# The Problem Corner

Edited by Pat Costello

*The Problem Corner* invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before November 1, 2019. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2019 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 829-839

**Problem 829**. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Let $\Omega_n = \binom{n}{7} + 2\binom{n-1}{7} + 3\binom{n-2}{7} + \cdots (n-6)\binom{7}{7}$ for all n≥7.

Find $\Omega = \lim_{n\to\infty} \sqrt[n]{\Omega_n}$ .

**Problem 830**. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

If $x \in (0, \frac{\pi}{2})$, prove that $2(\sin x)^{1-\sin x} \cdot (1-\sin x)^{\sin x} \le 1$.

**Problem 831**. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

If $\Delta ABC \sim \Delta A'B'C'$, prove that $\sum \frac{(a'+b')(a'+c')}{b'c'} + 3 \ge \frac{15(b+c)(c'+a')(a'+b')}{8ab'c'}$ .

**Problem 832.** *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Prove that in any triangle ABC the following holds

$$\frac{a}{a+b+c} \geq \frac{2\sqrt{3}}{9} \sin A.$$

**Problem 833.** *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Show that the equation $x^6 - 5x^5 - 6x^4 + 2x^3 + 9x^2 - 17x + 1 = 0$
has no negative roots.

**Problem 834**. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let $(F_n)$ be the Fibonacci sequence, i.e. $F_0 = 0$, $F_1 = 1$, $F_{n+2} = F_{n+1} + F_n$ for all $n \geq 0$.
If $e_n = \left(1 + \frac{1}{n}\right)^n$, prove that $\left(\sum_{k=1}^{n} e_k F_{2k-1}\right)\left(\sum_{k=1}^{n} \frac{F_{2k-1}}{e_k}\right) \leq \frac{(e+2)^2}{8e} F_{2n}^2$

**Problem 835**. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let $(L_n)$ be the Lucas sequence, i.e. $L_0 = 2$, $L_1 = 1$, $L_{n+2} = L_{n+1} + L_n$ for all $n \geq 0$. Prove that
$n^{n-2}(n-1)\sum_{k=1}^{n} L_k^n + n^{n-1}\prod_{k=1}^{n} L_k > (L_{n+2} - 3)^n$ for all $n \geq 2$.

**Problem 836**. *Proposed by Abhijit Bhattacharjee (student), Banaras Hindu University, India.*

Prove that the equation $1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} = 0$ has exactly one real root if $n$ is odd and no real root if $n$ is even.

**Problem 837.** *Proposed by Pedro H.O. Pantoja, University of Campina Grande, Brazil.*

Evaluate $\lim_{n\to\infty} \int_{-1}^{1} \left( x^{2n+1} + \frac{1}{x^{2n+1}} \right) \ln(1 + e^{nx}) dx$

**Problem 838.** *Proposed by Mathew Cropper, Eastern Kentucky University, Richmond, KY.*

Let $A_n$ be the number of *n*-bit strings of zeros and ones that contain at least one sequence of three consecutive ones (111) and no sequence of four or more consecutive ones. The sequence starts $A_1 = 0$, $A_2 = 0$, $A_3 = 1$, $A_4 = 2$. Using the well-known tribonacci sequence $T_0 = 0$, $T_1 = 0$, $T_2 = 1$, $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ which counts the number of (n-3)-bit strings that contain NO sequence of three consecutive ones, develop a recursive formula for $A_n$ and use it to compute $A_{15}$.

**Problem 839.** *Proposed by the editor.*

A recurrence is defined in the following way: $c_1 = 3$, $c_n = 4 + \sum_{i=1}^{n-1} c_i$ for all $n \geq 2$.

Find a formula for $c_n$ for $n \geq 2$ that just involves $n$.