The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before April 15, 2021. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2020 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 859-869

Problem 859. *Proposed by the editor*.

A regular n-gon is inscribed in a circle which is inscribed in a square with side length x. Find the length of one side of the n-gon in terms of x. [The particular case where $n = 6$ and $x = 25$ was a theMathContest.com problem.]

Problem 860. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain*.

Find out whether or not there is a prime p such that $p!$ ends in exactly 2020 zeroes. Is there a corresponding prime q such that $q!$ ends in exactly 2019 zeroes?

Problem 861. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain*.

Let a_0, a_1, a_2, a_3 be any distinct nonzero real numbers. For any integer $n \geq 3$, prove that the zeroes of the polynomial $A(x) = x^{2n} + a_3 x^3 + a_2 x^2 + a_1 x + a_0$

cannot all be real.

Problem 862. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Romania.*

If $a,b \in \mathbb{C}$ such that $|a^2 + 25| \leq 5$; $|b^2 + 36| \leq 6$; $|a + 5| \leq \sqrt{5}$; $|b + 6| \leq \sqrt{6}$, then

$$|a + b|^2 + |a - b|^2 \leq 4$$

Problem 863. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Romania.*

If $a, b, c > 0$ and $a + b + c = 6$, then

$$\frac{a^2}{\sqrt{b^2+6bc+5c^2}}+\frac{b^2}{\sqrt{c^2+6ca+5a^2}}+\frac{c^2}{\sqrt{a^2+6ab+5b^2}}\geq\sqrt{3}$$

Problem 864. *Proposed by Toyesh Prakash Sharma (student) St. C.F Andrews School, Agra, India.*

Evaluate the following sum

$$\sum_{z=0}^{\infty}\sum_{y=0}^{\infty}\sum_{x=0}^{\infty}\frac{x+y+z}{2^{(x+y+z)}}$$

Problem 865. *Proposed by Pedro H.O. Pantoja, Natal/RN, Brazil.*

Let a, b, c be positive real numbers such that $a + b + c = 1$. Prove that

$$27(a^5+b^5+c^5)+13\leq 40(a^2+b^2+c^2)$$

Problem 866. *Proposed by Dorin Marghidanu, Colegiul National 'A. I. Cuze', Corabia, Romania.*

If a, b, c are the lengths of the sides of a triangle, prove the following

$$\sqrt{\frac{-a+b+c}{a}}+\sqrt{\frac{a-b+c}{b}}+\sqrt{\frac{a+b-c}{c}}>2\sqrt{2}$$

Problem 867. *Proposed by George Stoica, Saint John, New Brunswick, Canada.*

Let a, b, c be positive real numbers and $x, y, z \geq 0$ such that

$$x+aby\leq a(y+z); y+bcz\leq b(z+x); z+cax\leq c(x+y)$$

Prove that $x=y=z=0$. Is the conclusion still true if one assumes that $a, b, c \geq 0$?

Problem 868. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let the sequence (a_n) be defined by $a_n=\sum_{k=1}^{n} arctan\frac{1}{k^2-k+1}$ Compute the following:

(i) $\lim_{n\to\infty} a_n = a$

(ii) $\lim_{n\to\infty} (a-a_n)n$

Problem 869. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

If $\gamma_n = -\ln n + \sum_{k=1}^{n} \frac{1}{k}$ with $\lim \gamma_n = \gamma$, the Euler-Mascheroni constant, compute the following:

(i) $\lim_{n\to\infty} (\gamma_n - \gamma)n$

(ii) $\lim_{n\to\infty} (\gamma_n \gamma_{n+1} - \gamma^2)n$