

THE PENTAGON



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KAPPA MU EPSILON

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FROM PRESIDENT PYTHAGORAS

E. R. SLEIGHT

Following out the wartime policy of many national organizations, Kappa Mu Epsilon did not hold its regular biennial convention in 1943. For this reason a committee, consisting of Dr. C. V. Newsom, New Mexico Alpha (chairman), Dr. C. C. Richtmeyer, Michigan Beta, and Dr. C. B. Tucker, Kansas Beta, was appointed to submit nominations for the various national officers. In accordance with the suggestions of this committee, and the vote of the various chapters, the following are now your officers for a two-year period:

E. R. Sleight, Michigan Alpha, President;
F. W. Sparks, Texas Alpha, Vice-President;
E. Marie Hove, Iowa Alpha, Secretary;
L. F. Ollmann, New York Alpha, Treasurer;
Sister Helen Sullivan, O.S.B., Kansas Gamma,
Historian.

The officers thus elected now constitute the National Council, and will do their best in meeting the problems of the present difficult situation.

Without the inspiration of a convention, the Council is experiencing some difficulties in formulating a program that will be adequate for the present, as well as sound and workable for the future. Without the helpful suggestions and wise counsel of our secretary, Miss Hove, the difficulties would be even greater. Under her guidance, combined with helpful letters from the past presidents, a program is being developed, and we wish to make the PENTAGON a means of transferring something of this program to the chapters.

Reports reaching the president's office indicate that many of the chapters are having difficulty in recruiting new members. Some chapters have found it necessary to suggest an inactive status for the present. The Council is

very anxious that each chapter remain active, even though only the faculty members remain. In this way contact with the national organization may be maintained; the chapter may initiate the occasional candidate who meets the requirements, and in every way retain its rights and privileges in the National Fraternity. The founders of Kappa Mu Epsilon showed wisdom in placing a great deal of the chapter responsibility upon faculty members, and we trust that this wisdom will be demonstrated in a 100% Active Chapter Roll.

Some college organizations have found it expedient to discontinue expansion for the present. Such is not the case with respect to Kappa Mu Epsilon. Mathematicians are still being trained in our institutions of higher learning. Although opportunities for expansion may be somewhat restricted, nevertheless, we hope to add a limited number of chapters to our chapter roll even in the face of difficulties.

Each member of the Council extends greetings to all chapters of Kappa Mu Epsilon as well as to each of the men and women who constitute its membership. We solicit your advice and coöperation, and in turn, pledge you our best efforts in endeavoring to maintain the high standards which have made Kappa Mu Epsilon a strong factor on many a college campus.

THE GOLDEN SECTION

R. F. GRAESSER

University of Arizona

The division of a line segment into extreme and mean ratio is called the Golden Section. By this we mean so dividing the segment that the whole segment shall have the same ratio to its larger part that its larger part has to its smaller part. In other words, the larger part is the mean proportional between the whole segment and its smaller part. The Golden Section has some interesting geometric implications. The same is true if we treat it algebraically. I shall devote about half of this paper, then, to the mathematical aspects of the Golden Section, but I shall use only very simple mathematics. In the remainder of the paper, I shall try to explain why it is called the Golden Section. Most of the writers on the Golden Section have been interested in this latter aspect; i. e., its association with metaphysics, magic, natural science, and the graphic and plastic arts.

GEOMETRY OF THE GOLDEN SECTION

In our school days we learned a ruler and compass construction for the Golden Section. I should like to show one

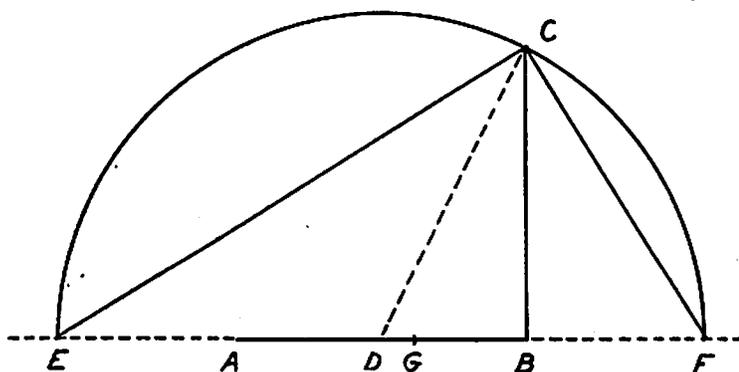


Fig. 1

of these constructions. Let AB be the segment to be divided by the Golden Section (Fig. 1). At B erect BC equal and perpendicular to AB . Let D be the center of AB , and with DC as a radius draw an arc cutting AB produced in E and F . Then BF laid off on AB gives G , the Golden Section. To prove this, recall that the perpendicular to the diameter from a point on the circumference is the mean proportional between the segments of the diameter; that is, $EB/BC = BC/BF$. Subtracting unity from both members, we have $(EB-BC)/BC = (BC-BF)/BF$. Hence, substituting equals, $AG/AB = GB/AG$, or, inverting, $AB/AG = AG/GB$.

If we take AB as the radius of a circle, then AG can be stepped off as a chord exactly ten times. In other words, AG is the side of an inscribed decagon. In order to establish this, lay off AG along AB (Fig. 2). Draw CG . Then $AC/CB = CB/GB$ by the Golden Section, and angle 1 is equal to angle 2 as base angles of an isosceles triangle. Then $\triangle ACB$ is similar to $\triangle CGB$, and hence angle 3 is

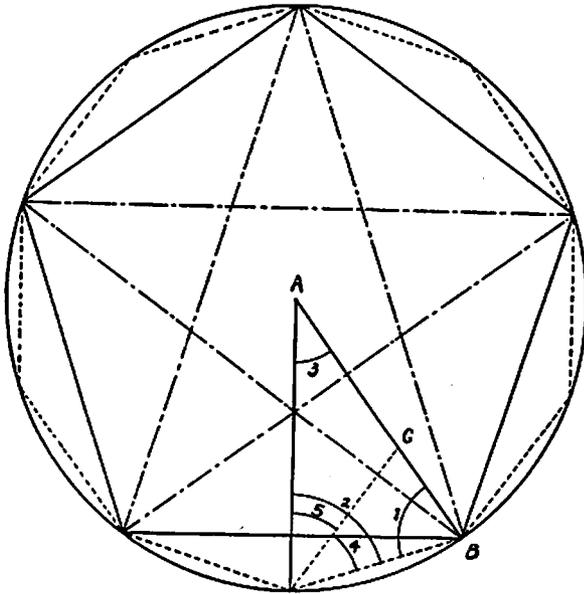


Fig. 2

equal to angle 4 being corresponding angles of similar triangles. Also, angle 5 equals angle 3 since they are base angles of an isosceles triangle. Hence, the sum of the angles of $\triangle ABC$ is equal to five times angle 3, so that angle 3 is equal to 36° . Thus, CB is the side of a regular inscribed decagon.

Return to Figure 1. I wish to state some facts which I shall not take time to prove. Consider a circle of radius AB . Then BF is the side of a regular inscribed decagon, BC is the radius or also the side of a regular inscribed hexagon, and CF is the side of a regular inscribed pentagon. From the Pythagorean theorem we have: *Given a circle, the sum of the square of the side of the regular inscribed decagon and the square of the side of the regular inscribed hexagon equals the square of the side of the regular inscribed pentagon.* This rather pretty theorem is due to Eudoxus, a contemporary of Plato, in the fourth century before Christ. Proclus, whose works inform us concerning the history of Greek geometry, says that Eudoxus "greatly added to the number of the theorems which Plato originated regarding the section," meaning, of course, the Golden Section. Again, EC is the side of the regular inscribed star pentagon or pentagram; BF is the Golden Section of CB ; CB is the Golden Section of EB ; and finally CF is the Golden Section of EF .

Later we want to discuss the so-called Golden Rectangle so perhaps I should define it before leaving geometric considerations. If the sides of a rectangle are in the ratio of the Golden Section, then we have a Golden Rectangle. Such a rectangle can be divided into a square and another Golden Rectangle which later may be again divided into a square and another Golden Rectangle, and so on ad infinitum. For this reason a Golden Rectangle forms in a sense a complete unit.

ALGEBRA OF THE GOLDEN SECTION

We now consider the Golden Section algebraically. Let a line be divided in the Golden Section, and let x be the

ratio of the longer segment to the whole line, the so-called ratio of the Golden Section. If we let the length of the line be unity when x is the longer segment, then $1 - x$ is the shorter segment, and $1/x = x/(1 - x)$, or $x^2 + x = 1$. Solving this equation we find the ratio of the Golden Section to be $x = \frac{1}{2}(-1 \pm \sqrt{5})$. Since x is positive, we select the plus sign, and $x = \sqrt{1.25} - 0.5 = 0.618$, approximately. If we write $x^2 = 1 - x$ so that $x = \sqrt{1 - x}$, and then replace x in the right member by its value, $\sqrt{1 - x}$, we obtain $x = \sqrt{1 - \sqrt{1 - x}}$. Continuing this process indefinitely we secure the value of x as an infinite radical, $x = \sqrt{1 - \sqrt{1 - \sqrt{1 - \dots}}}$. The validity of this result I shall not stop to prove, but shall present it only as a formal result. This seems to be the simplest infinite radical obtainable, and it is curious that this should give the Golden Section.

Starting again with $x^2 + x = 1$, we can write

$$x(x + 1) = 1, \text{ or } x = 1/(1 + x).$$

Replacing the x in the right member by its value, $1/(1 + x)$, and continuing this process we obtain

$$\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

which for convenience we write

$$\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

This is called an infinite continued fraction. Again it is hard to imagine any simpler such fraction, and it is striking that the simplest infinite continued fraction, like the simplest infinite radical, should give us the Golden Section. If as a sequence of approximations to this infinite continued fraction we take

$$\frac{1}{1}, \frac{1}{1+\frac{1}{1}}, \frac{1}{1+\frac{1}{1+\frac{1}{1}}}, \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}}, \dots,$$

and simplify these complex fractions, we obtain 1/1, 1/2, 2/3, 3/5, etc. The n th fraction may be obtained by adding unity to the $(n - 1)$ st fraction and taking its reciprocal. This sequence of fractions is technically known as the successive convergents of the infinite continued fraction, but I am not assuming that my readers know anything of the theory of continued fractions. Another law of formation for these convergents is readily seen. Add two consecutive numerators for the numerator of the next fraction, and do the same with the two consecutive denominators to obtain the next denominator. The sequence of numerators is the same as the sequence of denominators; viz., 1, 1, 2, 3, 5, 8, 13, Each term of the sequence is obtained by adding the two preceding terms. Any term divided by the one following gives an approximation to the Golden Section, and the further out we go the better the approximation. This sequence is well known and is named for the Italian who is considered by some to be the greatest mathematician of the Middle Ages, Leonardo Fibonacci, sometimes called Leonardo Pisano or Leonardo the Pisan. It is called the Fibonacci series, also sometimes the Lamé series. Among the miscellaneous arithmetical problems in his *Liber Abaci*, Fibonacci gives the following: How many pairs of rabbits can be produced from a single pair in a year if it is supposed (1) that every month each pair begets a new pair which, from the second month on, becomes productive, and (2) no deaths occur? This problem leads to Fibonacci's series. We shall notice this series further in our applications.

THE GOLDEN SECTION IN HISTORY AND PHILOSOPHY

What we have just been discussing seems to me to be an interesting topic in elementary mathematics, but there is nothing mysterious nor miraculous about it. Yet, from time immemorial it has been given an enchantment of mystery. It has been called the Golden Section, the Golden Mean, the

Divine Proportion, the Divine Section. Perhaps we can now understand a little of the reason for this. Mankind seems possessed of an innate urge to associate the mystic and the supernatural with phenomena that he cannot explain. From this urge arises the significances attributed to mathematical forms. The belief in such significances commenced with the ancient Babylonians and Egyptians and became a leading principle for the explanation of the universe with the members of the Pythagorean brotherhood.

Pythagoras of Samos is perhaps the most picturesque and interesting figure in Greek mathematics. He lived in the sixth century B. C. and established at Crotona, in Magna Grecia, a secret brotherhood, the so-called Pythagoreans, which became the model of secret societies from that day to this. The Pythagoreans persisted as an organization for nearly two centuries. It was a society for the pursuit of knowledge, for the study of mathematics, philosophy, and science. The Pythagoreans discovered the Golden Section. Their symbol was the pentagram or five-pointed star, whose connection with the Golden Section we have already seen, for the pentagram cannot be constructed without the use of the Golden Section. The pentagram is not only a part of the seal of Kappa Mu Epsilon, but it is also the star in the American flag, the star of the P.E.O. society, of the Eastern Stars, etc. Shades of Pythagoras!

The Pythagoreans believed that the essence and explanation of the universe lay in number and form, and that the universe was the incarnation of all wisdom and beauty. The explanation of beauty was to be found in simple ratios. This idea was later greatly strengthened by the Pythagoreans' discovery that the harmony of musical sounds depends upon simple numerical ratios among the vibration frequencies of the notes. Then what would be more natural than to seek the explanation of the beauty of proportion and form in simple ratios? What held for the ear ought also to hold for the eye. The Pythagoreans sought this explanation in the ratio of the Golden Section, and this has been going on nearly ever since. The *Art Digest* published

in New York City has had several articles in the last few years on the Golden Section. Witness also the books of Mr. Jay Hambidge, Yale University Press, entitled, *Principles of Dynamic Symmetry*, *The Parthenon and other Greek Temples and their Dynamic Symmetry*, and *Dynamic Symmetry of the Greek Vase*. The Golden Section is a special case of what Mr. Hambidge calls dynamic symmetry.

According to the Pythagoreans, if divine harmony was to be realized on this imperfect earth then harmonious ratios must exist among earthly things. To find these harmonious ratios we must seek geometric figures making the impression of greatest perfection. These are the regular figures. Regular figures are either plane or solid, and between these two kinds of regular figures there is a strange difference. Of the regular plane figures, the regular polygons, there is an infinite number, while of the regular solid figures, the regular polyhedrons, there are but five, the so-called Platonic bodies. Plutarch tells us that the Pythagoreans believed that these five regular polyhedrons were fundamental forms in the structure of the universe. There were four elements (instead of 92) in the material world; viz., earth, fire, air, and water. The nature of these elements depended on their forms. The smallest constituent particle of earth (its atom) was hexagonal or cubical, the atom of fire was a tetrahedron, that of air an octahedron, and that of water an icosahedron. These polyhedrons are constructed of comparatively simple figures, the square and the equilateral triangle which are easily obtained. The first three were known to the Egyptians. The dodecahedron with its twelve pentagonal faces depending on the Golden Section was much harder to obtain. When discovered it was taken as the symbol of the supernatural, the heavenly domain. As it contains the Golden Section, this ratio was supposed to be the dominating one in the realm of the spirit. And the star pentagram has played an important role in magic and as a talisman ever since. In various European regions it is used as a protection against evil spirits and nightmares. In Goethe's *Faust*, the devil, Mephistopheles,

is prevented from escaping from Faust's chamber by a pentagram on the threshold of the door.

We find the Golden Section appearing again with the revival of Platonic philosophy in the early Renaissance. In the latter half of the fifteenth century one of the most important European writers on mathematics was Luca Paciola. Being a monk he was also known as Fra Luca di Borgo. His *Suma* was the first printed work dealing with arithmetic and algebra. It had a wide circulation and much influence. Paciola also wrote *Divina Proportione*, the first work devoted entirely to the Golden Section. Paciola found miraculous attributes in the Golden Section which alone could belong to God. I quote,

“The first is, that this proportion is unique. It is not possible to derive other proportions or variations from it. According to both theological and philosophical doctrine this unity is an attribute of God alone. The second divine property is that of the Holy Trinity. As the Father, Son, and Holy Ghost are one and the same, likewise must one and the same ratio obtain among the three quantities no more and no less. The third attribute is that just as God cannot be defined or made comprehensible to us through words, neither can this ratio be expressed by a rational number but remains always secreted and hidden and is called by mathematicians an irrational. Fourthly, as God cannot change and is the same in all his parts and the same everywhere, so is our proportion always the same and unchanging be it evident in large or small quantities nor can it be understood in any other way. The fifth attribute can with justice be added to the preceding; viz., as God creates divine virtues (the so-called fifth element) and by means of this creates the four other elements, earth, water, air, and fire, and by means of these gives existence to every other thing in nature, so, according to Plato in his *Timaeus*, does our divine proportion give formal existence to Heaven

itself, as it gives to Heaven the form of a dodecahedron which cannot be constructed without our proportion."

Such metaphysical significance attributed to the Golden Section will meet with little favor in our modern eyes. It smacks too much of superstition and too little of science. As H. E. Timerding, a recent German author on the Golden Section remarks,

"The Golden Section has again and again enticed men to seek the road into the enchanted land of metaphysics."

THE GOLDEN SECTION IN NATURE

We shall not, however, be engaged with a metaphysical proposition if we seek to establish the Golden Section as a norm in nature. Let us consider the subject of phyllotaxy, the system of leaf arrangement in plants. It has been found that the seemingly innumerable leaf arrangements can be reduced to comparatively few oft recurring cases. We have the whorled and the spiral arrangements. In the former the leaves in a given plane are equally spaced in a whorl or verticil about the stem or stock. We may think of the spiral arrangement as the result of displacing the leaves of the whorl vertically. The leaves are then no longer arranged in a circle but are equally spaced along a so-called genetic spiral or more properly a cylindrical helix. The spiral or helix winds around the stem of the plant. In this case one obtains a fraction representing the leaf arrangement by wrapping a string about the stem to represent the helix on which the leaves are located. Any leaf is numbered 0. The others then proceed 1, 2, 3, etc., along the string until a leaf is reached which is directly above the leaf numbered zero. Suppose this leaf has the number n . Then n is the denominator of the fraction. The numerator is given by the number of complete revolutions of the string between the zeroth and n th leaves. For example, if the leaf numbered 8 is directly above the zeroth leaf and the string has encircled the stem three times between the zeroth and

the eighth leaves then the fraction expressing the leaf arrangement is $3/8$. Proceeding in this way we find that the two ranked leaves of all grasses, Indian corn, basswood, and the horizontal branches of the elm and other trees have the fraction one half. One third belongs to all sedges, alder, birch, white hellebore; two fifths (a very common ratio) applies to the willow, rose, drupe (i.e., plum, cherry, apple, apricot, peach, poplar, almond); three eighths to cabbage, asters, hawkweed, holly, plantain; five thirteenths to needles of various conifers, houseleek, and to mulleins; eight twenty-firsts to the scales of spruce and fir cones; and thirteen thirty-fourths belongs to scales of cones of the *pinus larico*.

Consider this sequence of fractions, $1/2$, $1/3$, $2/5$, $3/8$, $5/13$, $8/21$, and $13/34$. The law of formation is obvious: Add any two numerators for the next numerator; add any two denominators to form the next denominator. Higher members of the sequence belong to flowers and involucre of the composites, such as the sunflower and the thistle, and in both leaves and cones of the pine family.

If the string is wrapped about the stock in the opposite direction (to left instead of to the right) then in place of the arrangement quotient m/n one obtains the arrangement quotient $(n - m)/n$, or the series $1/2$, $2/3$, $3/5$, $5/8$, $8/13$, $13/21$, etc. The law of formation for this sequence is the same as for the former sequence. This latter sequence we have seen converges to the ratio of the Golden Section. It can easily be shown that the former sequence also converges to the same limit. This regularity of leaf arrangement is sometimes called Ludwig's Law. It is so striking that it was observed as early as 1834, and it led to mystical speculations concerning the so-called "spiral tendency of vegetation." It might be noted also that the great German poet, Goethe, was much fascinated by this subject.

A German writer, Zeising, about the middle of the last century, has perhaps the most successfully and consistently supported the Golden Section as a relationship of universal occurrence in natural forms. He says:

"The fundamental principle underlying all forms of Nature and Art which approach beauty and perfection (*totalitat*) is to be found in the relationship of the Golden Section and the Golden Section has been from the beginning (*Uranfang*) the highest goal and ideal of all forms and ratios, the cosmic as the individual, the organic as the inorganic, the acoustic as the optical, but which has been realized most completely in the human figure."

Here are a few occurrences of the Golden Section in the ideal human figure:

1. The division of the stature by the waist line is a Golden Section.
2. The point of the middle finger, when the arms hang naturally, divides the height in the Golden Section.
3. The well-known rule that the forehead, nose, and lower portion of the face should be equal can be supplemented by the statement that the lower portion of the face should be divided by the mouth in the Golden Section.
4. The eyebrows should divide the whole height of the head in a Golden Section.

Zeising gives other instances of its occurrence in the human form. The title of his book translated is *New Theory of Proportions in the Human Figure*, (Leipzig, 1854).

THE GOLDEN SECTION IN ART

Does the Golden Section give us a division which is pleasing to the eye? H. E. Timerding (*Der Goldene Schnitt*, Berlin, 1937) says that this question is to be answered with an unconditional yes. From time immemorial it has been a rule that pictures with pronounced horizons should be divided in a Golden Section by those horizons. It cannot be said, however, that this rule has been universally used; it has even been deliberately and intentionally ignored by the realists of the last century.

The Golden Rectangle has proportions particularly pleasing to the eye. This was first tested experimentally by psychologist Fechner in 1876. Fechner's procedure was to place ten different rectangles before a person, and to ask that person to choose the rectangle which had for him the most pleasing proportions. Among a large number of persons, the Golden Rectangle was the one most frequently chosen; it seemed to be the norm about which all the choices clustered.

The ratio $5/8$ is one of the convergents approximating the ratio of the Golden Section. A beautiful example of the use of this approximate value is found in the division of the height of a certain urn from the Salem cathedral. The urn consists of a base, bowl, cover, and cover ornament. If the bowl is divided into eight parts, the base and the cover each measure five such parts and the cover ornament three parts. Thus, the cover and the bowl are in the ratio of $5/8$, while the cover and the cover ornament together bear a ratio of $8/13$ to the bowl and base together. The ratio $8/13$ is the next convergent in the sequence of convergents approaching the ratio of the Golden Section. The effect of these proportions is exceptionally pleasing.

Wherever the requirements of stability or other reasons do not enter, the modern architect determines the proportions of his buildings from his own artistic or aesthetic sense, and is glad of this freedom. The ancient architect sought for fixed rules that would eliminate individual judgment. These rules took on a mystical and religious significance. They were looked upon as incarnations of divine order; to them were ascribed wonder-working powers. The measurements and proportions of the temples of the Egyptians and Babylonians had sacred significances. Division by the Golden Section was such a rule. The Golden ratio was used in the façade of the Parthenon and in façades and floor plans of other Greek temples. A passage from Herodotus leads us to believe that the Great Pyramid of Gizeh, tomb of Cheops, was constructed with the area of each face equal to the altitude. Modern measurements

confirm this. If it be true, then the relations between the altitude, slant height, and side of the base can be expressed with the Golden Section.

The literature of the Golden Section is surprisingly extensive. Literally dozens of books, pamphlets, and articles have been written about it. It is said to have other connections which I have not had time to verify, such as a relationship with the periodic table in chemistry and with the distribution of prime numbers. So this paper is far from exhaustive, but constitutes merely an introduction to the subject.

θ . Select an arbitrary length on OA as the unit distance and draw the parallel AC to OT , meeting OB extended at C . Then $\angle DCO = \theta$. Now draw OD equal to the unit length so that triangle AOD is isosceles with base angles 2θ (since they are alternate interior angles, $\angle DAO = \angle AOT$). It is evident, since $\angle ADO$ is the sum of the opposite interior angles of triangle DCO and $\angle DCO = \theta$, that $\angle DOC = \theta$. Thus, triangle DCO is isosceles and $DC = DO = 1$. Let x denote the distance OC , $2y$ the distance AD , and a the projection of OA upon the side OB (i.e., $\cos 3\theta$). From the similar right triangles CMD , CNA , and CLO , we have:

$$x/2 = (x + a)/(1 + 2y) = (1 + y)/x.$$

The elimination of y here produces:

$$x^3 - 3x - 2a = 0.$$

Since we may construct the quantity $a = \cos 3\theta$, we may think of this as being given with the angle AOB . If the point C , or its distance x along OB , can be determined, the problem is at once solved by connecting C to A and then constructing the trisecting parallel OT . Thus, the geometrical solution of the problem is equivalent to the algebraic solution of the corresponding trisection equation. This cubic equation cannot in general be decomposed into factors

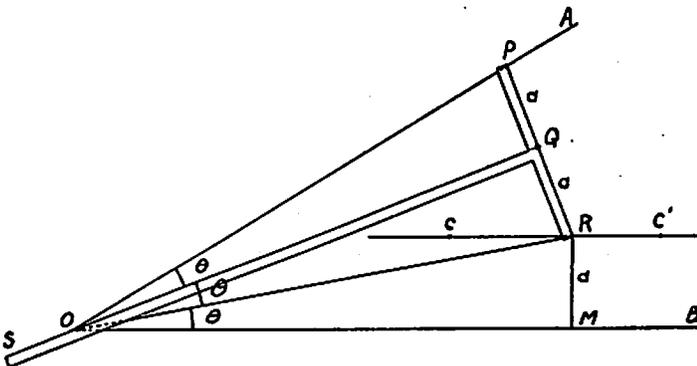


Fig. 2

with real constructible coefficients [5, 15], and thus its roots cannot be obtained by straightedge and compasses.

The Carpenter's Square [12, 13] is essentially a "T" formed by joining rigidly together a segment and its perpendicular bisector. Let $\angle AOB$ be the angle to be trisected (Fig. 2). Place the edge QS of the "T" along one side of OB of the angle and mark two distinct points, C and C' , at a distance $PQ (= a)$ from OB . Draw CC' . Insert the segment PR so that P falls upon OA , R upon CC' , with the edge QS passing through O . Then $\angle AOB$ is trisected by QS . For, from the congruent right triangles POQ , ROQ , ROM , it follows that $\angle POQ = \angle ROQ = \angle ROB$.

The Tomahawk [6, 15] is similar to the Carpenter's Square. It is formed by rigidly attaching a tangent line to a semicircular disk whose diameter is extended the length of the radius as shown (Fig. 3). Its application to the

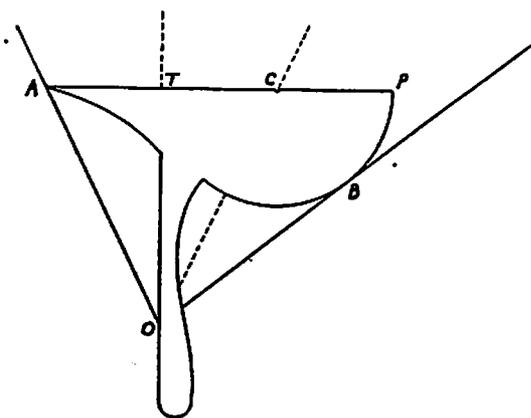


Fig. 3

trisection of a given angle AOB is immediate.

The two foregoing instruments may also be used to draw the cissoid and duplicate the cube (i.e., extract the cube root of a selected segment) [17].

A single straightedge carrying two arbitrarily placed marks, P and Q , will trisect the general angle if assisted by the compasses [3, 8, 15, 17]. Let the distance PQ be $2a$

(Fig. 4). Upon one side OB of the given angle establish $OT = a$. Draw TX and TY perpendicular and parallel respectively to OA . Place the ruler through O so that P and Q fall on these lines as shown. Then OPQ trisects $\angle AOB$.

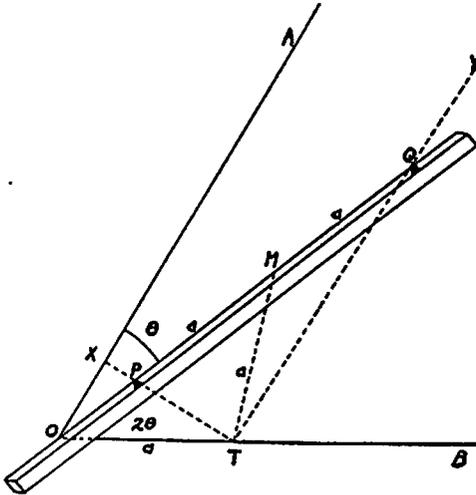


Fig. 4

For, if M be the midpoint of PQ , then $PM = MQ = MT = a$, and $\angle MQT = \angle MTQ$. Since $\angle OMT$ is an exterior angle of triangle MQT , and further since $MT = OT = a$, then $\angle MOT = \angle OMT = 2 (\angle MQT)$. But since OQ traverses two parallel lines, $\angle AOQ = \angle OQT$. Thus, $\angle MOT = 2 (\angle AOQ)$, and OQ is a trisector of $\angle AOB$.

It should be noted that as P travels along TX with the straightedge always through O , the path of Q is one branch of the conchoid of Nicomedes, one of the curves prominent in the history of trisection [15, 16].

As P and Q travel upon the fixed lines TX and TY , the path of any selected point of PQ is an ellipse having the fixed lines as axes of symmetry. The envelope of the lines PQ , and of the family of ellipses thus formed, is the astroid, the four-cusped member of the hypocycloid family [16, 17].

The paper and cone [1, 15] combination affords an

interesting trisection arrangement. A right circular cone is constructed having its slant height equal to three times the base radius (Fig. 5). The cone is placed so that the

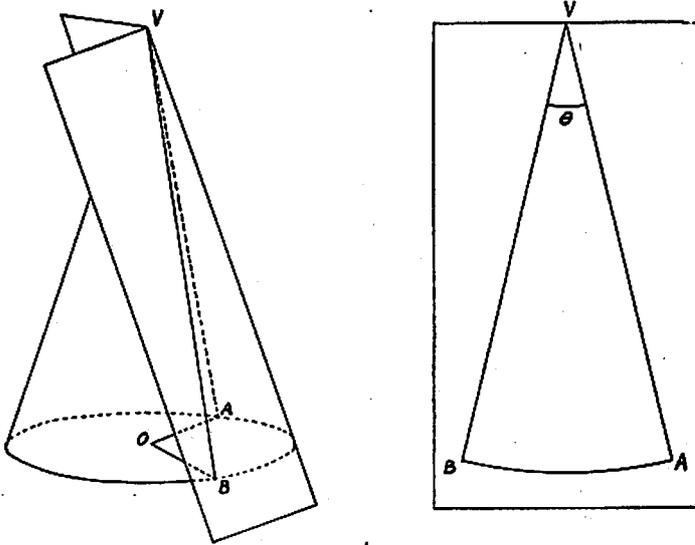


Fig. 5

center of the base is coincident with the vertex of the given angle $AOB = 3\theta$. Then arc $AB = 3r\theta$. A sheet of paper is now wrapped around the cone and the points A , B , and V are marked on it as shown. When the sheet is removed and flattened out, the angle AVB is one-third angle AOB . For, since $AV = 3r$, arc $AB = 3r (\angle AVB)$, and thus $\angle AVB = \theta = (\angle AOB)/3$.

The instrument of Pascal [1, 15, 16] is composed of three bars and incorporates a straight groove, or slot, in which a point F slides (Fig. 6). The two bars OE and OF are taken equal in length and hinged together at O . The grooved bar ED is extended to C so that $CE = OE = OF = a$. To trisect a given angle AOB , fix the bar OF upon a side OB of the angle and move the point C until it falls on the extension of OA . Now, since $\angle OCE = \angle COE = \theta$, $\angle OEF = \angle OFE = 2\theta$, then $\angle AOF = 3\theta$.

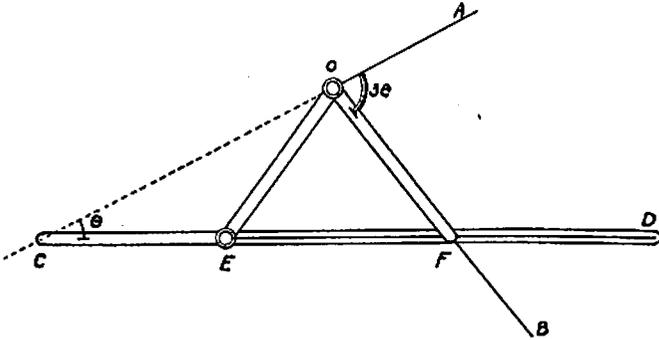


Fig. 6

Other features of this mechanism are noteworthy. If the bar OF is fixed, as is the case in trisecting the angle, the path of C is a limaçon of Pascal. If, instead, CD is fixed, any selected point of OF (or any point rigidly attached to OF) describes an ellipse symmetrical about CD . If C is fixed and O be moved along a fixed line CA , then F describes the cycloidum anomalarum of Ceva, a curve resembling the lemniscate.

The common pantograph [4, 9, 15, 16] is formed from a hinged parallelogram with extended legs. If the Pascal device of Fig. 6 is reflected in the bar CD the result (if the bars OF and $O'F$ are extended) is the apparatus of Fig. 7. Let X and Y be the points of intersection of the sides of a given angle AEB with the circle having E for center and EO for radius. Fix the diagonal groove along the bisector of the given angle and let the point F move until the bars OF and $O'F$ pass through X and Y . Then $\angle OFO' = (\angle AEB)/3$. For, it can be shown that arc $XP = \text{arc } OO' = \text{arc } P'Y$, and since arc $OO' = \text{arc } PP'$, then $\angle EXP = \angle PEP' = \angle P'EY = \angle OFO'$. The grooved bar may be discarded of course, if some other arrangement is made to guide the point F along the bisector.

Crossed parallelograms [8, 15] form an entertaining trisector. Let $OABC$ be constructed from four bars, equal in pairs and crossed (Fig. 8). That is, $OA = BC = a$ and $OC = AB = b$, with the bars hinged at $O, A, B,$ and C .

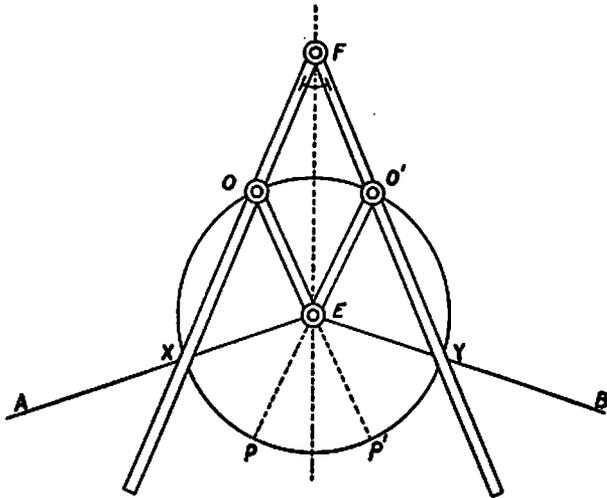


Fig. 7

It is evident that the angles at O and B remain equal through all deformations of the crossed parallelogram. A second crossed parallelogram is joined to the first as shown so that $DE = OC = b$ and $OD = CE = c$. Generally, $\angle DOC$ does not equal $\angle COA$. However, if these angles are to be equal, the two crossed parallelograms must be similar and have proportional sides, and conversely so. Thus, if $\angle DOC = \angle COA$, then $a/b = b/c$ or $b^2 = ac$.

If a third crossed parallelogram be joined to the second, in similar fashion, three equal angles will be formed at O

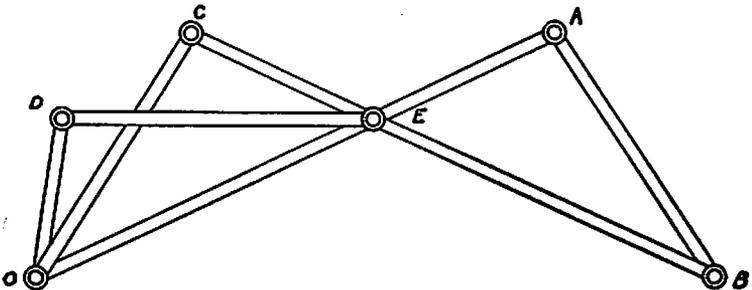


Fig. 8

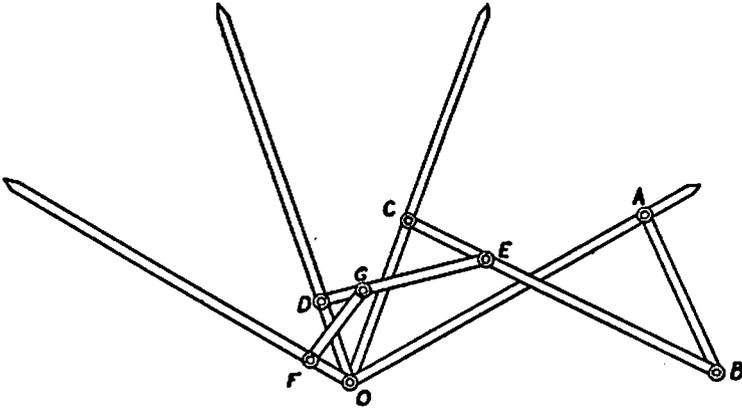


Fig. 9

as shown in Fig. 9. The four lengths involved form a geometric progression. The bars OA , OC , OD , etc., may be extended to convenient lengths for the sake of appearance.

BIBLIOGRAPHY

1. Aubry: *Journal de Mathématiques Spéciales* (1896), pp. 76- 84; pp. 106-112.
2. Breidenbach, W.: *Die Dreiteilung des Winkels*, Leipzig, (1933).
3. Bussey, W. H.: *American Mathematical Monthly*, Vol. 43 (1936), pp. 265-280.
4. Ceva, Th.: *Acta Erud.* (1695), p. 290.
5. Dickson, L. E.: *Elementary Theory of Equations*, New York, (1914).
6. Good, A.: *Scientific Amusements* (no date: reprinted about 1937).
7. Hudson, H. P.: *Ruler and Compasses*, London, (1916).
8. Kempe, A. B.: *Messenger of Mathematics*, NS IV (1875), pp. 121-124.
9. Lagarrigue, J. F.: *The Trisection Compass*, New York, (1831).
10. *Liouville's Journal*, Vol. 2, p. 366.
11. Mitzcherling, A.: *Das Problem der Kreisteilung*, Leipzig, (1913).
12. Nicholson, T. W.: *The Analyst*, Vol. 10 (1883), pp. 41-43.
13. Scudder, H. T.: *American Mathematical Monthly*, Vol. 35 (1928) pp. 250-251.
14. Vahlen, T.: *Konstruktionen und Approximationen*, Leipzig, (1911).
15. Yates, R. C.: *The Trisection Problem*, Baton Rouge, (1942).
16. Yates, R. C.: *The Mathematics Teacher*, Vol. 37 (1944), pp. 23-26.
17. Yates, R. C.: *Tools, A Mathematical Sketch and Model Book*, Baton Rouge, (1941).
18. General Bibliography: *L'Intermédiaire des Mathématiciens*, supplements of May and June, (1904).

THE NEWTON-LEIBNIZ CONTROVERSY

MURIEL THOMAS, A. B., '42

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"Roughly speaking, we may say that there are five distinct stages in the history of modern mathematics."¹

The first stage was the invention of analytic geometry by Descartes in 1637. The second stage was marked by the invention of the fluxional or differential calculus. The second was made possible by the first, and together these two subjects became the chief means of further progress in mathematics.

"All the mathematical analysis was leading up to the ideas and methods of the infinitesimal calculus. Fore-shadowings of the principles and even of the language of that calculus can be found in the writings of Napier, Kepler, Pascal, Fermat, Wallis, and Barrow. It was Newton's good luck to come at a time when everything was ripe for the discovery and his ability enabled him to construct almost at once a complete calculus."²

However, the question, "Who invented the calculus, or rather, to whom do the rights of priority belong?", has been the subject of a long and bitter controversy between Isaac Newton and Wilhelm Leibniz. The heated and widespread controversy of the 18th century has today led to an almost universal agreement among mathematical historians that each of the two invented it independently.

A third viewpoint is expressed by W. P. Milne and G. J. B. Wescott when they quote J. M. Child who has done recent historical research on the problem. Child says, "Isaac Barrow was the first inventor of the infinitesimal calculus; Newton got the main idea of it from Barrow by personal communication; and Leibniz also was in some measure indebted to Barrow's work, obtaining confirmation of his

¹ W. W. R. Ball, *A Short History of Mathematics*, p. 264.

² *Ibid.*, p. 347.

own original ideas and suggestions for their further developments from the copy of Barrow's book that he purchased in 1673." Milne and Wescott go on to say, "The general conclusion seems to be that Barrow reached his results in geometrical form, while Newton and Leibniz developed Barrow's ideas analytically by means of algebra."³

Isaac Barrow, as Isaac Newton's professor at Cambridge, was certainly in a position to communicate ideas to his student. However, Isaac Barrow, whose problems on tangents are mentioned by various authors, didn't present the calculus as the algebraic, analytic method we know today even though there is the possibility that from him Newton received the basis for his invention. Much work had been done on the solution of the problems of inverse and direct tangents. "It is evident that the lines of approach to the calculus in general have been two in number, one representing the static phase as seen in the measurements of fixed lengths, areas, or volumes, and in the making use of such ideas as those of the infinitesimals and indivisibles, the other representing the dynamic phase as seen in motion of a point."⁴ Kepler and Cavalieri worked along the first, and Newton and Leibniz followed the second trend of thought.

"The early distinction between the systems of Newton and Leibniz lies in this, that Newton holding to the conception of velocity or fluxions, used the infinitely small increment as a means of determining it, while with Leibniz the relation of the infinitely small increments is itself the object of determination. The difference between the two rests mainly upon a difference in the mode of generating quantities."⁵

A short history of the development of the calculus by Newton and by Leibniz is in order. In 1665, Newton began work on the calculus, which was described by him as the theory of fluxions, and he used this theory in finding the tangent and radius of the curvature at any point on a curve.

³ W. P. Milne & G. J. B. Westcott, *A First Course in The Calculus*, p. xiii.

⁴ David E. Smith, *History of Mathematics*, Vol. II, p. 691.

⁵ F. Cajori, *A History of Mathematics*, p. 197.

In 1666, he applied the method of fluxions to the study of equations.⁶ The independent variable—*fluents*—were represented by x , y , z , and were defined as quantities that could be considered as gradually and indefinitely increasing.

Fluxions were the “velocities by which every fluent is increased by its generating motion,”⁷ and were represented by the letters x , y , and z with dots above them. Newton’s moments of fluxions corresponded to Leibniz’s differentials.

Newton assumed that all geometrical magnitudes might be considered as generated by continuous motion; thus a line may be considered as generated by the motion of a point, and a plane may be considered as being generated by the motion of a line, etc. The quantity thus generated was defined by him as the fluent or flowing quantity. The velocity of the moving magnitude was defined as the fluxion of the fluent. The fluxion is defined as the ratio of the rates at which the related quantities of a function change one with regard to the other. This seems to be the earliest definite recognition of the idea of a continuous function, though it had been foreshadowed in some previous papers.⁸ Newton worked with two kinds of problems, the object of the first was to find the fluxion of a given quantity, or more generally, “The relation of the fluents being given, to find the relation of their fluxions,”⁹ which is equivalent to differentiation; the second was in general, “An equation being proposed exhibiting the relation of the fluxions of quantities, to find the relations of those quantities, or fluents, to one another,”¹⁰ which is equivalent to integration.

Newton and Leibniz both based their systems on the infinitesimals. The fundamentals of the calculus were set forth by Newton in the *Principia*. This invention was one of the great achievements of the 17th century. This method of analysis, expressed in the notation of fluxions and fluents, was used by Newton in or before 1666, but no account of it

⁶ David E. Smith, *History of Mathematics*, Vol. I, p. 400.

⁷ Cajori, *op. cit.*, p. 194.

⁸ Ball, *op. cit.*, p. 344.

⁹ *Ibid.*

¹⁰ *Ibid.*

was published until 1693, though its general principles were known to his friends and pupils long before it was published. No complete exposition of it was given before 1736. Newton never published any of his work, with the exception of two works on optics, without being practically forced to do so by his friends. He hadn't any desire for fame and he disliked the controversies that resulted from the publication of a new theory.

Leibniz, on the other hand, had published various philosophical and mathematical works, and had enjoyed much prominence on the continent as an intellectual aristocrat. Starting out as a lawyer, Leibniz was sidetracked to Mathematics after a visit to Paris where he met Huygens, under whom he studied Cartesian geometry and the direct and inverse problems of the tangent. In employing the "infinitely small triangle between the infinitely small part of the curve coinciding with the tangent, and the differences of the ordinates and abscissas"¹¹ that both Barrow and Pascal had used in studying tangents, Leibniz saw that the problem of tangents was connected to the quadrature of curves. In this connection he first introduced a new notation. He says, "It will be useful to write sl for the sum of the l 's."¹² "From this he deduced that since the symbol for summation raises the dimension, the opposite calculus, or that of differences, d , would lower them. Therefore if $sl = ya$, then $l = ya/d$. The symbol d was first placed in the denominator."¹³ Cajori says that this is taken from a manuscript dated October 29, 1675. Leibniz does not use the term *differential* but always uses *difference*. He does not explain the significance of dx and dy except in a Latin marginal note until ten years after he first started using the notation. It took him considerable thought and effort to determine whether or not $dx dy$ and $d(xy)$, and dx/dy and $d(x/y)$ were equivalent forms. He concluded that they were not equivalent but he could not give the true value for each.

¹¹ Cajori, *op. cit.*, p. 207.

¹² *Ibid.* (quoted in Cajori).

¹³ *Ibid.* (quoted from C. J. Gerhardt, *Entdeckung der hoheren Analysis*. Halle, 1855, p. 125).

The problem of the inverse method of tangents was easily solved by integral calculus, and within half a year Leibniz had discovered that the direct problem of tangents also yielded to his new instrument—the calculus. Before he left Paris, he had most of the elementary rules and formulae of the infinitesimal calculus in his possession. "In 1684, or nine years after the new calculus first dawned upon the mind of Leibniz, and 19 years after Newton first worked at fluxions, and three years before the publication of Newton's *Principia*, Leibniz published, in the *Acta eruditorum*, his first paper on the differential calculus."¹⁴ This paper is brief and vague, no proofs of the rules of calculation are given, and the meaning of dx and dy is not made clear. Two years later he published another paper setting forth the rudiments of the integral calculus. John and James Bernoulli became interested in the new analysis and studied Leibniz's papers; thus they came in for their part in the famous controversy.

As to the actual controversy—it was not a battle of "wits" but a battle of great intellectual leaders and, in fact, a battle of nations as the friends and countrymen of the two men rallied to their support and the dispute was long and bitter. Smith says, "The dispute between the friends of Newton and those of Leibniz as to the priority of discovery was bitter and profitless . . . The facts are that Leibniz knew of Barrow's work on the 'differential triangle' before he began his own investigations, or could have known of it, and that he was also in a position to know something of Newton's work. The evidence is also clear that Newton's discovery was made before Leibniz entered the field; that Leibniz saw some of Newton's papers on the subject as early as 1677; that he proceeded on different lines from Newton's and invented an original symbolism; and that he published his results before Newton's appeared in print."¹⁵

Another author presents the facts in a somewhat different manner. Evidence that is to be found in the notes of

¹⁴ *Ibid.*, p. 210.

¹⁵ Smith, *op. cit.*, Vol. II, p. 699.

Leibniz on his visit to England (1673) lends support to the possibility of his having read Isaac Barrow's lectures. In these notes he does not refer to Newton except in connection with optics. Evidently Leibniz did not obtain a knowledge of fluxions during this visit to London, nor is it claimed by any of his opponents that he did. "Various letters of I. Newton, J. Collins, and others up to the beginning of 1676, state that Newton invented a method by which tangents could be drawn without the necessity of freeing their equations from irrational terms. Leibniz announced to H. Oldenburg, secretary of the Royal Society, in 1674, that he possessed very general analytical methods, by which he had found theorems of great importance on the quadrature of the circle by means of series. Oldenburg replied that Newton and Gregory had also discovered methods of quadrature which extended to the circle. Newton, at Oldenburg's request, wrote two letters to Leibniz."¹⁶ It is upon these two letters that Newton's supporters based their claim that Newton had communicated his method to Leibniz who had stolen the invention. Nothing material happened until about 1684, when Leibniz published his first paper on the differential calculus. While the priority of invention undoubtedly belongs to Newton, Leibniz was the first to give the full benefit of the calculus to the world. "In 1695, John Wallis informed Newton by letter that he had heard that his notions of fluxions passed in Holland with great applause by the name *Leibniz Calculus Differentialis*."¹⁷ Wallis stated in a preface to one of his own works that the *calculus differentialis* was Newton's method of fluxions which had been communicated to Leibniz in Newton's letters.

In 1699, a Swiss, Fatio de Duillier, stated in a paper which he presented to the Royal Society that Newton was the first inventor, and that, whether Leibniz was the second inventor or had borrowed his ideas from Newton, he would leave to be established by Newton's papers and manuscripts. This was the first direct hint of plagiarism, but English mathe-

¹⁶ Cajori, *op. cit.*, p. 214.

¹⁷ *Ibid.*

maticians had obviously been cherishing suspicions against Leibniz. These insinuations brought the matter to an open dispute. Leibniz replied to the accusation and complained to the Royal Society of the injustice done him. John Keill, a professor at Oxford, undertook the defense of Newton and claimed that Leibniz had published the calculus as invented by Newton, changing only the name and notation. The Royal Society was appealed to as a judge and in their report their final conclusion was that Newton was the "first inventor."¹⁸ But this was not the point. The question was not whether Newton was the first inventor, but whether Leibniz had stolen the method. The committee had not formally ventured to assert their belief that Leibniz was a plagiarist. They insinuated that Leibniz did take or might have taken his method from that of Newton when they said, "And we find no mention of his [Leibniz's] having any other *Differential Method* than Mouton's before his letter of 21st of June, 1677, which was a year after a copy of Mr. Newton's letter of the 10th of December, 1672, had been sent to Paris to be communicated to him; and about four years after Mr. Collins began to communicate that letter to his correspondents in which letter the method of fluxions was sufficiently described to any intelligent person."¹⁹

Although it was later shown that Leibniz never received the letter in question but only excerpts made by Oldenburg, the action of the Royal Society prolonged the dispute. John Bernoulli, in a letter to Leibniz, also helped to prolong the dispute. The letter, as unfair to Newton as any Englishman had been to Leibniz, was published as an anonymous tract and was answered by John Keill. Newton and Leibniz soon appeared as mutual accusers in several letters addressed to third parties.

National pride and party feeling prevented the adoption of impartial opinions in England, but it is now generally admitted by nearly all who are familiar with the topic that Leibniz really was an independent inventor. C. J. Gerhardt

¹⁸ *Ibid.*

¹⁹ *Cajori, op. cit., p. 215.*

points out in his study of Leibniz's mathematical papers that there was a gradual and natural evolution of the rules of the calculus in his own mind.²⁰

According to Ball it is quite possible that Leibniz had access to some of Newton's papers on the subject. Notes have been found among his papers on "De Anlysi per Equationes Numero Terminorum Infinitas" by Newton. He was working, in 1675, with Tschirnhausen who possessed a copy of Newton's manuscript. However, it is equally possible that he made the extracts after Newton printed the book in 1704.²¹ Whether or not he had the extracts and used them in his invention or made them later it is impossible to determine. The whole question is a difficult one. It is a case where the evidence is conflicting and circumstantial, and the distance in time adds another problem to be faced by the investigator. It is essentially a case of Leibniz's word against a number of suspicious details pointing against him. His word becomes less reliable when it is considered that on more than one occasion he deliberately altered or added to important documents and falsified a date on a manuscript.²² However, it must also be remembered that it was not generally claimed that the entire system was stolen by Leibniz but only that he received several suggestions which formed the basis of his later work. The notation he introduced was his own, as well as his development of the subject, so perhaps he regarded the original suggestions as immaterial and considered the work to be entirely his own.

"The controversy is to be regretted on account of the long and bitter alienation which it produced between English and Continental Mathematicians. It stopped almost completely all interchange of ideas on scientific subjects. The English adhered closely to Newton's methods until about 1820, and remained, in many cases, ignorant of the brilliant mathematical discoveries that were being made on the

²⁰ Ball, *op. cit.*, p. 359.

²¹ *Ibid.*, p. 360.

²² Ball, *op. cit.*, p. 360.

continent. The loss in point of scientific advantage was almost entirely on the side of Britain. The only way in which this dispute may be said to have furthered mathematics is through the challenge problems by which each side attempted to annoy its adversaries."²³

During the 18th century, the majority of opinion was against Leibniz, but today the inclination of mathematical historians as a whole is to believe that the calculus was invented independently by Isaac Newton, the Englishman, and Gottfried Wilhelm Leibniz, the German.

²³ Cajori, *op. cit.*, p. 217.

BIBLIOGRAPHY

- Ball, W. W. R., *A Short History of Mathematics*. Macmillan and Co., 1908.
- Cajori, F., *A History of Mathematics*. Macmillan, 1919.
- Milne, W. P., and J. G. B. Wescott, *A First Course in the Calculus*. London, 1919.
- Smith, D. E., *History of Mathematics*, Vol. I and II. Ginn and Co., 1925.

THE MATHEMATICAL SCRAPBOOK

If you strike

*Upon a thought that baffles you, break off
From that entanglement and try another.
So shall your wits be fresh to start again.*

—ARISTOPHENES.

==▽==

Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not. 'Immortality' may be a silly word, but probably a mathematician has the best chance of whatever it may mean.

—G. H. HARDY.

==▽==

If we start with any number and add the number obtained by reversing the digits, and continue this process as often as necessary, a number is reached eventually which reads the same from either end. For example,

$$\begin{array}{r} 64 \\ 46 \\ \hline 110 \\ 011 \\ \hline 121 \end{array}$$

==▽==

Can you decode the following problem in addition by assigning digits to each letter?

$$\begin{array}{r} SEND \\ MORE \\ \hline MONEY. \end{array}$$

==▽==

According to Stobaeus, someone who began to read geometry with Euclid had no sooner learned the first

theorem, than he asked, 'What shall I get by learning these things?' Whereupon Euclid called a slave and said, 'Give him three-pence, since he must need make gain out of what he learns.'

=▽=

Mathematics is the science which uses easy words for hard ideas.

—EDWARD KASNER.

=▽=

Bodacious was born on July 1, 20 B. C., and died on July 1, 20 A. D. How old was he when he died?

=▽=

Six monkeys set to strum unintelligently on six typewriters for millions of years would be bound in time to write all the books in the British Museum.

—HUXLEY.

=▽=

State and prove the general law illustrated by the following relationships:

$$1^3 = 1,$$

$$2^3 = 3 + 5,$$

$$3^3 = 7 + 9 + 11,$$

$$4^3 = 13 + 15 + 17 + 19.$$

=▽=

A road is one mile uphill and one mile downhill. An auto travels up the hill at 25 miles per hour. How fast must it travel downhill to maintain an average speed of 50 miles per hour?

=▽=

In 1640 Pascal stated the remarkable theorem known as the 'Mystic Hexogram': If any six points are chosen on a conic section, numbered 1, 2, 3, 4, 5, 6, then the intersection of the sides 12.45, 34.61, 56.23 will lie in a straight line.

—J. B. SHAW.

Was it not Felix Klein who remarked that all analysis was centered here? Every symbol has its history—the principal whole numbers 0 and 1; the chief mathematical relations + and =; π the discovery of Hippocrates; i the sign for the 'impossible' square root of minus one; and e the base of Napierian logarithms. —H. W. TURNBULL.

=▽=

3.141592653589793238462643383279

Sir, I send a rhyme excelling
In sacred truth and rigid spelling
Numerical sprites elucidate

=▽=

For me the lessons's dull weight
If nature gain
Not you complain
Tho Dr. Johnson fulminate.

=▽=

Let one small formula be quoted as an epitome of what Euler achieved:

$$e^{\pi i} + 1 = 0.$$

A baker sent a boy to deliver an order for nine doughnuts. The baker placed the doughnuts in a box, and wrote IX on the cover to indicate the number of doughnuts contained therein. On the way, the boy ate three of the doughnuts, and then changed the number on the cover accordingly. His pencil had no eraser. How did he change the number?

=▽=

Mathematics can never tell us whether any alleged fact is true or false. Mathematics, in short, has no more to do with truth than logic has. To say that something is mathematically proved is tantamount to saying that it cannot possibly be true. —E. V. HUNTINGTON.

Professor Prim makes it a point to arrive at his classes precisely on time. Yesterday, in setting out for his 9:00 o'clock class, he indulged in some mental arithmetic. He figured that if he walked at the rate of 3 miles per hour he would be 15 minutes late, whereas if he walked at 5 miles per hour he would arrive 15 minutes early. How fast did he walk?

=▽=

Multiply 6×7 , 66×67 , 666×667 , etc., and establish the general rule involved.

=▽=

A small boy looked longingly at the luscious bananas, tagged 4 for 3 cents. Then he reasoned: Four for 3 cents, 3 for 2 cents, 2 for 1 cent, 1 for nothing. Guess I'll take one.

=▽=

The following examples illustrate a mental method for squaring numbers of 3 digits which are near 500:

$$(512)^2 = (512 - 250) \times 1000 + 12^2;$$

$$(486)^2 = (486 - 250) \times 1000 + 14^2.$$

=▽=

The certainty of mathematics depends on its complete abstract generality. —WHITEHEAD.

=▽=

If 1 is added to the product of any four consecutive positive integers, the result is always a square. Thus,

$$1 \cdot 2 \cdot 3 \cdot 4 + 1 = 5^2;$$

$$2 \cdot 3 \cdot 4 \cdot 5 + 1 = 11^2;$$

$$3 \cdot 4 \cdot 5 \cdot 6 + 1 = 19^2; \text{ etc.}$$

=▽=

The following four incorrect statements are written with the aid of matches. Change each one into a true equation by moving just one match.

$$\text{VI} - \text{IV} = \text{IX} \quad (\text{twelve matches});$$

$$\text{VI} - \text{X} = \text{IV} \quad (\text{eleven matches});$$

$$\begin{aligned} V - IV &= VII && \text{(twelve matches)}; \\ MV - I &= IV && \text{(thirteen matches)}. \end{aligned}$$

=▽=

If a , b , and c are rational, and $a + b + c = 0$, then the quadratic equation $ax^2 + bx + c = 0$ always has a rational root.

=▽=

$$\log 1.371288575 = .1371288575.$$

=▽=

A farmer used to bring watermelons up to his house and sell them to passing motorists. He always stocked with a dozen small melons and a dozen large ones. He sold the small melons at three for a quarter; the large ones, two for a quarter. The farmer's problem was that most persons wanted only one watermelon, and didn't bother much about sizes. So, since three for a quarter and two for a quarter made five for fifty cents, he began to sell his melons at a dime apiece. Counting the receipts after he had tried his new system, the farmer found he had taken in two dollars and forty cents, while on previous days he had taken in two dollars and fifty cents. He looked all over the barnyard for the dime that he had lost, but never found it. What happened to the farmer's dime?

=▽=

The day preceding the duel [which resulted in his death], Galois penned three letters. One of these, addressed to his friend, Auguste Chevalier, contained a résumé of the whole of the young man's contribution to mathematics. That letter, fortunately, is preserved. Brief as it is (7 pages), it is one of the most precious documents in science. . . . It was only years after . . . that his work began to be recognized and acclaimed for something of its true worth. More than a generation later, a French mathematician, C. Jordan, undertook the task of 'discovering' Galois to the scientific world. Jordan found he had to devote a lifetime to the task

. . . and still had to leave it unfinished, although he wrote a ponderous commentary of several hundred pages in an attempt to elucidate what Galois, at nineteen, had set down on a few sheets of foolscap.

—GUSTAV DAVIDSON.

=▽=

$$1^3 + 2^3 + 3^3 + \dots + n^3 = (1 + 2 + 3 + \dots + n)^2.$$

=▽=

The following are five ways of writing 100 by means of the digits 1, 2, 3, . . . , 9 using each once and only once.

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \cdot 9;$$

$$1 + 2 \cdot 3 + 4 + 5 + 67 + 8 + 9;$$

$$1 \cdot 2 + 34 + 56 + 7 - 8 + 9;$$

$$123 - 45 - 67 + 89;$$

$$(1 + 2 - 3 - 4)(5 - 6 - 7 - 8 - 9).$$

—SPHINX.

=▽=

Hanging over a pulley there is a rope with a weight at one end; at the other end hangs a monkey of equal weight. The rope weighs 4 oz. per foot. The combined ages of the monkey and its mother are 4 yrs., and the weight of the monkey is as many pounds as its mother is years old. The mother is twice as old as the monkey was when the mother was half as old as the monkey will be when the monkey is 3 times as old as its mother was when she was 3 times as old as the monkey was. The weight of the rope and weight is half as much again as the difference between the weight of the weight and the weight of the weight plus the weight of the monkey. What is the length of the rope?

—SCH. SCI. AND MATH.

=▽=

A Hindu mathematician, Bhaskara Akarya, wrote a work, one part of which he named *Lilavati* (that is, the Beautiful) after his daughter, in order, so the story goes, to make her name immortal. At the close of this part he says of mathematics: 'Joy and happiness will ever increase

in this world for him who takes to his heart Lilavati; beautiful proportions are the ornament of her members; clear and complete are her solutions, her language eloquent with illustrations.'

—LA COUR.



There came three Dutchmen of my acquaintance to see me, being lately married; they brought their wives with them. The men's names were Hendriek, Claas, and Cornelius; the women's Geertruij, Catriin, and Anna; but I forgot the name of each man's wife. They told me they had been to market to buy hogs; each person bought as many hogs as they gave shillings for each hog; Hendriek bought 23 hogs more than Catriin, and Claas bought 11 more than Geertruij; likewise, each man laid out 3 guineas more than his wife. I desire to know the name of each man's wife.

—MISCELLANY OF MATHEMATICAL PROBLEMS, 1743.



—*draw the curtain close;
And let us all to meditation.*

—SHAKESPEARE.

FROM SECRETARY DIOPHANTUS

E. MARIE HOVE

At each national convention of Kappa Mu Epsilon it is the duty of the national officers to report the activities of the Fraternity for the preceding biennium. Since it seemed advisable not to hold a national convention in 1943, it is my pleasure to use the pages of the PENTAGON to report briefly the progress of Kappa Mu Epsilon since our last convention at Warrensburg, Missouri, on April 18 and 19, 1941.

The death in July, 1942, of Dr. Kathryn Wyant, founder of Kappa Mu Epsilon, brought a great loss to the Fraternity. During its first four years, Dr. Wyant served as president, and her counsel was always sought in what concerned the future of the Fraternity. Kappa Mu Epsilon is indeed deeply indebted to her for her untiring efforts.

Publication of the PENTAGON, official journal of Kappa Mu Epsilon, was authorized at the Warrensburg convention, and the first issue was published in the fall of 1941. Each initiate is given a one-year subscription. The PENTAGON is one of the big accomplishments of the biennium; it has done much to further the aims of Kappa Mu Epsilon, and to establish fraternal ties between the chapters. Much credit for this project is due Professor C. V. Newsom who served as editor during the first two years. Because of numerous other duties, Professor Newsom felt it impossible to continue as editor, and Professor H. D. Larsen is our newly appointed editor.

The 1941 convention program listed twenty-six active chapters. Since then, petitions have been approved and chapters established as follows:

New York Alpha, April 4, 1942; Michigan Beta, April 25, 1942; Illinois Gamma, June 19, 1942; New Jersey Beta, to be installed on April 21, 1944.

At the time of the 1941 convention, Kappa Mu Epsilon had a total membership of 2,857. Since that time 1,131

membership certificates have been issued, making a total of 3,988 members. The following summary may be of interest:

<i>Chapter</i>	<i>Initiated since 1941</i>	<i>Total Membership</i>	<i>Chapter</i>	<i>Initiated since 1941</i>	<i>Total Membership</i>
Alabama Alpha	15	63	Mississippi Beta	60	284
Alabama Beta	26	114	Missouri Alpha	30	231
Alabama Gamma	8	59	Missouri Beta	63	147
Illinois Alpha	34	236	Nebraska Alpha	24	163
Illinois Beta	14	115	New Jersey Alpha	13	27
Illinois Gamma	86	86	New Jersey Beta	12	12
Iowa Alpha	15	156	New Mexico Alpha	74	228
Iowa Beta	19	56	New York Alpha	42	42
Kansas Alpha	59	378	Ohio Alpha	26	103
Kansas Beta	41	193	Ohio Beta	33	56
Kansas Gamma	11	29	Oklahoma Alpha	34	299
Louisiana Alpha	150	317	S. Carolina Alpha	13	36
Michigan Alpha	37	157	Tennessee Alpha	32	32
Michigan Beta	55	55	Texas Alpha	43	79
Mississippi Alpha	28	154	Texas Beta	34	81

Business ordinarily cared for at the national convention has been transacted by correspondence. In the fall of 1943 a ballot was presented to the chapters for the election of national officers. The newly elected officers are already doing much to make the future of Kappa Mu Epsilon decidedly hopeful.

REPORT OF TREASURER NEWTON

Receipts	\$3,180.44
Disbursements	1,634.76
	<hr/>
Cash balance	1,545.68
U. S. bonds	775.00
	<hr/>
Total balance, July 1, 1943	\$2,320.68

KAPPA MU EPSILON NEWS

SISTER HELEN SULLIVAN, O.S.B.

Chapter 1. OKLAHOMA ALPHA, Northeastern State College, Tahlequah, Oklahoma.

President Eratosthenes ----- Miss Henrietta Stolper
Vice-President Napier ----- Mr. Elton J. Green, Jr.
Secretary Bernoulli ----- Miss Thelmajean Crane
Treasurer Leibniz ----- Miss Thelmajean Crane
Secretary Descartes ----- Miss Mary Katherine Stewart
Faculty Sponsor ----- Mr. W. S. Bishop

Throughout the entire year the members of Oklahoma Alpha have centered their discussions and activities on the study of astronomy. At one meeting graphs and projected slides of the heavenly bodies were shown by Dean L. P. Woods.

The teaching profession has attracted the following members of the chapter: Christia Carlton, an elementary teacher in Stillwell, Oklahoma; James Franklin, high school principal and mathematics teacher at Council Hill, Oklahoma; Anna Freedle, mathematics teacher at Central High School, Tahlequah, Oklahoma; Cleo Gray, mathematics teacher, Chelsea, Oklahoma; Fremont Harris, mathematics instructor for Naval Training Program at Oklahoma A & M. College, Stillwater, Oklahoma; G. C. Williams, instructor in physics, chemistry, and mathematics at Muskogee Junior College, Muskogee, Oklahoma. Dan McDonald who served as a meteorologist in Egypt, Arabia, and Eritrea was returned to the United States and spent six months in an advanced school of meteorologists. Mr. Leon Pense is enrolled as a regular student at the University of Arkansas, Fayetteville, Arkansas. Mr. Eugene Dooley is an A.S.T.P. student at Manhattan College in New York City.

Oklahoma Alpha is proud of the following members engaged in defense work: Betty Armstrong, accountant for Douglas Aircraft Corp., at Tulsa, Oklahoma; Marion Bogan, secretary Veteran's Administration, Muskogee, Oklahoma;

Faye McClure, clock-checker at Douglas Aircraft Corp., Tulsa, Oklahoma.

In the Service of the United States: Deryl Branscom, Charles McMurray, Winfield McMurtrey, Marion McQuigg, George Taylor, Daniel Vaughan, Warren Crane, O. D. Crane, A. E. Robertson, Jack Bliss, Ben Flanigan, Carl Holt, Albert Goodall, Edward Mann, Ray Stewart, George Atchley, Leo Harmon, Harmon Reeder.

Lt. Jack Brown, U.S.N.A.F., was killed in a mid-air crash on October 29, 1943.

Chapter 2. IOWA ALPHA, Iowa State Teachers College, Cedar Falls, Iowa.

A total absence of civilian student members makes it necessary for Iowa Alpha to carry on with the faculty members only. The mathematics department at Iowa State Teachers College is composed of the following: Mr. Henry Van Engen, Head of Department, Secretary Descartes of Iowa Alpha; Mr. Ira S. Condit; Mr. H. C. Trimble; Mr. E. E. Watson; Miss E. Marie Hove, National Secretary of Kappa Mu Epsilon.

An Army Air Corps unit of five hundred men on the campus keeps the entire mathematics staff well employed.

Chapter 3. KANSAS ALPHA, Kansas State Teachers College, Pittsburg, Kansas.

President Archimedes	-----	Miss Helen Kriegsman
Vice-President Plato	--	Mrs. Mildred Martin Bradshaw
Secretary Lagrange	-----	Miss Mary Lou Ralston
Treasurer Thales	-----	Miss Virginia Prussing
Secretary Descartes	-----	Mr. W. H. Hill
Faculty Sponsor	-----	Mr. J. A. G. Shirk

Mrs. Mildred Bradshaw, '34, whose husband is stationed in the Aleutians, is working towards her master's degree in mathematics. At the same time she holds the office of Vice-President of the Kansas Alpha Chapter.

The following members are engaged in teaching: Anna

Rupert Lanier, instructor in radio at Scott Field, Belleville, Illinois; Miss Billie Sherwood, president of Kansas Alpha 1942-1943, teacher of mathematics at Douglas, Kansas.

In the Service of the United States: Franklin Lanier in O.C.S. at Quantico, Virginia; Thomas Stallard, meteorologist in India; Harvey Lanier, studying meteorology at N. Y. U. for Army Air Corps.

Chapter 4. MISSOURI ALPHA, Southwest Missouri State Teachers College, Springfield, Missouri.

President Archimedes	Miss Isabel Bayless
Vice-President Galileo	Miss Elsie Mantels
Secretary Ahmes	Miss Dotty Carter
Secretary Descartes	Mr. Carl V. Fronabarger
Faculty Sponsor	Mr. L. E. Pummill

Three members of the chapter are teaching school: Elizabeth Burke, Zedna Miles, and Dale Woods.

In the Service of the United States: Jerome Twitty, teacher of navigation at Naval Reserve Midshipman's School, Northwestern University, Chicago, Illinois; Elliot A. Dewey, Midshipman's School, Notre Dame; Joseph F. Wilson, Subchaser Training Center, Miami, Florida; Odus L. Fronabarger, Army Engineering School, North Carolina State College; Richard L. Matthews, Second Lieutenant in the Marine Corps, Cherry Point, North Carolina; John M. Teem, Jefferson Barracks, Missouri; Wallace Stone, U. S. Naval Training Station, Great Lakes, Illinois.

Chapter 5. MISSISSIPPI ALPHA, Mississippi State College for Women, Columbus, Mississippi.

President Gauss	Miss Frances Grossnickle
Vice-President Stevin	Miss Frances Husbands
Secretary-Treasurer Desargues	Miss Carolyn Sessions
Secretary Descartes	Mr. R. L. Grossnickle
Faculty Sponsor	Mr. R. L. Grossnickle

Concerning last year's members Mississippi Alpha reports this news: Miss Mary Cliet is employed in teach-

ing. Miss Frankie Stephens and Miss Sue Worley have engineering jobs. Miss Emily Gilmore, Miss Frances Randle, and Miss Jimmie Mae Ward are working for T. V. A. Ex-president Love McKinstry is working in the code department of the Signal Corps. Miss Frances Grossnickle, chapter president, was recently honored by Mortar Board for the highest scholastic average in the junior class. Mortar Board makes an annual award of twenty-five dollars. For the purpose of keeping the old members in touch with the chapter activities, Mississippi Alpha established a novel "Round Robin" letter. M.S.C.W. has reached its peak enrollment in mathematics classes this year.

Chapter 7. NEBRASKA ALPHA, Nebraska State Teachers College, Wayne, Nebraska.

President Leibniz	Miss Mary Anita Pape
Vice-President Archimedes	Mr. Paul Peterson
Secretary Galileo	Miss Claire Jordan
Treasurer Einstein	Miss Barbara Hook
Historian Pascal	Mr. Joseph Scollard
Reporter Gauss	Mr. Harold Foecke
Secretary Descartes	Mr. C. H. Lindahl
Faculty Sponsor	Miss Jessie W. Boyce

Mr. C. H. Lindahl has been appointed to the mathematics faculty to succeed Miss E. Marie Hove who accepted a position on the staff of Iowa State Teachers College at Cedar Falls. Professor Lindahl holds his B. S. degree from Nebraska State Teachers College, Kearney, and his M. S. from the University of Colorado at Boulder. He has done additional graduate work at the University of Colorado and Colorado State College of Education.

Miss Jessie W. Boyce and Mr. C. H. Lindahl participated in a panel discussion at the district meet of the Nebraska State Education Association in October, 1943. The panel was presented by the mathematics section of the Association and the subject discussed was "Wartime and Post-War Mathematics in Our Schools."

Chapter 8. ILLINOIS ALPHA, Illinois State Normal University, Normal, Illinois.

President Gauss ----- Mr. Amber Grauer
 Vice-President Pascal ----- Mrs. Mildred Wunderlich
 Secretary Ahmes ----- Miss Eleanor Rae Lower
 Treasurer Napier ----- Miss Frances Marie Cyrier
 Historian Cajori ----- Miss LaNora Hood
 Social Chairman Lilavati ----- Miss Edith Robinson
 Secretary Descartes ----- Dr. C. N. Mills
 Faculty Sponsor ----- Miss Edith Irene Atkins

Illinois State Normal has a Navy V-12 unit on the campus with two hundred fifty men enrolled. There are fifteen mathematics classes composed entirely of navy men. Dr. Larsen and Dr. Decker of the education department have been loaned to the regular mathematics staff to help carry this increased teaching load. The Homecoming breakfast held in early fall attracted thirty mathematics lovers. Rose Marie Morisy was voted Homecoming Queen.

Illinois Alpha sent each of its fifty-two members in service a chapter letter at Christmas time. The Chapter is represented in the officer's groups as follows: fourteen ensigns, six lieutenants (j. g.), one major, three captains, twelve lieutenants (1st and 2nd), and one wave (ensign).

Chapter 9. KANSAS BETA, Kansas State Teachers College, Emporia, Kansas.

President Pascal ----- Mr. Daryl Dale Errett
 Vice-President Gauss ----- Mr. Richard Wayne Lee
 Secretary Eratosthenes ----- Miss Wanda Rector
 Treasurer Bhaskara ----- Miss Lenore Zimmerman
 Historian Ahmes ----- Miss Irene Hageberg
 Secretary Descartes ----- Mr. Alfred W. Philips
 Faculty Sponsor Thales ----- Mr. O. J. Peterson

Professor Charles B. Tucker, formerly Secretary Descartes of Kansas Beta, has left State Teachers College and gone to Johns Hopkins University for the duration. The

mathematics staff is very busy teaching Army Air Corps Students. In spite of the decreased civilian enrollment Kansas Beta is functioning with nine faculty and five student members.

Mr. Harold Bird, in the service of the U. S. Navy, lost his life in one of the North African battles.

In the service of the United States: Mr. Frank Faulkner, Navy V-12 Unit, Ann Arbor, Michigan; Mr. Voots, student of meteorology at M.I.T., Boston; Mr. Veal and Mr. Provost, students of meteorology at Chicago University; Mr. Charles and Mr. Vernon Wells, U. S. Army.

Chapter 10. ALABAMA ALPHA, Athens College, Athens, Alabama.

President Carmichael	_____	Miss Oleta Garrison
Vice-President Dickson	_____	Miss Hazel Ruth Harrison
Secretary Hedrick	_____	Miss Opal Whitten
Treasurer Veblin	_____	Miss Thelma Long
Secretary Descartes	_____	Miss Florence Tillman
Faculty Sponsor	_____	Miss Florence Tillman

The theme for discussion in the meetings of the chapter for the current year is "Mathematics and Everyday Life."

Chapter 11. NEW MEXICO ALPHA, University of New Mexico, Albuquerque, New Mexico.

President Benjamin Peirce	_____	Miss Elena Davis
Vice-President E. H. Moore	_____	Mr. Leslie Wheeler
Secretary Cajori	_____	Miss Esther Barnhart
Treasurer Bocher	_____	Prof. C. B. Barker
Secretary Descartes	_____	Prof. C. A. Barnhart
Faculty Sponsor	_____	Prof. C. A. Barnhart

The V-12 Navy unit which has been established on the campus has proved a great boom to New Mexico Alpha. Since the arrival of these service men twenty-seven new members have been initiated into the chapter.

Mr. Frank Fischer is doing graduate work in the

research unit of Westinghouse Electric and Manufacturing Company. Miss Ruth Barnhart is a junior mathematician with the Langley Memorial Aeronautical Laboratory at Hampton, Virginia. Miss Roberta Warren entered the U. S. Indian Service as dietitian. Miss Marie Phillips is teaching secondary school mathematics at Carlsbad, New Mexico. Miss Reka Lois Black is doing graduate work leading to the A.M. degree at Texas Christian University.

The following papers were presented at the meetings of New Mexico Alpha: "The Squares Escribed to a Triangle," by Dr. Frank Gentry; "The Relationship Between Mathematics and Logic," by Dr. H. G. Alexander; "The Witch of Agnesi," by Dr. H. D. Larsen. Dr. C. V. Newsom discussed the new Karpinski Collection which has recently been added to the department of mathematics.

Chapter 12. ILLINOIS BETA, Eastern Illinois State Teachers College, Charleston, Illinois.

President Thales	Miss Ruth Ellis
Secretary Khayyam	Miss Lillian Fagen
Treasurer Archimedes	Mr. Richard Bidle
Secretary Descartes	Mr. H. F. Heller
Faculty Sponsor	Mr. E. H. Taylor

During the spring term the Illinois Beta Chapter plans to sponsor an all-school activity in honor of Dr. E. H. Taylor who is retiring from active duty on the mathematics faculty. Dr. Taylor has been head of the department of mathematics since 1899.

Chapter 13. ALABAMA BETA, Alabama State Teachers College, Florence, Alabama.

President Pascal	Mr. Lilbourne Hall
Vice-President Leibniz	Mrs. Eleanor Hanlin Vernon
Secretary DeMoivre	Miss Elizabeth Gray
Treasurer Euler	Mr. Thomas Sherer
Reporter	Miss Corinne Dickson

Secretary Descartes ----- Miss Orpha Ann Culmer
 Faculty Sponsor ----- Miss Orpha Ann Culmer

Alabama Beta Chapter was grieved to hear of the death of Lt. (j.g.) D. Jones. He was killed in aerial combat as he piloted a pursuit plane from a Navy aircraft carrier. He had seen action in the Pacific against the Japs.

At the August commencement, Bernice Posey was awarded the Turris Fidelis award, highest honor that can be given an S.T.C. student. J. Y. Moultrie was given the award in June. Both of these students and Thomas Sherer were in *Who's Who in American Colleges and Universities 1942-1943*. Milton Holmes and Corinne Dickson are in *Who's Who* for 1943-1944.

Cecil Gregg has been elected superintendent of schools at Carbon Hill. Caroline Wilson, Mary Ann Green, Helen Chandler, Mildred Simpson, Bernice Posey, and Geraldine Keith are employed by the Tennessee Valley Authority. Hays Hanlin is with the U. S. Postal Service in London. Thomas Sherer and Lilbourne Hall, after being with the army for several months have been given honorable discharges and have returned to school.

Chapter 14. LOUISIANA ALPHA, Louisiana State University, University, Louisiana.

President Gauss ----- Mr. Leland E. Morgan
 Vice-President Poincare ----- Miss Nina Nichols
 Secretary Fermat ----- Miss Gloria McCarthy
 Treasurer Galois ----- Miss Julia Weil
 Secretary Descartes ----- Mr. Houston T. Karnes
 Faculty Sponsor ----- Miss Marlena White

Three of last year's members are doing graduate work leading to the A.M. degree. One former member is employed at T.V.A., Nashville, Tennessee.

It is the policy of Louisiana Alpha to hold semi-monthly meetings at which time a staff member presents a paper on some phase of mathematics. In October, Dr. K. L. Nelsen's paper was "Probability and Computations." At the Novem-

ber meeting Dr. Frank Rickey gave a paper on "Base of the Natural Logarithms."

Each year the chapter gives two awards. An award of fifteen dollars goes to the graduating senior who has proved superior according to the rules set up in the local chapter by-laws. A second award is conferred on the freshman who ranks highest in a special K.M.E. examination. The winners for this year have not yet been named.

Chapter 15. ALABAMA GAMMA, Alabama College, Montevallo, Alabama.

President Archimedes	-----	Miss Ann Cooper
Vice-President Appollonius	-----	Miss Lerah Sterling
Secretary Bernoulli	-----	Miss Carolyn Irvin
Treasurer Agnesi	-----	Miss Virginia Windham
Secretary Descartes	-----	Miss Rose Lea Jackson
Faculty Sponsor	-----	Miss Rose Lea Jackson

Alabama Gamma is actively functioning with thirteen regular student members. The general topic for discussion for the year is "Mathematics and the War." Last year's chapter members are employed as follows: Nelladeane Chandler, chemist for Tennessee Coal and Iron Co., Fairfield, Alabama; Olivia Hines, Curtiss Wright Engineering Student, Purdue University; Doris Kilgo, married; Peggy Kirk, chemist for International Paper Co.; Burke Land, O.S.U., Brookley Field, Mobile, Alabama; Clio Reed, Teaching in Gadsden, Alabama.

Chapter 16. OHIO ALPHA, Bowling Green State University, Bowling Green, Ohio.

Secretary Descartes	-----	Mr. Harry R. Mathias
Faculty Sponsor	-----	Mr. F. C. Ogg

Ohio Alpha has no civilian student members but is operating with four faculty members. A Navy V-12 Unit has been set up on the campus which absorbs the time and energy of the mathematics staff.

Chapter 17. MICHIGAN ALPHA, Albion College, Albion, Michigan.

President Townsend	Mr. Kenneth Ferguson
Vice-President Slaught	Mr. John Barcroft
Secretary-Treasurer Agnesi	Betty Hossfeld
Secretary Descartes	Mr. E. R. Sleight
Faculty Sponsor	Mr. E. R. Sleight

Student members of Michigan Alpha are doing their patriotic duty as assistants to the mathematics staff since the establishment of the pre-flight unit on the Albion College Campus. Those engaged in this work are: Kenneth Ferguson, Joseph Hill, Clare Stanford, Eugene Van Osdal, Richard Hadley, William Vogleson.

A survey on the employment of last year's members reveals the following: Miss Betty Davis and Miss Alice Gibb are married; Miss Helen Shephard is employed at Elgin Water Softener Corp.; Robert Ballard is in defense work; Andrew Sunyar is doing graduate work at the University of Illinois.

In the Service of the United States: Le Roy Benjamin, Robert Benjamin, Dale Buerstetta, Joseph Hill, James Hollingsworth, Leon McClellan, Robert Maynard, Russel Meier, Robert Murch, Ralph Powers, Howard Sherman, William Steers, John Telander, Lester Thayer, James Tobias, Eugene Van Osdal, John Waite, Morris Wetters, Richard Zeiss. This list includes only those who have been active members of Michigan Alpha since the Pearl Harbor Attack.

Chapter 19. SOUTH CAROLINA ALPHA, Coker College, Hartsville, South Carolina.

President Leibniz	Miss Dorothy Boykin
Vice-President Pascal	Miss Betty McIntosh
Secretary Thales	Miss Catherine Still
Treasurer Gauss	Miss Elinor McIntosh
Secretary Descartes	Miss Caroline Reaves
Faculty Sponsor	Miss Caroline Reaves

The theme for general discussion in meetings throughout the year has been "Mathematics Relative to War Work." Each member devotes two hours a month to Red Cross work.

Chapter 20. TEXAS ALPHA, Texas Technological College, Lubbock, Texas.

President Lobatchewsky -----	Miss Virginia Bowman
Vice-President Agnesi -----	Mr. Brac Biggers
Secretary Noether -----	Miss Betty Grace Pugh
Treasurer Cayley -----	Miss Betty Grace Pugh
Secretary Descartes -----	Miss Lida B. May
Faculty Sponsor -----	Mr. D. L. Webb

Lee Michie, son of Professor J. N. Michie, head of the mathematics department at Texas Technological, is a prisoner of the Japanese. Both father and son are members of Texas Alpha Chapter of Kappa Mu Epsilon. Ex-president Mary Sparks, who has been working the past year with Eastman Kodak Company in Rochester, New York, has been transferred to Hollywood, California, to work with a film company. Ex-president Nancy Ann Miller is doing graduate work at Brown University. She will receive her A.M. degree in June. Ruth Keeter Frazine has returned to Texas Technological College as a member of the pre-flight mathematics faculty. Blanche Zeman, member of the O.P.A. staff for the Panhandle District, has been transferred to Lubbock, Texas, Headquarters.

Of the twenty-one members who were initiated into Texas Alpha on November 18, 1943, three are from countries other than the United States: Carlos Rios, Mexico City, Mexico; Guido Rodrigues and Fabio Urbina, Alajuela, Costa Rica.

In the service of the United States are the following last year's members: Orville Alderson, Charles Britton, Charles Freeman, and Elmer Jameson. All are in A.S.T.P. at Texas Technological.

Chapter 21. TEXAS BETA, Southern Methodist University, Dallas, Texas.

President Galois	Mr. Milton Drandell
Vice-President Abel	Mr. Ray Schumacher
Secretary-Treasurer Pascal	Mr. John W. Fisher
Secretary Descartes	Mr. Ray Q. Seale
Faculty Sponsor	Mr. Gerald B. Huff

Dr. P. K. Rees, former Secretary Descartes at Texas Beta, resigned his position on the mathematics faculty at Southern Methodist University and is teaching in a college in Louisiana. Dr. Ray Q. Seale replaced Dr. Rees at S.M.U. Ex-president Marian Weaver is teaching at North Texas Agricultural College. The establishment of a U. S. Navy Unit on the campus keeps the mathematics staff members busily occupied.

Chapter 22. KANSAS GAMMA, Mount St. Scholastica College, Atchison, Kansas.

President Leibniz	Miss Virginia Meyers
Vice-President Sylvester	Miss Rosemary Solas
Secretary Abel	Miss Mary Lou Maloney
Treasurer Euler	Miss Margaret Mary Wolters
Secretary Descartes	Sister Helen Sullivan, O.S.B.
Faculty Sponsor	Sister Helen Sullivan, O.S.B.

The chapter's central topic for research and discussion this past year has been "Mathematics as It Is Studied and Taught in Other Countries." Preparatory to the open meeting which was held in January on "Mathematics in Spanish-speaking Countries," the chapter president had direct correspondence with Bernardo Baidaff, editor of *Boletin Mathematico*. Guest speakers for this meeting were Misses Mary Rachel Robleda and Fanny Mary Durantes of Mexico City, Mexico.

Kansas Gamma now claims two former members in the service of the United States and four employed in war work. Those in service are: Marjorie Dorney, class of 1939, in the

Ferry Command; ex-president Margaret Molloy, class of 1943, ensign in the U.S.N.R., now studying aerology at Massachusetts Institute of Technology in Boston. In war work: Florence Kramer, class of 1932, supervisor in an ordnance plant in Milan, Tennessee; ex-president Bobbe Powers, class of 1942, instructor at Chicago Aircraft Instrument School; Mary Flaherty, class of 1942, statistician in the office of the Provost Marshall at the Signal Corps Office in Chicago; ex-president Sarah Alice Woodhouse, employee at Navy Research Division of University of California at San Diego.

Miriam Powers, class of 1939, has recently taken over the teaching of four mathematics classes at the Kelryn Park High School, Chicago, Illinois. In June, 1943, Margaret Mary Kennedy, a graduate of last year, entered the Benedictine Novitiate at Mt. St. Scholastica College, Atchison, Kansas. She is now known as Sister Malachy.

Three members of Kansas Gamma were married since the last issue of THE PENTAGON was released. Lucille Laughlin, '39, Imogene, Iowa, married Robert Jordon of Baldwin City, Kansas. Betty Moore, '39, Yukon, Oklahoma, married Patrick J. Kelley, Ensign in U.S.N.R. Muriel Thomas, '42, Antonito, Colorado, married Howard Meier of Basehor, Kansas.

Kansas Gamma is happy to announce the reelection of Virginia Meyers, Chapter President, to *Who's Who in American Colleges and Universities*. Mary Margaret Downs, '43, vice-president of Kansas Gamma for two consecutive years, is employed as a research chemist with Swift Co. in Chicago. Helen Newport, '43, holds a fellowship and is doing graduate work in social science at St. Louis University. Marge Barry, '43, is employed at American Telephone & Telegraph Co. in Chicago as Junior Engineering Data Clerk.

Chapter 23. IOWA BETA, Drake University, Des Moines, Iowa.

President Alchwarizmi ----- Miss Julia Rahm
Vice-President Aryabhata

Miss Mary Catherine Spencer

Secretary Brahmagupta ----- Miss Ardis Ferguson

Treasurer Leonardo ----- Miss Julia Roberts

Secretary Descartes ----- Miss Floy Woodyard

Faculty Sponsor ----- Mr. I. F. Neff

With the single exception of Lois Jean Kerr, who is teaching mathematics at Waukee, Iowa, all of last year's members are in the service.

Chapter 24. NEW JERSEY ALPHA, Upsala College, East Orange, New Jersey.

President Thales ----- Miss Zelda Meisel

Vice-President Appollonius ----- Mr. Joseph Prieto

Secretary Abel ----- Miss Elizabeth Ebel

Treasurer Fibonacci ----- Miss Lillian Meisel

Historian Gauss ----- Miss Marjorie Wolfe

Secretary Descartes ----- Mr. M. A. Nordgaard

Faculty Sponsor ----- Mr. M. A. Nordgaard

Miss Anne Zmurkiewicz resigned her position on the mathematics faculty at Upsala College to take up a defense research job in chemistry and mathematics. Mr. Ingold Opsal is in the Navy V-12 Unit at Drew University. Miss Phyllis Gustafson has a civil service position in Washington, D. C.

Chapter 26. TENNESSEE ALPHA, Tennessee Polytechnic Institute, Cookeville, Tennessee.

President Leibniz ----- Mr. Charles Tabor

Vice-President Euler ----- Miss Wilma Leonard

Secretary Napier ----- Miss Frances Hollis

Treasurer Kepler ----- Mr. Robert Johnson

Secretary Descartes ----- Mr. R. H. Moorman

Faculty Sponsor ----- Mr. R. O. Hutchinson

A special feature of the meetings of Tennessee Alpha was the study of topics relating to Engineering. Although a great number of the members have not returned to school this fall, the following members are in attendance: Dr. Richard Moorman, Dr. Robert Hutchinson, Charles Tabor, Wilma Leonard, Frances Hollis, Mildred Murphy, Don Ferguson, and Robert Johnson. Miss Margaret Plumlee, last year's president, is now in war work at Langley Field, Virginia. Harry Grisham is also in war work at Clinton, Tennessee.

In the Service of the United States: Ollie James Agee, Army Air Corps; W. J. Blevins, Navy; Pierce Brown, Army Air Corps; Charles Cagle, Working on Clinton Government Project; William Fitzgerald, Norfolk Navy Yard; Harvy Grisham, Army; Howard Herndon, Navy V-7; Cooper Loftis, Army, A.S.T.P.; Cordell B. Moore, Army, Meteorological Division; Frederick D. Paulk, Navy V-12; John F. Roy, Navy V-7.

Chapter 27. NEW YORK ALPHA, Hofstra College, Hempstead, New York.

President Gauss	Miss Edith Hufman
Vice-President Pascal	Mr. Harry Durham
Secretary Fermat	Mr. E. R. Stabler
Treasurer Laplace	Miss Jean Ruppel
Historian Jordan	Miss Jean Ruppel
Secretary Descartes	Mr. E. R. Stabler
Faculty Sponsor	Mr. Albert Capuro

Mario Juncosa, president last year, is doing graduate work in mathematics and physics, and assisting in physics at Cornell University. Edward Ryder, Nicholas Vogel, Seymour Yuter, Robert Mulford, and Russell Terry are in various branches of the armed services. Professor H. Hunter Smith, on leave of absence, is with the physics department at Amherst College. Lectures given at the meetings of New York Alpha include the following: "Mechanics of Structural Design" by Professor Albert Capuro, and "Postulational Methods" by Professor E. R. Stabler.

Chapter 28. MICHIGAN BETA, Central Michigan College, Mount Pleasant, Michigan.

President Appollonius	Mr. Paul Brown
Vice-President Menelaus	Miss Zelda Montague
Secretary Laplace	Mr. Robert Mark
Treasurer Tartaglia	Mr. Kenneth Miller
Secretary Descartes	Miss Nikoline Bye
Faculty Sponsors, Mr. C. C. Richtmeyer, Mr. J. W. Foust	

Miss Jennie Masters, 1942-43 president of Michigan Beta, was awarded the scholarship for graduate study at University of Michigan this year and is majoring in statistics.

Chapter 29. ILLINOIS GAMMA, Chicago Teachers College, Chicago, Illinois.

President Archimedes	Miss Agnes Houlihan
Vice-President Euler	Miss Ruth Thometz
Secretary Galileo	Miss Mary C. Cooke
Treasurer Kepler	Miss Elaine Drews
Secretary Descartes	Mr. Ralph Mansfield
Faculty Sponsor	Mr. Ralph Mansfield

Mr. Ralph Mansfield has been appointed to succeed Mr. J. J. Urbancek in the capacity of faculty sponsor for Illinois Gamma. Illinois Gamma is proud to report that thirty-two of its last year's members are in the armed services of the U. S. Thirty former members are teaching in the public schools of Chicago. Interesting papers presented at the meetings of Illinois Gamma include the following: "A Synthetic Approach to the Fourth Dimension," by Bernard Malina; "Three Famous Problems of Antiquity," by Sam Altshuler.

Chapter 30. NEW JERSEY BETA, New Jersey State Teachers College, Montclair, New Jersey.

President Abel ----- Miss Florence Wirsching
 Vice-President Einstein ----- Mr. Ernest Yeager
 Secretary Appolonius ----- Miss Hazel Petrie
 Treasurer Fourier ----- Miss Harriet Dresdner
 Secretary Descartes ----- Mr. David R. Davis
 Faculty Sponsor ----- Dr. Virgil S. Mallory

Installation ceremonies for New Jersey Beta Chapter are scheduled for April 21, 1944. Professor L. F. Ollman of Hofstra College, the National Treasurer of Kappa Mu Epsilon, will be the installing officer. He will be assisted by Dr. E. R. Stabler and Mr. Albert Capuro, also of Hofstra College.

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J. A. G. Shirk, *The Early Years of Kappa Mu Epsilon*

F. W. Sparks, *The Theory of Numbers*

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