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CONTENTS

THE HYPATIA OF THE NINETEENTH CENTURY

MARY LOU MALONEY
Mount St. Scholastica College

Few people understand why women's achievements in science, compared with those of men, have been so few and of so small import, or why it is that we hear so little of her in times past. They ask why she so very seldom appeared in the scientific world before the second half of the nineteenth century. It is these people who do not realize the intensity of woman's age-long struggle for freedom and justice in things of the mind.

As in all countries, French women too felt the hampering hand of prejudice. Even up to the age of Louis XIV women were much retarded in their quest for education by an environment which was becoming daily more and more unfavorable to the higher education of women. A young girl's education was deemed complete if she was able to read, write, dance, and play some musical instrument. Anything more was superfluous and deserving of censure and ridicule rather than praise.

During this period appeared what were perhaps to become the two greatest factors in deterring the education of women: the two plays of Molière, *Les Femmes Savants*, and *Les Précieuses Ridicules*. They were ample arsenals which supplied the opponents of education-for-women with the arms needed to decide, in their favor, the long warfare against the gentler sex. Replete with great sarcasm, the plays were merely an expression of prevailing opinion aptly worded by Molière:

"It is not seemly, and for many reasons,
That a woman should study and know so many things."[1]

1. Jean Baptiste Paquelin Molière, *Les Femmes Savants*, Act II, Scene 7.

Few women had the courage to defend their sex against such condemnations as found in the words of Molière's "aide de campe", the comic dramatist, Destouches:

> "A learned woman ought—so I surmise—conceal
> her knowledge or she'll be unwise.
>
> Must keep the level of the common kind,
> To subjects commonplace devote her mind.
>
> That knowledge shall not make her seem unwise,
> She must herself in foolishness disguise."[2]

However, France did produce a few exceptions like Mme. du Châlelet, Sophie Germain, and Mme. Lepaut; women who dared to face the ridicule which was inevitable if they devoted themselves to science or philosophy.

Sophie Germain is perhaps one of the most interesting of the French women students. She was a profound scholar and was called by De Prony "the Hypatia of the nineteenth century." The world can acclaim her as one of the founders of mathematical physics. It has been said that the theory of elasticity belongs to the nineteenth century and that the establishment of the broad outlines of this theory was accomplished almost exclusively by French writers, principal among whom was Mademoiselle Sophie Germain. Her success, however, was the result of a struggle of aspiring and acquisitive young womanhood in opposition to many obstacles, the chief among which was the opposition of her family.

Perhaps we can best realize her determination to continue the study of this science which had become her passion if we consider a typical evening in France about the year 1791. A young girl is being fondly put to bed by her parents. But we note that before leaving the room the father and mother thoroughly and systematically gather all the clothing in the room, extinguish the fire, and leaving, lock the door securely. The girl appears resigned, but the house is scarcely still before she rises, wraps herself

2. Néricault Destouches, *L'Homme Singulier*, Act III, Scene 7.

in quilts and blankets, and applies herself to some books in the corner. Thus Sophie Germain spent many nights in spite of the intense cold which we are told often froze even the ink in her inkwell. Perhaps her parents would find her in the morning, chilled through, and would say again, "Of what use is Geometry to a girl?"[3]

But in 1816, twenty-five years later, we find a pleasant bustle in a large room filled with the savants of France. It is a public séance of the *Institut de France.* The last memoir of Mlle. Germain on vibrating surfaces is to be crowned publicly. It is the triumph of eight years of work by this profound scholar on the mathematical theory of the vibrations of elastic surfaces, a work which won for Sophie Germain the "Grand Prix" of the French Academy. It is from this memoir that we have the often quoted line, "Algebra is but written geometry and geometry is but figured algebra."[4]

Mlle. Germain entered the philosophical as well as the mathematical field. In addition to her *Memoire sur la surfaces elastiques,* she published a philosophical work, *Considerations Generales sur L'Etat des Sciences et des Lettres aux Différentes Epoques de Leur Culture.*

Considering all points of the question she was probably the most profoundly intellectual woman which France can boast; and yet, strange to say, even her death certificate did not give her the distinction of "mathematicienne." When the Eiffel Tower was erected and inscribed with the names of seventy-two savants, the name of France's daughter of genius, Sophie Germain, was not included.[5] Is it because she was a woman? Past history would seem to indicate that this is woman's fate. But if this be the case, the more ungrateful are those responsible, for this woman deserved well of science. We of today cannot hesitate to give her an enviable place in the Hall of Fame. May she serve as a well-cast model for the "Hypatias of Today".

3. H. J. Mozans, *Woman in Science,* p. 154.
4. Robert Edouard Moritz, *Memorabilia Mathematica,* p. 276.
5. H. J. Mozans, *Woman in Science,* p. 156.

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"These two good friends, and two of Britain's most famous scientists, Newton a mathematician and physicist, and Locke a great philosopher, were walking along the English country side one day when Newton, as was his frequent custom, began working mentally on a problem which was interesting him. After carrying out the investigation he obtained a result, and turning to his friend, Locke, he said, 'And that gives us x'. Just to carry on the conversation, and appear interested, Locke replied, 'Does it?' Newton, surprised and worried, cried, 'Doesn't it?' At once he plunged into the solution again, found that he had made a mistake, and said to Locke, 'You were right, it does not give us x, it gives us y'. Forever after, Sir Isaac Newton regarded John Locke the most erudite of philosophers, never dreaming that Locke had no idea of the problem which Newton had solved."

—NATIONAL MATHEMATICS MAGAZINE.

ON TRISECTING AN ANGLE[1]

BARBARA STEINBERG

Queen's College

There is a proof, or so I'm told
By scientist respected,
That certain angles stubbornly
Refuse to be trisected
With compass, straight edge, pen and ink.—
And that's the only ticket,
For Euclid says that other tools
Simply are not cricket.
Yet every year some hardy souls
Will publish refutations
Of scientific treatises
And learned dissertations.
They send out methods by the score
And hope that by insistence
They will succeed in lowering
The scientists' resistance.
Some hint that mathematicians
(Ugly insinuation)
Claim no solution's possible
Out of sheer desperation.
You think you've found a method, eh,
Which cannot be refuted?—
I tell you, sir, I know it's right!
Aha, but is it Euclid?

1 From *The American Mathematical Monthly*, Vol. 51, p. 398 (Aug.-Sept., 1944).

MAGIC SQUARES

FRANK LANE

University of New Mexico

A magic square is defined as a square which is divided into n^2 sub-squares arranged in n columns and n rows, each sub-square containing such a number that the sum of the numbers in each column, each row, and each diagonal is the same. Each number associated with a sub-square is known as an element of the square. The number n is called the order of the square; thus, a magic square with four columns and four rows will be called a magic square of fourth order.

Magic squares were known in China and India before the Christian era. Like much of the mathematics known to the ancients, magic squares were associated with mysticism; some were given the power to expel devils and others the power to protect the possessor from mishaps. A magic square of fourth order is engraved on the gate of the fort of Gwalior, India. Another magic square of fourth order appears on the painting "Melancholy" by Albert Dürer.

Much of the work on the theory of magic squares was done by a group of French mathematicians, the most important contributions being made by De La Hire. In fact, the method of constructing magic squares which will be discussed in this paper is attributed to De La Hire.

Probably the most interesting magic squares are those composed of the first n^2 integers arranged in n rows and n columns. Others, such as those composed of the first n integers each occurring n times, are so much simpler that in general they can be constructed at will with little knowledge of the theory of magic squares. The following discussion will be devoted entirely to finding a means of constructing some of the nth order squares composed of the first n^2 integers, although squares of other types will be required in the construction. With the suggested restric-

tion, there is one magic square of first order, none of second order, and only one distinct magic square of third order; that is, all other squares of third order can be derived from one basic form by rotation or reflection. Beyond the third order, the number of possible squares increases rapidly. There are 880 squares of the fourth order. No enumeration of possible squares of order higher than the fourth has been made, but it is known that those of the fifth order number in the hundreds of thousands.

Before beginning the solution of the problem, it seems worth while to know whether there is some law that governs the manner in which the n^2 numbers must be distributed within the columns and rows of the square; that is, is the sum of the rows and columns already determined when n is known or can the sum be made any desired number? There are n^2 numbers to be placed in n rows in such a way that the sum of each row shall be the same. The sum of the first n^2 integers is given by $n^2(n^2+1)/2$, which means that the sum of the numbers in each row must be $n(n^2+1)/2$. The columns and diagonals of a magic square must also have this same sum, so the sum is completely determined by the value of n.

Notice, now, that all of the numbers $1, 2, \cdots, n^2$ which are to be the elements of the nth order square can be written in the form $nA+B$, where A is one of the numbers $0, 1, \cdots, (n-1)$ and B is one of the numbers $1, 2, \cdots, n$. As an example, when n is three, $1 = 0 \cdot 3 + 1$, $6 = 1 \cdot 3 + 3$, $7 = 2 \cdot 3 + 1$, and so on for all the numbers 1 through 9.

Now the problem is broken in half. Instead of an nth order square with n^2 different integers, two auxiliary squares of nth order will do provided the first auxiliary square contains in each row, column, and diagonal numbers which are possible for A, and the second auxiliary square contains numbers which are possible for B. The auxiliary squares will be constructed in such a way that they are magic; that is, the sum of the elements of each row, the sum of the elements of each column, and the sum of the elements of each diagonal are all equal.

After two suitable auxiliary squares have been selected, all that need be done to form the main square is to multiply an element of the first square by n, add to it the element of the second square which holds a like position, and place the sum as the corresponding element of the main square. The main square thus formed will always be magic for the auxiliary squares are magic, and the sum of the elements of each row, column, and diagonal will be $n^2(n-1)/2 + n(n+1)/2$, which is $n(n^2+1)/2$. Care must be taken, however, that the auxiliary squares are such that the n^2 numbers which form the elements of the main square are distinct, for the elements of the auxiliary squares may be in such a position that the same number will appear as two or more elements of the main square. An example of this is given for a third order square. Use for the first auxiliary square: first row 1, 0, 2; second row, 2, 1, 0; third row, 0, 2, 1. For the second auxiliary square use: first row 2, 1, 3; second row 3, 2, 1; third row 1, 3, 2. The resulting main square will be: first row 5, 1, 9; second row, 9, 5, 1; third row 1, 9, 5. This is certainly undesirable from our point of view.

A magic square of nth order is a very complex structure, so it seems necessary to establish some sort of reference device as an aid in discussing such a structure. The scheme used will be similar to the latitude-longitude system used for position measurements on the surface of the earth. The numbers $0, 1, \cdots, (n-1)$, in order, will be associated with the columns from left to right. Similarly, the same n numbers will be associated with the rows from top to bottom. The column of the square will be indicated first; thus, the ijth element will mean the element of the square which appears in the ith column and the jth row. As further conventions of notation that will be used here, the first auxiliary square will be filled with the n distinct numbers $a_0, a_1, \cdots, a_{n-1}$ where each a_k will be one of the numbers $0, 1, \cdots, (n-1)$; similarly, the second auxiliary square will be filled with the n distinct numbers $b_0, b_1, \cdots, b_{n-1}$ where each b_k will be taken from the list $1, 2, \cdots, n$.

Inasmuch as there are only n distinct numbers from which to select a_k or b_k, a number such as a_{n+k} shall mean the same thing as a_k, and b_{n+k} shall mean b_k. Always the a's and b's will have meaning no matter what their subscripts might be.

Magic squares of nth order are usually divided into three classes. In increasing order of difficulty of construction these are: class 1, n odd; class 2, n even and divisible by four; class 3, n even but not divisible by four. Squares of class 3 are constructed principally by empirical rules and will not be considered in this paper.

CLASS 1 (n odd).

Fill the first auxiliary square in such a way that the ijth element will be a_{i+j}. The elements of the square will appear as in Fig. 1. Each row and each column contains all of the numbers $a_0, a_1, \cdots, a_{n-1}$ in some order so that the rows and columns are magic. The sum of these n numbers is $n(n-1)/2$.

On the descending diagonal $i=j$, whence the ijth element is a_{2i}. But the numbers a_{2i}, where $i=0, 1, \cdots, (n-1)$, are equal in some order to the numbers $a_0, a_1, \cdots, a_{n-1}$. Hence, the descending diagonal has the magic sum $n(n-1)/2$.

The ascending diagonal is composed entirely of the element a_{n-1} written n times. In order to have the square magic, na_{n-1} must be equal to $n(n-1)/2$, or $a_{n-1}=(n-1)/2$.

Now that the first auxiliary square has been constructed, the second auxiliary square is filled with the elements b_{i-j}, as shown in Fig. 2. Notice that, as in the first auxiliary square, the elements in each of the rows and columns are, in some order, the numbers $b_0, b_1, \cdots, b_{n-1}$ and thus the

$$\begin{array}{ccccc} a_0 & a_1 & a_2 & \cdots & a_{n-1} \\ a_1 & a_2 & a_3 & \cdots & a_0 \\ a_2 & a_3 & a_4 & \cdots & a_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n-1} & a_0 & a_1 & \cdots & a_{n-2} \end{array}$$

FIG. 1.

$$\begin{array}{ccccc} b_0 & b_1 & b_2 & \cdots & b_{n-1} \\ b_{n-1} & b_0 & b_1 & \cdots & b_{n-2} \\ b_{n-2} & b_{n-1} & b_0 & \cdots & b_{n-3} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ b_1 & b_2 & b_3 & \cdots & b_0 \end{array}$$

FIG. 2.

rows and columns are magic. The sum of these numbers is $n(n+1)/2$.

Along the ascending diagonal $i = n - j - 1$ so the elements of this diagonal are b_{n-2j-1}, $j = 0, 1, \cdots, (n-1)$. These are, in some order, the numbers $b_0, b_1, \cdots, b_{n-1}$. Thus, the ascending diagonal contains the magic sum $n(n+1)/2$. On the other hand, the descending diagonal contains the number b_0 written n times, so in order for the square to be magic, nb_0 must be equal to $n(n+1)/2$, or $b_0 = (n+1)/2$.

One problem remains. Is it possible to prove that the elements $na_{i+j} + b_{i-j}$ of the resulting magic square are distinct? Let us assume that there are two such elements which are equal; that is, assume $na_{i+j} + b_{i-j} = na_{p+q} + b_{p-q}$. This can be written as $b_{i-j} = b_{p-q} + nP_1$ where P_1 is an integer. Now b_{i-j} and b_{p-q} are among the integers $1, 2, \cdots, n$ so their difference cannot exceed n. Therefore $P_1 = 0$ and $b_{i-j} = b_{p-q}$. Also $na_{i+j} = na_{p+q}$, so $a_{i+j} = a_{p+q}$. This means that $i + j = p + q + nP_2$ and $i - j = p - q + nP_3$ where P_2 and P_3 are integers. Solvin terms of p and q, we find $i = p + n(P_2 + P_3)/2$ and $j = q + n(P_2 - P_3)/2$. But both i and j are less than n, so that $P_2 + P_3 = 0$ and $P_2 - P_3 = 0$. Thus, $P_2 = P_3 = 0$, whence $i = p$ and $j = q$. We conclude that the numbers which make up the elements of the main square are distinct. The magic square so formed must be composed of the first n^2 integers.

Perhaps the method will become evident with an example of a fifth order square formed this way. With

4	1	0	3	2
1	0	3	2	4
0	3	2	4	1
3	2	4	1	0
2	4	1	0	3

First Auxiliary Square

3	4	1	5	2
2	3	4	1	5
5	2	3	4	1
1	5	2	3	4
4	1	5	2	3

Second Auxiliary Square

23	9	1	20	12
7	3	19	11	25
5	17	13	24	6
16	15	22	8	4
14	21	10	2	18

Resulting Magic Square

FIG. 3.

$n = 5$, a_{n-1} which is a_4 must be 2, but a_0, a_1, a_2, and a_3 may be associated with the numbers 0, 1, 3, 4 in any way desired, such as $a_0 = 4$, $a_1 = 1$, $a_2 = 0$, and $a_3 = 3$. Similarly, b_0 must be 3, but the other elements may be picked at will, say $b_1 = 4$, $b_2 = 1$, $b_3 = 5$, $b_4 = 2$. With this choice, the element in the first column and first row of the main square will be $1 \cdot 5 + 4 = 9$, and so on for all 25 elements. The auxiliary squares and the resulting magic square appear as in Fig. 3.

CLASS 2 (n divisible by 4).

Fill the first auxiliary square with the numbers a_{i+2mj} where $m = n/4$. With this rule for forming the elements, each row consists of the numbers $a_0, a_1, \cdots, a_{n-1}$ in some order, and so has the magic sum $n(n-1)/2$.

Along the descending diagonal $i = j$, so the elements are of the form $a_{i(1+2m)}$, $i = 0, 1, \cdots, (n-1)$. Since $1 + 2m$ and n are relatively prime, these n numbers will be distinct and therefore equal, in some order, to $a_0, a_1, \cdots, a_{n-1}$. Thus, the descending diagonal has the desired sum.

Along the ascending diagonal $i = n - j - 1$, so the general element will be $a_{n-j-1+2mj}$ which is equivalent to $a_{(2m-1)j-1}$. Now, since $2m - 1$ and n are relatively prime, $a_{(2m-1)j-1}$ will have n distinct values as j takes on the values $0, 1, \cdots, (n-1)$. It must be true, then, that the ascending diagonal contains all of the numbers $a_0, a_1, \cdots, a_{n-1}$ and so has the desired magic sum.

Each column, however, contains the elements a_i and a_{i+2m} a total of $2m$ times each. In order for the columns to be magic, $2m(a_i + a_{i+2m})$ must equal $n(n-1)/2$, or $a_i + a_{i+2m} = n - 1$. With this one restriction the first auxiliary square is magic.

The second auxiliary square is filled with the numbers b_{2mi+j}. This differs from the first auxiliary square only in that the i and j have been interchanged, so the reasoning used in the discussion of the first auxiliary square will apply if i is read as j, j as i, row as column, column as row, descending diagonal as ascending diagonal, and ascending diagonal

as descending diagonal. The rows will be magic only if $b_j + b_{2m+j} = n + 1$.

The remainder of the problem is to prove that the resulting n^2 elements in the main square are distinct. Assume that the ijth element is equal to the pqth element. Then by an argument similar to that used in the preceding case, it can be shown that $i = p$ and $j = q$. Hence, if two elements are equal they must have the same position in the square and consequently are the same element. Therefore, the n^2 elements of the resulting square are distinct.

This method may now be applied to a fourth order magic square. When $n = 4$ and therefore $m = 1$, we must have $a_i + a_{i+2m} = 3$ so that if $a_0 = 3$ and $a_1 = 2$, a_2 must be 0 and a_3 must be 1. Also, $b_i + b_{i+2m} = 5$, so $b_0 = 2$ and $b_1 = 4$ requires that $b_2 = 3$ and $b_3 = 1$. These values give the auxiliary squares and final magic square shown in Fig. 4. There are, of course, other values that could be

3	2	0	1
0	1	3	2
3	2	0	1
0	1	3	2

First Auxiliary Square

2	3	2	3
4	1	4	1
3	2	3	2
1	4	1	4

Second Auxiliary Square

14	11	2	7
4	5	16	9
15	10	3	6
1	8	13	12

Resulting Magic Square

FIG. 4.

assigned to a_0, a_1, b_0, and b_1: a_0 can be any of the permitted values, but a_2 is then fixed; a_1 can be either of the two remaining permitted values, but a_3 is then fixed. A similar process is used in finding combinations of the b's.

The method outlined above is fairly general and will permit construction of numerous magic squares, but still only a fraction of the total number of squares possible. The method can be generalized by filling the auxiliary squares with a_{i+rj} and b_{si+j} or some such general elements, but even then the totality of squares will not be represented. In fact, no general formula for constructing all magic squares has yet been devised.

TOPICS FOR CHAPTER PROGRAMS—II

The first of a series of articles presenting bibliographies on subjects which are suitable for chapter programs was published in the Spring, 1946, number of the PENTAGON. The subjects covered in the first article were 1) Women as Mathematicians, 2) The Cattle Problem of Archimedes, and 3) Paper Folding. In continuing this series, the editor again urges each person who presents a paper on a chapter program to submit a bibliography. Any suggestions for making this series as complete and useful as possible will be appreciated.

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"Above all, adepts find in mathematics delights analogous to those that painting and music give. They admire the delicate harmony of numbers and of forms; they are amazed when a new discovery discloses for them an unlooked-for perspective; and the joy they thus experience, has it not an esthetic character, although the senses take no part in it? Only the privileged few are called to enjoy it fully, it is true; but is it not the same with all the noblest arts?"

—HENRI POINCARE.

A BRIEF HISTORY OF THE FOURTH DIMENSION

HELENA WEIGAND

College of St. Francis

The geometry of more than three dimensions is a modern branch of mathematics which had its beginning in the first part of the nineteenth century. However, before this time there were some early references to the number of dimensions of space. In the first book of *Heaven*, Aristotle writes, "The line has magnitude in one way, the plane in two ways, and the solid in three ways, and beyond these there is no other magnitude because the three are all." [*4*, p. 1].* Ptolemy pointed out in his book, *On Distance*, that three mutually perpendicular lines could be drawn in space, but a fourth perpendicular to these would be without measure or definition [*3*, p. 130]. Thus, for centuries equations of degree higher than the third were regarded as unreal. For example, Stefel (1486-1567) in the *Algebra* of Rudolph speaks of "going beyond the cube just as if there were more than three dimensions. Which is," he adds, "against nature." [*4*, p. 3]. Also, John Wallis in his *Algebra* says, "Length, Breadth, and Thickness take up the whole of Space. Nor can Fansie imagine how there should be a Fourth Local Dimension beyond these three." [*4*, p. 5]. However, Ozanam (1640-1717) does admit the imaginary existence of more than three dimensions when he says in his *Dictionaire mathematique*, "A product of more than three letters will be a magnitude of as many dimensions as there are letters, but it will only be imaginary because in nature we do not know of any quantity which has more than three dimensions." [*4*, p. 6].

In the writings of some philosophers we find references to a space of four dimensions. In a book published

* Numbers in brackets refer to the literature cited at the end of this paper.

in 1671, Henry More says that spirits have four dimensions, and Kant (1724-1804) has several references in his works to the dimensions of space [*4*, p. 7].

After much discussion and disputation, it was suggested by several writers in the latter part of the eighteenth century that mechanics be considered a geometry of four dimensions with time as the fourth dimension. Lagrange advanced this theory in 1797 in his book, *Theorie des fonctions analytiques*, and he is usually given credit for the idea. The same idea was also expressed by d'Alembert in an article on "Dimension" published in 1754. These are almost the only instances in which we find the subject of the fourth dimension referred to before 1827 [*4*, p. 8].

We may distinguish definite contributions to the subject in the period beginning with 1827. As far as we know, the first contributions to the geometry of four dimensions was made in 1827 by Möbius who pointed out that symmetrical figures could be made to coincide if there was a space of four dimensions. He explained the reason that two solid figures do not coincide, even though they are equal and similar, is that beyond the solid space of three dimensions there is none of four dimensions. He explained further that we could make two solid triangles coincide by letting one triangle make a half revolution around one of its sides through a space of four dimensions. However, since such a space is not present, coincidence is impossible [*5*, p. 526].

In 1846, Cayley made use of geometry of four dimensions to investigate certain configurations of n points situated in any manner in space. By passing lines through all the combinations of two points and planes through all the combinations of three points, and then cutting these lines and planes by any plane, Cayley developed the following theorem. "We can form a system of N_2 points situated 3 at a time on N_3 lines, to wit, representing the points by 12, 13, 23, etc., and the lines by 123, etc., the points 12, 13, 23 will be situated on the line 123, and so on." He adds that the above theorem may be considered as the expression of an analytical fact which will hold also in considering four

coordinates instead of three. However, in supposing four dimensions of space it is necessary to consider lines determined by two points, half-planes determined by three points, and planes determined by four points [*5*, pp. 527-8].

The next contribution to our subject came from James Joseph Sylvester (1814-1897). He wrote several papers on the study of geometries. In 1851 in a paper on homogeneous functions, Sylvester discussed polar and tangent forms in n-dimensional geometry [*4*, p. 5]. In 1859 in some lectures on partitions, he made an application of hyperspace and showed how to picture the fourth dimension geometrically [*1*, p. 399]. In a memoir written in 1863, "On the Centre of Gravity of a Truncated Triangular Pyramid," he considered the corresponding figures in four and n dimensions and proved his theorems for all of these figures [*4*, p. 5]. A contemporary of Sylvester, William Clifford, in 1866 made an application of the higher geometry to a problem in probability [*5*, p. 540].

Geometry of the fourth dimension was slow in gaining recognition even by leading mathematicians. Gradually, however, mathematicians began to apply the language of geometry to the processes of algebra and analysis. An example of this theory is furnished by Cauchy in 1847 in his "Memoir on Analytic Loci." He says, "We shall call a set of n variables an analytical point, an equation or system of equations an analytical locus." [*4*, p. 6].

One of the most important contributions of this period was made by George Bernard Riemann in his paper, "On the Hypotheses which Lie at the Foundation of Geometry." In this paper Riemann builds up the idea of multiply extended manifolds and their measure relations [*4*, p. 6]. Our present conception of the fourth dimension is based upon Riemann's theory. A modern condensed definition of a four-dimensional Euclidean manifold in terms of analytical geometry, where the word manifold or class is used instead of space, reads, "A four-dimensional Euclidean manifold is the class of all number quadruples: (x, y, z, u), (x', y', z', u'), $(x'', y'', z'', u'',)$, etc., to any two of which

there may uniquely be assigned a measure, (called the distance between them) defined by the formula

$$\sqrt{(x-x')^2+(y-y')^2+(z-z')^2+(u-u')^2}.$$

Certain subclasses of this class are called points, lines, planes, and hyperplanes. Analytical four-dimensional Euclidean geometry is the system formed by theorems derived from these definitions." [*3*, p. 124].

Numerous contributions and papers on the fourth dimension have been written in recent years. Many of the leading physicists have propounded d'Alembert's idea of time as the fourth dimension. Among these are F. Klein, A. Brill, L. Heffter, and H. Minkowski [*2*, p. 480]. Another is Sir William Rowan Hamilton, who in speaking of his contribution to quaternions, remarked, "Time is said to have only one dimension, and space to have three dimensions. . . . The mathematical quaternion partakes of both these elements: in technical language it may be said to be time plus space; or 'space plus time,' and in this sense it has or at least involves a reference to four dimensions." [*6*, p. 257]. Time as a fourth dimension furnishes the simplest statement of the physical principle of relativity. However, Edward Kasner says, "Physicists may consider time to be a fourth dimension, but not the mathematician. The physicist, like other scientists, may find that his latest machine has just the right place for some new mathematical gadget; that does not concern the mathematician. The physicist can borrow new parts for his changing machine every day for all the mathematician cares." [*3*, p. 119].

Geometry of four dimensions is now recognized as an indispensable part of mathematics. It is of special use in connection with two complex variables, both in the study of one variable as a function of the other, and to study functions of both variables considered as independent variables. Four-dimensional geometry is much more extensive than three-dimensional geometry. It has various applications; it enables us to prove theorems in geometry of three dimensions. Although all of us are not in agreement as to what

the fourth dimension is and most of us cannot picture ourselves living in a space of four dimensions, we can reason logically about it and proceed to build up a geometry of four dimensions without a realization of it.

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"Wolfgang Bolyai asked to be buried without a marker on his grave. An apple tree would do, he said, to remind him in his long sleep of the three apples of history: the apple which Eve foisted off on Adam, the apple which Paris awarded Helen of Troy as the fairest of the fair, and the apple whose fall inspired Newton to his law of universal gravitation. The first two, he remarked, made earth a hell; the third restored the earth to its dignity among the heavenly bodies."

—E. T. BELL.

THE MATHEMATICAL ROMANCE OF POLY —1— AND RAY —2—[1]

Poly was a —3— yet rather —4— girl for her features were —5— though —6— and her —7— —8— while Ray was a —9— faced young man of —10— build and —11— purpose and —12— opinions. They lived in —13— —14— and their —15— often —16—.

Poly was a —17— young woman of —18— temper, who trusting her —19— —20— over Ray, kept his hopes —21— from —22— to —23—. But at last she became so —24— that he reached the —25— and flew off at a —26—, a very —27— Ray —2—. Still his love was —28—and the —29— of their estrangement was of —30— duration.

One day a —31— escaped her lips, and Ray who was far from —32—, noticed the —33—and asked, "Will you give me a —34— of your love by a sweet —35—?" She was an —36— girl, but so —37— were their —38— that she did not —39— when she was —40— by his arms.

Mr. I. Cosa —1— was a —41— business man —42— —43— in affairs of the heart, to whom the —44— —45— of life were —46—. He and Ray held —47— opinions, so when Ray came to the —48— of the matter he received a —49— answer and Polly's father, growing —50—, called him a —51— for stealing a girl's affections before he had —52— the —53— of the H.C.L. However, Ray inherited a valuable —54— of land and Mr. I. Cosa —1—, who could —55— between —56— and —57— wealth, gave his consent.

A large —58— of —59— attended the wedding. There was the uncle whose —60—nearly —61— his —62— and the cousin who was a —63— favorite and the —64— minded old maid saying "prunes and —65—." The bride was the —66— of all attention as she appeared in a gown —67— on

1 Reprinted from the *American Mathematical Monthly*, Vol. XLIII, Jan. 1936, p. 41. The Mathematical Romance was first used at a Christmas party of the Cornell Parabola Club in 1919.

—68— —69 with a veil falling —70—to her —71—. As Ray slipped the —72— on her third —73—, he whispered, "The world is —74— for me with you as the —75— of my existence." Throwing her bouquet in a —76— —77—to her attendants, the happy —78— sped to the land of the —79— for a honeymoon, far from the icy —80—as they had previously —81—. They lived happily since they had —82— dispositions and Polly stayed in her proper —83—.

"Anyone who looked for a source of power in the transformation of the atoms was talking moonshine."

—RUTHERFORD (about 1920).

= ▽ =

"Then in 1940 the great reservoir of the physicists of America and England broke loose to bring about in five years a result, the achievement of which was not expected for 100,000,000 years—the production of the atomic bomb."

—GORDON FERRIE HULL.

THE MATHEMATICAL SCRAPBOOK

Have you summoned your wits from woolgathering?
—THOMAS MIDDLETON.

=▽=

If one bird can sing 80 half-notes in one-half minute and a second bird can sing one-half as many in twice as much time, how many half-notes could they both sing together in one-quarter minute?

=▽=

Prof: Why are you late, Pat?

Pat: 'Twas like this, professor. When I left my room, it was so slippery that if I took one step forward, I went two backward.

Prof: Then how in the world did you get here?

Pat: Sure and I turned the other way.

= ▽ =

Decode the following addition.

```
 AHAHA
   TEHE
 -----
 TEHAW
```

= ▽ =

A tramp picked up 36 cigarette butts. If he could make one whole cigarette from six butts, how many whole cigarettes did he smoke?

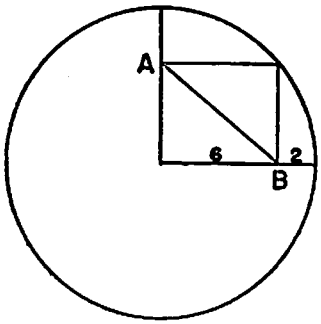
= ▽ =

A says: "I am a skeptic. I am certain of nothing." B counters: "There is no such thing as a true skeptic. What you have just said proves it."

"Angling may be said to be so like mathematics that it can never be fully learnt." —IZAAK WALTON.

= ∇ =

What is the length of AB in the figure at the right?



= ∇ =

= ∇ =

3.14159265358979323 84

Now, I have a score notations
Of digits large and small,
Teaching diameter's precise relations,
And we can remember 'tall.

—G. E. GUDE.

= ∇ =

To find $\sqrt{N}$, let $N = a \times b$ where a and b are nearly equal. Then find successively the arithmetic means of a and b, of $(a + b)/2$ and $N \div (a + b)/2$, etc. For example, let us find $\sqrt{20}$. Since $20 = 4 \times 5$, we compute $(4 + 5)/2$ and $20/4.5 = 4.444$. Then $\sqrt{20} = (4.5 + 4.444)/2 = 4.472$ correct to three decimal places.

= ∇ =

$$371 = 3^3 + 7^3 + 1^3$$

= ∇ =

An empty barrel weighs 10 lb. What can you put in the barrel to make it weight 9 lb.?

Here is wisdom. Let him that hath understanding count the number of the beast: for it is the number of a man; and his number is six hundred threescore and six. (Revelations 13:18.)

Let $A = 100$, $B = 101$, $C = 102$, $\cdots$, $Z = 125$. Then

$$H + I + T + L + E + R = 666.$$

$= \nabla =$

Three persons sold eggs at precisely the same rate. A sold 50 eggs, B sold 30 eggs, and C sold 10 eggs. What was the selling price if each person received the same amount from his sale?

$= \nabla =$

444	555		999
444	555		999
16	25		81
1616	2525	etc	8181
161616	252525		818181
1616	2525		8181
16	25		81
197136	308025		998001

$= \nabla =$

Who can mistake great thoughts?
They seize upon the mind—arrest,
and search,
And shake it. —PHILIP JAMES BAILEY.

$= \nabla =$

From a report on Russia's state of naval preparedness: "That Russia was amply prepared in the event of war was indicated by the announcement her naval forces had increased 1700% since 1933. Admiral Ivan Orloff, chief of naval forces, listed the following increases: Submarines 715%; warships 300%; coast artillery 75% anti-aircraft guns 100%; marine aviation 510%." It adds up correctly to 1700%.

"The most distinct and beautiful statement of any truth must take at last the mathematical form." —THOREAU.

= ▽ =

"The mathematician's best work is art, a high and perfect art, as daring as the most secret dreams of imagination, clear and limped. Mathematical genius and artistic genius touch one another." —MITTAG-LEFFLER.

= ▽ =

An army L miles long advances M miles while a despatch rider goes from the rear to the front and returns to the rear. How far did he travel?

= ▽ =

Let $a - b = c$. Then

$$\begin{aligned} (a-b)^2 &= c(a-b) \\ a^2 - 2ab + b^2 &= ac - bc \\ a^2 - ab - ac &= ab - bc - b^2 \\ a(a-b-c) &= b(a-b-c) \\ a &= b. \end{aligned}$$

= ▽ =

Two players sit at solitaire, each with a pack of 52 cards. Each deals himself a hand of 13 cards. A, showing his hand, remarks, "I have the ace of hearts." B, doing likewise, replies, "I also have an ace." Show that A has more chance than B of having another ace.

= ▽ =

If 14 dogs with 3 legs each can catch 48 rabbits with 76 legs in 25 minutes, how many legs must 24 rabbits have to get away from 3,000 dogs with no legs at all?

= ▽ =

The quarrel is a very pretty quarrel as it stands; we should only spoil it by trying to explain it.

—RICHARD BRINSLEY SHERIDAN.

CHAPTERS OF KAPPA MU EPSILON

ALABAMA ALPHA, Athens College, Athens.
ALABAMA BETA, Alabama State Teachers College, Florence.
ALABAMA GAMMA, Alabama College, Montevallo.
ILLINOIS ALPHA, Illinois State Normal University, Normal.
ILLINOIS BETA, Illinois State Teachers College, Charleston.
ILLINOIS GAMMA, Chicago Teachers College, Chicago.
ILLINOIS DELTA, College of St. Francis, Joliet.
IOWA ALPHA, Iowa State Teachers College, Cedar Falls.
IOWA BETA, Drake University, Des Moines.
KANSAS ALPHA, Kansas State Teachers College, Pittsburg.
KANSAS BETA, Kansas State Teachers College, Emporia.
KANSAS GAMMA, Mount St. Scholastica College, Atchison.
LOUISIANA ALPHA, Louisiana State University, Baton Rouge.
MICHIGAN ALPHA, Albion College, Albion.
MICHIGAN BETA, Central Michigan College, Mount Pleasant.
MISSISSIPPI ALPHA, State College for Women, Columbus.
MISSISSIPPI BETA, Mississippi State College, State College.
MISSOURI ALPHA, Missouri State Teachers College, Springfield.
MISSOURI BETA, Missouri State Teachers College, Warrensburg.
NEBRASKA ALPHA, Nebraska State Teachers College, Wayne.
NEW JERSEY ALPHA, Upsala College, East Orange.
NEW JERSEY BETA, New Jersey State Teachers College, Montclair.

NEW MEXICO ALPHA, University of New Mexico, Albuquerque.

NEW YORK ALPHA, Hofstra College, Hempstead.

OHIO ALPHA, Bowling Green State University, Bowling Green.

OHIO BETA, College of Wooster, Wooster.

OKLAHOMA ALPHA, Northeastern State College, Tahlequah.

SOUTH CAROLINA ALPHA, Coker College, Hartsville.

TENNESSEE ALPHA, Tennessee Polytechnic Institute, Cookeville.

TEXAS ALPHA, Texas Technological College, Lubbock.

TEXAS BETA, Southern Methodist University, Dallas.

Answers for the Mathematical Romance: 1, Hedron; 2, Show (ratio); 3, plane; 4, a-cute; 5, regular; 6, angular; 7, form; 8, symmetrical; 9, round; 10, square; 11, set; 12, fixed; 13, adjacent; 14, regions; 15, paths; 16, intersected; 17, complex; 18, variable; 19, increasing; 20, power; 21, oscillating; 22, zero; 23, infinity; 24, arbitrary; 25, limit; 26, tangent; 27, cross; 28, constant; 29, period; 30, infinitesimal; 31, loci (low sigh); 32, obtuse; 33, sign; 34, demonstrations; 35, osculation; 36, independent; 37, similar; 38, inclinations; 39, object; 40, encircled; 41, solid; 42, 43, everywhere dense; 44, 45, ideal elements; 46, unknown; 47, opposite; 48, point; 49, negative; 50, irrational; 51, lo-cus; 52, solved; 53, problem; 54, section; 55, differentiate; 56, imaginary; 57, real; 58, number; 59, relations; 60, perimeter; 61, equalled; 62, height; 63, prime; 64, vacuous; 65, prisms; 66, focus; 67, cut; 68, straight; 69, lines; 70, perpendicularly; 71, feet; 72, circle; 73, digit; 74, pi (e); 75, center; 76, 77, parabolic arc; 78, pair; 79, pyramids; 80, poles; 81, projected; 82, complementary; 83, sphere.