

# THE PENTAGON



FALL, 1941

Volume I

Number 1

*Published semi-annually by*  
KAPPA MU EPSILON

# Kappa Mu Epsilon

## NATIONAL HONORARY FRATERNITY IN MATHEMATICS

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## Greetings

O. J. PETERSON

As president of Kappa Mu Epsilon, I send best wishes to each chapter and to each individual member of our fraternity. In these days of uncertainty, one thing remains clear, scholarship is needed as never before. Persons disciplined in the ways of correct thinking and possessing mathematical imagination will be the leaders of the future. Thus the obligations upon our own society are very great. It is our hope that the ideals of Kappa Mu Epsilon will be an inspiration to hundreds of students throughout the country.

Through the medium of the PENTAGON, the influence of Kappa Mu Epsilon should be multiplied many fold. The publication of such a journal is probably the most significant project ever undertaken by the fraternity. It deserves the support of every member. To Professor Newsom and his editorial staff we extend good wishes for the success of their efforts.

## From the Editor

C. V. NEWSOM

With this issue of the PENTAGON, Kappa Mu Epsilon embarks upon a new enterprise. An official journal for the fraternity was authorized at the fifth biennial convention held in Warrensburg, Missouri, last April, and preliminary plans for its publication were made only three months ago. Thus the present issue is offered to you with a certain amount of apology. Nevertheless, the editorial staff has high hopes for the future, and plans are under way to make the magazine a journal of distinction. It should be understood that the PENTAGON is a project of Kappa Mu Epsilon, and the success of the project will depend to a great extent upon the amount of coöperation given by the organized chapters and the individual members of the fraternity.

For years, many persons have believed that there is a place in this country for a magazine catering to the needs of college students of mathematics, and featuring expository articles of a semi-popular type. Moreover, there is a serious need for a medium through which outstanding student papers can be made available. These are precisely the thoughts behind the present editorial policy of the PENTAGON. In each issue, there will be selected papers presenting a variety of mathematical topics. In addition, the section bearing the title, "The Mathematical Scrapbook," will contain an assortment of brain-teasers for persons interested in mathematics. Of course, as the official Kappa Mu Epsilon publication, an important feature of each issue will be the news items from the various chapters.

Contributions for each section of the PENTAGON are solicited from each member of Kappa Mu Epsilon, student or faculty. In general, any contribution may be sent to the editor, but the work of the editorial board will be facilitated if the particular member of the board is contacted who is

in charge of the section in which the contribution is to appear. Professor E. R. Sleight, Albion College, Albion, Michigan, will edit all student papers submitted for publication. Professor E. A. Hazlewood, Texas Technological College, Lubbock, Texas, is making a collection of items for "The Mathematical Scrapbook." News Notes should be submitted to Miss Orpha Ann Culmer, Alabama State Teachers College, Florence, Alabama, or to Miss E. Marie Hove, Nebraska State Teachers College, Wayne, Nebraska.

Material submitted for publication in the PENTAGON should be written with care so that a minimum of editing is required. Manuscripts should be typewritten double-spaced with wide margins, and the original copy should be submitted. It is desirable that footnotes be reduced to a minimum, and they should be used essentially for bibliographical material. Figures and diagrams must be drawn on plain white paper with black India ink, and should be constructed twice the size that they are to be printed. Authors are requested to keep in mind typographical difficulties involved in setting up complicated mathematical formulas.

# Mathematics and National Defense

WILLIAM L. HART<sup>1</sup>

*University of Minnesota*

In the past, the United States has always been geared to a peace-time economy with only brief intermissions when military affairs were reckoned of importance. Hence, it is natural to find a large element of surprise in the public reaction to the fact that military science in its most essential branches is a mathematical science. In this connection, however, it is interesting to note the following authoritative statement by a non-mathematical group;<sup>2</sup> "The mechanization of military defense and the expansion of industry require increasing emphasis upon mathematics, science, and technical skills."

In considering the role of mathematics in national defense, we shall be interested equally in the industrial and military phases of the situation. Let us pass by those facts which are common knowledge: thus, we shall not allude again to the well-known importance of mathematics in the engineering profession or in advanced physical science, all of which is essential in times of peace and doubly so in times of war. Let us devote our attention to those mathematical activities which are peculiar to the present emergency or are definitely associated with the services of the army and navy.

First, it is of interest to notice some steps which have been taken by mathematical organizations as their contribution to the national program. About two years ago, the American Mathematical Society and the Mathematical Asso-

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<sup>1</sup> Professor W. L. Hart is chairman of the Subcommittee on Education for Service of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America.

<sup>2</sup> From the report of a conference of college and university presidents and representatives of national defense agencies of the Government in July, 1941; Bulletin 13, *Higher Education and National Defense*, by the American Council on Education, Washington, D. C.

ciation of America created a joint War Preparedness Committee which has been functioning actively under the chairmanship of Professor Marston Morse of the Institute for Advanced Study, of Princeton, New Jersey. This Committee now is divided into three subcommittees: one on Research, a second on Preparation for Research, and a third on Education for Service. Also, the National Council of Teachers of Mathematics, a national organization composed principally of teachers of secondary mathematics, recently appointed a War Preparedness Committee to give guidance and leadership in curricular actions in the field of secondary mathematics during the emergency. A comprehensive picture of the mathematical activities associated with the present situation is given by a consideration of the program of the subcommittees of the War Preparedness Committee of the Society and the Association.

A major achievement of the Subcommittee on Research, under Professor Dunham Jackson, of Minnesota, as chairman, has been the creation of a board of consultants in mathematical fields of present national importance. Chief consultants have been appointed in each of the following fields: aeronautics, ballistics, computation, cryptanalysis, industry, probability, and statistics. Some of these consultants have associated with themselves other mathematicians expert in the specific fields involved. Through the efforts of Professor Morse and various other individuals, contacts have been established between the consultants and those people in the army, navy, or other agencies of the Government who face difficulties requiring highly skilled mathematical treatment. As a result of such contacts, the consultants have been called upon considerably for help in their fields. Also, various mathematicians not associated with the board of consultants are engaged in mathematical work for industry or the Government in the defense program. Since practically all of this mathematical activity at advanced levels deals with confidential material related to national defense, we must remain in ignorance of the nature

of the service until a time when the requirements of military secrecy are no longer operative. When that time arrives, it is likely that an interesting panorama of the national utility of mathematics will be unfolded.

The Subcommittee on Preparation for Research, under Professor M. H. Stone, of Harvard, as chairman, is concerned with the professional education of mathematicians to fit them for research on problems associated with defense. This Subcommittee is calling for substantially increased emphasis on training in applied mathematics, particularly at advanced levels. Certain mathematicians feel that the recommended shift in emphasis also is highly desirable from a long range viewpoint, apart from present emergency considerations. Concrete evidence of increased emphasis on applied mathematics is furnished by the recent inauguration of an extensive program of advanced instruction in mathematical mechanics at Brown University.

In this connection we note that the position of mathematical research in industry has just been discussed admirably by Dr. Thornton C. Fry in a chapter<sup>3</sup> of the book entitled *Research—A National Resource*, a House Document of the Seventy-seventh Congress. Dr. Fry's position as Director of Mathematical Research of the Bell Telephone Laboratories, and his general activity in mathematical circles, fit him uniquely to give authoritative opinions on the subject which he treated. The following quotations from Fry's article emphasize the importance, from a long range as well as from an emergency viewpoint, of the recommendation about applied mathematics presented by Stone's Subcommittee:

(1) Because of its general significance as the language of natural science, mathematics always pervades the whole of industrial research.

(2) Its field of usefulness is nevertheless growing, partly through the development of new indus-

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<sup>3</sup> This was published also as a pamphlet supplement to the *American Mathematical Monthly*, v. 48, No. 6, June-July, 1941.

tries, such as the aircraft business, and partly through the incorporation of new scientific developments into industrial research, as in the application of quantum physics in chemical manufacturing and statistical theory in the control of manufacturing processes.

(3) The need for professional mathematicians in industry will grow as the complexity of industrial research increases, though their number will never be comparable to that of the physicists or chemists.

(4) There is a serious lack of university courses for the graduate training of industrial mathematicians.

(5) Management, which is already keenly alive to the importance of mathematics, is also rapidly awakening to the value of mathematicians and the peculiar relationship which they bear to other scientific personnel.

The Subcommittee on Education for Service, with the author of the present paper as its chairman, set up the following program as a principal objective:<sup>4</sup>

1. To investigate what mathematics is of prime utility in industry below the level of the trained engineer, and in the army and navy, for the program of national defense.

2. In accordance with the results of this investigation, to make recommendations concerning mathematical instruction in the secondary schools, at the undergraduate level in colleges, and in centers of adult education.

The Subcommittee's evaluation of the mathematical needs of the army and navy were made in the light of the emergency situation, with a subjective view as to the level of training which can be attained if full advantage is taken

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<sup>4</sup>The Subcommittee has published a report of its activities and recommendations concerning this objective in the *American Mathematical Monthly*, v. 48, June-July, 1941, pp. 353-362.

of our extensive educational system. The peace-time levels of mathematical training for regular officers at West Point and Annapolis were not given undue weight in the evaluation just referred to. For certain classifications of relatively small size, in the army and navy, the Subcommittee agreed with existing rules which practically require the officers to be trained engineers or men with proper backgrounds in advanced physical science and mathematics, as in the signal corps, the meteorological section of the air corps of the army, the technical sections related to army or navy ordnance, etc. Outside of the activities of these highly specialized technicians, the Subcommittee found evidence supporting the following opinions as to emergency minima in mathematical preparation:

In all combat units of the army and in the navy, a mathematical background including algebra and geometry would be an asset for enlisted men and officers. In the navy and in important branches of the army, the officers and a substantial percentage of the enlisted men should possess mathematical training through trigonometry.

Information at hand relating to industry indicates the need for skilled mechanical workers, draftsmen, workers in lofting departments, and many others below the level of trained engineers, with a knowledge of trigonometry. This need, in addition to the personnel requirements of the army and navy, is creating a demand in excess of the supply for men with mathematical training through trigonometry. The importance thus ascribed to such an elementary subject as trigonometry is of considerable significance because this low but satisfactory level of training for the necessary number of men obviously can be attained if proper steps are taken. It is not complimentary to our educational system, at the high school and college levels, that at this moment there exists a shortage of men with the desired elementary knowledge of mathematics. To correct the situation so far as it relates to young men out of school, we must rely on various measures of adult education. For example, in the various

centers of the Engineering Defense Training Program of the United States Office of Education we find that many courses in mathematics are being given for workers in defense industries. In some cases these courses involve content beyond the level of elementary calculus. The director of these defense mathematics courses in one large center of the aircraft industry remarks that, in the first offering of his program, the content ranges from a review of secondary mathematics through the elements of calculus, and that the classes have been swamped with students.

To eliminate the mathematical shortage which has just been mentioned, with a view to conditions as they will exist one, two, or many years from now, proper actions should be taken in the high schools. More of the intelligent boys should be studying substantial mathematics in high school, and more should be continuing at least through the stage of trigonometry in high school or in college. The preceding opinion is of importance not only in connection with the need for more trigonometry in industry, the army, and the navy. Beyond this, it is essential for the high schools to graduate greater numbers of students with considerable training in mathematics in order to provide a proper reservoir of candidates for college instruction in the physical sciences, applied mathematics, and other technical fields where a shortage of trained men exists at present. In addition, of course, such high school graduates are needed also as candidates for the various professions and general fields of learning, which, although not usually labeled as mathematical, nevertheless require substantial high school mathematics for efficient progress.

A major section of the recent report of the Subcommittee on Education for Service consists of recommendations designed to eliminate future shortages in the supply of men and women with training in mathematics through various appropriate stages. In order to influence corresponding actions in the high schools, this report has been published not only in the *American Mathematical Monthly* but also in

the two principal periodicals relating to the field of secondary mathematics. Moreover, copies of the report will be sent to superintendents of schools in all cities of over 2,500 population.

In a consideration of the utility of our field of learning in the present program of national defense, it is well to avoid a selfish attitude. Thus, we should not take the present emergency as an opportunity for artificially enhancing the position of mathematics. Instead, our full attention should be devoted to unselfish efforts to advance all plans which might increase the available supply of mathematical talent for national service.

## Dyadic Arithmetic

HAROLD D. LARSEN

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We are all familiar with the child who determines the number of apples in a bowl by counting them on his fingers. Strangely enough, there exist in the darkest parts of Africa and Australia savage tribes so low in the scale of civilization that they have not yet reached this childish stage of counting. One writer reports that a certain tribe of the Torres Straits makes use of the following rather awkward number names in counting objects:

- 1 = urapun
- 2 = okosa
- 3 = okosa-urapun
- 4 = okosa-okosa
- 5 = okosa-okosa-urapun
- 6 = okosa-okosa-okosa.

If the number of objects being counted exceeds six, these natives simplify matters by calling it a heap.

Although this system of numeration is rather limited in scope, it does have some points of interest. One notes that these natives do not count by tens as we do in our every-day arithmetic, but count by twos. We can state this fact in mathematical language by saying that they use a binary, or dyadic, system of numeration, whereas we use a decimal system.

The aforementioned writer does not reveal whether or not this particular tribe possesses a set of symbols for recording the results of their counting efforts. Nevertheless, it is possible to construct such a system of symbols which will possess all of the essential properties of our own decimal system of notation. Such a system of symbols is known as a binary system of notation, and it has several interesting applications

in mathematics, particularly in the solution of certain problems usually associated with mathematical recreations. Before discussing some of these problems, however, it might be well first to explain in some detail the principles involved in the binary system. In order to facilitate this explanation, let us begin by reviewing briefly the principles involved in the familiar decimal system of notation.

In the first place, we recall that when we write such a number as 38,306 we are actually abbreviating the quantity,

$$3 \times 10^4 + 8 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 6 \times 10^0.$$

Thus each digit appearing in the number 38,306 has in reality two values. First, it has an absolute value; e. g., the digit 3 has the same value as that brought to mind when one speaks of 3 apples, or 3 fingers, or 3 pounds of beans. Secondly, each digit has a place-value; thus, the position in which the first 3 is written gives that 3 the value of ten-thousands; the position in which the second 3 is written gives that 3 the value of hundreds. Furthermore, the successive place-values are determined by a simple law; viz., the value of any particular place is always ten times that of its neighbor to the right. Thus, ten units make a ten, ten tens make a hundred, ten hundreds make a thousand, and so on. We note, finally, that in our decimal system of notation, we have need of ten and only ten different symbols; namely, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. All numbers greater than nine are obtained by the proper combination of these ten symbols, following the principles outlined above. In particular, it should be noted that the notation 10 for ten is actually a combination of 1 and 0, and merely stands for  $1 \times 10^1 + 0 \times 10^0$ .

Many persons have the impression that we use the decimal system of notation in our arithmetic because of some inherent properties of the number ten. The convenient rule for multiplying a number by 10, by 100, by 1000, etc., is pointed out as a property of the base ten which is not possessed by any other number, and hence is sufficient reason in itself for counting by tens. As a matter of fact, any other

base could be used with equal facility. The only reason we use the base ten in our scale of notation is that each of us has ten fingers. If the mathematician by some miracle could scrap the arithmetic used today, and start all over, he would choose some scale other than ten. His choice would likely be twelve, which has several advantages over the decimal scale.

Whatever base is chosen for the system of notation, it is to be noted that the number of different symbols required is equal to the base used. For the base ten we need ten symbols; for the base twelve we would need twelve symbols; and for the base twenty we would need twenty different symbols. The binary system of notation, in which we are particularly interested, requires but two different symbols, 0 and 1. All numbers in the binary scale are expressed as combinations of these two symbols. The law which determines the successive values of the places takes this form: each place-value is two times that of its neighbor to the right. Thus, as we proceed from right to left, the place-values in order are  $2^0$ ,  $2^1$ ,  $2^2$ ,  $2^3$ ,  $2^4$ , etc. For example,

$$101011 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0.$$

The following table gives the first thirty-two numbers as they are written in the binary scale.

Decimal	Binary	Decimal	Binary
1	1	13	1101
2	10	14	1110
3	11	15	1111
4	100	16	10000
5	101	17	10001
6	110	18	10010
7	111	19	10011
8	1000	20	10100
9	1001	21	10101
10	1010	22	10110
11	1011	23	10111
12	1100	24	11000

25	11001	29	11101
26	11010	30	11110
27	11011	31	11111
28	11100	32	100000

We might note in passing that if an integer in the binary system of notation ends in 0, it is an even number; if it ends in 1, it is an odd number. Moreover, we immediately observe a simple rule for multiplying by two: merely annex a zero to the multiplicand. This statement can be verified by an inspection of the table above. For example, two  $\times$  five =  $10 \times 101 = 1010 =$  ten. Similarly, to multiply by four, one annexes two zeros to the multiplicand; to multiply by eight, one annexes three zeros, etc.

In order to perform the four fundamental operations of arithmetic, namely, addition, subtraction, multiplication, and division, it is first necessary that one memorize sets of facts commonly called the primary combinations. For the binary system, these reduce to the following:

ADDITION:

$$\begin{array}{r} 0 \quad 1 \quad 0 \quad 1 \\ 0 \quad 0 \quad 1 \quad 1 \\ \hline 0 \quad 1 \quad 1 \quad 10 \end{array}$$

MULTIPLICATION:

$$\begin{array}{r} 0 \quad 1 \quad 0 \quad 1 \\ 0 \quad 0 \quad 1 \quad 1 \\ \hline 0 \quad 0 \quad 0 \quad 1 \end{array}$$

The various operations are performed in the dyadic arithmetic in precisely the same manner as in the ordinary decimal arithmetic; only the primary combinations have changed. The following are examples illustrating addition and multiplication in the binary scale. The reader might like to verify the results.

Add:	1101	Multiply:	1011
	110		101
	1011		<u>1011</u>
	1111		1011
	<u>101101</u>		<u>110111</u>

There are available rather simple methods for changing from a number in the decimal scale to the equivalent number in the binary scale, and vice versa. The proofs of these methods are quite elementary, and will not be given. It suffices to give some examples of the methods.

As an illustration, let us change thirteen from the decimal scale to the binary scale. As shown to the right, we divide thirteen in the decimal notation successively by 2 until the quotient 0 is reached. The remainders in order give the successive digits in the binary representation of thirteen, the last remainder being the left-hand digit. In order to change a number from the binary scale to the decimal scale, we use a method which is reminiscent of synthetic division, a process which is explained in any college algebra text. The following illustration shows how 110101 is changed from the binary system to the scale of ten.

$$\begin{array}{rcccccc}
 1 & 1 & 0 & 1 & 0 & 1 & (2 \\
 - & 2 & 6 & 12 & 26 & 52 & \\
 \hline
 1 & 3 & 6 & 13 & 26 & 53 & 
 \end{array}$$

Thus 110101 in the binary system is equivalent to 53 when the base is ten.

Many mathematicians in the past have waxed eloquent over the beauty and simplicity of the binary system of notation. Leibnitz, enraptured by the mystic elegance of the dyadic arithmetic, exclaimed, "One suffices to derive all out of nothing." Laplace commented as follows on Leibnitz's enthusiasm for the binary system: "Leibnitz saw in his binary arithmetic the image of Creation. . . . He imagined that Unity represented God, and Zero the void, just as unity and zero express all numbers in his system of numeration. This conception was so pleasing to Leibnitz that he communicated it to the Jesuit, Grimaldi, president of the Chinese tribunal for mathematics, in the hope that this

emblem of creation would convert the Emperor of China, who was very fond of the sciences."<sup>1</sup>

Without stopping to investigate the many other interesting questions connected with the dyadic arithmetic (such as the notation for common and radix fractions, expressions for  $\pi$  and  $e$ , etc.), we proceed to observe some of the many applications of the binary system of notation.

A TABLE FOR FINDING A PERSON'S AGE

The accompanying table illustrates an old method for determining the age of a person. Ask him to tell you in which columns his age is found. The sum of the numbers at the tops of the columns named will be his age. Thus, if the person announces that his age is found in columns A, D, and E, he is  $1 + 8 + 16 = 25$  years old.

A	B	C	D	E
1	2	4	8	16
3	3	5	9	17
5	6	6	10	18
7	7	7	11	19
9	10	12	12	20
11	11	13	13	21
13	14	14	14	22
15	15	15	15	23
17	18	20	24	24
19	19	21	25	25
21	22	22	26	26
23	23	23	27	27
25	26	28	28	28
27	27	29	29	29
29	30	30	30	30
31	31	31	31	31

This simple trick depends on the fact that any integer can be represented uniquely in the binary scale. Each col-

<sup>1</sup> Dantzig, Tobias, *Number the Language of Science*, Macmillan, New York, 1989, p. 15.

umn in the table can be interpreted as having a place-value in such a scale. Thus, column A represents units, column B represents twos, column C denotes fours, etc. It is left to the reader to complete the analysis.

#### THE PROBLEM OF BACHET

A large number of the elementary problems commonly included among arithmetical recreations are to be found in Bachet's *Problèmes plaisans et délectables*, which was first published in 1612. Among the more difficult problems proposed by Bachet was the determination of the least number of weights which would serve to weigh any integral number of pounds from 1 lb. to 40 lbs., inclusive.<sup>2</sup> Under the assumption that the weights are to be unequal and that they may be placed in only one of the scale-pans, Bachet showed that the series of weights of one, two, four, eight, sixteen, and thirty-two lbs. would be sufficient.

It is clear that the solution of Bachet's problem depends upon the fact already noted that any integer can be represented uniquely in the binary scale. In order to determine the particular weights necessary to weigh a given number of pounds, it is only necessary to express the number in the scale of two. Thus, for example, thirteen = 1101; hence the weights one, four, and eight lbs. are used to weigh thirteen pounds.

There are several variations of Bachet's classical problem. The following problem is one of the more interesting. The solution is left to the reader.

A man has one thousand dollars in dollar bills, and ten bags. How should he distribute the money in the bags so that he can pay out any amount in dollars from one to one thousand without having to take any money out of any of the bags?

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<sup>2</sup>Ball, W. W. R., and Coxeter, H. S. M., *Mathematical Recreations and Essays*, Eleventh Edition, Macmillan, London, 1939, pp. 50-53.

THE RUSSIAN PEASANT METHOD OF MULTIPLICATION

There is a remarkable method of multiplication which requires no more knowledge than that of addition, multiplication by two, and division by two. This method was used at one time by peasants in various sections of Russia, and in some respects resembles a method of multiplication employed by the ancient Egyptians. The process is best explained by means of an example. Let us consider the problem of multiplying fourteen by twenty-five. We begin by writing the two numbers side by side

14	25
28	12
56	6
112	3
224	1

in the decimal scale; then multiply 14 by 2 and divide 25 by 2. In this division, the remainder is simply ignored. The process is continued; at each step the number in the first column is multiplied by 2, and the number in the second column is divided by 2, all remainders being dropped. We now look at the second column and pick out the even numbers, in this case, 12 and 6, and cross out the rows containing these even numbers. The sum of the numbers remaining in the first column is the desired product; i. e.,  $14 \times 25 = 350$ .

The reason why the Russian peasant method of multiplication always gives the correct product is readily explained by means of the binary system of notation. Let us analyze the above example. We begin by expressing the numbers, fourteen and twenty-five, in the binary scale, and find their products in the ordinary way. For this purpose we use the primary combinations listed earlier, and otherwise proceed exactly as in the decimal scale:

1110	(14)
11001	(25)
<u>1110</u>	
1110	
1110	
<u>101011110</u>	(350)

It is to be noted that the following partial products are required in obtaining the desired product:

$$\begin{aligned} 1 \times 1110 &= 1110 \\ 1000 \times 1110 &= 1110000 \\ 10000 \times 1110 &= 11100000. \end{aligned}$$

Let us next arrange the given numbers according to the Russian peasant method, successively multiplying the numbers in the first column by two and dividing the numbers in the second column by two:

1110	11001
11100	1100
111000	110
1110000	11
11100000	1

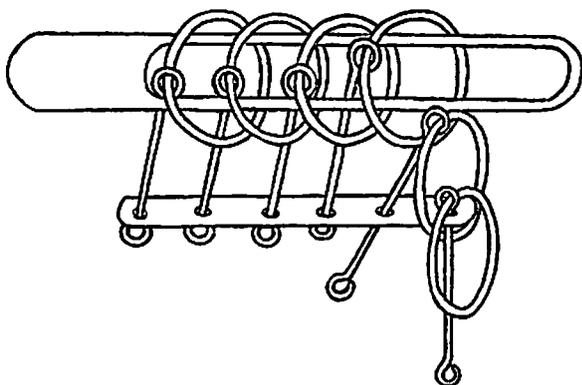
Now consider any particular digit 1 of the multiplier, 11001, which appears at the head of the second column. To be definite, let us take the fourth digit counting from the right. Each time the number in the second column is divided by two, this particular digit moves one place to the right. Simultaneously, the number in the first column is doubled by the annexation of a zero. After three steps, the chosen digit 1 falls in the right-hand, or units, position, and three zeros have been annexed to the multiplicand to form the number opposite in the first column. But this number in the first column is precisely the partial product contributed to the total product by this particular digit. The same holds true for any other digit 1 of the multiplier. The number of steps required to bring any digit 1 to the units place is equal to the number of zeros to be annexed to the multiplicand to form the partial product contributed by that digit. By the time any given 1 turns up in the units place, we find opposite it the corresponding partial product.

Since the digit 0 in the multiplier does not contribute anything to the product, we ignore those rows in which a 0 occupies the units place in the second column. But if a number in the binary scale ends in 0, it is an even number.

Thus, we can say that each row is erased in which the number in the second column is even. All of the numbers remaining in the first column are seen to be partial products, and their sum is, therefore, the desired product.

### THE CHINESE RINGS

An ingenious application of the binary system of notation is to be found in the solution of a problem connected with the puzzle known as the Chinese rings.<sup>3</sup> This interest-



ing puzzle is considered by some authorities to be the greatest ever devised. Certainly its ancient origin entitles it to a top ranking among mechanical puzzles. Just when and where it was invented we do not know, but as early as 1550, Cardan described the puzzle, and later in 1693, it was discussed by Wallis.

The Chinese rings which are represented in the figure are currently manufactured by the Mecanno Company, and are on sale at toy-counters for five cents. The puzzle consists of six rings which are hung upon a bar in such a manner that the ring at the end can be taken off or put on the bar at pleasure, but the other rings are so intertwined that they cannot be removed directly. Of course, the puzzle is to remove all the rings from the bar.

<sup>3</sup> Ball, W. W. R., and Coxeter, H. S. M., *op. cit.*, pp. 305-310.

The secret of the puzzle is rather simple. Any ring after the first can be taken off or put on only when the ring next to it towards the end is on the bar, and all the rest towards the end are off the bar. Except for the first two rings, only one ring can be taken off or put on at a time. To simplify the following discussion, we shall assume that in every case, only one ring is taken off or put on at a time, and such an act will be called a step. Immediately, we encounter the interesting problem: "How many steps are required to remove all six rings?"

The more general problem of finding the number of steps necessary to remove  $N$  rings was attacked unsuccessfully by both Cardan and Wallis. Subsequently, the problem was solved in different ways by various mathematicians. The solution given below is due to a French mathematician, Gros, who published his solution in 1872.

The following table is a schematic representation of the first few steps in removing six rings from the bar.

Step	Position	Number
0	<u>000000</u>	101010
1	<u>0000 0</u> 0	101001
2	<u>0000</u> 00	101000
3	<u>00 0</u> 0 00	100111
4	<u>00 0 0</u> 0 0	100110
5	<u>00 000</u> 0	100101
6	<u>00 00</u> 0 0	100100
7	<u>00 0</u> 00 0	100011
n	<u>000000</u> 000000	000000

The rings are indicated by circles. A circle drawn above the line represents a ring on the bar; a circle drawn below the line represents a ring off the bar. To each position of the rings there is associated a number which is determined according to the following rule. Let the rings on the bar be denoted alternately by the digits, 1 and 0, counting from the left; let a ring off the bar be denoted by the same digit assigned to that ring on the bar which is nearest to it on the left, or by a 0 if there is no ring to the left of it. In this way, every position of the rings has an associated number written in the binary scale.

If we examine closely the successive numbers in the table above, it is observed that each number after the first is obtained by subtracting unity from the preceding number. That this is true, in general, becomes apparent after some thought. In arriving at this conclusion, it should be observed that the end ring is removed first in case there are an odd number of rings, and that the ring next to the end is removed first in case there are an even number of rings. Furthermore, it should be observed that every ring on the bar gives a variation from 1 to 0, or from 0 to 1, and every ring off the bar gives a continuation, so that each step results in the subtraction of unity from the associated number. Now when all the rings are on the bar, the number is 101010; when all the rings are off the bar, the number is 000000. Hence, to change from the original position to the final position requires a number of steps equal to the difference between these numbers; namely,

$$\begin{array}{r}
 101010 \\
 -000000 \\
 \hline
 101010 = \text{thirty-two} + \text{eight} + \text{two} = \text{forty-two steps.}
 \end{array}$$

A similar analysis will yield the number of steps required to remove any number of rings.

## THE GAME OF NIM

One of the most interesting applications of the binary system of notation is to be found in the analysis of the game of nim.<sup>4</sup> Although nim is frequently played for money, it can hardly be classed as a gambling game, for, as we shall see, a player who knows the secret of the game can virtually always win.

The game of nim is usually played with matches. Any number of matches are divided into any number of piles; the piles need not contain the same number of matches. Players, A and B, move alternately. Suppose it is A's turn. He may choose any one pile, and take from it all or only some of the matches, but he must take at least one. Then it is B's turn, and he does likewise. Then it is A's turn, and so on. The player who succeeds in picking up the last match wins the game.

The secret of the game of nim is quite simple, but one needs a little practice to perform mentally the arithmetic which is involved, and it is best to start out with a small number of matches. The winning technique depends on the fact that if A can make his drawings so that eventually B must draw from two piles of two matches each, designated symbolically by (2) (2), or from four piles of one match each, (1) (1) (1) (1), then A is certain to win no matter how B plays. This fact is very easily verified.

Before explaining the winning technique, it is convenient to introduce the terms, *odd set* and *even set*. Suppose, for example, that the matches are divided into three piles, (14) (7) (13). Let each of these numbers be expressed in the binary scale, and then let the digits in each column be added in the same manner as when the decimal base is used; thus,

$$\begin{array}{r}
 \text{Fourteen} = 1110 \\
 \text{Seven} = 111 \\
 \text{Thirteen} = 1101 \\
 \text{Sums} = \underline{2322}.
 \end{array}$$

<sup>4</sup>C. L. Boulton, *Annals of Mathematics*, vol. II, series 2, 1903, pp. 35-39.

Since at least one of these individual sums, or digits, is odd, this particular distribution of matches is called an odd set. Suppose, on the other hand, that the matches are divided into the three piles, (9) (13) (4). Proceeding as before, we obtain:

Nine	=	1001
Thirteen	=	1101
Four	=	<u>100</u>
Sums	=	2202.

In this case, each individual sum is even, and the given distribution is designated as an even set. The reader may like to verify that the set, (11) (7) (9) (13), is odd, whereas the set, (11) (7) (9) (5), is even.

If a player draws from any even set, he necessarily must leave an odd set, for, considering the representation of the set in the binary scale, any drawing whatever will remove a digit 1 from at least one column, and the sum of that column will no longer be even. On the other hand, if a player draws from an odd set, he can leave either an odd set or an even set, depending on his particular move. However, there are usually only one or two moves which can be made which will change an odd set into an even set. Thus a drawing from an odd set which is made merely at random will very likely result in leaving an odd set.

Let us consider again the two winning end-distributions, (2) (2) and (1) (1) (1) (1). Each of these is an even set. It therefore follows that the winning technique in the game of nim is to try to force your opponent to draw from an even set. If you succeed in leaving your opponent an even set, his drawing will leave an odd set. Thereupon, after a little mental arithmetic, you can draw in such a way as to leave an even set once again. Continuing this process, you are able eventually to force your opponent to draw from one of the two even sets, (2) (2) or (1) (1) (1) (1), and the game is won.

If at the start of the game you have an even set before

you, perhaps the best procedure is to draw a single match from the largest pile, of course, leaving an odd set. But if your opponent does not know the secret of the game, the chances are very high that he will misplay and leave an odd set, whereupon you are able to force a win.

It might be of interest to follow the moves in an illustrative game. Suppose it is A's turn, and that he has before him the three piles, (7) (6) (3). Expressing this set in the binary scale, we obtain:

$$\begin{array}{r} \text{Seven} = 111 \\ \text{Six} = 110 \\ \text{Three} = \underline{11} \\ \text{Sums} = \underline{232}, \text{ (an odd set).} \end{array}$$

To leave an even set, A can draw two matches from any pile. Suppose he draws two matches from the first pile, thus leaving the set:

$$\begin{array}{r} \text{Five} = 101 \\ \text{Six} = 110 \\ \text{Three} = \underline{11} \\ \text{Sums} = \underline{222}, \text{ (an even set).} \end{array}$$

No matter how B moves, he is forced to leave an odd set. Assume that B removes three matches from the second pile, leaving the set:

$$\begin{array}{r} \text{Five} = 101 \\ \text{Three} = 11 \\ \text{Three} = \underline{11} \\ \text{Sums} = \underline{123}, \text{ (an odd set).} \end{array}$$

At this point, A has no choice other than to draw all five matches from the first pile. There remains the set:

$$\begin{array}{r} \text{Three} = 11 \\ \text{Three} = \underline{11} \\ \text{Sums} = \underline{22}, \text{ (an even set).} \end{array}$$

Now, regardless of how B draws, A is able to take the last match.

There is a method of finger-reckoning which can be used to simplify the arithmetic involved in the winning technique in the game of nim. In the process of determining whether a given distribution of matches forms an odd set or an even set, it is not necessary to know the actual sum of each column, but only that it is odd or even. The character of each sum is readily determined with the aid of the fingers. Let the fingers of one hand represent successive places in the binary scale; i. e., let the little finger represent units, the next finger twos, the middle finger fours, etc. Then the unit digits in any particular column can be added, without carrying, by alternately raising and lowering the appropriate finger. An example should make the method clear. The following steps illustrate the determination of the character of the set, (6), (7), (5). At the start, all fingers are lowered.

	Addend	Middle Finger	Fourth Finger	Little Finger
1.	110	raised	raised	lowered
2.	111	lowered	lowered	raised
3.	101	raised	lowered	lowered

A set is even if, and only if, all the fingers in the result are lowered. Since, in this example, the middle finger remains raised, the given distribution of matches forms an odd set.

After an opponent has been defeated several times at nim, he will no doubt become a little suspicious, and might suggest that the game be changed so that the person who picks up the last match should lose instead of win. If so, make the change, but continue using the same technique, for if you can force your opponent to draw from the set, (2) (2), you can make him take the last match. If, however, he draws from the set, (1) (1) (1) (1), he wins. But this last situation does not turn up, on the average, more than once in three games, so you can still safely bet your shirt.

## Robert Record's Whetstone of Witte

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At the time of the invention of printing, arithmetic was considered both an art and a science. Accordingly there were two types of arithmeticians, the algoristic and the Boetian. The former were concerned primarily with calculation, while the latter were interested in the properties of the subject. Tonstoll and Record<sup>1</sup> were the preservers of the algoristic method in England, and before the end of the sixteenth century the ordinary style of commercial arithmetic, which has prevailed ever since, was in course of establishment. From the time of Robert Record, England was always conspicuous in numerical skill as applied to money. DeMorgan says, "Nothing could arise to alter my conviction that the efforts which were made in this country towards the completion of the logarithmic tables in the seventeenth century were the results of the superiority in calculation."

By some historians, Robert Record is given credit for having written the first text on the subject of arithmetic in the English language, and they all agree that this was the first text to receive general use in England. In Ball's *History of Mathematics in Cambridge*, there appears this statement: "This text was the earliest English scientific work of any value, and is the best treatise on arithmetic produced in the sixteenth century."

Robert Record was born at Tenby, England, about 1510. He was educated at Oxford, and later taught mathematics at Cambridge. The fact that he and Tonstoll, the two outstanding mathematicians of that century, both taught at Cambridge would indicate that this university was becoming an important school of mathematics even at that early

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<sup>1</sup> Sometimes spelled Recorde.

date. It is said of him concerning his teaching that "he rendered clear to all capacities to an extent wholly unprecedented." He was also a doctor of medicine, and later in life became court physician to the King and Queen of England.

The *Ground of Artes*, as Record named his arithmetic, made its first appearance about 1540. This, as well as other texts which he wrote, is written in the dialogue style. The fact that the last known edition appeared in 1673 is a testimonial to the hold that it had upon the schools of England. It even found its way across the waters, and was used in the American colonies along with Dilworth, Cocker, and other English arithmetics.

The *Whetstone of Witte* appeared in 1556, and according to the author "this is the Second Parte of Arithmetic: containing the extraction of Roots, the cossike practice with the rules of equations and the works of surd Numbers." Sir Walter Scott immortalized this text in one of his novels. A traveler was spending the night at a certain inn. Desiring something to read for the evening he called the landlord's daughter and asked her if the inn contained any reading matter. She replied that there were only two books in the house, the Bible which her father would loan to no one, and Record's *Whetstone of Witte*.

This work of Record's is decidedly an algebra, and the greater part of it is devoted to the solution of equations, which he called the cossike art. It is my purpose to omit the solution of equations and review the part of the text devoted to properties of numbers, extraction of roots, and surds.

Under the head of properties we find a discussion of the so called diametral numbers. According to the definition, "A diametral number is such a number as both two parts of that nature that if they be multiplied together they will make the aforesaid number. And the squares of those two numbers being added together will make a number whose root is the diameter of the diametral number." For example, twelve is considered a diametral number, and five its diam-

eter, since twelve is made up of the two parts (factors) three and four, and five represents the number whose square is the sum of the squares of these two numbers. This relation is represented by a rectangle whose area is the diametral number, the two parts being the dimensions of the rectangle, and the diagonal being the diameter of the diametral number.

Five rules are given for determining diametral numbers:

1. A diametral number must end in 0, 2, or 8.
2. It must be divisible by 12.
3. A given diametral number has only one diameter.
4. Conversely, one diameter may belong to more than one diametral number.
5. No square number can be a diametral number.

The method of procedure for finding whether a number is diametral or not resolves itself into separating the number into various factors, then examining each pair of factors in the light of the definition by applying the five rules previously stated. As an example, 108 has the following pairs of factors:

2	3	4	6	9
54	36	27	18	12

It is found by inspection that 9 and 12 respond to the tests, and therefore, 108 is a diametral number. The two factors are called the sides of the diametral number.

In the text we find a table of these numbers.

3	4	5	12		11	60	61	660
5	12	13	60		12	35	47	420
6	8	10	48		16	63	65	1008
7	24	25	168		20	99	101	1980
8	15	17	120					
9	{ 12	15	108		40	75	85	3000
	{ 40	41	360			96	104	3840
						198	202	7920
						399	401	15960

If from the table, ratios of the sides be formed by using the odd numbers of the first column as the numerators, we obtain the following results:

$$3/4, 5/12, 7/24, 9/40, \dots$$

While if the even numbers be used as numerators, we have,

$$8/15, 12/35, 16/63, 20/99, \dots$$

The first set is called the first order ratios while the second is designated the second order. Inverting both sets, we obtain, 1  $1/3$ , 2  $2/5$ , 3  $3/7$ , 4  $4/9$ , ---, and 1  $7/8$ , 2  $11/12$ , 3  $15/16$ , 4  $19/20$ , ---. "Where in the first order you see both in the whole numbers, and also in the numerators of the fractions, the natural order of numbers. And in the denominators the natural progression of odd numbers. But in the second order the whole numbers are in natural order while the numerator and denominator keep an Arithmetical Progression with equal difference of .4. Save that in the numerators all the numbers be odd, while in the denominators they are all even."

These observations are used to determine whether a given number is diametral. Thus for 43,200, two of its factors are 180 and 240. Forming the fraction  $180/240$  and reducing to lowest terms, "here is obtained  $3/4$ , which is the first ratio in the first order, and 43,200 is a diametral number."

The outstanding contribution to mathematics made by Robert Record is his development of the methods which we now use for extracting roots. Three definite steps are given as *first steps* in the process of extracting square roots. Quoting directly from Record, these steps are:

1. Write the number as it is, then set a point (prick) under each odd place (counting from the right) and so shall every point have .2. numbers except the last, which may have only one.

2. Mark the numbers that belong to the last point

to the left hand, and whether it hath one number, or two. Look what the root may be of that number, if it be a square. And that root set by a crooked line, as you place the quotient in division. Cancel all that square number.

3. But if that number belonging to the last point be not a square, then take the root of the greatest square, which is contained in it, and place the root as I said before. And the square of it shall you take from the number, and let the rest be set over this number as you would in division. And so have you ended your work for that point.

Before proceeding to a discussion of extracting roots, it will be helpful to note Record's plan of division, using the dialogue method.

*Master.* I would divide 365 by 28. Then set I those two sums thus:  $\begin{array}{r} 365 \\ 28 \end{array}$ . Now I look how many times I

may find 2 (which is the last figure of the divisor) in 3, (which is the last of the number to be divided), and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers and within it I set the 1, and that is called the quotient number. Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 over 3 then cancel the 3 and the 2, then will the figures stand

$\begin{array}{r} 1 \\ (3)65 \\ (2)8 \end{array}$  thus (3)65[1.\* Then come I to the next figure of the

divisor, and take it so many times out of the figures that be over it, and look what doth remain, that I must write over them and cancel them as in this example. Therefore I will now take once 8 out of 16, and there remaineth 8, which I must set over the 6,

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\* Figures which appear canceled in Record's original work appear here in parentheses.

and cross out the 16, and the 8 of the divisor; and

then will the figures stand thus:  $\begin{matrix} (1) & 8 \\ (3) & (6) & 5 & [1. \end{matrix}$  So I

have thus wrought once. When you have thus wrought then must you begin again and write your divisor anew, nearer toward the right hand by one place, as in this example you shall set 2 under 8 and 8 under

5, thus:  $\begin{matrix} & 1 & 8 \\ (3) & (6) & 5 & [1. \\ (2) & (8) & 8 \\ & 2 \end{matrix}$  Then as before seek how many

times you may take your divisor out of the number over him now.

*Scholar.* That may I do here 4 times.

*Master.* True it is that you may find 2 four times in 8; but then mark whether you can find the figure following so many times in the other that is over him. Can you find 8 four times in 5?

*Scholar.* No, neither yet once.

*Master.* Therefore take 2 out of 8 once less. Well, then 3 times 2 makes 6. If I take 6 out of 8, there remaineth 2; which 2 with the 5 following makes 25, in which sum I find 8 three times also, and therefore I take 3 as the true quotient and write it in the crooked

line before the 1, thus:  $\begin{matrix} & & (2) \\ (1) & (8) & 1 \\ (3) & (6) & (5) & [13 \\ (2) & (8) & (8) \\ & (2) \end{matrix}$

By this process it has been discovered that when 365 is divided by 28 the quotient is 13 and the remainder is 1. It might be observed that Record has used this example to illustrate how many months there are in a year, provided there were exactly 28 days in each month. Following this illustrative example it is suggested that 365 be divided by 52, evidently having something of the same idea in mind.

To illustrate the instructions of the Master concerning square root, we may use 110,224. Applying the three suggestions given, the following results:  $(1) \overset{2}{(1)}0 \overset{2}{2}24[3.$

The Master then continues,

Now you shall double your root, and put that double under the next space toward the right hand that is behind the next point. Always supposing that if the double contains more figures than one, that the first shall be set under that place, and the second under the next figure toward the left hand. Then seek a quotient as you do in division, which shall show how often that double number may be found in that that is over it, which quotient you shall set before the first root within the quotient line. But this regard must you have, that you may leave over the next point, toward the right hand, as much as the square of that quotient with which you work, for from that the square of that quotient must be subtracted. And here work both subtractions at once.

The original example now appears in the form, 20,224, to which the Scholar applies these later instructions obtaining  $(2) \overset{1}{(0)} \overset{3}{(2)}24[3.$  This now leaves 1,324. The trial division is 66, and the problem is written  $(1) (3) (2) (4) [2.$  In an attempt to display these operations as a sequence of what has come before, Record presents the following picture:

$$\begin{array}{ccccccc} & & & & (1) & & \\ & & & & (1) & (3) & \\ (1) & (1) & (0) & (2) & (2) & (4) & [332. \\ & & & & (6) & (6) & \end{array}$$

Every digit being crossed off (that is, inclosed in parentheses here) indicates that there is no remainder, and therefore the number selected is a perfect square. The 1 above

the 3 indicates the remainder when  $2 \times 6$  is subtracted from 13, as is indicated by a left to right process of division. But the 1 is offset by the next operation.

Record's plan for extracting cube roots is very similar to the method now in use, except that he uses a *stagger form* to present the results. The trial divisor is found by "taking the square of the quotient 300 times." Then instead of adding the corrections to the trial divisor and multiplying this sum by the new digit in the root he multiplies the trial divisor and each correction separately, and adds the three results. For example, Record's plan of procedure as compared with the one now generally used might thus be illustrated:

Record's Plan	Present Plan
2	26463592[29
(1) (8) 0 7 4	8
(2) (6) (4) (6) (3) 592[29	1200
1 2 0 0	540
<hr style="width: 100%; border: 0.5px solid black;"/>	81
1 0 8 0 0	16389
4 8 6 0	<hr style="width: 100%; border: 0.5px solid black;"/>
7 2 9	1821
<hr style="width: 100%; border: 0.5px solid black;"/>	2074
1 6 3 8 9	

The line is always drawn under the trial divisor as above indicated. After a thorough discussion of the method the Master makes this statement, "And if you understand this, there resteth no more difficulty."

Evidently the Scholar had no more difficulties when the proposed number was a perfect cube. But what to do with the remainder caused him to inquire, "And now for the remainder, how shall I have to bring it unto a fraction that may aptly express the root?" To which the Master replies,

There are as many ways as there are writers, almost. Whereof Cardan, his rule is this. Multiply the root squarely, and again by 3, and that shall be the divisor under the remainder. Scheube informeth

another order. Triple the root, and the square of it also and add both these numbers together and .1. more: and so have you a denominator for your numerator.

These two rules are applied later in discussing the method for "duplicating the cube." "Grecians given to idle banqueting did procure thereby such mortal sickness that the quick were scarce able to bury the dead. Wherefore consulting their gods for redress thereof they received answer that when they would double the alter, which was cubic in form, they should be delivered from that plague. Meaning that learning is a due means to deliver realms from such a plague. But what say you? If the side of the cube be .3. feet as well it may, how many feet shall the side be of that cube, which must be double unto it? The side of the cube is .3. and therefore the whole cube is .27., whose double is .54. And the cube root is 3 and  $27/27$  by rule of Cardan. That is .4. which is plainly false. But by Scheube's Rule it will be .3 and  $27/37^*$  or .3 and  $3/4$ , which is nearer the truth." Record then proceeds to extract the root by his own method obtaining 3.77 which he asserts "is equivalent to .3 and  $77/100$  and more."

In regard to finding roots of fractions the Master states, "For the roots of fractions I shall be able to say no more but this: that if the numerator and denominator both be squares or cubes then may you find in that fraction no such root. As  $16/27$  is neither cubic nor square, because the parts do not agree in square name nor in cubic name."

Record's treatment of surds is interesting and quite modern. Here for the first time he uses positive and negative signs in their present form. In the discussion of these signs he informs the Scholar that the negative sign augments the number, as well as the positive, "for (-) doeth ever abate the quantity of the number though it do increase the name. Yet for a difference the numbers that be compounded with

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\* No dot follows the fraction.

(+) shall be called Bimedialles and those that be compounded with (-), be named Residualles.”

The word, *more*, was applied to addition, while subtraction was designated by use of such terms as *save*, *abate*, and *less*. Thus  $a + b - c$  might be designated as *a more b less c*, or *a more b save c*. In some cases the Master uses *and* for addition, so that  $a + b - c$  is read *a and b less c*.

Record indulged in rather fanciful names for certain types of numbers. For instance a *zenzizenzike* number stands for the square root of the square root, while the “square of a square squared” (8th power) is called a *zenzizenzizenzike* number. The last in the group of such numbers refers to the “square of a squared cube” (12th power) to which is applied the term a *zenzizenzicubic*. These names suggest some such flights of fancy as are found in that new book, *Mathematics and the Imagination*.<sup>4</sup>

Conforming to the author’s usual practice we find an exaggerated use of the dot. However there seems to be no consistency, for .26, .12., 38. are all found in the discussion on surds although some of these forms may have resulted from typographical errors.

Some ingenious devices were used for operating upon surds.<sup>5</sup> Thus in adding the square root of .384. and the square root of .150. we find the following.

	.384.		.150.
	64		25
6)	8		5
		13	
		13	
		<u>169</u>	
		6	
		<u>1014</u>	

From this he concludes that the result is the square root of .1014. Record then uses quite another plan, but as both

<sup>4</sup> Kasner and Newman, *Mathematics and the Imagination*, Simon and Schuster, New York, 1940.

<sup>5</sup> Special symbols were used by Record to denote the square root and the cube root.

processes yield the same result, the Master remarks, "I judge them both to be correct."

"Cubic roots" are treated in the same manner. For example:

$$\begin{array}{r}
 \sqrt[3]{.81.} \quad + \quad \sqrt[3]{.24.} \\
 27 \qquad \qquad \qquad 8 \\
 3) \qquad \qquad 3 \qquad \qquad 2 \\
 \qquad \qquad \qquad 5 \\
 \qquad \qquad \qquad 5 \\
 \qquad \qquad \qquad \hline
 \qquad \qquad \qquad 25 \\
 \qquad \qquad \qquad 5 \\
 \qquad \qquad \qquad \hline
 \qquad \qquad \qquad 125 \\
 \qquad \qquad \qquad 3 \\
 \qquad \qquad \qquad \hline
 \qquad \qquad \qquad 375
 \end{array}$$

Therefore the original sum is equivalent to the cube root of .375.

Much confusion in notation exists. For instance,  $\sqrt[3]\{108 + \sqrt{[29 + \sqrt{(760)}]}\}$  is represented by  $\sqrt[3]{(108) + \sqrt{[\sqrt{(29)} + \sqrt{(760)}]}}$ . This confusion of symbols seems to perplex the Scholar for at the close of the subject of subtraction he states, "This is as easy as addition, for by the art of simple surds I do see that the square root of 10 plus the square root of 19 does make  $\sqrt{[29 + \sqrt{(760)}]}$ . But when  $\sqrt{[29 + \sqrt{(760)}]}$  is set as a total and the square root of 19 is to be subtracted out of it, how I shall work that and have the square root of 10 for the remainder I see not." That the Master did not expect the Scholar to understand this process is evidenced by the final conversation which seems to have been prompted by some unusual occurrence in the life of Record.<sup>6</sup>

The dialogue follows:

*Master.* You say truth. But hark what meaneth that hasty knocking at the door.

<sup>6</sup> Record died not long after the completion of the *Whetstone of Wits*.

*Scholar.* It is a messenger.

*Master.* What is the message: tell me in mine ear. Yea sir is that the matter. Then there is no remedy, but that I must neglect all studies and teaching, for to withstand those dangers. My fortune is not so good to have a quiet time to teach.

*Scholar.* But my fortune is much worse that your unquietness so hindereth my knowledge. I pray God amend it.

*Master.* I am forced to make an end of the matter, but yet will I promise you, that which you shall challenge of me, when you see me at better leisure, that I will teach you the whole art of *Universalle roots*.<sup>7</sup> And the extraction of roots in all square surds, with the demonstration of them and all the former works. If I might have been quietly permitted to rest but a little longer I had determined not to cease till I had ended all these things at large. But now farewell. And apply your study diligently in this that you have learned. And if I may get any quietness reasonable I will not forget to perform my promise with an augmentation.

*Scholar.* My heart is so oppressed by this sudden unquietness that I can not express my grief. But I will praise with all them that love honest knowledge that God in his mercy will soon end your troubles and grant you such rest as your trouble doeth merrit. And all that love learning say thereto Amen, Master, Amen.

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<sup>7</sup>Such as  $\sqrt{5+2\sqrt{6}}$

## The Mathematical Scrapbook

*thoughts, that shall not die, and cannot be  
destroyed.—Wordsworth*

As Newton neared the end of his career, his meditations prompted him to say,

I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore, and diverting myself now and then finding a smoother pebble or a prettier shell than ordinary, while the great ocean of truth lay all undiscovered before me.

If I have seen farther than Descartes, it is by standing on the shoulders of giants.



Kepler's long and patient efforts before demonstrating the ellipticity of the orbit of Mars are well known. His enthusiasm as he neared the completion of his work caused him to write,

What I prophesied two and twenty-years ago, as soon as I discovered the five solids among the heavenly orbits, what I firmly believed long before I had seen Ptolemy's *Harmonies*, what I had promised my friends is the title of this book, which I named before I was sure of my discovery, what sixteen years ago I urged as a thing to be sought, that for which I joined Tycho Brahe, for which I settled in Prague, for which I have devoted the best part of my life to astronomical contemplation, at length I have brought to light, and recognized its truth beyond my most sanguine expectations. It is not eighteen months since I got the first glimpse of light, three months since the dawn, very few days since the unveiled sun, most admirable to gaze upon, burst upon me. Nothing holds

me; I will indulge my sacred fury; I will triumph over mankind by the honest confession that I have stolen the golden vases of the Egyptians to build up a tabernacle for my God far away from the confines of Egypt. If you forgive me, I rejoice; if you are angry, I can bear it; the die is cast, the book is written, to be read either now or by posterity, I care not which! it may well wait a century for a reader, as God has waited six thousand years for an observer.

—▽—

It is the merest truism, evident at once to unsophisticated observation, that mathematics is a human invention.—P. W. Bridgman.

How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?—Albert Einstein.

—▽—

A mathematician who is not also something of a poet will never be a complete mathematician.—Weierstrass.

A scientist worthy of the name, above all a mathematician, experiences in his work the same impression as an artist; his pleasure is as great and of the same nature.—Henri Poincare.

—▽—

Sixty is among all the numbers the most convenient, because, being the smallest among all those which have the most divisors, it is the easiest to handle.—Theon.

—▽—

I put them aside to finish later in the year, and in the meantime, deserving, as I thought, a little restful luxury, devoted myself to Differential and Integral Calculus.—De Morgan.

—▽—

*What is the hardest task in the world? To think.*

—Emerson

One of the great mathematicians of the middle ages was the Italian, Fibonacci, sometimes known as Leonardo of Pisa. In his *Liber Abaci* appears the following problem:

"How many pairs of rabbits can be produced from a single pair in a year if it is supposed that every month each pair begets a new pair which, from the second month on, becomes productive; and no deaths occur?"

This problem leads to the famous sequence of numbers known as Fibonacci's progression; it is 1, 1, 2, 3, 5, 8, 13, 21,

---

—▽—

In approximately 1350, Narayana stated the problem: "A cow gives birth to one calf every year. The calves become young and themselves begin giving birth to calves when they are three years old. O learned men, tell me the number of progeny produced during twenty years by one cow."

—▽—

In Euclid there appears a proof of the statement that there is an infinite number of prime numbers. The greatest prime number known at the present time, however, is  $2^{127} - 1$  = 170,141,183,460,469,231,731,687,303,715,884,105,727.

—▽—

A strip of paper which has been twisted through an angle of 180 degrees, and the ends gummed together (the so-called *Möbius ribbon*), is a unilateral surface. A pencil mark may be drawn from one point on the surface to any other point without the inconvenience of crossing its edge. Moreover, the edge forms a closed, but not knotted, curve. Such a Möbius band, cut along a middle stripe, does not fall apart, but forms a bilateral surface. The new surface, when cut along a middle stripe, gives rise to two strips of paper which are interwoven with each other.

—▽—

During 1939, the question was raised during a radio program<sup>1</sup> in regard to the number of different ways of changing a dollar. The correct answer is 292.

—▽—

Kelly, Simpson, and Davis are members of a vaudeville troupe; one of the men is a dancer, another is the featured

<sup>1</sup> Vox Pop, the Columbia network.

singer, and the third is the violinist. One day, as a coincidence, their audience contained three men with the same names whom we shall designate as Mr. Kelly, Mr. Simpson, and Mr. Davis.

- (1) Mr. Simpson is 60 years of age.
- (2) The violinist of the vaudeville troupe is 45 years old.
- (3) Mr. Davis earns \$10,000 per year.
- (4) Kelly always defeats the featured singer at checkers.
- (5) The violinist earns \$150 per month.
- (6) The man in the audience of age nearest to that of the violinist earns three times as much as the violinist.
- (7) The man in the audience with the same name as the violinist is 30 years of age.

As a consequence of these statements, what is the name of the dancer?

=▽=

A cow is tied by a rope of length  $b$  to a silo of radius  $a$ . What is the length of the boundary that defines her grazing range?

=▽=

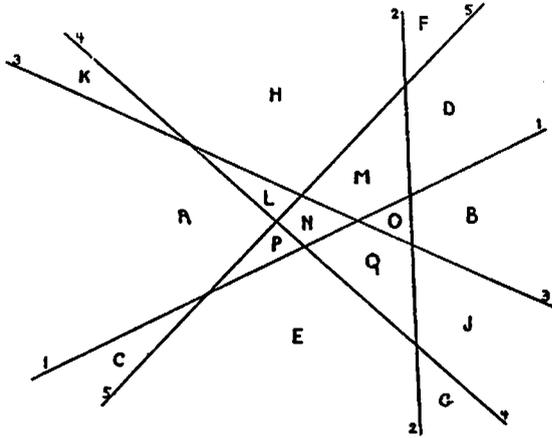
So, naturalists observe, a flea  
 Hath smaller fleas that on him prey;  
 And these have smaller still to bite 'em;  
 And so proceed ad infinitum.—Jonathan Swift.

Now, if a flea upon the bottom weighs  $\sqrt{2}$  grams, and any flea weighs  $\sqrt{[2-f(n-1)]}$  grams, where  $f(n-1)$  is the weight of the flea under him, how much does the flea on top weigh?

=▽=

Let us consider a set of  $n$  straight lines,  $n$  greater than 2, lying in a plane, with the understanding throughout the discussion that no three lines of the set are concurrent in a

point. One sees that the lines separate the plane into regions. If the plane were the plane of Euclidean geometry, some of the regions would not be completely bounded, but would extend out indefinitely like regions A and B in the accompanying figure of 5 lines. But in the *projective* plane, A and B to-



gether would make up one pentagonal region, bounded by segments of the lines 1, 2, 3, 4, 5 in that order. From the projective point of view this whole figure consists of

- 1 pentagonal region, A-B;
- 5 quadrilateral regions, M, N, Q, E-F, G-H;
- 5 triangular regions, L, P, O, C-D, J-K;

eleven regions in all. For any set of 5 lines we will always get this same result. Similarly for any set of 3 lines we will always have 4 regions, all triangular; and for any set of 4 lines we will always have 4 triangular regions and 3 quadrilateral regions, 7 in all.

For a figure of  $n$  lines, let  $a_3$  be the number of triangular regions,  $a_4$  the number of quadrilateral regions, ---, and  $a_n$  the number of  $n$ -sided regions. For  $n=3, 4,$  and  $5,$  we have pointed out that the only possible values for the  $a$ 's are as follows:

$$n=3; a_3=4.$$

$$n=4; a_3=4, a_4=3.$$

$$n=5; a_3=5, a_4=5, a_5=1.$$

But for a value of  $n$  greater than 5, it has been found that there are different figures with different sets of values of the  $a$ 's; and a number of interesting problems arise such as the following:

I. Show that for a given value of  $n$  the total number of regions,

$$a_3 + a_4 + \dots + a_n,$$

is always the same, and find the expression for this number in terms of  $n$ .

II. Show that for a given value of  $n$  the total number of all the sides of all the regions,

$$3a_3 + 4a_4 + \dots + na_n,$$

is always the same, and find the expression for this number in terms of  $n$ .

III. For  $n=6$ , what are the sets of values of  $a_3$ ,  $a_4$ ,  $a_5$ , and  $a_6$  which correspond to possible figures?

IV. For values of  $n$  greater than 6, what are the sets of values of the  $a$ 's which correspond to possible figures?

The formulas called for in I and II are known, and are not difficult to derive. The answer to III is known. The answer to IV is known for  $n=7$ , and is partially known for  $n=8$ , but almost nothing is known about it for cases where  $n$  is greater than 8.

[EDITORIAL NOTE: The above discussion upon the separation of the projective plane into regions by a set of straight lines was submitted by Professor W. B. Carver, of Cornell University. His address as retiring president of the Mathematical Association of America, given at Chicago upon September 3, 1941, treated the same topic. For a further discussion of the problem and references to earlier papers, see a

paper by Professor Carver in a forthcoming issue of the *American Mathematical Monthly*, vol. 48, 1941.]



A simple but unsolved problem is the four-color problem. First proposed by Cayley<sup>2</sup> before the London Mathematical Society, the problem pertains to the number of colors necessary for the coloring of a map so that any two countries or states which touch along a line will have different colors. Heawood<sup>3</sup> showed in 1890 that five colors will be sufficient, but no maps have been constructed that require five colors. The four-color problem, then, is merely that of showing that four colors will always suffice. Much progress has been made in recent years, but the problem still remains one of the unsolved problems of mathematics.



From a point outside a sphere, lines are drawn to all points of a circle upon the sphere. It can be proved the lines will meet the sphere again in a circle.



Let  $x$  be a number that satisfies the equation,

	$e^x = -1$ .
After squaring each member,	$e^{2x} = 1$ .
But,	$e^0 = 1$ .
Therefore it follows that	$2x = 0$ ,
and	$x = 0$ .
Thus,	$e^x = e^0$ .

But,  $e^x = -1$ , and  $e^0 = 1$ , so  $-1 = 1$ . Where is the fallacy?



From an old French source comes the study of the equation,

one-half full bottle equals one-half empty bottle.

<sup>2</sup> A. Cayley, "On the Colouring of Maps," *Proceedings of the London Mathematical Society*, v. 9, 1878, p. 148.

<sup>3</sup> P. J. Heawood, *Quarterly Journal of Mathematics*, v. 24, 1890, p. 332.

If each member of this equation is multiplied by 2, there results,

full bottle equals empty bottle.

[EDITORIAL NOTE: It is obvious that such mathematics would be highly regarded in certain quarters.]

## Kappa Mu Epsilon News

### *Chapter 1. OKLAHOMA ALPHA, Northeastern State College, Tahlequah, Oklahoma.*

President Eratosthenes ----- Mr. Eugene Dooley  
Vice-President Napier ----- Mr. Leo D. Harmon  
Secretary Bernoulli ----- Miss Wanda Crispin  
Treasurer Leibnitz ----- Miss Thelma McClure  
Secretary Descartes ----- Miss Mary K. Stewart  
Faculty Sponsor ----- Mr. Noble Bryan

Dr. Paul Lewis, one of the founders of Kappa Mu Epsilon while a student at Oklahoma Alpha, has recently joined the mathematics department of the Oklahoma Agricultural and Mechanical College. Other alumni of the chapter who have entered educational work include W. W. Dolan, now president of Bacone University, Bacone, Oklahoma, and G. H. Peeler, who is completing the course of study and dissertation for a doctor's degree at Columbia University, New York. John M. West, class of 1936, recently resigned a position at the University of Iowa to accept a position in research with the DuPont Chemical Company.

### *Chapter 2. IOWA ALPHA, Iowa State Teachers College, Cedar Falls, Iowa.*

President Pascal ----- Mr. Ralph Aschenbrenner  
Vice-President Archimedes ----- Mr. August Ebel  
Secretary Leibnitz ----- Miss Dorothy Clark  
Treasurer Gauss ----- Mr. G. Robert Kurtz  
Secretary Descartes ----- Mr. H. Van Engen  
Faculty Sponsor ----- Mr. H. Van Engen

John Wahl and Travis Phillips, members of the chapter last year, are now employed by the Sylvania Radio Corporation at Emporium, Pennsylvania.

A public meeting has been planned for November under the auspices of Kappa Mu Epsilon and the local science fra-

ternity. The speaker will be Dr. O. J. Perrine, of the American Telephone and Telegraph Company, who will demonstrate "Pedro the Voder."

*Chapter 3. KANSAS ALPHA, Kansas State Teachers College, Pittsburg, Kansas.*

President Archimedes ..... Mr. Mack McCormick  
Vice-President Plato ..... Mr. Leslie W. Baxter  
Secretary Lagrange ..... Miss Roberta W. Newcom  
Treasurer Thales ..... Mr. Richard Alsop  
Secretary Descartes ..... Mr. W. W. Hill  
Faculty Sponsor ..... Mr. J. A. G. Shirk

Several members of the chapter are now fellows in other institutions. Among them are William Wyatt, who is studying chemistry at Iowa State College, John Wagoner, who is a chemist at Kansas State College, and Ralph Overman, who has a teaching appointment at Louisiana State University. Also, Franklin Lanier has a scholarship at Ohio State University, and Mack McCormick will return to the Kansas State Teachers College to do graduate work as a fellow.

*Chapter 4. MISSOURI ALPHA, Southwestern Teachers College, Springfield, Missouri.*

President Archimedes ..... Mr. Robert Karch  
Vice-President Galileo ..... Miss Christine Radley  
Secretary Ahmes ..... Miss Elizabeth White  
Treasurer Napier ..... Mr. Jerome Twitty  
Secretary Descartes ..... Miss Louise Stockard  
Faculty Sponsor ..... Mr. L. E. Pummill

*Chapter 5. MISSISSIPPI ALPHA, Mississippi State College for Women, Columbus, Mississippi.*

President Gauss ..... Miss Annie Dorman  
Vice-President Stevin ..... Miss Dorothy Wallace  
Secretary-Treasurer Desargues, Miss Esther Mosley  
Secretary Descartes ..... Mr. R. L. Grossnickle

Faculty Sponsor ----- Mr. R. L. Grossnickle  
 Historian Sanford ----- Miss Maizie Higgenbotham

Mississippi Alpha held its first meeting of the present academic year in September. Ten new members were elected, namely, Mary Cliett, Emily Gilmore, Love McKinstry, Esther Mosley, Frances Randle, Frankie Stephens, Jimmie Ward, Claudia Massie, Ruth Johnson, and Maizie Higgenbotham.

Ten members of the chapter graduated last year. Of these, five completed their studies with honors. Laura Ruth Pitts and Anna Rose Crawford received the award, *Magna Cum Laude*, and Virginia Livingston, Elizabeth Moore, and Virginia House were given the designation, *Cum Laude*. Every graduate has a position, and eight of them are teaching mathematics or science.

Jessie E. Grossnickle, of the class of 1940, is now an instructor at the Texas State College for Women. She was vice-president of the local chapter of Kappa Mu Epsilon during her senior year.

*Chapter 6. MISSISSIPPI BETA, Mississippi State College, State College, Mississippi.*

President D. E. Smith ----- Mr. Harrison C. Leake  
 Vice-President H. L. Reitz, Mr. J. S. Williford, Jr.  
 Secretary G. D. Birkhoff ----- Mr. William O. Pepple  
 Treasurer L. E. Dickson -- Mr. Hoyt B. Wilder, Jr.  
 Secretary Descartes ----- Mr. C. R. Stark  
 Faculty Sponsor ----- Mr. W. O. Spencer  
 Publicity Director ----- Mr. Felix J. Lann

*Chapter 7. NEBRASKA ALPHA, Nebraska State Teachers College, Wayne, Nebraska.*

President Leibnitz ----- Miss Margie Morgan  
 Vice-President Archimedes ----- Mr. Don Strahan  
 Secretary Galileo ----- Mr. Homer Scace  
 Treasurer Einstein ----- Mr. Russell Vlaanderen

Secretary Descartes ----- Miss E. Marie Hove  
 Faculty Sponsor ----- Miss E. Marie Hove  
 Reporter Gauss ----- Miss Marjorie Johnson

Van Beringer, class of 1941, has a graduate assistantship at Iowa State College, where he is doing graduate work in physics, and Flaven E. Johnson, class of 1939, has a graduate assistantship at the University of Nebraska.

John Jones and Quentin Whitmore, recent alumni, received government appointments to do graduate work in meteorology, preparatory to entering the service of the weather bureau. John Jones is studying at the University of Southern California, and Quentin Whitmore is studying at the Massachusetts Institute of Technology.

Jack Morgan, class of 1937, who is employed as a chemist in the defense laboratories at the University of Nebraska, has been advanced to director of the research laboratory. He is also working on his doctor's degree in organic chemistry.

*Chapter 8. ILLINOIS ALPHA, Illinois State Normal University, Normal, Illinois.*

President Gauss ----- Miss Shirley Isaacson  
 Vice-President Pascal ----- Miss Nancy Hightower  
 Secretary Ahmes ----- Miss Geneva Meers  
 Treasurer Napier ----- Mr. Leo Montgomery  
 Secretary Descartes ----- Mr. C. N. Mills  
 Faculty Sponsor ----- Miss Edith Irene Atkin  
 Social Chairman Lilavati -- Miss Dorothy Johnson

Several members of Illinois Alpha have received graduate appointments to other institutions for the present year. Among them are Stanley Breen, who is a fellow in physics at the University of Wisconsin, and Max Chiddix, who is a fellow in chemistry at the University of Illinois. Also, Bill Staker will be an assistant in the physics department at the University of Iowa for his second year of graduate study, and Philip Malinberg has an assistantship in physics in the same institution. Violet Hachmeister received the distinc-

tive honor of being elected "scholar in mathematics" at the University of Illinois, where she is beginning her graduate study.

Last year, Illinois Alpha developed the theme, mathematics in action, in its programs, and lectures upon the applications of mathematics were featured.

*Chapter 9. KANSAS BETA, Kansas State Teachers College, Emporia, Kansas.*

President Pascal	-----	Mr. Warren Burns
Vice-President Gauss	-----	Mr. Alfred Freeman
Secretary Eratosthenes	-----	Miss Daisy Wheeler
Treasurer Bhaskara	-----	Miss Rosemary Haslouer
Secretary Descartes	-----	Mr. Charles B. Tucker
Faculty Sponsor	-----	Mr. O. J. Peterson
Historian Ahmes	-----	Mrs. Ruby Norris

Kansas Beta reports the following alumni who now hold positions of distinction:

Vernon Boger, chemical engineer with the Goodrich Rubber Company,  
 Aldro Bryan, research associate at Northwestern University,  
 Russell Byall, research chemist for the Goodyear Rubber Company,  
 Frank Faulkner, graduate student and instructor in mathematics at Kansas State College,  
 Edison Greer, instructor in mathematics at Wichita University,  
 Virgil Kinnamon, instructor in an aircraft company in Kansas City,  
 Harold McFarland, fellow in physics at the University of Wisconsin,  
 Charles Rickart, Ph.D., Benjamin Pierce Instructor at Harvard University,  
 Raymond Shobe, graduate student and instructor in mathematics at the University of Kansas,

- Worth Seagondollar, fellow in physics at the University of Wisconsin,
- Glenn Sheppard, research worker for the Radio Corporation of America,
- Harold Stout, research worker at the Massachusetts Institute of Technology,
- Hoyt Warren, research worker for the Radio Corporation of America.

*Chapter 10. ALABAMA ALPHA, Athens College, Athens, Alabama.*

- President Carmichael ----- Mr. Lloyd Stone
- Vice-President Dickson -- Mr. William Whittenberg
- Secretary Hedrick ----- Miss Edna Cox
- Treasurer Veblen ----- Miss Leanne Gunter
- Secretary Descartes ----- Miss Mary E. Renich
- Faculty Sponsor Wyant ----- Miss Mary E. Renich

Members of Kappa Mu Epsilon will be interested to learn that Dr. Kathryn Wyant, founder of Kappa Mu Epsilon, and first sponsor of Alabama Alpha, left Athens recently for Rochester, Minnesota, where she will receive treatment at the Mayo Clinic.

*Chapter 11. NEW MEXICO ALPHA, University of New Mexico, Albuquerque, New Mexico.*

- President Benjamin Peirce ----- Mr. John Coy
- Vice-President E. H. Moore, Mr. Lawrence Williams
- Secretary Cajori ----- Miss Ruth Barnhart
- Treasurer Bocher ----- Mr. C. B. Barker
- Secretary Descartes ----- Miss Eupha Buck
- Faculty Sponsor ----- Miss Eupha Buck
- Student Senate Representative --- Mr. Bruce Clark

Of the four students who were officers last year in New Mexico Alpha, three are doing graduate work this year as fellows in other institutions. C. D. Firestone was awarded the Brooks Fellowship at Cornell University, where he is

continuing his study of mathematical logic. Frank Lane is a fellow in mathematics at the Illinois Institute of Technology, and Anna Vallevik is a fellow in chemistry at Mills College. Bruce Clark, the remaining officer, is a student assistant in mathematics this year at the University of New Mexico, and is president of the student senate.

Abraham Franck, president of the chapter two years ago, had a scholarship in mathematics at the University of Michigan last year, and has a fellowship at Brown University for the present year. Some of his research done in collaboration with Dr. C. V. Newsom was recently published in *Boletin Matematico*.

Wade Ellis, distinguished negro mathematician, and an alumnus of New Mexico Alpha, is continuing his mathematical research this year under a Rockefeller grant.

C. B. Barker, business manager of the PENTAGON, was a charter member of New Mexico Alpha when he was a student. Recently he completed the work for the doctorate at the University of California, and now is a member of the mathematics staff at his Alma Mater.

Miss Eupha Buck, New Mexico's representative at Warrensburg, Missouri, last spring, is now a teaching assistant. Her duties include the sponsorship of Kappa Mu Epsilon activities, and the study of problems arising in connection with secondary teaching.

*Chapter 12.* ILLINOIS BETA, Eastern Illinois State Teachers College, Charleston, Illinois.

President Thales .....	Miss Maxine Rennels
Vice-President Apollonius .....	Mr. Carroll Endsley
Secretary Khayyam .....	Miss Sylvia Diel
Treasurer Archimedes .....	Mr. Edward Wilson
Secretary Descartes .....	Mr. Hobart F. Heller
Faculty Sponsor .....	Mr. E. H. Taylor
Reporter .....	Mr. David Fisher

**Chapter 13. ALABAMA BETA, Alabama State Teachers College, Florence, Alabama.**

President Pascal .....	Miss Dorothy Burgess
Vice-President Leibnitz .....	Miss Dorothy Denman
Secretary DeMoivre .....	Miss Caroline Wilson
Treasurer Euler .....	Miss Geraldine Keith
Secretary Descartes .....	Miss Orpha Ann Culmer
Faculty Sponsor .....	Miss Orpha Ann Culmer
Historian .....	Miss Mildred Simpson

The following item appeared in a Florence, Alabama, newspaper for June 5, 1941:

Alabama Beta Chapter of Kappa Mu Epsilon, national honorary mathematics fraternity, was invited by the National Council to install the Tennessee Alpha Chapter, at Tennessee Polytechnic Institute early in the summer.

Marie Lacefield, Mary Ann Green, Robert Bliss, Charles Barr, and Miss Orpha Ann Culmer made the trip to Cookeville.

Robert is president of the local chapter for next year and Miss Culmer is national historian.

This is the second chapter Alabama Beta chapter has installed. The other was at Coker College, Hartsville, S. C.

Several members of Alabama Beta have accepted positions with the Tennessee Valley Authority or with the Reynolds Metals Company. Many of these are undergraduates who expect to return later to finish their college courses.

**Chapter 14. LOUISIANA ALPHA, Louisiana State University, University, Louisiana.**

President Gauss .....	Mr. John Laufer
Vice-President Poincare .....	Mr. Mark Carrigan
Secretary Fermat .....	Miss Yvonne Jones
Treasurer Galois .....	Mr. Ralph Shapiro

Historian Cajori ----- Miss Marion Leach  
 Secretary Descartes ----- Mr. H. T. Karnes  
 Faculty Sponsor ----- Miss M. White

*Chapter 15. ALABAMA GAMMA, Alabama College, Montevallo, Alabama.*

President Archimedes ----- Miss Mattie Sue Oden  
 Vice-President Apollonius ----- Mr. Burke Land  
 Secretary Bernoulli ----- Miss Alice Yarbrough  
 Treasurer Marie Agnesi, Miss Nelladeane Chandler  
 Secretary Descartes ----- Miss Rosa Lea Jackson  
 Faculty Sponsor ----- Miss Rosa Lea Jackson

Alabama Gamma is beginning the fifth year of its existence. The program for the year will be inaugurated with a spend-the-night party at the college camp. Meetings will be held each month; at each of these, two papers will be presented, one of a recreational nature, and the other dealing with a mathematical topic of some difficulty. Also, there will be special meetings of a social nature distributed through the year in addition to the very important initiation banquet.

*Chapter 16. OHIO ALPHA, Bowling Green State University, Bowling Green, Ohio.*

President Archimedes ----- Mr. George John  
 Vice-President Leibnitz ----- Mr. Richard J. Camp  
 Secretary Vieta ----- Miss Mary D. Percy  
 Treasurer Napier ----- Mr. James Stearns  
 Secretary Descartes ----- Mr. Harry R. Mathias  
 Faculty Sponsor ----- Mr. F. C. Ogg

*Chapter 17. MICHIGAN ALPHA, Albion College, Albion, Michigan.*

President Townsend ----- Miss Lorna Betz  
 Vice-President Slaught ----- Mr. Richard Hadley  
 Secretary-Treasurer Agnesi ---- Miss Betty Evans

Secretary Descartes ----- Mr. E. E. Ingalls  
Faculty Sponsor ----- Mr. E. R. Sleight

The department of mathematics of Albion College maintains the tradition of hanging in the office of the department a picture of every alumnus who receives the Ph.D. degree. The most recent addition to the collection is a picture of David Wunchel Lee, class of 1922. Robert Gaskell, class of 1934, also received the doctorate a year ago from the University of Michigan.

Among the alumni of Michigan Alpha who have graduate appointments in other institutions are Norman Sleight, who is a fellow in chemistry at Iowa State College; Gerald Allen, who is a graduate assistant at the University of Iowa; and Paul Dunn, who is Bartell Memorial Fellow at Ohio State University. Also David Lawler is doing graduate work at the Massachusetts Institute of Technology.

Robert Esling, 1940, who is now a graduate assistant in the department of physics at Michigan State College, spent the summer teaching at Old Forge, one of the engineering defense training centers of the Pennsylvania State College. Norman McCarty, 1931, is also upon the Michigan State College campus as a member of the chemistry staff.

Gladys A. Swanson, class of 1940, is teaching mathematics in Coldwater, Michigan, and Margaret Ingram, class of 1941, is teaching in Detroit.

The following members of Michigan Alpha are now at the University of Michigan studying in the College of Engineering: Robert Ballard, Ernest Longman, Donald Murch, Ralph Fischer, Robert Maynard, Mark Putnam, Jack Telander, and Emmet Ward.

Owen A. Emmons, class of 1931, was selected as one of the outstanding alumni of Albion College to be honored with election to the chapter of Phi Beta Kappa recently installed on the Albion Campus. He also received an honorary degree last June.

Judson Foust and Cleon Richtmeyer, members of the

class of 1923, are co-authors of a text, *Business Arithmetic*. They are members of the department of mathematics at Central State College of Education at Mount Pleasant, Michigan.

Cecil Sessions, class of 1940, is teaching in the department of physics at Albion College while Dr. Spencer, the head of the department, is doing research work at the Illinois Institute of Technology. Marvin Pahl, 1930, is now assistant to the president of Albion College, and Dorothy Rafter, 1940, is also a member of the administrative staff.

*Chapter 18. MISSOURI BETA, Central Missouri State Teachers College, Warrensburg, Missouri.*

President Laplace	-----	Mr. James McCloud
Vice-President Pascal	-----	Miss Ruby E. Karrick
Secretary Gauss	-----	Miss Doris Bush
Treasurer Galois	-----	Mr. William Tracy
Secretary Descartes	-----	Mr. Emmett Ellis
Faculty Sponsor	-----	Mr. Paul A. DeVore

Missouri Beta maintains its activities during the summer session. This past summer, twenty-three students were initiated in a ceremony directed by the chapter president, Kenneth Martin, at Montserrat Federal Park. Previous to the initiation, which was held out of doors, there was a fried chicken picnic.

At the first regular meeting of the present year, members of the chapter listened to talks on the constitution and by-laws of Kappa Mu Epsilon, the national history of the society, and a resume of the life of Missouri Beta. The first social activity of the year was a steak fry held Friday night, October 17.

The faculty sponsor, F. W. Urban, will be on a leave of absence during the winter quarter.

A number of alumni have fellowships and teaching assistantships in graduate schools this year. A majority of the others are teachers in secondary schools.

**Chapter 19. SOUTH CAROLINA ALPHA, Coker College, Hartsville, South Carolina.**

President Leibnitz \_\_\_\_\_ Miss Dorris Seidenspinner  
Vice-President Pascal \_\_\_\_\_ Miss Theresa Waters  
Secretary Thales \_\_\_\_\_ Miss Frances Carey  
Treasurer Gauss \_\_\_\_\_ Miss Margaret Ellen Greyard  
Secretary Descartes \_\_\_\_\_ Miss Caroline M. Reaves  
Faculty Sponsor \_\_\_\_\_ Miss Caroline M. Reaves

At the alumna meeting of last year, Miss Caroline M. Reaves, sponsor of South Carolina Alpha, was presented a gold watch by Miss Bonnie Cone, a graduate of Coker College, in behalf of South Carolina Alpha. This gift was given to Miss Reaves in appreciation of her twenty-five years of service and guidance to Coker's mathematics club.

Miss Almena Workman, who was secretary of South Carolina Alpha last year, is teaching in Cope, South Carolina, this year, and is sponsoring a mathematics club in the high school there.

**Chapter 20. TEXAS ALPHA, Texas Technological College, Lubbock, Texas.**

President Lobatchewsky \_\_\_\_\_ Mr. Don Shepherd  
Vice-President Agnesi \_\_\_\_\_ Miss Nancy Ann Miller  
Secretary Noether \_\_\_\_\_ Miss Ruth Keeter  
Treasurer Cayley \_\_\_\_\_ Mr. Allen Smith  
Secretary Descartes \_\_\_\_\_ Mrs. Opal L. Miller  
Faculty Sponsor \_\_\_\_\_ Mr. Raymond K. Wakerling  
Reporter Einstein \_\_\_\_\_ Mr. John Ely

Texas Alpha was granted a charter on April 5, 1940. There were twenty-five charter members. On February 15, 1941, eleven new members were initiated. This membership has been drawn from the mathematics faculty, from those who are majoring in mathematics or the mathematical sciences, and from the division of engineering. Of the thirty-six initiates of the chapter, twenty-three are active members this year.

The chapter opened its program for the year in September with a meeting which featured a talk by Dr. Fred D. Rigby on the subject, "The Luxury of a Space of Infinitely Many Dimensions."

Three members of the fraternity received the M.A. degree in mathematics during the summer of 1941. They were Mr. William Wallis, Miss Hardy Masters, and Mrs. Roberta Willingham Kincaid. Mr. Wallis, who was president of the chapter last year, is now teaching mathematics in the high school at Artesia, New Mexico.

*Chapter 21. TEXAS BETA, Southern Methodist University, Dallas, Texas.*

President Galois .....	Mr. Merle Mitchell
Vice-President Abel .....	Mr. Roland Porth
Secretary-Treasurer Pascal .....	Mr. Billy Parham
Secretary Descartes .....	Mr. Paul K. Rees
Faculty Sponsor .....	Mr. Kenneth Palmquist

Julia Smith, president of the chapter in 1940-'41, is now attending the University of Iowa, where she was awarded a scholarship to do graduate work. She is studying actuarial theory and practice, and related courses.

Kenneth Cole, secretary of the chapter in 1941-'42, is studying chemical engineering this year at the Georgia School of Technology.

*Chapter 22. KANSAS GAMMA, Mount St. Scholastica College, Atchison, Kansas.*

President Tartaglia .....	Miss Bobbe Powers
Vice-President Cauchy, Miss Mary Margaret Downs	
Secretary Galileo ...	Miss Margaret Mary Kennedy
Treasurer Napier .....	Miss Mary Hughes
Secretary Descartes ...	Sister Helen Sullivan, O.S.B.
Faculty Sponsor .....	Sister Helen Sullivan, O.S.B.

At the end of its first year in Kappa Mu Epsilon, Kansas Gamma had thirteen active members, six graduate mem-

bers, and six pledges. These came from nine different states.

Mary Agnes Schirmer, of Elizabeth, New Jersey, graduate of the class of 1939, is now teaching mathematics at an academy in Paterson, New Jersey. She is also doing graduate work towards the M.A. degree at Columbia University, in New York.

Marjorie Dorney, of Monte Vista, Colorado, graduate of the class of 1939, is now a government student of meteorology at the University of California at Los Angeles. At the expiration of the intensive course, she will be eligible for employment in the weather bureau.

Among the other alumnae, Mary Donahoe is teaching in Mount Olive, Illinois, and Lucille Laughlin is a teacher in Nebraska. Betty Moore is employed by a milling company in Oklahoma, and Miriam Powers is a service representative for the Bell Telephone Company in Chicago. Sarah Woodhouse, valedictorian of the class of 1941, is employed in Kansas City as a computing machine operator.

Sister Jeanette Obrist, O.S.B., Ph.D., a member of the college faculty since 1932, completed her work for the doctorate in 1941, and is now a full time professor in the department of mathematics. Her research problem for the degree was entitled, "A Problem Arising from the Special Symmetric Correspondence,  $C_2$ , Set up by the Rational Quartic Curve with Two Cusps."

Several active members of Kansas Gamma are well known for their contribution to student affairs. Bobbe Powers, president of the chapter, is also president of the student council, the highest office upon the campus. Mary Margaret Downs was queen of the Mardi Gras two years ago, and Margaret Mary Kennedy was awarded the alumnae scholarship which is given biannually to the student whose high scholastic rating merits it.

Three members of Kansas Gamma have recently been listed in *Who's Who Among Students in American Colleges and Universities*. They are Bobbe Powers, Margaret Mary Kennedy, and Mary Flaherty.

The central theme that will dominate the activities of Kansas Gamma for the present academic year pertains to the part to be taken by American college women in national defense.

*Chapter 23. IOWA BETA, Drake University, Des Moines, Iowa.*

President Alchwarizmi .....	Mr. Robert Goss
Vice-President Aryabhata ...	Mr. Norman Landess
Secretary Brahmagupta .....	Miss Julia Rahm
Treasurer Leonardo .....	Mr. Bob Lambert
Secretary Descartes .....	Mr. Floy Woodyard
Faculty Sponsor .....	Mr. I. F. Neff

Three senior members of Iowa Beta were elected last year to Phi Beta Kappa; they were Floyd Beasley, Charles Miller, and Bernard Smith. This year, Mr. Beasley is taking graduate work in actuarial science at the University of Iowa, Mr. Smith is a fellow at Iowa State College, and Mr. Miller is studying law.

A large number of alumni are now associated with insurance companies. Among them are Virginia MacLennan, Floyd Bash, Frank Oshlo, and Jane Bush; the first two are actuaries.

Kenneth Austin, a member of the class of 1941, is a teaching fellow this year at the University of Iowa.

*Chapter 24. NEW JERSEY ALPHA, Upsala College, East Orange, New Jersey.*

President Thales .....	Miss Anne Zmurkiewicz
Vice-President Apollonius .....	Mr. George Robbin
Secretary Abel .....	Miss Phyllis Gustafson
Treasurer Fibonacci .....	Miss Edith Olson
Historian Gauss .....	Mr. Bernard Morrow
Secretary Descartes .....	Mr. M. A. Nordgaard
Faculty Sponsor .....	Mr. M. A. Nordgaard

New Jersey Alpha had the annual initiation in May, and

installed new officers for the period from June, 1941, to June, 1942. Professor H. Fehr, of the department of mathematics at Montclair State Teachers College, gave the address of the evening. Four new members were initiated at this meeting, making a total of seven for the year.

One of the features of the meetings this year will be a series of papers, one at each meeting, in which each officer will discuss the life and contributions of the mathematician for whom his office is named. Along with these expositions of a historical nature will be a study of problems pertaining to applied and to pure mathematics.

Kermit Carlson, class of 1939, a charter member, obtained his M.A. in mathematics from the University of Iowa last June. His thesis was on ballistics. Mr. Carlson held a graduate assistantship at the University of Iowa for the years, 1939-'41.

*Chapter 25.* OHIO BETA, College of Wooster, Wooster, Ohio.

Vice-President Abel (acting president)

	Mr. Erdine Maxwell
Secretary Gauss .....	Miss Marjorie Owen
Treasurer Leibnitz .....	Mr. James Halkett
Secretary Descartes .....	Mr. Melcher F. Fobes
Faculty Sponsor .....	Mr. C. O. Williamson

Dr. L. F. Ollmann, former sponsor of Ohio Beta, resigned during the past summer to become head of the mathematics department of Hofstra College, at Hempstead, New York. Dr. Ollmann was active in the affairs of Kappa Mu Epsilon, and he had much of the responsibility leading to the installation of a chapter of the fraternity in the College of Wooster.

Miss Florence Edgerton is teaching mathematics in the junior high school of Denison, Ohio.

The following members of Ohio Beta are doing graduate work this year in mathematics or science: Doane Gero at

Harvard, Donald Grove and James Campbell at the Massachusetts Institute of Technology, Robert Rice at Ohio State University, and Frank Niuman at Syracuse University. Also, Kenneth Yates and Montford Smith are graduate assistants at Ohio State University, and Andrew Sharkey is a graduate assistant at the Case School of Applied Science.

*Chapter 26. TENNESSEE ALPHA, Tennessee Polytechnic Institute, Cookeville, Tennessee.*

President Leibnitz .....	Mr. Kent Walthall
Vice-President Euler .....	Mr. Joseph E. Lane, Jr.
Secretary Napier .....	Miss Margaret Plumlee
Treasurer Kepler .....	Mr. William Fitzgerald
Secretary Descartes .....	Mr. R. H. Moorman
Faculty Sponsor .....	Mr. R. O. Hutchinson

Tennessee Alpha was installed at Tennessee Polytechnic Institute upon June 5, 1941. The installation was conducted by Alabama Beta, and Miss Orpha Ann Culmer makes the following report:

Tennessee Alpha of Kappa Mu Epsilon was installed, June 5, 1941, at Tennessee Polytechnic Institute, located in Cookeville, Tennessee. Dr. R. O. Hutchinson, Dr. R. H. Morgan, and Mr. H. D. Duncan, members of the faculty, were among the charter members. The installation took place at the home of Prof. R. O. Hutchinson.

Alabama Beta installed Tennessee Alpha, with Miss Orpha Ann Culmer, Historian Hypatia of Kappa Mu Epsilon, as installing officer. Four student members of Alabama Beta accompanied Miss Culmer to Cookeville, and assisted with the installation service.

Tennessee Polytechnic Institute offers a well balanced four year curriculum in mathematics. The new members, both faculty and students, are enthusiastic, and are happy to be associated with Kappa Mu Epsilon. I am convinced that our twenty-sixth

chapter will prove to be an asset to our mathematical fraternity.

Of the eleven charter members, only six remain in school this year, namely, the officers listed above and the faculty members. Harold Duncan, James Foster, and William Painter, of the original class, are now all employed by the Tennessee Valley Authority. John Killian and Robert Tate have industrial positions.

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