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# CONTENTS

	Page
Guidance in the Field of Mathematics for High School and College Students By <i>James H. Zant</i> .....	45
A Historical Introduction to the Calculus of Variations By <i>Rodney T. Hood</i> .....	51
Topics for Chapter Programs — I .....	66
The Mathematical Scrapbook.....	69
Kappa Mu Epsilon News By <i>Sister Helen Sullivan, O.S.B.</i> .....	76



# GUIDANCE IN THE FIELD OF MATHEMATICS FOR HIGH SCHOOL AND COLLEGE STUDENTS<sup>1</sup>

JAMES H. ZANT

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Teaching is an ancient and honorable profession and the teacher of mathematics occupies an exalted position among his fellows. It is a position of trust that has many compensations. I am not unmindful of any of them. It is also a position which to be filled adequately requires the best type of mind and training. Our best young people may aspire to a useful life of service in that capacity. I have spent most of my professional life training and encouraging these young people for the profession of teaching in the field of mathematics.

However, it must be admitted that not all the young people in high school and college who are capable and interested in the study of mathematics are also interested in teaching it. Hence, it is necessary for the mathematics teacher to have a wider knowledge of the vocational possibilities of mathematics than most of us have had in the past.

The teacher of mathematics in the public schools and colleges have only a vague and indefinite idea of how their subject is used or what the employment opportunities of a well-trained student are. The students have a right to expect their teachers to know what types of jobs and professions require a knowledge of mathematics, the specific mathematics needed for many types of jobs, and what courses the student should take to gain this knowledge. Moreover, the teachers should have contacts with various phases of business, industry, and government service to such an extent that they can tell a student just where to go to

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<sup>1</sup> Read before the Kappa Mu Epsilon chapter of the Chicago Teachers College, November 26, 1945.

obtain these jobs. If possible, in addition to the things mentioned, the teacher of mathematics should have work experience at some job or jobs which makes use of mathematics. This should not be hard, since most teachers have long vacations. Employers in industry and business particularly seem anxious to cooperate in such a program so that they may gain early contact with employable well-trained young people.

The jobs available for students of mathematics will be discussed only briefly. They have been discussed more fully in a previous paper.<sup>1</sup> For college students who have a major in mathematics including a few specialized courses and a minor in related fields there seem to be many sorts of jobs open. There are positions as arithmetical clerks and computers, junior mathematicians, junior engineers, junior statisticians, statisticians, research statisticians, actuaries and actuarial mathematicians, and astronomers. The teaching field is much broader than is usually considered. We need secondary and college teachers, but we will need more and more teachers of engineering mathematics and applied mathematics.

Jobs available to high school students are of a lower grade and perhaps not so specific. However, larger and larger numbers of high school students are going to college, and, as college students, will wish to prepare for vocations and professions. Hence, it will be necessary for the public school teachers of mathematics to know the mathematical prerequisites of these vocations and professions so that prospective college students can be properly advised. This is one of the tasks which the Commission on Post-War Plans of the National Council of Teachers of Mathematics has set for itself. This will be discussed more fully below.

It is not always possible to say just what specific training in mathematics it is necessary to have. Often employers will give more emphasis to the general background of the student and his native ability than they will to the specific

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<sup>1</sup> James H. Zant, "Must a Mathematician Teach?," *The Phi Delta Kappan*, Vol. 27, pp. 13-15 (Sept., 1945).

courses he has had. In other professions like engineering and actuarial work the requirements are very definite as concerns hours of work and courses. The college graduate who expects to hold a job requiring a knowledge of mathematics should have completed calculus and then have three or four specialized courses in algebra, statistics, mathematics of finance, life insurance, etc., depending on the particular sort of job he expects to seek. For the high school student, whether he is going into college and a profession like engineering, government service, science, etc., or expects to go to work at a mathematical job at the end of the high school period, his training should include all of the mathematics he can get. Such students need to acquire by the end of the high school period an ability to think mathematically, that is, in terms of numbers, formulas, and algebraic symbols. Without this power of thought or analysis or whatever it may be called a person cannot use mathematics effectively either in studying for a profession or on a job. This power is gained by the majority of people only by continued experience with the various phases of mathematics. Hence, the students training should be varied and should extend over as long a period as is feasible.

Getting such information to the student requires, of course, that the teachers, or some teacher at least, have the information. This may be gained by personal investigation, or reading investigations made by other groups, or it may be gained by first-hand information on the job. Ideally there should be individuals on the school staff who have had job experience in many types of work. They must also know the mathematics needed and the courses which will give these required skills. Sample programs should be available which show just what courses should be taken and the sequence in which they should be taken. Statements giving the intellectual prerequisites and the training demanded should be available to the students.

Finally a more complete analysis should be available in the form of a pamphlet or brochure which the student may read and study at his leisure. This last suggestion has

been compiled with for the college student in a small bulletin issued by the Department of Mathematics of the Oklahoma A. & M. College.<sup>1</sup> This bulletin includes a complete discussion of six different classifications of jobs, duties involved, training required, and a suggested plan of study which will fit a student for each type of job. This plan of study includes the actual courses in mathematics required and those suggested as well as definite suggestions regarding the student's minor and electives.

The bulletin also includes a discussion of the opportunities which are available for the employment of mathematics majors and gives a list of twenty-one individuals and business firms all of whom have been contacted personally and have expressed a desire to cooperate in placing graduates. Several students have been placed already in good paying positions.

The last section of the bulletin describes the work of the Department of Mathematics, and lists the staff with their educational qualifications and also designates the members of the staff who are especially qualified to advise students in particular fields.

This bulletin is designed to be used in the guidance of prospective mathematics majors in the college. It will be sent to every high school principal in the state of Oklahoma and made available to mathematics teachers and students. Through it, teachers and students will know what particular sorts of jobs are available for a student who has graduated with a major in mathematics, and they will know what the course requirements are for one who wishes to qualify himself for one of these jobs. Finally, they will have a limited number of business firms who have been contacted by members of the mathematics staff and have expressed a belief in the project and a willingness to cooperate in finding jobs for qualified students. The project is still new and we are not sure, of course, just how it will work. Only a few students have graduated since we have had this data col-

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<sup>1</sup> *Mathematics—Its Vocational Aspects*, Bulletin, Oklahoma A. & M. College, Vol. 42, No. 13 (May, 1945).

lected. However, we have gone far enough to feel that we are on the right track and that such a procedure can be definitely helpful to the students.

As stated above, the Commission on Post-War Plans has set out to do at the high school level something similar to what the Oklahoma A. & M. College has done at the college level. "The Board of Directors are agreed that the next job of the Commission is to create, or cause to be written, a relatively small but effective pamphlet on mathematics to be used in the guidance of the junior and senior high school pupil. Its purpose will be to make clear what mathematics has to offer. The idea is that it be directed to the student, but widely distributed to mathematics teachers who in turn may transmit the pamphlet or at least the ideas to home room teachers and other persons with responsibilities for guidance."<sup>1</sup>

The Commission is making every effort to compile a reliable worthwhile pamphlet for use with secondary students. For this reason we need the help of all forward looking mathematics teachers.

At our last meeting in Washington, D. C., in October, 1945, a number of categories were set up to work on. These categories or types of jobs are: (1) aviation, navigation and surveying, (2) civil service, (3) business and accounting, (4) engineering, (5) actuarial work, (6) agriculture, (7) mechanical trades, (8) statistician, (9) nursing, and (10) scientists and mathematicians. Suggestions as to others will be welcome.

The preliminary write-ups of these categories will attempt to set forth two aspects of vocational guidance: (1) the typical mathematical needs or demands made on a worker in a job or profession (for example, engineering), and (2) the nature and amount of mathematics a student should take while in high school in terms of courses and/or topics.

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<sup>1</sup> James H. Zant, "The Next Step in Planning for Post-War Mathematics," *The Mathematics Teacher*, Vol. 28, p. 276 (Oct., 1945). [Collaborators in this project are Professor W. D. Reeve of Columbia University, Professor Raleigh Schorling of the University of Michigan, and Professor C. V. Newsom of Oberlin College.—Ed.]

If definite and reliable information can be obtained on these and perhaps other topics, the mathematics teacher's job can be made easier and vastly more effective. There is no question that the information is available in part but some of it is still a matter of opinion. It is the task of the Commission to make the information available in as definite a form as possible and in language which can be understood by high school students. We need all the help we can get. If you have information about specific jobs or if you are in a position to obtain such information from first hand sources, it will be a definite contribution to send the data to us.



"The more advanced the sciences have become, the more they have tended to enter the domain of mathematics, which is a sort of center toward which they converge. We can judge of the perfection to which a science has come by the facility, more or less great, with which it may be approached by calculation."

—ADOLPHE QUETELET.

# A HISTORICAL INTRODUCTION TO THE CALCULUS OF VARIATIONS

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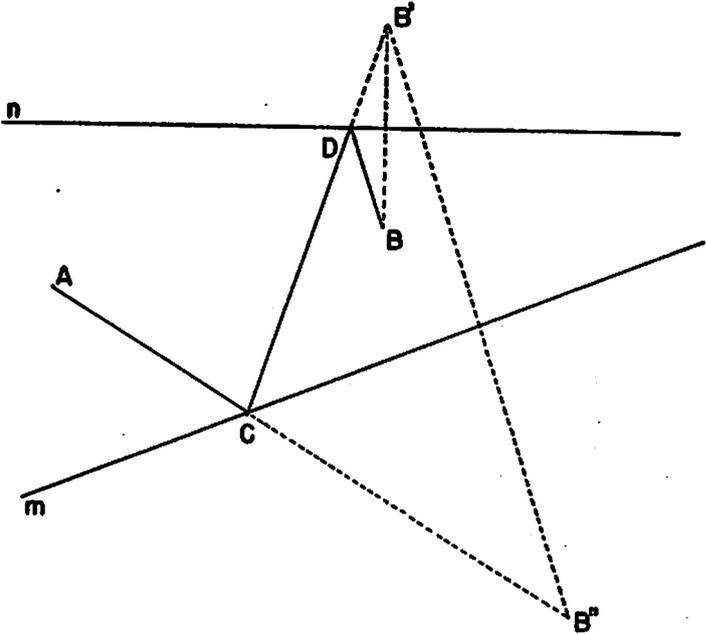
Since the time of the ancient Greeks, mathematicians and scientists have been interested in problems whose solution requires the obtaining of a maximum or a minimum. A. R. Forsyth expresses himself thus: ". . . it may suffice to mention the mathematical-physical conception denoted by the word Action. Philosophers have been fain to deduce the mechanical movements of a dynamical configuration, even much of the physics of the universe, from the single property, that the Action between any two states of the configuration in continuous change is a minimum; and the property has been elevated in postulation to the Principle of Least Action."<sup>1</sup>

Maximum and minimum problems have often been quite practical and often very easily solved. For example, the problem of finding the maximum area a triangle can have, where two of its sides are known, is a very old problem. Clearly, the solution here is a right triangle whose shorter sides are the sides given, as may be seen from the following argument. Let the two given sides be  $a$  and  $b$ . Considering  $a$  as the base, the formula for the area is  $A = \frac{1}{2} ah$ , where  $h$  is the altitude of the triangle. Since  $a$  is fixed, the maximum  $A$  occurs when  $h$  is a maximum. This is a maximum when  $h = b$ . Therefore, the maximum area occurs when  $a$  and  $b$  are perpendicular to each other.

Another simple problem is the problem of Heron of Alexandria (second century B.C.). He showed that the path of a ray of light from a point  $A$  to a mirror to another point  $B$  is such that the distance traversed is a minimum. The problem can be generalized to include several mirrors.

<sup>1</sup> A. R. Forsyth, *Calculus of Variations*. Cambridge, The University Press, 1927, p. 3.

This is the problem met by the Arab who is standing in a desert bounded on one side by a meadow and on the other side by a river. He desires to know the shortest path by which he can ride his horse to the pasture, then to the river, and lastly to his tent which is also in the desert. The



problem is solved by reflecting  $B$  (the position of the tent) in the line  $n$  (representing the river bank), giving the point  $B'$ .  $B'$  is then reflected in the line  $m$  (representing the edge of the pasture), giving the point  $B''$ .  $A$  (the original position of the Arab and his horse) is now connected to  $B''$ , intersecting the line  $m$  in the point  $C$ .  $CB'$  is drawn, intersecting  $n$  in the point  $D$ .  $DB$  is drawn. The path  $ACDB$  is the path of the shortest length which satisfies the problem.

These problems belong to the most simple type of maximum and minimum problems. They may be solved by the methods of elementary geometry and algebra. When, in the seventeenth century, such men as René Descartes

(1596-1650), Giles Persone de Roberval (1602-1675), Pierre de Fermat (1601-1665), Isaac Barrow (1630-1677), Isaac Newton (1642-1727), and Gottfried Wilhelm Leibniz (1646-1716) developed slowly the branch of mathematics known as differential and integral calculus, they brought into play a new tool which could be used to maximize and minimize functions. This was a great advance over what had previously been known. In 1629 Fermat devised a theorem which is given in present-day calculus books as Fermat's theorem. The theorem is as follows: If a function  $f(x)$  assumes a maximum or a minimum value at an interior point  $c$  of the interval  $(a,b)$ , and if  $f(x)$  is differentiable at  $x = c$ , then  $f'(c) = 0$ .

Naturally, Fermat's formulation of the theorem and the notation he used are different from the above. Newton expressed the same notion in a paper written in 1671 (but not published until 1736), as did Leibniz, from another point of view, in 1684. However, although the vanishing of the first derivative of a function at a point is a *necessary* condition for a maximum or a minimum at that point, it is not sufficient. Leibniz in his paper of 1684 showed how  $f''(x)$  could be used to distinguish between maximum and minimum values. Colin Maclaurin (1698-1746) later showed how to obtain both necessary and sufficient conditions for extrema by the use of still higher derivatives.

With this new technique, a tremendous new class of problems requiring the determination of extreme values could now be formulated mathematically and solved. These problems are all of the type where it is required to maximize quantities which can be expressed as functions of one or several variables. A great many of these problems are of a practical nature where the quantities considered are such things as distances, areas, volumes, pressures, temperatures, velocities, and costs.

For a while it seemed that the calculus was the last word in methods for solving maximum and minimum problems. However, before long it became apparent to mathematicians that there were problems of this general sort

which the calculus could not solve. To treat these more general and more difficult problems, there developed gradually a new branch of mathematics which we now know as the calculus of variations. As Bolza says, "The Calculus of Variations deals with problems of maxima and minima. But while in the ordinary theory of maxima and minima the problem is to determine those values of the independent variables for which a given function of these variables takes a maximum or minimum value, in the Calculus of Variations *definite integrals* (or functions defined by differential equations) involving one or more unknown functions are considered, and it is required so to determine these unknown functions that the definite integrals shall take maximum or minimum values."<sup>1</sup>

One of the most ancient problems of this type is the so-called isoperimetric problem (from the Greek words *isos* meaning equal and *perimetron* meaning circumference), that is, to find among all closed curves which have a given length  $L$  the one which incloses the greatest area. The origin of this problem is not known, but Zenodorus (who lived about 150 B.C.) attempted to prove it, and so did Archimedes (287?-212 B.C.). The proof of Zenodorus has been preserved in the writings of Pappus of Alexandria (fourth century A.D.) A completely rigorous proof was not given until the latter half of the nineteenth century, and the majority of proofs have appeared since 1900. Many important men in the calculus of variations have given proofs of this theorem.

The problem can be formulated mathematically as follows: Let  $x = x(t)$  and  $y = y(t)$  be the parametric equations of a closed curve, where  $t$  varies from 0 to 1 and  $x(0) = x(1)$  and  $y(0) = y(1)$ . The area of this curve may be expressed as a line integral,

$$A = \frac{1}{2} \int (x dy/dt - y dx/dt) dt,$$

while the length of the curve is

$$L = \int \sqrt{(dx/dt)^2 + (dy/dt)^2} dt,$$

<sup>1</sup> Oskar Bolza, *Lectures on the Calculus of Variations*. Chicago, University of Chicago Press, 1904, p. 1.

the limits of integration in each case being from 0 to 1. In the isoperimetric problem the first integral must be maximized, keeping the second integral constant. It must occur to all that the circle encloses the largest area among all closed curves with a given perimeter. However, this is one of the apparently self-evident facts for which the proof has been late in appearing. Jakob Steiner (1796-1863) has given a relatively simple proof of a part of the problem which does not use the calculus of variations.

One of the earliest problems of the calculus of variations is due to Newton, and it appears in his *Philosophiæ Naturalis Principia Mathematica*. The problem is to find the form of the surface of revolution which, having its axis lying in a fixed direction, offers the least resistance in moving through a liquid in the direction of the axis, it being supposed that the resistance of an element of surface is proportional to the square of the component of velocity in the direction of its normal. Let the  $y$ -axis be the axis of the solid, and let the surface be given by the parametric equations  $x = x(t)$ ,  $y = y(t)$ . Then, neglecting a constant factor, the problem of Newton reduces to the problem of minimizing the definite integral

$$\int [xx'^3 / (x'^2 + y'^2)] dt,$$

the limits of integration being from  $t_1$  to  $t_2$ .

Newton's problem attracted little attention at the time it appeared (1686) since in the *Principia* he merely stated the results of his investigations. The methods by which he arrived at these results have been reconstructed from a letter written by Newton to Professor David Gregory in 1694. Newton's solution to the problem was unsatisfactory because his laws of resistance were incorrect. It has since been shown that neither a maximum nor a minimum solution to the problem exists. However, in spite of the difficulties connected with it, it is still an important problem in the study of projectiles, ballistics, and the shapes of airplane wings.

Another of these early problems was that of finding

the curve formed by a hanging chain which is completely flexible; that is, to find the curve of length  $L$  with extremities at the points  $P_1(a,c)$  and  $P_2(b,d)$  and with its center of gravity a minimum. In the language of the calculus of variations, the problem is to minimize the integral

$$I = \int y \sqrt{1 + (dy/dx)^2} dx,$$

at the same time keeping the integral

$$L = \int \sqrt{1 + (dy/dx)^2} dx$$

constant.\* This problem was considered by Galileo Galilei (1564-1642) in 1638, reformulated by Jacob Bernoulli in 1691, and solved by Christian Huygens (1629-1695) and Leibniz. The solution is the catenary (from the Latin *catena*, meaning chain).

In 1696 Johann Bernoulli (1667-1748) called attention to one of the problems which has become a classic in the field and which gave a real impetus to research in the calculus of variations. In the June, 1696, issue of *Acta Eruditorum*, a scientific journal of that time, he announced his *Problema novum ad cuius solutionem mathematici invitantur*. The announcement reads as follows:

"Given two points  $A$  and  $B$  in a vertical plane, to assign to the moveable point  $M$  a path  $AMB$ , on which, leaving  $A$ , it may of its own weight, reach  $B$  in the shortest time.

"In order that lovers of such things may enjoy attempting the solution of this problem, let them not think that it is mere speculation and that it has no practical use, as might appear to be the case. On the contrary, strange as it seems, it proves very useful also in other branches of knowledge than mechanics. In order to put an end to a rash judgment, let it be noticed that the straight line  $AB$  is certainly the shortest between  $A$  and  $B$ , however, it is not traveled in the shortest time. However, the curve  $AMB$  is very well-known to geometers, which I shall announce, if no one else has named it by the end of this year."

This problem was discussed by Galileo in his *Dialog über die beiden hauptsächlichsten Weltsysteme* (1630)

\* In each case the limits of integration are from  $a$  to  $b$ .

where he comes to the erroneous conclusion that a circular arc is the path of shortest time, after comparing this path with the path composed of sides of an inscribed polygon. The problem is known in the literature as the brachistochrone (from the Greek *Brachistos* meaning shortest and *chronos* meaning time.)

In response to this public announcement of the problem, Jakob Bernoulli (1654-1705), a brother of Johann, published a solution in May, 1697, as did also his brother who proposed it. Speaking about his solution Johann says, "With justice we admire Huygens because he first discovered that a heavy particle falls down along a *common cycloid* in the same time no matter from what point on the *cycloid* it begins its motion. But you will be petrified with astonishment when I say that precisely this *cycloid*, the *tautochrone of Huygens* is our required *brachistochrone*. I arrived at this result along two different paths, one indirect and one direct. When I followed the first I discovered a wonderful accordance between the curved orbit of a ray of light in a continuously varying medium and our *brachistochrone curve*. I also observed other things in which I do not know what is concealed which will be of use in dioptrics."<sup>1</sup>

Jakob's enthusiasm is also quite evident from the following: "Thus for this curve, which has been investigated by so many mathematicians that apparently nothing further concerning it could remain to be discovered, we find a new property, as it were an indication of its desire that no obligation might be incurred to future centuries but that it might attain the pinnacle of perfection at the end of the present one at whose beginning its birth was celebrated and among whose researches there have fallen to its lot the discovery of all of its mensurable properties and many other beautiful characteristics."<sup>2</sup>

The solution of Johann was quite elegant, but the method he used was of a very special type and thus could

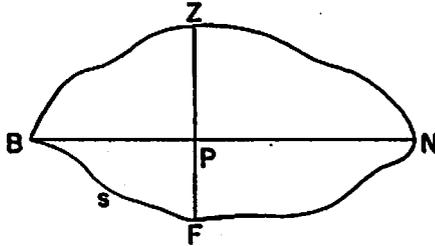
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<sup>1</sup> D. E. Smith, *A Source Book in Mathematics*. New York, McGraw-Hill, 1929, pp. 649-650.

<sup>2</sup> G. A. Bliss, *Calculus of Variations*. Chicago, Open Court Publishing Co., 1925, p. 54.

not be applied more generally. Jakob's method was more general, although his solution was more laborious and not at all rigorous. The problem was also solved by Newton, Leibniz, and Guillaume Francois Antoine l'Hospital (1661-1704).

At the end of his paper Jakob proposed a more difficult problem, to find the curve of quickest descent from a fixed point to a fixed vertical line, and also two other problems which were still more difficult. It was required to find among all curves  $BFN$ , two such that the area  $BNPZ$  is a maximum when  $PZ = PF^n$ , or when  $PZ = s^n$ , where  $s$  is the length of arc  $BF$ . These problems were a challenge particularly to his brother Johann, who had great difficulty in



solving them. Jakob published his solution in 1701 in the *Acta Eruditorum*. Later Johann simplified the proofs.

The work of the Bernoulli brothers may be said to have initiated the calculus of variations as an actual branch of mathematics. From this time on it was studied by some of the great mathematicians of the world. We have here something essentially different from what was encountered in previous maximum and minimum problems where it was required to find a *point* on a given *curve*. Here it is required to find a whole curve with a maximum or minimum property. Needless to say, the solution of this more general problem is much more difficult and quite often impossible.

Another writer who contributed to the subject in its earliest phase was Christian Huyghens whose book, *Theorie de la Lumiere* (1690), can now be considered as a genuine book on the subject. Had it not been for the criticism which

this book received from Newton on account of Huyghens' *wave theory* of light, this book might have been very influential.

The papers of the Bernoullis of 1701 and 1718 greatly influenced Leonard Euler (1707-1783), a great Swiss mathematician (and probably the most prolific the world has ever known) and a pupil of Johann Bernoulli. They formed the starting point for his notable work. Euler systematized the methods of the Bernoullis. He replaced the integrals of the problem by finite sums, thus making them functions of a finite number of ordinates of the curve  $y = y(x)$ . Then he varied one or more ordinates and calculated the resulting variations of the integrals. The variation of the integral to be minimized was to be zero, giving as a result, difference equations. Then by a limiting process, these difference equations become differential equations which characterize the curve sought.

Joseph Louis Lagrange (1736-1813), a young contemporary of Euler, made several important contributions to the calculus of variations. In particular, he introduced the symbol  $\delta$ . He replaced the function  $y(x)$  by  $y(x) + \delta y(x)$ , where  $\delta y(x)$  is an increment of the function of  $x$ . Then the integral  $I'$  whose first order terms in  $y$  and its derivatives were called  $\delta I$ , was the original integral plus an increment. The symbol  $\delta$  was used to indicate smallness, probably in imitation of the notation for a differential. The new quantity  $\delta y(x)$  was called by Euler a *variation* of the function  $y(x)$ . From this time on the subject was called the Calculus of Variations.

This notion of Lagrange proved very useful. In working with his new integrals (those containing the variation), he was able to integrate by parts and find conditions which must be satisfied by the curve  $y = y(x)$ . In addition, Lagrange applied his methods of analysis to problems where there were two independent variables. He is regarded as the founder of the theory of minimal surfaces for he obtained the partial differential equation of the second order which holds for such surfaces. After Lagrange's work on

variations appeared, Euler again wrote on the subject, giving a geometrical interpretation of them. Euler considered  $\delta y$  as a vertical change in position of a point on an original curve along its ordinate to a corresponding point on the varied curve.

These variations are the simplest type of a large class of variations which have since been found useful. To get from a point on the original curve to the corresponding point on the varied curve, Lagrange merely moved up a small distance on the ordinate of the point. Other possibilities are moving the point sideways for a short distance, or first sideways and then upwards, or in an oblique direction, or moving a portion of the curve so that its slope changes or so that its curvature changes. All these are other types of variations.

Lagrange also extended the brachistochrone problem by considering the case of variable end points to the curve. Such a problem would be to find the curve of quickest descent from a point to a given curve, or from a given curve to a point, or from one of two given curves to another.

The work of Euler and Lagrange was to give qualifying tests for extreme values of certain integrals, but whether these were maxima or minima was often left to intuition. In 1786 Adrien Marie Legendre (1752-1833) published a paper in the *Mémoires de l'Académie Royale des Sciences* entitled, "Mémoire sur la manière de distinguer des maxima et minima dans le calcul des variations," in which he sought criteria for distinguishing between maximum and minimum solutions. He did this by studying the so-called second variation of an integral, written §21. This work of Legendre, while not entirely rigorous, was a decided advance. Until that time the only tests available were the Euler test for one independent variable and the Lagrange test for two independent variables. These were concerned only with the first variation. It will be noticed that there is quite an analogy between the *variation* in the calculus of variations and the *differential* in the differential calculus.

Carl Gustav Jacob Jacobi (1804-1851) extended the re-

searches of Legendre by studying more thoroughly the second variation. He obtained more general forms for the tests which were already known and finally developed an additional test. This test may be illustrated easily by the problem of the geodesic of the sphere. If two points are given on the surface of the sphere, the shortest arc connecting them is an arc of a great circle. However, if the two points are diametrically opposite, the geodesic between them is not unique (although it is still a minimum), and if the portion of the great circle between the two points is greater than 2 radians, it is certainly not a minimum. Jacobi's test introduces the important notion of *conjugate points* (in this problem, two diametrically opposite points on the sphere are conjugate points). It indicates the limit to the range along the curve within which the integral being considered has a maximum or a minimum value.

Another man who did work in the calculus of variations who should be mentioned is Sir William Rowan Hamilton (1805-1865). Known primarily for his quaternions, he left another important discovery, his principal function. It is singular that none of the books dealing with historical aspects of the calculus of variations mention Hamilton. However, J. L. Synge remarks, "Hamilton was, in fact, a great contributor—probably the greatest contributor of all time—to the calculus of variations."<sup>1</sup> Hamilton's principle is also a relevant statement here to indicate the nature of his interest in the calculus of variations. It is this: if a system of particles is moving subject to their own gravitational attraction, their paths will be minimizing curves for the integral of the difference between the kinetic and potential energies of the system.

During the next thirty-five years, a period in which outstanding developments were taking place in other branches of mathematics, the calculus of variations remained relatively at a standstill. There were many minor contributors, but no essential departure was made from what was already

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<sup>1</sup> J. L. Synge. "The Life and Early Work of Sir William Rowan Hamilton," *Scripta Mathematica*, vol. 10 (1944), p. 15.

known. For a long time mathematicians had been dissatisfied with much of the work done in the calculus of variations. There were errors even in the best memoirs and the lack of rigor made many things unsatisfactory. It was time for some first-class mathematician to put the subject on a sound foundation and make it rigorous. The man who did this was Karl Weierstrass (1815-1897).

Although Weierstrass published very little during his lifetime, he was very influential in the modernization of the calculus of variations through his students to whom he lectured on the subject at the University of Berlin in 1872 and later years. He initiated a method which was new in form and new in substance. What he did was to consider variations of the dependent variable as well as the independent variable. Weierstrass used parametric equations in setting up integrals which must be maximized or minimized. Naturally, this was more general than anything which had previously been done. He found a new test, using his so-called excess-function and the notion of a field.

Up to this time of Weierstrass, students of the calculus of variations had not been concerned with discriminating between necessary and sufficient conditions. As soon as they had found a new test, they took it for granted that the conditions then known were sufficient for an extremum in the problems they were studying. Of course, this is quite unjustified. Ten cents is necessary to enter the Apollo Theater but it is not sufficient. On the other hand, \$5.00 is sufficient but not necessary. The mere fact that new conditions kept cropping up apparently convinced Weierstrass of the need for a sufficiency theorem. Here Weierstrass hit upon his ingenious device called a field, a region  $F$  of the plane in which there is a one-parameter family of extremal arcs all of which intersect a fixed curve  $D$  and such that through each point  $(x,y)$  of  $F$  there passes only one extremal of the family.

For the next thirty years there was undertaken a critical revision and "rigorizing" of the foundations of the older theories. This resulted in a clearer formulation of prob-

lems which were encountered, rigorous proofs for the necessary conditions of Euler, Legendre, and Jacobi, and a rigorous proof of their sufficiency under certain circumstances. During this time Weierstrass extended the theory of the first and second variations to the case in which the curves and integrals are represented parametrically. Also, Weierstrass discovered a fourth necessary condition and a sufficiency proof which gave a complete solution (for simple types of problems) for the first time. The entirely new method by which he obtained complete solutions is called *Weierstrass' construction*. Of course, the giant of this period was Weierstrass, but other men such as Erdmann, DuBois-Reymond, Schaeffer, and Schwarz were important.

One of the problems which had been known for a long time entered the picture again after 1873 when Joseph Plateau (1801-1883), in his *Statique experimentale et theoretique des Liquides*, described soap film experiments. Using a solution of glycerine water in which wire models were dipped, Plateau produced soap films which were minimal surfaces bounded by the wire curves. The subject of minimal surfaces had previously been studied by Legendre, Karl Friedrich Gauss (1777-1855), Gaspard Monge (1746-1818), and later Hermann Amandus Schwarz (1843-1921). Mathematically, the minimal surface problem leads to the rather difficult problem of minimizing the integral

$$\iint \sqrt{1 + (dz/dx)^2 + (dz/dy)^2} \cdot dx dy$$

over the area in the  $xy$ -plane which is the projection of the boundary curve. General existence proofs were developed only recently (in the last twenty years) by Rado, Douglas, McShane, Garnier, and Courant. The soap film, however, is a physical method of solution which can throw light on the three general questions which may arise in the study of minimal surfaces. These are 1) the question of the existence of a solution of a given type, 2) the question of the uniqueness of the solution, and 3) the question of the dependence of the solution on the prescribed data, particularly the question whether or not the solution depends continu-

ously on the prescribed boundaries. Many interesting forms and configurations can be obtained by means of the soap films, and by them it is easy to see that occasionally there may be more than one solution to the problem.

Another development of the end of the nineteenth century was the work of Jean Gaston Darboux (1842-1917) whose *Theorie des Surfaces* appeared in 1894. In it he made an elaborate study of the problems of finding geodesics, or shortest arcs, on a surface. Zermelo and Kneser later generalized the work of Darboux.

In 1899 David Hilbert (1862-1943) stated and proved an existence theorem for a minimizing arc. The theorem has been reproved and stated more simply since. With this as a starting point, Leonida Tonelli (1855-) in 1921-1923 made a revision of the entire subject, concerning himself chiefly with existence theorems. Tonelli's revision is published in his book *Calcolo delle Variazioni*. In this book the most modern analysis is used.

One of the students of Weierstrass was Oskar Bolza (1857-1942) who not only obtained a fifth necessary condition for a minimum, but also wrote several excellent works on the calculus of variations. As one of the professors of mathematics at the University of Chicago in its early days, Bolza was responsible for a considerable amount of interest in the calculus of variations in this country. One of his students was Miss Mary Emily Sinclair (1878-), who made some interesting discoveries on surfaces of revolution of minimum area of various types. Another of his students was Gilbert Ames Bliss (1876-) who has continued to do research and to inspire research. Another contemporary American working in the field is Marston Morse (1892-). Indeed, the calculus of variations has been a favorite field of research among mathematicians in this country, especially at the University of Chicago as the four volumes of *Contributions to the Calculus of Variations* show.<sup>1</sup>

Throughout the development of this subject one can

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<sup>1</sup> These volumes are sponsored by the Department of Mathematics of the University of Chicago. They contain the theses on topics in the calculus of variations accepted for the Ph.D. degree at the University of Chicago during the years 1931-1941.

note an ever-increasing generalization. From Johann Bernoulli's problem concerning an arc with two fixed points, his brother Jakob proposed one with variable endpoints. Later on, the type of integral to be considered was generalized. More recently still, a very general type of integral is considered, as well as more generalized end conditions on the integral. The problem of Bolza (1913) was a problem of great generality to which methods in the calculus of variations may be applied. This problem includes as a special case the problem of Adolf Mayer (1839-1908) of 1878 and also that of Lagrange of 1770.

In conclusion, let me also emphasize the applicability of the calculus of variations to other fields of research. It has already been seen in this paper that it has applications to mechanics, optics, and dynamics. It also has applications to the field of differential geometry, to astronomy, and other subjects. Two quite striking illustrations in the twentieth century of its use other than as an end in itself are its applications to the famous relativity theory of Albert Einstein and the quantum theory of E. Schrödinger.



"Leibniz was a universal genius. Philosopher, mathematician, theologian, jurist, statesman, physicist, philologist, and historian, he was a dilettante at nothing to which he turned his great mind. In the press of affairs in his superfilled life he lacked the time to write books, and his discoveries and inventions were given to the world in articles often so brief and fragmentary as to be intelligible or even readable to but a few."

—R. E. LANGER.

## TOPICS FOR CHAPTER PROGRAMS—I

What topics are most suitable for chapter programs? In the words of the late Professor H. E. Slaught, a chapter program should "afford an opportunity to all students primarily interested in mathematics, whether undergraduates or graduates, to present before a sympathetic audience the results of reading and investigation, especially along lines not likely to be included in regular class work . . . [The papers presented by students should have for their purpose] (1) accumulation of information by the members in many lines that they might otherwise not find time or inclination to look up for themselves; (2) stimulation to activity on the part of individuals who might otherwise be content to do the required class-room work and nothing more; (3) cultivation of independence in study, and ease and clearness in presenting the results of study to an audience; (4) preparation of students either for teaching or for further advanced study, in many ways which would not otherwise be realized in the regular college and university work."<sup>1</sup>

The arrangement of interesting and instructive programs is not an easy matter. Too often the student is unfamiliar with the wealth of topics which are available in the literature. Moreover, once he has chosen a topic for presentation to his chapter, the student usually requires aid in locating sources of information. It appears worthwhile, therefore, to prepare a series of bibliographies on subjects which are suitable for chapter programs.

The topics discussed below have been chosen more or less at random. They are topics which have been tested time and again before student mathematics clubs. This list will be continued in future numbers of *THE PENTAGON*. In order to make the bibliographies as complete and useful as possible, the editor will welcome any and all assistance. Each student who presents a paper on a chapter program is urged to submit a bibliography.

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<sup>1</sup> *The American Mathematical Monthly*, vol. 25, pp. 34-35 (Jan., 1918).

It is hoped that these series of bibliographies not only will form the basis of many successful chapter programs but will also result in future articles for THE PENTAGON.

### 1. WOMEN AS MATHEMATICIANS

- E. W. Carter, "Sophie Kowaleski," *Fortnightly Review*, vol. 63, pp. 767-783 (1895).
- Encyclopaedia Britannica*: See articles on Maria Gaetana Agnesi, Caroline Lucretia Herschel, Hypatia, and Sophie Kowaleski.
- C. A. Lubbock, *The Herschel Chronicle. The Life-Story of William Herschel and his Sister Caroline Herschel*. New York, the Macmillan Company, 1933.
- Sister Mary Thomas A Kempis, "The Walking Polyglot," *Scripta Mathematica*, vol. 6, pp. 211-217 (Dec., 1939).
- Harold D. Larsen, "The Witch of Agnesi," *School Science and Mathematics*, vol. 46, pp. 57-62 (Jan., 1946).
- H. J. Mozans, *Women in Science with an Introductory Chapter of Woman's Long Struggle for Things of the Mind*. New York, Appleton, 1913.
- New York Times*: See issues of April 15, 1933 (p. 19, col. 3) and May 4, 1935, (p. 12, col. 5) for items on Amalie Emmy Noether.
- Alfred Noyes, *The Torch-Bearers. Watchers of the Sky*. New York, Frederick A. Stokes Company, 1922. (Contains information on life of Caroline Herschel.)
- A. Rebière, *Les Femmes dans la science*. Paris, Novy, 1897. (Contains sketches of Maria Agnesi, Marquis du Châtelet, Sophie Germain, Caroline Herschel, Dorothy Klumpe, Sophie Kowaleski, Christine Ladd-Franklin, Charlotte Scott, and others.)
- A. W. Richeson, "Mary Somerville," *Scripta Mathematica*, vol. 8, pp. 5-13 (1941-1942).
- M. E. Tabor, *Pioneer Women. Fourth Series: Caroline Herschel, Sarah Siddons, Maria Edgeworth, Mary Somerville*. London, The Sheldon Press, 1933.
- Hermann Weyl, "Emmy Noether," *Scripta Mathematica*, vol. 3, pp. 201-220 (July, 1935).
- "Women as Mathematicians," *The American Monthly Review of Reviews*, vol. 23, pp. 618-619 (Nov., 1903).

### 2. THE CATTLE PROBLEM OF ARCHIMEDES

- A. H. Bell, "On the celebrated 'Cattle Problem' of Archimedes," *The Mathematical Magazine*, vol. 2, pp. 163-164 (Jan., 1895).
- T. L. Heath, *The Works of Archimedes*. Cambridge, 1897, pp. xxxiv-xxxv, 319-326.
- , *Diophantus of Alexandria, A Study in the History of Greek Algebra*, 2d edition, Cambridge, 1910, pp. 11, 12, 121-124, 279.
- B. Krumbiegel, "Dos problema bovinum des Archimedes." *Zeitschrift für Mathematik und Physik*, hist. literar. Abt., 1880, vol. 25, pp. 121-136. (This is followed by a mathematical discussion by A. Amthor, pp. 153-171.)

- H. E. Licks *Recreations in Mathematics*. New York, D. Van Nostrand, 1917, pp. 33-39.
- M. Merrimam, "The Cattle Problem of Archimedes," *Popular Science Monthly*, vol. 67, pp. 660-665 (1905).
- Ivor Thomas, *Selections Illustrating the History of Greek Mathematics*. Cambridge, Harvard University Press, 1941, vol. 2, pp. 203-5.

### 3. PAPER FOLDING

- A. J. Lotka, "Construction of Conic Sections by Paper Folding," *School Science and Mathematics*, vol. 7, pp. 595-597. (Oct., 1907). Reprinted in the *Scientific American Supplement*, vol. 73, p. 112 (Feb. 17, 1912).
- T. Sundara Row, *Geometric Exercises in Paper-Folding*. Edited and revised by W. W. Beman and D. E. Smith. Chicago, Open Court Publishing Company, 1901.
- C. A. Rupp, "On a Transformation by Paper Folding," *The American Mathematical Monthly*, vol. 31, pp. 432-435 (November, 1924).
- R. C. Yates, "Folding the Conics," *The American Mathematical Monthly*, vol. 50, pp. 228-230 (April, 1943).



"(De Moivre) happened to be at the home of the Earl of Devonshire (no doubt in the position of tutor) when Newton presented the latter with a copy of his *Principia*. Deceived by the apparent simplicity of the work, De Moivre was surprised to find it beyond his powers. He procured the book, and tore out the leaves, carrying a few of them about with him at a time to study in the intervals of his work, until he had mastered its argument. Grandjean de Fouchy remarks that he could not have offered the great mathematician a more dignified or more flattering homage than that which he rendered by thus tearing up his book."

—HELEN M. WALKER.

## THE MATHEMATICAL SCRAPBOOK

*See one promontory (said Socrates of old), one mountain, one sea, one river, and see all.*

—ROBERT BURTON.

= ▽ =

The loci of the points of intersection of the

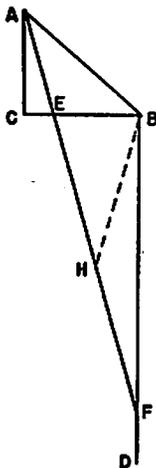
- 1) medians,
- 2) perpendiculars constructed to the midpoints of the sides,
- 3) bisectors of the angles,
- 4) altitudes

of all triangles having the same area and the same base are respectively

- 1) a straight line,
- 2) a straight line,
- 3) an ellipse,
- 4) a hyperbola.

= ▽ =

I have two current United States coins in my hand. Together they total 55 cents. One is not a nickel. What are the coins?



= ▽ =

An acute angle  $BAC$  may be trisected by the following construction.

Draw  $BC$  perpendicular to  $AC$ . Draw  $BD$  parallel to  $AC$ . Let  $AEF$  be a variable ray meeting  $BC$  in  $E$  and  $BD$  in  $F$  so that  $EF = 2AB$ . Then  $\angle EAC = \frac{1}{3} \angle BAC$ . Why?

"Probably most students meet the name of Diophantus for the first time in connection with a certain method, known as Diophantine Analysis, of solving indeterminate equations. It is a singular fact that this method, which admits only integral values for the unknowns, is not the one commonly employed by Diophantus. In fact, Diophantus sought solutions involving rational fractions as well as integers, but he did not give negative numbers as solutions."

—G. A. MILLER.

= ∇ =

On a certain pasture the grass grows at an even rate. It is known that 70 cows can graze on it for 24 days before the grass is exhausted, but 30 cows can graze there only 60 days. For how many cows will this pasture last for 96 days? (Ans. 20 cows.)

= ∇ =

The principle of mathematical induction may be illustrated very neatly by a box of dominoes.

1. Place the dominoes on end in a row so that each, in falling, will overturn the next dominoe.
2. Knock over the first dominoe.

= ∇ =

Although he was born in France, De Moivre spent 66 years of his life in England and is properly considered a member of the English school of mathematicians. The formula which bears his name,

$$(\cos x + i \sin x)^n = \cos nx + i \sin nx,$$

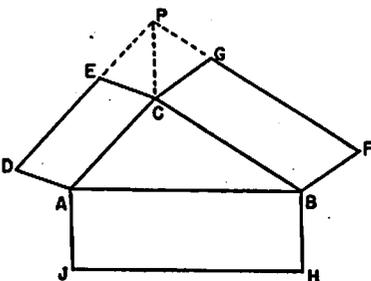
is not explicitly stated in any of his writings, although there is evidence that he was thoroughly familiar with the formula.

= ∇ =

If the thermometer registers 0° F., what will it register if it gets twice as cold?

= ∇ =

The Pythagorean theorem is a special case of the following theorem of Pappus. Let  $ABC$  be any triangle and let  $ACED$ ,  $BCGF$  be any parallelograms. Let  $DE$  and  $FG$  meet at  $P$ . Construct the parallelogram  $ABHJ$  so that  $AJ$  is equal to and parallel to  $PC$ . Then  $ABHJ = ACED + BCGF$ .



$$= \nabla =$$

$$1^3 + 5^3 + 3^3 = 153$$

$$= \nabla =$$

The product of two "teen" numbers may be obtained mentally by use of the identity,

$$(10 + a)(10 + b) = (10 + a + b) \times 10 + ab.$$

Thus,

$$17 \times 16 = (17 + 6) \times 10 + 7 \times 6 = 230 + 42 = 272.$$

$$= \nabla =$$

If a pair of pants is worth five dollars, what is a pant worth?

$$= \nabla =$$

The February, 1946, number of *The American Mathematical Monthly* has an interesting note by N. S. Mendelsohn on "A Psychological Game." The following excerpt describes the game:

"Players A and B each choose a positive integer simultaneously; the player whose number is the smaller scores one point, unless the other player chooses a number exactly one greater, in which case the latter scores two points. I have called this a psychological game because in practice each player attempts to guess his opponent's next move, and usually, after a short while, one of the players gains

a psychological ascendancy and piles up a huge score, in spite of the fact that the game is equitable.

"This game is subject to a mathematical analysis. It can be shown that the best strategy for a player A is to choose numbers 1, 2, 3, 4, 5, with frequencies, 1, 5, 4, 5, 1; the choice being made in random order. The strategy is best in the following sense; (a) there is no strategy which can beat it in the long run; (b) corresponding to any other strategy, a winning counter strategy can be devised. In particular, any strategy which incorporated the number six or higher numbers would be a losing strategy."

= ∇ =

How many ways can you find to arrange eight 8's to get 1000? Here are a few:

$$\begin{aligned} & 888 + 88 + 8 + 8 + 8 \\ & (8 \times 8 \times 8 - 8) (8 + 8) / 8 - 8 \\ & 8888 \div 8.888 \\ & 888 + (888 + 8) / 8 \\ & (88/.88) \times (88/.88) \end{aligned}$$

= ∇ =

The radical sign ( $\sqrt{\quad}$ ) originated as the initial letter of the word radical.

= ∇ =

If a woman can boil an egg in three minutes, how many women would it take to boil an egg in one minute?

= ∇ =

*Sir, he made a chimney in my father's house, and the bricks are alive at this day to testify it.*

—SHAKESPEARE.

= ∇ =

"I know of scarcely anything so apt to impress the imagination as the wonderful form of cosmic order ex-

pressed by the 'Law of Frequency of Error.' The law would have been personified by the Greeks and deified, if they had known it."

—SIR FRANCIS GALTON.

= ∇ =

Generalize the following identities:

$$3^2 + 4^2 = 5^2$$

$$10^2 + 11^2 + 12^2 = 13^2 + 14^2$$

$$21^2 + 22^2 + 23^2 + 24^2 = 25^2 + 26^2 + 27^2.$$

= ∇ =

"And how many hours a day did you do lessons?" said Alice, in a hurry to change the subject.

"Ten hours the first day," said the Mock Turtle: "nine the next, and so on."

"What a curious plan!" exclaimed Alice.

"That's the reason they're called lessons," the Gryphon remarked: "because they lessen from day to day."

This was quite a new idea to Alice, and she thought it over a little before she made her next remark. "Then the eleventh day must have been a holiday?"

"Of course it was," said the Mock Turtle.

"And how did you manage on the twelfth?" Alice went on eagerly.

"That's enough about lessons," the Gryphon interrupted in a very decided tone. "Tell her something about games now."

—LEWIS CARROLL.

= ∇ =

Arrange seventeen matches to form two rows of squares with three squares in each row, making a total of six squares. Now remove five of the matches and reduce the number of squares to three.

= ∇ =

Take any one of the quantities  $\sin^2 A$ ,  $\cos^2 A$ ,  $\tan^2 A$ ,  $\cot^2 A$ ,  $\sec^2 A$ , or  $\csc^2 A$ .

- a) Subtract from 1;
- b) find the reciprocal;
- c) subtract from 1;
- d) find the reciprocal;
- e) subtract from 1;
- f) find the reciprocal.

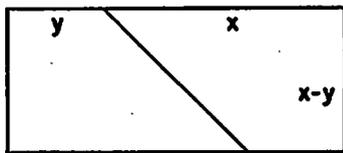
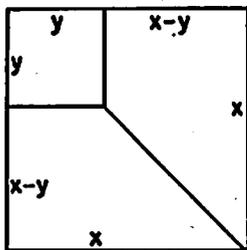
The result is the original quantity.

= ▽ =

Many feats of "lightning calculation" depend on tricks. The following simple trick is particularly impressive to an individual who is not mathematically minded. First have him choose any month in any year of any calendar. Next ask him to inclose a solid square block containing nine numbers. Then quickly tell him the sum of those numbers by adding 8 to the smallest number and multiplying the sum by 9.

= ▽ =

Many theorems of algebra may be established by means of geometry. The following figure demonstrates the identity,  $x^2 - y^2 = (x + y)(x - y)$ .



= ▽ =

Euler's form,  $x^2 + x + 41$ , is useful to illustrate the fact that a proposition in mathematics can not be established

merely by showing it is true for a large number of cases. Thus, suppose one advances the theorem that  $x^2 + x + 41$  is a prime number for all integral values of  $x$ . Then it will be found that the theorem is verified for  $x = 0, 1, 2, 3, \dots, 39$  but fails for  $x = 40$ .

Can 41 be replaced by a larger number to give a similar situation? It is known that there is at most one such number and it exceeds 1,250,000,000.

= ▽ =

The sides of a rectangular room are 39 ft. and 31 ft. respectively. A rectangular strip of carpet of area 225 sq. ft. is laid diagonally across the room in such a manner that each corner of the carpet touches one side of the room. Find the dimensions of the carpet.

—SCH. SCI. AND MATH.

= ▽ =

*The method employed I would gladly explain,  
While I have it so clear in my head,  
If I had but the time and you had but the brain—  
But much yet remains to be said.*

—LEWIS CARROLL.

# KAPPA MU EPSILON NEWS

SISTER HELEN SULLIVAN, O.S.B.

Early in the academic year of 1946-47, the news editor of THE PENTAGON will circulate special style sheets in order that the news of all chapters may be reported uniformly. It is earnestly requested that the corresponding secretary of each chapter study carefully this style sheet and prepare future news copy according to the suggested form.

This number of THE PENTAGON carries news reports from twenty chapters. It is hoped that all chapters will be able to respond with items for the next issue.

*Chapter 1. OKLAHOMA ALPHA, Northeastern State College, Tahlequah, Oklahoma.*

President.....Mr. James Harman  
Vice-President.....Mr. Warren Crane  
Secretary-Treasurer.....Miss Peggy Looper  
Secretary Descartes.....Miss Mary K. Stewart  
Faculty Sponsor.....Mr. Raymond Carpenter

Oklahoma Alpha has been very active in spite of its limited membership of eight. The theme for the year's discussions was "Men and Philosophy of Mathematics." Oklahoma Alpha again sponsored a week of popular astronomy in which an illustrated lecture was given each evening. The lecture was followed by an hour's observation of the constellations. The meetings were open to the public. The annual Founder's Day celebration will be held before the close of the term.

Mr. Nobel Bryan who joined the mathematics staff in 1937 has been appointed registrar of the college. He fills the vacancy caused by the death of Dean R. K. McIntosh. Mr. Bryan returned to the college in 1945 after spending three years in the service of the United States. Mr. Nobel Bryan, Jr., who spent five years with Boeing Aircraft in

Seattle, Washington, has returned to the college to finish work for his degree. Mr. Warren Crane and Mr. O. D. Crane who were recently discharged from military service are also back on the campus taking the college courses. Miss Thelma Jean Crane, chapter president '45, is secretary to the college president. Miss Penny Stolper, chapter president '44, was married to Mr. Albert Goodall in February at Muskogee, Oklahoma.

Oklahoma Alpha introduces its new faculty sponsor, Mr. Raymond Carpenter, who recently joined the mathematics staff. He is a native of Arkansas and holds the A.M. degree from Columbia University. Professor L. P. Woods, one of the co-founders of Kappa Mu Epsilon, is just completing his twenty-fifth year of service at Northeastern. In 1929 he was assigned the task of Dean of Men in addition to his regular mathematics teaching. During the past ten years he has given half time to directing student personnel.

**Chapter 3. KANSAS ALPHA, Kansas State Teachers College, Pittsburg, Kansas.**

President.....	Miss Hattie Highfill
Vice-President.....	Miss Nancy Horton
Secretary.....	Miss Patricia Marquardt
Treasurer.....	Mr. Jerome Degen
Secretary Descartes.....	Mr. W. H. Hill
Faculty Sponsor.....	Mr. J. A. G. Shirk

Kansas Alpha recently had initiation services for the induction of eight new members into the chapter. The total active membership numbers twenty-eight. Interesting and regular meetings were held throughout the term although there was no unifying theme for the year's activities.

Mr. Franklin Lanier, chapter president in '40-'41, is a Marine Corps liaison officer. He is located with the Navy department in Washington, D. C., where he is working on radar equipment. Professor R. W. Hart, on leave of ab-

sence from the College for the past three years, is in charge of Civil Readjustment at the Great Lakes Naval Training Station.

*Chapter 4. MISSOURI ALPHA, Southwest Missouri State Teachers College, Springfield, Missouri.*

President.....	Miss Anna Lee Taylor
Vice-President.....	Mr. Bob Stahl
Secretary.....	Miss Elizabeth Burke
Treasurer.....	Mr. William Compton
Secretary Descartes.....	Mr. Carl Fronabarger
Faculty Sponsor.....	Mr. L. E. Pummil

Missouri Alpha's twenty-one active members held initiation ceremonies for three new members on January 24th. No regular program has been planned for the present term.

*Chapter 7. NEBRASKA ALPHA, Nebraska State Teachers College, Wayne, Nebraska.*

President.....	Mr. Darold Bobier
Vice-President.....	Mr. Dean Sandahl
Secretary.....	Miss Margaret Macklin
Treasurer.....	Miss Vera Pedersen
Historian.....	Miss Betty Mae Griffon
Reporter.....	Miss Virginia Pape
Secretary Descartes.....	Mr. C. H. Lindahl
Faculty Sponsor.....	Miss Jessie W. Boyce

Nebraska Alpha is operating with ten active members. Mr. Paul Petersen, former vice-president of the chapter, has graduated and is now employed in teaching at Sutherland, Nebraska. Mr. Thomas Frentzel discontinued his studies at the mid-term to enter the service. Lt. Col. John J. Jones, former member of Nebraska Alpha, has recently been discharged from the Army. He has enrolled at the University of California in Berkeley where he is studying nuclear physics.

**Chapter 10. ALABAMA ALPHA, Athens College, Athens, Alabama.**

President.....Mr. Randall A. Freeman  
Vice-President.....Miss Florence Tilman  
Secretary.....Miss Mary Louise McCartney  
Treasurer.....Mr. Cleo Holcomb  
Secretary Descartes.....Miss Mary E. Renich  
Faculty Sponsor.....Miss Mary E. Renich

It is the opinion of the four active chapter members that the low point in membership has been passed. The vital theme of this group's activities has been "To Keep Alabama Alpha Alive." Due to the curtailed membership meetings were held but once each quarter.

On February 2nd, Alabama Alpha had initiation ceremonies for one active and three associate members. The formal ceremonies were followed by a waffle supper and a tour through the College Industrial Plant.

**Chapter 11. NEW MEXICO ALPHA, University of New Mexico, Albuquerque, New Mexico.**

President.....Mr. Bob Fox  
Vice-President.....Mr. William Dickerson  
Secretary.....Mr. Ted Hawley  
Treasurer.....Miss Merle Mitchell  
Secretary Descartes.....Miss E. Marie Hove

During the current term New Mexico Alpha has shared its programs with others who are interested in mathematics. Outstanding papers given at these meetings were "Our Calendar" by Dr. Martin Fleck and "How to Win on the Horses" by Dr. Harold Larsen.

Dr. Frank Gentry, former business manager for THE PENTAGON has resigned from the University staff to accept a position at Louisiana Technological College in Ruston. Dr. Charles Barker likewise has resigned from the University staff. He will continue to do scientific research for the

Navy at Massachusetts Institute of Technology. His latest assignment is that of chief of a crew of twenty scientists in the section of Guided Missile Antennae.

**Chapter 12. ILLINOIS BETA, Eastern Illinois State Teachers College, Charleston, Illinois.**

President.....Mr. Lyle Knott  
 Secretary.....Mr. Edward Wilson  
 Secretary Descartes.....Mr. H. H. Heller

At the present time Illinois Beta is trying to function with but two active members. Indications are strong that there will be candidates worthy of membership within the next few months.

**Chapter 13. ALABAMA BETA, Alabama State Teachers College, Florence, Alabama.**

President.....Miss June Campbell  
 Vice-President.....Mrs. Mary Evelyn Moomaw  
 Secretary-Treasurer.....Mr. Thomas Mitchell  
 Secretary Descartes.....Miss Orpha Ann Culmer  
 Faculty Sponsor.....Miss Orpha Ann Culmer

Alabama Beta has but six active members so was unable to carry on regular meetings. The new term gives promise of admitting at least six more members. The three student officers of the local chapter likewise hold office on the Student Council.

Dr. T. A. Bancroft who received his Ph.D. degree at Iowa State College, Ames, Iowa, is in Florence, Italy, where he holds a position on the staff of the Florence American University. Major C. B. Collier who has been teaching mathematics at the United States Military Academy at West Point for the past four years is now Counsellor for returning service men at the University of Tennessee.

**Chapter 15. ALABAMA GAMMA, Alabama College, Montevallo, Alabama.**

President.....Miss Annie Rives Dillard  
Vice-President.....Miss Franklee Gilbert  
Secretary.....Miss Betty Sue Wilhite  
Treasurer.....Miss Mary Havens  
Secretary Descartes.....Miss Rosa Lea Jackson  
Faculty Sponsor.....Miss Mamie Braswell

The twelve active members of Alabama Gamma arranged unusually interesting meetings during the year. Members of the chapter presented two papers at each meeting.

Rudy Renfro and Edith Foster have fellowships in Public Administration in the Southern Regional Training Program. Betty Perryman and Nelladean Chandler are working for the Southern Research Institute in Birmingham, Alabama. Carolyn Irwin has a position as bacteriologist with the State Health Department in Montgomery. Virginia Windham is a chemist for the Tennessee Valley Authority at Florence.

Virginia Jernigan, Juanita Jernigan, Sybil McCool, Helen Newton, and Lerah Sterling are now teaching. Peggy Kirk, who has been a medical student at Tulane University for the past two years, was married in November to Mr. Marion De Witt Shelton.

**Chapter 16. OHIO ALPHA, Bowling Green State University, Bowling Green, Ohio.**

President.....Miss Winifred Cole  
Vice-President.....Miss Lois Perrin  
Secretary.....Mr. Clayton McDole  
Treasurer.....Miss Zola Weaver  
Secretary Descartes.....Mr. Henry R. Mathias  
Faculty Sponsor.....Mr. Frank C. Ogg

When the Navy V-12 program terminated at Bowling Green State University, Ohio Alpha's membership decreased

to its present number, eleven. Prospects are good for a new pledge class and the possibility of regular meetings next semester.

Professor Wayne F. Cornell is on leave of absence from the mathematics staff and is working on his doctorate at Ohio State University.

*Chapter 17. MICHIGAN ALPHA, Albion College, Albion, Michigan.*

President.....	Miss Marie Shattuck
Vice-President.....	Miss Harriette Leonard
Secretary-Treasurer.....	Mr. Marion L. Bunte
Secretary Descartes.....	Mr. E. E. Ingalls
Faculty Advisor.....	Mr. E. R. Sleight

Michigan Alpha held initiation ceremonies on February 26th for ten new members, thus bringing the active membership to twenty-four. The regular meetings were devoted to a thorough study of the history, constitution, and ceremonies of Kappa Mu Epsilon and to the various applications of mathematics. As a special project Michigan Alpha has been assisting in the reorganization of the Michigan Undergraduate Research Program. Discharged from service and back on the campus are Mr. Robert Maynard and Mr. Harry Bleecker, members of Kappa Mu Epsilon.

*Chapter 20. TEXAS ALPHA, Texas Technological College, Lubbock, Texas.*

President.....	Mr. James Wanner
Vice-President.....	Miss Maisie Carter
Secretary.....	Miss Sarah Swafford
Treasurer.....	Mr. Ben Logan
Secretary Descartes.....	Miss Lida B. May
Faculty Sponsor.....	Miss Virginia Boman

Texas Alpha boasts thirty active members. Initiation ceremonies for eleven new members were held within recent months. The slogan for the term has been "Good Programs

and More Members." An unusual feature of the February meeting was the showing of the films "Atomic Bomb" and "Radar" by the Extension Department of the College.

**Chapter 22. KANSAS GAMMA, Mount St. Scholastica College, Atchison, Kansas.**

President.....	Miss Mary Lou Maloney
Vice-President.....	Miss Katherine Zeller
Secretary.....	Miss Mary Jane Fox
Treasurer.....	Miss Althea Rosmann
Secretary Descartes.....	Sister Jeanette Obrist
Faculty Sponsor.....	Sister Jeanette Obrist

Kansas Gamma is functioning with nineteen active members and thirteen active pledges. The latter will be eligible for initiation at the annual May ceremonies. The theme for the year's activities has been "Various Types of Mathematical Presentation." This topic was the unanimous choice of the chapter in order that their knowledge of the trends in mathematical writing might be advanced. During the first semester of the academic year the meetings dealt with the presentation to the general public. Meetings during the second semester dealt with the types of presentation for the practical mathematician, the scientist, and the theoretical mathematician. At an open meeting on March 11, the Reverend Pius Pretz, O.S.B., professor of mathematics at St. Benedicts College, Atchison, Kansas, gave a lecture entitled "Mathematics and the College Student." Kansas Gamma has continued the quarterly publication of the *Exponent*, the chapter paper.

Miss Patricia Warwick is employed in the office of the Economic Research Department of the Pure Oil Company, Chicago. Miss Margaret Mary Walters is a laboratory assistant in the department of chemistry at The Mount. Lt. (j.g.) Margaret Molloy, chapter president '42-'43, is soon to be discharged from the WAVES after having spent thirty months in the service. Miss Ann Hughes, chapter treasurer '44-'45, holds a position in the Office of Social

Service in Decatur, Alabama. Mrs. Bobbe Powers Beattie, chapter president '41-'42, is living in Atchison while her husband, Daniel F. Beattie, attends St. Benedicts College, Atchison. Miss Mary Catherine Donahoe, charter member of Kansas Gamma, is teaching mathematics in the Township High School, DeKalb, Illinois. Miss Mary Davis, of Elyria, Ohio, will be married in March to Mr. Richard Walsh, medical student at New York University. Miss Betty Sullivan is now a sophomore at Kansas City Junior College. Miss Clairece Hearing entered the congregation of the Sisters of St. Joseph, Wichita, Kansas, on September 3, 1945. She received the religious habit on March 19 of this year. Miss Mary Agnes Schirmer is employed by the Western Electric Company at Kearney, New Jersey.

Sister Helen Sullivan, O.S.B., is on leave of absence this year. She is teaching and carrying on research at St. Mary's High School, Walsenburg, Colorado. Sister Helen gave a paper entitled, "The Role of Mathematics in Women's Colleges," at the spring meeting of the Kansas Section of the Mathematical Association of America held on April 13th at Kansas State Teachers College, Emporia.

*Chapter 25. OHIO BETA, The College of Wooster, Wooster, Ohio.*

Due to the fact that there were no members of Ohio Beta during this term, we are unable to report any activities. Mr. Melcher P. Fobes, corresponding secretary for the chapter, is optimistic about membership possibilities in the near future.

**Chapter 26. TENNESSEE ALPHA, Tennessee Polytechnic Institute, Cookeville, Tennessee.**

President.....Mr. Wallace S. Prescott  
Vice-President.....Miss Ann Gwaltney  
Secretary.....Miss Mildred Murphy  
Treasurer.....Mr. Ollie James Agee  
Secretary Descartes.....Mr. R. K. Moorman  
Faculty Sponsor.....Mr. R. O. Hutchinson  
Tennessee Alpha is operating with ten active members.

Initiation ceremonies for six new initiates were held on February 21st. Professor Ray Kinslow has completed his research under the Oak Ridge Atomic Bomb project and has returned to the campus.

**Chapter 27. NEW YORK ALPHA, Hofstra College, Hempstead, New York.**

President.....Miss Ruth Mayer  
Vice-President.....Miss Wanda Scala  
Secretary.....Mr. Albert Capuro  
Treasurer.....Mr. J. B. Lutz  
Secretary Descartes.....Mr. E. R. Stabler  
Faculty Sponsor.....Mr. L. F. Ollmann

Active membership in New York Alpha numbers sixteen. The following talks were given by members during this academic year: "Famous Problems of Mathematics," by Miss Ruth Mayer; "Atomic Energy," by Miss Wanda Scala; "Proteins," by Professor J. G. Lutz; "Piloting and Tides," by Professor Marcus Old.

Three members have returned from service to continue their college work at Hofstra. They are Mr. Russell Terry, Mr. Nicholas Vogel and Mr. Edward Ryder. Mr. Robert Sherwood is employed as meteorologist with Colonial Airlines. Mr. Robert Beyer took the doctorate in physics at Cornell and is now a member of the staff of Brown University. Mr. Mario Juncosa is working towards the Ph.D. in mathematics at Cornell University.

**Chapter 28. MICHIGAN BETA, Central Michigan College, Mount Pleasant, Michigan.**

President.....	Miss Margaret Ketchum
Vice-President.....	Mr. Francis Teel
Treasurer.....	Miss Dorothy Sharrard
Secretary.....	Miss Dorothy Michener
Secretary Descartes.....	Mr. Lester H. Serier

Twenty-four active members in Michigan Beta are responsible for the unusually fine work being carried on there. Meetings are held regularly each month. Initiates are admitted each spring and fall. The chapter is compiling a pamphlet which will contain the mathematical requirements for all the professions. This pamphlet is to be placed in the hands of secondary school counsellors for use in their guidance programs.

A Physical Science and Mathematics Conference was held at Central Michigan College on February 9th. This conference was attended by approximately 250 of the better high school students of this area. The aim of the conference was to acquaint the students with the opportunities in the fields of mathematics and science. Outstanding speakers were: Professor Raleigh Schorling, "Mathematics as a Foundation;" Professor E. F. Barker, "The Future of Science."

**Chapter 29. ILLINOIS GAMMA, Chicago Teachers College, Chicago, Illinois.**

President.....	Miss Patricia Powers
Vice-President.....	Miss Jeanne Anderson
Secretary.....	Miss Lorraine Martinson
Treasurer.....	Miss Dorothy Tisevich
Secretary Descartes.....	Mr. J. J. Urbancek
Faculty Sponsor.....	Mr. J. J. Urbancek

The twenty-two active members in Illinois Gamma aim to make the regular monthly meetings both educational and

recreational. Outstanding meetings of the year included "Demonstration of the Slide Rule" which was attended by thirty-two students bringing their own slide rules. The lecture "Must a Mathematician Teach" by Professor James H. Zant of Oklahoma A. and M. College attracted eighty-three listeners. "Nomography" was the paper given by Dr. Joel S. Georges of Wright Junior College. The paper was clarified by the use of unusual and interesting computational charts. An unusual social meeting featured a dinner in town followed by Kodachrome movies of scenery in British Columbia and Alberta, Canada, and also of Yellowstone National Park. The movies were shown by Mr. and Mrs. Urbancek who also added interesting explanations and anecdotes.

*Chapter 80. NEW JERSEY BETA, New Jersey State Teachers College, Upper Montclair, New Jersey.*

President.....Miss Dorothy Pepoon  
Vice-President.....Miss Helga Schrank  
Secretary.....Miss Jane Wydeman  
Treasurer.....Miss Corinne Gerold  
Secretary Descartes.....Mr. Howard F. Fehr  
Faculty Sponsor.....Mr. Virgil S. Mallory

Eight active members make up this chapter. The general theme for the year's discussions and activities has been "Mathematics in its Application to Science." Outstanding papers of the year were "The Teaching of High School Mathematics" and "Probability and Games of Chance."

Ernest B. Yeager, chapter president for '44-'45, is a graduate assistant at Western Reserve University in Cleveland.

**Chapter 31. ILLINOIS DELTA, College of St. Frances, Joliet, Illinois.**

President.....Miss Lillian Koss  
 Vice-President.....Miss Helena Weigand  
 Secretary.....Miss Margaret Markiewicz  
 Treasurer.....Miss Jeanne Erlenborn  
 Secretary Descartes...Sister Claudia Zeller, O.S.F.  
 Faculty Sponsor.....Sister Claudia Zeller, O.S.F.

The fifteen active members of Illinois Delta met monthly during the past year. The following topics were presented: "Zeno's Paradoxes," by Sister Claudia; "Modern Geometry," by senior mathematics students; "Algebraic Discriminants," by junior mathematics students; "Applications of the Calculus," by sophomore mathematics students; "Threefold Aims in Teaching Mathematics," by teachers in the affiliated high school.

Miss Peggy Flynn is teaching mathematics and English at Woodson High School, Woodson, Illinois. Miss Mary Long has a full time position at the Joliet Post Office. Miss Gloria Urban is secretary for Lawyer Eva Shaw in Toledo, Ohio.



"Every tall can in every grocery store is a monument to the women 'who do not need to know any algebra.'"

—E. R. HEDRICK.