrapbook, Harold Larsen; Problem Corner, Frank Gentry have each carried out their responsibilities faithfully and well.

During the time that I have served as editor, five issues have appeared with a total of 25 articles. Seventeen of these articles have been by students—one by a high school student, and the other 16 by students from colleges having chapters of Kappa Mu Epsilon. These 16 articles represent contributions from 12 Kappa Mu Epsilon chapters: Colorado Alpha, Iowa Alpha, Iowa Beta, Kansas Gamma, Michigan Alpha, Michigan Gamma, Missouri Alpha, Missouri Beta, New York Alpha, Ohio Gamma, Texas Epsilon, and Wisconsin Alpha.

It is very gratifying to the editor to be able to report the large number of student contributions. It is my hope that in the years ahead the student contributions will continue and that each chapter can be counted among those who have had at least one article published in THE PENTAGON.

The pages of THE PENTAGON offer an opportunity for students to present results of reading and investigation, especially along lines not likely to be included in regular class work. Preparation of such a report is a valuable experience for the student writer because of (1) the mathematical information obtained, (2) the experience gained in manuscript preparation, and (3) the personal satisfaction of seeing in print his own creation. Directions for the preparation of manuscripts are given in the Spring, 1955, issue.

I will welcome for consideration not only student manuscripts but also manuscripts from faculty members or others, provided the material is within the understanding of undergraduate students of mathematics. It is the ambition of the editor that THE PENTAGON be recognized as the outstanding magazine for undergraduate mathematics students published in this country. With your assistance, this goal can and will be achieved.

—CARL V. FRONABARGER

#### **Report of the Business Manager of THE PENTAGON**

I am sure that you all have read your programs and have noticed how these reports of the National Council of KME are arranged in order of unimportance. As errand-boy for Kappa Mu Epsilon, I should like to call your attention to the following facts:

1. In returning home from this beautiful campus and interesting convention, you still have something to look forward to—your copy of the Spring, 1955, PENTAGON, which should be waiting for you.
2. I said, "should be" rather than "is" because many of the 1500 copies which Mrs. Sudborough and I mailed last Tuesday afternoon will come back to us as "undeliverable" and with 5 cents postage due. You must realize how discouraging it is to have this happen after we have exercised considerable care to place on the mailing envelopes the exact addresses which were sent to us by the chapter corresponding secretaries. . . .

In view of what I have just reported, I am sure that

you realize the importance of keeping your business manager informed of your correct address.

- 3. It should be well known, but apparently is not, that when a person joins Kappa Mu Epsilon he automatically receives a two-year subscription to THE PENTAGON. This includes four issues since THE PENTAGON is published in May and December of each calendar year. . . . Whenever a person enters our fraternity, his subscription to the official journal commences with the most recent issue. This means that a person joining in April, 1955, for example, will have his subscription started with the Fall, 1954, issue, unless so many copies have been sent out by that time that the only copies remaining in the business manager's office are the 100 in reserve.

. . . When the last copy for which you have paid arrives, there will be inserted in it a sheet of colored paper on which will be stated the pertinent facts.

So much for the more unpleasant remarks it has been my duty to report; I should now like to give you a few interesting items about our magazine.

- 4. Who are subscribers to THE PENTAGON? I shall attempt to answer that question by a break-down of the distribution of the current issue as an example.

Of the 1800 copies printed, 100 will be kept in reserve for various uses over a long period of time, 200 will be sent to new fraternity members as their names come into my office—and I might say now that if newcomers are initiated in as great numbers between now and next December as they have been during the last few months, the 200 copies will be "sold out" all too soon. It looks now as if we should order over 2,000 copies printed of the Fall, 1955, issue.

The 1500 Spring, 1955, PENTAGONS were distributed as follows:

|                     |     |
|---------------------|-----|
| Students and alumni | 73% |
| College professors  | 10% |
| College libraries   | 8%  |

(includes almost all colleges and universities in the U.S.A.)

|                       |    |
|-----------------------|----|
| Miscellaneous (paid)  | 5% |
| High School libraries |    |

|                                               |    |
|-----------------------------------------------|----|
| and teachers                                  | 3% |
| Book publishers, Balfour,<br>authors, editors | 1% |

5. Where are the subscribers to THE PENTAGON? They are in all of the 48 states and the District of Columbia, except Idaho, Montana, Nevada, Vermont, and West Virginia. They are in Canada, France, Greece, New Zealand, Argentina, Iran, Iraq, and Thailand, to mention a few.
6. In closing, I should like to point out that in view of the above distribution, the editor, associate editors, and all us members of Kappa Mu Epsilon have a right to be proud of our magazine. Incidentally, THE PENTAGON was chosen by a committee of mathematicians to be one of the American publications to be on display at the International Congress of Mathematicians in Amsterdam last summer. I thank you,

—DANA R. SUDBOROUGH



## Year 1955

$$\begin{aligned}
 & [(2h + k)^7 + (14h + k)^7 + (39h + k)^7 + (45h + k)^7 \\
 & \quad + (76h + k)^7 + (85h + k)^7 + (103h + k)^7] - \\
 & [(h + k)^7 + (19h + k)^7 + (28h + k)^7 + (59h + k)^7 \\
 & \quad + (65h + k)^7 + (90h + k)^7 + (102h + k)^7] = 1955, \\
 & \text{if } h = (23/1150269120)^{1/7} \text{ and } k \text{ is arbitrary.}
 \end{aligned}$$

—ALFRED MOESSNER

## Biographical Sketches

**Charles Bartlett Tucker**, Associate Professor of Mathematics at Kansas State Teachers College, Emporia, Kansas, is a native of Rhode Island. He received his B. S. degree from Antioch College, Yellow Springs, Ohio, and his M. S. from Brown University, Providence, R. I. He did additional graduate work at Brown University while serving as an instructor in mathematics there, and he has also done graduate work at the University of Minnesota. He has been a member of the mathematics staff at Kansas State Teachers' College, Emporia, since 1935. During World War II he was on leave from the college to serve as a physicist at the Johns Hopkins Applied Physics Laboratory, Silver Spring, Md. He was elected National President of Kappa Mu Epsilon in 1951 and again in 1953. He is now a member of the National Council as the Immediate Past President.

+ + +

**Cleon C. Richtmeyer**, Director of Instruction at Central Michigan College, Mount Pleasant, Michigan, has been a member of the National Council of Kappa Mu Epsilon since 1947. He served as National Historian from 1947 to 1951, then as Vice-President until 1955. He is now serving as President.

Dr. Richtmeyer received his bachelor's degree from Albion College, his master's from George Peabody, and his Ph.D. from Colorado State College of Education.

+ + +

**Laura Z. Greene** is a member of the Department of Mathematics of Washburn University of Topeka, Kansas. She did her undergraduate work at Washburn and received her S. M. degree at the University of Chicago.

Miss Greene served as Historian for Kappa Mu Epsilon from 1951 to 1955. She is now National Secretary.

+ + +

**M. Leslie Madison**, a native of Colorado, received his B. S. degree and his M. S. from Colorado University. He did additional graduate work at the University of California and has been on the staff of Colorado A & M since 1934 with the exception of five years which he spent with the army during World War II.

Professor Madison is serving his second term as Treasurer of Kappa Mu Epsilon.

+ + +

**Frank Hawthorne**, was born in a sod house in the sand hills of Nebraska. He took his bachelor's degree from Edinboro (Pa.) and a master's from Columbia. He also studied at Penn State and Allegheny. He is now Assistant Professor of Mathematics at Hofstra College. From 1952 to 1955 he edited the book shelf section for THE PENTAGON. He is now National Historian.

+ + +

**Jerome Sachs** took his bachelor's, master's, and doctor's degrees from the University of Chicago. He also attended Crane Junior College and the University of Illinois. He is now a member of the department of mathematics at Chicago Teachers College.

Prof. Sachs has written installation reports for THE PENTAGON and is now editing the scrapbook section for this magazine. He is a member of the National Council and is serving as Vice-President.

+ + +

**Carl V. Fronabarger**, of Southwest State College, Springfield, Missouri, received his bachelor's degree from Southeast Missouri State Teachers College, his master's from George Peabody, and his Ph.D. from the University of Missouri.

Dr. Fronabarger served as editor of the book shelf section of THE PENTAGON from 1950 to 1952. He has been editor of THE PENTAGON since the fall of 1952.

+ + +

**Dana R. Sudborough**, Business Manager of THE PENTAGON, is an Associate Professor of Mathematics at his alma mater, Central Michigan College, Mount Pleasant. Mr. Sudborough took his M.S. degree at the University of Michigan and has since done a considerable amount of graduate work in mathematics at Ann Arbor and also at Michigan State University.



$$\begin{aligned}
 & (1955^2 - 1955)/(1955^2 + 1) \\
 & + (1955^2 + 1955)/(1955^2 + 1) \\
 & = (1955^2 - 1955)^2/(1955^2 + 1)^2 \\
 & + (1955^2 + 1955)^2/(1955^2 + 1)^2
 \end{aligned}$$

—ALFRED MOESSNER



**Charles B. Tucker**  
Past National President



**Cleon C. Richtmeyer**  
National President



**Jerome M. Sachs**  
National Vice-President



**Laura Z. Greene**  
National Secretary



**M. Leslie Madison**  
National Treasurer



**Frank Hawthorne**  
National Historian

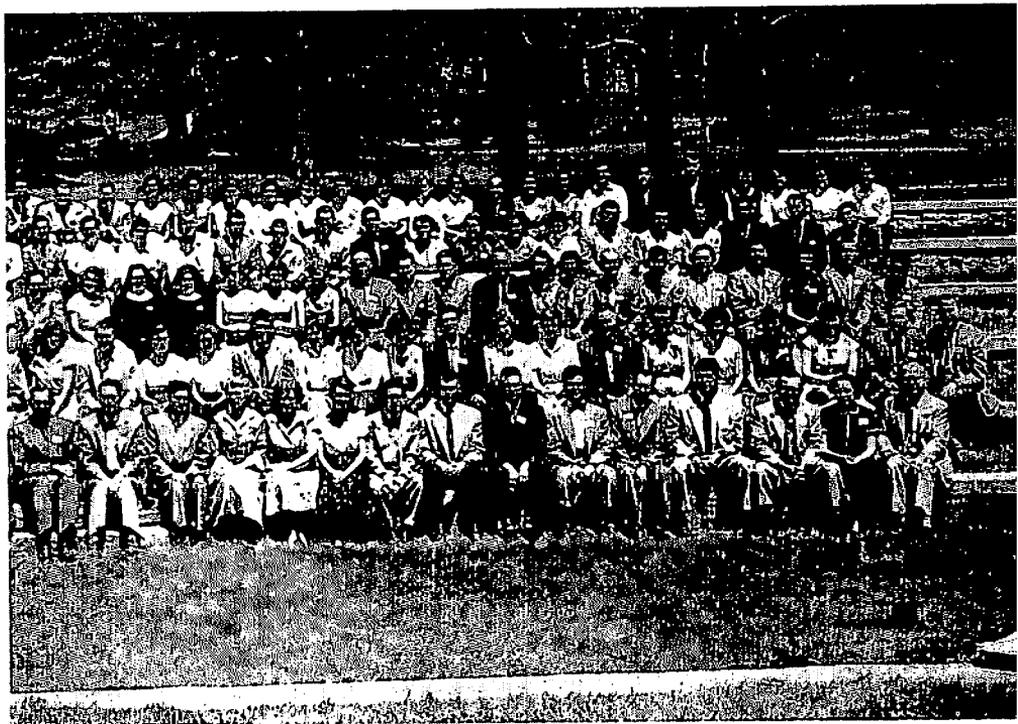
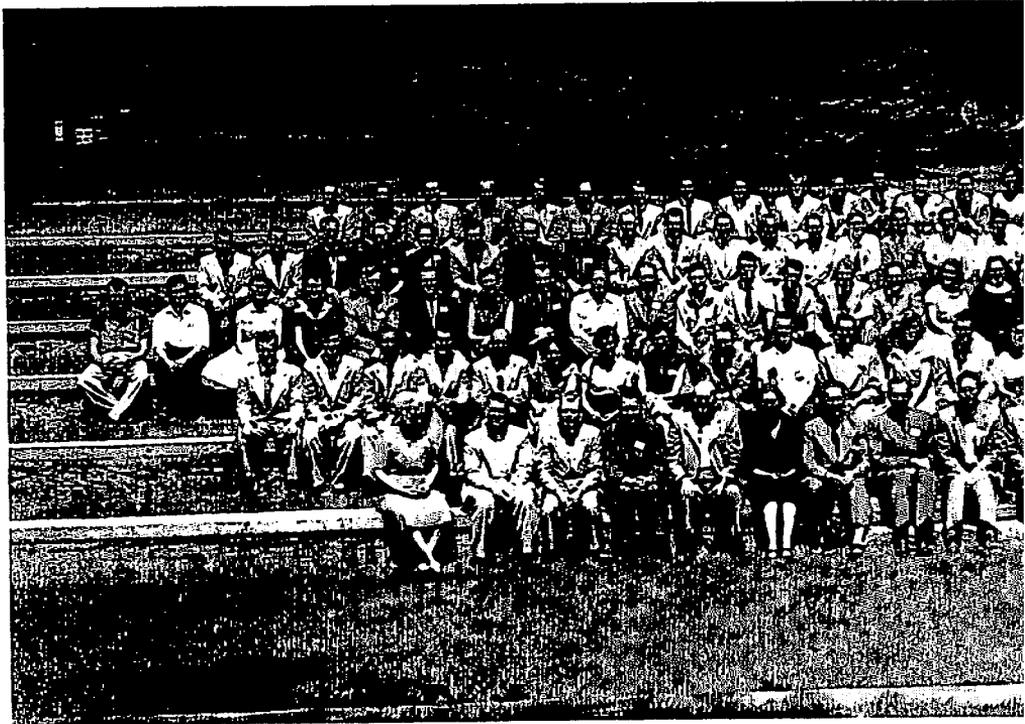


**Carl V. Fronabarger**  
Pentagon Editor

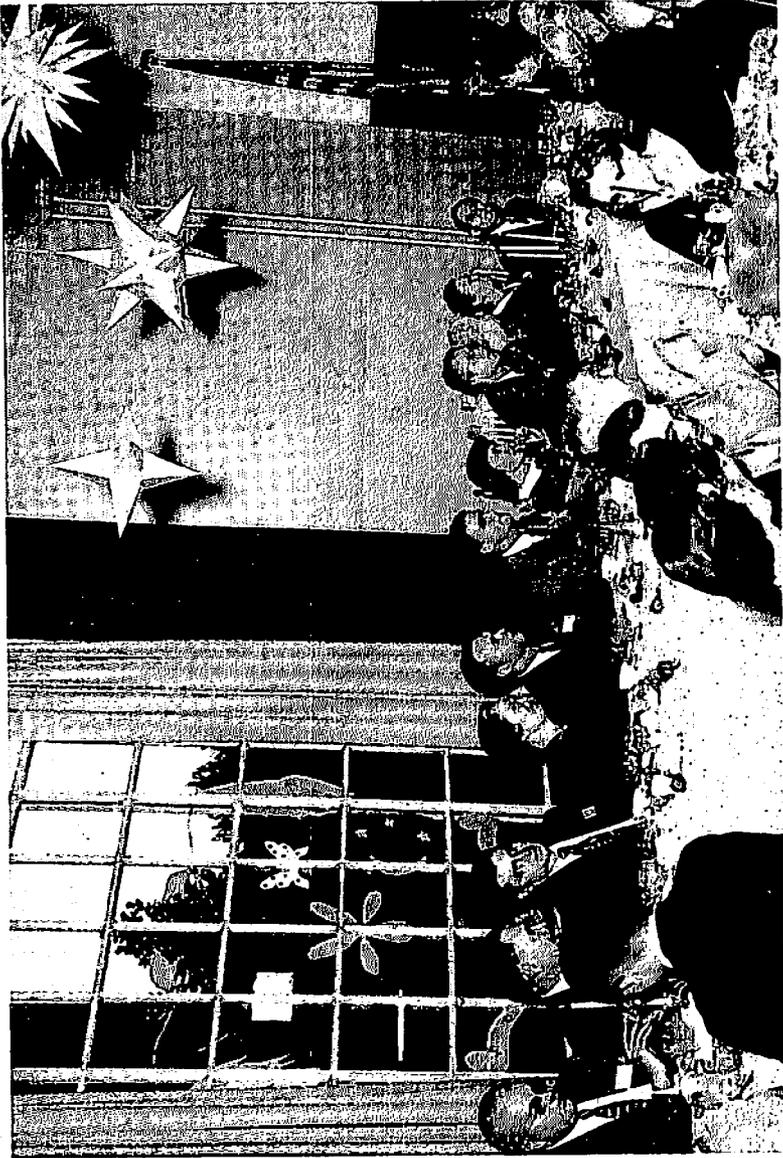


**Dana R. Sudborough**  
Business Manager of  
The Pentagon

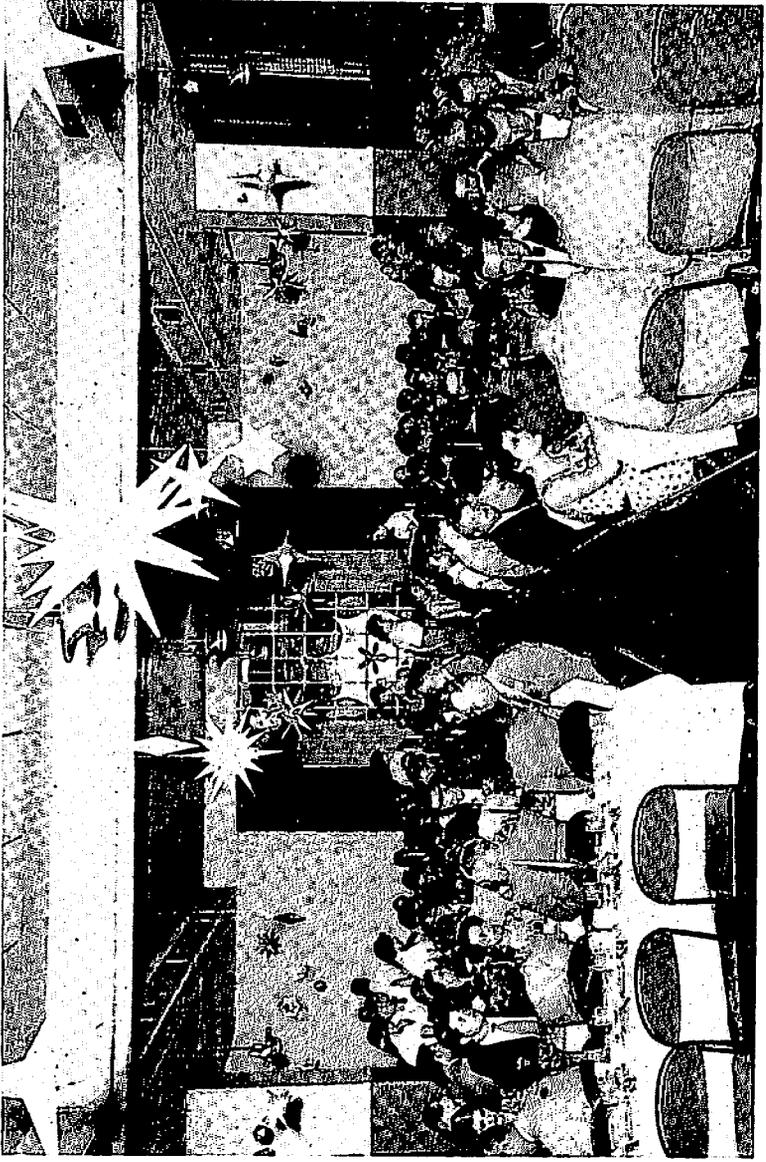
Biographical Sketches, pages 63-64



**1955 Convention**



**Speaker's Table, Convention Banquet 1955**



**1955 Convention Banquet**