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Morals in Arithmetic

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It may seem strange to discuss arithmetic in this journal; even stranger to put "morals" into the title of a mathematical article. But there is need to discuss certain improper practices in arithmetic, and mathematicians, if we may judge by their writings, are not free from reproach in this respect.

Let me introduce my topic by an example; I am not sure of the "grade-placement," but certainly the example belongs in the grades. The lengths of the sides of a rectangle are found by measurement to be 9.2 inches and 9.7 inches; what is the area? Surely, any beginner can answer; since

$$9.2 \times 9.7 = 89.24,$$

the area is 89.24 square inches. Yet, I assert this answer is "immoral," because it is deceitful: it pretends to an accuracy which it by no means possesses. In fact, as we shall see, the third and fourth digits in the result are completely worthless, while even the second digit is not entirely reliable.

I would not venture serious discussion of so elementary an example had not experience shown the necessity. In summer classes of high school teachers of mathematics, I have created dismay, in one case even tears, when I rudely destroyed faith in something long cherished, the sanctity of the accuracy of arithmetic.

Are there then two kinds of arithmetic, one for the schoolbook, and one for "life?" In a sense, there are. The arithmetic of the books is that of counting; that of life deals, in large part, with measurements, that is, with approximations. Even in counting, we must sometimes be content with less than perfect accuracy; could even the Bureau of the Census say what the population of New York is at a given instant?

If I look over my calculus class tomorrow morning, and

find 40 students present, that number, unless I have blundered, represents exactly what it pretends to. But if I measure a line with a scale, and find that the line is 9.2 inches long, that is a different matter. If I should use a more delicate method of measurement, I should find, perhaps, that the line is 9.24 inches, or 9.236 inches, long. And if, with my more sensitive scale, I should repeat the measurement, the two measurements would not be likely to agree. The inescapable conclusion is: the "true" length of the line is something unknown, and forever unknowable, to me. The best I can say, from my first, crudest measurement, is that the true length is somewhere, I know not where, between 9.15 and 9.25 inches. If I dislike this uncertainty, only one course is open to me: I can use more refined methods of measurement. But, even then, all I can accomplish is a diminution of the range of indeterminacy; I can never completely eliminate the uncertainty. Since an increase in accuracy usually means an expenditure of time and effort, I must frequently be content, and must use reasonable interpretation of my results. Good morals demand that I be honest in my interpretation.

Let us return to the rectangle. What can I say, from my measurements of the sides, about the true area? First, I must confess that I do not know, and never will know, the true area. All I can do is set limits within which it must lie. In this case, the area is greater than

$$9.15 \times 9.65, \text{ or } 88.2975 \text{ square inches,}$$

and less than

$$9.25 \times 9.75, \text{ or } 90.1875 \text{ square inches.}$$

I see now that anything to the right of the decimal points is worthless in the result; thus I shall report the area as 89 square inches. Even so, as even the most rigid of moralists must do at times, I am compromising with my (arithmetic) conscience, for I am not certain of the 9; it might be 8, it might be 10, for all I know.

But time is short, (this is written in 1942), and we cannot afford to spend so much time on a simple multiplication, three multiplications, really, to get one reasonably honest product. We need working rules to enable us to get fairly respectable results without too much effort. To derive such rules is not the purpose of this sermon; as with all such discourses, conversion is the aim. Once converted, the sinner may seek the materials of his salvation elsewhere.¹ To state such rules conveniently, it is necessary to discuss a matter of terminology.

The digits, 1, 2, . . . , 9, are called "significant." The digit, 0, may or may not be significant. If 0 comes between two non-zero digits, it is significant. If one or more zeros come to the right of the decimal point and before the first non-zero digit in a number between 0 and 1, they are not significant. Finally, if a number with no decimal part terminates in one or more zeros, no one but the writer of the number can tell whether or not the zeros are significant. To avoid this ambiguity, another instance of bad morals, it is increasingly becoming the practice to use "scientific" or "standard" notation. For example, a measurement of 27,000 will be written as

$$2.7 \times 10^3$$

if the method of measurement is such that all we can say is that the value of the quantity measured is between 26,500 and 27,500. But, if we can assert that the value sought is between 26,999.5 and 27,000.5, we write,

$$2.7000 \times 10^3.$$

In this notation, the decimal point is always placed at the right of the first significant digit.

¹ See, for example: Scarborough, *Numerical Mathematical Analysis*, Baltimore, The Johns Hopkins Press, 1940;

Bakst, *Approximate Computation*, The Twelfth Yearbook of the National Council of Teachers of Mathematics, New York, Bureau of Publications, Teachers College, Columbia University, 1937.

Now we can state a working rule for multiplication of approximate numbers; namely, the number of significant digits in the product of two or more approximate numbers is not greater than the number in that factor which has the smallest number of significant digits. To make this rule easier to state, let us introduce a definition:

The accuracy of an approximate number is the number of significant digits therein.

Now our rule reads: the accuracy of a product cannot exceed that of the least accurate factor. Note well: the rule says, "cannot exceed," not "equals." In any case of doubt, we can always return to first principles, as we did above in the case of the rectangular area.

Fortunately, the same rule holds for division also. For example,

$$\frac{7.96}{14.2} = 0.561.$$

$$\text{Check: } \frac{7.965}{14.15} = 0.5629^-, \quad \frac{7.955}{14.25} = 0.5582^+.$$

The rule thus gives one digit too many; the quotient should be 0.56.

For integral powers, we need no new rule, since this involution is but repeated multiplication. For example:

$$2.17^3 = 10.2.$$

$$\text{Check: } 2.165^3 = 10.147842125, \quad 2.175^3 = 10.289109375.$$

The last digit in 10.2 is in doubt.

For roots, and for functions in general, we turn to the calculus for aid. The general principle involved may be stated simply: if y is a function of x , and x is changed by an amount Δx , by how much will y be changed? The answer is Δy . But it is frequently laborious to calculate Δy , and we shall content ourselves with a good approximation. We know that the smaller Δx ($= dx$) is, the better an

approximation dy gives for Δy . In our present work, dx will always be relatively small, so we shall use dy instead of Δy . We shall illustrate the application of this principle with a few examples. The reader may construct as many more as he likes, and may succeed in upsetting some preconceived notions.

Example: $\sqrt{2.17} = ?$

The uninformed might say: in taking square roots, we mark off periods of two digits each, and each period gives one digit in the result. Here there are only two periods; hence we can trust only two digits in the result. Let's look at the facts:

Let $y = \sqrt{x} = x^{\frac{1}{2}}$; then,

$$dy = dx / (2\sqrt{x}) = 0.005 / (2\sqrt{2.17}) = 0.005 / 2.95 < 0.002.$$

We may therefore safely state: $\sqrt{2.17} = 1.47$.

As a check: $\sqrt{2.165} = 1.471^+$, $\sqrt{2.175} = 1.475^-$.

Example: $\sin 27^\circ 35' = ?$

Let $y = \sin x$; then, since x is expressed in minutes,

$$\begin{aligned} dy &= (\pi/10800) \cos x \, dx \\ &= (0.000291) (0.866) (0.5) < 0.0002. \end{aligned}$$

Therefore, $\sin 27^\circ 35' = 0.463$.

Check: $\sin 27^\circ 34.5' = 0.4629$, $\sin 27^\circ 35.5' = 0.4632$.

Example: $\ln 4.7 = ?$ (It is to be understood that \ln denotes a logarithm to the base e .)

Let $y = \ln x$; then,

$$dy = dx/x = 0.05/4.7 < 0.02.$$

Therefore, $\ln 4.7 = 1.5$.

Check: $\ln 4.65 = 1.537^-$, $\ln 4.75 = 1.558^+$.

There remains the question of addition and subtraction of approximate numbers. In this problem, the number of significant digits plays no part; instead, the number of decimal places is important. We use the word "precision," instead of "accuracy," to express the number of decimal places given. Then the rule is:

The precision of the algebraic sum of two or more approximate numbers cannot exceed that of the least precise item.

The same rule applies to subtraction. An example will make clear what is meant.

Example: $2.35 + 37.1 + 0.158 = ?$

$$\begin{array}{r}
 2.345 \\
 37.05 \\
 0.1575 \\
 \hline
 39.5525
 \end{array}
 \qquad
 \begin{array}{r}
 2.35 \\
 37.1 \\
 0.158 \\
 \hline
 39.608
 \end{array}
 \qquad
 \begin{array}{r}
 2.355 \\
 37.15 \\
 0.1585 \\
 \hline
 39.6635
 \end{array}$$

It follows from the rule that the answer should be stated to a precision of one decimal place, namely, 39.6; even so, the last digit is uncertain. To save time, we could have rounded off the more precise items to within one place of the least precise item, thus:

$$\begin{array}{r}
 2.35 \\
 37.1 \\
 0.16 \\
 \hline
 39.61
 \end{array}$$

The extra digit, which was carried to minimize cumulative errors of rounding off, is then dropped, giving 39.6 as the result.

In all calculations, where possible, one more place should be carried in the intermediate work, and then rounded off in the result. If a division is performed (whether of approximate or exact numbers), the work must be carried to one more place than is desired in the result, in order that rounding off may be done intelligently. Avoid, for example, the common blunder,

$$2/3 = 0.66; \text{ (it should be } 0.67\text{).}$$

In rounding off, the usual practice is: if the part discarded is less than $\frac{1}{2}$ unit of the last place retained, simply drop it; if it is more than $\frac{1}{2}$, increase by one unit the last digit retained. If the discarded part is exactly $\frac{1}{2}$ unit, use that one of the above statements which will make the final retained digit even. Thus

23.65, rounded to 1 decimal place, is 23.6;

23.75, rounded to 1 decimal place, is 23.8.

The above is an introduction to the principles of morality in the arithmetic of approximate numbers. May those who read these words, mark, learn, and inwardly digest, to the end that those who teach may bring a reform; may the day come when we shall no longer be subjected to arithmetic atrocities in the pages of mathematics texts, to say nothing of those in engineering and the other sciences!

The Early Years of Kappa Mu Epsilon

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Organizations that rise and flourish do so because of two factors; first, there are individuals deeply interested in their development, and second, there have already existed needs for such organizations. Many times attempts to launch new enterprises have met with disappointing results because of the lack of response, despite the untiring efforts of their sponsors. The history of the origin and early development of any organization would be incomplete without portraying both of the factors mentioned. This brief history of Kappa Mu Epsilon will therefore recognize the labors of its founders, and will also attempt to show the conditions which favored its reception. This latter phase will be considered first.

The rapid growth of the universities and colleges in the United States during the latter part of the past century and the early part of the present century led to the organization and extensive development of professional societies in every field. Law, medicine, science, engineering, teaching, and other professional fields, developed societies with memberships numbering in some cases into the thousands. Local clubs were formed in most of the larger educational institutions for the purpose of promoting interest in special departmental objectives. The natural desire for affiliation with other groups with similar ideals led to the establishing of national and state organizations for mutual stimulation and encouragement.

In the field of mathematics, Pi Mu Epsilon became the national fraternity for instructors and advanced students who were in institutions offering graduate work in mathematics. Other fields of college study, in which more students usually specialize, took the lead in establishing na-

tional fraternities essentially for undergraduate students. The first fraternities open to mathematics students seem to have been primarily for science students, but these organizations did not appeal strongly to those whose chief interest was in symbolic thinking. From the letter files of the author and of Dr. Kathryn Wyant, it would seem that preliminary discussions about the need for a national mathematics fraternity were made during 1929-1930 in at least four separate localities, namely, the Iowa State Teachers College at Cedar Falls, the Kansas State Teachers College at Pittsburg, the Mississippi State College for Women at Columbus, and the University of New Mexico at Albuquerque. The Mississippi State College was also interested in this matter, but it is difficult to determine from the records which of the Mississippi colleges had the idea first, since the young men at State College with true southern courtesy always defer to the young ladies at Columbus when it comes to questions of priority.

In the fall of 1930, Dr. Kathryn Wyant went to the Northeastern Oklahoma State Teachers College at Tahlequah as an instructor in the mathematics department. Being a member of Pi Mu Epsilon and other professional fraternities, she went to work with vigor and enthusiasm to transform the mathematics club at Tahlequah into the first chapter of a national fraternity. Professor L. P. Woods, who was head of the department and dean of men, was a valuable co-worker in the perfection of the many details incident to the project. He was largely responsible for the completed rituals used for the initiation of members and the installation of officers. These two faculty members of the mathematics department, together with twenty-two other faculty members and students, became the charter members of Oklahoma Alpha Chapter on April 18, 1931.

One can easily imagine the tremendous amount of work necessary for the successful inauguration of such an enterprise. It is most interesting to read the records pertaining to the selection of the name of the fraternity, its motto,

emblem, colors, names for officers, and other necessary matters. Dr. Wyant took a most important part in this work. Also she conducted an extensive correspondence to find where other groups were located that had been contemplating a like move, and who would therefore be receptive to the new organization.

The author, who was head of the department of mathematics at the Kansas State Teachers College of Pittsburg, was away on sabbatical leave during that year, but Professor W. H. Hill, who was acting head, informed Dr. Wyant of his favorable attitude towards a national fraternity.

Dr. Ira S. Condit and other members of the staff of the Iowa State Teachers College at Cedar Falls had also contemplated the desirability of such a move. Thus the second chapter was formed, and the installation took place on May 27, 1931.

In the fall of 1931, Dr. Wyant again presented the plan of Kappa Mu Epsilon to the mathematics staff of Pittsburg with the result that this group applied for a chapter, and was installed as the third chapter on January 30, 1932.

During the time that Kappa Mu Epsilon was being developed at Tahlequah, the two Mississippi colleges previously named were making progress with their plans for inaugurating a mathematical fraternity. The name, "Phi Mu Epsilon," had been selected by them, and a pin had been devised by the group at Mississippi State College for Women at Columbus. When Kappa Mu Epsilon learned that the Mississippi State College and the State College for Women were proceeding with their plans for a national fraternity, the officers of Kappa Mu Epsilon and other interested persons urged that the groups in Mississippi give up their contemplated organization and that they become affiliated with the society whose organization was entirely completed.

Consequently, the southern groups agreed to unite with Kappa Mu Epsilon in the formation of one national mathematics fraternity of collegiate rank. Dr. C. D. Smith at State College and Professor R. L. Grossnickle at Columbus

are especially to be commended for their breadth of view in supporting one organization rather than continuing with another separate fraternity. The establishment of two different organizations would probably have led to each one being only sectional in scope.

The unity of purpose that proceeds from the knowledge that twenty-eight chapters are now located in the nation will animate the thousands of young men and young women who have promised to maintain a scholarly interest in the development and use of mathematics. History renders the ultimate verdict as to the value of any movement, and the growth and the influence of Kappa Mu Epsilon in a little over a decade give a portent of its greater contributions in the decades yet to come.

The Theory of Numbers

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Introduction. The theory of numbers deals with the properties of integers, and it is one of the oldest as well as the most difficult branches of mathematics. It differs from the other branches in the lack of general methods, and even comprehensive theorems seem more difficult to devise than in algebra or in analysis. On the other hand, its problems frequently may be stated so simply that they can be understood by one with little or no mathematical training. The challenge presented by this fact has attracted hordes of investigators including the greatest of the great mathematicians. It is noteworthy that this interest has been motivated solely by intellectual curiosity because few practical applications have been found for the discoveries in this field. Kummer is reported to have said that the theory of numbers is the only branch of mathematics not yet sullied by contact with application. Recently, however, articles have appeared in the *American Mathematical Monthly*¹ showing applications of the theory of numbers to gear ratios and to the splicing of telephone cables.

In this paper we cannot hope to do more than introduce the reader to the aims and to some of the methods of number theory. For this purpose we shall present a few of the important theorems of the subject together with their proofs, and shall illustrate them with simple arithmetical problems.

Prime numbers. From the standpoint of divisibility, integers may be divided into two classes, prime numbers and composite numbers. An integer is *prime* if it has no factor other than itself and unity, and it is *composite* when it is not prime. The ancients called the former strong or masculine

¹ *American Mathematical Monthly*, v. 42, pp. 81-91; v. 45, pp. 22-31.

numbers and the latter, weak or feminine numbers. These interesting questions have arisen in connection with prime numbers: How many are there? How are they distributed? What are the tests of primality?

The first of these questions was answered by Euclid, who used a very ingenious method to prove that the number of primes is infinite. Since his proof is an excellent illustration of the type of thinking used in number theory, we shall give it here. We shall let p represent a prime number, and shall let P represent the product of all primes less than and including p . Thus if $p = 7$, $P = (1) (2) (3) (5) (7) = 210$. We next consider $N = P + 1$. If N is a prime it is greater than p . If it is composite it can be factored into primes in one and only one way.² Since any prime less than or equal to p divides P , it cannot divide N . Hence, if N is composite, its prime factors are greater than p . Therefore, in either case, there exists a prime number greater than p however large p may be.

The question of the distribution of primes in our system of integers presents many interesting aspects. An examination of a table of prime numbers reveals the fact that there is no regularity in their occurrence. Of the primes less than 5000, there are 168 less than 1000, 135 between 1000 and 2000, 127 between 2000 and 3000, 120 between 3000 and 4000, and 119 between 4000 and 5000. One can think of several pairs of primes less than 100 which differ by 2, such as 3 and 5, 17 and 19, 41 and 43, and an examination of the most comprehensive table of primes reveals that such pairs recur frequently throughout the table. Is there an infinitude of such pairs? This is one of the unanswered questions of number theory. Another unsolved problem in connection with prime numbers concerns their form. The prime integers, 5, 17, and 37, are primes less than 100 that are of the form $n^2 + 1$ where n is an integer. It is not known whether

² For a proof of this statement see L. E. Dickson, *Introduction to the Theory of Numbers*, University of Chicago Press, 1930, p. 3.

or not the number of such primes is infinite. This is an easily formulated problem that has not been solved.

Since all primes greater than 2 are odd, they can be expressed in the form $2n + 1$, and since the numbers of this type form an arithmetic progression as n runs through the positive integers, we have an infinitude of primes in such a progression. What about other arithmetic progressions which contain an infinitude of primes? Dirichlet has proved that if a and b are relatively prime integers, the progression, $ax + b$, (as x runs through the integers), contains an infinitude of primes.

The question of the number of primes in certain intervals is still a will of the wisp. Landau has proved that there exists at least one prime between any number n and its double $2n$, $n > 1$. These limits have since been narrowed to n and $3n/2$, $n > 2$, but this still leaves much to be desired. Much work has been done on the problem of establishing the existence of at least one prime between n^2 and $(n + 1)^2$, but this has not been solved.

No convenient or easily applied test for primality has been discovered. One of the earliest methods of sifting out the primes less than a given number is known as the *Sieve of Eratosthenes*. It consists of writing all the integers in order beginning with unity and terminating with the given number. Then, starting with the integer 2, we cancel all its multiples in the set. We repeat the procedure for 3, and so on. The integers left uncanceled when the process is completed are prime numbers. The following array shows the Sieve of Eratosthenes applied to the set of consecutive integers from 1 to 23. Numbers inclosed in parentheses have been cancelled. 1 2 3 (4) 5 (6) 7 (8) (9) (10) 11 (12) 13 (14) (15) (16) 17 (18) 19 (20) (21) (22) 23. The Sieve of Eratosthenes is historically interesting, but it has no practical value in the matter of testing large numbers for primality. The labor involved is entirely too great.

Possibly the most practical method of testing for primality, if the integer is not too large, is simply to attempt to

divide it by smaller primes. Fortunately, if the number n is the product of two integers, a and b , with $a < b$, then a is less than \sqrt{n} , since $a^2 < (ab = n)$. Hence in our series of trials, it is not necessary to use numbers greater than the square root of n .

Leibnitz³ and others noticed that n is prime if and only if $1 + (n - 1)!$ is divisible by n . This test is certainly general, but, just as certainly, the labor required to apply it to large numbers is prohibitive. The same statement can be made regarding any other general test which is available, and hence, if one has access to a well equipped library, the easiest way for him to tell whether or not a given number is prime is to refer to the tables prepared by D. N. Lehmer,⁴ which lists the primes from 1 to 10,006,721.

Perfect numbers. It is not surprising that an integer, such as $6 (= 1 + 2 + 3)$ or $28 (= 1 + 2 + 4 + 7 + 14)$, which is equal to the sum of all its factors less than itself should arouse the attention of the mystically minded ancients. Such numbers are called *perfect*, and we know the identity of only a few. Euclid proved that $2^{p-1} (2^p - 1)$ is a perfect number if $2^p - 1$ is a prime, and Euler proved that if an even integer is perfect, it is of the above type. By means of this formula the existence of the twelve perfect numbers associated with $p = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107,$ and 127 has been established. The difficulty in the use of this formula lies in establishing whether $2^p - 1$ is prime or composite. For $p = 137, 139, 149, 157, 167, 193, 199, 227, 229, 241,$ and 257 the question is still doubtful.

The existence or the non-existence of an odd perfect number has not been established, although Sylvester has found that there exists no odd perfect number with fewer than six prime factors, and none not divisible by 3 with fewer than eight prime factors.

Amicable numbers. The two integers, 220 and 284, have

³ L. E. Dickson, *History of the Theory of Numbers*, Carnegie Institution of Washington, Washington, 1919, v. I, p. 426.

⁴ D. N. Lehmer, *Carnegie Institute Publications No. 165*, 1914.

the property that the sum of the aliquot⁵ divisors of each is equal to the other. Such pairs are called *amicable*, and they appealed to the mysticism of the people of ancient times. Among some people, in fact, these numbers were associated with love and friendship. We know that the properties of the above pair were known to the Pythagoreans, and from the references to 220 and 284 found in earlier religious literature, we assume their properties were known by people antedating the Pythagoreans. Almost one hundred pairs of amicable numbers are known today, and among the discoverers we find the names of Euler, Legendre, Descartes, Seelhoff, Dickson, and a sixteen year old Italian boy, Paginni.⁶

Brief theory of congruences. We shall now turn from the descriptive to the more technical aspect of our discussion. A mathematical relationship that belongs particularly to number theory, and that plays a role here somewhat analogous to that of the equation in algebra, is known as the *congruence*. If $a - b$ is divisible by the integer m , we say that a is congruent to b modulo m , or symbolically, $a \equiv b \pmod{m}$. We shall present a few of the fundamental theorems pertaining to congruences, and shall show some elementary but interesting results of them.

Theorem I. If $a \equiv b \pmod{m}$ and $a \equiv c \pmod{m}$, then $b \equiv c \pmod{m}$.

Proof. Since $a - b$ is divisible by m , we have $a = b + rm$, where r is an integer. Similarly, $a = c + sm$, with s being an integer. Thus $b + rm = c + sm$, or $b - c = m(s - r)$. Since the right member of the last relation is divisible by m , $b - c$ is also, and we have $b \equiv c \pmod{m}$.

Theorem II. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $a \pm c \equiv b \pm d \pmod{m}$.

Proof. By definition,

⁵ If $a < n$ is a factor of n , it is an aliquot divisor of n .

⁶ Tobias Dantzig, *Number the Language of Science*, Macmillan, New York, 1939, p. 44.

- (1) $a - b = rm$, and
 (2) $c - d = sm$, r and s being integers.

Adding and subtracting the right and the left members of (1) and (2), we get

$$\begin{aligned} (a - b) \pm (c - d) &= rm \pm sm, \text{ or} \\ (a \pm c) - (b \pm d) &= m(r \pm s). \end{aligned}$$

Hence by the definition of a congruence, we have

$$a \pm c \equiv b \pm d \pmod{m}$$

Theorem III. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv bd \pmod{m}$.

Proof. Obviously, relations (1) and (2) given in the previous proof are true in this case. Multiplying both members of (1) by c and of (2) by b , and adding, we obtain

$$\begin{aligned} ac - bd &= (cr + bs)m, \text{ or} \\ ac &\equiv bd \pmod{m}. \end{aligned}$$

An immediate consequence of theorem III is the following:

Corollary. If $a \equiv b \pmod{m}$, then $a^n \equiv b^n \pmod{m}$.

By a repeated application of the above theorems and corollary, we may prove

Theorem IV. If $a_i \equiv b_i \pmod{m}$, $i = 0, 1, 2, 3, \dots, n - 1$, and if $x \equiv y \pmod{m}$, then

$$\begin{aligned} a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n &\equiv b_0y^n + b_1y^{n-1} \\ &+ \dots + b_{n-1}y + b_n \pmod{m}. \end{aligned}$$

Another theorem of interest is

Theorem V. If $na \equiv nb \pmod{m}$ and n and m are relatively prime, then $a \equiv b \pmod{m}$.

Proof. By the conditions of the theorem, $n(a - b) = tm$. Then, since n and m are relatively prime, $a - b$ is divisible by m . Hence, $a \equiv b \pmod{m}$, and the proof is complete.

Since any number N may be written in the form

$$(3) \quad N = a_0 10^n + a_1 10^{n-1} + \dots + a_{n-1} 10 + a_n,$$

we may obtain several tests of divisibility of N by a one digit divisor. For example, since $10 \equiv 0 \pmod{2}$, we have by theorem IV, $N \equiv a_n \pmod{2}$, and, therefore, N is divisible by 2 if and only if its last digit is even. Furthermore, since $10 \equiv 1 \pmod{3}$ and 9 , we have as a corollary to theorem IV,

$$(4) \quad N \equiv a_0 + a_1 + a_2 + \dots + a_n \pmod{3 \text{ and } 9}.$$

Therefore N is divisible by 3 or by 9 if the sum of its digits is divisible by 3 or by 9, respectively. Moreover, permuting the digits of an integer divisible by 3 or 9 does not affect its divisibility by these integers.

A very old result mentioned in Cajori's *History of Mathematics*, page 65, states that if we add any three consecutive integers the greatest of which is a multiple of 3, then add the digits of the sum obtained, next add the digits of that sum, and so on until a number less than 10 is obtained, the final sum will be 6. In order to prove this we shall need the following:

Lemma 1. If a and b are positive integers and each is less than 9, and if $a \equiv b \pmod{9}$, then $a = b$.

Proof. Since $a \equiv b \pmod{9}$, we have $a - b = 9k$, with k being an integer. Obviously $a - b < 9$; hence $k = 0$.

To continue with the theorem given by Cajori, we let N , equal the sum of the digits in N , N_2 equal the sum of the digits in N_1 , and so on until we obtain $N_r < 10$. Then we let $N_r = N'$. By (4) we have,

$$(5) \quad N \equiv N_1 \equiv N_2 \equiv \dots \equiv N_r \equiv N' \pmod{9}.$$

If $N = 9k + t$, $t < 9$, we have by (5), by theorem I, and by lemma 1, $N' = t$. Since any integer divisible by 3 can be expressed in the form $3n + 3$, we may write the sum of the three consecutive integers of our problem in the form $(3n + 1) + (3n + 2) + (3n + 3) = 9n + 6$, and we have $(9n + 6)' = 6$.

We may use the foregoing results to derive a method for detecting possible errors in addition, multiplication, subtraction, and division. By (5) and theorem III, we have

$$MN \equiv (MN)' \equiv (M'N')' \pmod{9}.$$

Thus, by lemma 1, $(MN)' = (M'N')'$. Using the last result to test the product, $(867)(345) = 299115$, we have $[(867)'(345)']' = (3')(3') = 9$, and $(299115)' = (27)' = 9$.

For addition, we have by (5) and theorem II,

$$(M + N)' \equiv M' + N' \equiv (M' + N')' \pmod{9},$$

which when applied to the sum, $862 + 1347 + 782 = 2991$, yields, by further use of (5),

$$(862' + 1347' + 782')' = 2991',$$

or $(16' + 15' + 17') = 21',$

or $(7 + 6 + 8)' = 3.$

Similar tests can be worked out for subtraction and division. They are all very old, and they comprise the method known as "casting out nines." The name is derived from the fact that in obtaining M' we may strike out the digit 9 and each combination of digits whose sum is 9. Thus $297456' = (2 + 9 + 7 + 4 + 5 + 6)' = 6$. It should be noted that the satisfaction of these tests is necessary for the correctness of the computation, but is not sufficient; for instance, the obviously false equation, $(3)(2) = 69$, meets the test.

Fermat's Theorem; Euler's Theorem. When an integer a is divided by m until a remainder r less than m is obtained, r is called the least residue of a modulo m . Evidently r must be one of the numbers,

$$(6) \quad 0, 1, 2, 3, \dots, (m - 1),$$

and hence the integers (6) are called a *complete set of least residues modulo m* . No two integers, a and b , that are incongruent, modulo m , can have the same least residue r , modulo m , for in such a case we would have $a = tm + r$ and $b = am$

$\dagger r$. Hence a and b are each congruent to r , and thus by theorem I, they are congruent to each other, modulo m . Two successive powers of a , $a > 1$, will not be congruent, modulo m , to the same integer of (6), and it is an interesting question to ascertain what powers of a are congruent to 1, modulo m . The investigation of this question has led to two very important theorems of number theory which we shall discuss next.

We shall first consider the case in which m is a prime integer p . If a is not divisible by p , obviously none of its multiples,

$$(7) \quad a, 2a, 3a, \dots, (p-1)a,$$

are so divisible. Furthermore, if $sa \equiv ta \pmod{p}$ where $s < p$ and $t < p$, then $a(s-t) \equiv 0 \pmod{p}$, $s-t \equiv 0 \pmod{p}$, and hence, $s = t$. Thus, no two integers of (7) have the same residue, modulo p , and are congruent, modulo p , to the integers in some permutation of the sequence,

$$(8) \quad 1, 2, 3, \dots, (p-1).$$

Thus by theorem III, the product of the integers listed in (7) is congruent to the product of those of (8), or

$$(p-1)! a^{p-1} \equiv (p-1)! \pmod{p}.$$

Hence, by theorem V, $a^{p-1} \equiv 1 \pmod{p}$, and we have

Theorem VI (Fermat's Theorem). If p is a prime integer and if a is not divisible by p , then $a^{p-1} \equiv 1 \pmod{p}$.

In the case of a composite modulus m , we shall need a notation known as Euler's ϕ -function. We shall denote the number of integers not exceeding m that are prime to m by the symbol $\phi(m)$. Thus $\phi(5) = 4$, and $\phi(6) = 2$. For a prime p , $\phi(p) = p - 1$, but for a composite number m , we determine the value of $\phi(m)$ by means of a theorem which we shall state without proof.[†]

[†] See L. E. Dickson, *Introduction to the Theory of Numbers*, University of Chicago Press, 1930, p. 7.

Theorem VII. If a and b have no common factors, then $\phi(ab) = \phi(a)\phi(b)$.

Thus $\phi(15) = \phi(3)\phi(5) = (2)(4) = 8$.

We are now in a position to prove Euler's Theorem. We shall state it as

Theorem VIII (Euler's Theorem). If a and m have no common factors, then $a^{\phi(m)} \equiv 1 \pmod{m}$.

Proof. We shall designate the $\phi(m) = k$ integers that are less than and prime to m by

$$(9) \quad a_1, a_2, a_3, \dots, a_k,$$

and shall consider the set,

$$(10) \quad aa_1, aa_2, aa_3, \dots, aa_k$$

Evidently the difference $aa_1 - aa_j = a(a_1 - a_j)$ is not divisible by m , since no two integers of (9) are congruent, modulo m , and a and m have no common factors. Furthermore, we may express the integers listed in (10) in the form, $aa_i = \delta_i m + r_i$, $i = 1, 2, 3, \dots, k$, where $r_i < m$. Since a and m have no common factors, and a_1 is prime to m , r_1 is also prime to m . Hence r_1 is one of the integers of (9), and the integers of (10) are congruent, modulo m , to those of (9) rearranged. Thus by theorems II and V we have

$$(aa_1)(aa_2)(aa_3) \dots (aa_k) \equiv a_1 a_2 a_3 \dots a_k \pmod{m}$$

or

$$(a_1 a_2 a_3 \dots a_k) a^k \equiv a_1 a_2 a_3 \dots a_k \pmod{m}$$

and finally,

$$a^k \equiv 1 \pmod{m}.$$

We shall next illustrate the use of the previous theorems to show that $n^{273} \equiv n \pmod{273}$ for n equal to any integer. We shall need two lemmas for the completion of the problem.

Lemma 1. If n, a, b, t, r , and s are integers with a and

b relatively prime and with $r = ts$, and if $n^r \equiv 1 \pmod{a}$ and $n^s \equiv 1 \pmod{b}$, then $n^r \equiv 1 \pmod{ab}$.

Proof. By the conditions of the lemma, $n^r = n^{ts}$ and $n^s = ub + 1$ where u is an integer. Thus,

$$n^r = (ub + 1)^t = u^t b^t + tu^{t-1} b^{t-1} + \dots + tub + 1.$$

Evidently the left member of the above equality is congruent to 1, modulo b , and we have $n^r \equiv 1 \pmod{b}$. Hence $n^r - 1$ is divisible by the relatively prime integers, a and b , and consequently by their product. Thus, $n^r \equiv 1 \pmod{ab}$.

Lemma 2. If a and m are relatively prime integers, and if $a^n \equiv 1 \pmod{m}$, then $a^{n+p} \equiv a^p \pmod{am}$.

Proof. Since $a^n - 1 = tm$, we have $a^{n+p} - a^p = a^p tm$.

Evidently the right member is divisible by am . Thus, $a^{n+p} \equiv a^p \pmod{am}$.

In order to solve our problem we notice that $273 = (13)(7)(3)$, and that any integer n may be expressed in the form $n = (13^a)(7^b)(3^c)(p)$ where p is not divisible by 13, 7, or 3. By Fermat's theorem,

$$p^{12} \equiv 1 \pmod{13}, p^6 \equiv 1 \pmod{7}, p^2 \equiv 1 \pmod{3},$$

and by a repeated application of lemma I, we have $p^{12} \equiv 1 \pmod{273}$.

It follows at once that

$$(i) \quad p^{13} \equiv p \pmod{273}.$$

By Euler's theorem $(13^a)^{\exp \phi(21)} \equiv 1 \pmod{21}$, or

$$(13^a)^{12} \equiv 1 \pmod{21}, \text{ since } \phi(21) = \phi(7)\phi(3) = 12.$$

Thus by lemma 2, we have

$$(ii) \quad (13^a)^{13} \equiv 13^a \pmod{273}.$$

Again using Fermat's theorem, it follows that

$$(7^b)^{12} \equiv 1 \pmod{13}, (7^b)^2 \equiv 1 \pmod{3},$$

$$(3^c)^{12} \equiv 1 \pmod{13}, (3^c)^6 \equiv 1 \pmod{7}.$$

The application of lemma 1 to these congruences yields $(7^b)^{12} \equiv 1 \pmod{39}$ from the first pair, and $(3^c)^{12} \equiv 1 \pmod{91}$ from the second pair. Then lemma 2 gives us

$$(iii) \quad (7^b)^{13} \equiv 7^b \pmod{273},$$

$$(iv) \quad (3^c)^{13} \equiv 3^c \pmod{273}.$$

Then, multiplying the right and left members, respectively, of (i), (ii), (iii), and (iv), we have

$$[(13^a)(7^b)(3^c)(p)]^{13} \equiv (13^a)(7^b)(3^c)(p) \pmod{273}$$

or
$$n^{13} \equiv n \pmod{273}.$$

Linear congruences. The expression,

$$(11) \quad ax \equiv n \pmod{m},$$

is called a linear congruence in one variable, and is satisfied by the integer c if $ac \equiv n \pmod{m}$. One's experience with linear equations in algebra would lead him to suspect that congruence (11) has one solution, and this is true under certain restrictions. If c is a solution of the congruence under consideration, and if $b \equiv c \pmod{m}$, then it is easily verified that b is also; so we make the restriction that c and all integers congruent to it, modulo m , shall count as one solution. Now if a and m are relatively prime, we shall show that there exists one and only one solution of (11). By Euler's theorem, if $\phi(m) \equiv k$, we have $a^k \equiv 1 \pmod{m}$, and thus $a^kn \equiv n \pmod{m}$. Hence $x = a^{k-1}n$ is a solution of (11). Furthermore, any other solution b of (11) is congruent to the above value of x , since we have $ab \equiv n \pmod{m}$, and consequently, $a^kn = ab \pmod{m}$. Thus, by theorem V, $a^{k-1}n \equiv b \pmod{m}$.

Now we let the g.c.d. of a and m be $d > 1$. If a solution c of (11) exists, we have $ac - n = tm$; then, since d is a divisor of a and m , it must also be a divisor of n . We let

$$(12) \quad a = a'd, n = n'd, m = m'd;$$

then a' and m' are relatively prime, and there exists a solution X of

$$(13) \quad a'x \equiv n' \pmod{m'}.$$

We shall show that the set of numbers,

$$(14) \quad X + im', i = 0, 1, 2, \dots, (d-1),$$

constitutes d solutions of (11). In the first place, no two of the integers (14) are congruent, modulo m , for the difference between any two of them is obviously less than m . In the second place, if $x = X + im'$,

$$\begin{aligned} ax &= a'dx = a'd(X + im') = da'X + a'dm'i \\ &= d(tm' + n') + a'dm'i, \\ &\quad \text{[by (13)]} \\ &= tm + n + a'mi, \text{ [by (12)].} \end{aligned}$$

Obviously the right member of the last equation is congruent to n modulo m . Hence each of the integers (14) is a root of (11). As a consequence, we have

Theorem IX. A linear congruence $ax \equiv n \pmod{m}$ has exactly one solution when a and m are relatively prime, no solution when the g. c. d., d , of a and m does not divide n , and exactly d solutions when d is a divisor of n .

A very old theorem known as the Chinese Remainder Theorem deals with the simultaneous solution of several linear congruences with different moduli. It is stated as follows:

If $m_1, m_2, m_3, \dots, m_r$ are relatively prime in pairs, there exist integers x which satisfy

$$(15) \quad x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2}, \dots, x \equiv a_r \pmod{m_r}.$$

These integers are congruent, modulo $m_1 m_2 m_3 \dots m_r$.

Proof. Let

$$(16) \quad m_1 m_2 m_3 \dots m_r = m_1 M_1 = m_2 M_2 = m_3 M_3 = \dots = m_r M_r.$$

Then m_i and M_i , $i = 1, 2, 3, \dots, r$, are relatively prime and the congruences,

$$(17) \quad \begin{array}{l} M_1 u_1 \equiv 1 \pmod{m_1}, \\ M_2 u_2 \equiv 1 \pmod{m_2}, \\ M_3 u_3 \equiv 1 \pmod{m_3}, \\ \dots \\ M_r u_r \equiv 1 \pmod{m_r} \end{array}$$

have solutions $a_1, a_2, a_3, \dots, a_r$, by theorem IX. Then

$$x = a_1 u_1 M_1 + a_2 u_2 M_2 + a_3 u_3 M_3 + \dots + a_r u_r M_r,$$

is a solution of (15) as the reader may verify by a consideration of (16) and (17).

If x' is another integer satisfying (15), then $x' \equiv x \pmod{m_i}$, $i = 1, 2, 3, \dots, r$, by theorem I. Consequently, $x - x'$ is divisible by m_i , and, therefore, by the product $m_1 m_2 m_3 \dots m_r$, since the latter integers are relatively prime in pairs.

We shall illustrate this theorem by solving a problem proposed by Bramaguptra in the seventh century. *Find the number having the remainders 5, 4, 3, 2 when divided by 6, 5, 4, 3, respectively.* This is equivalent to finding the solution of the congruences,

$$(18) \quad \begin{array}{l} x \equiv 5 \pmod{6}, \\ x \equiv 4 \pmod{5}, \\ x \equiv 3 \pmod{4}, \\ x \equiv 2 \pmod{3}. \end{array}$$

Any value of x satisfying the above congruences must satisfy the last three in which the moduli are relatively prime in pairs. We may, therefore, apply the Chinese Remainder Theorem to the last three, and obtain $m_1 m_2 m_3 = 60 = 5(12) = 4(15) = 3(20)$. Thus $M_1 = 12$, $M_2 = 15$, and $M_3 = 20$. The congruences,

$$12u_1 \equiv 1 \pmod{5}, \quad 15u_2 \equiv 1 \pmod{4}, \quad 20u_3 \equiv 1 \pmod{3},$$

yield the solutions $u_1 = 3$, $u_2 = 3$, $u_3 = 2$. Then

$$x = (4)(3)(12) + (3)(3)(15) + (2)(2)(20) = 359 \\ = 5(60) + 59.$$

Thus, $x = 59$ is a solution of the last three congruences of (18), and by trial we see that it also satisfies the first.

The theory of quadratic residues, or of admissible values of n in congruences of the type $x^2 \equiv n \pmod{m}$, constitutes one of the most important divisions of number theory, but space forbids our entry into this field. Neither can we discuss the solution of the general congruence of degree greater than one.

Diophantine equations. We shall conclude this paper with a discussion of two important theorems or problems in the field of Diophantine equations. A Diophantine equation is one which must be satisfied by integral values of the unknown. Equations of this type are named after Diophantus, a Greek mathematician, and the solution of them constitutes a study which has attracted the interest of thousands; the field still contains many unsolved problems. As a simple illustration of a Diophantine problem, we shall solve the following:

A man buys a certain number of cows, sheep, and pigs, 100 animals in all, for \$300. If he pays \$25 for each cow, \$6 for each sheep, and \$2 for each pig, how many of each does he buy?

Evidently the solution of this problem must be in integers. If x represents the number of cows, y the number of sheep, and $100 - (x + y)$ the number of pigs, the equation,

$$25x + 6y + 2[100 - (x + y)] = 300,$$

or

$$23x + 4y = 100,$$

must be satisfied. Since 23 is prime to 4, and 100 is divisible by 4, we have at once that $x \equiv 0 \pmod{4}$, and since $0 < x < 6$, $x = 4$. Then $y = 2$, and $100 - (x + y) = 94$. Hence the man buys 4 cows, 2 sheep, and 94 pigs.

The two theorems which are offered in conclusion deal with the sums of like powers of integers. Triads of integers

satisfying $x^2 + y^2 = z^2$ interested those of a mathematical mind in very ancient times, and numerous formulas have been proposed which yield solutions to the equation. Kronecker is responsible for the formula,

$$x = 2pqt, y = t(p^2 - q^2), z = t(p^2 + q^2),$$

which yield all the solutions to the problem.

The obvious generalization of this problem is that of finding solutions of the equation, $x^n + y^n = z^n$. Fermat made this notation on the margin of his copy of *Arithmetica* by Diophantus: "It is impossible to partition a cube into two cubes, or a biquadrate into two biquadrates, or generally any power of higher degree into two powers of like degree. I have discovered a truly wonderful proof of this, which, however this margin is too narrow of hold." This statement is the famous "Fermat's Last Theorem," and since Fermat's time, number theorists have tried, but have failed, to prove the theorem either to be true or false. Euler proved the theorem for n equal to 3 and to 4; Dirichlet, for n equal to 5 and to 14; Lamé, for $n = 7$; Sophie Germain, for $n < 100$; Maillet, for $n < 223$; Miriamoff, for $n < 257$; and Dickson, for $n < 7000$.

Frequently, research dealing with the solution of a difficult problem leads to discoveries as by-products which may be more important than the problem itself. Probably the most important contribution to mathematics resulting from the problem of Fermat is the invention and development of the theory of ideal numbers by Kummer.

If integral solutions of the equation,

$$x_1^n + x_2^n + x_3^n + \dots + x_r^n = p,$$

exist, we say that the integer p may be partitioned, or represented by, r n th powers. The ancients grappled with the problem of the representation of integers as the sum of squares, and it was known as early as 1640 that any integer may be expressed as the sum of four squares.

Edward Waring made the conjecture in 1770 in connec-

tion with this latter problem that has never been completely demonstrated. His formulation stated that every positive integer is the sum of four squares, or of nine cubes, or of nineteen fourth powers. No proof was given, and mathematicians are of the opinion that his statement was a plausible guess. Proofs of the first statement were given by Euler, by La Grange, and by Legendre in the 18th century.

With regard to the representation of every integer as the sum of cubes, Maillet proved in 1895 that every integer is the sum of 21 such powers. Fleck reduced the number to 17 cubes in 1906, Weifrigh gave an argument in 1909 which was completed in 1919 by Kempner showing that every number is the sum of 9 cubes. This number cannot be reduced since it takes 9 cubes, $2(2^3) + 7(1^3)$, to represent 23.

With respect to the last part of Waring's Theorem, Liouville proved, about 1859, that it requires at most 53 fourth powers to represent any integer. This limit was reduced to 47 by Realis in 1878, to 41 by Lucas in the same year, to 39 by Fleck in 1906, to 38 by Landau in 1907, and to 37 by Weifrigh in 1909. Since no integer is known that requires more than 19 fourth powers, it is thought that Waring was right.

The Mathematical Scrapbook

*The words of a man's mouth are as deep waters,
and the wellspring of wisdom as a flowing brook.—*
Proverbs 18:4.

As mathematics evolves it both expands and contracts, somewhat like one of Lemaitre's models of the universe. At present the phase is one of explosive expansion, and it is quite impossible for any man to familiarize himself with the entire inchoate mass of mathematics that has been dumped on the world since the year 1900. But already in certain important sectors a most welcome tendency toward contraction is plainly apparent.—E. T. Bell.

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All the effects of nature are only the mathematical consequences of a small number of immutable laws—Laplace.

A geometer like Riemann might almost have foreseen the more important features of the actual world.—Eddington.

The profound study of nature is the most fecund source of mathematical discoveries.—Fourier.

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If geometry is to be really deductive, the deduction must everywhere be independent of the *meaning* of geometrical concepts, just as it must be independent of the diagrams; only the *relations* specified in the propositions and definitions employed may legitimately be taken into account. During the deduction it is useful and legitimate, but in *no* way necessary, to think of the meanings of the terms; in fact, if it is necessary to do so, the inadequacy of the proof is made manifest. If, however, a theorem is rigorously derived from a set of propositions—the *basic* set—the deduction has a value which goes beyond its original purpose. For if, on replacing the geometric terms in the basic set of propositions by certain other terms true propositions are obtained, then

corresponding replacements may be made in the theorem; in this way we obtain new theorems as consequences of the altered basic propositions without having to repeat the proof.—Pasch.

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The ordinary mathematical treatment of any applied science substitutes exact axioms for the approximate results of experience, and deduces from the axioms rigorous mathematical conclusions. In applying this method, it must not be forgotten that mathematical developments transcending the limits of exactness of the science are of no practical value.—Felix Klein.

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God created the integers, the rest is the work of man.—Kronecker.

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The essence of mathematics is its freedom.—George Cantor.

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Recently, the Bureau of Ships of the United States Navy published a memorandum concerning the research being conducted at the David W. Taylor Model Basin. The following quotation from the publication possesses considerable significance for the mathematician:

A mathematical physicist who is confronted with a problem (pertaining to the research at the David W. Taylor Model Basin) finds himself from the very beginning of his work in a rather embarrassing situation. The definiteness of problems to which he grew accustomed in his academic work does not exist in these naval problems; a considerable amount of data which he would like to have in order to start his differential equations is not available, and he cannot wait until it will be available. It is necessary thus to start an approximate theory, to form a provisional hypothesis in the hope that in this manner he will reach at least a *first approximation* rather than a wrong guess. A broad mathematical training helps considerably in such a case; as mathematicians say, it is a *necessary* condition, but by no means a *sufficient* one.

Problems of naval design, particularly those involving

the ship as a whole, are of such extreme complexity on account of so many variables that unless one is prepared to simplify the problem of neglecting a considerable number of these variables, or parameters, the mathematical end of the problem simply cannot be started at all. At this point a mathematical expert encounters a most difficult point in his line of attack; which parameters can he safely set aside in the beginning of the work? If the answer depends solely on the order of magnitude of the different factors, this is not yet a difficult problem provided that this order of magnitude can be ascertained from the experimental data and that the problem is considered within the range in which it does not exhibit any critical or "threshold" conditions; or, if one wishes to express it mathematically, when a reasonable amount of "linearization" is permissible.

If the mathematician is lucky enough to simplify the problem in this way, the first approximation is thus obtained; if not, he simply makes a wrong guess and from this moment on theory and practice begin to diverge. He has to have the courage in this case to admit that he was wrong and to apply some other methods of procedure.

It appears, thus, that in addition to a broad mathematical training, a mathematical expert called on to attack such complicated problems must possess also a kind of a combined physico-mathematical common sense—simplifying the problem when it can be simplified and facing difficult situations when a simplification is dangerous on account of the possibility of losing contact with the practical problem. In this connection an ample mathematical training is helpful in so far as a man possessing it can rapidly change his mathematical tactics in his endeavor to conquer the difficulties which he is facing; this, however, is not a sufficient condition unless it is coupled with a searching mind, initiative, perseverance and absence of any preconceived ideas which might force him to subordinate a given problem to mathematics rather than to subordinate the latter to the former. These qualities probably cannot be acquired in school; they are undoubtedly inherent in an individual and when a man possesses them, such a man will be particularly suited for attacking the problems within the scope of activity of the David W. Taylor Model Basin.

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Let every man prove his own work, and then shall he rejoice in himself alone, and not in another.—Galatians 6:4.

Consider any number that is one less than a multiple of 24. If this number is expressed as the product of two factors, their sum will always be a multiple of 24.

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If a coin, starting with heads up, is rolled along the circumference of another coin of the same size, the point of contact will move along the circumference of the moving coin and along the fixed coin. Since the circumferences of the two coins are equal, it follows that the point should have passed along half the circumference of the moving coin after passing along half the circumference of the fixed coin. But on testing this, it is found that the moving coin again lies heads up. (Why is this so?)

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We have a piece of string just long enough to exactly encircle the globe at the equator. We take the string and fit it snugly around, over oceans, deserts, and jungles. Unfortunately, when we have completed our task we find that in manufacturing the string there was a slight mistake, for it is just a yard too long. To overcome the error, we decide to distribute this 36 inches evenly over the entire distance. How far will the string stand off from the ground at each point?

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A magic square, by definition, is a square array of numbers such that the sum down any column is constant and equal to the sum across any row, and also is equal to the sum along a diagonal. Although the Arabs worked with magic squares as early as the tenth century, A. D., the first treatise upon the subject appears to have been written by Manuel Moschopoulos, who according to Heath, lived during the reign of Emperor Andronicus II, (1282-1328 A. D.).

Many persons have played with magic squares as a hobby. As a young man, Benjamin Franklin worked with them. One magic square of which he was especially proud is the following:

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

In Franklin's words, the properties of this square array are:

(1) that every straight row, horizontal or vertical, of 8 numbers added together makes 260, and half each row half 260. (2) That the bent row of 8 numbers, ascending and descending diagonally, viz., from 16 ascending to 10, and from 23 descending to 17, and every one of its parallel bent rows of 8 numbers, make 260. Also the bent row from 45 to 43 descending to the left, and from 23 to 17 descending to the right, and every one of its parallel bent rows of 8 numbers, make 260. Also the bent row from 52 to 54 descending to the right, and from 10 to 16 descending to the left, and every one of its parallel bent rows of 8 numbers, make 260. Also the parallel bent rows next to the above-mentioned, which are shortened to 3 numbers ascending and 3 descending, etc., as from 53 to 4 ascending, and from 29 to 44 descending, make, with the 2 corner numbers, 260. Also the 2 numbers, 14, 61 ascending, and 36, 19 descending, with the lower four numbers situated like them, viz., 50, 1 descending and 32, 47 ascending, make 260. And, lastly, the 4 corner numbers, with the 4 middle numbers, make 260.



The present national emergency has served to emphasize two systems of angular measurement usually not taught in elementary courses in mathematics. The first is the mil system which is used extensively by the mobile artillery and infantry of the United States Army.

By definition, a mil is a unit of angular measurement equal to 1/6400 of a complete revolution. Such a definition has been chosen because it gives an angle which subtends

an arc approximately equal to $1/1000$ of the radius. Thus, in actual practice, 1 mil is taken as the angle which subtends a chord of 1 yard at a distance of 1000 yards, or a chord of 1 rod at a distance of 1000 rods. So, an artillery unit equipped with quadrants graduated in mils can make some very rapid computations.

The air forces of the United States Army employ the modified time system for measuring certain angles employed in celestial navigation. The same system is used in recording the time of events. The system regards 24 hours as equivalent to one complete revolution, and the abbreviations, A.M. and P.M., are discarded. As an illustration, 9:15 A.M. becomes 0915 hour, and 11:27 A.M. is given as 1127 hour. For times after noon, the value is increased by 1200, so 1:22 P.M. becomes 1322 hour, and 6:55 P.M. is listed as 1855 hour. It has been found that many types of error are eliminated through the use of the modified hour system.

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Important definitions in air navigation:

1. True Course.—Direction flown over the surface of the earth expressed as an angle with respect to true north.

2. True Heading.—Angular direction of the longitudinal axis of an aircraft taken with respect to true north.

3. Drift.—Angular difference between the true heading and the true course. If the true course is greater than the true heading, the drift is characterized as right drift. Similarly, left drift occurs when the true course is less than the true heading.

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Some of the most disturbing problems in modern mathematics, especially set-theory, have to do with paradoxes. All have heard of the ancient paradox of Epimenides the Cretan, who said that all Cretans were liars. This paradoxical statement appeared in many forms during ancient and medieval times, and all of them were designated as

"insolubilia." An interesting variation of the paradox was constructed by Jourdain in 1913. In the Jourdain paradox, there exists a card on the front of which appears the sentence, "On the other side of this card is written a true statement" whereas the other side of the card states, "On the other side of this card is written a false statement." The difficulty is apparent. If the statement on the front is true, then the statement on the back is true; consequently the statement on the front must be false. But if the statement on the front is false, then the statement on the back must be false; so the statement on the front must be true.

The importance of such paradoxes in mathematics is indicated by the fact that Burali-Forti published one concerning the greatest ordinal number in 1897; Russell announced his pertaining to the greatest cardinal number in 1903; König gave one relative to the least undefinable ordinal number in 1905; and Richard stated the famous paradox concerning definable and undefinable real numbers, also in 1905. There still is no universal agreement among mathematicians in regard to the proper solution of these and other paradoxes.

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It can be shown that $\sin k\pi x / \sin \pi x = 1 + 2 \cos 2\pi x + 2 \cos 4\pi x + \dots + 2 \cos (k-1)\pi x$, if k is odd. There exists a similar relation if k is even; what is it?

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Many good geometers are unfamiliar with the principles of conformal geometry. In fact, several important problems still remain unsolved. The rudiments of the subject enter into many fields of both pure and applied mathematics, including the science of navigation.

Conformal geometry originated with man's desire to map the earth's curved surface upon a plane. By conformal mapping is meant any continuous one-to-one mapping of the points of one surface upon those of another in such a manner that corresponding angles in the two surfaces are equal. The term, conform, is due to Gauss who was inter-

ested in the property, which follows as an immediate consequence of the definition just given, that corresponding triangles of the two surfaces tend to conform, that is, approach similarity, when the lengths of their sides approach zero.

As a consequence of this latter property, small triangles upon the surface of the earth will appear as similar triangles upon a plane map of the earth's surface if the representation has been conformal. Of course, a similar statement will also be true of other small figures. Thus, Ireland upon a conformal map will really resemble the shape of Ireland, and the sketch of Iceland will look like Iceland, even though the scale for Ireland and the scale for Iceland may be quite different.

The earliest known conformal mapping is the stereographic projection of Hipparchus (140 B.C.) and Ptolemy (150 A.D.). In this type of mapping, a plane is taken tangent to a sphere, and corresponding points are obtained by drawing straight lines from the point diametrically opposite the point of tangency; that is, the point of intersection of such a line and the sphere corresponds to the point of intersection of the line and the plane. It is a matter of elementary geometry to prove that such a mapping is conformal.

The best known example of conformal mapping, however, is Mercator's projection promulgated by G. Kremer in 1569. This type of projection when applied to the mapping of the earth assumes that the earth is a perfect sphere. A cylinder is then imagined tangent to the earth along the equator. Consequently, meridians upon the earth's surface may be projected into elements of the cylinder, and, by the same process, parallels of latitude become circular cross-sections. The cylinder can now be cut along an element, and rolled out on a plane; whereupon the resulting map may be reduced to any desired scale. Such a Mercator's Chart possesses the important property that the meridians and the parallels of latitude appear as two orthogonal

families of parallel straight lines. Moreover, a rhumb line on the earth takes the form of a straight line upon the map; this property, of course, is most important in navigation.

In spite of the development of a great variety of aeronautical charts, the Mercator projection still remains the most popular among aeronautical men. The 1941 British R. A. F. Air Navigational Manual even states that Mercator's Charts are the only ones that are suitable. This conclusion confirms the judgment of seamen who have used this type of map almost exclusively for centuries.

Kappa Mu Epsilon News

Chapter 1. OKLAHOMA ALPHA, Northeastern State College, Tahlequah, Oklahoma.

Oklahoma Alpha held its annual Founders' Day banquet on April 18, the exact date of the founding of Kappa Mu Epsilon.

Many alumni of the chapter are now engaged in activities in connection with the defense program. Leo Harmon and Joe Culver are both instructors in the Lafayette Radio School, Lexington, Kentucky; Miss Bobbie Lou Vaught is a Spanish translator for the Government in San Antonio, Texas; Noble Bryan, Jr., is with the Boeing Aircraft Corporation in Seattle, Washington; Bill Forney is with the Dow Magnesium Corporation, Tulsa, Oklahoma; and John West is a research physicist with the DuPont Chemical Corporation.

Other alumni are in active military service. Alfred Southworth is at the United States Naval Academy; Dick Woods was with the Air Corps in Java; A. T. Moore is a ground school instructor at Shepard Field, Wichita Falls, Texas; James Carter is teaching mathematics at the Spartan School of Aeronautics, Tulsa, Oklahoma; Dan McDonald is an army meteorologist in Tampa, Florida; Bill Mitchell is in the Navy, and is now stationed at Dearborn, Michigan; Russell Walker, past president of the chapter, is an instructor in aviation at Chanute Field, Illinois; and Charley Burns is in the Naval Aviation Corps.

Oklahoma Alpha has the distinction of having a woman member who is an instructor in aviation. She is Miss Ann Fogle, Okmulgee, Oklahoma.

Dr. J. M. Hackler, the first instructor in mathematics at Northeastern State College, (1909), is a member of Oklahoma Alpha.

Chapter 3. KANSAS ALPHA, Kansas State Teachers College, Pittsburg, Kansas.

Mr. Joe L. Campbell of the class of 1939, and Mr. Louie Rogers of the class of 1940 are now teaching mathematics in the Coast Artillery Training School at Camp Wallace, Texas.

Chapter 4. MISSOURI ALPHA, Southwestern Teachers College, Springfield, Missouri.

During the year, several resignations have made it necessary to elect a new set of officers. They are as follows:

President Archimedes	-----	Miss Christine Radley
Vice-President Galileo	-----	Mr. Herbert Hodges
Secretary Ahmes	-----	Miss Elizabeth White
Treasurer Napier	-----	Mr. Jerome Twitty
Secretary Descartes	-----	Mr. Carl V. Fronabarger
Faculty Sponsor	-----	Mr. L. E. Pummill

Myrle Johnson is now in the graduate school of the University of Chicago as a consequence of an appointment he received in the Weather Bureau. Thomas Butler and Chester Hamilton are studying at Iowa State College. Mr. Hamilton has a teaching fellowship.

Ivan Dean Calton, Loran Blain, Robert Dewitt, and Ruth Parks are now teaching in high school. The last three graduated "with distinction" at the end of the summer term. Also, Mary Lee Mires and Bernice Williams graduated "with high distinction" at the end of the summer term. Six more members graduated this year at the end of the spring term; they all received the designation, "with high distinction" or "with distinction". Of these, Roberta Gene Thompson and Mabel Morris are teaching elsewhere in the state, Francis B. Belshe is teaching in the Junior High School in Springfield, and John Ellis is teaching at the Senior High School in Springfield. Jack Wommack, Jr., is continuing his education, and Warren Hitt is in the Air Corps.

John Hoey, Billy Joe Compton, and Ashlie Ellis are

now attending the Missouri School of Mines at Rolla, Missouri.

Miss Dorothy Martin, daughter of Dr. Robert W. Martin of the faculty, has been awarded a position on the faculty of the University of Illinois. The appointment came as a result of her high score on a qualifying examination given to all students at the University of Illinois. The instructorship in chemistry at the University of Illinois will be a half-time position. The rest of Miss Martin's time will be devoted to a continuation of her studies.

Chapter 7. NEBRASKA ALPHA, Nebraska State Teachers College, Wayne, Nebraska.

Charles Winter has a graduate assistantship at the University of Nebraska.

James Ahern, past president of Nebraska Alpha, is a student at the Naval Academy at Annapolis, and will receive his commission in May.

Since January, Nebraska Alpha has been sponsoring weekly meetings giving instructions in the slide rule. The meetings have been quite popular.

Chapter 8. ILLINOIS ALPHA, Illinois State Normal University, Normal, Illinois.

Professor Henry Poppen has had a leave of absence during the present academic year, and has been studying at George Peabody Teachers College, Nashville, Tennessee. He expects to resume his teaching duties next September.

The annual Christmas party was held at the home of Dr. C. N. Mills, head of the mathematics department. Santa Claus, (alias the genial host), officiated at an exchange of toys which were later given to the Odd Fellows for distribution.

On March 19, an initiation service was held for four new members, thereby bringing the total active membership to 28. Fifteen persons became pledges of Illinois Alpha at a

special service on the same evening; as a token of pledge-ship, they wore pink and white pledge ribbons for one week.

On April 3, a group of members presented a fifteen minute radio skit entitled "Mathematics for Victory" over WJBC, the local station.

Alumni members who are in active service or are in civilian defense work were special guests at the annual banquet held in the spring. Seventeen former students are in some kind of defense work.

Chapter 9. KANSAS BETA, Kansas State Teachers College, Emporia, Kansas.

Kansas Beta has taken in twenty-two new members during the present academic year.

The chapter sponsored the mathematics exhibit for the "Science Open House" at the Kansas State Teachers College during April.

Ray Hanna is now an instructor at Wichita University. Edison Greer teaches mathematics in the same institution. Frank Faulkner recently accepted an instructorship at Brown University for next year; he has held a teaching assistantship at Kansas State College for the past two years.

John Zimmerman held an assistantship in the physics department at the University of Oklahoma the second semester of this year; next year he has an assistantship in physics at the University of Illinois. Also, Francis McGowan has accepted an assistantship in physics at the University of Wisconsin for next year, and Vaughn W. Edmonds is the recipient of a similar appointment at the University of Kansas.

Chapter 10. ALABAMA ALPHA, Athens College, Athens, Alabama.

Dr. Kathryn Wyant, former national president of Kappa Mu Epsilon, is again living in Athens. She is being cared for in the country home of Mrs. Richard Grubbs.

Because of the freezing of silk, the hosiery mill owned by Athens College and in which one hundred and fifty young people usually work had to close last August. It is now open on a part time schedule, and about fifty students employed in the mill are again back in College.

An initiation for associate members was held upon November 30th; the ceremony was followed by a breakfast. One of the members initiated was especially anxious to become an associate member before leaving for camp. He withdrew from college the following day.

The initiation upon March 21 was followed by the annual banquet. In former years, the banquet has been held off the campus, but because of limited funds, it was decided to hold the one this year in the building of the home economics department. The food was prepared by the members of the chapter, and the event proved to be a grand success.

Chapter 11. NEW MEXICO ALPHA, University of New Mexico, Albuquerque, New Mexico.

New Mexico Alpha inaugurated its activities for the year with a weiner roast at Doc Long's Picnic Grounds in the Sandia mountains. At the picnic, seventeen students were voted to membership in Kappa Mu Epsilon. Sixteen of these were initiated, and were honored with a banquet upon October 16.

During the first semester, several students presented papers at the monthly meetings. Student speakers were Miss Maxine Lind, Mr. Lawrence Williams, Mr. Vincent Brunelli, and Mr. Bruce Clark.

At the start of the second semester, a tea was given in honor of Dr. Arthur Rosenthal, former professor of mathematics and dean of the science faculty at Heidelberg University. Dr. Rosenthal has lectured at the University of New Mexico this past semester, and now becomes a permanent member of the staff. On February 19, Dr.

Rosenthal was initiated into Kappa Mu Epsilon as an honorary member; at the same time ten students were also initiated. A banquet followed the initiation.

New Mexico Alpha has sponsored several interesting projects this year. The chapter has given ten dollars toward the Traveling Lectureship of the Southwestern Section of the Mathematical Association of America; the lecturer this year is Dr. Roy MacKay of State College, New Mexico, an alumnus of New Mexico Alpha. Also, funds were solicited to assist Dr. Bernardo Baidaff in the publication of *Boletin Matematico* in Buenos Aires. Thanks to the assistance of several other chapters of Kappa Mu Epsilon, a total of eighteen dollars was raised.

Several members of this year's senior class have already left Albuquerque to take defense positions in different parts of the country. Mr. Bruce Clark, chapter president last year, left recently for the Massachusetts Institute of Technology to study meteorology, and Mr. Robert Reece has a similar appointment at the California Institute of Technology. Also, Miss Maxine Lind and Miss June Horn have become technical assistants in the United States Weather Bureau.

Chapter 12. ILLINOIS BETA, Eastern Illinois State Teachers College, Charleston, Illinois.

Illinois Beta has extended its activities considerably this year. Following the suggestion given by several other chapters during the national convention last year at Warrensburg, the fraternity has sponsored a mathematics club; the club and the active chapter hold joint meetings.

The first activity of the year was a picnic at the Fox Ridge State Park. The chapter has learned that an excellent spirit of fellowship results from such events.

During the year, an attempt has been made to unify the various divisions in the field of mathematics, and also to better understand the problems of teaching in the high

school. Outstanding among the programs have been the following: "Investigating the Fourth Dimension" by Dave Fisher, "Mathematics and Music" by Dean Heller, and "The Place of Logic in High School Geometry" by Miss Hendrix, critic teacher in the local high school. Also, there was a varied program by the Kappa Mu Epsilon pledges, and a demonstration of "Computing to the Base Eight" by high school students of mathematics.

The initiation banquet in February was held under the sign of the cardioid. Professor Pepper, an inspiring instructor from the University of Illinois, gave an interesting talk on the history of pi.

In April, a banquet was held honoring Professor Fiske Allen who is resigning this year. Special tribute was paid to Mr. Allen for his service in the field of mathematics and his contributions to the teaching profession in general.

Chapter 13. ALABAMA BETA, Alabama State Teachers College, Florence, Alabama.

Alabama Beta has been occupied with defense work. At the monthly meetings, defense stamps have been awarded as prizes in the mathematical contests. Instead of the annual banquet, Alabama Beta is using its banquet fund for the purchase of defense bonds.

Chapter 15. ALABAMA GAMMA, Alabama College, Montevallo, Alabama.

Alabama Gamma recently initiated six new members, and held its annual initiation supper. Officers for the coming year were selected at the regular meeting in April. It is planned to end the activities for this year with a picnic.

Chapter 20. TEXAS ALPHA, Texas Technological College, Lubbock, Texas.

Mrs. Opal Miller, former Secretary Descartes for Texas Alpha, has been on leave of absence for the second semester

of this year, and Lida B. May has been elected to take her place.

Chapter 21. TEXAS BETA, Southern Methodist University, Dallas, Texas.

Texas Beta has had several interesting meetings this year. Roland Porth, vice-president and program chairman, presented an illustrated talk on the subject, "Construction of the Logarithmic Scale with Straight-Edge and Compasses". Also, James Summer, one of the pledges, discussed, "The Limit Concept", and Merle Mitchell spoke on the topic, "An Interesting Algorithm". Most of the meetings have been held at the home of Dr. E. D. Mouzon where, after each program, the members enjoyed games and refreshments.

Mary Moseley, who was initiated last fall, has been elected to fill a vacancy in the office of secretary.

Yet this year, there will be spring initiation and a picnic; this latter event will occur sometime in May.

Chapter 22. KANSAS GAMMA, Mount St. Scholastica College, Atchison, Kansas.

Kansas Gamma now has a membership of twenty-six resident and seven non-resident members. The resident members include two faculty members, 12 active members, and 12 pledges.

Marjorie Dorney, class of 1939, received an appointment to the United States Hydrometeorological Service in Washington, D. C. The appointment became effective in February.

A brief summary of the activities of the chapter for the present year is as follows:

October 5: Initiation of pledges.

November 13: Round table discussion on "Mathematics and National Defense", with special emphasis on the role of the American college woman.

- December 14: Christmas party, the first social event of the year.
- January 17: Pop-corn sale, the proceeds from which were contributed for the support and continuance of *Boletin Matematico*.
- January 21: All-school assembly. The members of Kansas Gamma presented the short play "Modern Mathematics Looks Up Its Ancestors", by Marion Stark, and an original skit, "A Trip to Infinity", written by Tena Anders, a pledge.
- February 4: Officers approved the petitions of Hofstra College and of Central Michigan College of Education.
- February 11: Meeting in honor of Lincoln, with emphasis upon his accomplishments in mathematics and upon those of his contemporaries.
- March 15: The second social event of the year in recognition of St. Patrick. An original skit, "Irish Medley", written by Sister Helen Sullivan, O.S.B., faculty sponsor, was presented by members and pledges.
- April: Pledge day; on this day all pledges must prove their true worth as an indication of their value to the organization.
- May: Initiation of new members and installation of new chapter officers.

Recently it was announced that Bobbe Powers and Mary Flaherty were elected to membership in Kappa Gamma Pi. Both of these students have made outstanding records in Mount St. Scholastica College.

Chapter 23. IOWA BETA, Drake University, Des Moines, Iowa.

A summary of the program for the year is as follows:
October: Initiation of Julia Rahm and John McKiernan.
Pledging of Bill Banghart, Ragan Brock, Don Johnson, and Roderick Legg.

November: Picnic for all the members.

December: Mr. McLellan of the engineering department of the Bell Telephone Company gave a talk.

January: Professor Mehlin talked on the subject, "Aviation and Mathematics in the Present World Crisis".

February: Lieutenant Lauder discussed, "Mathematics in the Navy".

March: Initiation of Bob Hansen and Don Johnson. Pledging of Howard Braunstein, Bob Geist, Bill Meyer, James Mill, Garth Patterson, Julia Roberts, Richard Smith, Garth Spencer, Helen Stouffer, John Swanson, Dick Wendt, and Robert Johnson.

Chapter 24. NEW JERSEY ALPHA, Upsala College, East Orange, New Jersey.

Miss Anne Zmurkiewicz has been appointed instructor in mathematics and biology at Upsala College. In the meantime she is continuing her postgraduate courses at Columbia University.

Upon January 30th, New Jersey Alpha had a special mid-year program with Dr. Virgil Mallory of Montclair State Teachers College as invited speaker. The subject of his address was "The Mathematics of Defense". Dr. Mallory has made a special study of mathematics courses and their relation to defense, so he was able to give an illuminating exposition. Following the lecture, there was an interesting discussion upon the topic. Two such special programs are planned for each year.

Professor M. A. Nordgaard spoke before the college section of the New Jersey Association of Mathematics Teachers on March 7 in regard to the "Cultural and Historical Content that should go into a one year Course in College Mathematics".

Chapter 26. TENNESSEE ALPHA, Tennessee Polytechnic Institute, Cookeville, Tennessee.

Seven initiates were taken into Tennessee Alpha upon November 20, 1941. The following students composed the first class to be initiated since the installation of the chapter: Albert H. Bryan, Jr., James Fitzgerald, Howard Hernon, William Jarrell, Jr., Robert Johnson, Charles D. Tabor, and Thurman Webb. Charter members who attended the initiation ceremony and banquet were Dr. R. O. Hutchinson, Dr. R. H. Moorman, William Fitzgerald, Joseph Lane, Margaret Plumlee, and Kent Walthall. A second initiation of the year will be held the first part of June.

Albert H. Bryan left in February to take his place as a second lieutenant in the cavalry. Upon the petition of the college president, he was transferred to the engineering division of the Army.

**KAPPA MU EPSILON WELCOMES
TWO NEW CHAPTERS**

Chapter 27. NEW YORK ALPHA, Hofstra College, Hempstead, New York.

New York Alpha was installed upon April 4. Dr. Martin Nordgaard of New Jersey Alpha represented the National Fraternity, and presided at the ceremony of installation. The guest speaker at the installation dinner was Professor E. H. C. Hildebrandt, who is editor of the club section of the *American Mathematical Monthly*.

Hofstra College is a youthful institution, since it was only founded in 1937. Its growth and influence have been sufficiently great, however, that it was elected to the Middle States Association of Colleges in November of 1940, and to the American Association of Colleges in January of 1941. Dr. L. F. Ollmann, head of the mathematics department, and Dr. Leslie B. Poland, member of the faculty, were members of Kappa Mu Epsilon previous to the installation. The

enthusiasm of these men and their colleagues assures the membership of Kappa Mu Epsilon that New York Alpha will be a valuable addition to the Fraternity.

Chapter 28. MICHIGAN BETA, Central Michigan College of Education, Mount Pleasant, Michigan.

Michigan Beta was installed upon April 25. Professor E. R. Sleight of Albion College was the installing officer. The selection of Professor Sleight was especially appropriate inasmuch as Dr. C. C. Richtmeyer and Dr. Judson Foust, faculty members at Central Michigan College, are distinguished alumni of Michigan Alpha.

Central Michigan College of Education, formerly called the Central State Teachers College, was established in 1892. It is fully accredited, and possesses a graduate division. The resident enrollment during any one semester exceeds fifteen hundred.

This spring, the Michigan Undergraduate Conference is to meet upon the campus of Michigan Beta. This is merely an indication of the influence which the institution is already exerting in the field of undergraduate mathematics.

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